Labor Market Dynamics under Long Term Wage Contracting *

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Abstract

This paper investigates the effects of different wage contracting set ups for labor market dynamics. With incomplete markets and risk averse workers the labor market can behave in a way which provides workers income smoothing. I investigate whether this mechanism can help explain the failure of the matching model to account for the high cyclical volatility in vacancy-unemployment ratios that is observed. I find that both introducing risk aversion and further introducing limited commitment contracting, increase the volatility the model produces. *The quantitative assessment is not yet finished.*

1 Introduction

Labor income generally forms a large part of the income of consumers, so it is of central importance for their decisions. Labor productivity varies over time however, which means that in general consumers would wish to smooth their income in some way. This paper explores the implications of income smoothing which takes place through employment contracts. One reason why it would be natural to think that the employer would take the role of the insurer is that he probably can observe the productivity better than

* *This is a preliminary and unfinished version of the paper.* Please note that it includes color graphs. I am grateful to Fernando Alvarez, Francois Gourio and Robert Shimer for discussions.

outside parties. I will propose a model where there exist no asset markets, but wage contracts and labor market conditions reflect the preference for income smoothing.

In this model a set of agents face both cyclical and idiosyncratic labor productivity shocks and participate in a labor market where wage contracts offer insurance against this variation. The agents also face the risk of an employment contract ending and the labor market will provide a form of equilibrium insurance against this risk. The ending of an employment relationship means that the worker must spend a period of uncertain length suffering from reduced consumption, while he searches for a new job.

A key aspect of this model is that the labor market equilibrium involves a tradeoff between the present value the worker gets from a new wage contract and the probability of finding a job. One outcome of this is that when agents are more risk averse, the equilibrium shifts toward the latter as workers dislike the changes in consumption related to unemployment spells.

I consider alternative assumptions on the type of contract that is feasible, differing in enforceability conditions. Full commitment contracting implies that both employer and employee will honor the contract as long as the match remains productive. This will imply a constant wage within a contract. On the aggregate level wages nevertheless move cyclically due to the fact that wage contracts reflect the productivity conditions at the time of contracting.

I also consider limited commitment contracts, where both parties are free to leave the contract if searching for a new match is more attractive. This reduces the insurance that the contract can provide and implies procyclical movement in the wage during a contract. This type of contract compared to a full commitment one has the feature that the firm has less control over long run wages due to the participation constraints that constrain the contract. This turns out to have the effect that over the cycle there is less variation in the value newly hired workers receive from a contract. In turn there is more variation in job finding rates.

I use the model to quantitatively assess whether accounting for the long run contracting aspect of wages can improve on some reported empirical mismatch of the Diamond-Mortensen-Pissarides model when applied to a business cycle setting. The DMP-model was first considered in a cyclical setting in Mortensen and Pissarides (1994), who demonstrate that the model delivers many features of the data. The match has later been evaluated by various authors, but most recently Shimer (2005) (who provides an overview of this literature), points to a sizable mismatch between theory and data in the cyclical variability of the central variable of the model - the vacancy unemployment ratio. He states that, "In a reasonably calibrated model, the v-u ratio is only ten percent as volatile as in U.S. data."

A key feature that leads to reduced variation in the v-u ratio in the model is that the values that workers receive from a new contract at different phases of the cycle (which are the present values of wages and possible unemployment spells) vary quite freely. Hence firm values vary less and there is not much variation in the number of vacancies posted. In a bargaining setting this corresponds to the assumption that workers' bargaining power is constant over the cycle.

One way of creating less variability in worker values is wage rigidity across contracts. It has been investigated and confirmed by Hall (2004) and Shimer (2004) that this type of wage rigidity would solve this variability puzzle. The kind of wage rigidity that is meant is one where wages across contracts signed at different phases of the cycle are closer together than fixed weight bargaining would imply[1]. These papers impose an essentially exogenous fixed wage and find that the resulting increased variability in vacancies is consistent with that in the data.

This paper attempts to create the lack of variability in worker values endogenously through an explicit modelling of wage contracts as a form of income smoothing when agents are risk averse. The first departure from the standard DMP model is risk aversion, which will imply that the worker has preferences on the timing of wage payments in a contract and not only the present value. The second departure builds on this to consider how the equilibria look under efficient contracts with alternative assumptions on what is feasible.

The question I hope to answer is whether the increase in variability of the v-u ratio due to these considerations is quantitatively significant relative to the large disparity in the standard DMP model. *I find that introducing risk aversion alone will increase the variability of the v-u ratio and introducing limited commitment contracting amplifies the variation further. I have still to finish the quantitative assessment.*

Next I introduce the model and provide some characterization of the equi-

[1] Whether wages are rigid within a contract is less relevant.

librium. The proofs are in the appendix. I then present results of a computational experiment for a simplified case.

2 Model

Preferences and Technologies

There are N risk averse workers with preferences $E_t \sum_{\tau=0}^{\infty} \beta^{t+\tau} u(c_{t+\tau})$, where u is a CRRA utility function. They consume their income each period, so that consumption c equals the wage w if the agent is working and b if the agent is unemployed.[2]

There are a continuum of risk neutral entrepreneurs who also must consume their income each period, but they are assumed to have other endowments such that the income is not constrained to be non-negative. Their preferences are given by $E_t \sum_{\tau=0}^{\infty} \beta^{t+\tau} c^e_{t+\tau}$, where c^e is the sum of current period cash flows from firms that the entrepreneur is owns.

There are two technologies in this economy: a production technology and a search technology. The production technology is such that a single consumption good can be produced with labor as the only input and with constant returns to scale. In what follows I refer to a single worker production unit as a firm. Entrepreneurs have free access to this technology and one entrepreneur can operate an unlimited number of firms.

Output of an operating firm during a period is given by exz, where z is an aggregate shock, e is a match specific shock that governs whether the firm continues to operate and x is a match specific shock that captures idiosyncratic productivity variation. The aggregate shock $z > 0$ follows an n state Markov chain with transition probabilities π_{ij}, such that the transition matrix Π is monotonic. The match specific e is such that it is equal to one for all new matches and each period with probability δ it will remain at one and with probability $1 - \delta$ will switch to zero forever. The second match specific shock $x > 0$ is iid with two possible values $x_H > x_L$. New matches start at x_L and I denote the probability that $x_t = x_H$ by p. All shocks are independent of each other. The cash flow of an operating firm during a period is $c^f = xz - w$, as as the firm will shut down once $e \neq 1$.

[2] Unemployment consumption is taken as exogenous and reflects the value of leisure.

I denote a realization of shocks by $s_t = (x_t, z_t)$ and the history of shocks for a given production unit that at time t is in state s_t and that is still producing τ periods later by $s^{t+\tau}|s_t = (s_t, s_{t+1}, \ldots, s_{t+\tau})$. I restrict attention to wage contracts that 1) end once the match becomes unproductive and, 2)have bounded wages. A wage contract $\sigma(s) = \{w_\tau(s^\tau|s)\}_{\tau=0}^{\infty}$ specifies a wage payment for each continuation history after s as long as the match remains productive. Denote the set of feasible contracts by $\Sigma(s) = \{\{w_\tau(s^\tau|s)\}_{\tau=0}^{\infty}|w_\tau \in [\underline{w}, \overline{w}]\forall\tau\}$.

Suppose that the value of searching for a job for an unemployed worker given the aggregate state z is $V^u(z)$. The value a worker gets from a new contract σ in state s can then be computed as

$$V_\sigma(s) = u(w_0(s)) + \sum_{\tau=1}^{\infty} \beta^\tau \delta^{\tau-1} E_{s^\tau|s}[\delta u(w_\tau(s^\tau)) + (1-\delta)V^u(z_\tau)].$$

Because of free entry, the value of searching for a worker for an entrepreneur is driven to zero. Imposing this, the value a firm gets from a new contract σ in state s can be computed as

$$F_\sigma(s) = xz - w_0(s) + \sum_{\tau=1}^{\infty} \beta^\tau \delta^\tau E_{s^\tau|s}(x_\tau z_\tau - w_\tau(s^\tau)).$$

I will assume that all new contracts start with $x = x_L$ and hence I will drop the x from notation when considering new contracts.

The labor market is characterized by search frictions which are captured by a matching function. When there are N_u unemployed agents searching for jobs and N_v vacant jobs available, the number of matches taking place this period is assumed to be given by a CRS matching function[3] $M(N_u, N_v) = \frac{N_u N_v}{N_u + N_v}$. Defining $\theta = \frac{N_v}{N_u}$ as the labor market tightness or vacancy unemployment ratio, the probability for a worker to find a firm this period is $\mu(\theta) = \frac{\theta}{1+\theta}$ and the probability for a firm to meet a worker is $q(\theta) = \frac{\mu(\theta)}{\theta}$.

In addition to workers and entrepreneurs, there also exist profit maximizing market makers in the economy, who can open markets for specific wage contracts. Each such sub-market is characterized by a contract σ and a market

[3]This specification is used by den Haan, Ramey, and Watson (2000) and compared to the more common Cobb-Douglas form it has the advantage that $\mu, q \in [0, 1]$ for all $\theta > 0$, which is helpful for computations.

tightness θ and these are known by all parties. Workers and entrepreneurs must choose between the sub-markets.

Equilibrium

The labor market operates following the competitive search equilibrium concept of Moen (1997) and Shimer (1995). An equilibrium of the economy consists of: A set of wage contracts $\{\sigma(z)\}$ for all z, with $\sigma(z) \in \Sigma(x_L, z)$, with corresponding market tightness $\theta_\sigma(z)$, worker value $V_\sigma^0(z)$ and firm value $F_\sigma^0(z)$. In addition search values for unemployed workers $V^u(z)$ and $U(z)$ for all z such that

1. Entrepreneurs posting a vacancy decide which sub-market to search in to maximize $q(\theta_\sigma(z))F_\sigma^0(z) - c$. This implies that all sub-markets with firms searching must offer the same value.

2. Posting a new vacancy would imply zero profit to an entrepreneur so that $q(\theta_\sigma(z))F_\sigma^0(z) - c = 0$.

3. Unemployed workers decide which sub-market to search in to maximize $\mu(\theta_\sigma(z))(V_\sigma^0(z) - V^u(z)) + (1 - \mu(\theta_\sigma(z)))V^u(z)$. This implies that all sub-markets with workers searching must offer the same value $U(z)$. Fixing this value the equation $U(z) = \mu(\theta_\sigma(z))(V_\sigma^0(z) - V^u(z)) + (1 - \mu(\theta_\sigma(z)))V^u(z)$ defines a strictly decreasing mapping $\theta_\sigma(z) = g(V_\sigma^0(z), z)$ that holds across all sub-markets that are open.

4. The search values of workers satisfy

$$V^u(z) = u(b) + \beta \sum_{z'} \pi(z'|z)U(z').$$

5. Market makers decide on which contracts to open markets for such that there is no surplus left. The equilibrium set of wage contracts $\{\sigma(z)\}$ is such that there exists no contract $\hat{\sigma}(z) \in \Sigma(z)$ such that

$$q(g(V_{\hat{\sigma}}^0(z), z))F_{\hat{\sigma}}^0(z) - c > q(g(V_\sigma^0(z), z))F_\sigma^0(z) - c.$$

Specific Contracts

The last item in the definition of equilibrium implies that the equilibrium must lie in the set of Pareto efficient contracts. For any state s and a feasible worker values V, the Pareto frontier is defined as

$$f(V,s) = \sup_{\sigma \in \Sigma(s)} \{F_\sigma(s) | V_\sigma(s) \geq V\}.$$

So far the set $\Sigma(s)$ has only been restricted by bounded wages and the constraint that once e switches to zero the contract ends without further payments. How does the Pareto frontier look under these assumptions? I call contracts that attain the Pareto frontier under these conditions full commitment contracts.

Since a contract on the frontier cannot be Pareto dominated after any history, f should satisfy

$$\begin{aligned} f(V,s) = &\max_{w,\{V(s')\}} \{xz - w + \beta E_{s'|s} \delta f(V(s'), s')\} \\ \text{s.t. } & V = u(w) + \beta E_{s'|s}[\delta V(s') + (1-\delta)V^u(z')], \\ & w \in [\underline{w}, \overline{w}]. \end{aligned}$$

Proposition 2.1 *For any $V \geq V^u(z)$ there exists a unique efficient contract and $f(V,s)$ is differentiable, decreasing, and concave in V.*

Lemma 2.1 $\frac{\partial f}{\partial V}(V,s) = -\frac{1}{u'(w)}$, *where the wage is optimally chosen given state (V,s).*

Proposition 2.2 *The optimal contract under full commitment has a constant wage throughout the contract.*

Suppose now that contracts are not enforceable and both firm and worker are free to leave a contract at any time if they prefer to. If the contract yields the entrepreneur a negative value when the outside option yields zero, he would prefer to walk away. Similarly if the contract yields the worker a value that is less than the search value $V^u(z)$, he would prefer to walk away.

Define $V_\sigma(s^\tau)$ to be the value that an operating contract σ will deliver a worker from today on, given that the contract has been in operation $\tau > 0$ periods and the history of shocks is s^τ. We have

$$V_\sigma(s^\tau) = u(w_\tau(s^\tau)) + \sum_{\tilde{\tau}=\tau+1}^{\infty} \beta^{\tilde{\tau}-\tau}\delta^{\tilde{\tau}-\tau-1} E_{(s^{\tilde{\tau}}|s^\tau)}[\delta u(w_{\tilde{\tau}}(s^{\tilde{\tau}})) + (1-\delta)V^u(z_{\tilde{\tau}})].$$

For firms respectively

$$F_\sigma(s^\tau) = x_\tau z_\tau - w_\tau(s^\tau) + \sum_{\tilde{\tau}=\tau+1}^{\infty} \beta^{\tilde{\tau}-\tau}\delta^{\tilde{\tau}-\tau} E_{(s^{\tilde{\tau}}|s^\tau)}(x_{\tilde{\tau}} z_{\tilde{\tau}} - w_{\tilde{\tau}}(s^{\tilde{\tau}})).$$

In this case the set of feasible contracts must be restricted such that $\tilde{\Sigma}(z) = \{\sigma(z) \in \Sigma(z) | V_\sigma(s^\tau) \geq V^u(z_\tau), F_\sigma(s^\tau) \geq 0$ for all continuation histories s^τ of (x_L, z), $\forall \tau \geq 0\}$.

I denote the Pareto frontier with this smaller feasible set as the limited commitment frontier $\tilde{f}$. This frontier should satisfy the Bellman equation

$$\begin{aligned}
\tilde{f}(V,s) = & \max_{w,\{V(s')\}} \{xz - w + \beta E_{s'|s}\delta \tilde{f}(V(s'),s')\} \\
\text{s.t. } & V = u(w) + \beta E_{s'|s}[\delta V(s') + (1-\delta)V^u(z')], \\
& V(s') \geq V^u(z'), \forall s', \\
& \tilde{f}(V(s'),s') \geq 0, \forall s', \\
& w \in [\underline{w}, \overline{w}].
\end{aligned}$$

Proposition 2.3 *For each s, the set of values for which an efficient contract exists is a closed and bounded interval $[V^u(z), \overline{V}(s)]$. In this interval there is a unique efficient contract and $\tilde{f}(V,s)$ is decreasing and concave in V. In the interior of the interval $f(V,s)$ is differentiable with respect to V.*

Lemma 2.2 $\frac{\partial \tilde{f}}{\partial V}(V,s) = -\frac{1}{u'(w)}$, *where the wage is optimally chosen given state (V,s).*

Proposition 2.4 *The efficient wage contract is characterized by a wage which is constant unless either participation constraint binds. If $f(V) \geq 0$ binds the wage is adjusted down just enough to make it hold with equality. If $V \geq V^u$ binds the wage is adjusted up just enough to make it hold with equality.*

Adding the participation constraints to the feasibility conditions reduces the amount of income smoothing the firm can provide. Wages still remain constant whenever possible, but the outside options cause adjustments. The history dependence in wages is reduced by these constraints.

Using the properties of the two Pareto frontiers the market maker's problem can be posed as

$$\begin{aligned} \max_{V^0(z),\theta(z)} & \mu(\theta(z))V^0(z) + (1-\mu(\theta(z)))V^u(z) \\ \text{s.t. } & q(\theta(z))f(V^0(z), x_L, z) = c. \end{aligned}$$

Any contract (which we know must lie on the Pareto set of contracts) that there exists a sub-market for, must offer a $(V^0(z), \theta(z))$-pair that gives unemployed workers the maximum value given that firms obtain zero expected value.

Proposition 2.5 *For either of the Pareto frontiers considered above, the labor market characterized by a unique market tightness $\theta(z)$ and value of new contracts $V^0(z)$ for each z, satisfying the following conditions (substitute in $\tilde{f}$ for limited commitment)*

$$\frac{1}{f(V^0(z), x_L, z)} = \left(\frac{\mu(\theta(z))}{\theta(z)\mu'(\theta(z))} - 1\right)\frac{u'(w^0(z))}{V^0(z) - V^u(z)}, \tag{1}$$

$$c = q(\theta(z))f(V^0(z), x_L, z), \tag{2}$$

where $w^0(z)$ satisfies $\frac{\partial}{\partial V} f(V^0(z), x_L, z) = -\frac{1}{u'(w^0(z))}$.

Given a contract frontier f or $\tilde{f}$, computing an equilibrium of the economy involves finding $V^u(z)$, $V^0(z)$ and $\theta(z)$ such that

$$V^u(z) = u(b) + \beta \sum_{z'} \pi(z'|z)(\mu(\theta(z))V^0(z') + (1-\mu(\theta(z)))V^u(z')),$$

and equations (1) and (2) hold. Note that the contract frontiers depend on the equilibrium V^u.

3 A Two State Case

The purpose of this section is to demonstrate the effects of the different contracting on labor market equilibria. In particular I am interested in comparing three alternative settings: the linear utility setting which is standard in the matching literature, non-linear utility with full commitment (FC), and non-linear utility with limited commitment(LC).[4]

For simplicity I consider a case where there are only two possible aggregate states $z_H > z_L$ and idiosyncratic variation is only present through e. The computational approach builds on the fact that the contracts are characterized by wage intervals which move with the aggregate state. This procedure is explained in appendix B.

The calibration determines what type of wage movement we see in the limited commitment contracts. When labor market conditions make finding new matches hard, more insurance is possible through the wage contract, and wages will move less during a contract. When the labor markets allow to find a new match easily, wages move more. Things are complicated by the fact that the wage contracts are determined jointly with the labor market conditions.

The calibration presented here is chosen to highlight the effect of the contracting on the equilibrium. This means that it is a case where the outside option binds fairly often and hence wages within a contract move each time the aggregate state switches. I set the productivity states to $z = (1.1, 1)'$ so there is roughly 10% variation in productivity. Given that the Bellman equation imposes a mandatory unemployment spell after a match breaks up, I wish to have the time intervals fairly short. I consider a time interval of about $9-10$ days when setting the discount rate to $\beta = 0.999$. Transition probabilities are set to be persistent $\Pi = \begin{pmatrix} .99 & .01 \\ .01 & .99 \end{pmatrix}$. The separation rate as been estimated to be $2-3\%$ per month, so I set $\delta = 0.99$.

I am left with choosing b, c and the risk aversion. Risk aversion tends to make the limited commitment stand out more, so I pick a somewhat high number, 10. The model tends to produce very low unemployment through high probabilities of matching μ. To counter this I set the unemployment consumption and vacancy cost fairly high relative to productivity, $b = .9$ and $c = .9$.

Next I present some figures showing how the economy evolves for a simulation

[4]Under linear utility the timing of wage payments is not determined by the equilibrium, only the present values, so the distinction between full or limited commitment has no effects on the equilibrium θ or contract values.

of aggregate productivity in the three different equilibria.

Output and Wages

Figure 1 shows the average wages in the economy contrasted with productivity and per capita output. Output dynamics are determined by productivity z and unemployment N^u. The latter is a predetermined variable which follows $N^u_{t+1} = (1-\mu(\theta_t))N^u_t + (1-\delta)(N - N^u_t)$.

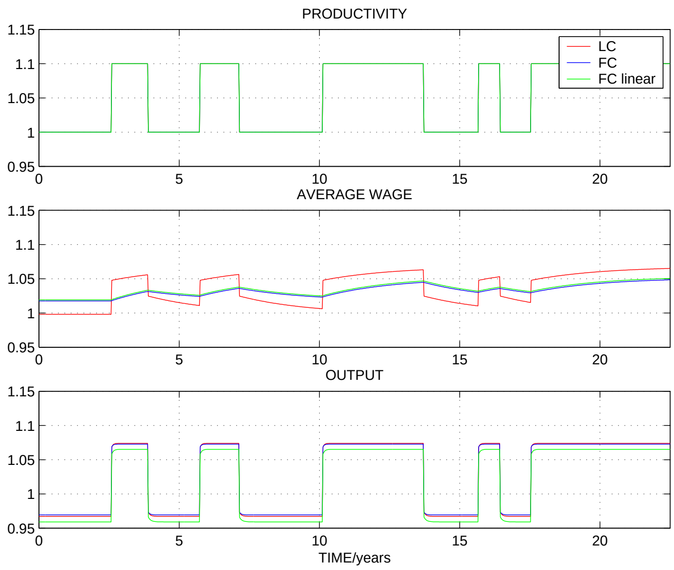

Figure 1: Productivity state, average wage and output for a simulation.

The dynamics for wages call for some explanation. (First, for the linear utility wages I have simply imposed constancy over time. This has no effect on other variables in the model and could be seen as a limiting case when risk aversion approaches zero.) Both the linear utility case and the full commitment case have a constant wage during a contract. The average wage changes due to a composition effect as contracts signed in different productivity states offer the worker different wage levels. Workers enter the labor force at different rates over the cycle and with different wage levels. Workers separate from a given match at a constant rate over the cycle. As a high state occurs, the probability of an unemployed agent finding a job increases and the wage associated will be higher. At the same time workers in old matches are entering the unemployment pool at a rate which is independent their wage. Therefore average wages rise. Additional dynamics come from the unemployment pool getting smaller while a good state lasts which reduces hiring and the number of operating firms increasing which increases separations.

The limited commitment contracts add to this by making within contract wages move. In the long run contracts will not depend on the time when they were signed, but solely on the current state. When z is high wages will jump up and when z is low they will be low. Additionally, the initial wages will reflect the state at time of contracting, such that initial wages (in contracts signed in the high and low state) will be more spread out than the long run ones. In all this means that the average wages jump when the state shifts. In addition the composition effect that was discussed above continues to be present causing the smooth adjusting seen while z is constant.[5]

Unemployment and Vacancies

Figure 2 shows time paths of unemployment, vacancies (number of firms searching for a worker), and the vacancy-unemployment ratio for a simulation. The market tightness θ reflects the aggregate state directly. Unemployment dynamics are driven mainly by job finding rates being higher in the high state than the low state. The other effect is that with a fixed separation rate there will be on average more people entering unemployment in the high state than the low state simply because there are more matches existing in the high state. The number of firms looking for workers is $\theta \times unemployment$ and so we see that it shows some history dependence coming from unemployment, but also discontinuity and the tendency of peaking right after a change in the state.

There are two things that this figure demonstrates. Firstly, the effect of linear vs. nonlinear utility on levels and variability and secondly, the effect of commitment on variability.

The figure shows that under risk aversion unemployment is lower and θ is higher and more variable. The labor market equilibrium delivers workers utility as a tradeoff between the value of a new contract and the ease of

[5] This is only one, and the most drastic, of possible limited commitment contract types. For different calibrations one can obtain more history dependence in the wage.

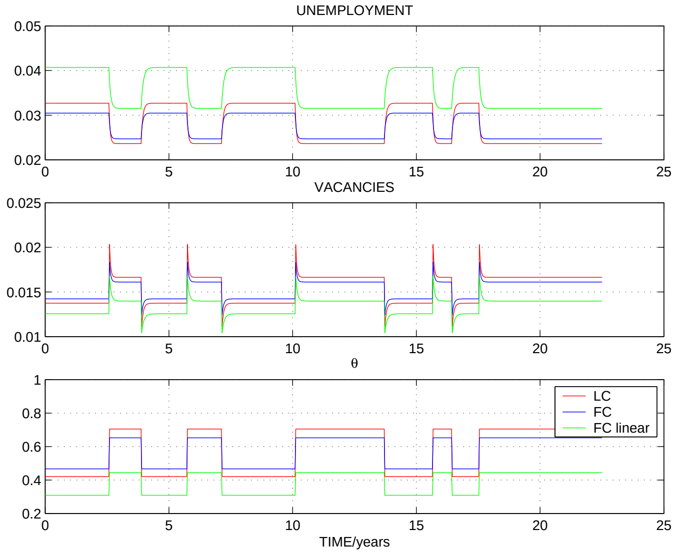

Figure 2: Simulation of unemployment, vacancies and market tightness for CRRA utility under limited commitment, CRRA-utility under full commitment and linear utility under full commitment.

finding a job. When workers are more risk averse, they dislike the change in the consumption associated with unemployment more. This leads to higher θ levels and lower contract values. Risk aversion will hence also imply lower unemployment. Due to decreasing marginal utility the nonlinear utility tends to produce some amplification in the θ as well.

The figure shows that limited commitment amplifies variation in θ over the cycle. The amplification in θ that results from limited commitment contracting is connected to the Pareto frontier becoming steeper for larger z as figure 3 shows. With limited commitment, the across z's variation in contract values must be created largely in the very beginning periods of a match since the wages are determined from outside the firm once z switches. If you contract at z_H you know your later wages will be exogenous due to participation constraints and lower than what you will initially get. To provide a worker value through this initial wage is more costly to the firm than if the wage was constant. The frontier $f(V, z_h)$ is steeper for LC than FC. If you contract at z_L you know your later wages will be exogenous due to participation constraints and higher than what you will initially get. To provide a worker value through this initial wage is less costly to the firm than if the wage was constant. The frontier $f(V, z_L)$ is flatter for LC than FC.

The variation in θ increases by 37% by going from linear utility to non-linear utility and by 53% by going from full to limited commitment. In total the variation is increased by 110% by going from linear utility to non-linear utility with limited commitment. Note however that these are different equilibria, so other variables differ as well.

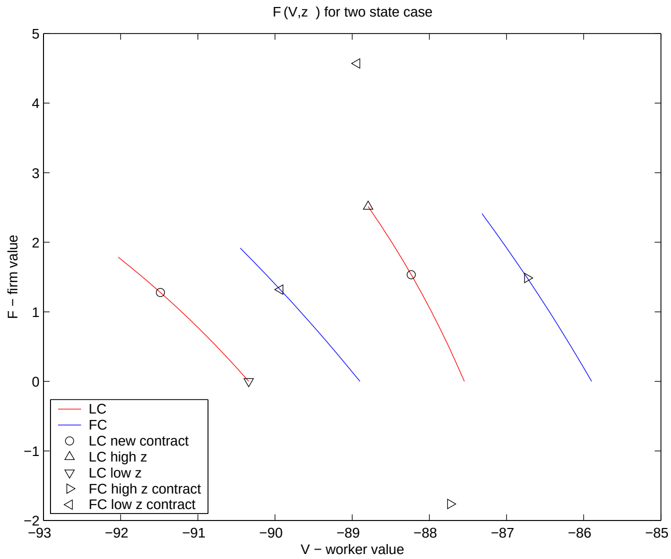

Figure 3: Value functions for full (FC) and limited commitment (LC) contracts with CRRA utility. LC contracts are plotted for the domain of definition and FC contracts for the domains that equilibria could be in. The figure shows equilibrium points as well. A FC contract features a constant wage, but the corresponding values change with the state. The wage depends on the initial state and hence there are two FC contracts shown in the figure. A LC contract in the type of equilibrium considered here has three possible wages: an initial wage and wage levels for each state z that are paid once the state changes once after contracting. The corresponding values change with the state. There are two LC contracts shown in the figure.

Figure 3 shows the frontiers of efficient contracts for the FC and LC equilibria. The key thing to note here is that in the high state the LC frontier is steeper than the FC and in the low state flatter. As the labor market picks a point on this frontier facing a tradeoff between θ and V, this curving will tend to bring $V^0(z)$ across the z closer together, while $\theta(z)$ are spread apart.

The limited commitment contract is defined on a bounded domain which depends on the state. The range of values is limited from below by the outside option V^u, which increases in productivity. At the upper bound the firm value equals zero. The full commitment contract can be defined on a larger set of values, but since in equilibrium the contract must supply the firm a non-negative value and the worker more than the search value, I plot only these ranges for full commitment contracts.

Note that the full commitment frontier is farther from the origin than the limited commitment frontier. If search values were the same in the figures this would reflect the gains from commitment. Here we consider separate equilibria, so one should take into account that the equilibrium search values are higher for the full commitment case which affects the figure as well.

The figure shows the equilibrium points in the two cases. Consider a FC contract signed in the high state. In the figure this starts off at the point $\triangleright$ on the outer FC frontier. This value is maintained until the state changes, at which time though the wage is fixed, changes in search values and transition probabilities imply moving to the $\triangleright$ which lies along the inner FC frontier, but at a point where the firm has negative profits. This wage is therefore too high to have been contracted in the low state. As the state changes the contract moves between the two points. A similar explanation goes for a FC contract signed in the low state with the points being denoted $\triangleleft$.

Consider a LC contract signed in the high state. This starts off at the point $\circ$ on the outer LC frontier. This value is maintained until the state changes to low, at which time the wage must be adjusted down to keep firms profits in the low state non-negative (compare to the FC case). In particular the wage is adjusted down just enough to put the contract at the point where firm value in the low state is zero, denoted by $\triangle$ on the inner LC frontier. This point is then maintained until the state changes, at which time the wage must be adjusted up such that the worker get his search value V^u. The corresponding high state point is $\triangle$ on the outer LC frontier. As the state changes again the contract moves between the two $\triangle$ points. A similar explanation goes for a LC contract signed in the low state starting at $\circ$ on the inner LC frontier and then moving between ∇'s.

4 Conclusions

I find that risk aversion and limited commitment do increase the variation in the vacancy unemployment ratio, but the magnitudes of increase may not be of the order that is called for. For the calibration presented the variation in θ was more than doubled, but this came at a cost of empirically too low unemployment levels and a wage level that was quite cyclical. A further investigation is underway to see whether adding idiosyncratic productivity shocks would improve the model. It is predicted to make the limited commitment constraints more binding and hence increase the volatility of θ, while not increasing the cyclicality of wages.

A Proofs

Full Commitment

Proof of Proposition 2.1 This Bellman equation can be treated directly using the techniques of Stokey, Lucas, Jr., and Prescott (1989). If wages are bounded, continuation values must be as well. If we impose bounds on the choice set $\{V(s)\}$ the Bellman equation can be written as

$$f(V,s) = \max_{\{V(s')\}\in[\underline{M},\overline{M}]} \{xz - u^{-1}(V - \beta E_{s'|s}[\delta V(s') + (1-\delta)V^u(z)]) \\ + \beta E_{s'|s}\delta f(V(s'), s')\}$$

The operator on the right hand side, T, is a contraction by the Blackwell sufficient conditions and therefore has a unique fixed point. The operator preserves continuity, decreasingness and concavity of f in promised value. Therefore, since the set of continuous, decreasing and concave functions is closed, the solution of the Bellman equation possesses these properties. The strictly increasing and concave CRRA utility further implies that the solution f is strictly decreasing and concave (since Tf maps weakly decreasing and concave functions to strictly decreasing and concave ones.) Differentiability applies Benveniste and Sheinkman's theorem. □

Proof of Proposition 2.2 The necessary conditions for an optimum are:

$$-\frac{1}{u'(w)} = \frac{\partial}{\partial V} f(V(s'), s') \ \forall s'.$$

The envelope condition implies $\frac{\partial}{\partial V} f(V,s) = -\frac{1}{u'(w)}$ as well. Therefore marginal values are equalizes across all states and this implies a constant wage. □

Limited Commitment

The Bellman equation is not a contraction and should not be expected to have a unique solution. The limited commitment Pareto frontier is unique however and can be obtained by applying the Bellman operator on the full commitment solution until pointwise convergence occurs. In what follows I start from defining the Pareto frontier and proceed to showing properties of the solution. The first part follows largely Thomas and Worrall (1988).

Lemma A.1 *The set of self-enforcing contracts $\tilde{\Sigma}(s)$ is convex.*

Pf. Consider two contracts $\sigma^1(s) = \{w^1_\tau(s^\tau)\}_{\tau=0}^\infty$ and $\sigma^2 = \{w^2_\tau(s^\tau)\}_{\tau=0}^\infty$ that are self enforcing. For $\alpha \in (0,1)$ define $\sigma^\alpha(s) = \{w^\alpha_\tau(s^\tau)\}_{\tau=0}^\infty$ such that $w^\alpha(s^\tau) = \alpha w^1(s^\tau) + (1-\alpha)w^2(s^\tau)$ for all s^τ.

Then

$$F_{\sigma^\alpha}(s^t) = \alpha F_{\sigma^1}(s^t) + (1-\alpha)F_{\sigma^2}(s^t) \geq 0,$$

$$V_{\sigma^\alpha}(s^t) > \alpha V_{\sigma^1}(s^t) + (1-\alpha)V_{\sigma^2}(s^t) \geq V^u(z_t),$$

imply that σ^α is self enforcing. □

Proof of Proposition 2.3

Claim: The set of V for which a self-enforcing contract exists is a closed and bounded interval $\Omega_s = [V^u(z), \overline{V}(s)]$.

Pf. Denote by $\Omega(s)$ the set V for which a self-enforcing contract exists in state s. If $V \in \Omega(s)$ then clearly $\tilde{V} \in \Omega(s)$ if $V^u \leq \tilde{V} < V$. Therefore $\Omega(s)$ forms a single interval. We need to show that it is closed.

Consider a sequence of promised utilities $V^k \in \Omega(s)$ that converges to a $\hat{V}$. We need to show $\hat{V} \in \Omega(s)$.

There is a corresponding sequence of contracts σ^k. Each contract specifies a wage for each history, so that there is a countable number of wages specified by each contract. Enumerate them by w^k_m. Each of these wages lies in a closed and bounded interval. Consider $m = 1$. The sequence w^k_1 lies in a bounded interval and so has a convergent subsequence. Drop the other k-terms and so form a new sequence of contracts σ^k where wages in $m = 1$ converge. Now consider $m = 2$ and repeat this procedure. Repeating this procedure in a countable way a limiting sequence will be obtained where wages converge for each history. Call this contract σ^∞.

The worker's value from the limiting contract equals the value of the limit $V_{\sigma^\infty}(s) = \lim V_{\sigma^k}(s) = \hat{V}(s)$ by the dominated convergence theorem. For the firm value similarly. As σ^k is self enforcing σ^∞ must be and since σ^∞ yields value $\hat{V}$, we have $\hat{V} \in \Omega(s)$. □

Claim: For each value $V \in \Omega(s)$, the efficient contract is unique.

Pf. The set of self-enforcing contracts is convex and u is strictly concave.□

Claim: The Pareto frontier is strictly decreasing, strictly concave and continuously differentiable on $(V^u(z), \overline{V}(s))$.

Pf. Consider the Bellman equation. If V were lowered, it would remain feasible to use the same next period promised values, while lowering the current period wage. This would lead to a strict rise in profits.

Suppose $\tilde{f}$ is concave in promised value. Consider V^1 and V^2 in $\Omega(s)$ with the corresponding optimal policies w^k, $V^k(s')$ for all s', $k = 1, 2$. Consider the convex combination of these: $\hat{V} = \alpha V^1 + (1-\alpha)V^2$, $\hat{w} = \alpha w^1 + (1-\alpha)w^2$, $\hat{V}(s') = \alpha V^1(s') + (1-\alpha)V^2(s')$, $\alpha \in (0, 1)$. Note that $\hat{V}(s')$ are feasible and that

$$
\begin{aligned}
\hat{V} =& \alpha u(w^1) + (1-\alpha)u(w^2) + \beta E_{s'|s}[\delta \hat{V}(s') + (1-\delta)V^u(z')] \\
=& u(\hat{\hat{w}}) + \beta E_{s'|s}[\delta \hat{V}(s') + (1-\delta)V^u(z')]
\end{aligned}
$$

implies that $\hat{\hat{w}} < \hat{w}$.

Then

$$
\begin{aligned}
(T\tilde{f})(\hat{V}) \geq & xz - \hat{\hat{w}} + \beta\delta E_{s'|s}\tilde{f}(\hat{V}(s'), s') \\
> & xz - \hat{w} + \beta\delta E_{s'|s}\tilde{f}(\hat{V}(s'), s') \\
= & \alpha(T\tilde{f})(V^1) + (1-\alpha)(T\tilde{f})(V^2).
\end{aligned}
$$

Thus Tf is strictly concave. It follows that iterations starting from the concave full commitment function must lead to a strictly concave limiting function.

To show differentiability one uses Benveniste-Sheinkman. Considering an interior point of $\Omega(s)$ there is an associated efficient contract σ. Perturbing this by a small constant ϵ for the current period only will keep the value interior. One can then associate a firm value to each perturbed worker value to obtain a function $f^\epsilon(V^\epsilon, s)$ which corresponds to $f(V, s)$ when $\epsilon = 0$. Otherwise f^ϵ is smaller than f. As u is strictly concave, f^ϵ is also strictly concave. As u is differentiable, f^ϵ is also differentiable. The Benveniste-Sheinkman theorem then implies differentiability of f in promised value. □

The necessary conditions for an optimum are written with the help of the

Lagrangian

$$
\begin{aligned}
L = xz - w + \beta \sum_{z'} \pi(z'|z)\delta(pf(V(x_H, z'), x_H, z') + (1-p)f(V(x_L, z'), x_L, z')) \\
+ \lambda\{u(w) + \beta \sum_{z'} \pi(z'|z)[\delta(pV(x_H, z') + (1-p)V(x_L, z')) + (1-\delta)V^u(z')] - V\} \\
+ \sum_{z'} \pi(z'|z)\beta\delta[\eta(x_H, z')p(V(x_H, z') - V^u(z')) + \eta(x_L, z')(1-p)(V(x_L, z') - V^u(z')) \\
+ \sum_{z'} \pi(z'|z)\beta\delta[\psi(x_H, z')pf(V(x_H, z'), x_H, z') + \psi(x_L, z')(1-p)f(V(x_L, z'), x_L, z')
\end{aligned}
$$

as follows

$$L_w = -1 + \lambda u'(w) = 0$$

$$L_{V(s')} = \frac{\partial}{\partial V} f(V(s'))(1 + \psi(s')) + \lambda + \eta(s') = 0 \ \forall s'$$

$$\frac{\partial}{\partial V} f(V, s) = -\lambda$$

Therefore we have

$$\frac{\partial}{\partial V} f(V, s) = \frac{\partial}{\partial V} f(V(s'), s')(1 + \psi(s')) + \eta(s'). \tag{3}$$

Note that there is a strictly increasing mapping between the promised value and wage for a given state: $\frac{\partial}{\partial V} f(V(s), s) = -\frac{1}{u'(w(s))}$.

Lemma A.2 *If the state does not change, the value or wage the worker receives does not change.*

Proof If V is feasible for state s', we may set the Lagrange multipliers in (3) to zero. Hence the value is unchanged. That wage is unchanged follows. □

Proof of Proposition 2.4

Suppose today's state is s and value V. Today's wage is then predetermined by $\frac{\partial}{\partial V} f(V, s) = -\frac{1}{u'(w)}$. For state s' tomorrow we have an interior optimum if $\frac{\partial}{\partial V} f(V, s) = \frac{\partial}{\partial V} f(V(s'), s')$ implies a feasible $V(s') \in \Omega(s')$. If this is the case wages are constant as marginal profits are.

If this is not the case there is a boundary solution in $V(s')$. If the upper bound of $\Omega(s')$ is binding, then $\psi(s') > 0$, $\eta(s') = 0$. This means that the $V(s')$ value implied by the original $\frac{\partial}{\partial V}f(V,s) = \frac{\partial}{\partial V}f(V(s'),s')$ suggestion must have been larger than the boundary solution. Similarly we argue that if it is the lower bound that binds, then $\psi(s') = 0$, $\eta(s') > 0$. This means that the $V(s)$ value implied by the original $\frac{\partial}{\partial V}f(V,s) = \frac{\partial}{\partial V}f(V(s'),s')$ suggestion must have been smaller that the boundary solution.

Hence we have that if it is feasible to have $\frac{\partial}{\partial V}f(V,s) = \frac{\partial}{\partial V}f(V(s'),s')$, this will happen. Then the wage will not change. If this is not feasible but would imply a $V(s')$ above $\Omega(s')$, $V(s')$ is adjusted down to the upper boundary of $\Omega(s')$. The wage is determined by $\frac{\partial}{\partial V}f(V(s'),s') = -\frac{1}{u'(w(s'))}$ so that adjusting the value down to the boundary implies adjusting the wage down to a boundary wage. Similarly if the value implied were too low, it would be adjusted to the lower bound along with the wage. The wage boundaries are defined by the value boundaries and $\frac{\partial}{\partial V}f(V(s'),s') = -\frac{1}{u'(w(s'))}$ (taking limits). $\square$

Labor Market Equilibrium

Proof of Proposition 2.5 The equality $q(\theta)f(V) = c$ can be inverted for a decreasing function $\theta = h(V)$. Plugging this into the maximand and differentiating we get

$$\mu(h(V))[1 + \frac{1}{h(V)}\frac{f'(V)}{f(V)}(V - V^u)].$$

When $V = V^u$ this is positive, as $V \in \Omega$ is increased it decreases and as θ approaches zero the value becomes negative. As the derivative is continuous it must have a unique zero.

For either of the commitment set-ups discussed above we can use the envelope condition to find the derivative of the value function. Using this and not plugging in the matching function we can state the first order condition as

$$\frac{1}{f(V,s)} = (\frac{\mu(\theta(s))}{\theta(s)\mu'(\theta(s))} - 1)\frac{u'(w(s))}{V(s) - V^u(z)}.$$

$\square$

B Two State Computations

For a given set of parameter values and functional forms, there are five possible configurations of how the the wage intervals in the two states align themselves and how the initial wages in the two states align themselves with respect to the intersection of the wage intervals. To enumerate the cases say that, the first case is when the wage intervals don't overlap. In this case the wage will jump every time the industry state changes. Initially it may jump more then after the initial change. This case is the only one with long run wage uncertainty due to the outside option of the agents.

The second case is one where the wage intervals overlap and the initial wages lie within this intersection. Then there is a constant wage throughout the contract, with contracts signed during the high state having a higher wage than those signed in the low state.

The third case is one where the intervals overlap, but the neither initial wage lies within the intersection. In this case there will always be a jump in the wage the first time during the contract when the industry state changes and after that wages are constant. If the contract is signed in the high state, the wage moves down when the first low state occurs and if the contract is signed in the low state, the wage moves up when the first high state occurs.

The fourth case would have the initial jump only happen for contracts signed in the high state and the fifth only for contracts signed in the low state.

Each of these cases translates into a set of equilibrium equations and I present the ones for the first case. In computing the solution one must consider each of the systems in turn and find the one which has a solution. Given that a solution to the general problem exists, it should be captured by one and only one of the cases (except for limiting cases where the two cases are the same)[6]. This approach to computing the limited commitment solutions was used by Alvarez and Jermann (2001).

Case of Non-overlapping Wage Intervals

Suppose there exists an equilibrium with non overlapping wage intervals

[6]It should be noted that each system is likely to have more than one solution. This is analogous to the limited commitment Bellman equation in endowment economies having autarky as one solution. One needs to select a solution that makes sense for the equilibrium.

$[\underline{w}_l, \overline{w}_l]$ and $[\underline{w}_h, \overline{w}_h]$. Here the lower bound corresponds to $V^0 = V^u$ and the upper bound to $F = 0$. Note that this is possible only if the full commitment solution is not incentive compatible.

Then, according to the optimal contract, each contract will eventually only feature $w_l := \overline{w}_l$ in the low state and $w_h := \underline{w}_h$ in the high state. In an equilibrium the initial wages will generally be interior to the intervals, since firms are receiving $\frac{c}{q(\theta)} > 0$ in discounted expected profit from new contracts. Denote the initial wage for contracts signed in the low state w_l^0 and similarly for the high state w_l^0.

We will have the following equations (note $F_L = 0$, $V_H = V_H^u$)

$$
\begin{aligned}
F_H &= z_H - w_H + \beta\delta\pi_{HH}F_H \\
0 &= z_L - w_L + \beta\delta\pi_{LH}F_H \\
F_H^0 &= z_H - w_H^0 + \beta\delta\pi_{HH}F_H^0 \\
F_L^0 &= z_L - w_L^0 + \beta\delta(\pi_{LL}F_L^0 + \pi_{LH}F_H) \\
V_H^s &= u(b) + \beta(\pi_{HH}(\mu_H V_H^0 + (1-\mu_H)V_H^u) + \pi_{HL}(\mu_L V_L^0 + (1-\mu_L)V_L^u)) \\
V_L^s &= u(b) + \beta(\pi_{LH}(\mu_H V_H^0 + (1-\mu_H)V_H^u) + \pi_{LL}(\mu_L V_L^0 + (1-\mu_L)V_L^u)) \\
V_H^0 &= u(w_H^0) + \beta(\delta(\pi_{HH}V_H^0 + \pi_{HL}V_L) + (1-\delta)(\pi_{HH}V_H^u + \pi_{HL}V_L^u)) \\
V_L^0 &= u(w_L^0) + \beta(\delta(\pi_{LH}V_H^u + \pi_{LL}V_L^0) + (1-\delta)(\pi_{LH}V_H^u + \pi_{LL}V_L^u)) \\
V_H^s &= u(w_H) + \beta(\delta(\pi_{HH}V_H^u + \pi_{HL}V_L) + (1-\delta)(\pi_{HH}V_H^u + \pi_{HL}V_L^u)) \\
V_L &= u(w_L) + \beta(\delta(\pi_{LH}V_H^u + \pi_{LL}V_L) + (1-\delta)(\pi_{LH}V_H^u + \pi_{LL}V_L^u)) \\
&q(\theta_H)F_H^0 = c \\
&q(\theta_L)F_L^0 = c \\
&\frac{1}{F_H^0} = \left(\frac{\mu(\theta_H)}{\theta_H\mu'(\theta_H)} - 1\right)\frac{u'(w_H^0)}{V_H^0 - V_H^s} \\
&\frac{1}{F_L^0} = \left(\frac{\mu(\theta_L)}{\theta_L\mu'(\theta_L)} - 1\right)\frac{u'(w_L^0)}{V_L^0 - V_L^s}.
\end{aligned}
$$

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