International Lending, Capital Controls and Wealth Inequality

Geoffrey Dunbar*
Department of Economics, Queen's University

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Abstract

In this paper I set out conditions under which a tax on capital inflows can be the optimal policy for a government to implement. In particular, market incompleteness, in the form of collateral constraints on domestic financial intermediation, can imply a positive measure of borrowing constrained agents in that market. When the measure of borrowing-constrained agents is sufficiently large, a capital inflow tax combined with a lump-sum transfer can increase aggregate social welfare. The intuition is simple: constrained agents do not adjust their consumption behaviour in response to small changes in intertemporal prices (*e.g.* interest rates) but strictly benefit from lump-sum transfers. When the measure of such agents is sufficient large, a tax and transfer scheme can thus raise aggregate social welfare. Also, I find evidence that the welfare gains to international capital markets are higher than the complete market model estimates of Lucas (1987) and Backus, Kehoe and Kydland (1992). In particular, open international capital markets can raise steady-state social welfare in the order of 2.5 percent when collateral constraints to borrowing exist.

*The author gratefully acknowledges financial support from SSHRC and OGS. Corresponding author: Geoffrey Dunbar, Department of Economics, Dunning Hall, Queen's University, Kingston, ON, K7L 3N6, Phone: 613-533-6660, email: dunbarg@qed.econ.queensu.ca

1 Introduction

The role of capital account restrictions on the social welfare of developing countries has come under scrutiny. In part, the scrutiny is a natural reaction to complaints that capital account liberalizations may have fanned the fires which led to the crises in many emerging markets towards the end of the 1990's. Other commentators appear to hold the opposite view, that capital account liberalizations were not proceeding quickly enough and that unhindered capital flows are essential to impose "market discipline" (see Forbes [16]) on otherwise susceptible markets. The goal of this paper is to examine the role of incomplete domestic credit markets on the welfare consequences of capital account restrictions.

In this paper, I propose a two-country, single-good model where individual agents borrow and lend with a domestic intermediary, the bank. The bank enforces borrowing contracts through collateral claims. That is, agents are required to post a sufficient amount of collateral to secure loans. Agents face a stochastic shock to their income which is private information and which renders state-contingent pricing infeasible. Hence, the bank operates as a credit market, buying and selling single-period uncontingent bonds.

International lending and borrowing in the model occurs through the banks and is secured by the government in both countries. In addition, the government can impose a per-unit tax on capital inflows. That only capital inflows may be taxed is assumed for two reasons. First, it simplifies the game-theoretic analysis of the model and second, it seems reasonable to consider capital account restrictions for those economies which typically impose them, namely developing countries with capital inflows. Finally, the usual Welfare Theorems do not hold in the current model and as such the government is not a social planner although it does act to maximize social welfare.

The main finding of the paper is that taxes on capital inflows which are remitted in a lump-sum fashion can be welfare-improving if a sufficient fraction of borrowers are collateral (or borrowing) constrained. The intuition is straightforward. Although a tax on capital inflows raises the cost of borrowing, constrained agents do not adjust their consumption decisions and hence strictly benefit as a result of the lump-sum transfer. If a sufficiently large measure of agents is constrained, then the welfare gains to the constrained agents (and lenders) can be large enough to outweigh the costs to other borrowers.

The literature on incomplete markets and international finance is extensive. Much of the literature focusses on international business cycles and cross-country correlations of consumption, investment and employment. Much of the motivation can be traced to Backus, Kehoe and Kydland [4] who develop a two-country complete-markets business cycle model and demonstrate that the model is unable to generate plausible cross-country correlations of output and consumption (among other statistics). While they do not address capital taxation problem directly, the results of their model are generally assumed to indicate the presence of market frictions. Baxter and Crucini [5], Stockman and Tesar [29], Kollmann [23], Heathcote and Perri [18] and Kehoe and Perri [19] extend the standard two-country model to examine incomplete markets. The results from the literature generally suggest that the presence of incomplete markets can generate correlations that are closer to those observed in the data. Specifically, [18] find that financial autarky, in which all international trade must be *quid pro quo*, tends to approximate observed data better than uncontingent bond or complete market models. In addition, [19] find that endogenous borrowing constraints can similarly explain observed employment-investment correlations. None of these papers examines explicitly the role, or consequences, of capital account restrictions.

There is also a burgeoning literature on endogenous borrowing constraints, both applied at the individual level and to international markets. Lacker [25] constructs a two-period model where output is only observable to the borrower and demonstrates that, when the borrower values collateral more than the lender, collateral ensures repayment. Kocherlakota [23] demonstrates that collateralized debt contracts are optimal in models where the ex-post value of collateral and the ex-post investment return are known only to the borrower. Geanakoplos and Zame [17] construct a two-period general equilibrium model where agents can default at any time and show that financial assets are only traded when backed by collateral. Andolafatto and Nosal [3] construct a model where agents endogenously circulate claims which are implicitly backed by collateral. Dubey, Geanakoplos and Shubik [11] construct a model of default and punishment in general equilibrium where separating equilibria can exist contingent on disutility from default. The current paper demonstrates that collateral is an endogenous borrowing constraint which restricts the amount of intermediation somewhat like endogenous solvency constraints. That is, the amount an agent may borrow is limited by the assets she may post as collateral. In this respect, the present paper is related to Kehoe and Levine [21] and Alvarez and Jermann [2] who study the effects of solvency constraints. One key difference of this paper from [21] and [2] is that I do not assume that agents determine borrowing constraints to ensure that repayment is individually rational for the borrower.

Finally, there is a literature on capital account restrictions and taxation. Eichengreen [14] surveys the literature and finds mixed support both theoretically and empirically, for capital account liberalization. In part, capital account liberalization can be costly because of its apparent links to the 'hot-money' outflows. Theoretical research on capital outflows restrictions has tended to focus on the role of restrictions in inefficiently allocating capital. As such, the non-optimality of restrictions usually follows from traditional competitive market and welfare theorems. Labán and Larraín [24] show that removing capital outflow restrictions initiates increases in capital inflows. Bartolini and Drazen [6] extend this literature to consider the signalling aspects of initiating/lifting capital controls. Finally, Lucas [26] and Backus, Kehoe and Kydland [4] show that there is little welfare gain to smoothing consumption international in complete markets models, an implication of which is that capital controls have (relatively) minor consequences on welfare. This paper purposefully abstracts from modeling capital account crises and as such does not address the question of the desirability of large changes to capital inflows. In addition, this paper finds that the presence of incomplete domestic credit markets implies a large welfare gains and cost to international capital markets than those found by [26] and [4].

The structure of this paper is as follows. Section 2 describes the model environment. Section 3 discusses the structure of the banking sector. Section 4 defines international capital flows and the international capital market clearing condition. Section 5 describes the role and objective of the government. Section 7 defines an individual agents' problem and Section **??** lays out the timing of the model. Section 9 defines the model equilibrium and, in particular, demonstrates that taxing capital inflows can be the optimal policy for a government. Section 10 presents results from model simulations which demonstrate that capital controls can be optimal for a utilitarian social-welfare maximizing government [Not quite yet]. Section 11 concludes [To come].

2 Model Environment

There are two countries, Country 1 and Country 2. Agents in each country are differentiated by a shock in their production function. Apart from the shock, the production technology is identical across all agents. In each Country $j = 1, 2$ there exist a continuum of agents whose measures are normalized to 1 and M respectively.[1]

Each agent in country j is initially endowed with $k^i_{j,0}$ units of the country-specific capital stock and an identical production technology:

$$y^i_{j,t} = \eta^i_{j,t}(k^i_{j,t})^{\alpha_j} \tag{1}$$

where $y^i_{j,t}$ is the output of the consumption good by agent i in country j in period t, $k^i_{j,t}$ is the capital stock used in production by agent i in country j in period t, $\alpha_j \in (0, 1)$ reflects the productivity of capital in the production function and $\eta^i_{j,t} \in N_j = \{\overline{\eta}_j, \underline{\eta}_j\}$; $\overline{\eta}_j > 1 > \underline{\eta}_j$, is a mean zero, idiosyncratic, stochastic shock to agent i in period t. Capital is assumed to depreciate by a fraction $\delta_j \in [0, 1)$ each period. I assume that agents cannot produce in the foreign country. The shock for agents in country j, $\eta^i_{j,t}$, follows a Markov process with transition probabilities $\pi_j(\eta'_j|\eta_j) = Prob(\eta^i_{j,t+1} = \eta'_j|\eta^i_{j,t} = \eta_j) > 0$ for $\eta_j, \eta'_j \in N$. The shock is intended to capture idiosyncratic elements of production, such as: drought, illness, median age, median experience, time constraints, *etc.*[2] The shock is assumed to be private information and to be revealed to agents once production has occurred.

Agents have the following preferences:

$$U^i_{j,t} = E_t[\sum_{t=0}^{\infty} \beta^t_j \frac{(c^i_{j,t})^{1-\gamma_j}}{1-\gamma_j} |\eta_{j,t}, \pi_j] \qquad j = 1, 2 \tag{2}$$

where $c^i_{j,t}$ is consumption at time t by agent i in country j, β_j is a time invariant discount factor for agents in country j and γ_j is a constant parameter of relative risk aversion for agents in country j. $\gamma_j \equiv \frac{1}{\omega_j}$ where ω_j measures an agent in country j's willingness to smooth consumption through time.

3 Banking

The idiosyncratic shock implies that intertemporal risk-sharing within countries can be Pareto-improving; hence contracting between agents is advantageous. The assumptions specified in this

[1] The model is qualitatively similar to the one-country model with endogenous collateral constraints developed in Dunbar (2004).

[2] Clearly, in an infinite horizon model, age and experience are identical across agents. However, infinite horizon agents may be considered also as dynastic households. Thus, the shock process also can be thought to represent the proportion of productive members of the household at any given point in time.

section are sufficient for collateralized lending and borrowing contracts to arise endogenously.[3]

Assumption 1 An agent's shock, $\eta_{j,t}$, is private information and output can be privately consumed.

Assumption 2 Capital stock holdings, $k^i_{j,t}$, are common knowledge to the bank (or costlessly verifiable).

Assumption 3 Contracts which specify a transfer, $b^i_{j,t}$, from (to) an agent i at a price $a^i_{j,t-1}b^i_{j,t}$ in period $t-1$ may be written costlessly. Contracts are assumed to be common knowledge and contracts may be breached.

Assumption 4 Agents may contract once per period through an intermediary in their country (henceforth referred to as the bank). There is assumed to be free-entry to banking.

Assumption 1 implies that agents cannot be differentiated by the bank by observation. In particular, agents with a high shock may consume their additional output without public observation and, more importantly, that the optimal consumption sequence for agent i, $\{c^i_{j,t}\}_{t=1}^{\infty}$, in any time period t is private information. Hence, the bank cannot infer the output or desired consumption sequence of any agent given the assumption of an infinity of agents. Assumption 3 characterizes the nature of the bargaining problem. In particular, Assumption 3 states that only one-period uncontingent bonds are traded. Assumption 4 rules out some contracting equilibria in that each agent can only bargain once per period.

Since Assumption 3 specifies that contracts may be breached, then any contracts must have an implicit or explicit enforcement technology to ensure that those contracts are repaid. The following enforcement technologies are assumed to be possible for a bank to offer:

Censure A bank may refuse to intermediate with a censured agent for a specified period of time, $\nu = \infty$;

Garnishment A bank may garnish a specific amount m of an agent's output in the period subsequent to default for a fixed cost, g;

[3]See Dunbar(2004) for a further discussion of environments where collateralized contracts can arise endogenously.

Collateral A bank may require collateral, q for a loan and may seize collateral costlessly upon default. Banks may disburse received collateral costlessly within the country it is seized. Banks cannot exchange capital internationally.

In addition, banks face a solvency constraint such that a bank cannot suffer a loss in any period. Since free-entry implies that banks cannot make positive expected profits in any period, then the solvency constraint implies that banks make zero profit in every period. The solvency constraint implicitly embeds an opportunity cost of zero to entry. That is, unless a bank can earn at least zero profits it has no incentive to enter. Define a country j bank's period t profit as:

$$\theta_{j,t} = \sum_j (-\sum_i b^i_{j,t} + \sum_i a^i_{j,t+1} b^i_{j,t+1} + \sum_i e^i_{j,t}). \tag{3}$$

where $\theta_{j,t}$ is the profit of a bank in country j in period t, $a^i_{j,t+1}$ is the price of a period $t+1$ bond in period t to agent i and $e^i_{j,t}$ is the net amount collected using the enforcement technology from agent i in period t contingent on default by agent i. It is important to note that the bank's profit function (3) includes the possibility of only one bank which operates in both countries. Thus, one could imagine a single international bank rather than independent, national, banks. Finally, I assume that banks are profit-maximizers.

The banks' problem then is to maximize their flow of profits, (3), subject to the choice of an enforcement technology.

3.1 Interest Rates and Bank Profit

The following proposition demonstrates that the price of the risk-free bonds is the same for all agents for a bond in country j. Intuitively, since the risk-free bond is the same instrument offered to all agents then the price of the bond, assuming market-clearing, must be the same for all agents.

Proposition 3.1.1 $a^i_{j,t} = a_{j,t} \qquad \forall i; \quad j = 1, 2.$

Proof: Given the environment, a bank could write contracts which are contingent on an agent's current period state, *i.e.* conditioning on an agent's capital holdings such that $a^i_{j,t} = a(k^i_{j,t})$, where $a(k^i_{j,t})$ is some pricing kernel. However, in equilibrium, it must be that $a^i_{j,t} = a_{j,t}$, $\forall i$. To see this, let $\hat{a}$ represent the price charged to an agent who borrows and $\tilde{a}$ represent the price offered to an agent who lends.

1. Suppose $\hat{a}^i_t < \tilde{a}^h_t$, Then an entrant bank can offer contracts $\hat{a}^i_{j,t} + \hat{\epsilon}^i$ and $\tilde{a}^h_{j,t} + \tilde{\epsilon}^h$, $\tilde{\epsilon}^h < 0 < \hat{\epsilon}^i$ which both agent i and agent h prefer. That is, if $\hat{a}^i_{j,t} < \tilde{a}^h_{j,t} < 1$ then the cost of borrowing for agent i is lower and the return for agent h is higher. If $1 < \hat{a}^i_{j,t} < \tilde{a}^h_{j,t}$ then the return to borrowing for agent i is higher and the cost of lending for agent h is lower. Moreover, $\hat{\epsilon}^i$ and $\tilde{\epsilon}^h$ may be chosen such that $\sum_i b^i_{j,t+1} + \sum_h b^h_{j,t+1} = 0$, *i.e.* such that market clearing is unaffected. Hence $\hat{a}^i_{j,t} < \tilde{a}^h_{j,t}$ cannot be an equilibrium.

2. Suppose $\hat{a}^i_{j,t} > \tilde{a}^h_{j,t}$. Then an entrant bank can offer contracts $\hat{a}^i_{j,t} + \hat{\epsilon}^i$ and $\tilde{a}^h_{j,t} + \tilde{\epsilon}^h$, $\tilde{\epsilon}^h < 0 < \hat{\epsilon}^i$ which both agent i and agent h prefer. That is, the borrowing agent i receives more in period t and the lending agent h is promised more in period $t+1$. With free-entry and price-competition, then the sequence of prices diverge such that: $\hat{a}^i_{j,t} \rightarrow \infty$ and $\tilde{a}^h_{j,t} \rightarrow 0$. It is trivial that market clearing in period t cannot hold and hence a stationary equilibrium cannot be sustained.

Hence $\hat{a}^i_{j,t} = \tilde{a}^h_{j,t} = a^i_{j,t} \quad \forall i, t$. To complete the proof consider the arguments of Rothschild and Stiglitz [28]. If $a^i_{j,t} \neq a^h_{j,t}$ for agents i and h then by the arguments above there exists an profitable deviation contract for a bank where agent i is a borrower and agent h is a lender (or vice versa). Hence $a^i_{j,t} \neq a^h_{j,t}$ cannot be an equilibrium. Thus the only equilibrium can be a pooling equilibrium where $a^i_{j,t} = a^h_{j,t} = a_{j,t}$ since there is no profitable deviation for a bank (any deviation by construction would entail a loss). $\blacksquare$.

Thus, a country j bank's period t profit can be re-written as:

$$\theta_{j,t} = \sum_j (-\sum_i b^i_{j,t} + \sum_i a_{j,t+1} b^i_{j,t+1} + \sum_i e^i_{j,t}). \tag{4}$$

The possibility of a single international bank entering both national banking sectors drives the structure of the banking sector. Country-specific idiosyncratic shocks provide a motive for international borrowing and lending but the possibility of an international bank enforces international bond market clearing. Suppose, for instance, that the banking sectors operated independently.

Then each bank's profit would be:

$$\bar{\theta}_{j,t} = -\sum_i b^i_{j,t} + \sum_i a_{j,t+1} b^i_{j,t+1} + \sum_i e^i_{j,t}; \quad j = 1, 2. \tag{5}$$

As long as $a_{1,t+1} \neq a_{2,t+1}$ an international entrant could offer a profitable deviation. Suppose $a_{1,t+1} < a_{2,t+1} \leq 1$. Then an international entrant could offer to borrow bonds from Country 2 at a price $a_{2,t+1} - \epsilon_2$, $\epsilon_2 \to 0$, and lend bonds to Country 1 at a price $a_{1,t+1} + \epsilon_1$, where $\epsilon_2 > \epsilon_1$, such that the entrant makes a profit. Similar analysis holds when $a_{1,t+1} > a_{2,t+1} \geq 1$. Hence, the possible entry of an international entrant forces the world bond market to clear and, in particular, that $a_{j,t} = a_t; \ \forall j$.

3.2 Equilibrium Enforcement Mechanism

Although the bank can choose among the feasible enforcement mechanism, in equilibrium banks will choose to enforce contracts using collateral requirements. Free-entry implies that, whichever enforcement mechanism chosen, the bank cannot expect to earn profits from the enforcement mechanism. The solvency constraint implies that banks will choose an enforcement mechanism which does not earn negative profits. The following proposition demonstrates that collateral emerges as the equilibrium enforcement mechanism.[4]

Proposition 3.2.1 *In equilibrium, banks choose collateral as the enforcement mechanism for contracts*

Proof: It is immediate that, in equilibrium, banks will not choose censure as the enforcement mechanism for contracts since they cannot enforce the cost of censure to individual agents. With free-entry into banking and the opportunity to offer a competing contract, autarky is not the relevant punishment scheme since an entrant bank could offer either collateral or garnishment contracts without any loss of profits. Hence, the value of the outside option of an agent is always (weakly) greater than autarky and, as a result, the optimal strategy of a borrower is to borrow the maximum amount allowed under censure and then default. The continuation payoff is thus (weakly) greater than that of continuing under censure. Since default implies negative profits, which banks

[4]See Dunbar (2004) for a further discussion on the emergence of collateral contracts in a model with assymmetric information.

cannot support, censure contracts cannot be an equilibrium when either collateral or garnishment are feasible enforcement mechanisms.[5]

Both garnishment and collateral enforcement mechanisms are feasible for a bank in the sense that they can recoup losses suffered through default. However, given the specification of the production function, any garnishment contract may be written equivalently as a collateral contract, since there exists a one-to-one mapping between capital and output. However, the fixed cost of garnishment implies a deadweight loss which is avoidable using collateral contracts. For instance, given an incumbent bank offering garnishment contracts, an entrant bank could offer exactly the same contracts to agents using collateral as the enforcement mechanism and earn the deadweight costs as profit. Hence, no bank has an incentive to offer garnishment contracts if collateral contracts are feasible. ■

Proposition 3.2.1 states that banks optimally choose to offer collateral contracts. The next proposition, Proposition 3.2.2, demonstrates that banks choose to offer collateral contracts where the value of collateral posted is equal to the face-value of the bond obligation.

Proposition 3.2.2 *Banks set the collateral requirement, $q^i_{j,t+1}$, for a loan, $b^i_{j,t+1}$ to an agent i in country j in period t such that $q^i_{j,t+1} = b^i_{j,t+1}$.*

Proof: Essentially, this is a Bertrand result. If the bank sets the collateral requirement higher than the face value of the loan then a competing bank may enter and offer a marginally lower collateral requirement without any concommitent loss of profits (no agent would default if $q^i_{t+1} > b^i_{t+1}$). Since collateral constrains consumption smoothing, any relaxation of the collateral requirement will attract all the agents to the new bank. In addition, no bank will set the amount of collateral lower than the face value of the loan. To see this, consider the arguments of Rothschild and Stiglitz [28]. Suppose the bank solves the individual problem of both high shock and low shock agents with given capital stocks, k^i_t. The enforcement value of collateral to an agent is the expected future period, $t+1$, marginal value of capital to that agent. Suppose the bank sets the borrowing constraint

[5] I note that it is possible for a bank to write a censure contract where the outside option is not permanent autarky but instead the continuation payoff of the competing mechanism. However, the general equilibrium consequences of such contracts are unclear since the borrowing constraint must, by necessity, be zero for any agent who would prefer to borrow, default and exercise the outside option. Moreover, no agent can be allowed to borrow to that level. Hence the borrowing constraints are much tighter than simply Kehoe-Levine style contracts. In addition, the value of the outside option is difficult to specify since it depends on a and thus, by construction, on the measure of agents at a given point in time. The exact solution to this problem is beyond the scope of this paper.

for all agents with k_t^i at the level of borrowing at which the agent with the highest future period marginal value of capital is indifferent. Call this agent the low type. By the arguments of [28] this cannot be a pooling equilibrium since all high types would borrow to the level of the low type and then default. Moreover, when a low type contract is offered, all high types have an incentive to act as a low type. Thus, there can be no separating equilibrium. The only other option is a pooling equilibrium where the borrowing constraint is set at the level of the high type. However, by [28] this can not be a pooling equilibrium in a competitive market since there exists a profitable deviation (contract) for a low type agent who has a higher future period marginal value of capital (*i.e.* relaxing the borrowing constraint). Hence, only high type borrowing constraints cannot be an equilibrium. Thus, the only equilibrium which exists is where $q_{t+1}^i = b_{t+1}^i$. ■

Finally, the solvency constraint and free-entry imply banks earn zero profits. Moreover, since only risk-free bonds are traded then a direct implication is that all contracts are zero-profit. The following proposition demonstrates that all contracts are zero-profit.

Proposition 3.2.3 $q_{j,t+1}^i = b_{j,t+1}^i \Rightarrow \sum_i e_{j,t}^i = 0$ *and* $\theta_{j,t} = 0 \Rightarrow \sum_j(-\sum_i b_{j,t}^i + \sum_i a_{j,t+1} b_{j,t+1}^i) = 0$.

Proof: That $q_{j,t+1}^i = b_{j,t+1}^i$ implies $\sum_i e_{j,t}^i = 0$ is immediate since the collateral requirement means no bank can earn a profit from disbursing received collateral. That $\theta_{j,t} = 0$ implies $\sum_j(-\sum_i b_{j,t}^i + \sum_i a_{j,t+1} b_{j,t+1}^i) = 0$ follows from the possibility of an international bank. If $\sum_j(-\sum_i b_{j,t}^i + \sum_i a_{j,t+1} b_{j,t+1}^i) > 0$ then the international bank can earn a profit in period t which cannot occur with free-entry into banking. Hence $\sum_j(-\sum_i b_{j,t}^i + \sum_i a_{j,t+1} b_{j,t+1}^i) = 0$. ■

4 International Lending and Borrowing

National banks are assumed to be able to borrow and lend internationally, in essence replicating the structure of an international bank. International lending and borrowing between national banks is assumed to be unsecured, in the sense that a bank cannot expropriate capital posted as collateral across borders. In addition, garnishment mechanisms are also assumed to be infeasible, since banks cannot forcibly seize foreign output. As a result, international lending and borrowing must be enforced through other means.

International borrowing and lending between banks necessarily implies a slackness in the feasible sets of national intermediary contracts. Where $b^i_{j,t+1}$ is the borrowing (lending) of an agent i at time $t+1$ in country j at a price $a_{j,t}$. Then, for a bank in country j:

$$a_{j,t}\sum_i b^i_{j,t+1} = 0$$

in a equilibrium without international borrowing or lending. However, this need not be the case with international borrowing and lending. Where a_t is the world price of a bond $b^i_{j,t+1}$ by agent i at time $t+1$ the the world equilibrium bond market clearing condition is:

$$a_t \sum_i b^i_{1,t+1} + a_t \sum_i b^i_{2,t+1} = 0. \tag{6}$$

The capital account for country j at time t, $KA_{j,t}$ includes flows from both net bond sales, $a_t \sum_i b^i_{j,t+1}$ and net bond repayments, $\sum_i b^i_{j,t}$. Hence, the capital account for country j may be defined as:

$$KA_{j,t} = \sum_i b^i_{j,t} - a_t \sum_i b^i_{j,t+1}. \tag{7}$$

In any stationary equilibria then bond flows are constant, *i.e.* $\sum_i b^i_{j,t} = \sum_i b^i_{j,t+1} = \sum_i b^i_j;\quad \forall t$, and the price of bonds is constant, $a_t = a;\quad \forall t$. Then, in a stationary equilibrium, there is an equivalence between the world capital account clearing and world bond market clearing:

$$\sum_j KA_j = 0 \Leftrightarrow (1-a)\sum_j \sum_i b^i_j = 0 \Rightarrow \sum_j \sum_i b^i_j = 0; \tag{8}$$

which states that the world capital account must clear whenever the world bond market clears. It is important to note that in any stationary equilibria then $KA_j < 0$ implies an excess demand of loans in the current period bond sales, *i.e.* $a_t \sum_i b^i_{j,t+1} < 0$, whenever $0 < a < 1$.

5 The Government

I assume that there exists a national authority, the government, in each country j which has preferences over the aggregate social welfare in j. The government is assumed to have preferences over social welfare such that:

$$W_{j,t} = \sum_i \zeta^i_j \mu^i_{j,t} U^i_{j,t} \tag{9}$$

where $W_{j,t}$ is the utility of the government in country j in period t, ζ_j^i is the weight the government in country j assigns to agent(s) i and $\mu_{j,t}^i$ is the measure of agents of type i in country j in period t. For the model considered in this paper, I assume that the government weights each agent equally so that $\zeta_j^i = 1$. Although the government cannot observe each agents' utility, $U_{j,t}^i$, since the shock is private information, I assume that a law of large numbers applies such that the government can determine $W_{j,t}$ to an arbitrarily close degree.

The government can restrict or tax any agent or bank who borrows or lends internationally. It is this assumption that permits foreign exchange. The government has an incentive to allow international lending and borrowing since the world bond market constraint implies greater risk-sharing and thus higher welfare. The government in Country i can enforce bank repayment of any international obligations by threatening to bar any bank, or even all agents from a country, which has reneged on a past debt. Essentially, the government can impose a Kehoe-Levine type punishment by instituting international autarky for any defaulting bank which creates a wedge for an international entrant. Moreover, the government faces no *ex-post* incentive to renegotiate since it can allow future intermediation with an international entrant. Thus, a bank which has defaulted on debt repayment faces insolvency since an international entrant could use capital inflows to offer contracts which are strictly preferred by domestic agents. Moreover, no government would want a purely national bank to operate (even one that had previously defaulted and temporarily enriched its citizens) since international intermediation is welfare improving. Hence, international default ensures insolvency for the offending bank since both governments then have an incentive to allow an international bank to enter. Finally, I assume the government cannot arbitrarily seize any repayment flows. That is, a government cannot force a bank to become bankrupt by forcing it to default on any bond repayments.

The government in country j has complete control over the flow of goods across its border and can impose a per-unit tax, $\phi_{j,t}$, on capital account inflows, $a_t \sum_i b_{j,t+1}^i < 0$ or $\sum_i b_t^i > 0$.[6] The level of $\phi_{j,t}$ is assumed to be freely observable to all agents in either economy. Any revenues obtained from the imposition of $\phi_{j,t}$ are assumed to be distributed in a lump-sum fashion to all agents in

[6] I consider only taxes on capital account inflows for two reasons. The first reason is that it is more tractable in the context of the current model to tax only one flow since only one-period bonds are traded. Taxing both inflows and outflows would essentially double tax the same instrument. Secondly, taxes on capital inflows match the Chilean experience and are, at present, cautiously supported by the IMF according to Stanley Fischer [15], the former First Deputy Managing Director of the IMF.

country j. That is, all agents receive an amount $T_{j,t}$ from the government. In equilibrium, the government in country j chooses the level of $\phi_{j,t}$ to maximize the social welfare in country j. The government faces a budget constraint:

$$\sum_i T_{j,t} = \begin{cases} \phi_{j,t}\, |a_t \sum_i b^i_{j,t+1}| & \text{if} \quad a_t \sum_i b^i_{j,t+1} < 0 \\ \phi_{j,t} \sum_i b_{j,t} i & \text{if} \quad \sum_i b^i_{j,t} > 0. \end{cases} \tag{10}$$

Thus, the government's problem is to maximize social welfare:

$$W_j = \sum_i \zeta^i_j \mu^i_j U^i_j \qquad s.t. \ (10) \tag{11}$$

Finally, it is worth noting that I assume the government does not have taxation authority in the traditional sense. Since output is both private information and privately consumable, the government would not be able to tax consumable income without some type of enforcement mechanism. I abstract from these considerations and hence domestic income or capital taxes and any redistributive impacts they might have. One direct consequence for capital account taxation is that governments are restricted to positive tax levels, that is, they cannot offer subsidies.

6 Taxation and International Capital Flows

Without loss of generality, let $\sum_i b^i_{2,t+1} > 0 > \sum_i b^i_{1,t+1}$ which implies Country 1 has a net inflow from the sales of period $t+1$ bonds and Country 2 has a net outflow from the sales of period $t+1$ bonds. The imposition of a tax on capital inflows, $\phi_{j,t} > 0$ distorts the world capital account clearing, (8), such that in equilibrium it becomes:

$$\sum_i b^i_{1,t} - (1-\phi_{1,t}) a_t \sum_i b^i_{1,t+1} - \phi_{1,t} a_t \sum_i b^i_{1,t+1} + \\ (1-\phi_{2,t}) \sum_i b^i_{2,t} + \phi_{2,t} \sum_i b^i_{2,t} - a_t \sum_i b^i_{2,t+1} = 0. \tag{12}$$

A tax on capital inflows, $\phi_{1,t} > 0$, which is imposed by Country 1 would, in the absence of bond price changes, cause the world capital account to be in deficit. To see this, note that the world capital account clearing condition, (12), implies that an increase in ϕ_1 would reduce the net inflow in Country 1. As a result, the net world capital account outflows must be greater than the net inflows and hence the world capital account must be negative. Moreover, world bond market clearing for period $t+1$ bonds, (6), would also be affected. Specifically, the world bond market clearing condition for period $t+1$ bonds remains:

$$\sum_i b^i_{1,t+1} + \sum_i b^i_{2,t+1} = 0 \tag{13}$$

which implies that the imposition of the tax reduces the net inflow to the bank(s) in Country 1, *ceteris paribus*. Hence, the world bond price must adjust. An increase in the world bond price, a_t, increases the borrowing of Country 1, *i.e.* $-a_t(1-\phi_{1,t})\sum_i b^i_{1,t+1}$ increases. At the same time, an increase in a_t reduces the incentives to lending which reduces $-a_t \sum_i b^i_{2,t+1}$. Hence, an increase in a_t returns the economy to equilibrium. It is important to note that while the world bond price must increase, the borrowing costs in Country 1 will actually rise as a result of the tax. That is, the tax acts as a wedge between the bond prices in Country 1 and Country 2. Let $\hat{a}_t = (1-\phi_{1,t})a_t$ be the tax-adjusted bond price in Country 1.

Conversely, world capital account clearing, (12), imposes a time-inconsistency problem for a tax on capital inflows, $\phi_{2,t} > 0$. Consider a tax which is announced and imposed in the same period. The imposition of a tax on capital inflows by Country 2 in period t distorts the world bond market clearing of period t bonds. Specifically, the world bond market clearing condition for period t bonds becomes:

$$\sum_i b^i_{1,t} + (1-\phi_{2,t})\sum_i b^i_{2,t} = 0. \tag{14}$$

However, by construction, the bond market, (6), must have cleared in period $t-1$ when the period t bonds were traded. Hence, the imposition of $\phi_{2,t} > 0$ causes the net repayments from Country 1 to the bank(s) in Country 2 to fall short of its (their) net obligations. That is, the bank(s) in Country 2 will not receive sufficient after-tax repayments to pay off their bond obligations. Hence, the announcement and imposition of a tax in the same period must necessarily imply the insolvency of the bank(s) in Country 2. Next, consider the announcement of a tax in period $t-1$ which will be imposed in period t. Hence, in period t, the after-tax bond market clearing condition (14), must hold for an equilibrium to be sustained. However, this implies by necessity that in period $t-1$:

$$-a_{t-1}\sum_i b^i_{1,t} - a_{t-1}(1-\phi_{2,t})\sum_i b^i_{2,t} = 0 \Rightarrow -a_{t-1}\sum_i b^i_{1,t} - a_{t-1}\sum_i b^i_{2,t} < 0 \tag{15}$$

which cannot be an equilibrium.

7 Agent's Problem

Where Table 1 defines the parameters of the model then agents in country j face the following budget constraint:

$$
\begin{aligned}
c^i_{j,t} + \frac{k^i_{j,t+1}}{1-\delta} - (k^i_{j,t} - (1-\tau^i_{j,t})q^i_{j,t}) + a_{j,t}b^i_{j,t+1} &= \eta^i_{j,t}(k^i_{j,t})^\alpha + \tau^i_{j,t}b^i_{j,t} + T_j && (16)\\
b^i_{j,t+1} &\geq \underline{b}^i_{j,t+1} && (17)\\
\tau^i_{j,t} &= \{0,1\} && (18)
\end{aligned}
$$

where:

Table 1: **Parameter Definitions**

Parameter	**Definition**
$c^i_{j,t}$	consumption by agent i in period t
$k^i_{j,t+1}$	capital stock of agent i in period t
$\delta \in [0,1]$	depreciation rate on capital
$b^i_{j,t+1}$	bond savings at time t by agent i
$a_{j,t}$	the price of a bond at time t
$\underline{b}^i_{j,t+1}$	the endogenous collateral constraint on borrowing
$\tau^i_{j,t} = 0$	agent i defaults in period t
$\tau^i_{j,t} = 1$	agent i does not default in period t
$q^i_{j,t}$	collateral demanded for a loan $b^i_{j,t}$
T_j	lump sum transfer from the government

Under default agents are assumed to forgo repayment of a debt $b^i_{j,t}$ and forfeit their collateral. Hence the remaining capital stock of agent i after default is simply the difference between the capital stock at the beginning of the period and the amount claimed by the bank.[7]

Given that a_t, $q^i_{j,t}$ and $\phi_{j,t}$ are taken as exogenous by the agent in country j, the agent's problem can be written recursively in a stationary setting. Let $\Gamma^i_{j,t}$ represent the set of feasible choices for agent i in country j at time t. That is, $\Gamma^i_{j,t}$ is the set of all $(c^i_{j,t}, k^i_{j,t+1}, b^i_{j,t+1}, \tau^i_{j,t})$ vectors such that equations (16) - (18) are satisfied.

The agent's problem can be formulated as a dynamic program. Formally, given the state

[7]Default in the model is a binary choice variable. That is, agents cannot default on a fraction of their debt. This assumption is by construction restrictive but it serves the purpose of restricting renegotiation. However, the assumption of a binary choice over default seems justified since only one-period debt contracts are considered. Were this model extended to include multiple-period debt, then it would seem plausible to allow agents to default on their debt in some periods and not others. I leave this analysis to extensions of the current paper.

$S^i = (k^i_t, b^i_t, \eta^i_t)$ agent i chooses $(c^i_t, k^i_{t+1}, b^i_t, \tau^i_t)$ to satisfy:

$$V(S^i) = \max_{\Gamma^i_{j,t}} u(c^i_t) + \beta \sum_{\eta'} V(S^{i\prime})\pi(\eta'|\eta) \tag{19}$$

The choice over future period states reflects the future effects of default. S' denotes the next period state. Finally, $\mu(S^i)$ refers to the measure of agents in state (S^i).

8 Timing

The timing of the model is as follows. In each period, the following sequence of events occurs:

1. The government j reveals (chooses) the tax rate ϕ_j.
2. Agents produce using their available capital stocks.
3. Agents learn their shock and receive their output.
4. Agents settle their previous-period contracts, if such contracts exists and agents contract (or not) with the bank.
5. Banks exchange one-period payments and fulfill obligations.
6. Agents choose next-period capital holdings and consume.
7. Depreciation occurs.

The nature of the timing is not innocuous. In particular, the timing rules out a potential hold-up problem on the part of the lender. Second, the nature of the timing implies that depreciation affects the lender and the borrower in the same manner.[8]

9 Equilibrium

In this section, I describe the equilibrium of the model. In particular, I examine the three sectors of the model; banks, the government and agents, and define the economy-wide equilibrium. In addition, I demonstrate that taxing capital inflows can be an optimal policy for the government.

[8]The timing of the depreciation also simplifies the computational analysis in later sections. The difficultly with depreciation immediately following production is that it implies that lenders must discount collateral twice before they can consume it while borrowers only discount collateral once. For this reason, I assume that depreciation occurs at the end (or equivalently the beginning) of a period so that borrowers and lenders have the same discount period over collateral.

9.1 Equilibrium Definition

A stationary recursive competitive equilibrium in the model is defined as follows for $j = 1, 2$:

A set of functions, $V_j(S^i)$, $b_j(S^i)$, $k_j(S^i)$, $c_j(S^i)$, $y_j(S^i)$, $\tau_j(S^i)$, $\mu_j(S^i)$, *a tax rate* ϕ_j *and a bond price* a, *such that given* γ_j, β_j, δ_j, α_j, η_j *and* $\pi_j(\eta'|\eta)$*:*

1. The government in country j chooses ϕ_j subject to Equations (11) and (10).
2. Given ϕ_j, a is given by the world bond market clearing condition, equations (12) and (13).
3. Given ϕ_j and a, agent i in country j chooses $b_j(S^i), k_j(S^i), c_j(S^i), \tau_j(S^i)$ to maximize her dynamic problem $V_j(S^i)$ given by equation (19).
4. The agent's output is given by (1).
5. Aggregates result from individual behavior, $K_j = \sum_i \mu_j(S^i)k_j^i$ and $Q_j = \sum_i \mu_j(S^i)q_j^i$.
6. There exists an invariant probability measure P_j defined over the ergodic set of equilibrium distributions in country j.

9.2 Equilibrium Collateral Default

The bank determines q_{t+1}^i such that the expected value of the collateral q_{t+1}^i is equal to the expected value of the bond repayment, b_{t+1}^i. This implicitly determines the endogenous borrowing constraint for an agent i such that $|b_{t+1}^i| \leq (1-\delta)(k_t^i - (1-\tau_t^i)q_t^i) \equiv \underline{b}_{t+1}^i \;\; \forall b_{t+1}^i < 0$. Since no agent will refuse (default on) the repayment of a bond, then $q_{t+1}^i = 0 \;\forall\; b_{t+1}^i \geq 0$.

The price of capital is trivially determined in the model. The agents' budget constraint (16) implies that the price of capital is simply the numeraire price, 1, $\forall t$. The bank is assumed to sell off the received collateral so that all agents receive an identical share as long as they have sufficient resources to purchase capital.[9] However, in equilibrium, it must be that:

$$\sum_i \frac{k_{t+1}^i}{1-\delta} - (k_t^i - (1-\tau_t^i)q_t^i) \geq \sum_{\tau_t^i = 0} q_t^i \tag{20}$$

[9] Alternative disbursement programs won't affect the aggregate amount of net investment and thus the sale format has no impact on the equilibrium in this case.

If received collateral is such that (20) is not satisfied then the total amount of collateral to be sold is greater than the total amount of net investment and hence the price of capital must adjust.The next proposition demonstrates that Equation (20) must hold by construction.

Proposition 9.2.1 *In equilibrium, no agent defaults, i.e.* $\tau^i = 1\ \forall i$ *when (20) holds.*

Proof: Since the price of capital and the face value price of bonds are identical, then there is no effect on an agent's current period budget constraint of exchanging capital for bonds. Hence, no agent can be made better off by defaulting when the price of capital is 1. In addition, agents who face borrowing constraints are strictly better off by having more capital and thus prefer to pay off their bond debts using current period income and, if need be, some of their capital stock. By defaulting, they pay off their bond debts using just their capital stocks which means that the amount of capital available to collateralize loans would be less than if they used some current period income. Hence, default will not arise in equilibrium. ■.

9.3 Optimal Capital Inflow Taxation

The following proposition establishes the optimality of $\phi_j > 0$ for the government in country j.

Proposition 9.3.1 *A necessary condition for* $\phi_j > 0$ *to be optimal for a government in country* j *is that there exists a sufficient positive measure of borrowing-constrained agents.*

Proof: A borrowing agent in period t can be identified by two characteristics: they enter period t with insufficient wealth holdings to self-smooth their preferred consumption sequence, given a_t; and they expect to have a higher income (wealth) from which to borrow in period $t+1$. Hence, borrowers have relatively lower levels of wealth (in both capital and bonds) than lenders and constrained borrowers have relatively the lowest levels of wealth. To see this last part, note that a constrained borrower has debts which equal their collateralizable assets (capital) and hence can only consume and invest out of current income. In fact, depreciation implies their consumption must be strictly less than their income. No unconstrained borrower has as tight a period budget constraint. Carroll and Kimball [9] demonstrate that the consumption function is concave in wealth. Define $\hat{b}^i_{j,t+1} < 0$ as the optimal unconstrained borrowing choice of agent j in period t conditional on loans being repaid in period $t+1$. Let $\hat{\mu}_{j,t} > 0$ be the measure of agents for whom the constrained borrowing choice $0 > b^i_{j,t+1} > \hat{b}^i_{j,t+1}$. That is, $\hat{\mu}_{j,t}$ is the measure of agents who cannot borrow to the extent that they would prefer. Next, I note that the government's budget constraint, (10), implies that the revenues gained from the imposition of the tax must equal the lump-sum transfer in aggregate. Since the tax is collected from bond repayments ($\hat{a}_t < a_t$) and the lump-sum transfer is paid to all agents, then the implication is that the median borrower pays more in the tax than they receive from the transfer, so the tax in costly to most borrowers in a welfare sense.

In a slight abuse of notation, given a current period shock $\eta \in N_j$, let:

$$E(c^i_{j,t+1})^{-\gamma_j} = \sum_{\eta^i \in N_j} \pi_j(\eta^i|\eta)(c^i_{j,t+1}|\eta)^{-\gamma_j}.$$

Then the agents' problem, (19), implies the following Euler equation:

$$\left(\frac{\mathrm{E}(c^i_{j,t+1})}{(c^i_{j,t}|\eta)}\right)^{\gamma_j} \frac{a_t}{\beta_j} \geq 1, \tag{21}$$

with inequality for any constrained borrowing agent. One implication of the agents' Euler equation is that a constrained agent may not change her consumption behavior in response to a change in a_t.

I am now in a position to prove the proposition. The Benveniste and Scheinkman formula and the agents' Euler equation implies that for an unconstrained agent:

$$\frac{\partial V(S^i)}{\partial a} = -\frac{\partial c^i_{j,t+1}}{\partial a}(c^i_{j,t})^{-\gamma} \tag{22}$$

$$\frac{\partial V(S^i)}{\partial T_{j,t}} = (c^i_{j,t})^{-\gamma} \tag{23}$$

Hence, the effect of the tax is unambiguously beneficial to lenders and small borrowers but costly to most borrowers. However, for constrained borrowers $\frac{\partial c^i_{j,t+1}}{\partial a} = 0 = \frac{\partial V(S^i)}{\partial a}$ and the tax is unambiguously beneficial. Thus, whenever:

$$\sum_{i \in \hat{\mu}_{j,t}} \frac{\partial V(S^i)}{\partial T_{j,t}} > \sum_{i \ni \hat{\mu}_{j,t}} \left(\frac{\partial V(S^i)}{\partial a} + \frac{\partial V(S^i)}{\partial T_{j,t}}\right) \tag{24}$$

then a tax, $\phi_{1,t}$ is welfare improving. Where $c^i_{j,t}$ is lowest for constrained agents then the concavity of utility implies there exists a $\hat{\mu}_{j,t}$ such that a tax is optimal. ■

Corollary 9.3.2 *If a tax on capital outflows was permissible, then a tax, $\phi_{2,t} > 0$, and a lump-sum transfer, $T_{2,t} > 0$, would be welfare improving if there exists a sufficient measure of borrowing agents.*

Proof: The imposition of a tax, $\phi_{2,t} > 0$, on net bond outflows from Country 2 can be welfare-improving since the tax adjusted bond price in Country 2 must rise. Hence, the tax reduces the borrowing costs while concurrently increasing the costs of lending. Moreover, the lump-sum transfer $T_{2,t} < 0$ implies a benefit for all agents. Hence, a tax, $\phi_{2,t} > 0$, would redistribute wealth from lenders to borrowers and hence be welfare improving if the measure of borrowers is sufficient large. However, since Country 2 is a net lender then the measure of borrowers may not be sufficiently large. The presence of borrowing constrained agents does not improve the welfare-consideration because unconstrained borrowers actually benefit the most. ■

However, a tax on capital outflows is not feasible for the government to impose. I note also that Proposition 9.3.1 only requires that agents be borrowing constrained. It does not explicitly require that agents are *collateral* constrained. That collateral constrains borrowing in the model emerges from the model environment. However, alternative model environments where a positive measure of agents face borrowing constraints could similarly benefit from taxes on capital inflow combined with lump-sum transfers.

10 Numerical Examples

In this section, I present preliminary evidence on the welfare benefits of international intermediation and the benefits of capital inflow taxation [the latter part to come]. The examples are meant simply to provide some intuition as to the size of the welfare effects which are difficult to determine analytically.

The first example examines the impact of international capital flows with countries which are symmetric in terms of their size, $M = 1$, and the riskiness of the shock, $N_1 = N_2$ but which differ in the persistence of the shock, π_j. The second example examines the impact of international capital flows when the countries differ in size, $M < 1$ and persistence but where the riskiness of the shock remains symmetric. Thus, example 1 is intended to provide intuition on the benefits of capital inflow taxation on large open economies and example 2 is intended to provide intuition on the benefits of capital inflow taxation on small open economies. I note that the results presented are steady-state comparisons although the conditions under which a tax is welfare-improving do not hinge on stationarity. In particular, it is possible and indeed likely that the optimal welfare gains over the transition to a steady-state might require a time-varying tax.

Also, it is not *a priori* clear how best to measure the welfare gains from international capital markets. I present two measures. The first is a measure of the social welfare costs relative to the social welfare of a complete markets model. That is, the welfare cost is expressed as a percentage of the complete markets social welfare. The second measure reported, termed CV, is the lump-sum transfer of consumption required to attain the social welfare of a complete markets model. Hence, it is similar to a measure of compensating variation. It is important to note, however, that a lump-sum transfer to all agents is different from determining the lump-sum required to raise aggregate consumption to the complete markets level because of the heterogeneity of consumption levels.

The following sections present the results of the numerical examples. In particular, the results demonstrate that the welfare consequences of international capital markets can be substantial, in the order of 2.5 per cent of steady-state consumption. In addition, the results show that the relative sizes of the countries affects substantially the equilibrium world risk-free real interest rate. Figures 1-4 in the Appendix complement the quantitative results presented in the following sections.

10.1 Example 1

I set $\pi_1(\eta_1|\eta_1) = 0.9$ and $\pi_2(\eta_2|\eta_2) = 0.5$ for Country 1 and Country 2 respectively. That is, shocks in Country 1 are more persistent than shocks in Country 2. The remaining parameters of the model are identical across countries and are described below.

Parameter	**Definition**
α	0.4
$\bar{\eta}$	1.25
$\underline{\eta}$	0.75
β	0.96
γ	2
δ	0.1
M	1

Table 2 compares the steady-state equilibria between the open-economy and closed economy cases for both Country 1 and Country 2. By construction, open capital markets equate the real interest rate across countries. As should be expected, international capital flows cause the gap in the marginal product of capital across both countries to narrow. However, the gap does not entirely disappear, owing to the incomplete nature of the domestic bond markets. Indeed, the marginal product of capital in Country 1 actually increases slightly, indicating less capital over-accumulation. The change in output, Y, in Country 2 is more sizeable than in Country 1 as the fall in the marginal product of capital implies that the aggregate capital stock is increasing. Aggregate output in Country 1 actually falls slightly, as the increase in the marginal product of capital reduces steady-state capital. Finally, as one would expect, the increase in risk-sharing resulting from international capital flows increases steady-state consumption in both countries. The increase is sharpest in Country 2 where virtually all agents benefit from reducing the volatility of the random-walk shocks.

Table 2: Results

Country 1

Model	Y	C	K	B	KF	i	ROE
Open Economy	2.0404	1.4020	5.7498	1.9135	0.6376	2.50%	3.13%
Closed Economy	2.0417	1.3999	5.7590	1.5494	0	2.50%	3.05%
Closed Economy Autarky	1.9987	1.3748	5.58				3.72%

Country 2

Model	Y	C	K	B	KI	i	ROE
Open Economy	2.0348	1.3737	5.840	0.7057	−0.6336	2.50%	3.00%
Closed Economy	1.8857	1.3372	4.8554	1.3532	0	3.60%	4.43%
Closed Economy Autarky	1.861	1.3537	4.67				5.07%

Open Economy refers to the collateral model with free capital flows; Closed Economy refers to the collateral model with no international capital flows; Closed Economy Autarky refers to the case with no collateral trades and no international flows; Y refers to aggregate equilibrium output; C refers to aggregate equilibrium consumption; K is aggregate equilibrium capital stock; B is the face-value of all bonds traded; KF is the capital inflow (inflow if negative); i is the interest rate on the risk free bond; and ROE refers to the average expected return to capital;

Table 3 presents welfare comparisons of the open-economy and closed-economy models for both Country 1 and Country 2. In both cases, welfare is higher in the open-economy equilibrium than in the closed-economy equilibrium. Thus, the increase in steady-state aggregate consumption appears to imply a social welfare gain. In Country 1, the welfare gain is predominantly among borrowers who benefit from the lower cost of borrowing. In addition, there is a slight decrease in the measure of agents who are borrowing constrained which is strictly welfare improving. The indices of inequality suggest also that there is a slight decrease in inequality in Country 1 in the open-economy equilibrium.

However, it is in Country 2 that the welfare gains appear more pronounced. The welfare gains from international intermediation appear to halve the welfare costs associated with incomplete markets. The welfare gains are associated with a roughly 50 percent decrease in the measure of borrowing constrained agents, which suggests that international capital inflows can have tangible distributional consequences. The indices of inequality substantiate this point strongly, as both the Gini and the Generalised Entropy measures show sharp reductions in inequality. Thus, the presence of incomplete markets with collateral constraints appears to dramatically increase the welfare gains to international capital markets relative to the complete markets comparisons of [4] and [26]. In particular, the welfare gains are equivalent to roughly 2.5 per cent of steady-state consumption.

Table 3: Welfare Comparisons

Country 1

Model	Social Welfare	CV	Gini	GE	Top 30	Constrained
Open Economy	5.73	0.0697	0.3940	0.1105	56.61	17.24
Closed Economy	6.03	0.0731	0.4054	0.1190	57.44	18.27

Country 2

Model	Social Welfare	CV	Gini	GE	Top 30	Constrained
Open Economy	2.96	0.0389	0.1890	0.0265	43.00	5.92
Closed Economy	5.71	0.0731	0.3953	0.1100	59.28	10.00

Open Economy refers to the open-economy collateral model; Closed Economy refers to the closed-economy collateral model; Social Welfare refers to the difference between either model and a closed-economy complete market model (*i.e.* a Social Planner) where the number presented is the welfare cost is expressed as a percentage of the welfare in the closed-economy complete market model; CV is the per-capita lump-sum transfer of consumption required to attain the social welfare of the closed-economy complete market model; Gini refers to the Gini coefficient on the distribution of wealth, where wealth is defined as the sum of capital and bonds held by an agent; GE refers to the Generalised Entropy Measure on wealth using a weighting parameter of 0; Top 30 refers to the percentage of wealth held by the top 30 percent of agents; and Constrained refers to the percentage of agents who face a binding borrowing constraint.

10.2 Example 2

Again, I set $\pi_1(\eta_1|\eta_1) = 0.9$ and $\pi_2(\eta_2|\eta_2) = 0.5$ for Country 1 and Country 2 respectively. That is, shocks in Country 1 are more persistent than shocks in Country 2. The key difference is that I assume Country 1 is large relative to Country 2. Specifically, I set $M = 0.1$ so that the population of Country 1 is ten times larger than that of Country 2. The remaining parameters of the model are identical across countries and are described below.

Parameter	**Definition**
α	0.4
$\bar{\eta}$	1.25
$\underline{\eta}$	0.75
β	0.96
γ	2
δ	0.1
M	0.1

Table 4 compares the steady-state equilibria between the open-economy and closed economy cases for both Country 1 and Country 2. Again, in the open economy model, output and consumption are both higher than in the closed economy model. As well, the marginal products of capital in both countries tend to converge. However, the striking result from Table 5 is that the risk-free real interest rate is actually lower in the open economy model than the closed economy risk-free real interest rate in either country. That is, the risk-free real interest rate does not converge to some intermediate value as occurs in Example 1 above. The reason is that some lenders in Country 1 in the closed economy model who desire to lend are constrained because of the collateral requirements which prevent some borrowers from being able to borrow their optimal amount. However, with international lending and borrowing, these lenders in Country 1 can circumvent the domestic credit market and lend to agents in Country 2. Moreover, because the real interest rate at which these lenders in Country 1 are willing to lend is at or below 2.50 per cent then there is downward pressure on the real interest rate. Furthermore, a lower risk-free real interest rate in Country 2 induces relatively wealthier agents to borrow rather than lend which increases the demand for borrowing in Country 2 and leads the world bond market to clear. Hence, the relative size of Country 1 and Country 2 can have a significant impact on the world risk-free real interest rate as should be expected.

Table 5 presents welfare comparisons of the open-economy and closed-economy models for both Country 1 and Country 2. In both cases, welfare is higher in the open-economy equilibrium than in the closed-economy equilibrium. Thus, the increase in steady-state aggregate consumption appears to imply a social welfare gain. In Country 1, the welfare gain is predominantly among lenders and

Table 4: Results

Country 1							
Model	Y	C	K	B	KF	i	ROE
Open Economy	2.0527	1.4025	5.8389	1.4935	0.0835	2.33%	3.02%
Closed Economy	2.0417	1.3999	5.7590	1.5494	0	2.50%	3.05%
Closed Economy Autarky	1.9987	1.3748	5.58				3.72%

Country 2							
Model	Y	C	K	B	KI	i	ROE
Open Economy	2.0507	1.3774	5.9101	0.6668	-0.0827	2.33%	2.86%
Closed Economy	1.8857	1.3372	4.8554	1.3532	0	3.60%	4.43%
Closed Economy Autarky	1.861	1.3537	4.67				5.07%

Open Economy refers to the collateral model with free capital flows; Closed Economy refers to the collateral model with no international capital flows; Closed Economy Autarky refers to the case with no collateral trades and no international flows; Y refers to aggregate equilibrium output; C refers to aggregate equilibrium consumption; K is aggregate equilibrium capital stock; B is the face-value of all bonds traded; KF is the capital inflow (inflow if negative); i is the interest rate on the risk free bond; and ROE refers to the average expected return to capital;

unconstrained borrowers who benefit from the greater consumption smoothing which international lending provides. However, there is a slight increase in the measure of agents who are borrowing constrained which is costly in terms of social welfare. The indices of inequality suggest also that there is a modest decrease in inequality in Country 1 in the open-economy equilibrium. In addition, relative to the results in Example 1, the levels of inequality in Country 1 appear to decrease more. The reason is simply that the lower risk-free real interest rate increases the wealth of borrowers slightly, which reduces inequality.

Again, it is in Country 2 that the welfare gains appear more pronounced although not quite as large as in Example 1. The welfare gains from international intermediation also appear to roughly halve the welfare costs associated with incomplete markets. In particular, the welfare gains to international capital markets are equivalent, similar to the results in Example 1, with approximately a 2.5 per cent increase in steady-state consumption. The welfare gains are associated, also, with a roughly 40 percent decrease in the measure of borrowing constrained agents, which suggests that international capital inflows can have tangible distributional consequences. Once more, the indices of inequality substantiate this point strongly, as both the Gini and the Generalised Entropy measures show sharp reductions in inequality. Thus, the presence of incomplete markets with collateral constraints appears to dramatically increase the welfare gains to international capital markets relative to the complete markets regardless of the size of each country.

Table 5: Welfare Comparisons

Country 1

Model	Social Welfare	CV	Gini	GE	Top 30	Constrained
Open Economy	5.66	0.069	0.3836	0.1069	56.50	18.77
Closed Economy	6.03	0.0731	0.4054	0.1190	57.44	18.27

Country 2

Model	Social Welfare	CV	Gini	GE	Top 30	Constrained
Open Economy	2.86	0.0375	0.2058	0.0308	43.94	6.34
Closed Economy	5.71	0.0731	0.3953	0.1100	59.28	10.00

Open Economy refers to the open-economy collateral model; Closed Economy refers to the closed-economy collateral model; Social Welfare refers to the difference between either model and a closed-economy complete market model (*i.e.* a Social Planner) where the number presented is the welfare cost is expressed as a percentage of the social welfare in the closed-economy complete market model; CV is the per-capita lump-sum transfer of consumption required to attain the social welfare of the closed-economy complete market model; Gini refers to the Gini coefficient on the distribution of wealth, where wealth is defined as the sum of capital and bonds held by an agent; GE refers to the Generalised Entropy Measure on wealth using a weighting parameter of 0; Top 30 refers to the percentage of wealth held by the top 30 percent of agents; and Constrained refers to the percentage of agents who face a binding borrowing constraint.

11 Conclusion

This paper has investigated the effects of international lending and borrowing when domestic credit markets are incomplete. One implication of incomplete domestic credit markets is that a second-best result emerges. That is, a tax on capital inflows combined with a lump-sum transfer can be welfare-improving for a government to impose when some agents are borrowing constrained. The intuition is simply that constrained agents benefit from the transfer but do not suffer as a result of the tax.

A second finding from this paper is that international lending can have substantial implications for social welfare when domestic markets are incomplete. Specifically, if borrowers face collateral constraints on borrowing then, depending on the properties of the idiosyncratic shock, the aggregate social welfare gains can be in the neighborhood of 2.5 per cent of social welfare. Thus, the welfare gains to international lending may be much higher than previous complete market estimates have suggested.

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12 Appendix

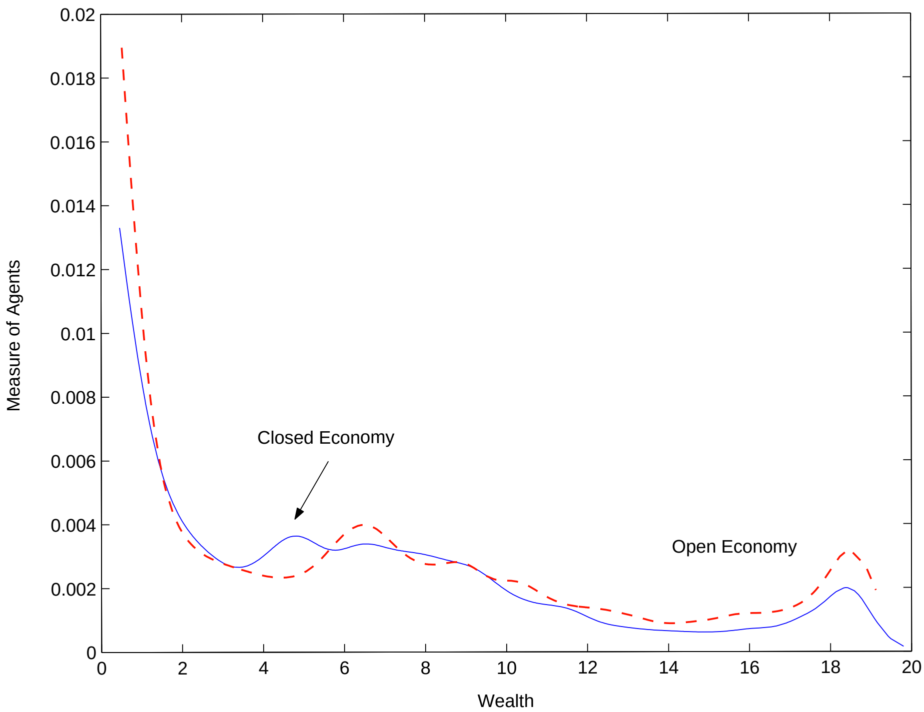

Figure 1: Example 1: Distribution of Wealth, Country 1

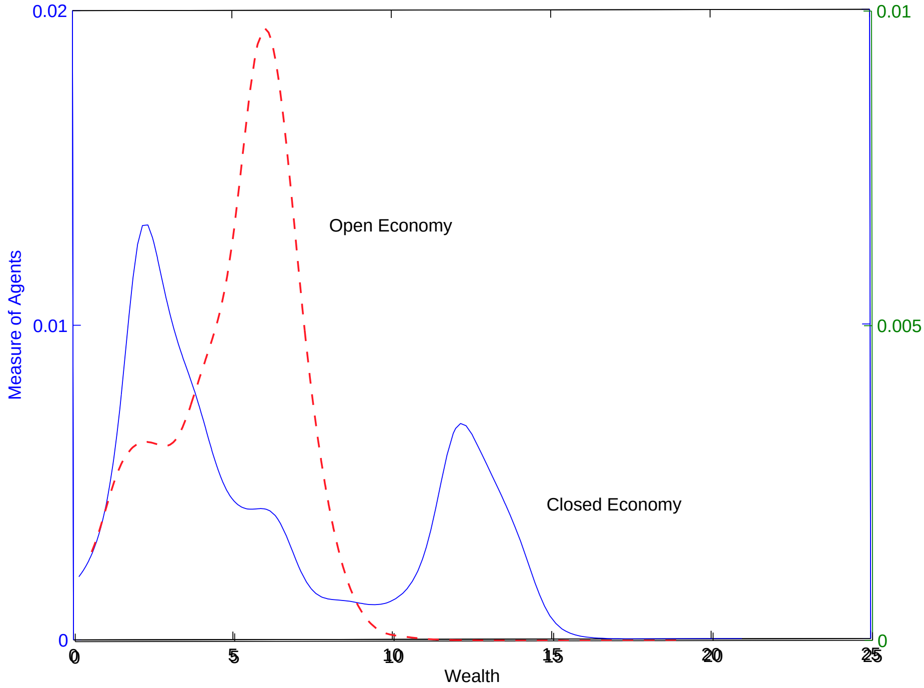

Figure 2: Example 1: Distribution of Wealth, Country 2

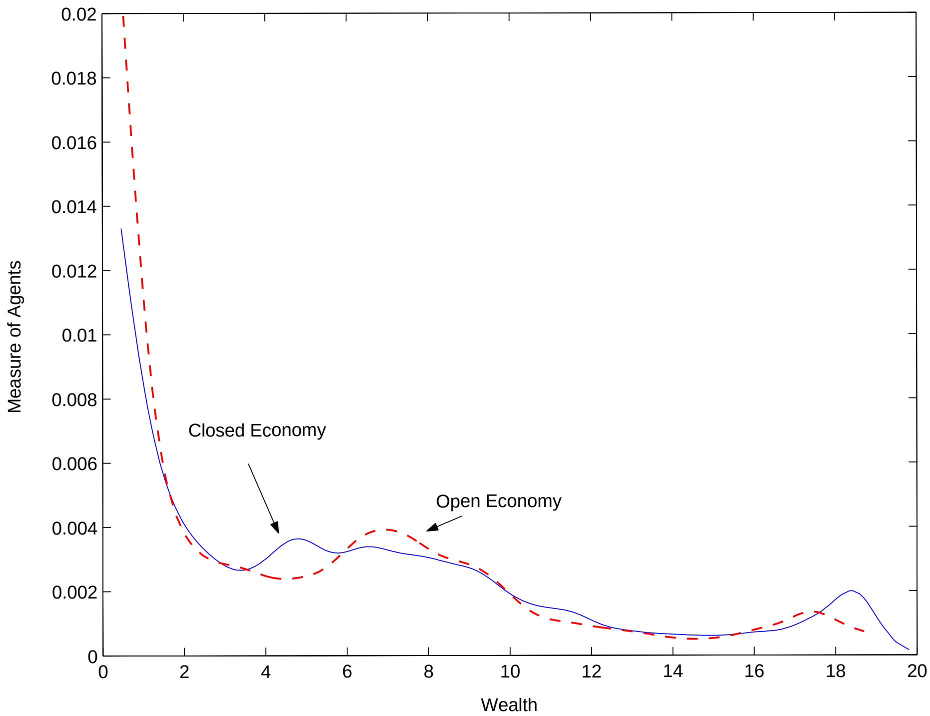

Figure 3: Example 2: Distribution of Wealth, Country 1

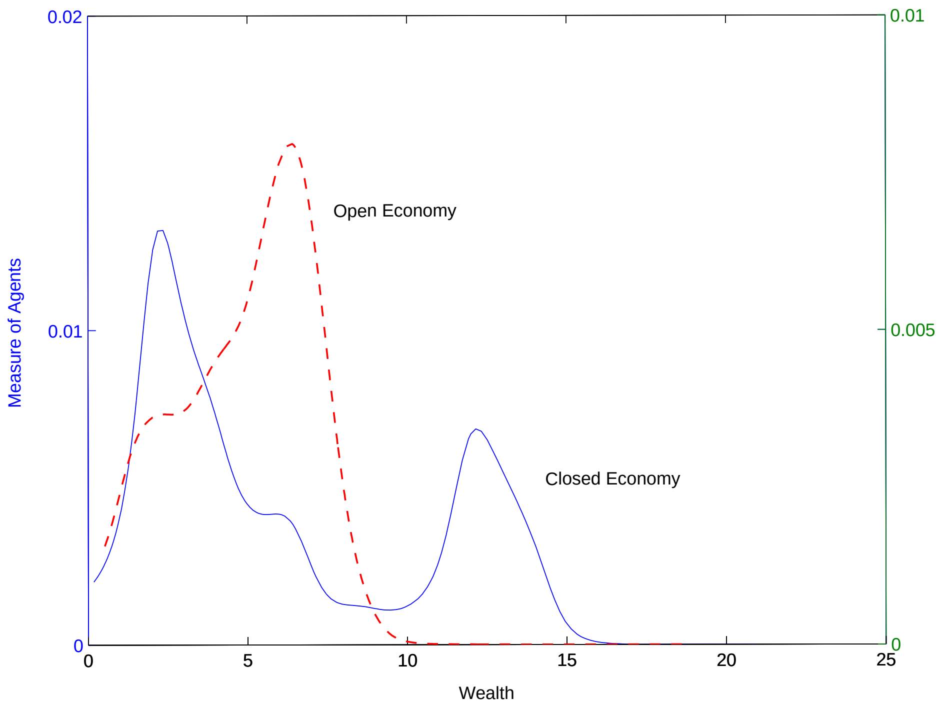

Figure 4: Example 2: Distribution of Wealth, Country 2