Heterogeneity within Communities: A Stochastic Model with Tenure Choice*

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Abstract

There is considerable income heterogeneity within neighborhoods. Standard explanations rely on differences of preferences across households and heterogeneity of the housing stock. We propose an alternative and complementary explanation. We construct a stochastic equilibrium sorting model where (1) income is the sole dimension of household heterogeneity, (2) households form state-contingent housing location plans that may involve moves over their lifetime, (3) households choose whether to own or rent their homes motivated by aversion to housing expenditure risk, (4) there is a probability that newcomer households move in and compete for homes with the native households. Income mixing arises for two reasons. First, allowing natives to form state-contingent housing location plans breaks the indivisibility of housing consumption implicit in the literature where households choose their location once and for all. Second, natives have the opportunity to insure themselves against rent fluctuations by buying their home prior to the realization of the shock; newcomers do not. As a result, poorer natives stay in the desirable community and only richer newcomers move in. Evidence from U.S. metropolitan areas provides empirical support for our arguments.

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1 Introduction

There is considerable income heterogeneity within neighborhoods. For example, Epple and Sieg (1999) find that 89 percent of the income variance in the Boston metropolitan area could be accounted for by within-community variance in 1980. Ioannides (2004) finds a correlation coefficient of only 0.3 between the income of a randomly chosen individual and the income of her ten closest neighbors. Hardman and Ioannides (2004) find that in 1993, more than two thirds of U.S. metropolitan neighborhoods surveyed by the American Housing Survey had at least one household with income in the bottom quintile of the metropolitan income distribution; more than half the neighborhoods had at least one household with income in the top quintile.

The empirical evidence stands in contrast to the prediction of early equilibrium sorting models where households were assumed to differ according to income only; e.g., Ellickson (1971), Henderson (1991). These models predict that each community consists of households whose income lies in a single interval and the set of communities partitions the support of the income distribution.

To address this discrepancy between theory and evidence, the literature has moved to models where households differ not only in terms of income but also in terms of preferences; e.g., Epple and Platt (1998), Epple and Sieg (1999). Equilibrium sorting models with these two dimensions of household heterogeneity still predict that the incomes of households within a community lie in a single interval. However, the intervals may overlap because households with the same level of income value differently the amenities offered by each community.

In this paper, we propose an alternative and complementary explanation for the income heterogeneity within neighborhoods. We construct an equilibrium sorting model where income is the sole dimension of household heterogeneity. Nevertheless, the equilibrium distribution of households across communities is not stratified according to income. The income of households within a community needs not to lie in a single interval. There are pairs of households such that the higher earner lives in the cheaper community.

We obtain this prediction because we set the community choice problem within a dynamic and stochastic environment where the cost of each location is determined in equilibrium. In particular, our model has the following three features: (1) households form state-contingent housing location plans that may involve moves over their lifetime, (2) households choose whether to own or rent their home motivated by aversion to housing expenditure risk, (3) there is a probability that new households move into the area and compete for homes with the earlier inhabitants.

We construct a two period model of a city with two communities. Households are risk averse. They all derive the same utility premium from living in the more desirable of the two communities. Housing in this community is available in fixed supply and can be either rented or purchased from competitive risk-neutral absentee landlords. For simplicity and tractability, we assume the supply of housing in the less desirable community to be perfectly elastic. In the second period, there is a positive probability that a group of newcomers move to the city. They enjoy the same utility from numeraire and location as the natives. Like the natives, they differ in income levels.

Income mixing arises for two complementary reasons. First, allowing native households to form state-contingent housing location plans breaks the indivisibility of housing consumption implicit in the literature where households choose their location once and for all. Consider two households with different incomes living in different neighborhoods. The lower earner may be living in the nicer neighborhood because this is part of a plan that involves moving to a cheaper neighborhood in the future, whereas the higher earner may have chosen to live in the same neighborhood for life.

Second, natives who want to remain in their neighborhood whatever happens to local rents have a chance to insure themselves against rent fluctuations by buying their home prior to the realization of the shock. Newcomers do not. This discrepancy between the housing opportunities of the natives and of the newcomers affects the income distribution within neighborhoods. In response to the arrival of newcomers, fewer natives move out of the desirable community because they benefit from capital gains on their home when the cost of housing increases. As a result, the income distribution of the newcomers who choose the same community is truncated at a higher level. The natives who stay in the desirable community are poorer, the newcomers who come in are richer than in a rental-only economy.

Gyourko, Mayer and Sinai (2004) study the differences in income between natives and newcomers across census-designated places within US metropolitan areas. They find that newcomers tend to be richer than natives in neighborhoods which have experienced price growth. Neighborhoods with the highest price growth attract a disproportionate share of the rich households moving to the metropolitan area. This finding supports the prediction of our model that variations in the time households move into a neighborhood contribute to income mixing within neighborhoods.

Ioannides (2004) reports evidence in support of the same idea. He finds, for example, that the coefficient of variation in neighborhood incomes increases with mean time since a household moved into the neighborhood.

Ioannides (2004) also finds that neighborhoods of renters are less heterogeneous than neighborhoods of owners, another fact which points to the relevance of the forces we identify in our model.

Davidoff (2004) finds that considerable income mixing remains even when one corrects for income reporting errors and the discrepancies between current and permanent income. The sense that income heterogeneity is not simply due to discrepancies between current and permanent income is reinforced by the evidence that households educational levels are as heterogeneous within neighborhoods as income levels (Ioannides 2004).

An alternative and complementary reason for income mixing within neighborhoods is that homes are heterogeneous. Keely (2004) explains why developers have incentives to develop subdivisions with heterogeneous homes. Nechyba (2000) presents a computational model with heterogenous housing stock within communities in order to analyze private-school voucher policies. We obtain our results without assuming any heterogeneity in the stock of homes within each location. Although heterogeneity of the housing stock must play a role in shaping the income distribution of a neighborhood, we note that Ioannides (2004) finds that both household income and education levels are more heterogeneous than property values.

Benabou (1996a, 1996b), Durlauf (1996), and Fernandez and Rogerson (1996, 1998) propose dynamic sorting models to analyze macroeconomic and policy issues. They assume that the benefits of living in a community depend on the make-up of the community. They are therefore determined endogenously. The same is true in the static models where the benefits of each community are determined by a political equilibrium; see, for example, Epple and Sieg (1999). Both sets of models also share the property that households make only one location decision in equilibrium, either by assumption or because of a focus on stationary environments. Here, we take the amenities of each community as given but we allow households to relocate and to choose whether to own or rent their property in the face of endogenous fluctuations in housing costs.

2 The model

To analyze the location choices of households in an environment where they may move in response to shocks we need a model with at least two locations and two periods. To analyze the housing market interaction between native households and newcomers, we assume that newcomers may appear in the second period. This population shock affects equilibrium housing rents and prices.

We focus on tenure choice driven by concerns over future housing expenditure risk. A number of papers provide evidence of the relevance of this driver of tenure choice; see, for example, Davidoff (2003), Diaz-Serrano (2004), Han (2004), and Hilber (forthcoming). With two periods, the model captures the idea that at short horizons, households' concerns over period-to-period rent risk are dominated by concerns over end-of-holding-period price risk, and vice versa at long horizons. Empirical support for this idea is provided in Sinai and Souleles (2004).

2.1 Economic environment

We consider a two-period model of an area with two communities, 0 and 1. In community 0, the supply of homes is perfectly elastic at a constant rent normalized to zero. In community 1, there is a measure S of identical homes owned initially by absentee landlords. For simplicity, the landlords are assumed to be risk neutral. They discount rents at the same exogenous interest rate, $r > 0$, at which households can borrow and save.

Initially, the area is populated by a measure one of households, the natives. They are distributed uniformly over the unit interval. Each one is identified by an index $i \in [0, 1]$. Household i receives a stream of income defined by $w_1(i)$ in period 1, and $w_2(i)$ in period 2, both increasing continuous functions of i. The capitalized value of the endowment of household i evaluated in period 2 is denoted $W(i)$, with $W(i) = w_1(i)(1 + r) + w_2(i)$.

Native households derive additively separable utility from the consumption of housing and the numeraire good. Location 1 is more desirable than location 0: housing utility derived from a home in location 0 is normalized to zero, whereas a home in location 1 yields an additive utility premium of $\mu > 0$ per period, independently of whether the home is owned or rented. There is no discounting of utility across periods. The numeraire good is enjoyed at the end of period 2 only. The utility derived from consumption of c units of the numeraire good is described by the constant absolute risk aversion function $U(c) = -e^{-ac}$ where $a > 0$ is the coefficient of absolute risk aversion.

There is uncertainty in period 2. With probability $\pi \in (0, 1)$, state H occurs: a measure ν of newcomer households moves to the city at the start of period 2. With probability $1-\pi$, state L occurs: nobody moves in. Although the shock is asymmetric by design, we will see later that from the point of view of the natives, it amounts to either a rent increase (state H), or a rent decrease (state L). Our specific modelling choice for the shock is motivated by our interest in the allocation of homes between households who had a chance to buy their homes early and those who move in later.

Newcomers are distributed uniformly over the unit interval; they are characterized by the index $n \in [0, 1]$. Their endowment is defined by the increasing and continuous function $\tilde{W}(n)$. Newcomers have the same utility function as natives except for the fact that they cannot obtain any utility from housing in period 1. The only decision they face is whether to live in location 0 or 1 if state H occurs and how much of the numeraire good to consume.

Within each period, households first receive their endowment. Second, the markets open and trade takes place. Third, households consume housing and the numeraire good.

For ease of exposition, we assume $0 < \nu < S < \frac{1}{2}$ throughout the paper. These assumptions limit the number of cases we will have to consider without taking anything away from our results.

2.2 Tenure choice

Whether a household owns or rents a home in community 0, the cost is nil by assumption. Since we also assume that housing utility does not depend on tenure, all households are indifferent between renting and owning a home in community 0.

Tenure matters for homes in community 1. We denote R_1 their rent in period 1, R_H in period 2 state H, and R_L in period 2 state L. In this subsection, we assume $R_L < R_H$. We will see later that this inequality must hold in any equilibrium where some newcomers choose community 1 in state H.

Arbitrage on the part of the landlords ensures that the price of a home in period 1, p_1, equals the first-period rent plus discounted expected second-period rent:

$$p_1 = R_1 + \frac{\bar{R}_2}{1+r} \tag{1}$$

where R_1 denotes the first-period rent and $\bar{R}_2 = \pi\, R_H + (1-\pi)\, R_L$ denotes the expected second-period rent. Since period 2 is the last period of the economy, renting a home in period 2 is equivalent to buying it, so the price of a home in period 2 coincides with the rental cost of that home in period 2.

Equation (1) highlights what ownership means in the model: by purchasing a home in the first period, a household effectively signs a two-period rental contract, locking in the second-period rent at its expected level. Whether buying is more or less risky rather than renting depends on the household's planned housing consumption in the second period. If the household plans to stay in community 1 in the second period, buying provides insurance against second-period rent risk. If the household plans to sell and move to community 0 in period 2, buying provides exposure to potential capital gains or losses; renting eliminates this risk.

Before proceeding further with the analysis of the native households' tenure choice, we need to establish some notation. A native household's *location plan* is denoted by the triple (h_1, h_H, h_L), where h_1, h_H and h_L take the value of 1 for community 1, and 0 for community 0. To indicate the tenure choice in the cases where $h_1 = 1$, we denote the combined *location-tenure plan* by $(1_B, h_H, h_L)$ if the household buys a home, and $(1_R, h_H, h_L)$ if it rents one. Figure 1 summarizes the location-tenure choices available to a native household.

Natives choose among twelve location-tenure plans. There are eight location plans as there are two alternatives for each of period 1, period 2 state H, and period 2 state L. In

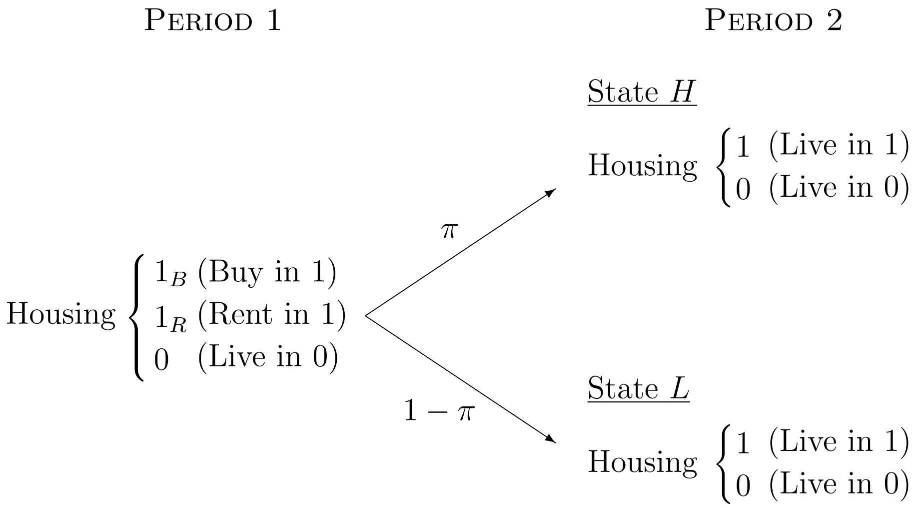

Figure 1: Native households' housing choices

addition, for the four location plans that involve living in location 1 in period 1, native households must decide whether to buy or rent.

The tenure choice affects how shocks to the housing markets translate into shocks to the household's cost of housing, through the budget constraint, into fluctuations in non-housing consumption. The stochastic properties of numeraire consumption are therefore what is at issue with regards to the choice of tenure.

For example, consider the expected numeraire consumption of a household that chooses to live in location 1 in period 1 and in period 2 whatever the shock. If the household rents in period 1, it pays first period rent and realized rent in period 2. Its expected numeraire consumption is

$$\pi(W - R_1 - R_H) + (1 - \pi)(W - R_1 - R_L).$$

If the household buys in period 1, it pays first period rent and expected second period rent. Its expected numeraire consumption is

$$\pi(W - R_1 - \bar{R}_2) + (1 - \pi)(W - R_1 - \bar{R}_2).$$

By equation (1), both expressions simplify to the same amount of expected numeraire consumption, $W - R_1 - \bar{R}_2$: expected numeraire consumption is independent of tenure choice. The same holds for every other plan which involves a tenure choice.

Because households are risk averse, this property of expected numeraire consumption implies that the tenure decision reduces to choosing the option that induces the smallest absolute difference between the numeraire consumption levels in the two states of the economy.

For the location plans with a deterministic horizon in the type 1 home, $(1,0,0)$ and $(1,1,1)$, the tenure choice is obvious since one of the tenure modes provides full insurance whereas the other does not. A household that rents in period 1 and moves to location 0 in period 2 does not suffer any shock to its consumption of numeraire. A household that buys in period 1 and remains in location 1 in period 2 does not face any numeraire consumption risk either. The location-tenure plans $(1_R,0,0)$ and $(1_B,1,1)$ therefore dominate the plans $(1_B,0,0)$, and $(1_R,1,1)$, respectively.

Under the location plans $(1,1,0)$ and $(1,0,1)$, by contrast, either tenure mode imposes some risk on the household. Under $(1,1,0)$, if the household rents, it pays the rent R_H in state H and no rent in state L; its numeraire consumption is lower by R_H in state H than in state L. If the household buys in the first period, it sells the home if the state L occurs. The price of a location 1 home in state L is R_L. The household's numeraire consumption is therefore lower by R_L in state H than in state L if it buys in period 1. Under $(1,1,0)$, buying is thus less risky given our working assumption that R_L is lower than R_H: the household that buys faces a smaller risk in terms of second-period numeraire consumption. The location-tenure plan $(1_B,1,0)$ therefore dominates the plan $(1_R,1,0)$.

Under $(1,0,1)$, the logic is reversed. If the household rents in the first period, numeraire consumption in state L is lower by R_L since the second-period rent is paid only in state L. If the household buys the home in the first period, numeraire consumption in state H is higher by R_H since the household sells its home in state H. Buying involves a greater difference in realized numeraire consumption. The location-tenure plan $(1_R,0,1)$ therefore dominates the plan $(1_B,0,1)$.

We summarize these findings in

Lemma 1 *If $R_L < R_H$, a native household wanting to live in location 1 in the first period prefers to own its home if and only if it plans to stay in location 1 should state H occur in the second period.*

2.3 Community choice

¿From the twelve location-tenure plans we started with, we have now shown that four are dominated; we are left with the following eight plans to consider: $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$, $(1_B,1,0)$, and $(1_B,1,1)$. Each one of these plans determines a curve in the plane with co-ordinates W (the household's capitalized endowment) and EU (the expected overall utility level). Determining the optimal plan for every W amounts to characterizing the upper envelope of these expected utility curves. In the following discussion, we maintain our working assumption $R_L < R_H$.

First, the CARA specification of non-housing utility implies that the expected utility of any location-tenure plan can be written as $EU = -A\, e^{-aW} + B$ with plan-specific constants $A > 0$ and $B \geq 0$, where $B \in \{\pi\mu,\ (1-\pi)\mu,\ \mu,\ (1+\pi)\mu,\ (2-\pi)\mu,\ 2\mu\}$ is the expected utility from housing. For example, for the plan $(1_B, 1, 1)$, the expected utility takes the form

$$EU_{(1_B,1,1)} = -\, e^{a\left[(1+r)R_1 + \bar{R}_2\right]}\, e^{-aW} + 2\mu.$$

It is easy to check that if the expected utility curves of two plans cross, the curve associated with the plan that promises a longer expected time in community 1 (and so has the higher B) is steeper at all endowment levels (greater A). Note also that the higher B, the greater the expected utility as W becomes large (the limit of EU as W tends to infinity is B). This immediately yields

Lemma 2 *The amount of housing a native household expects to consume in community 1 is weakly increasing in the household's endowment.*

Second, using CARA utility, it is easy to verify that the preference ranking of the plans $(1_R, 0, 0)$ and $(0, 1, 1)$ does not depend on the household's endowment W. In other words, the expected utility curves associated with these two plans are either identical or do not intersect. Both plans generate the same utility from housing, μ; their ranking is thus determined by the cost difference alone.

Lemma 3 *The plan* $(1_R, 0, 0)$ *weakly dominates* $(0, 1, 1)$ *if and only if*

$$e^{a(1+r)R_1} \leq \pi e^{aR_H} + (1-\pi)e^{aR_L}, \tag{2}$$

with a strict preference if the inequality is strict.

Third, building on what we have learned so far about households' tenure and location decisions, we find that the plans $(0, 1, 0)$ and $(1_B, 1, 0)$ are not chosen by anyone when $R_L < R_H$. Increasing the wealth of a household that chooses $(0, 0, 0)$ will eventually prompt this household to spend some time in community 1. The cheapest community 1 housing available is in period 2 state L. This is the one the household will choose first as its wealth rises. Conversely, decreasing the wealth of a household that chooses $(0, 1, 1)$ will prompt this household to eventually relinquish some time in community 1. As community 1 housing is most expensive in state H, this is the one the household will give up first as its wealth decreases. This is why no household ever chooses $(0, 1, 0)$. The same argument applies to $(1_B, 1, 0)$.

Lemma 4 *If* $R_L < R_H$*, a native household chooses a location-tenure plan from the following subset of alternatives:* $(0, 0, 0)$*,* $(0, 0, 1)$*,* $(0, 1, 1)$*,* $(1_R, 0, 0)$*,* $(1_R, 0, 1)$ *and* $(1_B, 1, 1)$*.*

Proof: See Appendix A.1. ■

Fourth, we study how optimal location-tenure plans differ between households with different endowment levels. We already know from Lemma 4 that the higher is a household's endowment, the more time it spends in community 1. Here, we ask whether we observe increases of the time spent in community 1 in the smallest possible steps. The answer is almost yes. More specifically, we prove the following.

Lemma 5 *Let $R_L < R_H$. Then,*

(i) *at least one of the plans* $(0,0,1)$ *and* $(1_R,0,1)$ *is preferred to both* $(0,0,0)$ *and* $(1_B,1,1)$ *at all endowment levels in some set of positive measure;*

(ii) *at least one of the plans* $(0,0,1)$ *and* $(1_R,0,0)$ *is preferred to both* $(0,0,0)$ *and* $(1_R,0,1)$ *at all endowment levels in some set of positive measure;*

(iii) *the plan* $(0,0,1)$ *is preferred to both* $(0,0,0)$ *and* $(0,1,1)$ *at all endowment levels in some set of positive measure;*

(iv) *the plan* $(1_R,0,1)$ *is preferred to both* $(1_R,0,0)$ *and* $(1_B,1,1)$ *at all endowment levels in some set of positive measure.*

Proof: See Appendix A.1. ■

The newcomers face a much simpler location choice problem. They appear in the second period only if state H occurs and therefore face a one-period deterministic consumption problem. They choose to move into community 1 if and only if the utility premium from living there more than outweighs the utility cost of the rent R_H; i.e. if

$$\mu \geq e^{-a(\tilde{W}(n)-R_H)} - e^{-a\tilde{W}(n)}. \tag{3}$$

Note that the right-hand side of this inequality is decreasing in $\tilde{W}$ because of the decreasing marginal utility of numeraire consumption. Therefore if a newcomer chooses community 1, any other newcomer with a greater income chooses community 1 as well.

2.4 Equilibrium

An equilibrium is a triple of rents, (R_1, R_H, R_L), and a period 1 price, p_1, for homes in community 1, together with a location-tenure plan for each native household and a location choice for each newcomer. The equilibrium price of homes in community 1 must be such that landlords are indifferent between selling a home in period 1 and renting it over both periods at the equilibrium rents. The equilibrium allocation must be such that housing markets clear and each household's utility is maximized given its budget constraint and the price and rents of homes in community 1.

To formulate the following proposition, we need to define $\underline{e} > 1$ as the unique real number satisfying the equality

$$2(1-S) = W^{-1}\left(\frac{1}{a}\ln\left(\frac{\underline{e}-1}{\mu}\right)\right) + W^{-1}\left(\frac{1}{a}\ln\left(\frac{\min\left\{\underline{e}\left(\underline{e}-1\right),\ \mu e^{aW(1)}\right\}}{\mu}\right)\right). \tag{4}$$

Proposition 1 *There exists a unique equilibrium. If*

$$\mu e^{a\tilde{W}(1)} > \underline{e} - 1, \tag{5}$$

a positive measure of newcomers choose community 1 in state H, the equilibrium prices satisfy $R_L < (1+r)R_1 < R_H$ and condition (2)*, and the location-tenure plans chosen by a positive measure of native households are* $(0,0,0)$, $(0,0,1)$, $(1_R,0,0)$ *plus either*

- $(0,1,1)$*, and possibly* $(1_R,0,1)$*, or*
- $(1_R,0,1)$ *and* $(1_B,1,1)$*, and possibly* $(0,1,1)$.

If (5) *does not hold, all newcomers choose community 0 in state H and the equilibrium prices satisfy $(1+r)R_1 = R_L = R_H = (\ln \underline{e})/a$, so tenure does not matter. The location plans chosen by a positive measure of native households are* $(0,0,0)$, $(0,1,1)$, $(1,0,0)$*, plus possibly* $(1,1,1)$.

PROOF: See Appendix A.1. ■

Condition (5) depends only on model parameters. Recall that $\tilde{W}(1)$ is the income received by the best-paid newcomer. The inequality ensures that the best-paid newcomer earns an income sufficiently high that he chooses community 1 in equilibrium.

The inequality $R_L < R_H$ reflects the price pressure newcomers exert when they appear in state H. That the opportunity cost of choosing community 1 in the first period, $(1+r)R_1$, lies strictly in between R_L and R_H is then dictated by market clearing. Intuitively, the cost of living in community 1 in period 1 cannot be too different from the cost of living in community 1 in period 2 for sure, a cost which lies in between R_L and R_H.

3 Income heterogeneity

In our two-period stochastic model, native households have the options to vary their housing consumption over time, to choose a state-contingent housing consumption plan, and to choose tenure more. In this section, we first explain how the behavior of natives generates income heterogeneity within communities. Second, we turn to the interaction between natives and newcomers and the effects of ownership on the allocation of community 1 homes. The fact that native households can own their home may add further to the heterogeneity of households within communities.

3.1 State-contingent plans and income mixing

As a preliminary step, note that if some natives choose the plan $(0, 1, 1)$, Proposition 1 implies that they are indifferent with the plan $(1_R, 0, 0)$ and that both plans are chosen in equilibrium. Consequently, two households with similar incomes may chose different communities in period 1. There are pairs of households in period 1 such that the one with the lower income lives in the nicer community. The households living in the nicer community in period 1 move to the cheaper community in period 2, and vice versa. Whatever the state in period 2, there are again pairs of households such that the one with the lower income lives in the nicer community. By definition, this means that communities are not perfectly stratified by income alone in both periods whenever some households choose the plan $(0, 1, 1)$.

Depending on parameters, either some newcomers move to community 1 in state H or they do not. If no newcomer chooses to live in community 1, Proposition 1 says that some natives choose the plan $(0, 1, 1)$. By the argument above, in both periods communities are not perfectly stratified according to income alone.

If some newcomers choose to live in community 1, they affect the distribution of income in that community. The newcomers' housing consumption is monotonic in their wealth which equals their income, by assumption. The native's choice of community is also monotonic in their wealth. However the wealth of the natives consists of accumulated incomes minus any housing costs incurred in period 1 plus, possibly, net capital gains that follow from the purchase of a home in period 1. In general, this implies that the income of the native household indifferent between living in either community is not the same as the income of the newcomer who is at the same point of indifference. This means that the communities are not stratified according to income when newcomers move in.

If state L occurs, there are always pairs of households such that the one with the lower income chooses the nicer community. Proposition 1 implies that the plans $(0, 0, 1)$ and $(1_R, 0, 0)$ are both chosen in equilibrium. Lemma 2 implies that the households that choose the first plan earn a higher income than the households that choose the second one. At the start of period 2, if state L occurs, the natives who chose $(1_R, 0, 0)$ move to the cheaper location, joining there the poorest natives in the area. At the same time, the natives who chose $(0, 0, 1)$ are joining the highest income natives in community 1 although they earn less than the households that chose $(1_R, 0, 0)$. Therefore, in state L, communities are not stratified according to income.

In all, we learn that optimal household choices yield an equilibrium where communities are not stratified according to income alone, for sure in period 2, possibly in period 1. The underlying reason for this result is that knowing a household's income is not sufficient to predict its housing consumption plan. First, past housing experiences affect the relationship between the household's wealth and its current income. More importantly, because of the diversity of equilibrium housing choices observed early on, the relationship between wealth and income does not remain monotonic over time. Furthermore, wealth itself is not sufficient to predict a household's community choice. Two households with identical wealth may choose different communities simply because they have different housing consumption plans for the future.

3.2 Tenure choice and income mixing

Proposition 1 implies that when the richest native households choose to live in community 1 for the duration of the model they buy their home in period 1. The next poorer natives rent in community 1 in period 1 and remain there only if state L occurs. The households indifferent between the two plans have accumulated earnings W such that the utilities they derive from each plan are equal; i.e.,

$$\begin{aligned} -e^{-a(W-(1+r)R_1-\bar{R}_2)} + 2\mu \quad &= \quad \pi\left(-e^{-a(W-(1+r)R_1)} + \mu\right) \\ &\quad + (1-\pi)\left(-e^{-a(W-(1+r)R_1-R_L)} + 2\mu\right), \end{aligned} \tag{6}$$

or

$$\mu e^{a(W-(1+r)R_1)} = e^{aR_H} - 1 + \frac{e^{a\bar{R}_2} - \pi e^{aR_H} - (1-\pi)e^{aR_L}}{\pi}. \tag{7}$$

If ownership were not an option, the plan $(1_R, 1, 1)$ would replace $(1_B, 1, 1)$ as the plan offering the greatest utility from housing. A derivation similar to the one above yields that the households indifferent between $(1_R, 1, 1)$ and $(1_R, 0, 1)$ would be the ones with accumulated earnings W defined by

$$\mu e^{a(W-(1+r)R_1)} = e^{aR_H} - 1. \tag{8}$$

The endowment level that solves equation (8) is higher than the endowment level that solves (7) because the third term on the right hand side of (7) is negative. This term captures the benefit of the insurance provided by a purchase in period 1: an owner pays expected second period rent, $\bar{R}_2$, instead of R_H with probability π and R_L with probability $(1-\pi)$. Therefore, at the equilibrium rents, if ownership were not an option, less native households would choose to stay in location 1 in state H.

This implies that if ownership were not an option, the equilibrium rents would be different. To analyze whether changes in rents would compensate the difference in natives' demand for community 1 in state H we compare the benchmark equilibrium to a rental-only equilibrium. We amend our benchmark model by removing the right to purchase a home. We prove in appendix that this rental-only economy has a unique equilibrium. Furthermore, if model parameters are such that a positive measure of newcomers choose community 1 in state H and a positive measure of native households choose the plan $(1_B, 1, 1)$ in the benchmark economy, then the equilibrium rents in the two economies display the following relationship:

$$R_1^r \leq R_1^b, \quad R_H^r < R_H^b, \quad R_L^r \geq R_L^b, \tag{9}$$

where we use superscripts "r" and "b" to distinguish variables in the rental-only economy from their counterparts in the benchmark economy, respectively.

The difference between the benchmark and the rental-only economy amounts to a downward shift in the natives' demand for community 1 in state H. This downward shift implies that R_H must be lower in the rental-only economy. A lower R_H raises the demand for community 1 not only from natives but also from newcomers. Therefore, when state H occurs, it cannot be that as many natives remain in community 1 than when ownership is an option.

Rents in period 1 and in state L may also differ between the rental-only and the benchmark economy. This is because the lower R_H in the rental-only economy may prompt some households who choose the plan $(1_R, 0, 0)$ in the benchmark economy to choose $(0, 1, 1)$ in the rental-only economy. In this case, the rents R_1 and R_L cannot be identical in the two economies. The shift from $(1_R, 0, 0)$ to $(0, 1, 1)$ implies a lower demand for location 1 in period 1 and a higher demand for location 1 in state L in the rental-only economy. This explains why R_1 may be lower and R_L may be higher in the rental-only economy. Alternatively, if no household chooses $(0, 1, 1)$ in the rental-only economy, then the same is true in the ownership economy. In this case, the effect on demands in period 1 and period 2 state L does not come into play, and the rents R_1 and R_L are the same in both economies. This explains the weak inequalities for R_1 and R_L in (9).

Overall, the difference in equilibrium rents between the benchmark and the rental-only economy does not fully compensate the drop in demand for location 1 in state H by native households. More newcomers move to location 1 in the rental-only economy than in the benchmark economy, taking advantage of the lower rent in state H.

Allowing households to own their home therefore increases the difference between the average income of the newcomers who move to location 1 and the average income of the natives who stay in location 1 in period 2 when state H occurs. Some poorer native households stay put in location 1 in state H when ownership is an option, while the income distribution of the newcomers who choose location 1 is truncated at a higher level. The newcomers who choose location 1 are richer on average. Unless the average income of newcomers located in 1 is lower than the average income of their native neighbors, this difference in averages implies greater income dispersion under homeownership than in the rental-only economy.

4 Concluding remarks

The standard explanation for the observed income mixing within US metropolitan neighborhood is that households differ in their preferences for local amenities. We propose a complementary explanation. By setting the community choice problem in a dynamic and stochastic environment, we show that income mixing arises even if households have identical standard preferences and the stock of housing within each neighborhood is homogeneous.

Our analysis sheds new light on policies that distort housing consumption. For example, when property taxes depend on the purchase price of a home and not its current value, buying a home provides a hedge not only against future rent risk but also against future tax liabilities risk. Such a policy reinforces the effects of ownership on the composition of neighborhoods that we identify in our model.

Our findings raise questions about the empirical research that relies on cross-sectional observations of household income and housing choice to estimate demand functions for local amenities. If the reason behind observed differences in location choices between households with identical income is not a difference in preferences, then observing a household's income and location choice is not sufficient to infer its preferences or its willingness to pay for local amenities.

Although we cast our discussion at the level of communities within the same urban area, our arguments apply readily to cities within the same region. What differentiates the cities in our model is the combination of their elasticity of supply of housing and their desirability. The relevance of such an interpretation is highlighted by the evidence reported by Gyourko, Mayer and Sinai (2004) who find that households that move to desirable cities with inelastic housing supplies tend to be richer than the households already living in these cities. This explains why housing price growth in such cities outpaces general income growth.

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Appendix

A.1 Proofs

To ease the notational burden, we define

$$e_1 = e^{a(1+r)R_1}, \quad e_H = e^{aR_H}, \quad e_L = e^{aR_L}, \quad e_2 = e^{a\bar{R}_2}. \tag{A.1}$$

Proof of Lemma 4: In view of Lemma 1, it is enough to show that the plans $(1_B, 1, 0)$ and $(0, 1, 0)$ are never optimal. We deal with $(1_B, 1, 0)$ first.

Suppose $\pi > \frac{1}{2}$. Let W_1 be the endowment level at which a native household would be indifferent between the plans $(1_R, 0, 1)$ and $(1_B, 1, 1)$, and W_2 the endowment level at which it would be indifferent between the plans $(1_R, 0, 1)$ and $(1_B, 1, 0)$. To show that the plan $(1_B, 1, 0)$ is never optimal, it suffices to show that $W_1 < W_2$. To see this, recall from Section 2.2 that if the expected utility curves of two plans cross, the curve associated with the plan that promises a larger amount of housing consumption in location 1 ex ante is steeper at all endowment levels. The curve associated with $(1_B, 1, 1)$ is above the curve associated with $(1_R, 0, 1)$ to the right of W_1, and the curve associated with $(1_R, 0, 1)$ is above the curve associated with $(1_B, 1, 0)$ to the left of W_2. If $W_1 < W_2$, this implies that the curve associated with $(1_B, 1, 0)$ is everywhere below the upper envelope of the curves associated with $(1_R, 0, 1)$ and $(1_B, 1, 1)$.

It is straightforward to verify that the endowment levels W_1 and W_2 are defined by

$$\mu e^{aW_1} = \frac{1}{\pi} e_1 \left[e_2 - \pi - (1-\pi) e_L \right], \tag{A.2}$$

$$\mu e^{aW_2} = \frac{1}{1-2\pi} e_1 \left[(1-\pi) \left(e_L - \frac{e_2}{e_L} \right) - \pi \left(e_2 - 1 \right) \right]. \tag{A.3}$$

It is easy to show that $W_1 < W_2$ if and only if $e_2 > e_L$, which in turn is equivalent to $R_L < R_H$.

Now suppose $\pi < \frac{1}{2}$. Let W_3 be the endowment level at which a native household would be indifferent between the plans $(1_R, 0, 1)$ and $(1_R, 0, 0)$:

$$\mu e^{aW_3} = e_1 \left(e_L - 1 \right). \tag{A.4}$$

The plan $(1_B, 1, 0)$ is never optimal if $W_2 < W_3$. This inequality is easily seen to be equivalent to $e_2 > e_L$.

In the case where $\pi = \frac{1}{2}$, a comparison of expected utilities shows that for $e_2 > e_L$, the plan $(1_R, 0, 1)$ is preferred to $(1_B, 1, 0)$ at all endowment levels. This completes the proof that $(1_B, 1, 0)$ is never optimal.

Turning to $(0, 1, 0)$, suppose $\pi > \frac{1}{2}$. Let W_4 be the endowment level at which a native household would be indifferent between the plans $(0, 1, 0)$ and $(0, 1, 1)$, and W_5 the endowment level at which it would be indifferent between the plans $(0, 0, 1)$ and $(0, 1, 0)$:

$$\mu e^{aW_4} = e_L - 1, \tag{A.5}$$

$$\mu e^{aW_5} = \frac{1-\pi}{1-2\pi} \left(e_L - 1 \right) - \frac{\pi}{1-2\pi} \left(e_H - 1 \right). \tag{A.6}$$

The plan $(0, 1, 0)$ is never optimal if $W_4 < W_5$. It is easy to verify that this inequality is equivalent to $e_L < e_H$.

Next suppose $\pi < \frac{1}{2}$. Let W_6 be the endowment level at which a native household would be indifferent between the plans $(0, 0, 0)$ and $(0, 0, 1)$, and W_7 the endowment level at which it would be indifferent between the plans $(0, 0, 0)$ and $(0, 1, 0)$:

$$\mu e^{aW_6} = e_L - 1, \tag{A.7}$$

$$\mu e^{aW_7} = e_H - 1. \tag{A.8}$$

The plan $(0, 1, 0)$ is never optimal if $W_6 < W_7$, which is obviously the same as $e_L < e_H$.

In the case where $\pi = \frac{1}{2}$, a comparison of expected utilities shows that for $e_L < e_H$, the plan $(0, 0, 1)$ is preferred to $(0, 1, 0)$ at all endowment levels. This completes the proof. ■

Proof of Lemma 5: Part (i): Let W_1 be the endowment level at which a native household would be indifferent between the plans $(0, 0, 0)$ and $(1_R, 0, 1)$, and W_2 the endowment level at which it would be indifferent between the plans $(1_R, 0, 1)$ and $(1_B, 1, 1)$. To show that the plan $(1_R, 0, 1)$ is preferred to both

$(0,0,0)$ and $(1_B,1,1)$ on a set of endowment levels of positive measure, it is enough to show that $W_1 < W_2$. To see this, recall from Section 2.2 that if the expected utility curves of two plans cross, the curve associated with the plan that promises a larger amount of housing consumption in location 1 ex ante is steeper at all endowment levels. The curve associated with $(1_R,0,1)$ is above the curve associated with $(0,0,0)$ to the right of W_1, and above the curve associated with $(1_B,1,1)$ to the left of W_2. If $W_1 < W_2$, therefore, $(1_R,0,1)$ is preferred to both $(0,0,0)$ and $(1_B,1,1)$ at all wealth levels strictly between W_1 and W_2. It is straightforward to verify that the endowment levels W_1 and W_2 are defined by

$$\mu e^{aW_1} = \frac{1}{2-\pi}\left[\pi e_1 + (1-\pi)e_1 e_L - 1\right], \tag{A.9}$$
$$\mu e^{aW_2} = \frac{e_1}{\pi}\left[e_2 - \pi - (1-\pi)e_L\right]. \tag{A.10}$$

It is easy to see that $W_1 < W_2$ if and only of $2(1-\pi)e_1(e_2 - e_L) + \pi\left[e_1 e_2 - 2e_1 + 1\right] > 0$. As $e_L < e_H$, we have $e_2 > e_L$. If $e_2 \geq e_1$, we also have $e_1 e_2 - 2e_1 + 1 \geq (e_1 - 1)^2$, so $e_2 \geq e_1$ is a sufficient condition for $W_1 < W_2$, and hence for $(1_R,0,1)$ to be preferred to $(0,0,0)$ and $(1_B,1,1)$ on some open interval of endowment levels.

Next, let W_3 be the endowment level at which a native household would be indifferent between the plans $(0,0,0)$ and $(0,0,1)$, and W_4 the endowment level at which it would be indifferent between the plans $(0,0,0)$ and $(1_B,1,1)$:

$$\mu e^{aW_3} = e_L - 1, \tag{A.11}$$
$$\mu e^{aW_4} = \frac{1}{2-\pi}\left[e_1 e_2 - \pi - (1-\pi)e_L\right]. \tag{A.12}$$

It is easy to see that $W_3 < W_4$ if and only of $e_1 e_2 - 2e_2 + 1 + 2(e_2 - e_L) > 0$. As $e_L < e_H$, we have $e_2 > e_L$. If $e_1 \geq e_2$, we also have $e_1 e_2 - 2e_2 + 1 \geq (e_2 - 1)^2$, so $e_1 \geq e_2$ is a sufficient condition for $W_3 < W_4$, and hence for $(0,0,1)$ to be preferred to $(0,0,0)$ and $(1_B,1,1)$ on some open interval of endowment levels.

Part (ii): An argument similar to the one used for part (i) shows first that for $e_1 \leq e_L$, $(1_R,0,0)$ is preferred to $(0,0,0)$ and $(1_R,0,1)$ on some open interval of endowment levels; and second, that for $e_1 \geq e_L$, $(0,0,1)$ is preferred to $(0,0,0)$ and $(1_R,0,1)$ on some open interval of endowment levels.

Part (iii): Let W_5 be the endowment level at which a native household would be indifferent between the plans $(0,0,0)$ and $(0,1,1)$, and W_6 the endowment level at which it would be indifferent between the plans $(0,0,1)$ and $(0,1,1)$:

$$\mu e^{aW_5} = \pi e_H + (1-\pi)e_L - 1, \tag{A.13}$$
$$\mu e^{aW_6} = e_H - 1. \tag{A.14}$$

It suffices to show that $W_5 < W_6$. This is easily seen to be equivalent to $e_L < e_H$.

Part (iv): Let W_7 be the endowment level at which a native household would be indifferent between the plans $(1_R,0,0)$ and $(1_B,1,1)$, and W_8 the endowment level at which it would be indifferent between the plans $(1_R,0,1)$ and $(1_B,1,1)$:

$$\mu e^{aW_7} = e_1\,(e_2 - 1), \tag{A.15}$$
$$\mu e^{aW_8} = e_1\left[e_2 - 1 + \frac{1-\pi}{\pi}\,(e_2 - e_1)\right]. \tag{A.16}$$

It suffices to show that $W_7 < W_8$. This is easily seen to be equivalent to $e_L < e_2$, which in turn is the same as $e_L < e_H$. ■

Proof of Proposition 1: Lemma A.6 shows that in equilibrium, second period rents satisfy $R_L < R_H$ if a positive measure of newcomers choose location 1, and $R_L = R_H$ otherwise. Lemma A.11 shows that (2) holds if $R_L < R_H$. Lemmas A.8 and A.9 show that equilibrium configurations must be as stated in the proposition. This implies that the relevant market clearing conditions are (A.27)–(A.30). Lemma A.12 shows that these conditions are equivalent to the system of equations (A.31)–(A.34). Lemmas A.13 and A.14 show that this system admits a unique solution with $R_L \leq R_H$ and the properties stated in the proposition. Lemma A.15 shows that this solution yields an equilibrium. ■

A.2 Existence and uniqueness of equilibrium in the rental-only economy

Replicating the arguments we used in the benchmark economy with homeownership, it is easy to verify that in a rental-only equilibrium where some newcomers choose location 1 in state H, the rental prices satisfy $R_L < R_H$ and native households will choose housing consumption plans from the following subset of alternatives: $(0,0,0)$, $(0,0,1)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$ and $(1_R,1,1)$. Again, $(0,1,1)$ is weakly dominated by $(1_R,0,0)$ and can arise as a native household's equilibrium choice only if (2) holds as an equality. So there are again four critical indices that characterize marginal households. The indices i_1, i_2, i_3 are defined exactly as in the benchmark economy. For indifference between $(1_R,0,1)$ and $(1_R,1,1)$, however, the critical index is now defined by

$$\mu e^{aW(i_4)} = e^{a(1+r)R_1}\left(e^{aR_H} - 1\right). \tag{A.17}$$

Proposition 2 *There is a unique equilibrium in the rental-only economy. If a positive measure of newcomers choose location 1 in state H and a positive measure of native households choose the plan $(1_B,1,1)$ in the economy where homeownership is allowed, then the equilibrium prices in the rental-only economy compare as follows with those in the ownership economy:*

$$R_1^r \le R_1^b, \quad R_H^r < R_H^b, \quad R_L^r \ge R_L^b.$$

Proof of Proposition 2: Existence and uniqueness of equilibrium are shown along exactly the same lines as in the proof of Proposition 1. Note in particular that equations (A.31) and (A.33) are the same in both economies, so Lemma A.13 with its description of e_1 and e_L as continuous monotonic functions of e_H carries over without any modification. Given a value for e_H, we thus have the same values for e_1, e_L, i_2 and n_1 in both economies. In contrast, we have $i_4^b < i_4^r$ at any given value of e_H that is assumed common to both economies, different from e_L and such that $0 < i_4^b < 1$. By the definitions of these indices, the stated inequality is equivalent to $e_2 - (1-\pi)e_L < \pi e_H$, which always holds by the convexity of the exponential function. As a function of e_H, therefore, the right-hand side of (A.32) is strictly larger in the rental-only economy over the range where $0 < i_4^b < 1$. This implies that if $i_4^b < 1$ in the ownership equilibrium, then the equilibrium rental prices in state H satisfy $e_L < e_H$. The remaining comparison results now follow from Lemma A.13. ∎

A.3 Auxiliary results on household behavior

If $R_L > R_H$, the roles of the two states in period 2 are reversed. The following three results are therefore just mirror images of Lemmas 1, 4 and 5, respectively, and do not require proofs of their own.

Lemma A.1 *If $R_L > R_H$, a native household wanting to live in location 1 in the first period prefers to own its home if and only if it plans to stay in location 1 should state L occur in the second period.*

Lemma A.2 *If $R_L > R_H$, a native household chooses a location-tenure plan from the following subset of available options:* $(0,0,0)$, $(0,1,0)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,1,0)$ *and* $(1_B,1,1)$.

Lemma A.3 *Let $R_L > R_H$. Then:*

- **(i)** *at least one of the plans* $(0,1,0)$ *and* $(1_R,1,0)$ *is preferred to both* $(0,0,0)$ *and* $(1_B,1,1)$ *at all endowment levels in some set of positive measure;*
- **(ii)** *at least one of the plans* $(0,1,0)$ *and* $(1_R,0,0)$ *is preferred to both* $(0,0,0)$ *and* $(1_R,1,0)$ *at all endowment levels in some set of positive measure;*
- **(iii)** *the plan* $(0,1,0)$ *is preferred to both* $(0,1,1)$ *and* $(0,0,0)$ *at all endowment levels in some set of positive measure;*
- **(iv)** *the plan* $(1_R,1,0)$ *is preferred to both* $(1_B,1,1)$ *and* $(1_R,0,0)$ *at all endowment levels in some set of positive measure.*

If $R_H = R_L$, the tenure mode is irrelevant, so native households' decisions concern location only.

Lemma A.4 *If $R_L = R_H$, each of the location plans $(1,1,0)$, $(1,0,1)$, $(0,1,0)$ and $(0,0,1)$ is optimal for a native household at precisely one endowment level, and suboptimal at all other endowment levels. Thus, only the plans $(0,0,0)$, $(0,1,1)$, $(1,0,0)$ and $(1,1,1)$ may be chosen by a positive measure of native households.*

PROOF: The first statement follows if we let R_L tend to R_H in Lemmas 4 and A.2. The second statement follows trivially from the first. ■

Lemma A.5 *Let $R_L = R_H$. If the location plans $(1,1,1)$ and $(0,0,0)$ are optimal at some endowment levels, then one of the plans $(1,0,0)$ and $(0,1,1)$ is preferred to both $(1,1,1)$ and $(0,0,0)$ on a set of endowment levels of positive measure.*

PROOF: As $R_L = R_H$, we have $e_L = e_H = e_2$, for which we shall write e_*.

Let W_1 be the endowment level at which a native household would be indifferent between the plans $(0,0,0)$ and $(1,0,0)$, and W_2 the endowment level at which it would be indifferent between the plans $(1,0,0)$ and $(1,1,1)$:

$$\mu e^{aW_1} = e_1 - 1, \tag{A.18}$$
$$\mu e^{aW_2} = e_1(e_* - 1). \tag{A.19}$$

Thus, $W_1 < W_2$ if and only if $e_1(e_* - e_1) + (e_1 - 1)^2 > 0$, a sufficient condition for which is $e_* \geq e_1$.

Next, let W_3 be the endowment level at which a native household would be indifferent between the plans $(0,0,0)$ and $(0,1,1)$, and W_4 the endowment level at which it would be indifferent between the plans $(0,1,1)$ and $(1_B,1,1)$:

$$\mu e^{aW_3} = e_* - 1, \tag{A.20}$$
$$\mu e^{aW_4} = e_*(e_1 - e_*). \tag{A.21}$$

Thus, $W_3 < W_4$ if and only if $e_*(e_1 - e_*) + (e_* - 1)^2 > 0$, a sufficient condition for which is $e_1 \geq e_*$. This implies that no matter what e_1 and e_* are, at least one of the inequalities $W_1 < W_2$ and $W_3 < W_4$ holds. ■

A.4 Auxiliary results on equilibrium prices and configurations

In the following, we shall write D_1, D_H and D_L for native households' aggregate demand for housing in location 1 in period 1, period 2 state H, and period 2 state L, respectively.

Lemma A.6 *In equilibrium, second period rents satisfy $R_L < R_H$ if a positive measure of newcomers choose location 1, and $R_L = R_H$ otherwise.*

PROOF: Suppose that $R_L \geq R_H$ with a positive measure of newcomers choosing location 1 in state H. Then, Lemmas A.2 and A.4 imply that $D_L \leq D_H$. Aggregate demand for housing in location 1 by native households and newcomers is therefore higher in state H than in state L. Given that the supply of housing in location 1 is the same in both states, this is incompatible with market clearing. This proves the first part of the lemma.

Next, suppose that $R_L < R_H$ with all newcomers choosing location 0 in state H. Then, market clearing implies $D_H = D_L$, which in turn implies that the plans $(1_R,0,1)$ and $(0,0,1)$ are not chosen by any native households. This contradicts parts (ii) and (iii) of Lemma 5 unless either $(1_R,0,0)$ and $(0,1,1)$ are the only plans chosen (in which case they are chosen in equal measure), or $(1_B,1,1)$ and $(0,0,0)$ are the only plans chosen. The first alternative contradicts our assumption that $S < \frac{1}{2}$, the second contradicts part (i) of Lemma 5. A similar argument involving Lemma A.3 instead of Lemma 5 shows that the inequality $R_L > R_H$ is incompatible with no newcomers choosing location 1 in state H. This proves the second part of the lemma. ■

Lemma A.7 *In any equilibrium, the plan $(0,0,0)$ is chosen by a positive measure of native households.*

PROOF: From Lemma A.6, we know that $R_L \leq R_H$. From Lemmas 4 and A.4, we know that the only housing consumption plans that may be chosen by a positive measure of native households are $(1,1,1)$, $(1,0,1)$, $(1,0,0)$, $(0,1,1)$, $(0,0,1)$ and $(0,0,0)$. We write m_{111} for the measure of native households choosing $(1,1,1)$, m_{101} for the measure of native households choosing $(1,0,1)$ etc.

Now suppose $m_{000} = 0$. Then, market clearing in period 1 implies $m_{001} + m_{011} = 1 - S$; market clearing in period 2 state L implies $m_{100} = 1 - S$. Adding up these two equations yields $m_{001} + m_{011} + m_{100} = 2(1 - S) > 1$, which contradicts the fact that the total native population has size 1. ■

Lemma A.8 *In an equilibrium where a positive measure of newcomers choose location 1 in state H, the location-tenure plans chosen by a positive measure of native households are* $(0,0,0)$, $(0,0,1)$, $(1_R,0,0)$ *plus either*

- **(a)** $(0,1,1)$, *or*
- **(b)** $(0,1,1)$ *and* $(1_R,0,1)$, *or*
- **(c)** $(0,1,1)$, $(1_R,0,1)$ *and* $(1_B,1,1)$, *or*
- **(d)** $(1_R,0,1)$ *and* $(1_B,1,1)$.

PROOF: From Lemma A.6, we know that $R_L < R_H$. From Lemma 4, we know that the only plans that may be chosen by a positive measure of native households are $(1_B,1,1)$, $(1_R,0,1)$, $(0,1,1)$, $(1_R,0,0)$, $(0,0,1)$ and $(0,0,0)$. Because of our assumption that $\nu < S$, there must be a positive measure of native households consuming housing in location 1 in state H. This means that at least one of the plans $(1_B,1,1)$ and $(0,1,1)$ must be chosen.[1]

Case 1: $(0,1,1)$ is not chosen, so $(1_B,1,1)$ must be chosen. We want to show that $(1_R,0,1)$, $(1_R,0,0)$ and $(0,0,1)$ are chosen as well. Market clearing requires $D_1 = D_L > D_H$, so $(1_R,0,1)$ must be chosen, or both $(1_R,0,0)$ and $(0,0,1)$ must be chosen. By part (iv) of Lemma 5, $(1_R,0,1)$ is chosen whenever $(1_R,0,0)$ is chosen. So $(1_R,0,1)$ must be chosen. Next, Lemma A.7 implies that $(0,0,0)$ is chosen, so by part (ii) of Lemma 5, at least one of $(0,0,1)$ and $(1_R,0,0)$ is chosen. As $D_1 = D_L$, one cannot be chosen without the other.

Case 2: $(1_B,1,1)$ is not chosen, so $(0,1,1)$ must be chosen. First, note that $(1_R,0,0)$ must be chosen as well; otherwise, D_1 cannot equal D_L. Next, Lemma A.7 implies that $(0,0,0)$ is chosen, so by part (iii) of Lemma 5, $(0,0,1)$ is chosen.

Case 3: Both $(1_B,1,1)$ and $(0,1,1)$ are chosen. Arguing as in the previous case, we see that $(1_R,0,0)$ $(0,0,1)$ and $(0,0,0)$ are chosen as well. Finally, part (iv) of Lemma 5 implies that $(1_R,0,1)$ is also chosen. ■

Lemma A.9 *In an equilibrium where all newcomers choose location 0 in state H, the location plans chosen by a positive measure of native households are* $(0,0,0)$, $(0,1,1)$, $(1,0,0)$ *plus possibly* $(1,1,1)$.

PROOF: From Lemma A.6, we know that $R_L = R_H$. For this case, Lemma A.4 implies that the only housing consumption plans possibly chosen by a positive measure of native households in equilibrium are $(0,0,0)$, $(0,1,1)$, $(1,0,0)$ and $(1,1,1)$. Lemma A.7 implies that $(0,0,0)$ is chosen. Lemma A.5 implies that the configuration cannot just consist of $(0,0,0)$ and $(1,1,1)$. Market clearing implies that if the equilibrium configuration contains $(0,1,1)$, it must also contain $(1,0,0)$, and vice versa. ■

By Lemma 3, this immediately implies

Lemma A.10 *In an equilibrium where all newcomers choose location 0 in state H, equation* (2) *holds with equality; as $R_L = R_H$, this means $(1+r)R_1 = R_L = R_H$.*

Lemma A.11 *In an equilibrium where a positive measure of newcomers choose location 1 in state H, the measure of native households who choose the plan* $(1_R,0,0)$ *is at least as large as the measure of native households who choose the plan* $(0,1,1)$. *As a consequence,* $(0,1,1)$ *cannot dominate* $(1_R,0,0)$, *so* (2) *holds.*

PROOF: Suppose to the contrary that fewer native households choose $(1_R,0,0)$ than $(0,1,1)$. In view of Lemmas A.6 and 4, this implies that $D_H \geq D_L$, which is incompatible with the premise that a positive measure of newcomers choose location 1 in state H. ■

[1]Here and in what follows, we always understand the word "chosen" to mean "chosen by a positive measure of native households".

A.5 Auxiliary results on existence and uniqueness of equilibrium

It will be convenient to work with e_1, e_L and e_H instead of R_1, R_L and R_H, respectively. We define $\psi = \mu e^{aW(1-S)} + 1$.

Four critical endowment indices fully characterize native households' choices. For indifference between $(0,0,0)$ and $(0,0,1)$, the critical endowment index is i_1 with

$$\mu e^{aW(i_1)} = \max\left\{\min\left\{e_L - 1,\ \mu e^{aW(1)}\right\},\ \mu e^{aW(0)}\right\}. \tag{A.22}$$

For indifference between $(0,0,1)$ and $(1_R,0,0)$, the critical endowment index is i_2 with

$$\mu e^{aW(i_2)} = \max\left\{\min\left\{\frac{e_1 - \pi - (1-\pi)e_L}{\pi},\ \mu e^{aW(1)}\right\},\ \mu e^{aW(0)}\right\}. \tag{A.23}$$

For indifference between $(1_R,0,0)$ and $(1_R,0,1)$, the critical endowment index is i_3 with

$$\mu e^{aW(i_3)} = \max\left\{\min\left\{e_1\left(e_L - 1\right),\ \mu e^{aW(1)}\right\},\ \mu e^{aW(0)}\right\}. \tag{A.24}$$

For indifference between $(1_R,0,1)$ and $(1_B,1,1)$, the critical endowment index is i_4 with

$$\mu e^{aW(i_4)} = \max\left\{\min\left\{\frac{e_1\left(e^{a\bar{R}_2} - \pi - (1-\pi)e_L\right)}{\pi},\ \mu e^{aW(1)}\right\},\ \mu e^{aW(0)}\right\}. \tag{A.25}$$

Given our results on the set of possible equilibrium configurations, these critical indices satisfy the following conditions in equilibrium: $0 < i_1 \leq i_2 < i_3 \leq i_4$.

Newcomers face a deterministic one-period problem since they only enter if state H occurs. Given the continuity and monotonicity of the endowment function $\tilde{W}$, we obtain a critical index n_1 such that the newcomers with index $n > n_1$ prefer location 1 over location 0. This index is implicitly defined by the equation

$$\mu e^{a\tilde{W}(n_1)} = \max\left\{\min\left\{e_H - 1,\ \mu e^{a\tilde{W}(1)}\right\},\ \mu e^{a\tilde{W}(0)}\right\}. \tag{A.26}$$

Using the definition of the five critical indices, the market clearing conditions for housing in location 1 period 1, period 2 state H and period 2 state L take the following form

$$S = 1 - i_3 + \rho(i_3 - i_2), \tag{A.27}$$
$$S = 1 - i_4 + (1-\rho)(i_3 - i_2) + (1 - n_1)\nu, \tag{A.28}$$
$$S = 1 - i_3 + (1-\rho)(i_3 - i_2) + i_2 - i_1, \tag{A.29}$$

where ρ is the fraction of households with indices between i_2 and i_3 who choose $(1_R,0,0)$. By definition, $0 \leq \rho \leq 1$. Lemma 3 implies that

$$(1-\rho)\left(e_1 - \pi e_H - (1-\pi)e_L\right) = 0. \tag{A.30}$$

Note that our use of min and max operators in the definitions of the critical indices allows us to write each demand for location 1 housing as a single expression for all possible equilibrium configurations.

Lemma A.12 *The system of equations* (A.27)–(A.30) *is equivalent to the following system of equations*

$$2(1-S) = i_1 + i_3, \tag{A.31}$$
$$2(1-S) + \nu = i_2 + i_4 + \nu n_1, \tag{A.32}$$
$$e_1 = \pi \min\left\{e_H,\ \mu e^{aW(1-S)} + 1\right\} + (1-\pi)e_L, \tag{A.33}$$

and

$$\rho = \frac{i_3 - (1-S)}{i_3 - i_2}. \tag{A.34}$$

PROOF: Adding up equations (A.27) and (A.28), we obtain (A.32). Adding up equations (A.27) and (A.29), we obtain (A.31). Now, if $e_1 < \pi e_H + (1-\pi)e_L$ then $\rho = 1$. Equation (A.27) then implies $i_2 = 1 - S$, which by the definition of i_2 yields

$$e_1 = \pi\psi + (1-\pi)e_L < \pi e_H + (1-\pi)e_L. \tag{A.35}$$

Then, since $i_2 = 1 - S$, equation (A.34) simply becomes $\rho = 1$. If $e_1 = \pi e_H + (1-\pi)e_L$ then $\rho \leq 1$ and the definition of i_2 becomes $\mu e^{aW(i_2)} = e_H - 1$. Moreover, (A.27) implies $i_2 \leq 1 - S$, hence $\mu e^{aW(1-S)} \geq e_H - 1$. Therefore,

$$e_1 = \pi e_H + (1-\pi)e_L \leq \pi\psi + (1-\pi)e_L. \tag{A.36}$$

Therefore equation (A.33) holds. Using (A.27), we obtain (A.34).

Conversely, equation (A.33) gives us two possible cases. First, if $\mu e^{aW(1-S)} + 1 < e_H$, then (A.33) plus the definition of i_2 imply $i_2 = 1 - S$, which yields $\rho = 1$ by equation (A.34) and implies that equations (A.27) and (A.30) hold. Then, replacing one term $1 - S$ by i_2 in equations (A.32) and (A.31) yields equations (A.28) and (A.29) for the case $\rho = 1$. Second, if $\psi \leq e_H$, then (A.33) implies that (A.30) holds. Using (A.34) to replace one $1 - S$ term in equations (A.32) and (A.31) yields equations (A.28) and (A.29). Rearranging (A.34) yields (A.27). ■

For our next result, recall the definition of $\underline{e}$ in Section 2.4. It is straightforward to see that $\underline{e} < \psi$.

Lemma A.13 *Equations* (A.31) *and* (A.33) *yield e_1 and e_L as continuous monotonic functions of e_H, with the first weakly increasing and the second weakly decreasing in e_H. The inequality $e_L < e_H$ holds if and only if $e_H > \underline{e}$. More precisely, $\mu e^{aW(1-2S)} + 1 < e_L < \underline{e} < e_1 < e_H$ if $\underline{e} < e_H < \psi$, and $\mu e^{aW(1-2S)} + 1 < e_L < e_1 < \psi \leq e_H$ if $e_H \geq \psi$. Finally, $e_L = e_1 = e_H$ if and only if $e_H = \underline{e}$.*

PROOF: Equation (A.31) implies that neither i_1 nor i_3 can be zero, and at most of them can assume the value one. By the definitions of i_1 and i_3, the right-hand side of (A.31) is strictly increasing in e_L, and weakly increasing in e_1. This defines e_L as a weakly decreasing function of e_1 which assumes the value ψ at $e_1 = 1$ and tends to $\mu e^{aW(1-2S)} + 1$ as e_1 goes to infinity. Rearranging equation (A.33) into

$$(1-\pi)e_L = e_1 - \pi \min\{e_H,\ \psi\} \tag{A.37}$$

defines e_L as a strictly increasing function of e_1, given e_H. This function assumes a value of at most 1 at $e_1 = 1$ and tends to infinity as e_1 does. This implies that for any given e_H, (A.31) and (A.33) determine unique values of e_1 and e_L with $\mu e^{aW(1-2S)} + 1 < e_L < \psi$. An increase in e_H either leaves both functions unchanged, or shifts the second function down and leaves the first unchanged. Continuity is obvious.

Next, note that in the (e_1, e_L)-plane, the graph of the function defined by (A.37) cuts the 45 degree line from below at $e_1 = \min\{e_H,\ \psi\}$, while the graph of the function defined by (A.31) cuts the 45 degree line from above at $e_1 = \underline{e}$. Using these facts, it is now easy to verify the statements about the ranking of e_1, e_H and e_L. ■

An immediate corollary of the above proof is that if no household chooses $(0, 1, 1)$ in the rental-only economy, then the same is true in the ownership economy.

Lemma A.14 *The system of equations* (A.31)–(A.33) *has a unique solution with $e_H \geq \underline{e}$, and $e_H = \underline{e}$ if and only if $\mu e^{a\tilde{W}(1)} \leq \underline{e} - 1$.*

PROOF: We want to to establish that equation (A.32) admits a unique solution e_H once e_1 and e_L are solved for as functions of e_H according to Lemma A.13. First, we note that i_2 is weakly increasing in e_1 and weakly decreasing in e_L. This implies that i_2 is weakly increasing in e_H. Second, n_1 is also weakly increasing in e_H. Third, the definition of i_4 can be rearranged into

$$\mu e^{aW(i_4)} = \max\left\{\min\left\{e_1 e_L - e_1 + e_1 e_L z,\ \mu e^{aW(1)}\right\},\ \mu e^{aW(0)}\right\}, \tag{A.38}$$

where $z = \left[(e_H/e_L)^{\pi} - 1\right]/\pi$ is strictly increasing in e_H and non-negative when $e_H \geq \underline{e}$. We know from the proof of Lemma A.13 that $i_3 > 0$. If $i_3 < 1$, then $\mu e^{aW(i_3)} = e_1 e_L - e_1$, which is weakly increasing in e_H by Lemma A.13 and equation (A.31) because i_1 is weakly decreasing in e_H. This in turn implies that

$e_1 e_L - e_1$ is weakly increasing in e_H. Given that e_1 is weakly increasing in e_H, $e_1 e_L$ is weakly increasing. So, if $i_3 < 1$, then i_4 is weakly increasing in e_H, and strictly increasing up to the level 1. If $i_3 = 1$, it is immediate that $i_4 = 1$ as well. This establishes that the right-hand side of (A.32) is strictly increasing in e_H up to a point and then possibly constant. The term $i_4 + \nu n_1$ becomes constant when e_H is so high that $i_4 = n_1 = 1$. In addition, when $e_H \geq \psi$, then equation (A.33) and the definition of i_2 implies that $i_2 = 1 - S$. So, if the right-hand side of (A.32) ever becomes flat as e_H increases, it does so at the level $2 - S + \nu$ which is greater than the left-hand side of (A.32). At $e_H = \underline{e}$, we have $i_2 = i_1$ and $i_4 = i_3$, so (A.31) implies that the right-hand side of (A.32) does not exceed the left-hand side. This establishes existence and uniqueness of a solution to the system of equations (A.31)–(A.33) with $e_H \geq \underline{e}$. It also shows that $e_H = \underline{e}$ if and only if n_1 equals 1 at $e_H = \underline{e}$, that is, if and only if $\mu e^{a\tilde{W}(1)} \leq \underline{e} - 1$. ■

Lemma A.15 *The solution to the system of equations* (A.31)–(A.33) *identified in Lemma A.14 constitutes an equilibrium.*

Proof: If $\mu e^{a\tilde{W}(1)} \leq \underline{e} - 1$, we have $e_1 = e_L = e_H = \underline{e}$ by Lemma A.13 and so $0 < i_1 = i_2 < i_3 = i_4$. If $\mu e^{a\tilde{W}(1)} > \underline{e} - 1$, Lemma A.13 implies that $0 < i_1 < i_2 < i_3 \leq i_4$. This shows that the ranking of the critical endowment indices i_1 through i_4 is the one that we assumed when formulating the market clearing conditions (A.27)–(A.30). So, the solution we identified constitutes an equilibrium. ■