Business Cycles and Firm Dynamics*

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May 19, 2005
VERY PRELIMINARY AND INCOMPLETE
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Abstract

This paper builds a framework for the analysis of macroeconomic business cycles that incorporates the endogenous determination of the number of producers over the business cycle. Economic expansions induce higher entry rates by prospective entrants subject to irreversible investment costs. The sluggish response of the number of producers (due to the sunk entry costs) generates a new and potentially important endogenous propagation mechanism for real business cycle models (which typically rely on the accumulation of physical capital by a fixed number of producers). Consistent with the data, our framework predicts a procyclical number of producers, and procyclical profits. We use the same modeling framework to analyze how endogenous entry affects the efficiency properties of business cycle models. We show that the market equilibrium of our model is efficient, even with prices above marginal costs, if labor supply is inelastic. When labor supply is endogenous, efficiency is restored by taxing leisure at a rate equal to the net markup in the market for consumption goods.

JEL Codes: E20; E32

Keywords: Business cycle transmission; Efficiency; Entry

*We thank participants in seminars at the ECB, Northeastern University, Princeton, and the University of Connecticut for helpful comments. We are grateful to Viktors Stebunovs and Pinar Uysal for excellent research assistance, and to Kolver Hernandez for help with Matlab codes. Remaining errors are our responsibility. Bilbiie thanks the ECB for hospitality in the summer of 2004. Ghironi and Melitz thank the NSF for financial support.

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1 Introduction

The number of firms in the economy varies over the business cycle. Figure 1 shows the quarterly growth rates of real GDP, profits, and net entry in the U.S. economy (measured as the difference between new incorporations and failures) for the period 1947-1998. Net entry is strongly procyclical and comoves with real profits, which are also procyclical. Figure 2 shows cross correlations between real GDP, profits, and net entry (Hodrick-Prescott filtered data in logs) at various leads and lags, with 95 percent confidence bands. The strong procyclicality of net entry and profits is evident, with net entry strongly correlated to profits. Importantly, Figure 2 shows that net entry tends to lead GDP and profit expansions, suggesting that firm entry in the expectation of future profits may play an important role in GDP expansions.[1]

This paper studies the role of firm entry in propagating business cycle fluctuations in a model with monopolistic competition and sunk entry costs. We seek to understand the contributions of the intensive and extensive margins to the response of the economy to changes in aggregate productivity, government spending, and market regulation (which affects the size of sunk entry costs). We also explore the consequences of introducing free entry for efficiency of the equilibrium of the economy.

In our setup, each individual firm produces using only labor. However, the number of firms that produce in each period can be interpreted as the capital stock of the economy, and the decision of households to finance entry of new firms is akin to the decision to accumulate physical capital in the standard real business cycle (RBC) model. In fact, we show that our model relates quite transparently to the traditional RBC model as laid out, for instance, in Campbell (1994).

There are significant differences though. We show that firm entry plays an important role in the propagation of responses to shocks. We start from a benchmark version of the model with inelastic labor supply. If aggregate productivity increases permanently, the expansion of aggregate GDP initially takes place at the intensive margin, with an increase in output of existing firms. Higher productivity makes entry more attractive and labor is reallocated to creation of new firms. Over time, the number of firms in the economy increases, and output per firm decreases. Further aggregate GDP expansion is the result of an increasing number of producers. In the long run, output per firm returns to the initial steady-state level and permanent GDP expansion is entirely driven by the extensive margin. These labor reallocation dynamics and intensive-extensive margin

[1] The procyclical pattern of net entry is the result of a strongly procyclical pattern of new incorporations and a countercyclical pattern of failures, which correlate negatively with GDP and profits.

effects are absent in the standard RBC framework. Importantly, even if total labor supply is fixed, and hence net job creation is absent, our model predicts sizeable gross job flows, precisely due to intersectoral reallocations.

We then introduce an endogenous labor supply decision and show that this further enhances the propagation mechanism of our model. In general, adjustment along this margin (total hours worked) amplifies the effects of shocks. Importantly, we show that government spending shocks have very different effects depending on whether labor supply is fixed or elastic enough. In the latter case, wasteful government spending can in fact increase private consumption, with a positive effect on welfare. In summary, we show that firm entry strengthens the notoriously weak endogenous propagation mechanism of business cycle models (see Cogley and Nason, 1995) and contributes significantly to both persistence and volatility of economic fluctuations.

The normative implications of our exercise are also significant. Importantly, despite prices being above marginal cost, the market equilibrium of our model with entry is efficient if labor supply is inelastic. We identify two mechanisms that ensure this result. First, since price adjustment is frictionless and producers are symmetric, markups in the pricing of all goods that bring utility to the consumer are synchronized. While this is also true in a model with monopolistic competition and a fixed number of firms when labor supply is inelastic, our model with entry has an important additional implication. Namely, although we let one factor of production (the number of firms) vary subject to a sunk entry cost, a time-to-build lag, and exogenous firm destruction, efficiency still holds. The resulting number of firms is socially optimal due to the key distinguishing feature of our framework – the entry mechanism based on C.E.S. preferences, as we explain below.[2]

Efficiency no longer holds when the other factor of production in our model varies, *i.e.*, when labor supply is endogenous. However, the relevant distortion is not the existence of a markup in the market for goods, but heterogeneity in markups between the "goods" the consumer cares about: consumption goods and leisure (priced at "marginal cost" in a competitive labor market). If the government taxes leisure (or subsidizes labor supply) at a rate equal to the net markup in consumption goods prices, efficiency is restored. While these results hold also in a model with a fixed number of firms, an equivalent optimal policy in that setup would have the markup removed by a distortionary tax on revenues. In our model, such a policy of inducing marginal cost pricing would eliminate entry incentives, since the sunk entry cost could not be covered in the absence

[2] Feenstra (2003) observes that a constant number of firms "violates the spirit of monopolistic competition."

of profits.[3] This shows that, in the presence of entry subject to a sunk cost, monopoly power is not a distortion and should in fact be preserved. Indeed, while markup synchronization is still a necessary condition for efficiency, sufficiency requires that markups be aligned to the relatively higher level.

[TO BE INCLUDED: SECOND-MOMENT PROPERTIES.]

Our paper contributes to several literatures. Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996*a*,*b*) documented the procyclical nature of entry and developed general equilibrium models with monopolistic competition to study the effect of entry and exit on the dynamics of the business cycle. However, entry is frictionless in their models: There is no sunk entry cost, and firms enter instantaneously in each period until all profit opportunities are exploited. A fixed period-by-period cost then merely ensures that the number of operating firms is finite; the free-entry condition sets profits to zero in all periods, and the number of firms that produce in each period is not a state variable. This is clearly inconsistent with two pieces of evidence presented above, namely the cyclical variation of profits (Figure 1) and the fact that net entry leads both output and profits (Figure 2). Moreover, this is also inconsistent with observed barriers to entry in most industries. In contrast, entry in our model is subject to a sunk entry cost and a time-to-build lag, so the free entry condition equates the expected present discounted value of profits to the sunk cost. Therefore, profits are allowed to vary and the number of firms is a state variable in our model, consistently with the evidence presented above and the widespread view that the number of producing firms is fixed in the short run.[4] Furthermore, Devereux, Head, and Lapham interpret the number of firms as an endogenous productivity shifter, whereas it is best interpreted as the capital stock of the economy in our model.[5]

By studying the efficiency properties of the model in dynamic, stochastic, general equilibrium (DSGE), our work contributes also to the literature on the efficiency properties of monopolistic competition started by the original work of Lerner (1934) and developed by Samuelson (1947), Spence (1976), Dixit and Stiglitz (1977), and Grossman and Helpman (1991), among others.[6]

[3] We are implicitly assuming that the government is not contemporaneously subsidizing the entire amount of the entry cost.

[4] In fact, our model features a fixed number of producing firms within each period and a fully flexible number of firms in the long run.

[5] Benassy (1996*b*) analyzes the persistence properties of the model developed by Devereux, Head, and Lapham. See also Hornstein (1993) and Kim (2004). The dynamics of firm entry and exit have recently received attention in open economy studies. See, for instance, Corsetti, Martin, and Pesenti (2005) and Ghironi and Melitz (2005).

[6] See also Mankiw and Whinston (1986) and Benassy (1996*a*). Kim (2004) also studies efficiency in his DSGE model with endogenous number of firms. However, the entry decision is not fully endogenous in his model, which reduces to a one-period structure. In addition, increasing returns can generate indeterminacy in his setup, whereas

The structure of the paper is as follows. Section 2 presents the benchmark model with inelastic labor supply. Section 3 analyzes the efficiency properties of the model and its solution. Section 4 illustrates the dynamic properties of the model for transmission of economic fluctuations by means of a numerical example. Section 5 extends the model to allow for endogenous labor supply. Section 6 completes the discussion of Section 4 by discussing impulse responses to exogenous shocks for different values of labor supply elasticity. [TO BE COMPLETED.]

2 The Benchmark Model

Household Preferences and the Intratemporal Consumption Choice

The economy is populated by a unit mass of atomistic households. All contracts and prices are written in nominal terms. Prices are flexible. Thus, we only solve for the real variables in the model. However, as the composition of the consumption basket changes over time due to firm entry (affecting the definition of the consumption-based price index), we introduce money as a convenient unit of account for contracts. Money plays no other role in the economy. For this reason, we do not model the demand for cash currency, and resort to a cashless economy as in Woodford (2003).

We begin by assuming that the representative household supplies L units of labor inelastically in each period at the nominal wage rate W_t. The household maximizes expected intertemporal utility from consumption (C): $E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}C_s^{1-\gamma}/\left(1-\gamma\right)\right]$, where $\beta\in(0,1)$ is the subjective discount factor and $\gamma>0$ is the inverse of the intertemporal elasticity of substitution.

At time t, the household consumes the basket of goods C_t, defined over a continuum of goods Ω: $C_t=\left(\int_{\omega\in\Omega}c_t\left(\omega\right)^{\theta-1/\theta}d\omega\right)^{\theta/(\theta-1)}$, where $\theta>1$ is the symmetric elasticity of substitution across goods. At any given time t, only a subset of goods $\Omega_t\subset\Omega$ is available. Let $p_t\left(\omega\right)$ denote the nominal price of a good $\omega\in\Omega_t$. The consumption-based price index is then $P_t=\left(\int_{\omega\in\Omega_t}p_t\left(\omega\right)^{1-\theta}d\omega\right)^{1/(1-\theta)}$, and the household's demand for each individual good ω is $c_t\left(\omega\right)=\left(p_t\left(\omega\right)/P_t\right)^{-\theta}C_t$.[7]

the equilibrium is always locally determinate in ours.

[7] An alternative setup would have the household consume a homogeneous good produced by a competitive sector that bundles intermediate goods using a production function that has the form of our consumption basket. All our results would hold also in that setup, though the interpretation would be different. In our setup, consumers derive welfare directly from availability of more varieties. In the alternative setup, an increased range of intermediate goods shows up as increasing returns to specialization. Empirical problems associated with increasing returns to specialization and a C.E.S. production function induce us to adopt the specification without intermediate varieties.

The Government

The government purchases consumption goods according to the same C.E.S. aggregator as households: $G_t = \left(\int_{\omega\in\Omega} g_t(\omega)^{\theta-1/\theta} d\omega\right)^{\theta/(\theta-1)}$. Government demand for each good is then $g_t(\omega) = (p_t(\omega)/P_t)^{-\theta} G_t$. We assume that the budget is balanced in each period and all spending is financed via lump-sum taxes $T_t = G_t$. Aggregate government consumption G_t is exogenous and follows an $AR(1)$ process (in logarithms).

Firms

There is a continuum of monopolistically competitive firms, each producing a different variety $\omega \in \Omega$. Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t, which represents the effectiveness of one unit of labor. Z_t is exogenous and follows an $AR(1)$ process (in logarithms). Output supplied by firm ω is $y_t(\omega) = Z_t l_t(\omega)$, where $l_t(\omega)$ is the firm's labor demand for productive purposes. The unit cost of production, in units of the consumption good C_t, is w_t/Z_t, where $w_t \equiv W_t/P_t$ is the real wage.[8]

Prior to entry, firms face a sunk entry cost of $f_{E,t}$ effective labor units, equal to $w_t f_{E,t}/Z_t$ units of the consumption good. There are no fixed production costs. Hence, all firms that enter the economy produce in every period, until they are hit with a "death" shock, which occurs with probability $\delta \in (0,1)$ in every period.[9]

Given our modeling assumption relating each firm to an individual variety, we think of a firm as a production line for that variety, and the entry cost as the development and setup cost associated with the latter (potentially influenced by market regulation). The exogenous "death" shock also takes place at the individual variety level. Empirically, a firm may comprise more than one of these production lines. Our model does not address the determination of product variety within firms, but our main results would be unaffected by the introduction of multi-product firms.

Firms set prices in a flexible fashion as constant markups over marginal costs. In units of consumption, firm ω's price is $\rho_t(\omega) \equiv p_t(\omega)/P_t = [\theta/(\theta-1)] w_t/Z_t$. The firm's profit in units of consumption, returned to households as dividend, is $d_t(\omega) = \rho_t(\omega)^{1-\theta}(C_t + G_t)/\theta$.

[8] Consistent with standard RBC theory, aggregate productivity Z_t affects all firms uniformly. We abstract from the more complex technology diffusion processes across firms of different vintages studied by Caballero and Hammour (1994) and Campbell (1998). We also do not address the growth effects of changes in product variety. Bils and Klenow (2001) document that these effects are empirically relevant for the U.S.

[9] For simplicity, we do not consider endogenous exit in this paper. Appropriate calibration of δ makes it possible for our model to match several important features of the data.

Firm Entry and Exit

In every period, there is a mass N_t of firms producing in the economy and an unbounded mass of prospective entrants. These entrants are forward looking, and correctly anticipate their future expected profits $d_s(\omega)$ in every period $s \geq t+1$ as well as the probability δ (in every period) of incurring the exit-inducing shock. Entrants at time t only start producing at time $t+1$, which introduces a one-period time-to-build lag in the model. The exogenous exit shock occurs at the very end of the time period (after production and entry). A proportion δ of new entrants will therefore never produce. Prospective entrants in period t compute their expected post-entry value $(v_t(\omega))$ given by the present discounted value of their expected stream of profits $\{d_s(\omega)\}_{s=t+1}^{\infty}$:

$$v_t(\omega) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} d_s(\omega). \tag{1}$$

This also represents the value of incumbent firms *after* production has occurred (since both new entrants and incumbents then face the same probability $1-\delta$ of survival and production in the subsequent period). Entry occurs until firm value is equalized with the entry cost, leading to the free entry condition $v_t(\omega) = w_t f_{E,t}/Z_t$. This condition holds so long as the mass $N_{E,t}$ of entrants is positive. We assume that macroeconomic shocks are small enough for this condition to hold in every period. Finally, the timing of entry and production we have assumed implies that the number of producing firms during period t is given by $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$. The number of producing firms represents the stock of capital of the economy. It is an endogenous state variable that behaves much like physical capital in the benchmark RBC model.

Symmetric Firm Equilibrium

All firms face the same marginal cost. Hence, equilibrium prices, quantities, and firm values are identical across firms: $p_t(\omega) = p_t$, $\rho_t(\omega) = \rho_t$, $l_t(\omega) = l_t$, $y_t(\omega) = y_t$, $d_t(\omega) = d_t$, $v_t(\omega) = v_t$. In turn, equality of prices across firms implies that the consumption-based price index P_t and the firm-level price p_t are such that $P_t = (N_t)^{\frac{1}{1-\theta}} p_t$, or $\rho_t = p_t/P_t = (N_t)^{\frac{1}{\theta-1}}$. An increase in the number of firms implies necessarily that the relative price of each individual good increases. When there are more firms, households derive more welfare from spending a given nominal amount, *i.e.*, *ceteris paribus*, the price index decreases. It follows that the relative price of each individual good must rise.[10] The aggregate *consumption* output of the economy is $Y_t^C = N_t \rho_t y_t$. Using the market clearing condition $y_t = c_t + g_t = (\rho_t)^{-\theta} (C_t + G_t)$ and $\rho_t = (N_t)^{\frac{1}{\theta-1}}$ yields $Y_t^C = C_t + G_t$.

Household Budget Constraint and Intertemporal Decisions

Households hold two types of assets: shares in a mutual fund of firms and risk-free bonds. (We assume that bonds pay risk-free, consumption-based real returns.) Let x_t be the share in the mutual fund of firms held by the representative household *entering* period t. The mutual fund pays a total profit in each period (in units of currency) equal to the total profit of all firms that produce in that period, $P_t N_t d_t$. During period t, the representative household buys x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_t + N_{E,t}$ firms (those already operating at time t and the new entrants). Only $N_{t+1} = (1-\delta) N_{H,t}$ firms will produce and pay dividends at time $t+1$. Since the household does not know which firms will be hit by the exogenous exit shock δ at the *very end* of period t, it finances the continuing operation of all pre-existing firms and all new entrants during period t. The date t price (in units of currency) of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms is equal to the nominal price of claims to future firm profits, $P_t v_t$.

The household enters period t with bond holdings B_t in units of consumption and mutual fund share holdings x_t. It receives gross interest income on bond holdings, dividend income on mutual fund share holdings and the value of selling its initial share position, and labor income. The household allocates these resources between purchases of bonds and shares to be carried into next period, consumption, and lump-sum taxes T_t levied by the government. The period budget constraint (in units of consumption) is:

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t + T_t = (1+r_t) B_t + (d_t + v_t) N_t x_t + w_t L, \tag{2}$$

where r_t is the consumption-based interest rate on holdings of bonds between $t-1$ and t (known with certainty as of $t-1$). The household maximizes its expected intertemporal utility subject to (2).

The Euler equations for bond and share holdings are:

$$(C_t)^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[(C_{t+1})^{-\gamma} \right], \quad \text{and} \quad v_t = \beta (1-\delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (v_{t+1} + d_{t+1}) \right].$$

[10] In the alternative setup with homogeneous consumption produced by aggregating intermediate goods, an increase in the number of intermediates available implies that the competitive sector producing consumption becomes more efficient, and the relative price of each individual input relative to consumption rises accordingly.

As expected, forward iteration of the equation for share holdings and absence of speculative bubbles yield the asset price solution in equation (1).[11]

Aggregate Accounting, Equilibrium, and the Labor Market

Aggregating the budget constraint (2) across households and imposing the equilibrium conditions $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1$ $\forall t$ yields the aggregate accounting identity $C_t + G_t + N_{E,t}v_t = w_t L + N_t d_t$: Total consumption (private plus public) plus investment (in new firms) must be equal to total income (labor income plus dividend income).

Different from the benchmark, one-sector, RBC model of Campbell (1994) and many other studies, our model economy is a two-sector economy in which one sector employs part of the labor endowment to produce consumption and the other sector employs the rest of the labor endowment to produce new firms. The economy's GDP, Y_t, is equal to total income, $w_t L + N_t d_t$. In turn, Y_t is also the total output of the economy, given by consumption output, $Y_t^C (= C_t + G_t)$, plus investment output, $N_{E,t}v_t$. With this in mind, v_t is the relative price of the investment "good" in terms of consumption.

Labor market equilibrium requires that the total amount of labor used in production and to set up the new entrants' plants must equal aggregate labor supply: $L_t^C + L_t^E = L$, where $L_t^C = N_t l_t = N_t (\rho_t)^{-\theta} (C_t + G_t) / Z_t$ is the total amount of labor used in production of consumption, and $L_t^E = N_{E,t} f_{E,t} / Z_t$ is labor used to build new firms.[12] In the benchmark RBC model, physical capital is accumulated by using as investment part of the output of the same good used for consumption. In other words, all labor is allocated to the only productive sector of the economy. When labor supply is fixed, there are no labor market dynamics in the model, other than the determination of the equilibrium wage along a vertical supply curve. In our model, even when labor supply is fixed, labor market dynamics arise in the allocation of labor between production of consumption and creation of new plants. The allocation is determined jointly by the entry decision of prospective entrants and the portfolio decision of households who finance that entry. The value of firms, or the relative price of investment in terms of consumption v_t, plays a crucial role in determining this allocation.[13]

[11] We omit the transversality conditions for bonds and shares that must be satisfied to ensure optimality. Note that the interest rate is determined residually in our economy (it appears only in the Euler equation for bonds and is fully determined once consumption is determined). This is due to the absence of physical capital. Indeed, what is crucial in our economy for the allocation of intertemporal consumption is the return on shares.

[12] We used the equilibrium condition $y_t = Z_t l_t = c_t + g_t = (\rho_t)^{-\theta} (C_t + G_t)$ in the expression for L_t^C.

[13] When labor supply is elastic, labor market dynamics operate along two margins as the interaction of household and entry decisions determines jointly the total amount of labor and its allocation to the two sectors of the economy.

Aggregate Production and the Role of Variety

Rearranging the labor market clearing condition and using $\rho_t = (N_t)^{\frac{1}{\theta-1}}$ yields the following expression for aggregate production of the consumption good:

$$Y_t^C = C_t + G_t = (N_t)^{\frac{1}{\theta-1}} \left(Z_t L - f_{E,t} N_{E,t} \right).$$

In turn, substituting this into $C_t + G_t + N_{E,t} v_t = Y_t$ and using firm pricing, free entry, and $\rho_t = (N_t)^{\frac{1}{\theta-1}}$ gives:

$$Y_t = (N_t)^{\frac{1}{\theta-1}} \left(Z_t L - \frac{1}{\theta} f_{E,t} N_{E,t} \right).$$

These expressions resemble analogous expressions in Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996*a*,*b*). An increase in the number of entrants $N_{E,t}$ absorbs productive resources in the form of effective labor and acts like an overhead cost. This cost is accounted for differently in GDP, since this recognizes that firm entry is productive. *Ceteris paribus*, when θ goes to infinity (goods are perfect substitutes), the expression for Y_t reduces to a familiar Cobb-Douglas production function with a zero share of capital: $Y_t = Z_t L$.

Chatterjee and Cooper (1993) and Devereux, Head, and Lapham (1996*a*,*b*) interpret the effect of changes in the number of active firms N_t in the expressions above as an endogenous aggregate productivity shifter. Since $\theta > 1$, an increase in the number of active firms N_t has a similar effect to that of an endogenous increase in productivity. As there is no sunk entry cost nor time-to-build lag in those studies, such endogenous productivity changes do not impart endogenous persistence in those models, in contrast to ours. If we think of the effect of entry as endogenously affecting productivity in our model, entry at time t affects labor demand at $t+1$ because it affects productivity of labor at $t+1$ by causing N_{t+1} to rise.

We prefer to interpret N_t as the stock of capital (production lines) of the economy during period t, treating aggregate productivity Z_t as exogenous in production of the consumption good, $Y_t^C = Z_t \rho_t N_t l_t$, and in production of new plants. There are two reasons for this. First, it is transparent to treat labor per firm and the number of firms as the factors of production in the aggregate function $Y_t^C = Z_t \rho_t N_t l_t$, in which Z_t is exogenous productivity, and the relative price ρ_t converts units of individual goods into units of consumption. In this interpretation, which does not hinge on the properties of C.E.S. demand to endogenize productivity relative to the number of firms, entry at t affects labor demand at $t+1$ because it increases the number of producing firms at $t+1$. This is akin to the benchmark RBC model, where investment at t affects labor demand at $t+1$ by increasing the capital stock used in production at $t+1$. Second, as argued in Ghironi and Melitz (2005), empirically relevant variables – as opposed to welfare-consistent concepts – net out the effect of changes in the range of available varieties. The reason is that construction of CPI data by statistical agencies does not adjust for availability of new varieties as in the welfare-consistent price index.[14] CPI data are closer to p_t than P_t. For this reason, when investigating the properties of the model in relation to the data (for instance, when computing second moments or impulse responses for comparison with empirical evidence), one wants to focus on real variables deflated by a data-consistent price index. For any variable X_t in units of the consumption basket, such data-consistent counterpart is obtained as $X_{R,t} \equiv P_t X_t / p_t = X_t / \rho_t = X_t / (N_t)^{\frac{1}{\theta-1}}$. Therefore, data-consistent measures of consumption output and GDP in our model are $Y^C_{R,t} = Z_t L - f_{E,t} N_{E,t}$ and $Y_{R,t} = Z_t L - \frac{1}{\theta} f_{E,t} N_{E,t}$, which remove the role of variety as an endogenous productivity shifter.[15]

Model Summary

Table 1 summarizes the main equilibrium conditions of the model. The equations in the table constitute a system of eight equations in eight endogenous variables: ρ_t, d_t, w_t, $N_{E,t}$, N_t, r_t, v_t, C_t. Of these endogenous variables, two are predetermined as of time t: the total number of firms, N_t, and the risk-free interest rate, r_t. Additionally, the model features three exogenous variables: government spending G_t, aggregate productivity Z_t, and the sunk entry cost $f_{E,t}$. The latter may be interpreted in at least two ways. Part of the sunk entry cost $f_{E,t}$ originates in the economy's technology for creation of new plants, which is exogenous and outside the control of policymakers. But another part of the entry cost is motivated by regulation and entry barriers induced by policy. Holding the technology component of $f_{E,t}$ given, we interpret changes in $f_{E,t}$ below as changes in market regulation facing firms.

3 Benchmark Model Properties and Solution

We can reduce the system in Table 1 to a system of two equations in two variables, N_t and C_t. To see this, write firm value as a function of the endogenous state N_t and the exogenous state

[14] Adjustment for variety, when it happens, certainly does not happen at the frequency represented by periods in our model.

[15] Treating aggregate productivity as exogenous in the absence of firm heterogeneity and endogenous exit is also consistent with Melitz (2003).

$f_{E,t}$ by combining labor market clearing, pricing, profits, and free entry, solving for $N_{E,t}$ and using aggregate accounting:[16]

$$v_t = \frac{\theta - 1}{\theta} f_{E,t} \left(N_t\right)^{\frac{1}{\theta-1}}. \tag{3}$$

Equation (3) and the free entry condition in Table 1 yield a first, important set of results: Since the number of producing firms is predetermined and does not react to exogenous shocks on impact, firm value and the real wage are also predetermined with respect to some exogenous shocks. Namely, firm value is predetermined with respect to all shocks but deregulation, while the real wage ($w_t = \left[\left(\theta - 1\right)/\theta\right] Z_t \left(N_t\right)^{\frac{1}{\theta-1}}$) is predetermined with respect to all shocks but productivity. A fall in the sunk entry cost encourages entry and decreases firm value since more firms start producing at $t+1$, which implies an expected decrease in demand for each individual firm. An increase in productivity results in a proportional increase in the real wage on impact through its effect on labor demand. Since the entry cost is paid in effective labor units, this does not affect firm value. An implication of the wage schedule $w_t = \left[\left(\theta - 1\right)/\theta\right] Z_t \left(N_t\right)^{\frac{1}{\theta-1}}$ is also that marginal cost, w_t/Z_t, is predetermined with respect to all shocks.

The number of new entrants as a function of consumption and number of firms is $N_{E,t} = Z_t L / f_{E,t} - \left(N_t\right)^{\frac{1}{1-\theta}} \left(C_t + G_t\right)/f_{E,t}$. Substituting this, equation (3), and the expression for profits ($d_t = \left(C_t + G_t\right)/\left(\theta N_t\right)$) in the law of motion for N_t and the Euler equation for shares yields:

$$N_t = (1-\delta)\left[N_{t-1} + \frac{Z_{t-1}L}{f_{E,t-1}} - \frac{1}{f_{E,t-1}}\left(N_{t-1}\right)^{\frac{1}{1-\theta}}\left(C_{t-1} + G_{t-1}\right)\right], \tag{4}$$

$$f_{E,t}\left(N_t\right)^{\frac{1}{\theta-1}} = \beta\left(1-\delta\right) E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[f_{E,t+1}\left(N_{t+1}\right)^{\frac{1}{\theta-1}} + \frac{1}{\theta-1}\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right]\right\}. \tag{5}$$

Equation (4) implies that the effect of shocks to the exogenous labor supply (if we allowed it to be time-varying) would be identical to that of productivity shocks – except for the real wage, as we explain below. For this reason, we do not consider the case of exogenously varying L.

Efficiency

We now prove that the market equilibrium of our benchmark model with fixed labor supply coincides with the solution of the problem that a social planner would solve to allocate resources optimally,

[16] The same relation can be obtained by using the equilibrium price index equation $\rho_t = \left(N_t\right)^{\frac{1}{\theta-1}}$ in conjunction with pricing and the free entry condition. Then one can back out the expression for the number of entrants from labor market clearing or from the aggregate accounting equation and $C_t + G_t = N_t \rho_t y_t = N_t \left(\rho_t\right)^{1-\theta}\left(C_t + G_t\right)$. It follows that, consistent with Walras' Law, labor market clearing – or aggregate accounting – is redundant if we include $\rho_t = \left(N_t\right)^{\frac{1}{\theta-1}}$ in the system to be solved.

guaranteeing efficiency of the market equilibrium.[17]

To solve for the planning optimum, we must write the problem in terms of quantities only. The simplest way is to think of the planner as choosing the amount of labor to be allocated to the sector producing consumption. Since the planner will allocate labor identically across symmetric firms, labor allocated to production of consumption is $L_t^C \equiv N_t l_t$. The rest of the labor endowment of the representative household, $L - L_t^C = L_t^E$, will then be allocated to the investment sector, covering the sunk entry cost for creation of new firms. For the purposes of this sub-section, we abstract from government spending, so that $C_t = Y_t^C$. The planner then chooses the sequence $\left\{L_s^C\right\}_{s=t}^{\infty}$ to maximize $E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}C_s^{1-\gamma}/\left(1-\gamma\right)\right]$ subject to the constraints

$$C_t = (N_t)^{\frac{1}{\theta-1}} Z_t L_t^C, \tag{6}$$

$$N_{t+1} = (1-\delta) N_t + (1-\delta) N_{E,t}, \tag{7}$$

$$L = L_t^C + N_{E,t}\frac{f_{E,t}}{Z_t}. \tag{8}$$

As we show in the appendix, the first-order condition for the planner's optimal choice of L_t^C is identical to equation (5). The constraints (6)-(8) can be consolidated into an equation identical to (4) – scrolled forward one period. Hence, the solution of the planner's problem coincides with the competitive equilibrium of the model, ensuring efficiency of the latter.[18]

This result stems from two features of our model economy: synchronization of markups and the entry mechanism under C.E.S. preferences, the role of which we shall now explain in detail.[19] The first piece of intuition, which we will refer to as "the Lerner-Samuelson intuition," concerns the synchronization of markups. Lerner (1934, p. 172) first noted that the allocation of resources is efficient when markups are equal in the pricing of all goods: "The conditions for that optimum distribution of resources between different commodities that we designate the absence of monopoly are satisfied if prices are all proportional to marginal cost." Samuelson (1947, p. 239-240) also makes this point clearly: "If all factors of production were indifferent between different uses and completely fixed in amount – the pure Austrian case –, then [...] proportionality of prices and marginal cost would be sufficient." This makes it clear that *equality* of prices to marginal cost is *not necessary* for achieving an optimal allocation, contrary to an argument often found in the

[17] In this subsection, we treat the sunk entry cost $f_{E,t}$ as a structural feature of the economy, akin to a characteristic of the production function, rather than as an instrument of policy that the planner may manipulate through regulation or deregulation.

[18] It is easy to verify that this result holds for a general period utility function $U(C)$ that has the standard properties.

[19] Our analysis below echoes points made by Grossman and Helpman (1991).

macroeconomic policy literature. This point is equally true in a model with a fixed number of firms N, where the planner merely solves a static allocation problem, allocating labor to the symmetric individual goods evenly.[20]

Our model has the important, additional property that the market allocation is efficient even when a dynamic allocation problem is solved under free entry subject to a sunk cost, a time-to-build lag, and exogenous exit. This is important because it implies that the allocation of labor to the two sectors of our economy is efficient, and it contradicts Samuelson's further claim that "If we drop these highly special assumptions [that factors of production are fixed –...], we should not have an optimum situation" (op. cit., p. 240). We let one factor of production (the number of firms, or the stock of production lines) vary and show that the market equilibrium is still efficient since all the new firms charge the same markup.[21] This brings us to the second feature of our economy that ensures efficiency.

Despite synchronized markups, entry could lead to inefficiency due to two other possible distortions – if new entrants ignore on the one hand the positive effect of a new variety on consumer surplus and on the other the negative effect on other firms' profits. Grossman and Helpman (1991) call these distortions the "consumer surplus effect" and the "profit destruction effect," respectively. With C.E.S. preferences, these two contrasting forces perfectly balance each other and the resulting equilibrium is efficient. However, when preferences do not take the C.E.S, inefficiency may arise.[22] We provide an example of this by considering a specification of consumption preferences that separates the degree of monopoly power from the consumer's taste for variety in an *ad hoc* fashion as, *e.g.*, in Benassy (1996a). In that case (explored in an appendix), the economy ends up with a suboptimally low (high) number of producing firms if the parameter governing the taste for variety is lower (higher) than the degree of monopoly power (the net price markup). Nevertheless, this preference specification implies that the consumer derives utility from goods that (s)he never consumes, and similarly is worse off when a good disappears even if consumption of that good was zero. This unappealing feature clearly drives the welfare conclusions, and induces us to adopt the C.E.S. specification in which when a good is produced, it will expand variety and hence increase utility only if it is consumed.

We have established that the competitive equilibrium of our benchmark model with fixed labor

[20] Notice, though, that the equilibrium of our model would be inefficient if, for some reason, the number of firms were fixed because agents are prevented from accessing the available technology for creation of new firms. Inefficiency would arise because the number of firms would be suboptimal.

[21] This result, however, does not hold if we relax the fixed-labor assumption, as shown in Section 5.

[22] See also Dixit and Stiglitz (1977) and Judd (1985) for a discussion of these issues.

is efficient and explained this result based on synchronization of markups and the entry mechanism under C.E.S. preferences. As should be intuitive by now, efficiency breaks down when there are differences in markups across firms or sectors of the economy, as is the case when firms are heterogeneous and/or price adjustment is not synchronized.[23] Moreover, as we show below, efficiency fails when labor supply is endogenous. But we shall argue that this inefficiency is induced by the *absence* of a markup in the pricing of leisure, and not by monopoly power (generating a markup in the consumption production sector). Indeed, we will argue that monopoly power should not be removed, since profit incentives are the driving force behind entry and production in our economy. Instead, a simple policy of subsidizing labor income can be designed that restores efficiency by effectively equalizing markups for all the goods the household cares about (including leisure).

The Steady State

Having established efficiency of the equilibrium of our benchmark model, we now turn to its solution, starting from the long-run equilibrium.

We assume that exogenous variables are constant in steady state and denote steady-state levels of variables by dropping the time subscript: $G_t = G$, $Z_t = Z$, and $f_{E,t} = f_E$. All endogenous variables are constant in steady state.

The steady-state interest rate is pinned down as usual by the rate of time preference, $1+r = \beta^{-1}$; the gross return on shares is $1 + d/v = (1 + r)/(1 - \delta)$, which captures a premium for expected firm destruction. The number of new entrants makes up for the exogenous destruction of existing firms: $N_E = \delta N/ (1 - \delta)$. We follow Campbell (1994) below and exploit $1 + r = \beta^{-1}$ to treat r as a parameter in the solution.

Calculating the shares of profit income and investment in consumption output and GDP allows us to draw another transparent comparison between our model and the standard RBC setup. The steady-state profit equation gives the share of profit income in consumption output: $dN/Y^C = 1/\theta$. Using this result in conjunction with those obtained above, we have the share of investment in consumption output:

$$\frac{vN_E}{Y^C} = \frac{\delta}{\theta (r + \delta)}.$$

This expression is similar to its RBC counterpart. There, the share of investment in output is

[23]For instance, the welfare costs of inflation in modern monetary policy analysis relying on staggered price adjustment (*e.g.* Woodford, 2003) can easily be explained in terms of the Lerner-Samuelson intuition. Imperfect price adjustment implies that *ex-post* markups are different across firms, and hence there is dispersion in relative prices. We explore the implications of imperfect price adjustment in Bilbiie, Ghironi, and Melitz (2005).

given by $s_K\delta/(r+\delta)$, where δ is the depreciation rate of capital and s_K is the share of capital income in total income. In our framework, $1/\theta$ can be regarded as governing the share of "capital" since it dictates the degree of monopoly power and hence the share of profits that firms generate from producing consumption output (dN/Y^C). Noting that $Y = Y^C + vN_E$, the shares of investment and profit income in GDP are $vN_E/Y = \delta/[\delta+\theta(r+\delta)]$ and $dN/Y = (r+\delta)/[\delta+\theta(r+\delta)]$, respectively. It follows that the share of private consumption in GDP is $C/Y = (1-\Gamma)\theta(r+\delta)/[\delta+\theta(r+\delta)]$, where Γ is the share of government consumption in total consumption output $\Gamma \equiv G/Y^C$ and is taken here as a parameter. The share of labor income in total income is $wL/Y = 1-(r+\delta)/[\delta+\theta(r+\delta)]$. Importantly, all these ratios are independent of the amount of labor L and are constant (which would hold also along a balanced growth path if we incorporated exogenous productivity growth) as consistent with the Kaldorian growth facts.[24]

Equations (3)-(5) allow us to solve for the steady-state levels of firm value, the number of firms, and consumption explicitly:

$$v = \frac{\theta-1}{\theta}\left[\frac{1-\delta}{\theta(r+\delta)-r}\right]^{\frac{1}{\theta-1}}\left(\frac{ZL}{f_E}\right)^{\frac{1}{\theta-1}} f_E, \tag{9}$$

$$N = \left[\frac{1-\delta}{\theta(r+\delta)-r}\right]\frac{ZL}{f_E}, \tag{10}$$

$$C = \left[\frac{(\theta-1)(r+\delta)}{1-\delta}\right]\left[\frac{1-\delta}{\theta(r+\delta)-r}\right]^{\frac{\theta}{\theta-1}}\left(\frac{ZL}{f_E}\right)^{\frac{\theta}{\theta-1}} f_E - G. \tag{11}$$

An increase in long-run productivity results in a larger number of firms, higher firm value, and higher consumption. Deregulation (a lower sunk entry cost) generates an increase in the long-run number of firms and consumption, and it increases firm value as a proportion of the sunk cost itself (v/f_E). The effect of deregulation on v depends on whether θ is larger or smaller than two. Empirically plausible values of θ, which satisfy $\theta > 2$, imply that deregulation has a negative effect on firm value. Government spending crowds out private consumption completely and has no effect on the economy. This is not surprising since, when labor supply is inelastic, the wealth effect of taxation on labor supply, which is central to fiscal policy transmission in the standard RBC model with endogenous labor supply, is absent.

Importantly, v, $C+G$, and N all tend to zero if θ tends to infinite. For firms to find it profitable to enter, the expected present discounted value of the future profit stream must be positive, so as

[24] Note that all ratios calculated above are identical if we compute them in terms of empirically relevant variables deflated by the average price p_t.

to offset the sunk entry cost. But profits tend to zero in all periods if firms have no monopoly power. This implies that no firm will enter the economy, driving N and $C+G$ to zero.

Of particular interest is the behavior of the real wage, given by:

$$w = \frac{\theta-1}{\theta} Z (N)^{\frac{1}{\theta-1}} = \frac{\theta-1}{\theta} \left[\frac{1-\delta}{\theta(r+\delta)-r} \right]^{\frac{1}{\theta-1}} \left(\frac{L}{f_E} \right)^{\frac{1}{\theta-1}} Z^{\frac{\theta}{\theta-1}}.$$

Both higher productivity and deregulation result in a higher wage, as a larger number of firms puts pressure on labor demand. Most importantly, deregulation and higher productivity cause steady-state marginal cost w/Z to increase (the long-run elasticity being $1/(\theta-1)$). This is in sharp contrast to models with a constant number of firms, where marginal cost would be constant relative to long-run changes in productivity. To see this, set $N=1$ for convenience and note that $w/Z = (\theta-1)/\theta$ in this case. Changes in productivity would be reflected in equal percentage changes in the real wage, so that marginal cost remains constant.[25] In a model with endogenous number of firms, higher productivity results in a more attractive business environment, which leads to more entry and a larger number of firms. This puts pressure on labor demand that causes w to increase by more than Z, so that the new long-run marginal cost is higher than the original one.[26] Entry is also crucial for the result that long-run consumption rises by more than a permanent increase in productivity in our model, the long-run elasticity being $\theta/(\theta-1)$. Again, this is different from what would happen if we had a constant number of firms. With $N_t = 1$, aggregate accounting would reduce to $C_t + G_t = w_t L + d_t$. Since $d_t = (C_t + G_t)/\theta$ in the absence of entry, it would be $C_t = Z_t L - G_t$, and consumption would increase by the same amount as productivity in all periods.[27]

Given solutions for v, C, N, and w/Z, it is easy to recover solutions for all other variables in Table 1, which we omit. To complete the information on the steady-state properties of the model, Table 2 reports the long-run elasticities of endogenous variables to permanent changes in Z, f_E, and Γ.

[25] In fact, marginal cost ($w_t/Z_t = [(\theta-1)/\theta](N_t)^{\frac{1}{\theta-1}}$) would be constant in all periods, in and out of the steady state, if the number of firms were constant – and $N_t = 1$ would imply $\rho_t = 1$, as in standard models without entry. In our model, it is the data-consistent measure of marginal cost $w_{R,t}/Z_t = (w_t/Z_t)/\rho_t$ that is constant, while the welfare-consistent marginal cost moves in response to changes in the number of producing firms.

[26] This mechanism is central for Ghironi and Melitz's (2005) result that a permanent increase in productivity results in higher average prices and an appreciated real exchange rate in the country that experiences such higher productivity relative to its trading partners.

[27] Of course, the results for the model with $N_t = 1$ are over-simplified as a consequence of the assumptions that labor is exogenous and production does not require capital, which imply that aggregate output is exogenous and determined by productivity. Nevertheless, the comparison is instructive to highlight the striking consequences of introducing entry in the model.

Dynamics

We solve for the dynamics in response to exogenous shocks by log-linearizing the model around the steady state obtained above. For simplicity, we assume $L = 1$ and initial steady-state levels of productivity and sunk cost $Z = f_E = 1$. We also set $G = 0$ (or a zero share of government spending in total consumption output, Γ). To have a system that can be written in the standard canonical form for application of Blanchard and Kahn's (1980) results, it is convenient to scroll the state equation (4) forward by one period, keeping in mind that N_{t+1} is predetermined. Using sans-serif fonts to denote percent deviations from steady-state levels (with the exception of G_t, which we define as $\mathsf{G}_t \equiv dG_t/C$), log-linearization under assumptions of log-normality and homoskedasticity yields:

$$\mathsf{N}_{t+1} = (1+r)\,\mathsf{N}_t - (\theta-1)(r+\delta)\,\mathsf{C}_t - \delta \mathsf{f}_{E,t} + [(\theta-1)\,r + \theta\delta]\,\mathsf{Z}_t - (\theta-1)(r+\delta)\,\mathsf{G}_t, \tag{12}$$

$$\mathsf{C}_t = \left[1 - \frac{r+\delta}{\gamma(1+r)}\right] E_t(\mathsf{C}_{t+1}) - \frac{1}{\gamma}\left[\frac{\theta}{\theta-1}\left(\frac{1-\delta}{1+r}\right) - 1\right]\mathsf{N}_{t+1} + \frac{1}{\gamma(\theta-1)}\mathsf{N}_t + \frac{1}{\gamma}\mathsf{f}_{E,t} \tag{13}$$
$$- \frac{1-\delta}{\gamma(1+r)}E_t\mathsf{f}_{E,t+1} - \frac{r+\delta}{\gamma(1+r)}E_t(\mathsf{G}_{t+1}).$$

Equation (12) states that the number of firms producing at $t+1$ increases if consumption at time t is lower (households save more in the form of new firms), if the sunk entry cost is below the initial level, or if productivity is higher. An increase in government consumption for given private consumption absorbs resources that would otherwise be invested in firm creation, and thus causes N_{t+1} to decrease. Equation (13) states that consumption at time t is higher the higher expected future consumption (if $\gamma \geq 1$) and the larger the number of firms producing at time t. Current deregulation lowers current consumption, because households save more to finance faster firm entry. However, expected future deregulation boosts current consumption as households anticipate the availability of more varieties in the future. The expectation of higher future government spending lowers consumption today as households anticipate the negative effect of future government consumption on firm entry and private consumption in the future. The effect of N_{t+1} depends on parameter values. For realistic values of θ, β, and δ, we have $\theta/(\theta-1) > (1+r)/(1-\delta)$. It follows that increases in the number of firms producing at $t+1$ are associated with lower consumption at t. (Higher productivity at time t lowers contemporaneous consumption through this channel, as households save to finance faster entry in a more attractive economy. However, we shall see below that the general equilibrium effect of higher productivity will be that consumption rises.)

We show in the appendix that the system (12)-(13) has a unique, non-explosive solution. To solve the system, we assume $\mathsf{Z}_t = \phi_Z \mathsf{Z}_{t-1} + \varepsilon_{Z,t}$, where $\varepsilon_{Z,t}$ is an i.i.d., Normal innovation with zero mean and variance $\sigma^2_{\varepsilon Z}$. A similar process $\mathsf{G}_t = \phi_G \mathsf{G}_{t-1} + \varepsilon_{G,t}$ governs the dynamics of government spending. Differently from productivity and government spending, we do not treat $\mathsf{f}_{E,t}$ as a stochastic process subject to random innovations at business cycle frequency. We think of changes in sunk entry costs as changes in market regulation, and we assume that market regulation is controlled by a policymaker, who can change it in more or less persistent fashion, so that $\mathsf{f}_{E,t} = \phi_{f_E} \mathsf{f}_{E,t-1}$ in all periods after an initial change. Subject to these assumptions, the unique solution to the system (12)-(13) takes the form:

$$\mathsf{N}_{t+1} = \eta_{NN}\mathsf{N}_t + \eta_{NZ}\mathsf{Z}_t + \eta_{Nf_E}\mathsf{f}_{E,t} + \eta_{NG}\mathsf{G}_t,$$
$$\mathsf{C}_t = \eta_{CN}\mathsf{N}_t + \eta_{CZ}\mathsf{Z}_t + \eta_{Cf_E}\mathsf{f}_{E,t} + \eta_{CG}\mathsf{G}_t.$$

where the η's are elasticities that we can obtain with the method of undetermined coefficients as in Campbell (1994). Table 3 summarizes the results, in a convenient order. The elasticity of the number of firms producing in period $t+1$ to its past level (η_{NN}) is such that $0 < \eta_{NN} < 1$.[28] It follows from the expression of η_{CN} in Table 3 that consumption is higher the larger the number of firms producing in period t. Plausible parameter values imply $0 < \eta_{CZ} < 1$ and $\eta_{NZ} > 0$: Consumption increases if productivity rises, as households have more resources to spend on consumption. However, the impact elasticity of consumption to productivity is smaller than one as households find it optimal to save part of the productivity gain in the form of more firms. The same plausible parameter values imply $0 < \eta_{Cf_E} < 1$ and $\eta_{Nf_E} > 0$: Deregulation lowers consumption on impact. As in the case of higher productivity, more firms enter the economy, but these firms are not more productive in the deregulation scenario. Thus, consumption must decrease to finance the entry of new firms. Government consumption crowds out private consumption: $\eta_{CG} < 0$ for plausible parametrizations. This effect is stronger: (i) the more persistent is government spending (for it is the present discounted value of future taxes that matters); and (ii) the higher is the elasticity of intertemporal substitution (the lower γ), since the consumer is then more willing to give up present consumption and postpone it to periods when taxes are lower. Government spending also crowds out investment in the short run ($\eta_{NG} < 0$). This effect is weaker when crowding out of consumption is stronger.

[28] See the appendix on equilibrium determinacy and non-explosiveness for details.

Some other results are worth mentioning. First, the value of η_{NN} is smaller the higher the probability of firm death δ. Intuitively, the faster firms die in the economy, the less persistent the deviations of N_t from the steady state. Second, if $\phi_Z = \phi_{f_E} = 1$ (permanent changes in productivity and regulation), $\eta_{CZ} + \eta_{Cf_E} = 1$ and $\eta_{NZ} + \eta_{Nf_E} = 0$. The same is true if $\gamma = 1$ (logarithmic utility) and $\phi_Z = \phi_{f_E} = \phi$ (equal persistence in productivity and regulation). Third, in general, η_{CZ} increases if ϕ_Z is higher and η_{Cf_E} decreases if ϕ_{f_E} is higher. Intuitively, the more persistent a productivity shock, the longer the amount of time during which households can enjoy higher income and the benefit of larger variety. Hence, consumption increases by more – and the number of firms correspondingly increases by less (η_{NZ} decreases). Similarly, the more persistent deregulation, the longer the horizon during which households can enjoy a larger range of varieties. This weakens the incentive to reduce consumption today to front-load firm entry – the absolute value of η_{Nf_E} becomes smaller. Fourth, if $\phi_G = 1$ (permanent changes in government spending), we obtain the same elasticities as in Table 2: Consumption is crowded out completely and the number of firms is not influenced.[29] This happens because there is no intertemporal substitution since taxation moves to a permanently higher level on impact and labor supply is inelastic.[30]

The solution for other endogenous, non-predetermined variables in the model is similar to that for C_t. For any variable x_t, it is:

$$\mathsf{x}_t = \eta_{xN}\mathsf{N}_t + \eta_{xZ}\mathsf{Z}_t + \eta_{xf_E}\mathsf{f}_{E,t} + \eta_{xG}\mathsf{G}_t.$$

Given the log-linear versions of equations in Table 1 and the solution for the elasticities in Table 2, one can easily recover the relevant elasticities. In particular, the solution for firm value is simply $\mathsf{v}_t = [1/(\theta-1)]\,\mathsf{N}_t + \mathsf{f}_{E,t}$ and that for the real wage is $\mathsf{w}_t = [1/(\theta-1)]\,\mathsf{N}_t + \mathsf{Z}_t$. Marginal cost ($\mathsf{w}_t - \mathsf{Z}_t$) and price ($\rho_t$ – now denoting percent deviation from steady state) are predetermined and equal to $[1/(\theta-1)]\,\mathsf{N}_t$. As we anticipated, the elasticities of endogenous variables to changes in L would be identical to those to Z_t. The only difference would be in the response of the real wage, which would of course be equal to the response of $\mathsf{w}_t - \mathsf{Z}_t$ to a productivity shock.

[29] The elasticity of consumption is of a different magnitude than in Table 2 ($\eta_{CG} = -1$) since we are using different measurement units due to the simplifying assumption that $\Gamma = 0$. However, $\Gamma/(1-\Gamma) = G/C$, which implies that a permanent change in G relative to C has a proportional effect on C.

[30] This differs from the cases of productivity and deregulation, where setting persistence to one does not deliver the long-run elasticities of Table 2, but simply delivers impact elasticities to permanent shocks. The reason is that productivity and deregulation shocks generate intertemporal substitution since the state variable (the number of firms) moves to the new, permanently higher level only gradually. The long-run elasticities of Table 2 take the long-run adjustment of the number of firms into account in computing the effect of the shock on consumption and other variables.

4 Business Cycles with Firm Dynamics, Part I

In this section we explore the properties of our benchmark model by means of a numerical example. We calibrate parameters to plausible values and compute impulse responses to productivity shocks. (We defer the responses to deregulation and government spending shocks to Section 6 for ease of comparison with the case of elastic labor supply.) The responses substantiate the results and intuitions in the previous section.

Calibration

We calibrate parameters as follows. We interpret periods as quarters and set $\beta = .99$ – a standard choice for quarterly business cycle models – and $\gamma = 1$. We use log utility as benchmark to facilitate comparison with the case of endogenous labor supply, where we restrict utility from consumption to be logarithmic.[31] We set the size of the exogenous firm exit shock $\delta = .025$ to match the U.S. empirical level of 10 percent job destruction per year.[32] We use the value of θ from Bernard, Eaton, Jensen, and Kortum (2003) and set $\theta = 3.8$, which was calibrated to fit U.S. plant and macro trade data.[33] The initial steady-state entry cost f_E does not affect any of the impulse responses.[34] We therefore set $f_E = 1$ without loss of generality.

Impulse Responses

Figure 3 shows the responses (percent deviations from steady state) to a permanent 1 percent increase in productivity. The number of years after the shock is on the horizontal axis. Consider first the long-run effects in the new steady state. As was previously described, the business environment becomes more attractive, drawing a permanently higher number of entrants, which translates into a

[31] The key qualitative features of the impulse responses below are unaffected if we set $\gamma = 2$.

[32] Empirically, job destruction is induced by both firm exit and contraction. In our model, the "death" shock δ takes place at the product level. In a multi-product firm, the disappearance of a product generates job destruction without firm exit. Since we abstract from the explicit modeling of multi-product firms, we include this portion of job destruction in δ. As a higher δ implies less persistent dynamics, our choice of δ is also consistent with not overstating the ability of the model to generate persistence.

[33] It may be argued that the value of θ results in a steady-state markup that is too high relative to the evidence. However, it is important to observe that, in models without any fixed cost, $\theta/(\theta - 1)$ is a measure of both markup over marginal cost and average cost. In our model with entry costs, free entry ensures that firms earn zero profits *net* of the entry cost. This means that firms price at average cost (inclusive of the entry cost). Thus, although $\theta = 3.8$ implies a fairly high markup over marginal cost, our parametrization delivers reasonable results with respect to pricing and average costs. The main qualitative features of the impulse responses below are not affected if we set $\theta = 6$, resulting in a 20 percent markup of price over marginal cost as in Rotemberg and Woodford (1992) and several other studies.

[34] The total number of firms in steady state is inversely proportional to f_E – and the size and value of all firms are similarly proportional to f_E. Basically, changing f_E amounts to changing the unit of measure for output and number of firms.

permanently higher number of producers. This induces marginal cost and the relative price of each product ρ to be higher. GDP Y and consumption also rise permanently, and they do so by more than the increase in productivity due to the expansion in the range of available varieties. Individual firm output y is not affected as the increase in the relative price offsets the larger demand resulting from higher consumption. Firm profit can be written as $d_t = \rho_t y_t / \theta$, which implies that profits and firm value are permanently higher.

Transition dynamics highlight the role of the number of firms as the key endogenous state variable in our model. Absent sunk entry costs, and the associated time-to-build lag before production starts, the number of producing firms N_t would immediately adjust to its new steady-state level. Sunk costs and time-to-build imply that N_t is a state variable that behaves very much like the capital stock in the standard RBC model: The number of entrants (new production lines) $N_{E,t}$ represents the consumers' investment, which translates into increases in the stock of production lines N_t over time. Marginal cost and the relative price ρ_t react to the shock with a lag and start increasing only in the period after the shock as a larger number of producing firms puts pressure on labor demand.

The responses of firm-level output and GDP highlight the different roles of intensive and extensive margins during economic expansions in response to permanent productivity improvements. Firm-level output booms on impact in response to larger consumption. Over time, the increase in ρ_t pushes firm-level output back to the initial steady state. Since output per firm returns to the initial steady state in the long run, the increase in productivity is offset by a matching decrease in firm-level employment as the cost of labor increases during the transition. Thus, our model predicts that the expansionary effect of higher productivity is initially transmitted through the intensive margin as output per firm rises, but it is the extensive margin that delivers GDP expansion in the long run. Over time, the expansion along the intensive margin is reabsorbed as the increase in the number of firms puts pressure on labor costs, and eventually the expansion operates only through the extensive margin.

Importantly, during the transition, there is a reallocation of the fixed labor supply from production of consumption to production of new firms, as implied by the increase in L_t^E and the decrease in L_t^C. As the increase in productivity boosts entry, labor shifts to the construction of new plants. Over time, the rising cost of effective labor – and thus the rising burden of the entry cost – redistributes this labor back to production of consumption. The gradual increase in the cost of effective labor explains why the number of new entrants overshoots its new long-run equilibrium in the short run.

The responses of several key macroeconomic variables deflated by average prices rather than with the consumption based price index are qualitatively similar to those in Figure 3.[35] Three key differences are worth mentioning: Once the variety effect and the implied increase in ρ_t are removed, firm real profits track firm output in Figure 3 and return to the original steady state in the long run. Aggregate real profits ($D_{R,t} \equiv N_t d_t / \rho_t$), however, increase in procyclical fashion, consistent with the evidence in Figure 1. Firm value is not affected at all by the shock, because the real wage $w_{R,t}$ increases exactly as much as the shock.

To further illustrate the properties of our model, Figure 4 shows the responses to a 1 percent productivity shock with persistence .9. The direction of movement of endogenous variables on impact is the same as in Figure 3, though all variables return to the steady state in the long run. Interestingly, firm level output is below the steady state during most of the transition, except for a short-lived initial expansion. Different from the permanent shock case, the relative price effect prevails on the expansion in consumption demand to push individual firm output below the steady state for most of the transition. In contrast to a model without entry and the traditional type of capital, the dynamics of firm entry result in responses that persist beyond the duration of the exogenous shock and, for some key variables, display a hump-shaped pattern.

5 Endogenous Labor Supply

In this section we consider a model with endogenous labor supply. The only modification with respect to the model of Section 2 is that now households choose how much labor effort to supply for production of the consumption good and to set up new firms. Consequently, the utility function features an additional term measuring the disutility of hours worked: $U(C_t, L_t) = \ln C_t - \chi (L_t)^{1+1/\varphi} / (1 + 1/\varphi)$, where φ is the Frisch elasticity of labor supply to wages, and the intertemporal elasticity of substitution in labor supply. As in Campbell (1994), our choice of functional form for the utility function in this case is guided by results in King, Plosser, and Rebelo (1988): Given separable preferences, log utility from consumption ensures that income and substitution effects of real wage variation on effort cancel out in steady state; this guarantees constant steady-state effort and balanced growth – if there is productivity growth.

[35] For instance, this is the case for $C_{R,t}$ and $Y_{R,t}$, even if the increase after the initial impact is muted and removal of the variety effect implies that these empirically-consistent variables do not increase by more than the size of the shock in the new steady state.

From inspection of Table 1, the only modifications to the existing equilibrium conditions are that γ is set to unity and L in the aggregate accounting identity now features a time index t. The new variable L_t is then determined in standard fashion by adding to the equilibrium conditions the intratemporal first-order condition of the household governing the choice of labor effort:

$$\chi \left(L_t\right)^{\frac{1}{\varphi}} = \frac{w_t}{C_t} \tag{14}$$

Combining this with the wage schedule $w_t = \left[\left(\theta - 1\right)/\theta\right] Z_t \left(N_t\right)^{\frac{1}{\theta-1}}$, which holds also with endogenous labor supply, yields the condition:

$$\chi \left(L_t\right)^{\frac{1}{\varphi}} C_t = \frac{\theta - 1}{\theta} Z_t \left(N_t\right)^{\frac{1}{\theta-1}}, \tag{15}$$

which can be solved to obtain hours worked as a function of consumption, the number of firms, and productivity. The number of new entrants is then:

$$N_{E,t} = \left(\frac{\theta - 1}{\chi\theta}\right)^{\varphi} \frac{\left(Z_t\right)^{1+\varphi} \left(N_t\right)^{\frac{\varphi}{\theta-1}}}{f_{E,t} \left(C_t\right)^{\varphi}} - \frac{\left(N_t\right)^{\frac{1}{1-\theta}} \left(C_t + G_t\right)}{f_{E,t}}.$$

The system (4)-(5) changes as follows. The Euler equation (5) changes only insofar as we set γ to one. The state equation (4) is now replaced by:

$$N_t = (1-\delta)\left[N_{t-1} + \left(\frac{\theta - 1}{\chi\theta}\right)^{\varphi} \frac{\left(Z_{t-1}\right)^{1+\varphi} \left(N_{t-1}\right)^{\frac{\varphi}{\theta-1}}}{f_{E,t-1} \left(C_{t-1}\right)^{\varphi}} - \frac{1}{f_{E,t-1}} \left(N_{t-1}\right)^{\frac{1}{1-\theta}} \left(C_{t-1} + G_{t-1}\right)\right].$$

Planning Optimum and Market Equilibrium

When labor supply is endogenous, the planner's problem becomes:[36]

$$\max_{\{L_s, N_{s+1}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \log\left[Z_s \left(N_s\right)^{\frac{1}{\theta-1}} \left(L_s - \frac{1}{(1-\delta)} \frac{f_{E,s}}{Z_s} N_{s+1} + \frac{f_{E,s}}{Z_s} N_s\right)\right] - \chi \frac{\left(L_s\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right\}.$$

The Euler equation for the optimal choice of N_{t+1} and the law of motion for the number of firms are identical to the case of fixed labor supply, except for labor being now indexed by time. The

[36] As before, we omit government spending from the planner's problem to simplify notation, and it is easy to verify that the results on efficiency hold for a general period utility function $U(C_t, L_t)$.

additional intratemporal condition for the planning optimum is:

$$\chi \left(L_t \right)^{\frac{1}{\varphi}} C_t = Z_t \left(N_t \right)^{\frac{1}{\theta - 1}} . \tag{16}$$

The only difference between the planning optimum and the competitive market equilibrium concerns the equations governing intratemporal substitution between consumption and leisure – equations (16) and (15). Comparing these two equations shows that the two equilibria differ as follows. At the Pareto optimum, the marginal rate of substitution between consumption and leisure ($\chi \left(L_t \right)^{\frac{1}{\varphi}} C_t$) is equal to the marginal rate at which hours and the consumption good can be transformed into each other ($Z_t \left(N_t \right)^{\frac{1}{\theta-1}}$). In the competitive equilibrium this is no longer the case: There is a wedge (equal to the reciprocal of the gross price markup) between these two objects that can be explained intuitively as follows. Since consumption goods are priced at a markup and leisure is not, the household is less willing than optimal to substitute from leisure into consumption. That is, a suboptimally high amount of leisure is purchased, since this is the relatively cheaper good (implying that hours worked and consumption are suboptimally low). This result conforms with the argument in Lerner (1934, p. 172) that "If the 'social' degree of monopoly is the same for all final products [including leisure] there is no monopolistic alteration from the optimum at all." The absence of a markup ('social' degree of monopoly) for the leisure good induces non-synchronization of relative prices which leads to an inefficient allocation.

Efficiency can clearly be restored by taxing leisure (or subsidizing labor supply) at a rate equal to the net markup in the pricing of consumption goods and applying a lump-sum transfer/tax to the households. This policy ensures equality of markups, consistent with the Lerner-Samuelson intuition described above (see the appendix for the proof). Note that while the same policy would also induce efficiency in a model with a fixed number of firms, there is an important difference concerning optimal policy between that framework and our model. When N is fixed, this policy is equivalent to one that induces marginal-cost pricing of consumption goods by taxing firm revenues (hence synchronizing relative prices).

This equivalence no longer holds in our framework with entry: Such a policy would remove the wedge from equation (15), but no firm would find it profitable to enter (in the absence of an additional entry subsidy) since there would be no profit with which to cover the entry cost. Therefore, while markup synchronization is *necessary* for efficiency, it is *not sufficient*. Absent an entry cost subsidy, the sufficient condition states that the planner needs to align markups to the *higher* (positive) level. Doing otherwise (inducing marginal-cost pricing) would make the economy stop producing altogether. This result highlights once more that monopoly power in itself is not a distortion and should in fact be preserved if firm entry is subject to sunk costs that cannot be entirely subsidized.

Solution

In what follows, we assume that the government does not have access to distortionary taxation of leisure that removes the inefficiency of the market equilibrium. We focus on the inefficient steady state, comparing it to the efficient one chosen by the planner, and we analyze fluctuations around this inefficient point.[37]

The Steady State

The steady state analysis is largely unmodified by the introduction of elastic labor supply. Due to the form of the assumed preferences, the shares derived in the exogenous labor supply case are exactly the same. Importantly, since steady-state hours are constant to variations in productivity and market regulation, the long-run elasticities to these shocks for the variable labor supply case are exactly the same as in Table 2. In particular, marginal cost, w/Z, increases in response to a long-run increase in productivity even when labor supply is endogenous. However, this equivalence does not hold for government spending shocks. To illustrate this, we use the steady-state version of (14) to obtain hours worked in steady state as a function of parameters:

$$L = \left\{ \frac{1}{\chi (1 - \Gamma)} \left[1 - \frac{r}{\theta (r + \delta)} \right] \right\}^{\frac{\varphi}{1+\varphi}} . \tag{17}$$

(Note that hours are indeed constant relative to variations in productivity and regulation.)

The planning optimum implies that the household supplies more labor effort than in the market equilibrium, consistent with the result that the latter becomes efficient if the government is taxing leisure. Setting $\Gamma = 0$, hours chosen by planner are:

$$L_P = \left\{ \frac{1}{\chi} \left[1 + \frac{\delta}{(\theta - 1)(r + \delta)} \right] \right\}^{\frac{\varphi}{1+\varphi}} > L.$$

Using this, it is also possible to verify that the number of varieties N and consumption C are

[37] We return to the implications of leisure taxation in Bilbiie, Ghironi, and Melitz (2005).

too small in the competitive steady state relative to the planning optimum. In the absence of a leisure tax, undersupply of labor results in too little investment in the creation of new firms and less consumption than optimal.

We can perform the same exercise as in Table 2, calculating the long-run elasticities of variables to a one percent permanent increase in the share of government spending Γ. Implicit log-differentiation of (17) yields $d\ln L/d\ln\Gamma = \Gamma\varphi/\left[(1-\Gamma)(1+\varphi)\right]$. This is a standard result in the RBC framework: An increase in government spending has a negative wealth effect due to taxation, inducing the agent to work more (and more so, the more elastic is labor supply) as (s)he feels effectively poorer. From (10), note that the effect on the steady-state number of firms is identical to the effect on hours worked.

The elasticity of real wage w and marginal cost w/Z to Γ is $d\ln w/d\ln\Gamma = \Gamma\varphi/\left[(\theta-1)(1-\Gamma)(1+\varphi)\right]$. Real wage and real marginal cost *increase* in response to government spending shocks.[38] The intuition for this result stems from entry dynamics. The wealth effect on labor supply would lead to a fall in the real wage and an increase in profitability for a given number of firms. With entry, however, profit opportunities attract new entrants, and this leads to an increase in labor demand such that the overall effect on real wage and marginal cost is positive. The long-run effect of Γ on private consumption is $d\ln C/d\ln\Gamma = \Gamma(1+\varphi-\theta)/\left[(\theta-1)(1-\Gamma)(1+\varphi)\right]$. Note that government spending can *crowd in* private consumption if labor supply is elastic enough and the degree of monopolistic distortion is high enough (θ low enough), namely if $\varphi > \theta - 1$. For such parameter values, the effect on the real wage is strong enough to compensate the negative wealth effect of taxation, as households substitute out of leisure and into consumption.

The crowding-in effect of government spending on consumption described above is different, however, from the expansionary consumption effect of government spending shocks estimated in empirical studies (Blanchard and Perotti, 2003, and Perotti, 2004, among others). The reason is that (real) consumption data reflect consumption deflated by the average price level $C_{R,t} \equiv C_t/\rho_t$ more than the consumption index C_t that properly accounts for variety in all periods. To explain the empirical observation, one would want C_R to increase in response to an increase in Γ.[39] It is

$$C_R = \frac{\theta-1}{\theta}\frac{Z}{\chi}L^{-\frac{1}{\varphi}}.$$

[38] The elasticities of profits and firm value are equal to that of the real wage.

[39] Since $\Gamma \equiv G/Y^C$, it is $\Gamma_R = \Gamma$, so that a shock to Γ corresponds to an increase in empirically measured government spending.

Therefore, an increase in Γ – which induces L to rise – causes C_R to decrease, in contrast with the empirical evidence.[40] Nevertheless, the welfare implications of government spending can be crucially different in our model from the standard RBC setup (which shares the same inability to generate data-consistent crowding-in). In the standard RBC setup, an increase in wasteful government spending crowds out consumption and lowers welfare. In our model, there can be combinations of parameter values for which a higher Γ crowds out data-consistent consumption, but welfare increases, as C rises enough to offset the welfare-reducing effect of increased labor effort.

Dynamics

As in the previous section, we log-linearize the system to study dynamics in the neighborhood of the steady state. We define a new parameter for analytical convenience, $\alpha \equiv \varphi \left[r + \delta + \delta / (\theta - 1)\right]$, where α is proportional to labor supply elasticity. Then,

$$\mathsf{N}_{t+1} = [1 + r + \alpha] \, \mathsf{N}_t - (\theta - 1)(r + \delta + \alpha) \, \mathsf{C}_t - \delta \mathsf{f}_{E,t} + [(\theta - 1)(r + \delta + \alpha) + \delta] \, \mathsf{Z}_t - (\theta - 1)(r + \delta) \, \mathsf{G}_t, \tag{18}$$

$$\mathsf{C}_t = \frac{1 - \delta}{1 + r} E_t (\mathsf{C}_{t+1}) - \left[\frac{\theta}{\theta - 1} \left(\frac{1 - \delta}{1 + r} \right) - 1 \right] \mathsf{N}_{t+1} + \frac{1}{\theta - 1} \mathsf{N}_t + \mathsf{f}_{E,t} - \frac{1 - \delta}{1 + r} E_t \mathsf{f}_{E,t+1} - \frac{r + \delta}{1 + r} E_t (\mathsf{G}_{t+1}) . \tag{19}$$

Compared to the fixed-labor-supply case (12)-(13), elastic labor supply implies a higher absolute value of the elasticities of number of firms producing at $t+1$ to its lagged value, consumption, and technology.

As in the inelastic-labor-supply case, the system (18)-(19) has a unique, non-explosive solution (see the appendix for the proof). The solution takes exactly the same form as in the inelastic-labor-supply case, with different elasticities depending on the labor supply elasticity. As before, the elasticity of the number of firms producing in period $t+1$ to its past level (η_{NN}) is such that $0 < \eta_{NN} < 1$. It follows from the expression of η_{CN} in Table 4 that consumption is higher the larger the number of firms producing in period t.

[40] Galí, López-Salido, and Vallés (2004) show that inclusion of "non-Ricardian" households who consume their disposable income in each period would ameliorate this problem. See also Bilbiie and Straub (2004).

6 Business Cycles with Firm Dynamics, Part II

This section completes the exploration of the properties of our model for business cycle transmission with firm dynamics by means of the numerical example we started exploring in Section 4. As in that section, we set $\beta = .99$, $\gamma = 1$, $\delta = .025$, $\theta = 3.8$, and initial steady-state levels $Z = f_E = 1$ and $G = \Gamma = 0$. We consider the following alternative values for labor supply elasticity (in addition to the inelastic case): $\varphi = 1, 2, 4, 20$.[41] We set the weight of the disutility of labor in the period utility function, χ, so that the steady-state level of labor effort in (17) is 1 regardless of φ, ensuring that the steady-state levels of all variables are the same as in the inelastic-labor case.[42]

Impulse Responses

Productivity

Figure 5 shows the responses (percent deviations from steady state) to a 1 percent permanent increase in productivity under alternative scenarios for labor supply elasticity. The number of years is on the horizontal axis. The round marker on the responses corresponds to inelastic labor (as in Figure 3), the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$.

The responses of most variables with elastic labor are qualitatively very similar to the inelastic-labor case. However, elastic labor implies that the household has an additional margin of adjustment in the face of shocks. This enhances the model's propagation mechanism and, as the figure shows, amplifies the impact responses of most endogenous variables with respect to the inelastic-labor case. Faced with an increase in the real wage, the household optimally decides to work more hours in order to attain a higher consumption level. Moreover, expectations of increased profitability, as before, make the household more willing to invest in new firms (and hence the impact responses of labor in the investment sector and investment in new production lines are correspondingly larger as labor supply becomes more elastic). This adds to the capital stock of the economy (the number of firms) and makes both GDP and consumption increase more quickly toward the new steady state as.φ increases. (The long-run responses are identical – independent of labor supply elasticity – and are explained in Section 2.) Except in the initial quarters, firm-level profits increase by less than in the inelastic-labor case since profit margins are eroded by the increased entry of new firms.

[41] We consider a very high value of φ for illustrative purposes. An infinite elasticity of labor supply corresponds to linear disutility of labor, as in Christiano and Eichenbaum (1992) and Devereux, Head, and Lapham (1996*b*).

[42] This requires $\chi = 924271$.

Figure 6 repeats the exercise for a shock with persistence .9. Responses display the same pattern, with labor supply elasticity amplifying the responses of most variables. Firm-level profits drop below the steady state during the transition, increasingly so as φ increases since more entry erodes individual firm profitability. However, aggregate real profits ($D_{R,t}$ – omitted) increase in procyclical fashion, the more so, the more elastic labor supply.

Deregulation

Figure 7 shows the responses to a 1 percent permanent deregulation shock. As for productivity, the long-run responses are identical regardless of φ. However, the transitional dynamics are different. Consider first the inelastic-labor case. Deregulation attracts new entrants and firm value decreases (the relative price of the investment good falls). Since investment is relatively more attractive than consumption, there is intersectoral labor reallocation from the latter to the former. Consumption falls initially as households postpone consumption to invest more in firms whose productivity has not increased. The number of firms starts increasing, but GDP initially falls as the decline in consumption dominates the increase in investment. All variables then move monotonically towards their steady-state values. In the endogenous labor case, while the mechanism above still applies, the additional margin of adjustment induces the consumer to decide optimally to supply more labor to both sectors and accommodate the extra labor demand generated by the increasing number of firms. The number of entrants (and hence investment) increase relatively more, and, as for a permanent productivity shock, the total number of firms, firm value, and the real wage all converge faster to their new steady-state levels, the more so the larger labor supply elasticity. GDP now increases, due to a larger increase in investment and a smaller decrease in consumption.

Government Spending

Figure 8 shows responses to a 1 percent permanent increase in government spending G. The figure allows us to emphasize the important effects of labor supply elasticity on the responses to government spending shocks. In the inelastic-labor case, the only effect of the shock is to crowd out consumption one for one. When labor supply is elastic, the household responds to the shock optimally by supplying more labor, which results in larger amounts of labor both in creation of new firms and production of consumption (more so in the investment sector initially as households front-load entry of new firms). Consequently, GDP increases, and consumption increases in the long run if φ is sufficiently large (and the short-run decrease in C becomes progressively smaller and shorter-lived as φ increases further).

Figure 9 repeats the exercise for a shock with persistence .9. This makes it possible to further highlight the consequences of labor supply elasticity. A persistent government spending shock has a negative wealth effect on the consumer. With inelastic labor supply, as a larger present discounted value of taxes induces the consumer to feel poorer, (s)he decreases both consumption and investment. The relative magnitude of the responses is dictated by the relative price of the investment good (the value of a firm). Since this is expected to fall, the household allocates relatively more hours out of the fixed labor endowment to the consumption sector. But since the number of entrants falls below the steady state, the capital stock of the economy is depleted (the total number of firms falls) and total demand for labor also falls inducing a lower real wage. When labor supply is sufficiently elastic, most responses have the opposite sign. The wealth effect is optimally accommodated largely through labor supply: The consumer decides to work more for a given real wage. Since firm value is expected to increase, much of the impact increase in hours goes to labor in the investment sector, and hence both investment (the number of entrants) and the number of firms increase. After the initial period, larger labor demand generates an increase in the real wage. For high enough elasticity of labor supply, this generates an increase in consumption after the impact response (since the consumer substitutes out of leisure and into consumption). When the relative price of investment is at its peak, the consumer optimally decides to restrain investment and the number of new entrants falls under the steady state. The extra labor is relocated to the consumption sector, and the transition to the steady state is monotonic thereafter.

[TO BE COMPLETED.]

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Table 1. Benchmark Model, Summary

Pricing	$\rho_t = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}$
Profits	$d_t = \frac{1}{\theta}\left(\rho_t\right)^{1-\theta}\left(C_t + G_t\right)$
Free entry	$v_t = w_t \frac{f_{E,t}}{Z_t}$
Number of firms	$N_t = (1-\delta)\left(N_{t-1} + N_{E,t-1}\right)$
Euler equation (bonds)	$(C_t)^{-\gamma} = \beta\left(1 + r_{t+1}\right) E_t\left[(C_{t+1})^{-\gamma}\right]$
Euler equation (shares)	$v_t = \beta\left(1-\delta\right) E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(v_{t+1} + d_{t+1}\right)\right]$
Aggregate accounting	$C_t + G_t + N_{E,t}v_t = w_t L + N_t d_t$
Labor market clearing	$L = N_t \frac{(\rho_t)^{-\theta}(C_t+G_t)}{Z_t} + N_{E,t}\frac{f_{E,t}}{Z_t}$

Table 2. Benchmark Model, Long-Run Elasticities

Elasticity of $\Downarrow$ w.r.t. $\Rightarrow$	Z	f_E	Γ
N, N_E	1	-1	0
w	$\frac{\theta}{\theta-1}$	$-\frac{1}{\theta-1}$	0
v, d	$\frac{1}{\theta-1}$	$\frac{\theta-2}{\theta-1}$	0
C, Y	$\frac{\theta}{\theta-1}$	$-\frac{1}{\theta-1}$	$-\frac{\Gamma}{1-\Gamma}$

Table 3. Benchmark Model, Log-Linear Solution

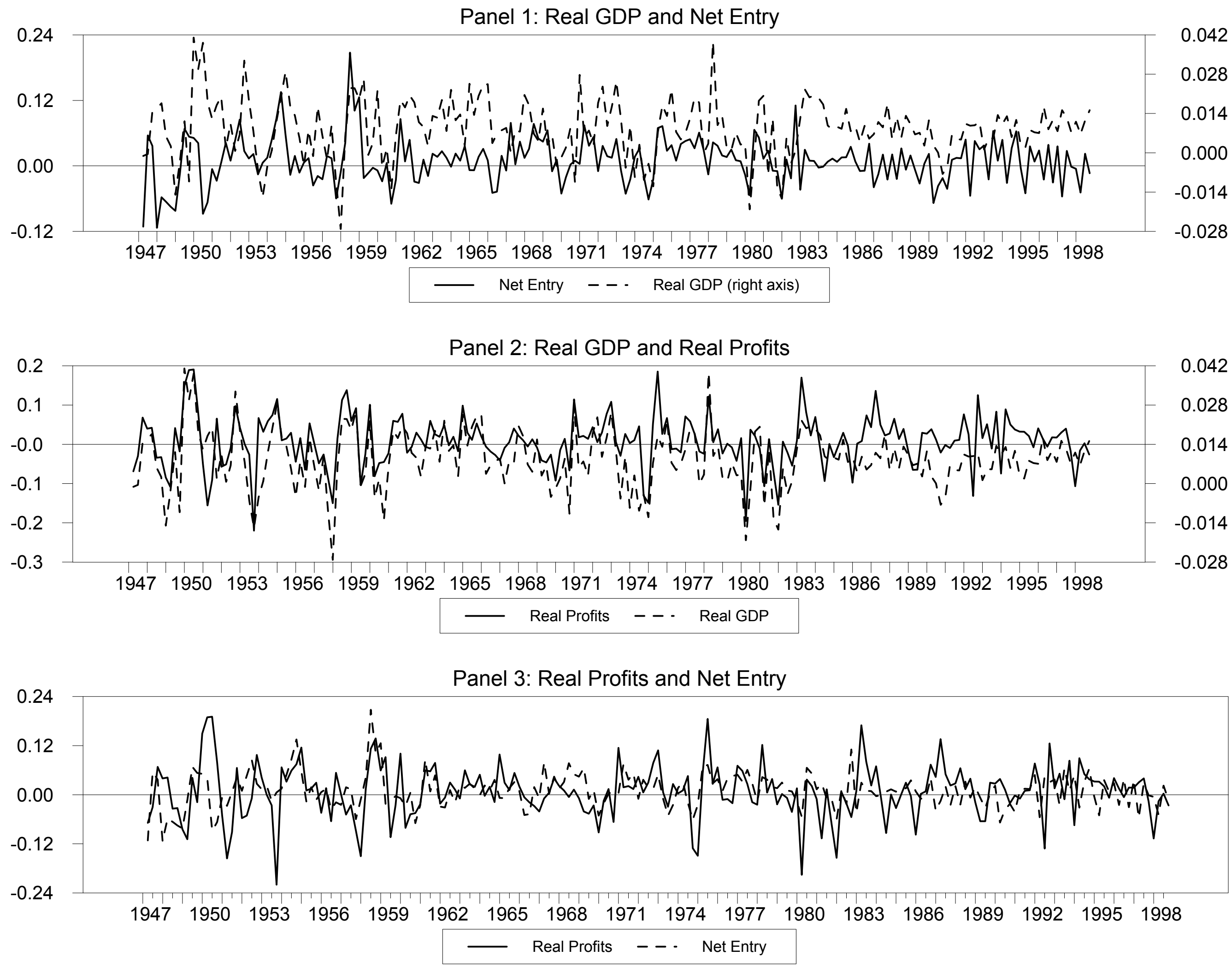
$0 < \eta_{NN} < 1$ (see the appendix for details)
$\eta_{CN} = \frac{1+r-\eta_{NN}}{(\theta-1)(r+\delta)}$
$\eta_{CZ} = \frac{[(\theta-1)r+\theta\delta]\left\{\gamma(1+r-\eta_{NN})-(r+\delta)\left[2-\frac{\theta(r+\delta)+\eta_{NN}}{1+r}\right]\right\}}{(\theta-1)[r+\delta]\left\{\gamma(2+r-\eta_{NN}-\phi_Z)-(r+\delta)\left[2-\frac{\theta(r+\delta)+\eta_{NN}+\phi_Z}{1+r}\right]\right\}}$
$\eta_{NZ} = (\theta-1)(r+\delta)(1-\eta_{CZ})+\delta$
$\eta_{Cf_E} = \frac{\delta(1-\delta)\theta+(\theta-1)(1-\delta)\left(1+r-\phi_{f_E}\right)+(1+r-\eta_{NN})\delta\left(1-\gamma\frac{1+r}{r+\delta}\right)}{(\theta-1)\left\{\gamma(1+r)\left(2+r-\eta_{NN}-\phi_{f_E}\right)-(r+\delta)\left[2(1+r)-\theta(r+\delta)-\eta_{NN}-\phi_{f_E}\right]\right\}}$
$\eta_{Nf_E} = -\left[(\theta-1)(r+\delta)\eta_{Cf_E}+\delta\right]$
$\eta_{CG} = -1+\frac{(1-\phi_G)\gamma}{\left(\gamma-\frac{r+\delta}{1+r}\right)(1+r-\phi_G-\eta_{NN})+\gamma+\left[\frac{\theta}{\theta-1}\left(\frac{1-\delta}{1+r}\right)-1\right](\theta-1)(r+\delta)}$
$\eta_{NG} = -(\theta-1)(r+\delta)(1+\eta_{CG})$

Table 4. Variable Labor Model, Log-Linear Solution

$0 < \eta_{NN} < 1$ (see the appendix for details)
$\eta_{CN} = \frac{1+r+\alpha-\eta_{NN}}{(\theta-1)(r+\delta+\alpha)}$
$\eta_{CZ} = \frac{\frac{\delta+(\theta-1)(r+\delta+\alpha)}{(\theta-1)(r+\delta+\alpha)}\left\{1+r+\alpha-\eta_{NN}-(r+\delta+\alpha)\left[\frac{1+r-\theta(r+\delta)}{1-\delta}\right]\right\}}{1+r+\alpha-\eta_{NN}-\phi_Z+\frac{1+r}{1-\delta}-(r+\delta+\alpha)\left[\frac{1+r-\theta(r+\delta)}{1-\delta}\right]}$
$\eta_{NZ} = (\theta-1)(r+\delta+\alpha)(1-\eta_{CZ})+\delta$
$\eta_{Cf_E} = \frac{\frac{\delta}{\theta-1}\left[\theta-\frac{1+r+\alpha-\eta_{NN}}{r+\delta+\alpha}\right]+1+r-\phi_{f_E}}{1+r+\alpha-\eta_{NN}-\phi_{fE}+\frac{1+r}{1-\delta}-(r+\delta+\alpha)\left[\frac{1+r-\theta(r+\delta)}{1-\delta}\right]}$
$\eta_{Nf_E} = -\left[(\theta-1)(r+\delta+\alpha)\eta_{Cf_E}+\delta\right]$
$\eta_{CG} = -1+\frac{(1-\phi_G)\frac{1+r}{1-\delta}+\frac{\alpha}{r+\delta+\alpha}(1+r+\alpha-\eta_{NN})-\alpha\left[\frac{1+r-\theta(r+\delta)}{1-\delta}\right]}{1+r+\alpha-\eta_{NN}-\phi_G+\frac{1+r}{1-\delta}-(r+\delta+\alpha)\left[\frac{1+r-\theta(r+\delta)}{1-\delta}\right]}$
$\eta_{NG} = -(\theta-1)\left[(r+\delta)(1+\eta_{CG})+\alpha\eta_{CG}\right]$

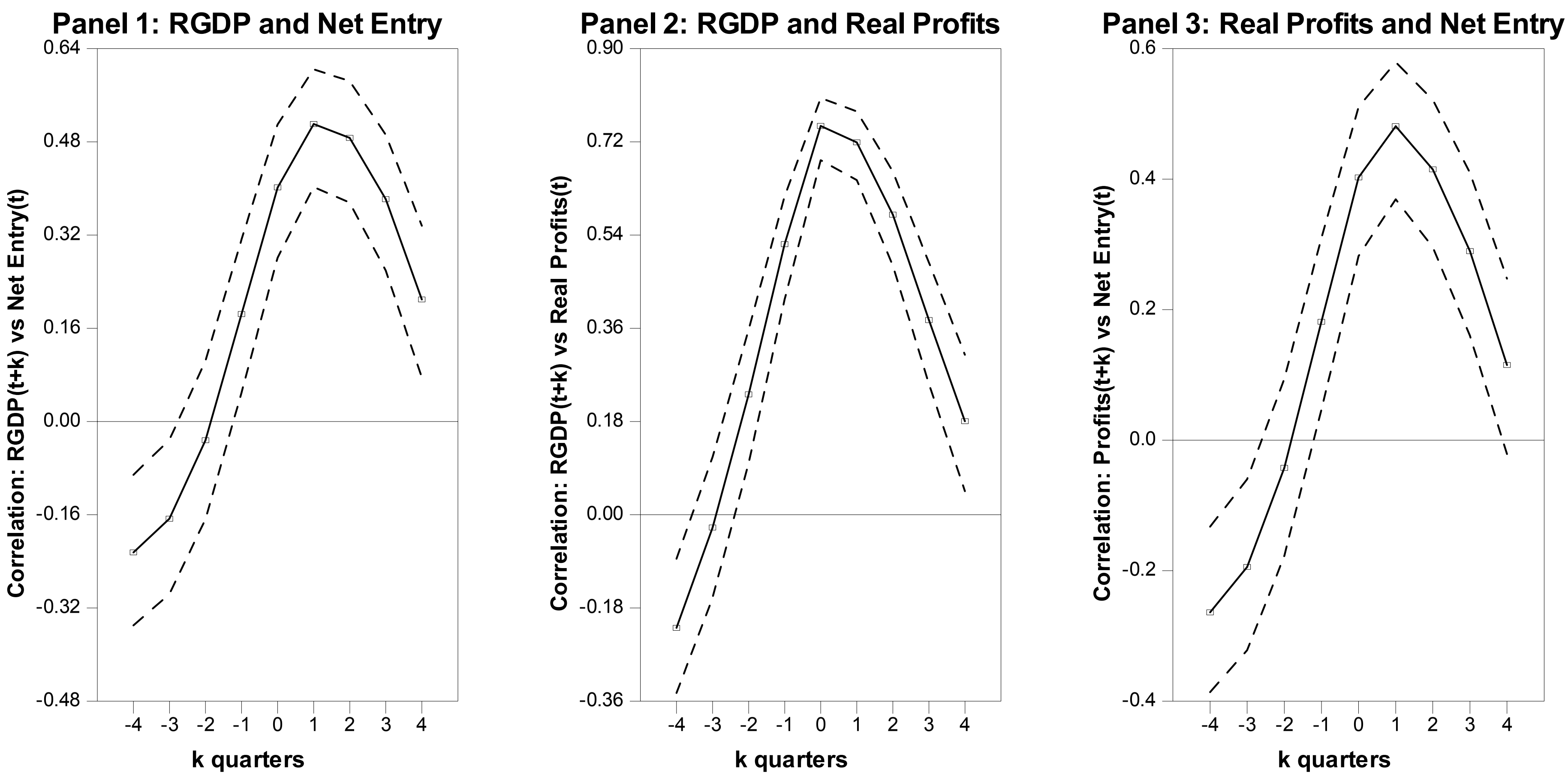
Growth Rates of Real GDP, Net Entry, and Real Profits

Sample period: 1947 - 1998



Cross Correlation: RGDP, Net Entry, and Real Profits

Hodrick-Prescott filtered data in logs, 95% confidence intervals, sample period: 1947 - 1998



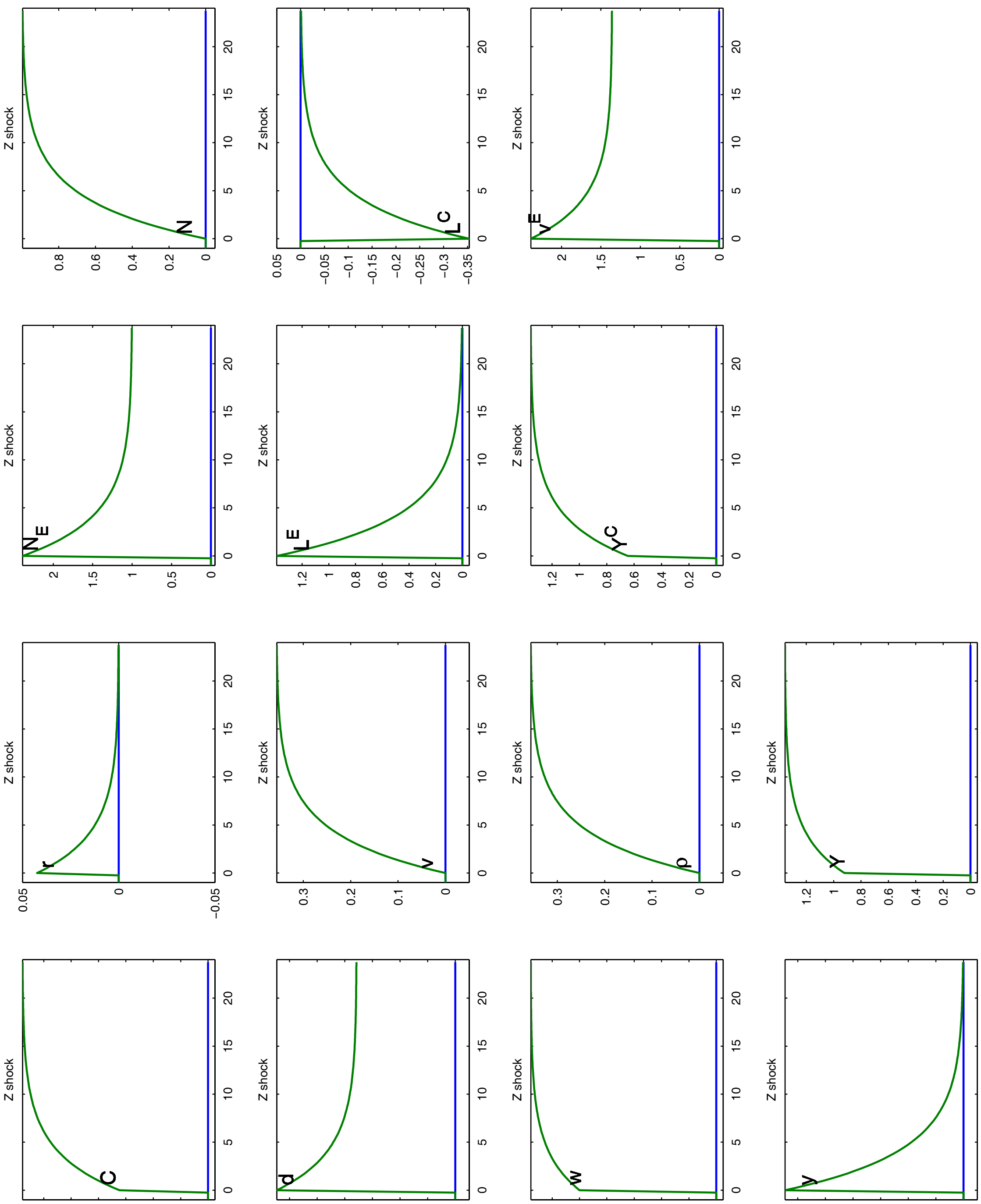


Figure 3. Permanent productivity shock, inelastic labor supply

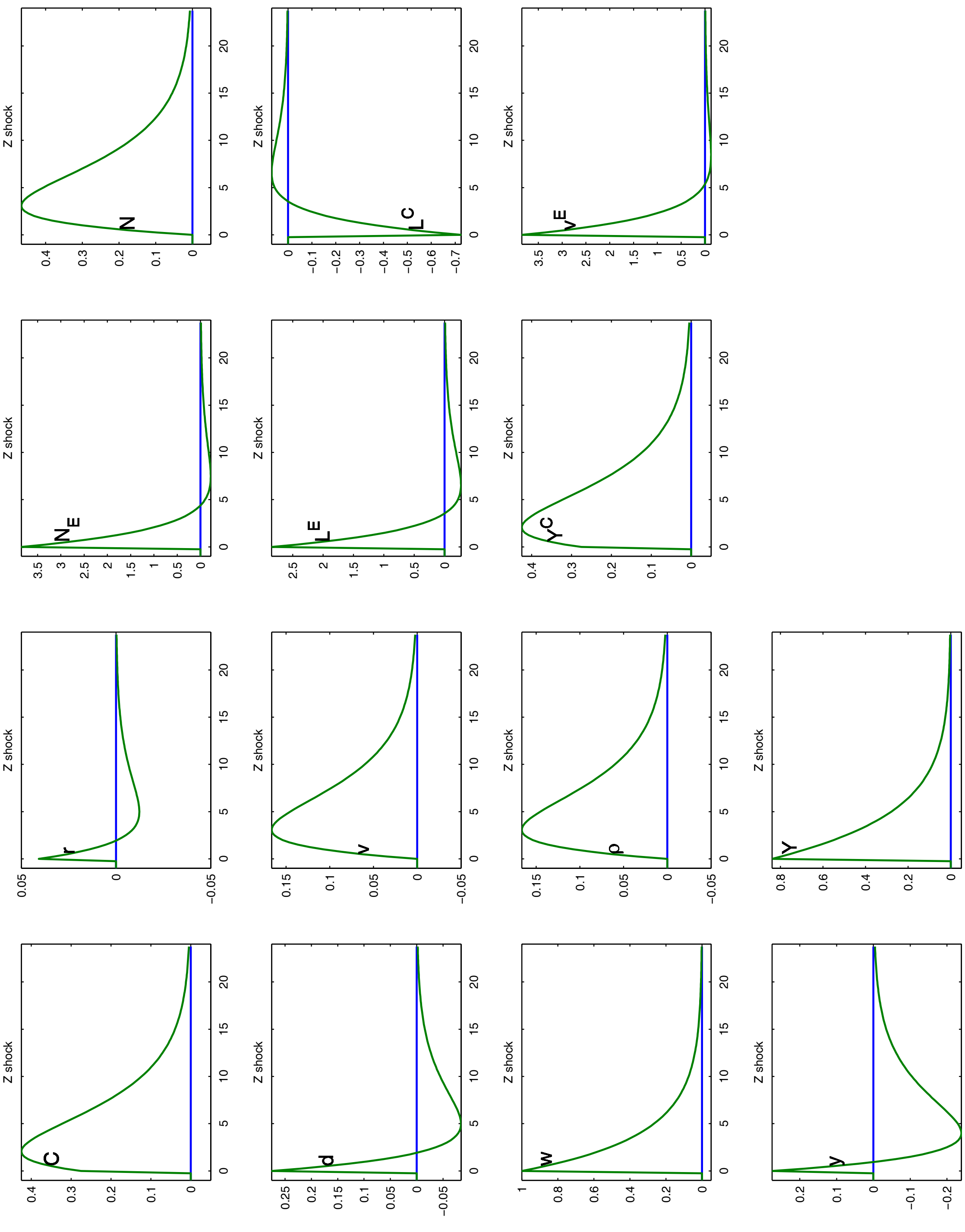


Figure 4. Productivity shock, persistence .9, inelastic labor supply

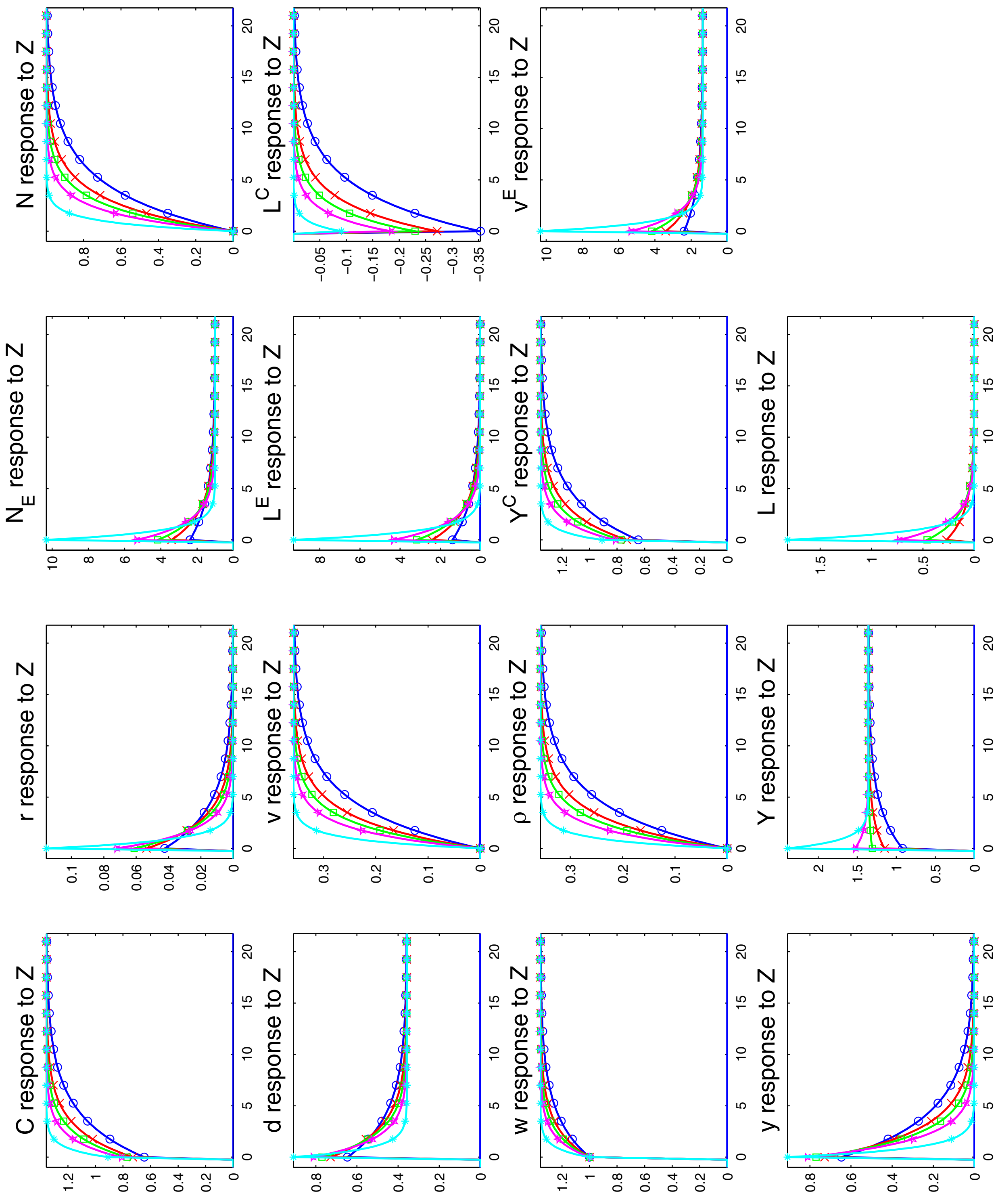


Figure 5. Permanent productivity shock, increasing labor supply elasticity

Legend: The round marker on the responses corresponds to inelastic labor, the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$

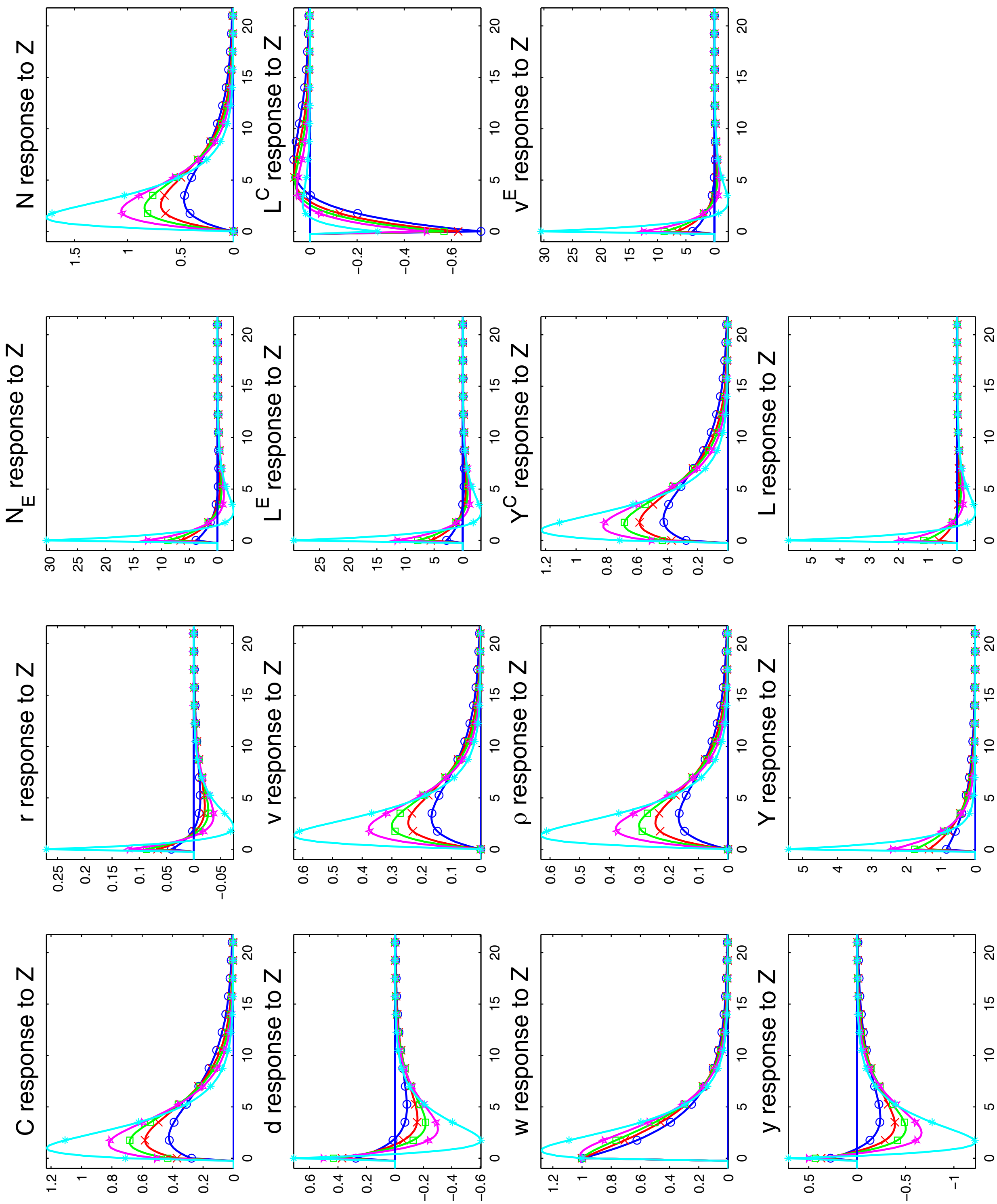


Figure 6. Productivity shock, persistence .9, increasing labor supply elasticity

Legend: The round marker on the responses corresponds to inelastic labor, the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$

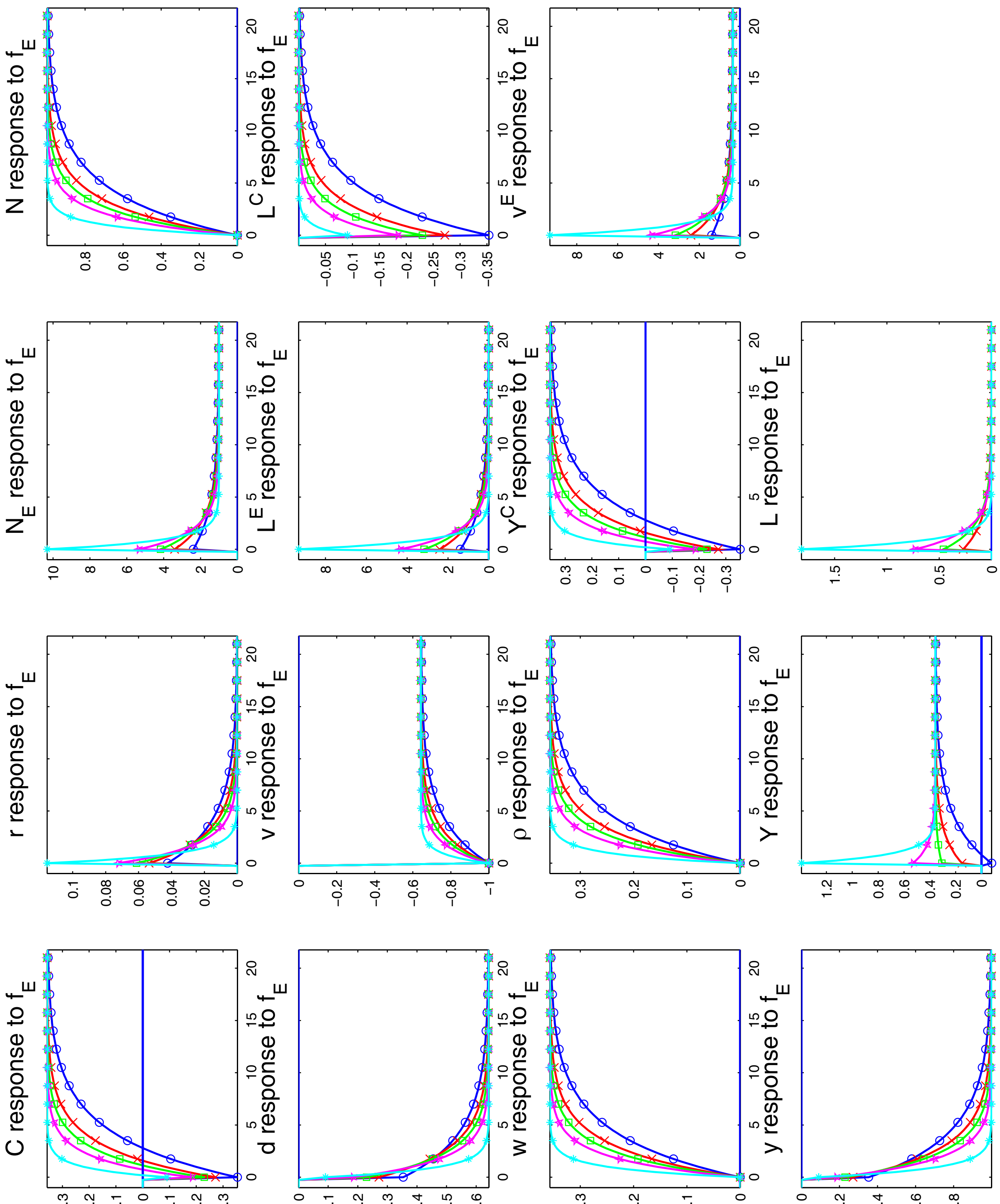


Figure 7. Permanent deregulation, increasing labor supply elasticity

Legend: The round marker on the responses corresponds to inelastic labor, the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$

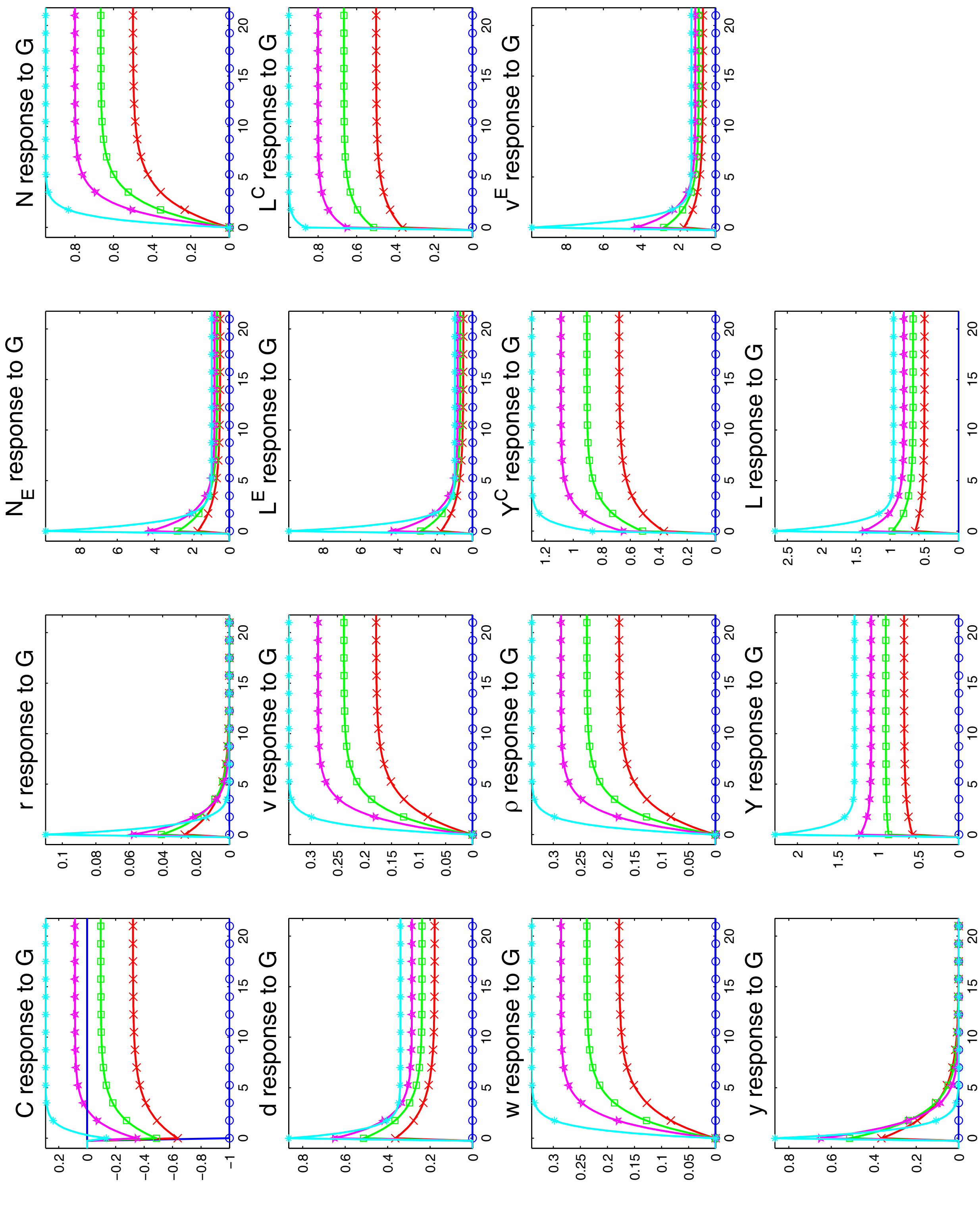


Figure 8. Permanent government spending shock, increasing labor supply elasticity

Legend: The round marker on the responses corresponds to inelastic labor, the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$

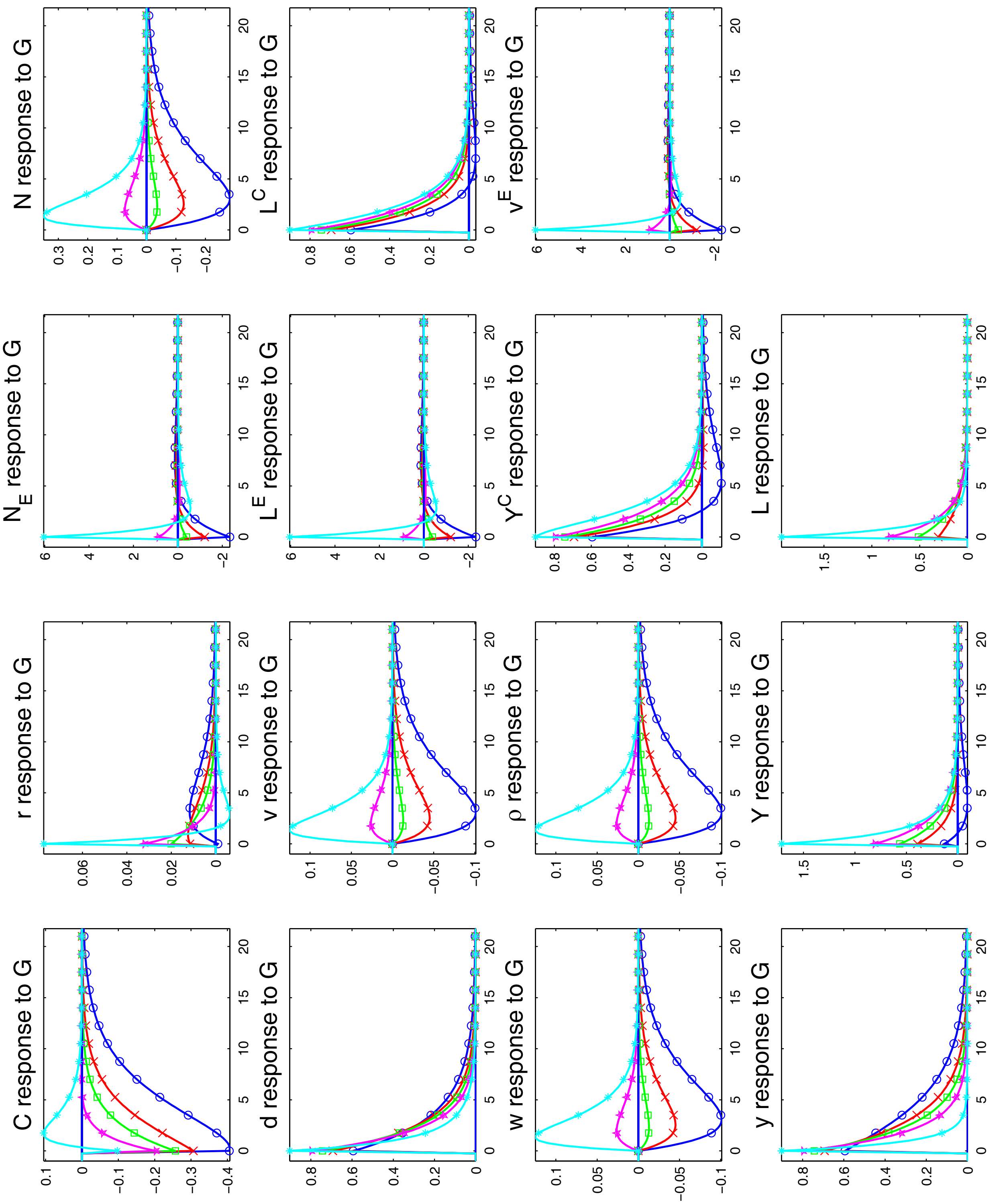


Figure 9. Government spending shock, persistence .9, increasing labor supply elasticity

Legend: The round marker on the responses corresponds to inelastic labor, the cross to $\varphi = 1$, the square to $\varphi = 2$, the pentagon to $\varphi = 4$, and the star to $\varphi = 20$

Appendix

The Benchmark Model

Equilibrium Efficiency

The planner's problem can be rewritten as:

$$\max_{\{L_s^C\}_{s=t}^{\infty}} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left[Z_s \left(N_s\right)^{\frac{1}{\theta-1}} L_s^C \right]^{1-\gamma}}{1-\gamma} \right\},$$

$$s.t.\ N_{t+1} = (1-\delta) N_t + (1-\delta) \frac{\left(L - L_t^C\right) Z_t}{f_{E,t}},$$

or, substituting the constraint into the utility function and treating next period's state as the choice variable:

$$\max_{\{N_{s+1}\}_{s=t}^{\infty}} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left[Z_s \left(N_s\right)^{\frac{1}{\theta-1}} \left(L - \frac{1}{(1-\delta)} \frac{f_{E,s}}{Z_s} N_{s+1} + \frac{f_{E,s}}{Z_s} N_s \right) \right]^{1-\gamma}}{1-\gamma} \right\}.$$

The first-order condition for this problem can be written

$$\begin{aligned} &(C_t)^{-\gamma} Z_t \left(N_t\right)^{\frac{1}{\theta-1}} \frac{1}{1-\delta} \frac{f_{E,t}}{Z_t} \\ &= \beta E_t \left\{ (C_{t+1})^{-\gamma} \frac{1}{\theta-1} Z_{t+1} \left(N_{t+1}\right)^{\frac{1}{\theta-1}-1} \left[L - \frac{1}{(1-\delta)} \frac{f_{E,t+1}}{Z_{t+1}} N_{t+2} + \theta \frac{f_{E,t+1}}{Z_{t+1}} N_{t+1} \right] \right\}. \end{aligned}$$

The term in square brackets in the right-hand side of this equation is:

$$L - \frac{1}{(1-\delta)} \frac{f_{E,t+1}}{Z_{t+1}} N_{t+2} + \frac{f_{E,t+1}}{Z_{t+1}} N_{t+1} + (\theta-1) \frac{f_{E,t+1}}{Z_{t+1}} N_{t+1} = L_{t+1}^C + (\theta-1) \frac{f_{E,t+1}}{Z_{t+1}} N_{t+1}.$$

Hence, the first-order condition becomes:

$$(C_t)^{-\gamma} Z_t \left(N_t\right)^{\frac{1}{\theta-1}} \frac{1}{1-\delta} \frac{f_{E,t}}{Z_t} = \beta E_t \left\{ (C_{t+1})^{-\gamma} \frac{1}{\theta-1} Z_{t+1} \left(N_{t+1}\right)^{\frac{1}{\theta-1}-1} \left[L_{t+1}^C + (\theta-1) \frac{f_{E,t+1}}{Z_{t+1}} N_{t+1} \right] \right\}.$$

Using equation (6) and rearranging yields equation (5). (Recall that we are abstracting from government spending.) Together with

$$N_{t+1} = (1-\delta)\left[N_t + \frac{Z_t}{f_{E,t}}L - \frac{1}{f_{E,t}}C_t\left(N_t\right)^{\frac{1}{\theta-1}}\right],$$

this implies that the planning optimum coincides with the competitive equilibrium.

Local Equilibrium Determinacy and Non-Explosiveness

To analyze local determinacy and non-explosiveness of the rational expectation equilibrium, we can focus on the perfect foresight version of the system (12)-(13) and restrict attention to endogenous variables. Rearranging yields:

$$\begin{bmatrix} \mathsf{C}_{t+1} \\ \mathsf{N}_{t+1} \end{bmatrix} = M \begin{bmatrix} \mathsf{C}_t \\ \mathsf{N}_t \end{bmatrix}, \quad M \equiv \begin{bmatrix} \frac{\gamma - \frac{r+\delta}{1+r}(1-\delta) + (\theta-1)\frac{(r+\delta)^2}{1+r}}{\gamma - \frac{r+\delta}{1+r}} & \frac{(1-\delta)\frac{\theta}{\theta-1} - \frac{1}{\theta-1} - (1+r)}{\gamma - \frac{r+\delta}{1+r}} \\ -(\theta-1)(r+\delta) & 1+r \end{bmatrix}.$$

Existence of a unique, non-explosive rational expectations equilibrium requires that one eigenvalue of M be inside and one outside the unit circle. Since the determinant $\det(M) = 1+r$ is also the product of the eigenvalues and is strictly greater than one, at least one of the roots will lie outside the unit circle; hence, equilibrium indeterminacy is never a problem in our model. The characteristic polynomial of M takes the form $J(\lambda) = \lambda^2 - (\text{trace}(M))\lambda + \det(M)$, where the trace is

$$\text{trace}(M) = 1 + r + \frac{\gamma - \frac{r+\delta}{1+r}(1-\delta) + (\theta-1)\frac{(r+\delta)^2}{1+r}}{\gamma - \frac{r+\delta}{1+r}}.$$

The condition for existence of a unique, non-explosive rational expectations equilibrium is then $J(-1)J(1) < 0$. It is straightforward to verify that: (i) $J(1) = 1 - \text{trace}(M) + \det(M) < 0$ if $\gamma > (r+\delta)/(1+r)$ as is the case for all reasonable parametrizations; (ii) if $J(1) < 0$, then $J(-1) > 0$. This proves determinacy and non-explosiveness of the rational expectations equilibrium. Since $J(0) > 0$, we can also conclude that both roots are positive. The elasticity of the number of firms producing in period $t+1$ to its past level (η_{NN}) in the solution of the model is then the stable root of $J(\lambda) = 0$, $\eta_{NN} = \left[\text{trace}(M) - \sqrt{(\text{trace}(M))^2 - 4\det(M)}\right]/2$.

Endogenous Labor Supply

Restoring Efficiency through a Leisure Tax

Suppose the government taxes leisure at the rate τ. In equilibrium, revenues from taxation of leisure are rebated to households through lump-sum transfers T_t^{LE}. The representative household's budget constraint becomes:

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t + T_t + T_t^{LE} = (1 + r_t) B_t + (d_t + v_t) N_t x_t + w_t L_t - \tau w_t \left(\bar{L} - L_t\right),$$

where $\bar{L}$ is the endowment of time in each period.

The first-order condition for the household's optimal choice of labor supply is the only equilibrium condition that is affected. It becomes:

$$\chi (L_t)^{\frac{1}{\varphi}} C_t = (1 + \tau) w_t.$$

Combining this with the wage schedule $w_t = [(\theta - 1)/\theta] Z_t (N_t)^{\frac{1}{\theta - 1}}$ yields:

$$\chi (L_t)^{\frac{1}{\varphi}} C_t = \frac{(1 + \tau)(\theta - 1)}{\theta} Z_t (N_t)^{\frac{1}{\theta - 1}}.$$

Comparing this equation to (16) shows that a constant rate of taxation equal to the net markup of price over marginal cost ($\tau = 1/(\theta - 1)$) restores efficiency of the market equilibrium.

Local Equilibrium Determinacy and Non-Explosiveness

Casting the system (18)-(19) in canonical form, the Jacobian matrix is now:

$$M \equiv \begin{bmatrix} \frac{1+r}{1-\delta} - (\theta - 1)(r + \delta + \alpha)\left[\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right] & -\frac{1}{\theta - 1}\frac{1+r}{1-\delta} + \left[\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right](1 + r + \alpha) \\ -(\theta - 1)(r + \delta + \alpha) & 1 + r + \alpha \end{bmatrix}.$$

The conditions for existence and uniqueness of a stable rational expectations equilibrium are the same as with inelastic labor supply. Also in this case $\det(M) = 1 + r$. The trace of M is:

$$\text{trace}(M) = 1 - \delta + \frac{1 + r}{1 - \delta} + (\theta - 1)(r + \delta + \alpha)\frac{r + \delta}{1 - \delta}.$$

Since $J(1) = -(r+\delta)[\delta + (\theta-1)(r+\delta+\alpha)]/(1-\delta) < 0$ and $J(-1) = 4 + 2r - J(1) > 0$, there exists a unique, stable rational expectations equilibrium for any possible parametrization. Note that, since $J(0) > 0$, both roots of M are positive. As with inelastic labor supply, the elasticity of the number of firms producing in period $t+1$ to its past level (η_{NN}) is the stable root of $J(\lambda) = 0$, $\eta_{NN} = \left[\text{trace}(M) - \sqrt{(\text{trace}(M))^2 - 4\det(M)}\right]/2$.

Taste for Variety and Monopoly Power

In our model, the same parameter $1/(\theta-1)$ governs the degree of monopoly power and consumer preference for variety. We verify that efficiency of the market equilibrium breaks down if we separate monopoly power and taste for variety in consumer preferences.[42] (We focus on the case of inelastic labor supply.) We disentangle monopoly power and taste for variety in *ad hoc* fashion as in Benassy (1996*a*). Using Benassy's specification, the consumption basket is $C_t = (N_t)^{\xi - \frac{1}{\theta-1}} \left(\int_{\omega\in\Omega} c_t(\omega)^{\theta-1/\theta} d\omega\right)^{\theta/(\theta-1)}$, where ξ now measures preference for variety and can be parametrized separately. When $\xi = 1/(\theta-1)$, we return to our benchmark.

Competitive Equilibrium

As before, we omit government spending. Many of the equations remain unchanged.

The relative price is now $\rho_t = p_t/P_t = (N_t)^{\xi}$, while prices are still set as a markup $\theta/(\theta-1)$ over marginal cost. The only equation in Table 1 that changes is the expression for profits. Profit for each firm is still given by $d_t = C_t/(\theta N_t)$.[43] However, using $\rho_t = (N_t)^{\xi}$, we now have $d_t = (\rho_t)^{-\frac{1}{\xi}} C_t/\theta$ instead of $d_t = (\rho_t)^{1-\theta} C_t/\theta$.

Performing the same exercise as in the main text to obtain a two-equation system, we get:

$$N_{t+1} = (1-\delta)\left[N_t + \frac{Z_t L}{f_{E,t}} - \frac{1}{f_{E,t+1}}(N_{t+1})^{1-\theta\xi} C_{t+1}\right],$$
$$f_{E,t}(N_t)^{\xi} = \beta(1-\delta)E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[f_{E,t+1}(N_t)^{\xi} + \frac{1}{\theta-1}\frac{C_{t+1}}{N_{t+1}}\right]\right\}.$$

[42] Kim (2004) obtains the same result in his model.

[43] Since $d_t = \rho_t Z_t l_t - w_t l_t = \rho_t l_t/\theta$, so that $N_t d_t = \rho_t N_t l_t/\theta = C_t/\theta$.

Planning Optimum

The planner's problem is the same as before, except the planner's constraint is now $C_t = Z_t (N_t)^\xi L_t^C$. Hence, the planner solves:

$$\max_{\{L_s^C\}_{s=t}^\infty} E_t \left[\sum_{s=t}^\infty \beta^{s-t} \frac{\left[Z_s (N_s)^\xi L_s^C \right]^{1-\gamma}}{1-\gamma} \right],$$

$$s.t.\ N_{t+1} = (1-\delta) N_t + (1-\delta) \frac{\left(L - L_t^C\right) Z_t}{f_{E,t}}.$$

This gives the Euler equation:

$$(C_t)^{-\gamma} (N_t)^\xi f_{E,t} = \beta (1-\delta) E_t \left\{ (C_{t+1})^{-\gamma} \left[\frac{1}{\theta - 1} \frac{C_{t+1}}{N_{t+1}} + f_{E,t+1} (N_{t+1})^\xi \right] \right\},$$

which is again the same as in the competitive equilibrium.

However, note that the dynamic constraint

$$N_{t+1} = (1-\delta) \left[N_t + \frac{Z_t L}{f_{E,t}} - \frac{1}{f_{E,t+1}} (N_{t+1})^{-\xi} C_{t+1} \right]$$

is now different and coincides with the competitive one if and only if $\xi = 1/(\theta - 1)$.

In order to gain some further intuition we focus on the steady state. The number of firms in the planning optimum and the competitive equilibrium is determined respectively by:

$$N^P = \frac{(1-\delta) L}{\delta + (\theta - 1)(r + \delta)}, \quad N^C = \frac{(1-\delta) L}{\delta + (\theta - 1)(r+\delta)(N^C)^{1+\xi(1-\theta)}}.$$

Direct comparison shows that (since $N > 1$) $N^P > (<) N^C$ if and only if $\xi < (>) 1/(\theta - 1)$. When $\xi < 1/(\theta - 1)$ the consumer derives disutility from the introduction of a new good and the economy ends up in the long run with too low a number of producers. However, as noted in the text, these preferences imply that the consumer is made worse off by the introduction of the good even if (s)he never consumes it. This induces us to adopt the C.E.S. specification as our benchmark.

Technical Appendix

A Solution, Log-Linear Inelastic-Labor Model

The system to be solved is:

$$\begin{aligned}
\mathsf{N}_{t+1} &= (1+r)\mathsf{N}_t - (\theta-1)(r+\delta)\mathsf{C}_t - \delta\mathsf{f}_{E,t} + [(\theta-1)r + \theta\delta]\mathsf{Z}_t - (\theta-1)(r+\delta)\mathsf{G}_t, \\
\mathsf{C}_t &= \left[1 - \frac{r+\delta}{\gamma(1+r)}\right] E_t(\mathsf{C}_{t+1}) - \frac{1}{\gamma}\left[\frac{\theta}{\theta-1}\left(\frac{1-\delta}{1+r}\right) - 1\right]\mathsf{N}_{t+1} + \frac{1}{\gamma(\theta-1)}\mathsf{N}_t + \frac{1}{\gamma}\mathsf{f}_{E,t} \\
&\quad - \frac{1-\delta}{\gamma(1+r)} E_t \mathsf{f}_{E,t+1} - \frac{r+\delta}{\gamma(1+r)} E_t(\mathsf{G}_{t+1}).
\end{aligned}$$

The canonical form of the model can be written as:

$$\begin{aligned}
\mathsf{N}_{t+1} &= b_{NN}\mathsf{N}_t + b_{NC}\mathsf{C}_t + b_{Nf}\mathsf{f}_{E,t} + b_{NZ}\mathsf{Z}_t + b_{NG}\mathsf{G}_t, \\
E_t(\mathsf{C}_{t+1}) &= d_{CC}\mathsf{C}_t + d_{CN}\mathsf{N}_t + d_{CZ}\mathsf{Z}_t + b_{Cf}\mathsf{f}_{E,t} + d_{CG}\mathsf{G}_t,
\end{aligned}$$

where:

$$\begin{aligned}
b_{NN} &\equiv 1+r, \\
b_{NC} &\equiv -(\theta-1)(r+\delta), \\
b_{NZ} &\equiv [(\theta-1)(r+\delta)+\delta], \\
b_{Nf} &\equiv -\delta, \\
b_{NG} &\equiv -(\theta-1)(r+\delta), \\
d_{CC} &\equiv \zeta\left[\frac{\gamma(1+r)}{1-\delta} - (\theta-1)(r+\delta)\left(\frac{\theta}{\theta-1} - \frac{1+r}{1-\delta}\right)\right], \\
d_{CN} &\equiv \zeta\left[\left(\frac{\theta}{\theta-1} - \frac{1+r}{1-\delta}\right)(1+r) - \frac{1}{\theta-1}\left(\frac{1+r}{1-\delta}\right)\right], \\
d_{CZ} &\equiv \zeta\left\{\left(\frac{\theta}{\theta-1} - \frac{1+r}{1-\delta}\right)[(\theta-1)(r+\delta)+\delta]\right\}, \\
d_{Cf} &\equiv \zeta\left[-\left(\frac{\delta\gamma}{\theta-1} + 1 + r\right) + \phi_{f_E}\right], \\
d_{CG} &\equiv \zeta\left[-(\theta-1)(r+\delta)\left(\frac{\theta}{\theta-1} - \frac{1+r}{1-\delta}\right) + \frac{r+\delta}{1-\delta}\phi_G\right].
\end{aligned}$$

and $\zeta \equiv \frac{1-\delta}{\gamma(1+r)-(r+\delta)}$.

The solution of the model be written as:

$$\mathsf{N}_{t+1} = \eta_{NN}\mathsf{N}_t + \eta_{NZ}\mathsf{Z}_t + \eta_{Nf_E}\mathsf{f}_{E,t} + \eta_{NG}\mathsf{G}_t,$$
$$\mathsf{C}_t = \eta_{CN}\mathsf{N}_t + \eta_{CZ}\mathsf{Z}_t + \eta_{Cf_E}\mathsf{f}_{E,t} + \eta_{CG}\mathsf{G}_t.$$

The elasticities η are found with the method of undetermined coefficients as follows:

$$\eta_{NN} = \text{stable eigenvalue of } \begin{bmatrix} d_{CC} & d_{CN} \\ b_{NC} & b_{NN} \end{bmatrix},$$
$$\eta_{CN} = \frac{\eta_{NN} - b_{NN}}{b_{NC}}.$$

For any $S \in \{Z, G, f_E\}$:

$$\eta_{CS} = \frac{d_{CS} + \frac{b_{NS}}{b_{NC}}(b_{NN} - \eta_{NN})}{\eta_{NN} - b_{NN} + \phi_S - d_{CC}},$$
$$\eta_{NS} = b_{NC}\eta_{CS} + b_{NS} = \frac{b_{NC}d_{CS} + b_{NS}(\phi_S - d_{CC})}{\eta_{NN} - b_{NN} + \phi_S - d_{CC}}.$$

B Solution, Log-Linear Elastic-Labor Model

The system to be solved is now:

$$\mathsf{N}_{t+1} = \left[1 + r + \varphi\left(\frac{\delta}{\theta - 1} + r + \delta\right)\right]\mathsf{N}_t - \{(\theta - 1)(r + \delta) + \varphi[(\theta - 1)(r + \delta) + \delta]\}\,\mathsf{C}_t -$$
$$- \delta\mathsf{f}_{E,t} + (1 + \varphi)[(\theta - 1)(r + \delta) + \delta]\,\mathsf{Z}_t - (\theta - 1)(r + \delta)\,\mathsf{G}_t,$$
$$\mathsf{C}_t = \frac{1 - \delta}{1 + r}E_t(\mathsf{C}_{t+1}) - \left[\frac{\theta}{\theta - 1}\left(\frac{1 - \delta}{1 + r}\right) - 1\right]\mathsf{N}_{t+1} + \frac{1}{\theta - 1}\mathsf{N}_t + \mathsf{f}_{E,t} - \frac{1 - \delta}{1 + r}E_t\mathsf{f}_{E,t+1} - \frac{r + \delta}{1 + r}E_t(\mathsf{G}_{t+1}).$$

The canonical form of the model can be written as:

$$\mathsf{N}_{t+1} = b_{NN}\mathsf{N}_t + b_{NC}\mathsf{C}_t + b_{Nf}\mathsf{f}_{E,t} + b_{NZ}\mathsf{Z}_t + b_{NG}\mathsf{G}_t,$$
$$E_t(\mathsf{C}_{t+1}) = d_{CC}\mathsf{C}_t + d_{CN}\mathsf{N}_t + d_{CZ}\mathsf{Z}_t + d_{Cf}\mathsf{f}_{E,t} + d_{CG}\mathsf{G}_t,$$

where:

$$
\begin{aligned}
b_{NN} &\equiv 1 + r + \alpha, \\
b_{NC} &\equiv -(\theta - 1)(r + \delta + \alpha), \\
b_{NZ} &\equiv [(\theta - 1)(r + \delta + \alpha) + \delta], \\
b_{Nf} &\equiv -\delta, \\
b_{NG} &\equiv -(\theta - 1)(r + \delta), \\
d_{CC} &\equiv \frac{1+r}{1-\delta} - (\theta - 1)(r + \delta + \alpha)\left(\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right), \\
d_{CN} &\equiv \left(\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right)(1 + r + \alpha) - \frac{1}{\theta - 1}\left(\frac{1+r}{1-\delta}\right), \\
d_{CZ} &\equiv \left(\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right)[(\theta - 1)(r + \delta + \alpha) + \delta], \\
d_{Cf} &\equiv -\left(\frac{\delta\theta}{\theta - 1} + 1 + r\right) + \phi_{f_E}, \\
d_{CG} &\equiv -(\theta - 1)(r + \delta)\left(\frac{\theta}{\theta - 1} - \frac{1+r}{1-\delta}\right) + \frac{r+\delta}{1-\delta}\phi_G.
\end{aligned}
$$

The solution of the model can be written as:

$$
\begin{aligned}
\mathsf{N}_{t+1} &= \eta_{NN}\mathsf{N}_t + \eta_{NZ}\mathsf{Z}_t + \eta_{Nf_E}\mathsf{f}_{E,t} + \eta_{NG}\mathsf{G}_t, \\
\mathsf{C}_t &= \eta_{CN}\mathsf{N}_t + \eta_{CZ}\mathsf{Z}_t + \eta_{Cf_E}\mathsf{f}_{E,t} + \eta_{CG}\mathsf{G}_t.
\end{aligned}
$$

The elasticities η are found with the method of undetermined coefficients as follows:

$$
\begin{aligned}
\eta_{NN} &= \text{stable eigenvalue of } \begin{bmatrix} d_{CC} & d_{CN} \\ b_{NC} & b_{NN} \end{bmatrix}, \\
\eta_{CN} &= \frac{\eta_{NN} - b_{NN}}{b_{NC}}.
\end{aligned}
$$

For any $S \in \{Z, G, f_E\}$:

$$
\begin{aligned}
\eta_{CS} &= \frac{d_{CS} + \frac{b_{NS}}{b_{NC}}(b_{NN} - \eta_{NN})}{\eta_{NN} - b_{NN} + \phi_S - d_{CC}}, \\
\eta_{NS} &= b_{NC}\eta_{CS} + b_{NS} = \frac{b_{NC}d_{CS} + b_{NS}(\phi_S - d_{CC})}{\eta_{NN} - b_{NN} + \phi_S - d_{CC}}.
\end{aligned}
$$