

Dynamics of Westminster Parliamentarism¹

by

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Abstract: We model the strategic interaction of two parties and a voter in a parliamentary system of government as a dynamic game of incomplete information. The parties' preferred policy is private information. A party's type (moderate or extreme) changes with positive probability following defeat in election, and partisan types are positively serially correlated. We study symmetric semi-Markovian Perfect Bayesian Nash equilibria. If parties assign high weight on office vs policy, extreme party types (when in control of the party) pursue extreme policies with positive probability only when the party is perceived less moderate than the opposition. Extreme policies occur in equilibrium when (a) both parties are perceived to be relatively extreme, and (b) neither party holds a significant advantage regarding its perceived extremism by the electorate. Equilibrium dynamics produce two qualitatively different adjustment paths. Either there is positive probability of extreme policies in the future for a protracted period of time; or there is zero probability of extreme policies along the equilibrium adjustment path. Thus, two-party parliamentary systems may display long phases of (relative) extremism or moderation. In the long-run, parties' perceived extremism converges to levels that guarantee moderate policies with probability one.

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“Butskelism’ is the, moderately satirical, term used in British politics to refer to the political consensus formed in the 1950s and associated with the exercise of office as Chancellor of the Exchequer by Rab Butler and Hugh Gaitskell.⁴

... The consensus dominated British politics until 1979 when the administration of Margaret Thatcher radically challenged accepted wisdom and institutionalized a greater emphasis on a free market approach to government.”⁵

1 Introduction

Few propositions in political theory combine the simplicity and appeal of Anthony Downs’ (1957) convergence to the median conclusion for two-party/candidate electoral competition in one dimension.⁶ Downs modeled parties or representatives that are purely motivated by the attainment of office, an assumption that allowed him (and the electorate) to equate post-election representative behavior to the fulfillment of a pre-election promise or platform. Given a pair of platforms in Downs’ world, the one closer to the median emerges victorious. Hence, the only equilibrium can arise at a pair of platforms set at the ideal policy of the median.

The ‘convergence to the median’ conclusion has left scores of graduate students gratified on the discipline’s level of intellectual attainment and generations of undergraduates leaving lecture rooms determined to communicate the exciting new insight to the outside world. Of course, many of these same students (and perhaps the reader) contemplated the pursuit of elective office *and*, on the same time, held ideal public policies quite likely different from those of the ‘median.’ If we admit that, no matter how strong the ambition for office, representatives also have genuine policy preferences, then the idyllic world of Downsian politics is disturbed.

To boot, in the one-period model of Downs, a representative or party has no incentive to deliver on pre-election platform promises once elected in office. This problem can be overcome in a model with a sufficient future horizon, assuming voters can maintain their focus and punish parties or candidates for not delivering on election promises. But in such models a lot of policy paths, not just repetitions of the median

⁴Author’s clarification: of the pre-1951 Labour and post 1951 Conservative governments, respectively. Butskelism from *Butler and Gaitskell*.

⁵Source: Wikipedia, The free Encyclopeida, <http://en.wikipedia.org/wiki/Butskelism>.

⁶For a recent review of this literature to date, see Duggan, 2004.

policy, obtain. For example, Duggan and Fey, 2004, demonstrate complete indeterminacy of equilibrium outcomes in a repeated elections setting, in fact assuming Downsian candidates/parties. Furthermore, historically, numerous parties/candidates that implemented policies quite distant from the median have moved on to win (consecutive) elections. Examples include the Conservative Margaret Thatcher's record-setting streak in the premiership of the UK, or George Bush's re-election to the US presidency in 2004.

Such examples can be attributed to electoral noise or 'probabilistic voting,' a reasonable assumption about the often capricious nature of elections. If we combine policy motivated candidates with probabilistic voting, we get equilibrium policies that are away from the median (Wittman, 1983).⁷ For that reason, Wittman's result has been welcomed by a number of scholars. It produces (possibly mild) policy differences among competing parties, so that 'politics matters' in equilibrium. Furthermore, it rationalizes data that seem to suggest that policies by competing parties/candidates are not identical.

Yet, important questions remain unanswered. On the theoretical front, Wittman assumes common knowledge of candidate preferences and maintains pre-election platforms that are delivered with disturbing precision. In reality, the true policy preferences of representatives are private information. Thus, both platform declarations or even past policies may be strategic choices to please or deceive the electorate regarding the party's/candidate's true intentions. Furthermore, both models produce trivial electoral dynamics in that they display levels of convergence or divergence that are constant over time.

Empirical observation suggests otherwise, as the introductory quotation illustrates. Indeed, Downsian convergence seems to be a fair approximation of the world of British politics in the 50's and 60's. But, by the late 70's and 80's, few can credibly make that claim. Importantly, the Downsian approximation is inaccurate both in terms of delivered policies, but also in spirit. For the most part, Margaret Thatcher's Conservatives made no effort to conceal their ideology, or pretend that their intentions were different than the actual policies of the time. Even when opponents attempted to appear more moderate, they were unsuccessful. In particular, in consecutive elections the Labour party attempted to present an ideologically reformed, more moderate facade, but these attempts (at least *ex post*) seem to have been discounted by the electorate which became convinced of the reform only recently. Today, after two decades of perceived and actual divergence, British politics seem to be once more close to the Downsian ideal.

Our goal in this paper is to develop a model of two-party competition that can account for this pattern

⁷Roemer 1999, 2000 proposes a model of party competition in multiple issue dimensions that is premised on the idea that disagreement within parties generates party competition equilibria when none of the parties can unanimously improve on their own platform given the platform of the opposition.

of dynamics in which protracted phases of divergent and convergent politics succeed one another. Unlike Downs or Wittman, we assume pre-election commitments are impossible and dispense with pre-election policy announcements. Instead, we assume that parties' true policy preferences are private information that is (possibly) revealed to the electorate through policy consequential choices while in government. Thus, our study is related to recent developments in electoral models of incomplete information such as those by Banks and Sundaram, 1993, 1998, Duggan, 2000, and Banks and Duggan, 2002.⁸

We differ from the above studies in that we do not assume that the incumbent in each period faces a challenger drawn from a common distribution. The assumption of an opponent drawn from a stationary distribution seems appropriate for individual candidacies, if we think of new challengers drawn from a common or identical pool of possible candidates. But such an assumption is particularly problematic when considering competition between political parties. This is because parties are collective organizations, and all organizations display inertia. As a result, if the electorate obtains new information about the prevalent ideological preferences of a party by observing its policy while this party is in government, then this information is likely to weigh on the voter's assessment about the ideological preferences of that party in future periods. We formalize this idea by assuming that following electoral defeat parties undergo an internal change that stochastically determines new partisan preferences, and that these new preferences are positively serially correlated.

In the model, the two parties and a median voter interact an infinity of periods. Aside from their policy preferences, parties also care about getting control of government. The voter's payoff depends on implemented policy following the election. Parties are more farsighted than the voter in that their payoff depends on the electoral and policy outcomes of two successive periods.

In each period the voter chooses one of the two parties, and may condition her choice on the rationally updated equilibrium beliefs about the extremism of the two parties, as well as on the policy implemented by the incumbent party prior to the election. Political parties/types condition their choice of policy while in government on the perception of the electorate about the extremism of the two competing parties.

We study symmetric semi-Markov Perfect Bayesian Nash equilibria. We show that if parties are impatient or place significant emphasis on office vs. policy, the only equilibrium involves party types implementing their ideal policy independent of the electorate's beliefs about the two parties. We also

⁸Other models of incomplete information focus on the fact that the incumbent's action while in office is unobserved (hidden action). Such models include Ferejohn, 1986, Rogoff and Sibert, 1988, Rogoff, 1990, and Meiwowitz, 2003. Banks and Sundaram, 1993, combine aspects of both models.

show that there does not exist a robust equilibrium in which the opposite holds and all parties implement moderate policies independent of beliefs. Thus, our setup does not sustain Downs' convergence to the median conclusion, no matter how strong parties' preference for office.

In the case when parties assign high weight on office utility vs policy, they moderate their policy choice when perceived less extreme compared to the opposition, and pursue extreme policies with positive probability when the opposite is true. Extreme policies occur in equilibrium when (a) both parties are perceived to be relatively extreme, and (b) neither party holds a significant advantage regarding its perceived extremism by the electorate. Equilibrium dynamics produce two qualitatively different adjustment paths. Either there is positive probability of extreme policies in the future for a protracted period of time, when beliefs about the two parties' extremism are above their long term steady state; or there is zero probability of future extreme policies in the opposite case. Thus, two-party parliamentary systems may display (relative) extremism or moderation in protracted periods. In the long-run parties' perceived extremism converges to levels that guarantee moderate policies with probability one.

Our presentation proceeds as follows. In section 2 we describe the model in detail and define the equilibrium concept used in our analysis. This analysis takes place in sections 3 and 4. Section 4 contains the main results of the paper concerning equilibrium in which parties value office more than policy. We characterize an equilibrium (proposition 3), discuss equilibrium properties (proposition 4), and equilibrium dynamics (proposition 5). In section 5 we extend our analysis to the case of probabilistic voting. We conclude in section 6.

2 Model

The game is played between the electorate represented by a moderate or median voter, M , and a set of partisan 'types' within each of two political parties. These players interact an infinity of periods $t = 1, 2, \dots$. We denote a generic party by P , which is either a left-wing party ($P = L$), or a right-wing party, ($P = R$). Each of the two parties contains individuals with two different ideological convictions, call them *moderates* and *extremists*. These two groups/types disagree as to the optimal government policy. In each period one of the two groups holds the prevailing ideological position of the party. Thus, in period t party $P \in \{L, R\}$ is either an extreme type, e , or a moderate type, m . We denote party P 's type in period t by τ_P^t with $\tau_P^t \in \{e, m\}$.

We assume that the type of the party varies over time because of the stochastic outcome of some internal battle between the groups of moderates and extremists for positions of influence within the party (MP candidacies, local and national party organization positions, union representation, lack of competent leadership of a group of faction despite arithmetic prevalence, etc.). Because of inertia in the manner in which partisan populations evolve, or due to the fact that the prevailing ideological group in the party commands resources and/or other institutional advantages, we assume that the prevailing type within the party is better positioned to fight the internal battle for control of the party in the next period. Thus, the type of the party is positively serially correlated (inequality (2) below). Furthermore, for similar reasons having to do with the control of resources used to influence within-party political battles, the prevalent partisan type is better able to maintain control of the party when elected in government rather than when in opposition (inequality (1) below).

Formally, we assume that partisan types τ_P^t follow a Markov chain the transition probabilities of which depend on whether the party is in government or not. Specifically, if the party's type is $\tau \in \{e, m\}$ in period t , it is of the same type in period $t + 1$, with probability π_τ^g or π_τ^o depending on whether the party is in government or in the opposition, respectively. In general, we assume

$$\pi_\tau^g > \pi_\tau^o, \tau \in \{e, m\}. \quad (1)$$

We also assume that

$$\pi_e^o > 1 - \pi_m^o \quad (2)$$

so that the probability of an extreme party type is higher if the party's type in the previous period is extreme. Finally, since (with the important exception of the discussion in subsection 5.2 ***to be added) our arguments do not depend on the exact values of π_τ^g , we set $\pi_e^g = \pi_m^g = 1$ in order to simplify the algebra.

Parties know the realization of their own type in each period, but that information is not revealed to other players except via policy consequential choices of the party/type while in government. Players hold (and rationally update) beliefs about the probability that each party is moderate or extreme. In particular, at each stage in the game there is a pair of probabilities $\mathbf{b} = (b_L, b_R) \in \mathcal{B}$, where $\mathcal{B} \equiv [0, 1]^2$, that represent the common beliefs of the voter about the two parties and of the parties for each other. Thus, probability b_L represents the belief of M , (and party R) that party L is extreme. Similarly, b_R is the corresponding belief that party R is of type e .

The voter and parties interact as follows. In each period, the voter elects one of the two parties to control the government. Following the election, nature chooses the type of the party according to the transition probabilities in (1). Then the party/type in government chooses and implements a policy $x^t \in X$. In general, there are four possible policies in each period, a left-wing policy, x_e^L , a moderate left-wing policy, x_m^L , and corresponding right-wing policies x_e^R and x_m^R . As will become evident by our assumptions on players' payoffs, we do not preclude the possibility that $x_m^L = x_m^R$ is a common policy. This permits a 'convergence to the median' equilibrium to occur. But we allow $x_m^L \neq x_m^R$, *i.e.* there may exist residual partisanship even if the moderates are the prevailing group within each party. In summary, we have $X = \{x_e^L, x_m^L, x_m^R, x_e^R\}$.

When it comes to the choice of government policies, we assume (naturally) that moderate types always implement the moderate policy x_m^P .⁹ The strategic burden in the model is born by the extreme partisan types. In particular, type $\tau_P^t = e$ may choose either x_e^P or x_m^P . The policy choice by the governing party is observed by all players and the game moves to the next period. In that period, the voter elects a new government, new partisan types are realized, the governing party implements a policy, etc.

Preferences Since moderate partisan types always pursue the same action,¹⁰ we only need state payoffs for the voter and the two extreme partisan types (left and right). The preferences of these players over policies in X are summarized by the following within period (or stage) payoffs:

	Payoff from policy:			
	x_e^L	x_m^L	x_m^R	x_e^R
voter M	v_e^L	v_m^L	v_m^R	v_e^R
Type e of Party L	u_e^L	u_m^L	a_m^L	a_e^L
Type e of Party R	a_e^R	a_m^R	u_m^R	u_e^R

We assume that $v_m^L = v_m^R > v_e^L = v_e^R$, *i.e.* the voter prefers moderate policies and parties are symmetrically located in each direction from the voter. For extreme partisan types $\tau^P = e$, we assume $u_e^P > u_m^P \geq a_m^P > a_e^P$, $P \in \{L, R\}$. The above preferences coincide with the intuitive interpretation of the different types: extremists of each party prefer the respective partisan policy most, moderate policies next, and they least

⁹Indeed, this is the behavior that would arise endogenously in an equilibrium of the type we characterize.

¹⁰In particular, this would be an optimal behavior in equilibrium for these types, even if they were allowed to choose among policies.

prefer the partisan policy of the other party. To preserve the symmetry of the game, we set $u_\tau^L = u_\tau^R$ and $a_\tau^L = a_\tau^R$, $\tau \in \{e, m\}$. A graphic rendition of admissible configurations of policies in the classical one-dimensional spatial model is given in figure 1.

[insert figure 1 about here]

While the voter only cares about the policy outcome, parties also prefer to control the government independent of the policy that the government pursues. In particular, partisan types receive utility $G \geq 0$ when their *party* (*i.e.* independent of prevailing party type) is in government. We assume that the voter is strategic but cares only about the policy outcome in the current period. Partisan types are (potentially) more farsighted and care about the electoral and policy outcome in two periods, the current period t as well as period $t + 1$. We parameterize the weight parties place in the outcome of the next period by assuming they discount that period's payoff by a factor $\delta \in [0, 1]$.

Strategies We shall focus our attention on Perfect Bayesian Nash equilibria in strategies that are appropriately Markovian, *i.e.* strategies that depend only on a summary of the history of the game in each period. Given the structure of the model, the payoff relevant strategic environment for the players is summarized by the commonly known beliefs about the probability that extremists prevail within the two parties, $\mathbf{b} \in \mathcal{B}$. Thus, a strategy for type e of party P is given by a function:

$$\sigma^P : \mathcal{B} \longrightarrow [0, 1], P \in \{L, R\}. \quad (3)$$

Accordingly, $\sigma^P(\mathbf{b})$ is the probability that type e of party P implements policy x_e^P .

In principle, we could similarly restrict the voter, M , to pursue a Markovian strategy that depends only on beliefs $\mathbf{b} \in \mathcal{B}$. Instead, we allow the voter's strategy to also depend on the policy choice of the party in government in the period prior to the election.¹¹ This allows us to build a retrospective element on voter's strategies, even though the voter is still prospective and strategic. Furthermore, this type of history dependence is necessary for existence of equilibrium. Thus, a voter strategy is given by a function

$$\Phi : X \times \mathcal{B} \longrightarrow [0, 1]. \quad (4)$$

Now, $\Phi(x^{t-1}, \mathbf{b}) = 1$ means voter elects party L in government in period t , while $\Phi(x^{t-1}, \mathbf{b}) = 0$ means a right-wing party is elected in government. $\Phi(x^{t-1}, \mathbf{b}) \in (0, 1)$ means the voter randomizes accordingly.

¹¹This is in addition to the indirect effect that these policies have on the voter's beliefs. In other words, the voter may choose a different voting action following two different policies, even if these two policies lead to the same posterior beliefs.

Evolution of Beliefs Players apply Bayes' rule to update their beliefs $(b_L, b_R) \in \mathcal{B}$ regarding partisan types. Specifically, if party P implements an extreme policy, it reveals its type and the updated belief about the probability the party is extreme is given by:

$$\beta^e(b_P) = 1, P \in \{L, R\}. \quad (5)$$

When observing a moderate policy, updated beliefs depend on the equilibrium probability with which party P chooses such a policy. If the probability of choosing an extreme policy is given by σ , then updated beliefs following a moderate policy are given by

$$\beta^m(\sigma, b_P) = \frac{(1 - \sigma)b_P}{1 - \sigma b_P}, P \in \{L, R\}. \quad (6)$$

Note that $\beta^m(0, b_P) = b_P$, *i.e.* beliefs remain unchanged if the party chooses an extreme policy with probability $\sigma = 0$. Similarly, we have $\beta^m(1, b_P) = 0$ if $\sigma = 1$.¹² No information is obtained for the party that is in opposition so that beliefs about this party remain unchanged.

Finally, according to our assumptions regarding the evolution of party types following elections, beliefs about party P that loses an election are given by

$$\beta^o(b_P) = \pi_e^o b_P + (1 - \pi_m^o)(1 - b_P), P \in \{L, R\} \quad (7)$$

Due to our assumption that $\pi_e^g = \pi_m^g = 1$, the corresponding beliefs about the winner of the election are obtained as

$$\beta^g(b_P) = \pi_e^g b_P + (1 - \pi_m^g)(1 - b_P) = b_P, P \in \{L, R\} \quad (8)$$

[insert figure 2 about here]

In our discussion of equilibrium dynamics, we will use the long-term steady state of the Markov chain induced by the internal re-structuring in the party following an electoral defeat. This long-term probability of the party being extreme is given by

$$b^o = \frac{1 - \pi_m^o}{2 - \pi_e^o - \pi_m^o} \quad (9)$$

and satisfies $\beta^o(b^o) = b^o$.¹³ It is straightforward to verify using (2) that

$$b_P > \beta^o(b_P) > b^o \iff b_P > b^o \quad (10)$$

¹²Even when $b_P = 1$.

¹³An analogous (unique) expression can be derived for the chain defined by the transition probabilities π_τ^g , if we relax the assumption that $\pi_\tau^g = 1$.

i.e. a party with a perceived extremism above (below) the long-term steady state is moving monotonically toward that steady state from either direction. In figure 2 we depict the evolution of beliefs about the two parties following an election in which either party wins (loses) the election. Note that the direction and magnitude of change in beliefs differs with the direction and distance of beliefs at the time of the election from the long-term steady state b^o .

Expected payoffs Given strategies $\Phi, \sigma^P, P \in \{L, R\}$, we may now derive expressions for players' expected payoffs. In particular, the expected utility of the voter M from choosing party P when beliefs are given by $\mathbf{b} \in \mathcal{B}$, is given by

$$V^P(\mathbf{b}) = \begin{cases} b_L \sigma^L(b_L, \beta^o(b_R)) (v_e^L - v_m^L) + v_m^L & \text{if } P = L \\ b_R \sigma^R(\beta^o(b_L), b_R) (v_e^R - v_m^R) + v_m^R & \text{if } P = R \end{cases} \quad (11)$$

Notice that (11) reflects the fact that, following an election, the losing party undergoes an internal shake up as determined by (7).

Likewise, the expected payoff of type e of party L from implementing a moderate policy is given by:

$$U_m^L(\mathbf{b}) = u_m^L + G \quad (12)$$

$$+ \delta \left(\begin{array}{l} \Phi(x_m^L, b'_L, b_R) [\sigma^L(b'_L, \beta^o(b_R)) (u_e^L - u_m^L) + u_m^L + G] \\ + (1 - \Phi(x_m^L, b'_L, b_R)) [b_R \sigma^R(\beta^o(b'_L), b_R) (a_e^L - a_m^L) + a_m^L] \end{array} \right)$$

where $b'_L = \beta^m(\sigma^L(\mathbf{b}), b_L)$. On the other hand, by implementing an extreme policy the party expects:

$$U_e^L(\mathbf{b}) = u_e^L + G \quad (13)$$

$$+ \delta \left(\begin{array}{l} \Phi(x_e^L, 1, b_R) [\sigma^L(1, \beta^o(b_R)) (u_e^L - u_m^L) + u_m^L + G] \\ + (1 - \Phi(x_e^L, 1, b_R)) [b_R \sigma^R(\beta^o(1), b_R) (a_e^L - a_m^L) + a_m^L] \end{array} \right)$$

The corresponding expressions for party R are obtained in an analogous fashion.

Equilibrium Concept With the above we can state the definition of our equilibrium concept as follows:

Definition 1 An equilibrium is a set of party strategies $\sigma^{P*}, P \in \{L, R\}$, and a voting strategy Φ^* such

that:

$$\Phi^*(x, \mathbf{b}) \begin{cases} = 1 & \text{if } V^L(\mathbf{b}) > V^R(\mathbf{b}) \\ \in [0, 1] & \text{if } V^L(\mathbf{b}) = V^R(\mathbf{b}) \\ = 0 & \text{if } V^L(\mathbf{b}) < V^R(\mathbf{b}) \end{cases}, \text{ for all } \mathbf{b} \in \mathcal{B} \quad (14)$$

and

$$\sigma^{P*}(\mathbf{b}) \begin{cases} = 1 & \text{if } U_e^P(\mathbf{b}) > U_m^P(\mathbf{b}) \\ \in [0, 1] & \text{if } U_e^P(\mathbf{b}) = U_m^P(\mathbf{b}) \\ = 0 & \text{if } U_e^P(\mathbf{b}) < U_m^P(\mathbf{b}) \end{cases}, \text{ for all } \mathbf{b} \in \mathcal{B}, P \in \{L, R\}. \quad (15)$$

Note that this equilibrium is semi-Markov Perfect Bayesian Nash.¹⁴ It shies away from being Markov Perfect Bayesian Nash because we allow the voter to condition her strategy on the policy that prevailed in the last period. This is a relatively mild deviation from Markovian strategies.

The history dependence of voting strategies allows us to incorporate a retrospective element on voting behavior. In particular, we say that a voting strategy is *retrospective* if the voter does not re-elect a party that pursued an extreme policy in the last period. Accordingly, we define a retrospective equilibrium.

Definition 2 *An equilibrium is retrospective if the voting strategy satisfies*

$$\Phi^*(x_e^P, \mathbf{b}) = \begin{cases} 1 & \text{if } P = R \\ 0 & \text{if } P = L \end{cases}, \text{ for all } \mathbf{b} \in \mathcal{B}. \quad (16)$$

Since parties only care about one future period, a retrospective voting strategy gives parties a strong incentive to pursue moderate policies. Thus, equilibria in which parties pursue extreme policies are significantly more credible when they are retrospective equilibria. We emphasize that we do not assume retrospective voting as a “hard-wired” behavioral trait of the electorate, *i.e.* voter’s strategy must still satisfy equilibrium condition (14) when retrospective. In other words, retrospective voting constitutes a best response if present in a retrospective equilibrium.

Our equilibrium definition leaves room for a further refinement on voting strategies. To motivate this refinement, note that when indifferent between the two parties the voter is allowed to randomize in an arbitrary fashion in choosing between these parties. This is not controversial in our symmetric setup if the beliefs about the two parties are identical. But if the beliefs about the two parties diverge, it seems intuitive that the voter may weakly favor the party that is perceived to be more moderate. This becomes

¹⁴Technically, it is a refined Perfect Bayesian Nash equilibrium because in the outset we resolve the question of possible out of equilibrium beliefs by setting $\beta^e(b_P) = 1$ and $\beta^m(1, 1) = 0$.

obvious if parties pursue pure strategies. In that case, the indifference of the voter is not robust to the possibility of a (small) exogenous probability $\varepsilon > 0$ that extreme types may ‘tremble’ and choose a policy different than the one intended. Thus we define a robust equilibrium as follows:

Definition 3 *An equilibrium is robust if there exists an $\bar{\varepsilon} \in (0, \frac{1}{2})$ such that for each $\varepsilon, \bar{\varepsilon} > \varepsilon > 0$, the voting strategy $\Phi^*(x, \mathbf{b})$ is a best response when party strategies are perturbed according to*

$$\sigma_{\varepsilon}^{P^*}(\mathbf{b}) = \begin{cases} 1 - \varepsilon & \text{if } \sigma^{P^*}(\mathbf{b}) > 1 - \varepsilon \\ \varepsilon & \text{if } \sigma^{P^*}(\mathbf{b}) < \varepsilon \\ \sigma^{P^*}(\mathbf{b}) & \text{otherwise} \end{cases}$$

A few remarks are in order concerning robust equilibria. First, there exists an obvious connection between our requirement and standard refinement arguments dating to Selten’s trembling hand perfect notion. It is important to emphasize that the concepts are also different. In particular, we consider one among a (very) large range of possible perturbations of partisan strategies. Furthermore, we do not consider the consequences of such perturbations on the optimality or robustness of parties’ strategies, even though that is an obvious avenue to pursue. Our goal with a robust equilibrium is more limited in that we simply seek to resolve the electorate’s indifference in a manner that is responsive to its beliefs about the relative extremism of the two parties.

There is a more direct (and apparently more restrictive) manner to impose such a refinement. In particular, it seems intuitive in our setup to conjecture that parties are *weakly* preferred by the voter when they are perceived to be less extreme than the opposition. Thus, if we require this intuitive property *and* resolve indifference in favor of the least extreme party, we may define an intuitive equilibrium as follows:

Definition 4 *An equilibrium is intuitive if the voting strategy satisfies*

$$\Phi^*(x, \mathbf{b}) = \begin{cases} 1 & \text{if } b_L < b_R \\ 0 & \text{if } b_L > b_R \end{cases} \quad (17)$$

for all $x \in X$.

Note that intuitive equilibrium implies retrospective equilibrium behavior whenever beliefs about the opposition are given by $b_{-P} < 1$ (because the posterior belief following an extreme policy is given by $\beta^e(b_P) = 1 > b_{-P}$), but condition (16) is not implied by condition (17). Thus, we will refer to an equilibrium as an *intuitive retrospective* equilibrium if the voter’s strategy satisfies both (16) and (17).

A second remark is that, in effect, condition (17) renders the surviving equilibria closer to genuine Markov Perfect equilibria. In particular, the voter is limited to (possibly) condition her action on past policy choices only in a set of payoff relevant states $\mathbf{b} \in \mathcal{B}$ such that $b_L = b_R$. Thus, an intuitive equilibrium involves voting strategies that are Markovian, except for a set of payoff relevant states $\mathbf{b} \in \mathcal{B}$ of measure zero.

In the next two sections we proceed to an analysis of the game. First we consider the analogues of ‘pooling’ and ‘separating’ equilibria in our dynamic game. In such equilibria, extreme partisan types pursue the same ‘pure’ action independent of the state $\mathbf{b} \in \mathcal{B}$, hence we call these equilibria simple. Our main results appear in section 4, where we consider robust retrospective equilibria that are not simple and involve parties that place high weight in office (high G) and in the future (high δ).

3 Simple Equilibria

Naturally, the primary focus of our analysis is in the dynamics induced by the strategic calculus of extreme partisan types when they contemplate the trade-off between a (preferable) extreme policy in the current period and the possible utility loss in the next period due to averse electoral consequences. In particular, we are interested in the range of the state space (the set of beliefs held by the electorate) in which the extreme partisan types pursue extreme policies or try to emulate moderate types (or not), and the policy dynamics these strategies generate.

Before we move to this more interesting analysis, we consider two simple types of equilibria in which parties’ strategy does not depend on the state of beliefs $\mathbf{b} \in \mathcal{B}$. First, in proposition 1, we give a precise range of parameters in which extreme partisan types implement extreme policies whenever in power, independent of beliefs $\mathbf{b} \in \mathcal{B}$. We have:

Proposition 1 *A robust retrospective equilibrium with $\sigma^P(\mathbf{b}) = 1$, for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$, exists if and only if*

$$\delta \leq \frac{u_e^P - u_m^P}{G + u_e^P - a_e^P} \quad (18)$$

This equilibrium is intuitive. Furthermore, when the inequality is strict, in all equilibria $\sigma^P(\mathbf{b}) = 1$, for all $\mathbf{b} \in \mathcal{B}$.

Proof. See the Appendix. ■

Note that (except for the case of equality) when condition (18) holds all equilibria of the game involve extreme partisan types pursuing their ideal policy. Thus retrospective voting is not sufficient to induce moderation when either (a) parties are impatient (low δ), or (b) parties place low value to office (low G), or (c) the loss in utility due to the policies pursued by the opposition party controlling the government is small (low $u_e^P - a_e^P$).

One may conjecture that when these conditions are reversed we may instead obtain a simple ‘pooling’ equilibrium in which extreme partisan types always pursue a moderate policy. It is possible to construct such equilibria (for high enough G & δ) exploiting voters’ indifference, but these equilibria are not robust. Indeed we can show that there does not exist a robust retrospective ‘pooling’ equilibrium:

Proposition 2 *There does not exist a robust retrospective equilibrium such that $\sigma^P(\mathbf{b}) = 0$, for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$.*

Proof. See the Appendix. ■

Thus the analogue to a ‘convergence to the median’ result is not attainable in our game in a robust equilibrium, despite retrospective voting. The reasoning behind proposition 2 is straightforward. If all party types moderate policies independent of the electorate’s beliefs, then the electorate is indifferent between the two parties. In a robust equilibrium, the voters then will elect that between the two parties that is perceived to be more moderate. Thus, a party that is in government, is controlled by extremists, and is perceived to be more extreme than the opposition, has no incentive to pursue a moderate policy. *This party faces electoral defeat independent of policy choice, so types in control of the party might as well pursue their ideal policy.*

Thus, in combination propositions 1 and 2 imply that when condition (18) fails, a robust retrospective equilibrium must involve some positive probability of moderate policies pursued by extreme types who anticipate a future electoral gain from doing so, as well as some positive probability of extreme policies pursued by these types. We take the analysis of such more interesting equilibria in the next section.

4 Equilibrium with Office Motivated Parties

Proposition 1 demonstrates that the strategic calculus of parties is trivial when partisans are primarily motivated by policy. Such parties/types simply pursue their ideal policy. Thus, the interesting strategic environment is one in which parties value office significantly compared to policy and are patient. Three

questions emerge in such an environment: (a) Does there exist a robust retrospective equilibrium in which extreme party types pursue extreme policies for some beliefs? (b) Are extreme policies observed along the equilibrium path?, and (c) What are the policy and electoral dynamics that prevail? In what follows, we answer question (a), (b), and (c) in subsections 4.1, 4.2, and 4.3 respectively.

4.1 Robust Equilibrium

Our goal in this section is to establish a robust retrospective equilibrium when condition (18) fails, and parties are sufficiently patient and motivated predominantly by office considerations (high G). Proposition 3 establishes such an equilibrium:

Proposition 3 *Assume*

$$\delta > \frac{u_e^L - u_m^L}{G + u_m^L - a_m^L}. \quad (19)$$

There exists a unique intuitive retrospective equilibrium with

$$\sigma^L(b_L, b_R) = \sigma^R(b_R, b_L) = \begin{cases} \frac{(b_L - b_R)}{b_L(1 - b_R)} & \text{if } b_L > b_R \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

This equilibrium is robust.

Proof. See the Appendix ■

Note that parties' equilibrium mixing probabilities are independent of G , δ , or of the players' payoffs u_τ^P , a_τ^P , v_τ^P , $P \in \{L, R\}$, $\tau \in \{e, m\}$. Furthermore, the equilibrium in proposition 3 holds for arbitrarily large values of G , as long as parties place some weight in the future ($\delta > 0$). Thus, no matter how office oriented parties are, there exists a configuration of beliefs by the electorate about the extremism of the two parties that makes it worthwhile for extreme partisan types to pursue extreme policies. This occurs when the party is disadvantaged electorally. Figure 3(a) displays the equilibrium probability of an extreme policy choice by extreme partisans of party L , $\sigma^L(\mathbf{b})$.

[insert figure 3 about here]

From the perspective of the electorate, the expected probability that, say, party L will pursue an extreme policy given beliefs $\mathbf{b} \in \mathcal{B}$, is given by $b_L \sigma^L(\mathbf{b})$. In figure 3(b) we plot this probability. In both cases of figure 3(a) and 3(b) it is straightforward to verify via calculus or visual inspection that

the probability of an extreme policy increases when the party is perceived to be more extreme than it's opposition.

To see why party L mixes when $b_L > b_R$, observe that the party loses the election if it pursues a moderate policy with probability one. This is because in that case (if $\sigma^L(\mathbf{b})$ was set to zero), the policy of the government conveys no information to the electorate and other players in the game. Thus, following a moderate policy it is still the case that posterior beliefs satisfy $b_L > b_R$, and party L still loses the election. Thus, pursuing a moderate policy with probability one is not an equilibrium.

What if the party implements an extreme policy with probability one ($\sigma^L(\mathbf{b}) = 1$)? Then, since the electorate expects party types to 'separate' the party has an incentive to deviate and implement a moderate policy instead. This would convince the voter that the party is moderate, when in fact it is extreme. Thus, the only possibility for an equilibrium is a mixed strategy, where the mixture probability is such that it makes the party barely competitive against its opponent when the realization of the party's randomization is a moderate policy.

Finally, when the party has an electoral advantage, it has no incentive to spoil its electoral prospects by implementing an extreme policy. In particular, the party wins the election whenever it sets a moderate policy. Thus, given that partisan types care sufficiently about office, the only equilibrium choice is to set $\sigma^L(\mathbf{b}) = 0$, *i.e.* chooses a moderate policy with probability one.

4.2 Extreme Policies Along the Equilibrium Path

The fact that extreme partisan types pursue their ideal policy with positive probability ($\sigma^P(\mathbf{b}) > 0$) for some beliefs $\mathbf{b} \in \mathcal{B}$ in proposition 3 is not sufficient to produce extreme policies along the equilibrium path. This is because these types pursue extreme policies only when the opposition party is perceived more moderate. But, parties that are perceived less moderate are not elected in government in the first place. In other words, along the equilibrium path, the probability that an extreme policy is observed is regulated via appropriate screening from the electorate.

Do we obtain extreme policies along the equilibrium path despite this screening by the electorate? The answer is in the affirmative and our analysis provides a precise mechanism for this to occur. Extreme policies are observed in equilibrium following elections in which: (a) both parties are perceived to be extreme (above their long-term level of extremism), and (b) the election is 'close'. Specifically, the set of beliefs at the election stage from which extreme policies are expected with positive probability is defined

as follows:

$$\tilde{\mathcal{B}} \equiv \{\mathbf{b} \in \mathcal{B} : b_L > \beta^o(b_R), b_R > \beta^o(b_L)\} \quad (21)$$

It is straightforward to verify that $\mathbf{b} \in \tilde{\mathcal{B}}$ implies both $b_L, b_R > b^o$.

The reason why extreme policies occur in this area of the space of possible beliefs involves a ‘*regression to the mean*’ effect. Parties that are perceived relatively more extreme and barely lose the election to the opposition undergo internal changes, or reforms following their electoral defeat with higher probability (assumption (1)). Because these parties’ perceived extremism is above their long-term equilibrium level, this internal shake up moves the party towards moderation closer to its long term perceived extremism (by condition (10)). As a result, a government that comes to power with a bare advantage, is perceived more extreme than the opposition immediately following the election. In these cases the government may pursue extreme policies. The area in which this is possible is depicted graphically in figure 4 for different values of the transition probabilities π_e^o and π_m^o .

[insert figure 4 about here]

We summarize our discussion of the equilibrium in the following proposition:¹⁵

Proposition 4 *The equilibrium in proposition 3 is such that:*

(a) *The (expected) probability that party L implements an extreme policy weakly increases with b_L , and weakly decreases with b_R ,*

(b) *The expected probability of an extreme policy following an election with beliefs $\mathbf{b} \in \mathcal{B}$, is weakly increasing with b_L when $b_L < b_R$, and is weakly decreasing with b_L when $b_L > b_R$. It is positive if and only if $\mathbf{b} \in \tilde{\mathcal{B}}$.*

We conclude our analysis in this section by considering the dynamics of beliefs and policies induced by the equilibrium in proposition 3.

4.3 Equilibrium Dynamics

Starting from any belief level $\mathbf{b} \in \mathcal{B}$, beliefs evolve over time via Bayes’ rule following government’s policy, and via the electorate’s anticipation of internal restructuring within parties that lose the election. It is

¹⁵The proof of this and the following proposition are straightforward and are omitted.

straightforward to verify that equilibrium beliefs remain unchanged if for some reason the system rests at belief points $(b_L, b^\circ) \in \mathcal{B}$ with $b_L \leq b^\circ$ and party L is in government¹⁶. In these cases the party in government is pursuing a moderate policy with probability one, and there are no changes in beliefs regarding the opposition because the opposition is already at its long-run level of beliefs.

Indeed, the equilibrium in proposition 3 is such that with probability one the political system is absorbed at one of these belief points, without any forces inducing a change in beliefs after that. At all these possible absorbing points, there is probability zero of an extreme policy. Both the eventual absorbing belief point and the path that leads to that point differ qualitatively depending on initial conditions. We summarize these dynamics in proposition 5:

Proposition 5 *The equilibrium in proposition 3 is such that when initial beliefs are $(b_L, b_R) \in \mathcal{B}$ and party L is in government:*

(a) *If $b_L \leq b_R$ and $b_L \leq b^\circ$ the system is absorbed at $(b_L, b^\circ) \in \mathcal{B}$ with probability one and there is zero probability of an extreme policy along the path of play,*

(b) *If $b_L > b_R$ and $b_R \leq b^\circ$ the system is absorbed at $(b_R, b^\circ) \in \mathcal{B}$ with probability $p = (1 - \sigma^L(b_L, b_R)) \Phi(x_m^L, b_R, b_R)$, or at (b°, b_R) with probability $1 - p$; there is probability $\sigma^L(b_L, b_R)$ of an extreme policy in the first period, and zero in the subsequent path of play, and*

(c) *If $b_L, b_R > b^\circ$ the system is absorbed at $(b^\circ, b^\circ) \in \mathcal{B}$ with probability one; for any point along the path of play, there is positive probability of an extreme policy in future periods and set $\tilde{\mathcal{B}}$ is visited infinitely often.*

The dynamics described in proposition 5 are illustrated graphically in figure 5. Cases (a) and (b) are very similar in that following the first election in these cases, the party that is elected in government is guaranteed to be perceived more moderate than the opposition. As a result, the government always implements a moderate policy and is re-elected with probability one. This process continues until players' beliefs about the extremism of the opposition party reach the long-run steady state b° given in (9).

[insert figure 5 about here]

The situation is much different when both parties are perceived to be above their long-term steady state level of extremism, b° . In these cases, we have one of two possibilities. Either the party in government

¹⁶Or, symmetrically if $(b^\circ, b_R) \in \mathcal{B}$ with $b_R \leq b^\circ$ and party R in government.

is perceived to be more extreme than the opposition in which case it implements an extreme policy with positive probability; or, the party is favored electorally and pursues a moderate policy. In the latter case, the governing party wins re-election until internal adjustments in the opposition ‘turn the tide,’ and the opposition is perceived more moderate than the government. Since both beliefs b_L, b_R exceed the long-term steady state level of perceived extremism b^o , such a situation will arise ‘infinitely often’ along the equilibrium path due to condition (10).¹⁷ As a consequence, if the system starts from a situation in which both parties are perceived to be relatively extreme, *extreme policies will occur in the future with strictly positive probability for every point along the path of adjustment to the long term absorbing state $(b^o, b^o) \in \mathcal{B}$.*

Importantly, the path to the long-run steady state may be quite long when $b_L, b_R > b^o$, depending on the values of π_τ^o , $\tau \in \{e, m\}$. Thus, even though in the long run the political system converges to a situation consistent with the predictions of Downsian competition, equilibrium adjustment dynamics may contain a significant number of electoral cycles away from that long-term steady state and with a positive expectation of extremism.

4.3.1 The i.i.d. Case

As we already mentioned, the exact speed of adjustment in beliefs is regulated by the values of the transition probabilities, π_τ^o . In a special case, the adjustment process is instantaneous. This occurs when the probability that a party is extreme following an internal shake-up is independent of the previous identity of the prevailing group in the party. Formally, this amounts to assuming $b^o = \pi_e^o = 1 - \pi_m^o$. This case is interesting because it corresponds to the assumption in existing electoral models with incomplete information (*e.g.* Banks and Sundaram, 1993, or Banks and Duggan, 2002). In these models, extremism is not serially correlated over time, and equilibrium is stationary along the equilibrium path. Thus, we do not observe paths of play with the qualitatively different dynamics described in proposition 5.

5 Probabilistic Elections

The model in the previous section constitutes a clean, baseline environment from which to evaluate the consequences of introducing more complicated assumptions. In this section we consider one such extension, namely the possibility of probabilistic elections.

¹⁷Of course, the probability of an extreme policy dissipates as beliefs approach the absorbing point (b^o, b^o) .

Even for the most tranquil political environments it is reasonable to assume that events out of the control of the players may influence the outcome of the electoral campaign and give a critical electoral advantage to one of the two parties contesting for power. Such exogenous events can be both favorable to the government (*e.g.* a victorious war or success in foreign policy) or the opposition, (*e.g.* scandal involving the government, etc.). They may simply represent a temporary swing on the electorate’s ideological convictions.

To incorporate this possibility, we assume that in each election period there is an (exogenous) probability w , where

$$w < \frac{(G + u_m^L - a_m^L) - (u_e^L - u_m^L)}{(G + u_m^L - a_m^L) + (G + u_e^L - a_e^L)} < \frac{1}{2} \quad (22)$$

that the incumbent government is re-elected or ousted, independent of the voter’s strategy. This amounts to assuming that the voter’s strategy is now given by:

$$\Phi : X \times \mathcal{B} \longrightarrow [w, 1 - w]. \quad (23)$$

with the obvious modifications on conditions (16) and (17).¹⁸

Of course, with this assumption a party that is perceived more extreme is no longer guaranteed defeat in elections. This has two main implications for our analysis. First, substantively we obtain behavior and outcomes that are closer to empirical observation. Just like Margaret Thatcher’s Conservatives won consecutive elections, in our analysis there is positive probability that a party may remain in government and implement an extreme policy in successive periods.

The second implication of our assumption on probabilistic voting has to do with equilibrium dynamics. In particular, it is straightforward to derive the following extension of propositions 3 and 5:¹⁹

Proposition 6 *Assume (23) and*

$$\delta > \frac{u_e^L - u_m^L}{(1 - w)(G + u_m^L - a_m^L) - w(G + u_e^L - a_e^L)}. \quad (24)$$

(a) *There exists a unique intuitive retrospective equilibrium with partisan strategies given by (20).*

This equilibrium is robust.

(b) *The expected probability of an extreme policy following an election with beliefs $\mathbf{b} \in \mathcal{B}$, is positive for both parties if and only if $\mathbf{b} \in \tilde{\mathcal{B}}$,*

¹⁸A slightly more complicated assumption in the same spirit is to assume w is an appropriate function of the electorate’s beliefs $\mathbf{b} \in \mathcal{B}$. This can be implemented in the analysis to follow, without any gain in insight.

¹⁹The proof is available upon request.

(c) For initial beliefs $\mathbf{b} \in \mathcal{B}^e = \{\mathbf{b} \in \mathcal{B} : b_L, b_R > b^o\}$ there is probability one that future beliefs remain in \mathcal{B}^e , and $\tilde{\mathcal{B}}$ is visited infinitely often prior to election,

(d) For initial beliefs $\mathbf{b} \in \mathcal{B}^m = \{\mathbf{b} \in \mathcal{B} : b_L > b^o \geq b_R \text{ or } b_L \leq b^o < b_R\}$ there is probability one that future beliefs remain in \mathcal{B}^m and $\mathcal{B}^m \cap \tilde{\mathcal{B}} = \emptyset$,

(e) Starting from any $\mathbf{b} \in \mathcal{B}$, the system is absorbed at $(b^o, b^o) \in \mathcal{B}$ with probability one.

Condition (24) is analogous to condition (19), adjusting for the probabilistic nature of elections. Because of the restriction on w in (22), there exists discount factor $\delta < 1$ that satisfies this condition. Furthermore, for any such discount factor, condition (24) is satisfied no matter how large G is, *i.e.* no matter how office oriented parties are.

Qualitatively, the equilibrium in proposition 6 is very similar to that in proposition 3. In particular, partisan strategies are identical, and equilibrium dynamics display similar properties. When belief about the extremity of at least one of the two parties is less than or equal to b^o , then extreme policies are observed only when contrary to the systematic²⁰ preference of the median voter (with probability w) a relatively moderate party loses the election. Furthermore, along the path of adjustment at least one of the two parties is always perceived to be extreme with probability less than b^o .

On the contrary, if beliefs about the extremity of both parties exceed the long-term level b^o there is probability of extreme policies by both contestants in the election along the path of adjustment. In other words, for beliefs that are visited infinitely often along the equilibrium path there are elections in which the winner implements extreme policies with positive probability, whether the winner is the relatively moderate party or not. Thus, in that sense it is still the case that the path of adjustment for initial beliefs $\mathbf{b} \in \mathcal{B}^e$ produces more policy polarization than is the case when $\mathbf{b} \in \mathcal{B}^m$.

Finally, unlike the equilibrium in proposition 3, with probabilistic voting the absorbing set does not depend on initial conditions. Beliefs are eventually absorbed at $(b^o, b^o) \in \mathcal{B}$ from any initial level $\mathbf{b} \in \mathcal{B}$.

6 Conclusions

We have analyzed electoral and policy dynamics in a two party parliamentary system of government. While the model we specified is in many regards coarse, it delivers a rich set of insights on the nature of two-party

²⁰*i.e.* when exogenous shocks in preferences such as scandals, foreign policy developments, etc. alter the median's ranking between the two contesting parties.

competition and the induced dynamics. On the same time, our setup leaves a number of open avenues for improvement. First, the equilibrium we characterize when parties are primarily office motivated involves a long run steady state level of beliefs about the two parties that is absorbing and involves moderate policies with probability one. This conclusion is qualified if we introduce additional noise in the system in one of two forms: (a) different transition probabilities for the type of the parties while they are in government, or (b) random (exogenous) shocks on the electorate's beliefs in any given period.

By way of generalization, we have made heavy use of symmetry and it is worth exploring equilibria in asymmetric settings. Unfortunately, the stumbling block in this case is the lack of analytical solutions, which is a very appealing feature of the current setup. Another more promising avenue for generalization concerns the extension of our current analysis to the case of more than two types and possible policies available to each of the two parties.

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8 APPENDIX

In this appendix we state the proofs of propositions 1 to 3.

Proof of Proposition 1. First, we show that if $\delta < \frac{u_e^P - u_m^P}{G + u_e^P - a_e^P}$ we must have $\sigma^P(\mathbf{b}) = 1$ for all $\mathbf{b} \in \mathcal{B}$ in all equilibria. Suppose, to get a contradiction, that there exists an equilibrium with $\sigma^P(\mathbf{b}) < 1$ for some $\mathbf{b} \in \mathcal{B}$. Then we must have $U_m^P(\mathbf{b}) \geq U_e^P(\mathbf{b})$ for these beliefs. Note that logically (independent of strategies) expected utilities must satisfy $U_e^P(\mathbf{b}) \geq u_e^P + G + \delta a_e^P$ and $U_m^P(\mathbf{b}) \leq u_m^P + G + \delta(u_e^P + G)$, for all $\mathbf{b} \in \mathcal{B}$. Thus, using these bounds, we deduce from $U_m^P(\mathbf{b}) \geq U_e^P(\mathbf{b})$ that $u_e^P + G + \delta a_e^P \leq u_m^P + G + \delta(u_e^P + G) \iff \delta \geq \frac{u_e^P - u_m^P}{G + u_e^P - a_e^P}$, a contradiction.

Next, we verify that $\sigma^P(\mathbf{b}) = 1$ for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$ is part of a robust retrospective equilibrium when (18) holds. From the above arguments, $\sigma^P(\mathbf{b}) = 1$ for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$ are (at least weak) best responses, independent of the voting strategy when $\delta \leq \frac{u_e^P - u_m^P}{G + u_e^P - a_e^P}$. As a result, we only need specify a voting strategy that constitutes a robust,retrospective best response. For perturbed party strategies $\sigma_\varepsilon^P(\mathbf{b}) = 1 - \varepsilon$ we calculate voter's expected utility as

$$V_\varepsilon^P(\mathbf{b}) = b_P(1 - \varepsilon)(v_e^P - v_m^P) + v_m^P, \quad P \in \{L, R\}$$

from which we verify that in a robust equilibrium the voting strategy must satisfy $\Phi(x, \mathbf{b}) = \begin{cases} 1 & \text{if } b_L < b_R \\ 0 & \text{if } b_L > b_R \end{cases}$.

Further set $\Phi(x_e^L, 1, 1) = 0$ & $\Phi(x_e^R, 1, 1) = 1$, so that Φ is also retrospective. We complete the specification of a robust voting strategy by setting $\Phi(x_m^L, 0, 0) = 1$, $\Phi(x_m^R, 0, 0) = 0$, and arbitrary values for $\Phi(x_m^P, b, b)$, $b \in (0, 1)$. We have established that $\sigma^P(\mathbf{b}) = 1$ for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$, is part of a robust, retrospective, and intuitive equilibrium.

Lastly, we show that $\sigma^P(\mathbf{b}) = 1$ for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$ are not part of a robust retrospective equilibrium when $\delta > \frac{u_e^P - u_m^P}{G + u_e^P - a_e^P}$. Without loss of generality assume party L is in government. In every robust retrospective equilibrium with these party strategies we must have $\Phi(x, \mathbf{b}) = \begin{cases} 1 & \text{if } b_L < b_R \\ 0 & \text{if } b_L > b_R \end{cases}$, $\Phi(x_e^L, 1, 1) = 0$, and $\Phi(x_e^R, 1, 1) = 1$. Given posterior beliefs $\beta^m(1, b_L) = 0$ and $\beta^e(b_L) = 1$, we calculate (except possibly for the case of beliefs $b_R = b_L$) expected utility when parties use the prescribed strategies

$$U_e^L(\mathbf{b}) = u_e^L + G + \delta(b_R(a_e^L - a_m^L) + a_m^L) \quad (25)$$

while a one-period deviation by extremists of party L to a moderate policy accrues

$$U_m^L(\mathbf{b}) = u_m^L + G + \delta(u_e^L + G) \quad (26)$$

where we have substituted for $\Phi(x, \cdot, \cdot)$, in (12) and (13). Now, comparing the two expected utilities we obtain

$$U_e^L(\mathbf{b}) \geq U_m^L(\mathbf{b}) \iff \delta \leq \frac{u_e^L - u_m^L}{b_R(a_m^L - a_e^L) + G + u_e^L - a_m^L}$$

Since $(a_m^L - a_e^L) > 0$, there exists $\hat{b}_R \in (0, 1)$ such that $U_e^L(\mathbf{b}) < U_m^L(\mathbf{b})$ when $b_R > \hat{b}_R$. As a consequence, $\sigma^P(\mathbf{b}) = 1$, for all $\mathbf{b} \in \mathcal{B}$ cannot be a robust retrospective equilibrium when (18) is violated. ■

We continue with the proof of proposition 2.

Proof of Proposition 2. If $\sigma^P(\mathbf{b}) = 0$, for all $\mathbf{b} \in \mathcal{B}$, $P \in \{L, R\}$, the voting strategy $\Phi(x, \mathbf{b})$ must satisfy condition (17) in a robust equilibrium since voter's expected utility calculated with perturbed strategies $\sigma_\varepsilon^P(\mathbf{b})$ must satisfy $b_L > b_R \implies V_\varepsilon^L(\mathbf{b}) < V_\varepsilon^R(\mathbf{b})$, for every $\varepsilon > 0$. Recall that posterior beliefs following moderate policies satisfy $\beta^m(0, b_P) = b_P$. Thus, for $\mathbf{b} \in \mathcal{B}$ such that, say, $b_L > b_R$ and party L is in power, we have from (17) that $\Phi(x_m^L, b_L, b_R) = 0$ in a robust equilibrium. In a retrospective equilibrium we also have $\Phi(x_e^L, 1, b_R) = 0$. Thus, substituting in the expected utility expressions (12) and (13) we get

$$U_m^L(\mathbf{b}) = u_m^L + G + \delta a_m^L < U_e^L(\mathbf{b}) = u_e^L + G + \delta a_m^L$$

Thus, $\sigma^L(\mathbf{b}) = 0$ is not optimal for party L and equilibrium condition (15) is violated. ■

Lastly we prove proposition 3.

Proof of Proposition 3. We shall prove the proposition using a few lemmas. We first show that in a robust equilibrium with the stated party strategies, voting strategy must be intuitive.

Lemma 1 *In a robust equilibrium with party strategies given by (20), the voting strategy satisfies condition (17).*

Proof. Without loss of generality assume $b_L < b_R$, and consider small $\varepsilon \geq 0$ ($\varepsilon < \frac{1}{2}$). We execute a direct proof starting with the true inequality

$$b_L \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R)) \leq b_R \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R))$$

This inequality is necessarily strict only when $\varepsilon > 0$ (because it is possible that $\sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R)) = 0$). Since $\sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R))$ is weakly decreasing in its second argument and $b_L < b_R \implies \beta^o(b_L) < \beta^o(b_R)$

(due to (2)) the above implies

$$b_L \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R)) \leq b_R \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_L))$$

Furthermore, since $\sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R))$ is weakly increasing in its first argument and $\beta^g(b_R) > \beta^g(b_L)$ (due to (2) and (1)) we must have

$$b_L \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R)) \leq b_R \sigma_\varepsilon^L(\beta^g(b_R), \beta^o(b_L))$$

By symmetry [$\sigma^L(b_L, b_R) = \sigma^R(b_R, b_L)$] the latter condition is equivalent to

$$\begin{aligned} b_L \sigma_\varepsilon^L(\beta^g(b_L), \beta^o(b_R)) &\leq b_R \sigma_\varepsilon^R(\beta^o(b_L), \beta^g(b_R)) \iff \\ V_\varepsilon^L(\mathbf{b}) &\geq V_\varepsilon^R(\mathbf{b}) \end{aligned}$$

where $V_\varepsilon^L(\mathbf{b})$, $V_\varepsilon^R(\mathbf{b})$ are voter's expected utilities from electing the left and right parties respectively given strategies $\sigma_\varepsilon^P(\mathbf{b})$. Thus, since the original inequality is strict when $\varepsilon > 0$ we deduce that the only robust equilibrium must involve intuitive voting strategies that satisfy condition (17). ■

Next, we show that given intuitive retrospective voting strategy, the only equilibrium party strategies are given by (20).

Lemma 2 *Assume (19) and a voting strategy that satisfies (17). In every retrospective equilibrium, party strategies are given by (20).*

Proof. Again without loss of generality we consider the strategy of the left party. For $\mathbf{b} \in \mathcal{B}$ with $b_L < b_R$, in a retrospective equilibrium (which we invoke for the case $b_R = 1$) we have

$$\begin{aligned} U_\varepsilon^L(\mathbf{b}) &= u_\varepsilon^L + \delta (b_R \sigma^R(\beta^o(1), b_R) (a_\varepsilon^L - a_m^L) + a_m^L) \\ &= u_\varepsilon^L + \delta (b_R \sigma^R(\pi_\varepsilon^o, b_R) (a_\varepsilon^L - a_m^L) + a_m^L). \end{aligned}$$

Since $\beta^m(\sigma, b_L) \leq b_L$ for all $\sigma \in [0, 1]$, we have $\Phi(x_m^L, \beta^m(\sigma, b_L), b_R) = 1$ from (17). So, the expected utility from pursuing a moderate policy is

$$\begin{aligned} U_m^L(\mathbf{b}) &= u_m^L + \delta (G + \sigma^L(\beta^m(0, b_L), \beta^o(b_R)) (u_\varepsilon^L - u_m^L) + u_m^L) \\ &= u_m^L + \delta (G + \sigma^L(b_L, \beta^o(b_R)) (u_\varepsilon^L - u_m^L) + u_m^L) \end{aligned}$$

We have

$$\begin{aligned} U_m^L(\mathbf{b}) &\geq U_e^L(\mathbf{b}) \iff \\ \frac{u_e^L - u_m^L - \delta(G + u_m^L - a_m^L)}{\delta(a_m^L - a_e^L)} &\leq \sigma^L(b_L, b_R^o) \frac{(u_e^L - u_m^L)}{(a_m^L - a_e^L)} + b_R \sigma^R(\pi_e^o, b_R) \end{aligned}$$

For the right hand side we have

$$\min \left\{ \sigma^L(b_L, b_R^o) \frac{(u_e^L - u_m^L)}{(a_m^L - a_e^L)} + b_R \sigma^R(\pi_e^o, b_R) \right\} \geq 0$$

while for the left hand side we get

$$\frac{u_e^L - u_m^L - \delta(G + u_m^L - a_m^L)}{\delta(a_m^L - a_e^L)} < 0 \iff \delta > \frac{u_e^L - u_m^L}{G + u_m^L - a_m^L}$$

But the latter is condition (19), so we conclude that $U_m^L(\mathbf{b}) > U_e^L(\mathbf{b})$ and $\sigma^L(b_L, b_R) = 0$ is the unique optimal strategy when (17) holds and $b_L < b_R$.

Next consider the case $b_L \geq b_R$. Again, by choosing an extreme policy, party L expects

$$U^e(\mathbf{b}) = u_e^L + G + \delta(b_R \sigma^R(\pi_e^o, b_R)(a_e^L - a_m^L) + a_m^L)$$

We can verify that for any probability σ of choosing an extreme policy by party L , beliefs following a moderate policy x_m^L are given according to

$$\beta^m(\sigma, b_L) \begin{cases} > b_R & \text{if } \sigma < \sigma^L(b_L, b_R) \\ = b_R & \text{if } \sigma = \sigma^L(b_L, b_R) \\ < b_R & \text{if } \sigma > \sigma^L(b_L, b_R) \end{cases}$$

i.e. $\sigma^L(b_L, b_R)$ is the (unique) mixing probability the induces the updated belief pair $(b_R, b_R) \in B$ when L chooses x_m^L . By choosing a moderate policy party L expects:

$$\begin{aligned} U_m^L(\mathbf{b}, \sigma) &= u_m^L + G \\ &+ \delta \left(\begin{aligned} &\Phi(x_m^L, b'_L, b_R) [\sigma^L(b'_L, \beta^o(b_R)) (u_e^L - u_m^L) + u_m^L + G] \\ &+ (1 - \Phi(x_m^L, b'_L, b_R)) [b_R \sigma^R(\beta^o(b'_L), b_R) (a_e^L - a_m^L) + a_m^L] \end{aligned} \right) \end{aligned}$$

where $b'_L = \beta^m(\sigma, b_L)$. Now, due to (17), we verify with straightforward algebraic manipulation that if (19) holds we get

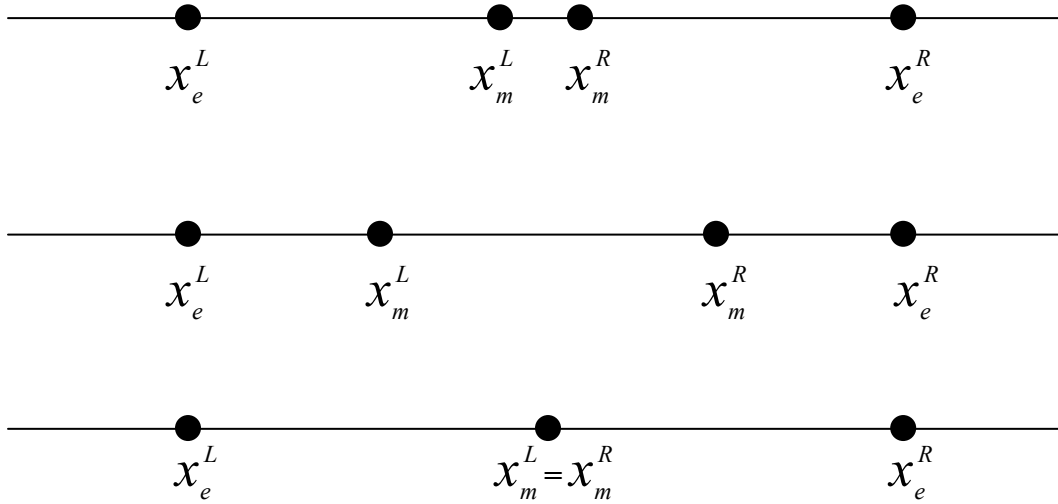
$$\begin{aligned} U_m^L(\mathbf{b}, \sigma) &< U^e(\mathbf{b}) \text{ if } \sigma < \sigma^L(b_L, b_R) \text{ (whence } \Phi(x_m^L, b'_L, b_R) = 0), \text{ and} \\ U_m^L(\mathbf{b}, \sigma) &> U^e(\mathbf{b}) \text{ if } \sigma > \sigma^L(b_L, b_R) \text{ (whence } \Phi(x_m^L, b'_L, b_R) = 1). \end{aligned} \tag{28}$$

As a consequence, equilibrium can only be attained when $\sigma = \sigma^L(\mathbf{b})$, in which case (because of the strict inequality in (28)) there exists $\Phi(x_m^L, b_R, b_R) \in (0, 1)$ such that $U_m^L(\mathbf{b}, \sigma^L(\mathbf{b})) = U^e(\mathbf{b})$.²¹ ■

To summarize we have shown that the only retrospective equilibrium with intuitive voting involves party strategies given by (20). Furthermore, the only robust voting strategy when party strategies are given by (20) is intuitive and $\Phi(x_m^P, b, b) \in (0, 1)$ is uniquely determined for $b \in [0, 1]$. Thus, there exists a unique intuitive retrospective equilibrium that is also robust. ■

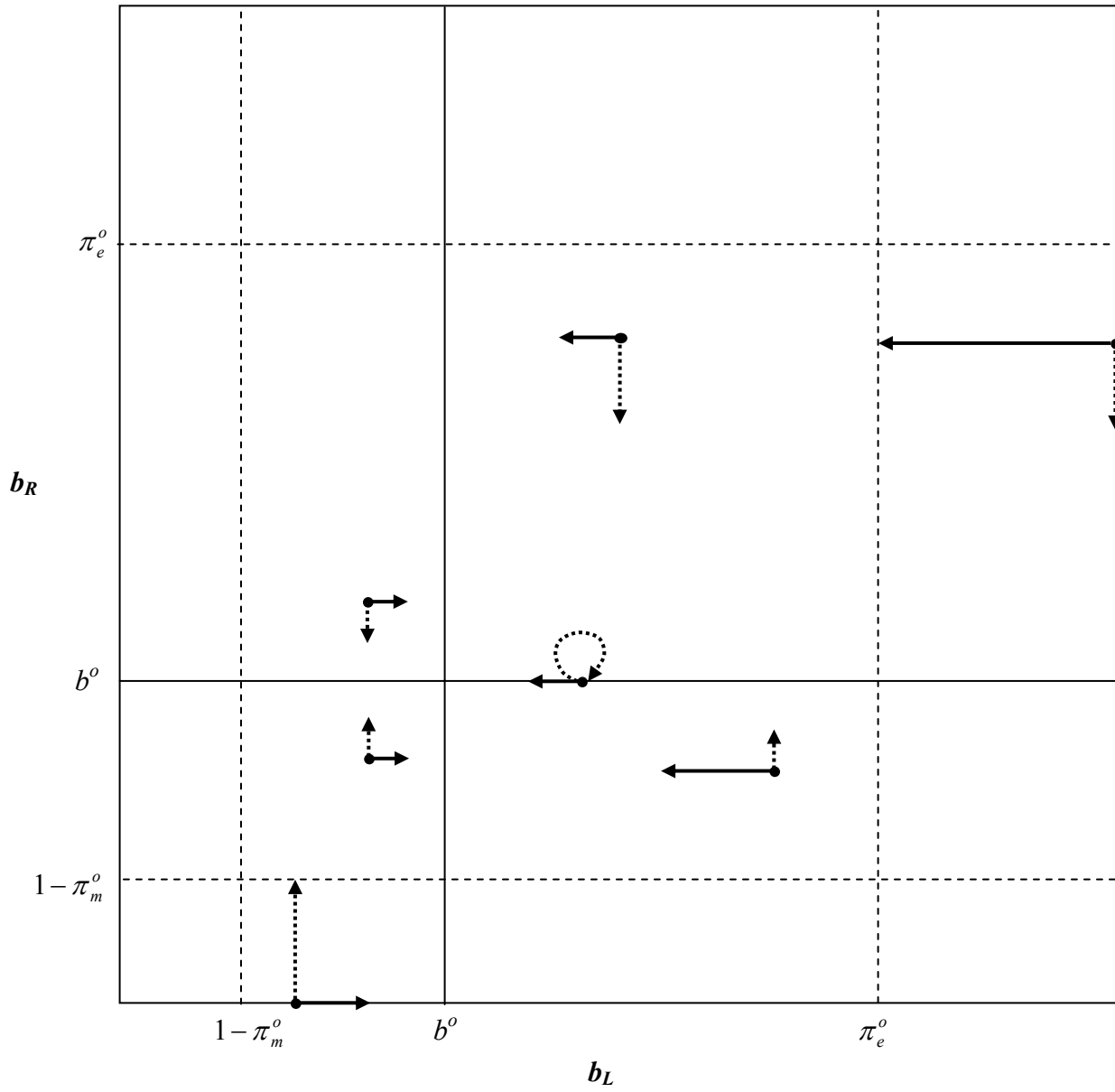
²¹The exact values of $\Phi(x_m^L, b_R, b_R)$ are available upon request.

Figure 1: Interpretation of Government Policies in the Spatial Model



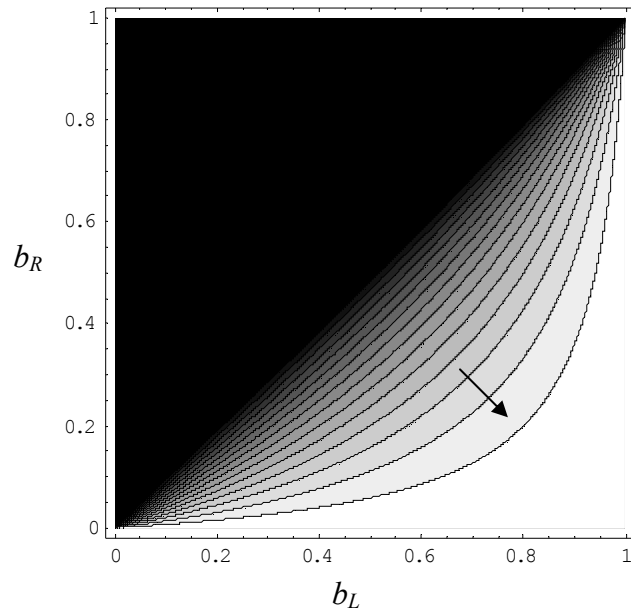
Key: *The ideal policy of moderate partisans may be identical (the median) in one dimension, or may reflect the bias of the party even when it is controlled by moderates.*

Figure 2: Evolution of beliefs following Electoral Defeat

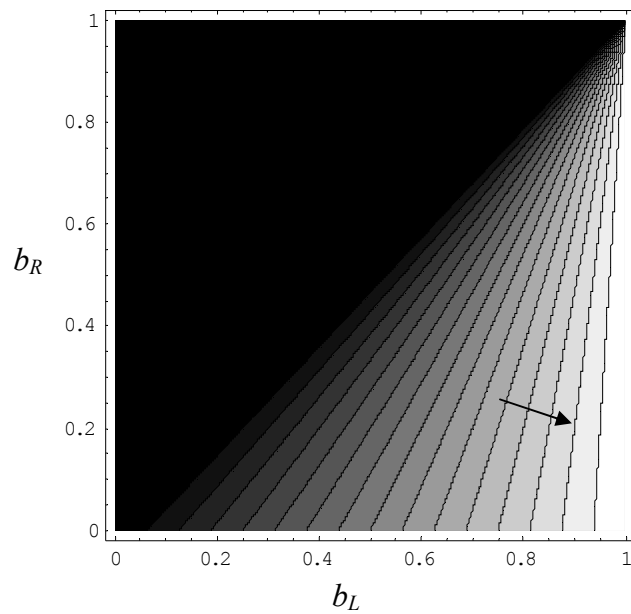


Key: \longrightarrow change in beliefs following defeat of the Left
 $\cdots\longrightarrow$ change in beliefs following defeat of the Right

Figure 3: Probability of Extreme Policy by Party L



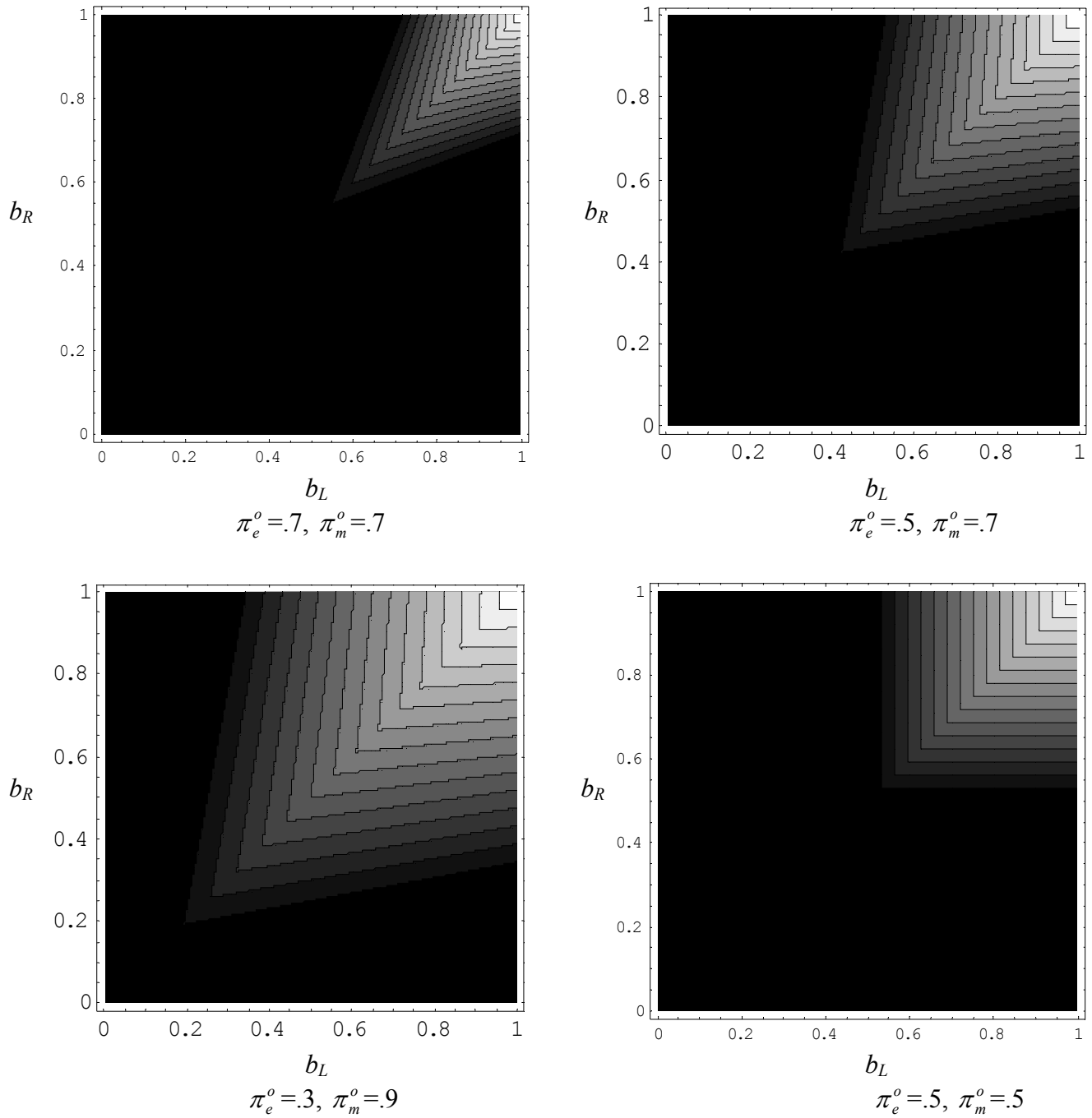
(a) Probability of extreme policy by extreme partisan of party L .



(b) Electorate's expected probability of extreme policy by party L .

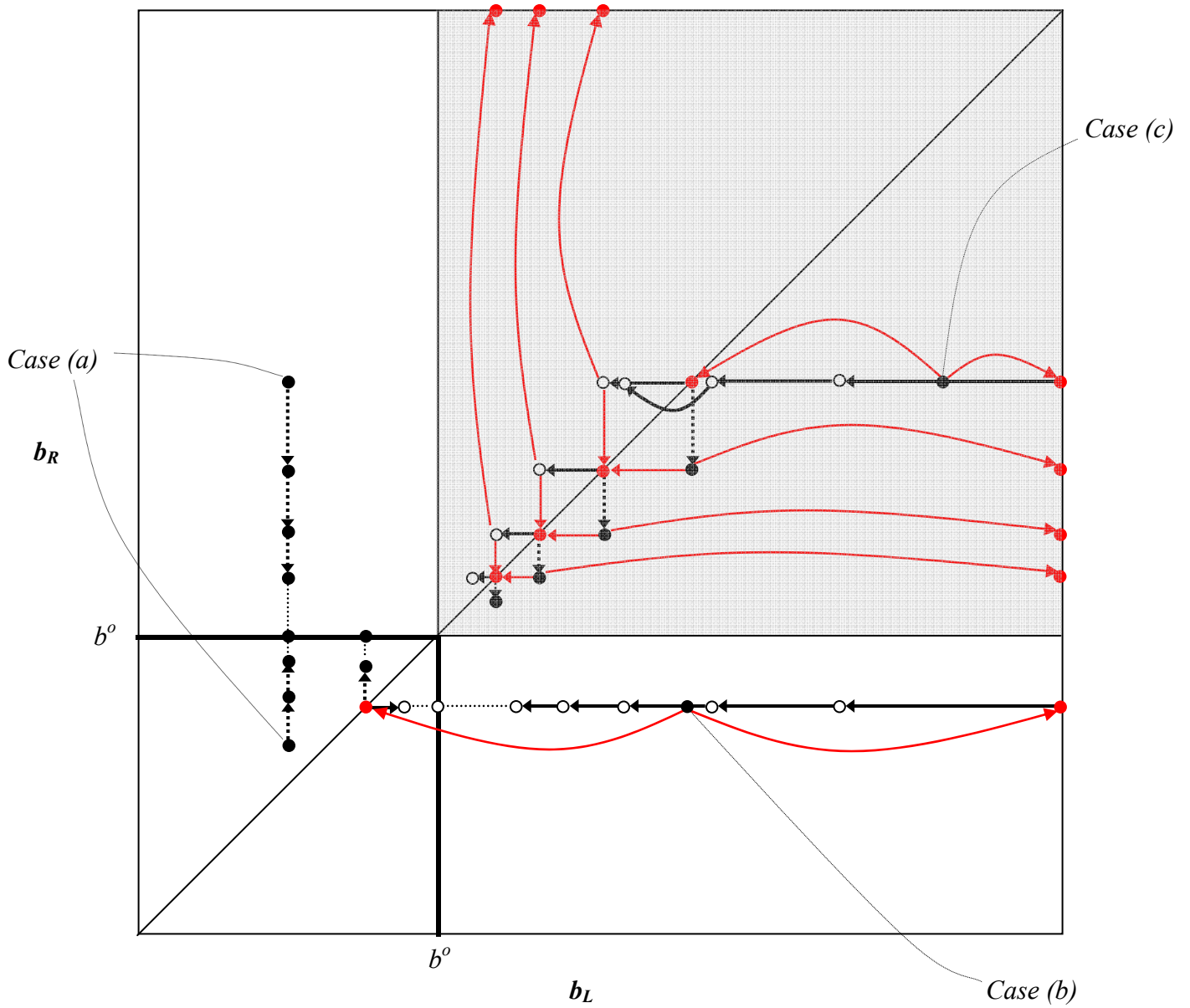
Key: Contour plots of probability of an extreme policy in the space of beliefs, $\mathcal{B} = [0,1]^2$. Lighter areas indicate higher probability. Probability is zero in black areas.

Figure 4: Equilibrium Expected Probability of Extreme Policy Prior to Election



Key: Contour plots of expected probability of an extreme policy prior to the election in the space of beliefs, $\mathcal{B} = [0,1]^2$. Lighter areas indicate higher probability. Probability is zero in black areas.

Figure 5: Equilibrium Dynamics



- Key:**
- change in beliefs following defeat of the Left
 - ⋯→ change in beliefs following defeat of the Right
 - change in beliefs following government policy
 - Left party in government
 - Right party in government
 - Beliefs following government policy & prior to election