Technology Shock and Employment: Do We Need Models with a Fall in Hours?

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Abstract

Recent empirical literature that uses the Structural Vector Autoregression (SVAR) approach has shown that a productivity shock identified using long–run restrictions leads to a persistent and significant decrease in hours worked. This evidence questions standard RBC models in which a positive technology shock leads to a rise in hours. In this paper, we estimates and test a standard RBC model using Indirect Inference on impulse responses of hours worked after a technology shock. We find that this model is not rejected by the data as it is able to produce – persistent and negative – impulse responses in SVAR from simulated data similar to impulse responses in SVAR from actual data. Moreover, technology shocks represent the main contribution to output growth fluctuations. Our results suggest that we do not necessary need models with a fall in hours.

Keywords: SVARs, Long–Run Restrictions, RBC models, Indirect Inference

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Introduction

Following Blanchard and Quah (1989) and Gali (1999), the recent empirical literature that uses the Structural Vector Autoregression (SVAR) approach has shown that a productivity shock identified using long–run restrictions lead to a persistent and significant decrease in hours worked. Using a difference specification (DSVAR), Gali (1999) shows that hours significantly decreases in the short rune in all G7 countries, with the exception of Japan. Gali (2004) also finds similar qualitative results for the euro area as a whole. Conversely, with the level specification[1] (LSVAR), the point estimate of the impact response becomes positive, but very small and not significantly different from zero. Despite a hump–shaped response of hours, the effect is not significantly different from zero for each horizon.[2] Conversely, the negative response of hours in DSVAR appears robust to various detrending methods of hours (see Gali and Rabanal (2004)) and to the inclusion of other variables in the VAR model (see Gali (1999), Francis and Ramey (2004)). If the measurement device, *i.e.* the DSVAR model, is taken seriously, this result is really challenging for a large part of the business cycle research program. Indeed, as pointed out by Gali and Rabanal (2004), the standard Real Business Cycle (RBC) model cannot reproduce this pattern, as hours worked increase after a positive technology shock. For Francis and Ramey (2004), these empirical evidences reject unambiguously the RBC model and thus constitute the death of the paradigm. For Gali (1999) and Gali and Rabanal (2004), these empirical findings suggest to abandon the frictionless approach in favor of models with nominal rigidities (stricky prices and/or sticky wages[3]). It is worth noting that flexible price models are able to reproduce a fall in hours following a technology shock, but they must include many frictions, such that habit persistence in consumption together with a high level of adjustment costs on physical capital (see Beaudry and Guay (1996), Boldrin, Christiano and Fisher (2001), Francis and Ramey (2004)).

However, recent contributions have questioned the ability of SVAR models to consistently measure the

[1]Christiano, Eichenbaum and Evans (2004) argue that the DSVAR may induce distortions if hours worked are stationary in level.

[2]Chari, Kehoe and McGrattan (2004b) find similar results using various US datasets, with the exception of Francis and Ramey (2004) dataset.

[3]Gali and Rabanal (2004) propose a structural model with real frinctions and nominal rigidities ("triple" sticky, following McGrattan (2004)) that is consistent with these evidences.

effect of a technology shock using long–run restrictions. Erceg, Guerrieri and Gust (2004) have shown using models calibrated on US data that the effect of a technology shock is not precisely estimated with SVAR. Moreover, when they adopt a DSVAR specification, the bias increases significantly.[4] Chari, Kehoe and McGrattan (2004b) provide similar results. They simulate a RBC model estimated by Maximum Likelihood on US data with two shocks (a permanent technology shock and a stationary labor tax shock) and they find that the DSVAR approach leads to a negative response of hours under a structural model wherein hours respond positively. These two papers cautions the use of SVAR in order to identify the effect of technology shocks as a model–independent approach. Moreover, these findings point out that it is not necessary to build macroeconomic model with a fall in hours after a technology shock.

This paper pursues this research line using another quantitative approach. Erceg, Guerrieri and Gust (2004) calibrate some models' parameters using a small set of moments and thus looks at their quantitative implications for the identification of technology shock using long–run restrictions. The two models under study (flexible and sticky) are not formally evaluated on their ability to reproduce other moments characterizing the US business cycle and impulse responses of various aggregate variables. Chari, Kehoe and McGrattan (2004b) estimate a flexible price model on US data. However, in order to solve some singularity problems, they add shocks that account for measurement errors. It will be thus hard to know what are the main sources of aggregate fluctuations in simulation experiments (structural shocks or measurement errors). Our approach differs in many ways. First, the model's parameters are estimated such that impulse responses from the model are as close as possible to the impulse responses from the actual data. Second, we introduce a large number of over–identifying restrictions. This allows us to test the hypothesis: *Do We Need Models with a Fall in Hours?*. Third, we use an original econometric approach in order to estimate and test the model. Rather than using a limited information strategy (see Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2004), Altig, Christiano, Eichenbaum and Linde (2005)), that estimate directly the structural parameter from impulse responses, we use an Indirect Inference

[4]This is true in the case of a RBC model. A Sticky Price/Wage model delivers better results, as the LSVAR and DSVAR provide a consitent estimate of the true (negative) response of hours.

approach (see Gouriéroux, Monfort and Renault (1993), Gouriéroux and Monfort (1996)). The idea of Indirect Inference typically is to use an auxiliary model (or an auxiliary criteria) in order to indirectly estimate and test a structural model. The empirical strategy adopted in this paper typically use the evidence from simulations experiments in Erceg, Guerrieri and Gust (2004) and Chari, Kehoe and McGrattan (2004b). As the DSVAR used by Gali can deliver downward biased responses of hours following a technology shock, we use DSVAR model as an auxiliary model. The model is not estimated directly from DSVAR on actual data, but indirectly using both DSVAR under the structural model and DSVAR under actual data. This indirect approach allows to correct for biases and distortions, if they exist. If we keep in mind one of the main results of Chari, Kehoe and McGrattan (2004b), this means that a model wherein hours worked increase after a technology shock is potentially able to match a DSVAR model where hours decreases. We thus estimate a RBC model in the line of Kydland and Prescott (1982). The model is simpler than Kydland and Prescott as time non–separability includes only one lag in leisures choices. However, we depart from Kydland and Prescott as we introduce an additional shock that shifts utility over periods. The preference shock accounts for persistent changes in the marginal rate of substitution between goods and work. This model can be viewed as "old–fashioned" and thus representative of the first generation of frictionless RBC models. By this mean, we want to evaluate if this type of model is really dead.

We complement this econometric approach by a simple model from which we can easily compute the impulse responses. Using this simple model as the Data Generating Process (DGP), we show analytically that DSVAR leads to biased estimated response of hours. In the original model, hours does not respond to a technology shock, whereas they persistently decrease in the DSVAR. Moreover, the bias increase with the variance and the persistence of the non–technology shock. When the persistence of this shock is very high, adding more lags in the DSVAR does not allows to correct the bias in the estimated response. These results shows that a direct quantitative evaluation of a structural model from DSVAR can be misleading. They also suggest an alternative quantitative method, *i.e.* an indirect approach that that consider the DSVAR model as an auxiliary model for estimating and testing a structural model.

As in Christiano, Eichenbaum and Vigfusson (2004), we first estimate impulse responses from DSVAR using alternative measures (in logs) of productivity and hours worked with quarterly U.S. data for the period 1948:1-2002:4. We then estimate the structural parameters using indirect inference on the impulse responses of hours to a technology and non–technology shock. For each dataset, the structural model is able to produce impulse responses very close to the ones obtained from actual data. This means that our structural model wherein hours worked persistently increases after a technology shock is consistent with the Gali's finding, *i.e.* a technology shock in DSVAR leads to a fall in hours. Moreover, from the parameter estimates, we compute the contribution of the two shocks to aggregate fluctuations. We find that the technology shock is the main source of output growth fluctuations, whereas the preference shock explains most of fluctuations in hours. We end our empirical exercice looking at the LSVAR approach. We show that the DSVAR specification, although bias induced, encompasses the LSVAR specification.

The paper is organized as follows. In a first section, we introduce a simple model that allows to clearly show the main sources of distortion with the DSVAR approach. In section 2, we briefly present a Kydland–Prescott type model. The third section is devoted to the econometric methodology. In section 4, we present the data and the results. A last section concludes.

1 Lessons from a Simple Model

In this introductory example, we consider a simple flexible prices equilibrium model without capital accumulation[5]. This model is deliberately stylized in order to deliver analytical results when a DSVAR model is estimated under this DGP. One can argue that the economy is highly stylized, so we cannot take its quantitative implications seriously. For example, the response of hours following a technology shock is zero. This is in contradiction with our results from SVARs in section 4 and previous quantitative findings (see Gali (1999), Gali and Rabanal (2004), Christiano, Eichenbaum and Vigfusson (2004)). This is not problematic for our purpose, as we simply try to evaluate the ability of SVARs (as a model–free statistical measurement method) to recover the effect of a technology

[5]A version of this model with capital accumulation is considered in the next section. Our main analytical findings are not qualitatively altered in this more general setup (See Erceg, Guerrieri and Gust (2004) and Chari, Kehoe and McGrattan (2004b)

shock. The lack of response of hours to a technology shock has only to be considered as a reference number for the analysis.

1.1 The Model

The representative household seeks to maximize

$$\log C_t + \chi_t(1 - N_t) \tag{1}$$

subject to the budget constraint

$$C_t \leq w_t N_t + \Pi_t \tag{2}$$

for every periods. The quantity of good consumed in period t is C_t. The variable N_t denotes hours, w_t is the real wage and Π_t represents the profit that household receives from the firm. The utility function is separable, logarithmic in consumption and following Hansen (1985), linear in leisure, thus implying an infinite labor supply elasticity. Without loss of generality, the time endowment is set to unity. χ_t is a random variable that shifts utility every periods. The logarithm of this variable is assumed to follows an AR(1)

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + \exp\left(\varepsilon_{\chi,t}\right)$$

where $\varepsilon_{\chi,t} \sim \mathcal{N}(0, \sigma_\chi^2)$. As noticed by Gali (2004b), this shock can be an important source of fluctuations, as it allows to represent persistent shifts in the marginal rate of substitution between goods and work (see Hall (1997)). Such shifts accounts for persistent fluctuations in labor supply that follow some changes in demography structure and/or labor market participation. Moreover, this preference shock allows to generate persistence in hours.[6] It is worth noting that our assumption of linear labor supply has no consequence on our results. In what follows, the formula would be exactly the same, except that we have to scale the variance of the preference shock by the square of ones plus the inverse of the Frishian labor supply elasticity. The first order conditions of the households problem (1)–(2) yield

$$\chi_t C_t = w_t$$

[6] Note that this shock is observationally equivalent to a tax on labor income (see Erceg, Guerrieri and Gust (2004) and Chari, Kehoe and McGrattan (2004b)). It allows to simply account for distortions on the labor market, *i.e. labor wedges* following Chari, Kehoe and McGrattan (2004a).

Consumption is an increasing function of real wage, whereas it decreases facing a positive preference shock on leisure.

The representative firm produces an homogenous good with a technology

$$Y_t = Z_t N_t^\alpha$$

where $\alpha \in (0, 1]$. The variable Z_t is the aggregate technology. The growth rate of Z_t is assumed to be *iid* and normally distributed

$$Z_t = Z_{t-1} \exp(\varepsilon_{z,t})$$

where $\varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$. The first order condition of the firm is

$$w_t = \alpha \frac{Y_t}{N_t}$$

From the households and firms optimality conditions and market clearing $Y_t = C_t = Z_t N_t^\alpha$, the equilibrium employment is given by $N_t = \alpha/\chi_t$, whereas labor productivity is directly deduced from the production function. Taking logs and without loss of generality ignoring constant terms, we obtain the following log–linear representation of the economy

$$n_t = -\chi_t \tag{3}$$

$$\Delta x_t = \varepsilon_t + (1-\alpha)\Delta\chi_t \tag{4}$$

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + \varepsilon_{\chi,t} \tag{5}$$

where lower letters represent the logarithm of each variables. In this economy, employment (3) does not react to a technological shock but decreases facing a preference shock. The technological shock increases positively and permanently – one–for–one – productivity (4), whereas the stationary preference shock (5) has no long–run effect.

1.2 Identification from SVAR(1)

We use the solution (3)–(5) as the DGP. Given the realization of the equilibrium, we will seek to evaluate the quantitative implications of SVARs when the econometrician uses long–run restrictions (on productivity or output growth) in order to recover the effect of a technology shock on employment. Note that we consider that fluctuations in hours can be highly persistent and undistinguishable from a unit root in small sample when ρ_χ is close to one. Indeed, many studies have suggested that hours can display non–stationarity (see Gali and Rabanal (2004) and Gali (2004b) among others). We consider the estimation of VAR models and the identification of technology shock using long–run restriction in a VAR model with a first difference specification. The VAR(1) model to be estimated has the following forms:

$$\mathbf{z}_t = \mathbf{A}_1 \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{z}_t$ includes the following variables

$$\mathbf{z}_t = \begin{pmatrix} \Delta x_t \\ \Delta n_t \end{pmatrix} \qquad \text{DSVAR(1)}(\Delta x, \Delta n) \text{ model}$$

In order to get analytical results, we only consider a VAR(1) model. Despite its simplicity, this assumption allows us to shed light on the main mechanisms at works. Increasing the number of lags does not modify the main results (especially when the preference is highly serially correlated ($\rho_\chi \approx 1$). Estimated VARs'parameters and associated IRF allows to determine the mapping between the structural parameters and the ones of the DSVAR. We impose the long–run restriction that only the productivity shock has a permanent effect on labor productivity (see the subsection 3.1 for more details about the identification using long–run restrictions). The following proposition determines impulse responses of hours to a technology shocks.

Proposition 1 *When $\alpha \in (0,1)$ and $\sigma_u, \sigma_\varepsilon > 0$, the impulse responses of hours worked to a technology shock under the structural model (3)–(5)is negative for each horizon in the DSVAR(1) $(\Delta x, \Delta n)$ model.*

Proposition 1 shows that the estimated response of hours in the DSVAR is downward biased. Note that these results are asymptotic and does not come from small sample biases. When the variance of the non–technological shocks is non–zero and $\alpha \in (0,1)$, this response is always negative. More precisely, the IRF at horizon k of the level of hours is given by:

$$\frac{\partial n_{t+k}}{\partial \boldsymbol{\eta}_{1,t}} = -\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}} \sum_{j=0}^{k} \left(-\left(\frac{1-\rho_\chi}{2}\right)\right)^j$$

where $\boldsymbol{\eta}_{1,t}$ is the productivity shock in DSVAR identified using long–run restrictions. We see that when the variance of the non–technology shock increases relatively to the the variance of the technology shock, the negative response is more pronounced. Moreover, when the labor share decreases the negative response is amplified. Conversely, when this share tends to one, the response is zero. In this latter case, the productivity growth depends only on the technology shock and the response of hours is zero. This simply means that when the productivity growth is an appropriate measure of total factor productivity growth, the DSVAR specification allows to recover the true IRF of hours that follows a technology shock. Another interesting result concerns the estimated response to a technology shock when hours display persistence. Proposition 1 shows that the persistence of the preference shock χ_t does not qualitatively affect the results in the DSVAR specification. The responses of hours is always negative in the DSVAR$(\Delta x, \Delta n)$ for any value $\rho \in [0,1]$. Note that when hours are non–stationary, the difference specification of employment in SVARs does not allow to recover the true response. In the limit case $(\rho_\chi \to 1)$, the impulse responses are given by:

$$\lim_{\rho_\chi \to 1} \frac{\partial n_{t+k}}{\partial \boldsymbol{\eta}_{1,t}} = -(1-\alpha)\sigma_\chi^2 \left(\sigma_z^2 + (1-\alpha)^2 \sigma_\chi^2\right)^{-1/2}$$

The response of hours is asymptotically biased in the DSVAR model even when the preference shock follows a random walk. This result can be easily explain, as the preference shock will have a permanent effect on productivity when $\rho_\chi = 1$. In this case, the long run restriction in the DSVAR model (only the technology shock has a permanent effect) is not satisfied. Finally, we have considered for simplicity only a VAR(1) model. The quantitative results will be affected in the case of DSVARs, has the first difference of hours introduces a unit root in the moving average of preference shock, especially when ρ_χ is close to zero. This point is illustrated by figure 1–(a) that reports the response of hours to a technology shock for various lags $(p = 1, ..., 12)$ when the preference shock is *iid*. In his case, increasing the number of lags allows to weaken the negative response of hours. Except at the impact where the response remains always negative, the response of hours is almost zero. This result does not hold when the preference shock (and thus hours) is persistent. Figure 1–(b) reports the impulse responses of hours for various lags when $\rho_\chi = 0.98$. As this figure shows, increasing the number of lags has a very small effect, especially in the short run. This result can be easily understood from the structural model when $\rho_\chi \approx 1$. In this case, employment and productivity in first difference are *iid* and estimated parameters from any $VAR(p)$ would be zero. The selection of the number of lags does not affect the response of hours. Finally, it is worth noting that despite its simplicity, the identification of technology shock from the model possesses some empirical contents. Indeed, the IRF from the DSVAR model under the model roughly match the IRF from the data. When hours are taken in first difference with productivity growth, the response is persistently negative as predicted by Proposition 1.

This latter remark suggests another way to quantitative evaluate business cycle models from DSVAR. Rather than a direct evaluation from impulse responses of hours in DSVAR, Proposition 1 suggests an indirect approach. Let $\widehat{\psi}_T$ the estimated impact response of hours to a technology shock using a DSVAR specification with actual data. Under the structural model, the impact response is

$$\psi(\alpha, \sigma_z, \rho_\chi, \sigma_\chi) = -\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}$$

Assume for simplicity that $\rho_\chi = 0$ and α and σ_z are set. One can thus determine a value of σ_χ such taht the following equality hold:

$$\psi(\sigma_\chi) = \widehat{\psi}_T$$

The binding function $\psi(\sigma_\chi)$ offers the opportunity to estimate a value of σ_χ, such that the impact response in a DSVAR model under the structural model is equal to the impact response in a DSVAR under actual data. Figure 2 illustrates this property. In this figure, we report the binding function is the $(\psi(\sigma_\chi), \sigma_\chi)$ plane.[7] This figure also illustrates the results of Proposition 1. When the standard–error of the non–technology shock increases, the negative response of hours in the DSVAR model is more pronounced. Now, we use the point estimate[8] $\widehat{\psi}_T \simeq -0.27$ of Gali and Rabanal (2004). From this point estimate, we can directly deduce the value of σ_χ such that $\psi(\sigma_\chi) = \widehat{\psi}_T$ using the binding function. This example simply show how to conduct a quantitative investigation about the ability of business cycle models to match impulse responses of hours in DSVAR model. We now introduce a RBC model in the line of Kydland and Prescott (1982)

[7] In this figure, we set $\alpha = 0.6$ and $\sigma_z = 0.025$.

[8] See also the subsection 4.1 and figure 3.

2 A Kydland–Prescott Type Model

We consider a simpler and modified version of the Kydland–Prescott model in the line of Wen (1998). We depart from Kydland and Prescott as the model includes two shocks: a random walk productivity shock (Z_t) and a stationary preference shock (χ_t). Second, we consider that intertemporal leisure choices are not time separable – as in Kydland and Prescott –, but we assume that the service flows from leisure are a linear function of current and one lag leisure choices. More precisely, the intertemporal expected utility function of the representative household is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \log(C_{t+i}) + \chi_{t+i} \log(L^\star_{t+i}) \right\}$$

where $\beta \in (0,1)$ denotes the discount factor and E_t is the expectation operator conditional on the information set available at time t. C_t is the consumption and leisure at time t and $L^\star_{t+i}$ represents some service flows from leisure L_t. As in the simple model, the labor supply $N_t \equiv 1 - L_t$ is subjected to a stochastic shock χ_t, that follows a stationary stochastic process:

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + \sigma_\chi \varepsilon_{\chi,t}$$

where $|\rho_\chi| < 1$, $\sigma_\chi > 0$ and $\varepsilon_{\chi,t}$ is *iid* with zero mean and unit variance. The servie flows is represented by

$$L^\star_t = B(L) L_t$$

where $B(L)$ is a defined as:

$$B(L) = 1 - bL$$

where L is a lag operator. This form of the utility function – although simpler – is very similar to Kydland and Precott[9]. The main difference with Kydland and Precott comes from the sign of b. Kydland and Prescott imposed that b is strictly negative, implying that current and future leisure choice are intertemporally substituable. We do not impose such a restriction and b will be chosen in order to match some moments.

[9]We consider only one lag whereas Kydland and Prescott assumes habit in leisure gradually reacts to past leisure choices.

The representative firm use capital K_t and labor N_t to produce a final good. The technology is represented by the following constant returns–to–scale Cobb–Douglas production function

$$Y_t = K_t^{\alpha} \left(Z_t N_t\right)^{1-\alpha}$$

where Z_t is assumed to follow an exogenous process of the form

$$\log(Z_t) = \gamma_z + \log(Z_{t-1}) + \sigma_z \varepsilon_{z,t}$$

where $\sigma_z > 0$ and $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The constant γ_z is a drift term in random walk process of Z_t. Capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta) K_t + I_t$$

where $\delta \in (0, 1)$ is a constant depreciation rate. Finally, the final output good can be either consumed or invested

$$Y_t = C_t + I_t$$

We first apply a stationnary–inducing transformation for variables that follows a stochastic trend. Output, consumption and investment are dived by Z_t, whereas the capital stock is dived by Z_{t-1}. The approximate solution of the model is computed from a log–linearization of stationary equilibrium conditions around the deterministic steady state using the numerical algorithm of Anderson and Moore (1985).

3 Econometric Methodology

In order to estimate and evaluate the RBC model, we use Indirect Inference (see Gouriéroux, Monfort and Renault (1993), Gouriéroux and Monfort (1996) and the basic intuitions of subsection 1.2). We then depart from a large strand of the literature that employs a limited information strategy (see Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2004) and Altig, Christiano, Eichenbaum, and Linde (2005), among others). The idea of this strategy is to estimate the structural parameters such that the impulse responses of some macroeconomic variables in the model directly match the impulse responses in a SVAR model from actual data. We do not employ this empirical strategy as we have shown in the introductory example of subsection 1.2 that the response of the

DSVAR can be severely downward biased. The idea of indirect inference is to use auxiliary criterium (or auxiliary model) in order to estimate the model. Rather than directly estimate the model using the theoretical impulse responses, we estimate a DSVAR model under the model and compute the responses of hours using long–run restrictions. These responses are then compared to the ones obtained from actual data and structural parameters are estimated such that the discrepancy between the two responses is as small as possible.

In order to present our econometric approach, we define the following VAR model:

$$\mathbf{z}_t = \mathbf{A}_1\mathbf{z}_{t-1} + \cdots + \mathbf{A}_p\mathbf{z}_{t-p} + \boldsymbol{\varepsilon}_t, \qquad \mathrm{E}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' = \boldsymbol{\Sigma}, \tag{6}$$

with $\mathbf{z}_t = (\Delta x_t, \Delta n_t)'$, where n_t is logged total hours worked per capita and x_t is the labor productivity. This specification with hours worked in first difference is used by Gali (1999), (2004a), (2004b), Galí and Rabanal (2004), Francis and Ramey (2004). We follow Galí and Rabanal (2004) and assume that $p = 4$.

3.1 Identification of Impulse Responses

Let us define $\mathbf{B}(L) = (\mathbf{I}_2 - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p)^{-1}$, so that

$$\mathbf{z}_t = \mathbf{B}(L)\,\boldsymbol{\varepsilon}_t,$$

where $\mathbf{I}_2$ is the identity matrix. Now, we assume that the canonical innovations are linear combinations of the structural shocks $\boldsymbol{\eta}_t$, i.e. $\boldsymbol{\varepsilon}_t = \mathbf{S}\boldsymbol{\eta}_t$, for some non singular matrix $\mathbf{S}$. As usual, we impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies $\mathrm{E}\boldsymbol{\eta}_t\boldsymbol{\eta}_t' = \mathbf{I}_2$. This gives us three constraints out of the four needed to completely identify $\mathbf{S}$. To setup the last identifying constraint, let us define $\mathbf{C}(L) = \mathbf{B}(L)\mathbf{S}$. Given the ordering of $\mathbf{z}_t$, we simply require that $\mathbf{C}(1)$ be lower triangular, so that only technology/supply shocks can affect the long-run level of labor productivity. This amounts to imposing that $\mathbf{C}(1)$ is the Cholesky factor of $\mathbf{B}(1)\boldsymbol{\Sigma}\mathbf{B}(1)'$. Given consistent estimates of $\mathbf{B}(1)$ and $\boldsymbol{\Sigma}$, we easily obtain an estimate for $\mathbf{C}(1)$. Retrieving $\mathbf{S}$ is then a simple task using the formula $\mathbf{S} = \mathbf{B}(1)^{-1}\mathbf{C}(1)$. Impulse responses are then deduced from the $VMA(\infty)$ representation:

$$\mathbf{z}_t = \mathbf{B}(L)\,\mathbf{B}(1)^{-1}\,\mathbf{C}(1)\,\boldsymbol{\eta}_t \tag{7}$$

where $\boldsymbol{\eta}_{1,t}$ is the identified technology shock, whereas $\boldsymbol{\eta}_{2,t}$ is the non–technology one. The standard–errors of IRFs are computed numerically using the δ-function method. [10]

3.2 Estimation Method

This section presents the econometric methodology. We partition the model parameters θ into two groups $\theta = \{\theta_1, \theta_2\}$.

The first group, denoted θ_1, is composed of γ_z, β, α and δ. The growth rate of Z_t is equal to 0.0036. We set $\beta = 1.03^{-0.25}$, which implies a steady state annualized real interest rate of 3 percent. We set $\alpha = 0.40$, that implies a steady state labor share equal to 60%. Finally, we set $\delta = 0.025$, which implies an annual rate of depreciation on capital equal to 10 percent.

The second group of model parameters is $\theta_2 = \{b, \sigma_z, \rho_\chi, \sigma_\chi\}$. These four parameters are estimated using Indirect Inference. The basic idea of the Indirect Inference is to use an auxiliary criteria (here, the IRFs computed from an estimated a DSVAR model) in order to estimate the parameter of interest. The DSVAR model is thus considered as an auxiliary model, that allows through simulations to identify and estimate θ_2. Note that when the DSVAR model provides consistent estimates of the true IRFs, the parameters ρ_χ and σ_χ does not matter and it would be impossible to identify them from the responses of hours to a technology shock. Conversely, when the DSVAR model induces large distortions in the estimated response of hours to a technological shock, this offers us with the opportunity to identify and estimate these parameters (see the discussion in subsection 1.2). In the empirical exercice, we consider the impulse responses of hours to a technology $\partial n_{t+k}/\partial \boldsymbol{\eta}_{1,t}$ and a non–technology shock $\partial n_{t+k}/\partial \boldsymbol{\eta}_{2,t}$, deduced from (7) for $k = 1, ..., h$ where h is the selected horizon. The estimation method is implemented as follows.

Step 1: Estimate a q-dimensional vector of IRFs, denoted $\widehat{\psi}_T$, from actual data, where $q =$ denotes the number of selected impulse responses (horizon of the responses h and/or the number of responses).

Step 2: From the model'solution, and given the vector of structural parameters, θ, and initial conditions on capital, labor and the shocks, S simulated paths for productivity and employment, denoted $\widetilde{x}^i_T(\theta)$, $\widetilde{n}^i_T(\theta)$, $i = 1, \cdots, S$, are performed.

[10] See appendix B for further details.

Step 3: From these simulations, we estimate a VAR model from the simulated data $\widetilde{\mathbf{z}}_t = (\Delta\widetilde{x}_t, \Delta\widetilde{n}_t)^{i\,\prime}$,

$$\widetilde{\mathbf{z}}^i_t = \widetilde{\mathbf{A}}^i_1\widetilde{\mathbf{z}}^i_{t-1} + \cdots + \widetilde{\mathbf{A}}^i_p\widetilde{\mathbf{z}}^i_{t-p} + \widetilde{\boldsymbol{\varepsilon}}^i_t, \qquad \mathrm{E}\boldsymbol{\varepsilon}^i_t\boldsymbol{\varepsilon}^{i\prime}_t = \widetilde{\boldsymbol{\Sigma}}^i, \qquad i = 1, \cdots, S$$

with the same number of lags ($p = 4$). We then compute the associated vector of IRFs, denoted $\tilde{\psi}^i_T(\theta)$ ($i = 1, \cdots, S$) using long–run restrictions as in step 1

$$\widetilde{\mathbf{z}}^i_t = \widetilde{\mathbf{B}}^i\,(L)\,\widetilde{\mathbf{B}}^i\,(1)^{-1}\,\widetilde{\mathbf{C}}^i\,(1)\,\widetilde{\boldsymbol{\eta}}^i_t$$

and we construct their average

$$\tilde{\psi}^S_T = \frac{1}{S}\sum_{i=1}^{S}\tilde{\psi}^i_T(\theta)$$

Step 4: A Indirect Inference estimate $\tilde{\theta}^S_{2,T}$ for θ_2 minimizes the quadratic form:

$$J(\theta_2) = g'_{T,S}W_T\,g_{T,S}$$

where $g_{T,S} = \left(\widehat{\psi}_T - \widetilde{\psi}^S_T(\theta_2)\right)$ and W_T is a symmetric nonnegative matrix defining the metric.

Steps 2 to 4 are conducted repeatedly until convergence — *i.e.* until a value of θ_2 that minimizes the objective function is obtained. Let denote ψ_0 the pseudo–true value of ψ and $\theta_{2,0}$ the pseudo–true value of θ_2, under standard regularity conditions, for S held fixed and as T goes to infinity, $\sqrt{T}(\tilde{\theta}^S_{2,T} - \theta_{2,0})$ is asymptotically normally distributed, with a covariance matrix equal to $\left(1 + \frac{1}{N}\right)(D'_\theta W_T D_\theta)^{-1}$ where $D_\theta = \partial g_{T,S}/\partial\theta_2$.

A preliminary consistent estimates of the weighting matrix W_T is required for the computation of $\tilde{\theta}^S_{2,T}$. It may be directly based on actual data, and corresponds to the inverse of the covariance matrix of $\sqrt{T}(\widehat{\psi}_T - \psi_0)$, which is obtained from step 1. Here, W_T^{-1} is a diagonal matrix with the sample variances of the $\widehat{\psi}_T$ along the diagonal. These variances correspond to the confidence intervals of impulse responses. So, with this choice of W_T, θ_2 is effectively chosen so that $\widetilde{\psi}^S_T(\theta_2)$ lies as much as possible inside these confidence intervals of $\widehat{\psi}_T$.

For identification sake, we impose that the number of IRFs exceeds the number of structural parameters. This enables us to conduct a global specification test in the lines of Hansen (1982), denoted $J - stat = TSJ(\theta_2)/(1 + S)$, which is asymptotically distributed as a chi–square, with a degree of freedom equal to the number of over–identifying restrictions ($q - \dim\theta_2$).

4 Empirical Results

In this section, we present the empirical results obtained with US data. We first document the data and discuss the impulse responses of hours to a technology shocks. Second, we present the estimation of the structural parameters using Indirect Inference on DSVAR. Finally, we investigate the ability of the structural model to ecompass LSVARs and DSVARs.

4.1 Data and the Responses of Hours

We first present results based on a simple bivariate VAR in a first difference specification. As in Christiano, Eichenbaum and Vigfusson (2004), we use alternative measures (in logs) of productivity and hours worked: (i) non–farm business output divided by non–farm business hours worked, non–farm business hours worked divided by civilian population over the age of 16 (NFB sector); (ii) business output divided by business hours worked, business hours worked divided by civilian population over the age of 16 (B sector); (iii) real GDP divided by non-farm business hours worked, non-farm business hours worked divided by civilian population over the age of 16 (mixed NFB) and (iv) real GDP divided by total business hours, business hours worked divided by civilian population over the age of 16 (mixed B). The empirical analysis uses quarterly U.S. data for the period 1948:1-2002:4.

The response of hours worked to a technology shock for each measure of productivity and hours are similar (see figure 3). For instance, as in Gali (1999), hours worked decrease significantly at the impact. The negative effect appears rather persistent. In the case of NFB sector data, as noticed by Gali and Rabanal (2004), hours do eventually return to their original level. Conversely, the response of hours is persistently bellow zero for the considered horizon. The main difference concerns the confidence interval. For measures (i) and (ii) (NFB sector and B sector), the negative response is not significantly different from zero after two periods, whereas the negative response remains significant at any horizon for measures (iii) and (iv). The impulse responses of hours to the non–technology shock[11] is persistant and hump–shaped. Moreover, the response of hours is precisely estimated for

[11]More precisely, figure 3 reports the responses of hours to a shock without long run effect on labor productivity. The shock is interpreted as a negative preference shock that reduces hours and increases labor productivity.

each horizon.

4.2 Estimation Results from DSVARs

The previous evidences does not support the empirical relevance of standard RBC models, as they cannot reproduce a persistent and negative response of hours after a transitory technology shock. We now investigate this issue using the econometric methodology discussed previously. Table 1 report the estimation resutls in four cases. Each case is associated to a particular mesure of productivity and hours worked. In each siutation, we use a bivariate VAR with a first difference specification and four lags. $S = 30$ simulations were used for a sample size equal to 192. Simulated values are redrawn from the same seed values for each function evaluation. In order to reduce the effect of initial conditions, simulated samples include 250 initial points which are subsequently discarded in the estimation. The minimization of the simulated criterion function is carried out using a Nelder-Meade method for minimization provided in the `Optim` matlab numerical optimization toolbox. At convergence of the Nelder-Meade method, a local gradient search method was used to check convergence. We estimate the four structural parameters $(b, \sigma_z, \rho_\chi, \sigma_\chi)$ using the responses of hours to technology and non–technology shock for an horizon equals to 21. By the mean, the model is estimated in order to match employment fluctuations generated by two shocks. We thus introduce $2 \times 21 - 4 \equiv 38$ over–identifying restrictions.

We first discuss the parameters estimates. The parameter of labor supply (b) is significantly positive, indicating that labor supply is subject to intertemporal complementarities. This result is in accordance with previous results of Eichenbaum, Hansen and Singleton (1988), Bover (1991) and Wen (1998). The estimated value is rather large (greater than 0.8 in each case) and very close to estimated values in Eichenbaum, Hansen and Singleton (see Table II, p. 65). Our results also contradict the calibration – not the specification of labor supply – of this parameter in Kydland and Prescott and clearly suggest that leisure today significantly reduces leisure services in the subsequent time period. Note that the estimated value is very similar in each case, *i.e.* the same degree of habit persistence in leisure habit allows to match different datasets. The estimated value of the standard–error of the technology shock σ_z is rather large (0.0256 and 0.0228), when indirect inference with DSVARs as auxiliary models is a applied on NFB or B sector data. This result comes in part from the high volatility of productivity and hours in these two sectors. When the data are mixed, the estimated value is close to previous estimates (0.0126 and 0.0137). This result can also been explained by the estimated value of b. When b is positive and large, the response of hours to any shock at impact will be small and increase gradually with the horizon. This is a direct consequence of habit in leisure that tends to smooth labor supply. The estimated value of the autoregressive parameter ρ_χ is not large, especially if we compare it to previous estimates. Our estimations suggests values between 0.65 and 0.70, whereas Chari, Kehoe and McGrattan report estimated values between 0.94 and 0.97.[12] The differences comes partly from the specification of the utility function. In Chari, Kehoe and McGrattan, the utility function is time separable, so most of the persistence in the fluctuations of hours worked is the result of a persistence property of the forcing variable. Conversely, when $b > 0$, hours can respond more persistently to a transitory shock. This explains that a large value of ρ_χ is unnecessary in order to reproduce persistent fluctuations in hours. Finally, the estimated value of σ_χ is rather large (between 0.025 and 0.034) compared to previous estimates.[13] As discussed in subsection 1.2, a large standard-error of the preference shock creates distortions in the DSVAR specification and thus allows to match more easily the negative response of hours.

Table 1 reports the global specification test statistic ($J - stat$). For each dataset, the model is not globally rejected by the data, as the p-values associated to the J-stat are large. One may argue that the standard–errors of the responses of hours to a technology shock are large and thus the structural model can easily match the data. For example, in the case of NFB and B sectors datasets, the response of hours is not significantly different from zero after 3 periods. But in the case of mixed datasets, the response of hours is significantly different from zero and the structural model matches well the data. Moreover, the four structural parameters are estimated in order to match simultaneously the responses of hours to a technology and non–technology shocks. The latter are precisely estimated and any departure from the response is highly penalized.

Figures 4–7 report the IRFs of hours to a technology and non–technology shocks under actual data

[12] Erceg, Guerrieri and Gust (2004) use $\rho_\chi = 0.95$.

[13] Chari, Kehoe and McGrattan report estimated values between 0.0076 and 0.0110, but they add measurement errors in the system.

and the model. These figures also include the true response of hours in the RBC model. As these figures shown, the response of hours to a technology shock is always positive. Note that the RBC model is able to reproduce a hump–shaped positive response of hours, as the maximal response is obtained after ten periods. The implied response of hours from DSVAR is negative and does not display an hump–shaped profile. These figure also illustrates the downward bias implied by DSVAR model. Another interesting quantitative feature of the RBC model is its ability to present a persistent response of hours to a non–technology shock.

Finally, we report in Table 1 the variance decomposition for output growth and employment. In each case, the fraction of variance of output growth explained by the technology shock always exceeds 70%. For instance, it exceed 90% with NFB sector data. It follows that a RBC model wherein the technology shocks represent the main source of output growth fluctuations is able to replicate the negative response of hours in DSVARs. It is worth noting that the preference shock explained most of employment fluctuations (between 70% and 92% of the variance). This is in accordance with the business cycle accounting exercice of Chari, Kehoe and McGrattan (2004a).

4.3 Estimation Results from SVARs

The estimation of impulse responses of hours critically depends on the SVAR specification. Christiano, Eichenbaum and Vigfusson (2004) argue that a difference specification of hours may create severe distortions in the DSVAR specification if hours are truly stationary. Using a LSVAR specification, they obtained a positive and hump–shaped response of hours following a technology shock, but the response is not precisely estimated (see Chari, Kehoe and Mac Grattan (2004b)). We report in Figure 8, the response of hours to a technology and non–technology shock in the LSVAR specification with NFB sector data. The response is always positive, but it is not significantly different from zero at each horizon. Conversely, the response of hours to a non–technology shock is persistent and significant.

We now investigate the ability of the structural model to replicate such patterns with NFB sector data. To do this, we conduct three experiments. In the first one, we compute the IRFs of hours in a LSVAR specification using the estimations of Table 1 (column NFB sector). This counterfactual experiment accounts for the ability of the model estimated from a DSVAR to replicate impulse responses from LSVAR under the structural model. In the second experiment, we estimate the structural model using the responses of hours to a technology and non–technology shocks in a LSVAR specification. We also compute the IRF of hours in a DSVAR specification under the estimated model. Finally, in a third experiment, we estimate the four structural parameters in order to match simultaneously the impluse responses of hours in a DSVAR and LSVAR specifications.

Table 2 report the empirical results. The first column is identical to the one of table 1. Given these parameters estimates, we compute the IRF from LSVAR under the structural model. Figure 9 reports the IRF from the actual data, from the simulated data and from the model. The right part of the figure 9 concerns the quantitative implications for LSVAR specification. As this figure shows, the structural model reproduces well the response of hours. The responses of hours from simulations present a similar hump–shaped to the ones obtained from the data. To test the match between the two IRFs, we compute the following Q_i for various horizon:

$$Q_i = \left(\widehat{\psi}_{[1:i],T} - \widetilde{\psi}^S_{[1:i],T}(\widehat{\theta}_2)\right)' W_{[1:i],T} \left(\widehat{\psi}_{[1:i],T} - \widetilde{\psi}^S_{[1:i],T}(\widehat{\theta}_2)\right)$$

where $W_{[1:i],T}$ is the inverse of the covariance matrix of $\widehat{\psi}_{[1:i],T}$. This simple test (see the Q_i ($i = 6, 11, 21$) statistic in Table 2) indicates that the structural model produce responses of hours that are not significantly different from the LSVAR model at horizon 6 and 11 (see the P-values of Q_6 and $Q_1 1$). However, for a longer horizon, the model faces some difficulties to reproduce the response of hours to a non–technology shock (see Q_{21} in the table). We now investigate if the model is able to match the response of hours in a LSVAR specification. The second column of Table 2 reports the parameter estimates. Note that the standard errors of the two shocks is larger than the ones of first column. The global specification tests indicates that the model easily matches the responses of hours. Given the large confidence interval of the response to a technology shock (see figure 10), this is not surprizing.[14] Moreover, the LSVAR specification is in accordance with the structural model. Using these estimated value, we simulate the model and compute the responses of hours using a DSVAR specification. The right part of figure 10 reports the impulse responses from DSVAR. The responses from simulated data depart significantly from the ones of the actual data. Indeed, the response

[14]However, the response to a non–technology shock is very precisely estimated, making any departure very penalizing.

of hours to a technology shock is zero at the impact and becomes persistently positive. Moreover, the response to a non–technology shock does not present a hump–shaped profile. The Q_i statistic (see the second column of table 2) indicates that the model estimated from a LSVAR specification fails to reproduce a DSVAR specification. This result, together with the previous one, show that the DSVAR specification, although providing biased responses of hours, indirectly encompasses the LSVAR specification. Finally, we estimate the structural model using the two specifications of the SVAR model as the auxiliary models, *i.e.* the responses of hours to a technology and non–technology shock with the hours in first difference and in level. The J-statistic in the third column of Table 2 is very small compared the critical value and the structural model is able to march very well the responses of hours. In the DSVAR specification, the response under the model is negative, whereas it is persistently positive in the LSVAR specification. These results show that a simple RBC model wherein hours increase after a technology shock easily encompasses SVAR models with contradictory results.

5 Concluding Remarks

The identification of the response of hours worked after a technology shock using SVAR has renewed the debate on the relative contributions of various shocks to the business cycle. The SVAR approach documents a striking evidence for the standard RBC model: after a positive technology shock, hours worked decrease. For researchers that use the SVAR approach, this evidence suggests to abandon the RBC model in favor of models with important (real) frictions and (nominal) rigidities.

This paper shows that DSVAR poorly identifies the impulse responses of hours and suggests another way to evaluate structural model. Using an indirect approach (Indirect Inference), we show that a Kydland–Prescott type model matches indirectly very well impulse responses of DSVAR. Moreover, the estimated technology shock account for a large part of output growth fluctuations. Finally, the model encompasses LSVAR and DSVAR models.

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Appendix

A Proof of Proposition 1

We consider the estimation of a VAR(1) model with data generated by the structural model of section 1 (see equations (3)–(5)). The VAR(1) model has the form:

$$\begin{pmatrix} \Delta x_t \\ \Delta n_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{pmatrix} \begin{pmatrix} \Delta x_{t-1} \\ \Delta n_{t-1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{1,t} \\ \boldsymbol{\varepsilon}_{2,t} \end{pmatrix}$$

The OLS regression from the first equation yields:

$$\begin{pmatrix} \widehat{\boldsymbol{a}}_{11} \\ \widehat{\boldsymbol{a}}_{12} \end{pmatrix} = \begin{pmatrix} V(\Delta x_t) & Cov(\Delta x_t, \Delta n_t) \\ Cov(\Delta x_t, \Delta n_t) & V(\Delta n_t) \end{pmatrix}^{-1} \begin{pmatrix} Cov(\Delta x_t, \Delta x_{t-1}) \\ Cov(\Delta x_t, \Delta n_{t-1}) \end{pmatrix}$$

whereas the OLS regression from the second equation yields

$$\begin{pmatrix} \widehat{\boldsymbol{a}}_{21} \\ \widehat{\boldsymbol{a}}_{22} \end{pmatrix} = \begin{pmatrix} V(\Delta x_t) & Cov(\Delta x_t, \Delta n_t) \\ Cov(\Delta x_t, \Delta n_t) & V(\Delta n_t) \end{pmatrix}^{-1} \begin{pmatrix} Cov(\Delta n_t, \Delta x_{t-1}) \\ Cov(\Delta n_t, \Delta n_{t-1}) \end{pmatrix}$$

The variances and covariances that enter in the B matrix are given by $V(\Delta x_t) = \sigma_z^2 + 2\sigma_\chi^2(1-\alpha)^2/(1+\rho_\chi)$, $V(\Delta n_t) = 2\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta n_t) = -2(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta x_{t-1}) = -(1-\rho_\chi)(1-\alpha)^2\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta x_t, \Delta n_{t-1}) = (1-\rho_\chi)(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$, $Cov(\Delta n_t, \Delta x_{t-1}) = (1-\rho_\chi)(1-\alpha)\sigma_\chi^2/(1+\rho_\chi)$ and $Cov(\Delta n_t, \Delta n_{t-1}) = -(1-\rho_\chi)\sigma_\chi^2/(1+\rho_\chi)$. The OLS estimator $\widehat{\boldsymbol{A}}$ of $\boldsymbol{A}$ is then deduced:

$$\widehat{\boldsymbol{A}}_1 = \begin{pmatrix} 0 & \frac{(1-\rho_\chi)(1-\alpha)}{2} \\ 0 & -\frac{1-\rho_\chi}{2} \end{pmatrix}$$

The residuals of each equation are given by;

$$\begin{aligned} \boldsymbol{\varepsilon}_{1,t} &= \Delta x_t - \frac{(1-\rho_\chi)(1-\alpha)}{2}\Delta n_{t-1} \\ \boldsymbol{\varepsilon}_{2,t} &= \Delta n_t + \frac{1-\rho_\chi}{2}\Delta n_{t-1} \end{aligned}$$

and the associated covariance matrix is

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_z^2 + \frac{(3-\rho_\chi)(1-\alpha^2)}{2}\sigma_\chi^2 & -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 \\ -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 & \frac{3-\rho_\chi}{2}\sigma_\chi^2 \end{pmatrix}$$

We thus compute the long–run covariance matrix

$$\begin{aligned} \left(\left(\boldsymbol{I}_2 - \widehat{\boldsymbol{A}}\right)^{-1}\right)\Sigma\left(\left(\boldsymbol{I}_2 - \widehat{\boldsymbol{A}}\right)^{-1}\right)' &= \begin{pmatrix} 1 & \frac{(1-\rho_\chi)(1-\alpha)}{3-\rho_\chi} \\ 0 & \frac{2}{3-\rho_\chi} \end{pmatrix} \begin{pmatrix} \sigma_z^2 + \frac{(3-\rho_\chi)(1-\alpha^2)}{2}\sigma_\chi^2 & -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 \\ -\frac{(3-\rho_\chi)(1-\alpha)}{2}\sigma_\chi^2 & \frac{3-\rho_\chi}{2}\sigma_\chi^2 \end{pmatrix} \\ &\quad \begin{pmatrix} 1 & 0 \\ \frac{(1-\rho_\chi)(1-\alpha)}{3-\rho_\chi} & \frac{2}{3-\rho_\chi} \end{pmatrix} \\ &= \begin{pmatrix} \sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2 & -\frac{2(1-\alpha)}{3-\rho_\chi}\sigma_\chi^2 \\ -\frac{2(1-\alpha)}{3-\rho_\chi}\sigma_\chi^2 & \frac{2}{3-\rho_\chi}\sigma_\chi^2 \end{pmatrix} \end{aligned}$$

The matrix $\boldsymbol{C}(1)$ is the Choleski decomposition of the long–run covariance matrix

$$\boldsymbol{C}(1) = \begin{pmatrix} \left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2} & 0 \\ -\frac{2(1-\alpha)\sigma_\chi^2}{(3-\rho_\chi)\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}} & \left(\frac{2\sigma_z^2\sigma_\chi^2}{(3-\rho_\chi)\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)}\right)^{1/2} \end{pmatrix}$$

The IRF for labor productivity and hours are then deduced from $\boldsymbol{C}(L)$

$$\boldsymbol{C}(L) = (\boldsymbol{I}_2 - \widehat{\boldsymbol{A}}L)^{-1}(\boldsymbol{I}_2 - \widehat{\boldsymbol{A}})\boldsymbol{C}(1)$$

The response at the impact of employment to a technology shock is negative:

$$-\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}$$

and the response at horizon k of the level of employment is:

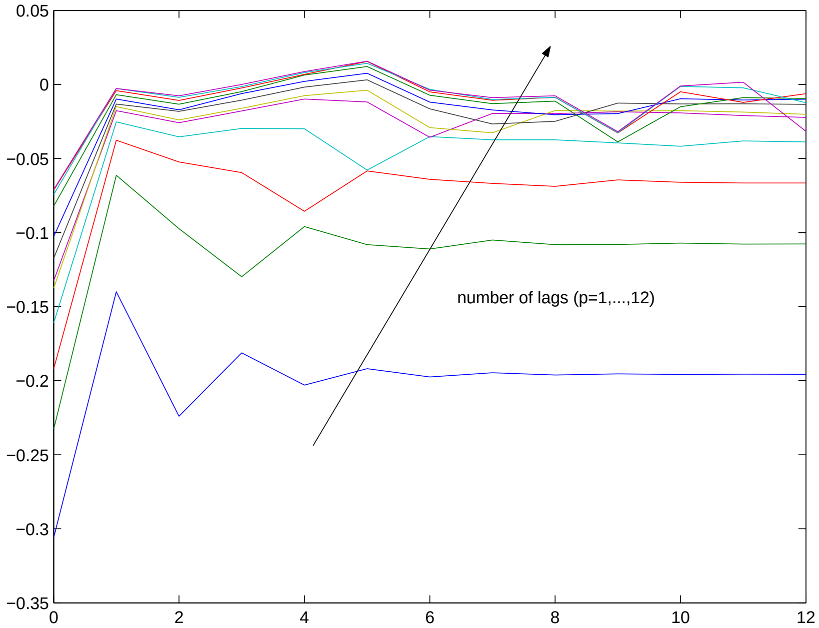
$$-\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}} \sum_{j=0}^{k} \left(-\left(\frac{1-\rho_\chi}{2}\right)\right)^j$$

When $k \to \infty$, the IRF of hours is:

$$-\left(\frac{2}{3-\rho_\chi}\right)\frac{(1-\alpha)\sigma_\chi^2}{\left(\sigma_z^2 + \frac{2(1-\alpha)^2}{3-\rho_\chi}\sigma_\chi^2\right)^{1/2}}$$

Figure 1: IRF of hours

(a) *iid* preference shock ($\rho_\chi = 0$)



(b) Persistent preference shock ($\rho_\chi = 0.98$)

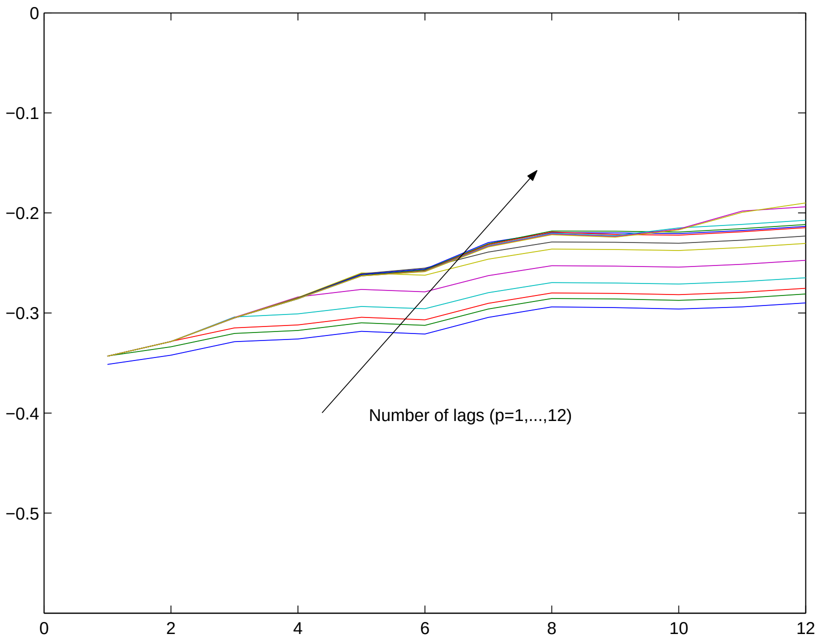
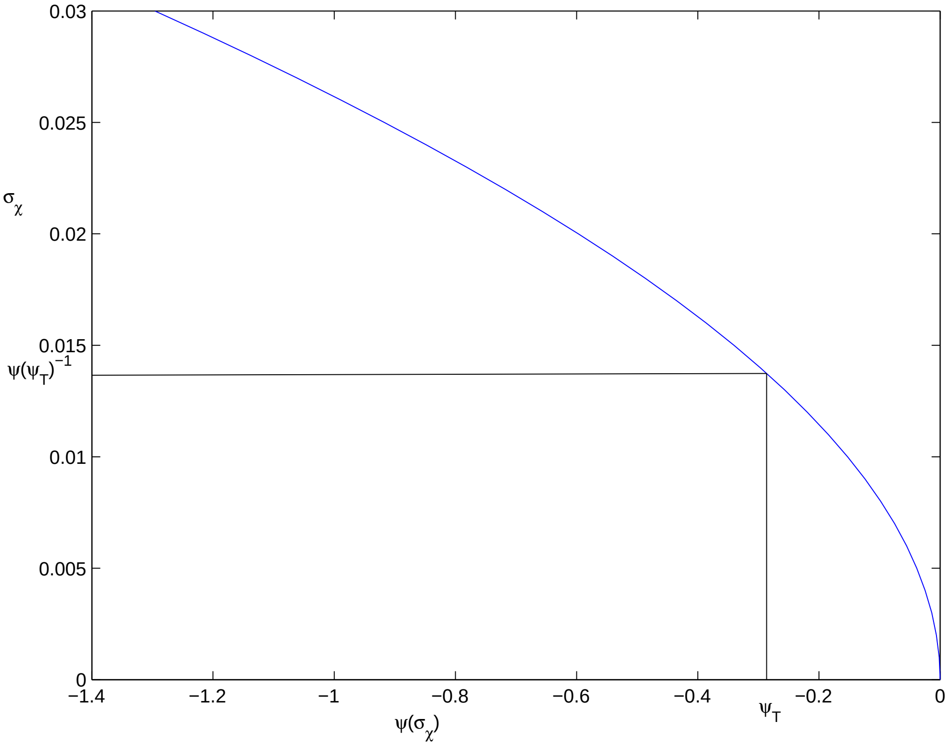


Figure 2: The Binding Function and Indirect Estimation



B Indirect Inference Weighting Matrix

This appendix describes how were computed the impulse response functions and their asymptotic confidence intervals. It is convenient to define

$$
\begin{aligned}
\mathbf{\Pi}' &= \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_\ell \end{pmatrix} \\
\mathbf{Q} &= \mathrm{E}\left(\begin{pmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \\ \vdots \\ \mathbf{z}_{t-p} \end{pmatrix} \begin{pmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \\ \vdots \\ \mathbf{z}_{t-p} \end{pmatrix}' \right)
\end{aligned}
$$

Now let $\hat{\mathbf{\Pi}}$ and $\hat{\mathbf{\Sigma}}$ denote the empirical estimates of $\mathbf{\Pi}$ and $\mathbf{\Sigma}$, respectively. We regroup the VAR parameters in the vector $\boldsymbol{\beta}$:

$$\boldsymbol{\beta} = (\mathrm{vec}(\mathbf{\Pi})', \mathrm{vech}(\mathbf{\Sigma})')', \quad \hat{\boldsymbol{\beta}} = (\mathrm{vec}(\hat{\mathbf{\Pi}})', \mathrm{vech}(\hat{\mathbf{\Sigma}})')',$$

where vec $(\cdot)$ is the operator transforming an $(n \times n)$ matrix into an $(n^2 \times 1)$ vector by stacking the columns, vech $(\cdot)$ is the operator transforming an $(n \times n)$ matrix into an $(n\,(n+1)/2 \times 1)$ vector by vertically stacking those elements on or below the principal diagonal. For later purpose, define $m = n\,(n\ell + (n+1)/2)$, so that $\boldsymbol{\beta}$ is an $(m \times 1)$ vector. Following Hamilton (1994) (proposition 11.2, page 301), it can be shown that

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \underset{a}{\sim} \mathrm{N}\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \mathbf{\Sigma}_\beta \right),$$

where T is the sample size and

$$\mathbf{\Sigma}_\beta = \begin{pmatrix} \mathbf{\Sigma} \otimes \mathbf{Q}^{-1} & 0 \\ 0 & \mathbf{\Sigma}_{22} \end{pmatrix},$$

with $\mathbf{\Sigma}_{22}$ defined as

$$\mathbf{\Sigma}_{22} = 2\left(\mathbf{D}_n^+\right)(\mathbf{\Sigma} \otimes \mathbf{\Sigma})\left(\mathbf{D}_n^+\right)'.$$

Here $\mathbf{D}_n^+$ is the unique matrix such that $\mathrm{vech}(\mathbf{\Sigma}) = \mathbf{D}_n^+ \mathrm{vec}(\mathbf{\Sigma})$. In practice, we replace $\mathbf{\Sigma}$ and $\mathbf{Q}$ in the above formula with their empirical counterparts

$$
\begin{aligned}
\hat{\mathbf{\Sigma}} &= \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_t \hat{\boldsymbol{\epsilon}}_t' \\
\hat{\mathbf{Q}} &= \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'
\end{aligned}
$$

We assume that the canonical innovations are linear combinations of the structural shocks $\boldsymbol{\eta}_t$, *i.e.*

$$\boldsymbol{\varepsilon}_t = \mathbf{S}\boldsymbol{\eta}_t,$$

for some non singular matrix $\mathbf{S}$. We impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies $\mathrm{E}\boldsymbol{\eta}_t\boldsymbol{\eta}_t' = \mathbf{I}_n$. Now, let us define

$$
\begin{aligned}
\mathbf{B}(L) &= (\mathbf{I}_n - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p)^{-1} \\
\mathbf{C}(L) &= \mathbf{B}(L)\,\mathbf{S}
\end{aligned}
$$

Now, let us define the vector collecting the dynamic response of the components of $\mathbf{z}_t$ to a technology/supply shock $\boldsymbol{\eta}_{1,t}$

$$\boldsymbol{\theta}_k = \frac{\partial \mathbf{z}_{t+k}}{\partial \boldsymbol{\eta}_{1,t}}.$$

Formally, $\boldsymbol{\theta}_k$ is the first column of $\mathbf{C}_k$, where $\mathbf{C}_k$ is the k-coefficient of $\mathbf{C}(L)$. In the sequel, we define $\boldsymbol{\theta}$ as

$$\boldsymbol{\theta} = \mathrm{vec}([\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_k]').$$

Recall that $\mathbf{C}_k = \mathbf{B}_k\mathbf{S}$, where $\mathbf{B}_k$ is the upper leftmost $(n \times n)$ block of $\mathbf{F}^k$ (Hamilton, 1994, p. 260), where

$$\underset{(np \times np)}{\mathbf{F}} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \cdots & \mathbf{A}_{p-1} & \mathbf{A}_p \\ \mathbf{I}_n & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{I}_n & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \mathbf{I}_n & \mathbf{0}_{n\times n} \end{pmatrix}$$

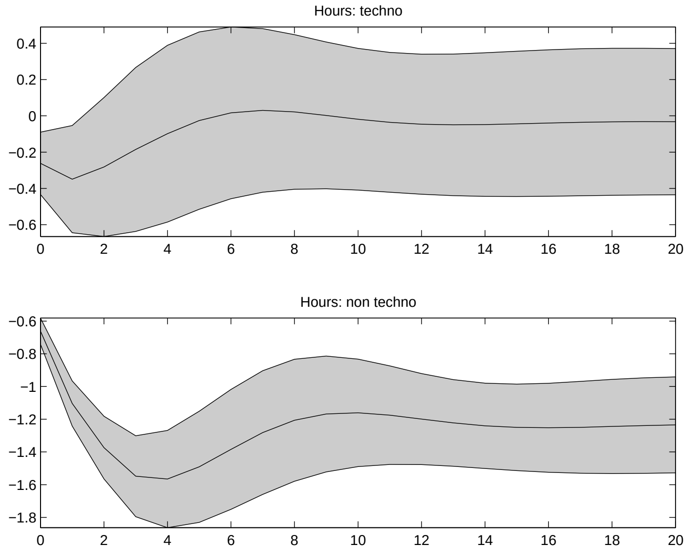
In practice, we use this formula with $\hat{\mathbf{\Sigma}}$, $\hat{\mathbf{A}}_1$,..., and $\hat{\mathbf{A}}_p$ substituted for $\mathbf{\Sigma}$, $\mathbf{A}_1$,..., and $\mathbf{A}_p$ to estimate the $\boldsymbol{\theta}_k$. In the sequel, we let $\hat{\boldsymbol{\theta}}_k$ denote the empirical estimates of $\boldsymbol{\theta}_k$ and $\hat{\boldsymbol{\theta}}$ denote the empirical estimate of $\boldsymbol{\theta}$. To compute the confidence intervals of $\hat{\boldsymbol{\theta}}$, we resort to the δ-function method. It can be shown that $\boldsymbol{\theta}$ is an implicit function of $\boldsymbol{\beta}$. Then, we obtain the formula

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \underset{a}{\sim} \mathrm{N}\left(0, \frac{\partial \boldsymbol{\theta}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}'} \mathbf{\Sigma}_\beta \frac{\partial \boldsymbol{\theta}(\boldsymbol{\beta})'}{\partial \boldsymbol{\beta}}\right).$$

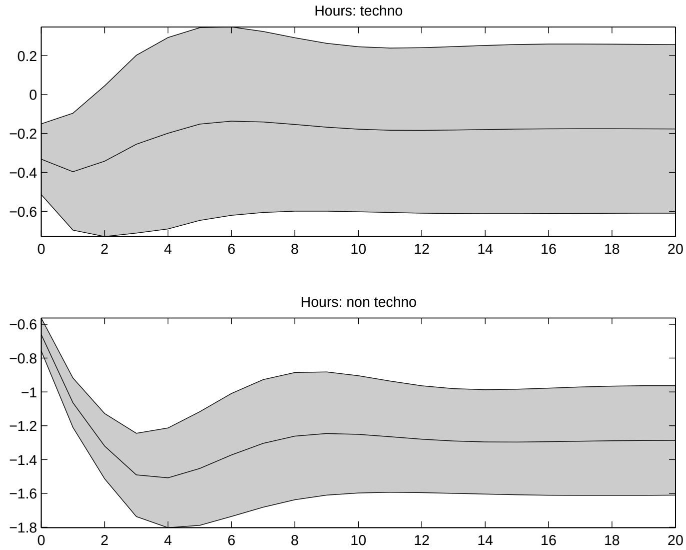
In practice, the derivatives $\partial \boldsymbol{\theta}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}'$ are computed numerically at the point estimate $\hat{\boldsymbol{\beta}}$.

Figure 3: IRF of hours

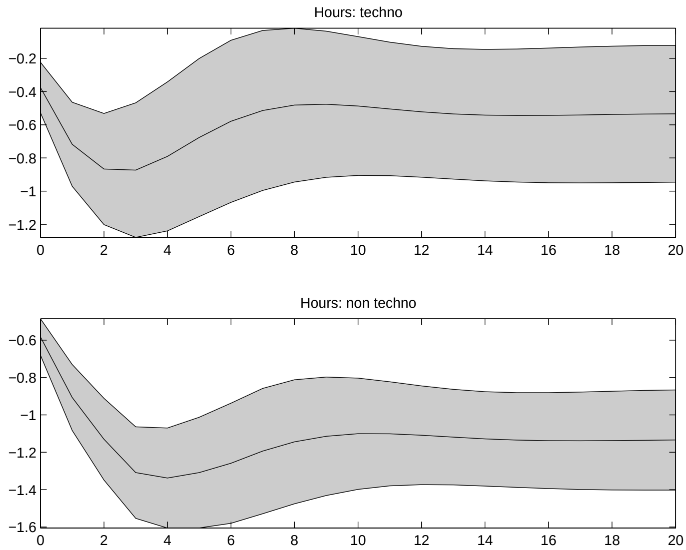
(a) NFB sector data



(b) B sector data



(c) Mixed NFB sector data



(d) Mixed B sector data

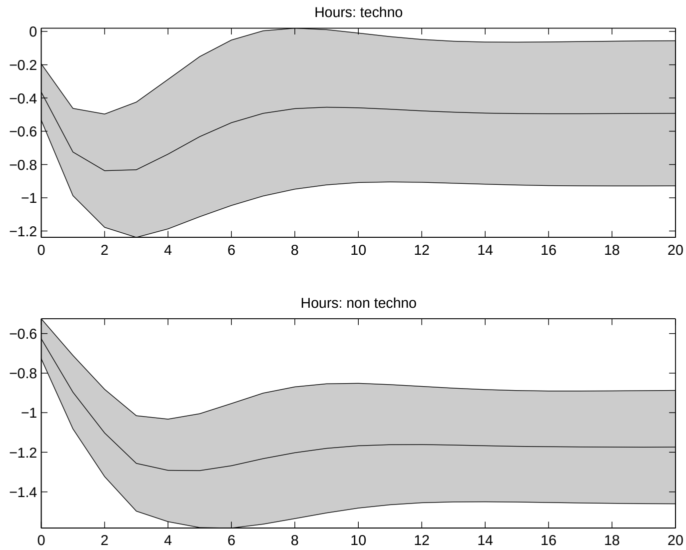


Table 1: Results from DSVARs

	NFB sector	B sector	Mixed NFB sector	Mixed B sector
b	0.8367	0.8563	0.8482	0.8816
	(0.2106)	(0.2100)	(0.2806)	(0.1678)
σ_z	0.0256	0.0228	0.0126	0.0137
	(0.0079)	(0.0065)	(0.0020)	(0.0017)
ρ_χ	0.6855	0.6837	0.7004	0.6507
	(0.2981)	(0.3073)	(0.4016)	(0.2636)
σ_χ	0.0255	0.0287	0.0270	0.03381
	(0.0281)	(0.0367)	(0.0422)	(0.0433)
$J-stat$	11.82	6.23	9.88	9.13
	[100]	[100]	[100]	[100]
$V(\Delta y/\varepsilon_z)$ (in %)	91.2	88.8	71.4	73.9
$V(n/\varepsilon_z)$ (in %)	30.7	22.3	8.4	8.2

Note: standard–errors in parentheses; P–values in brackets

Figure 4: IRF of hours (NFB sector data)

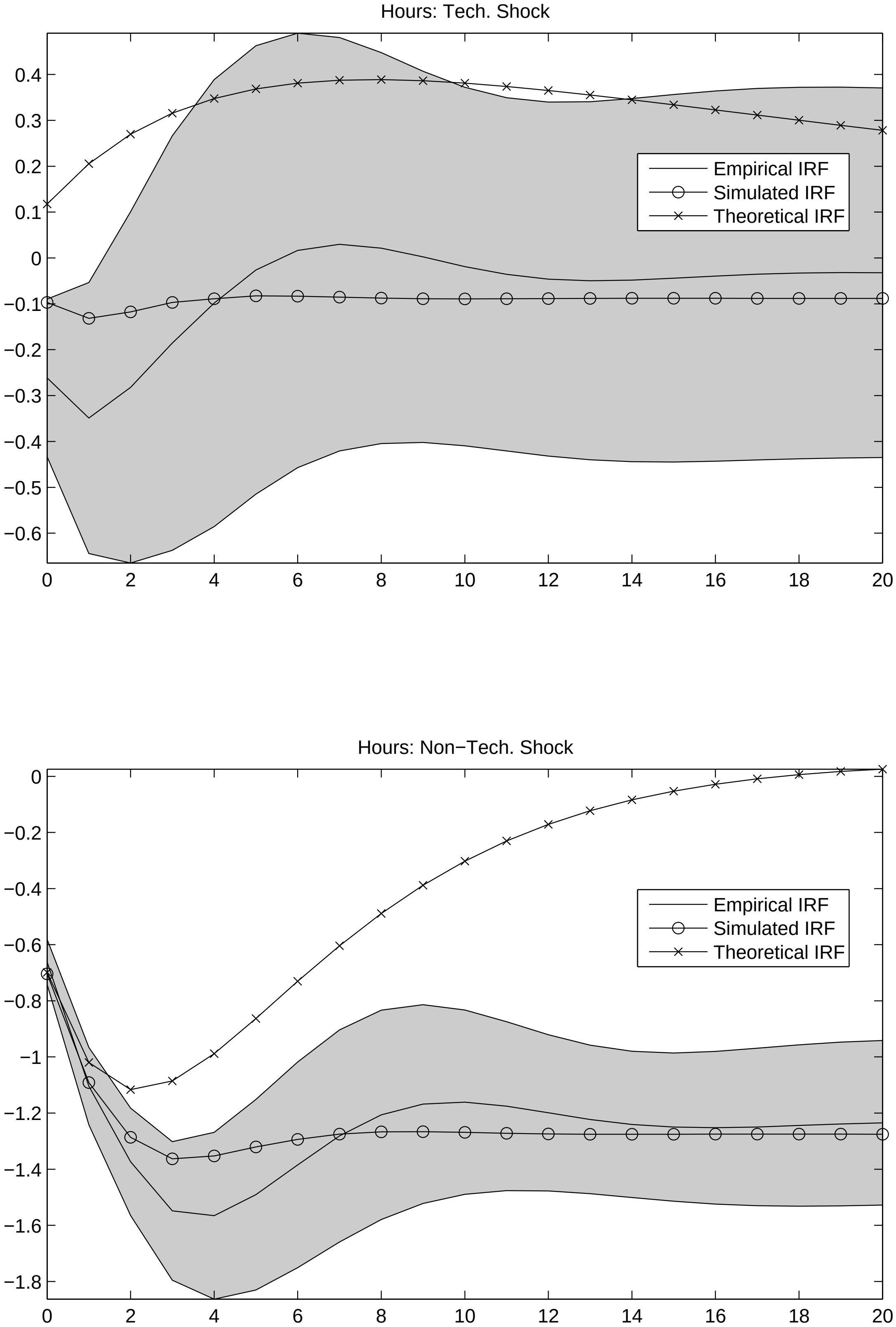


Figure 5: IRF of hours (B sector data)

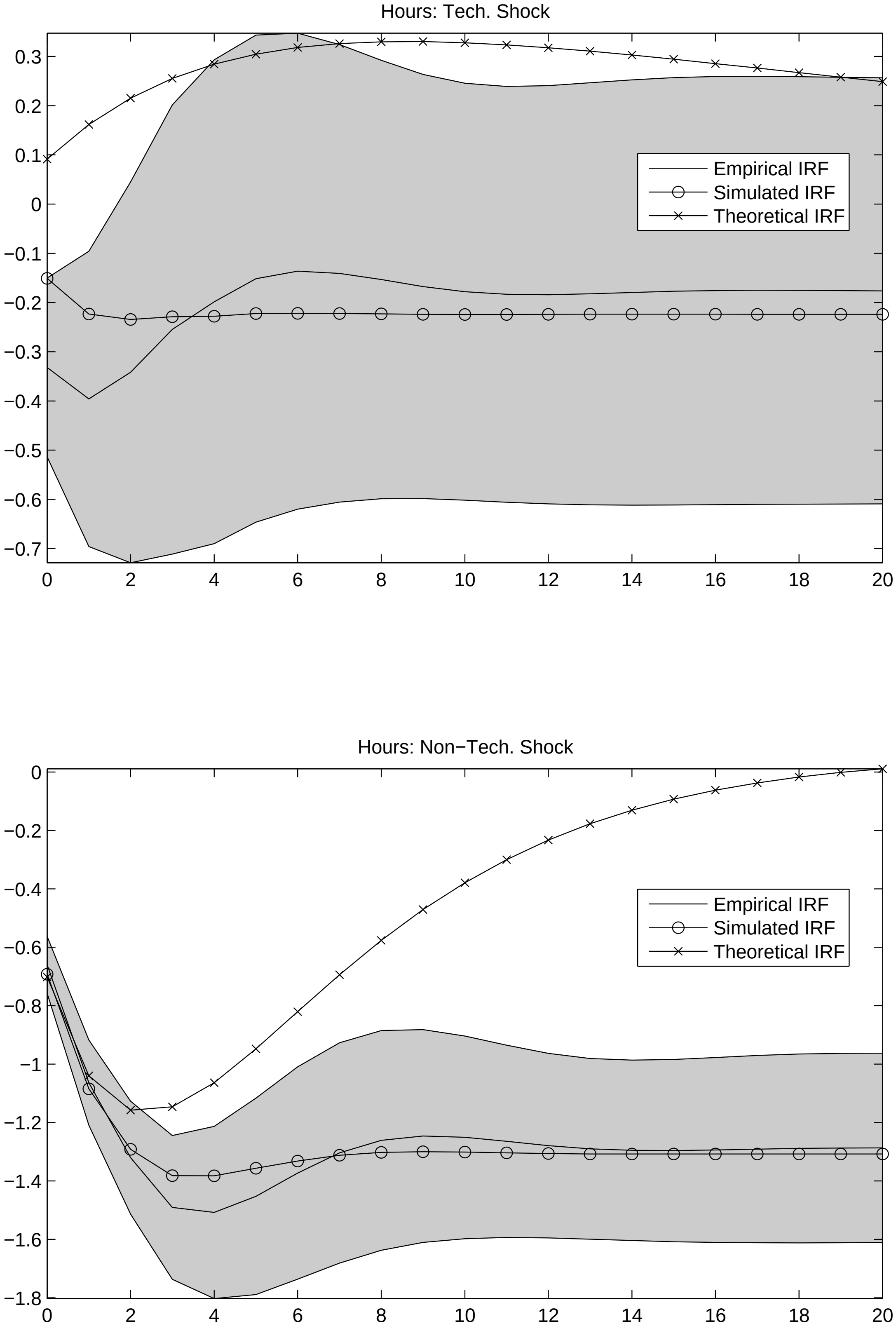


Figure 6: IRF of hours (mixed NFB sector data)

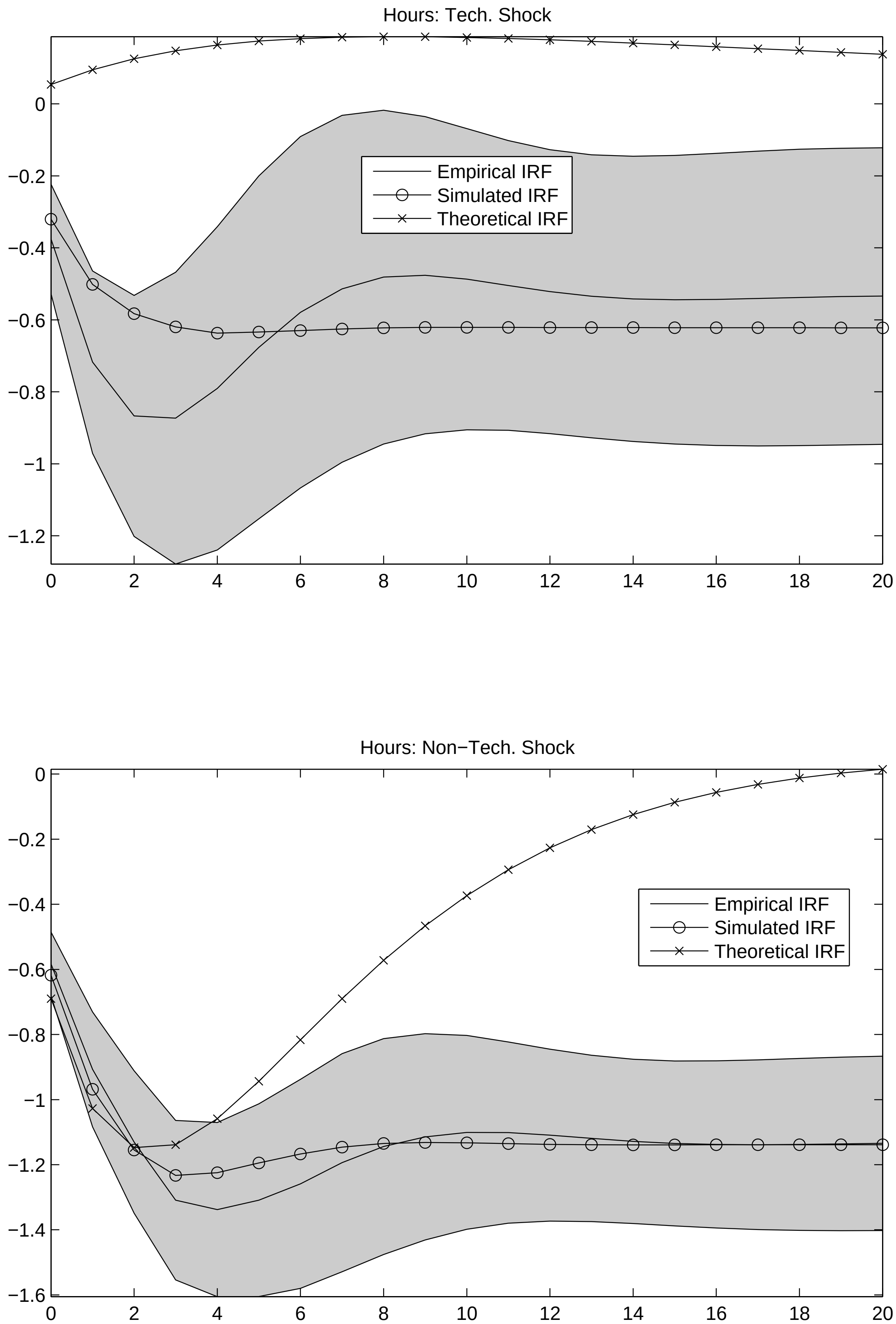


Figure 7: IRF of hours (mixed B sector data)

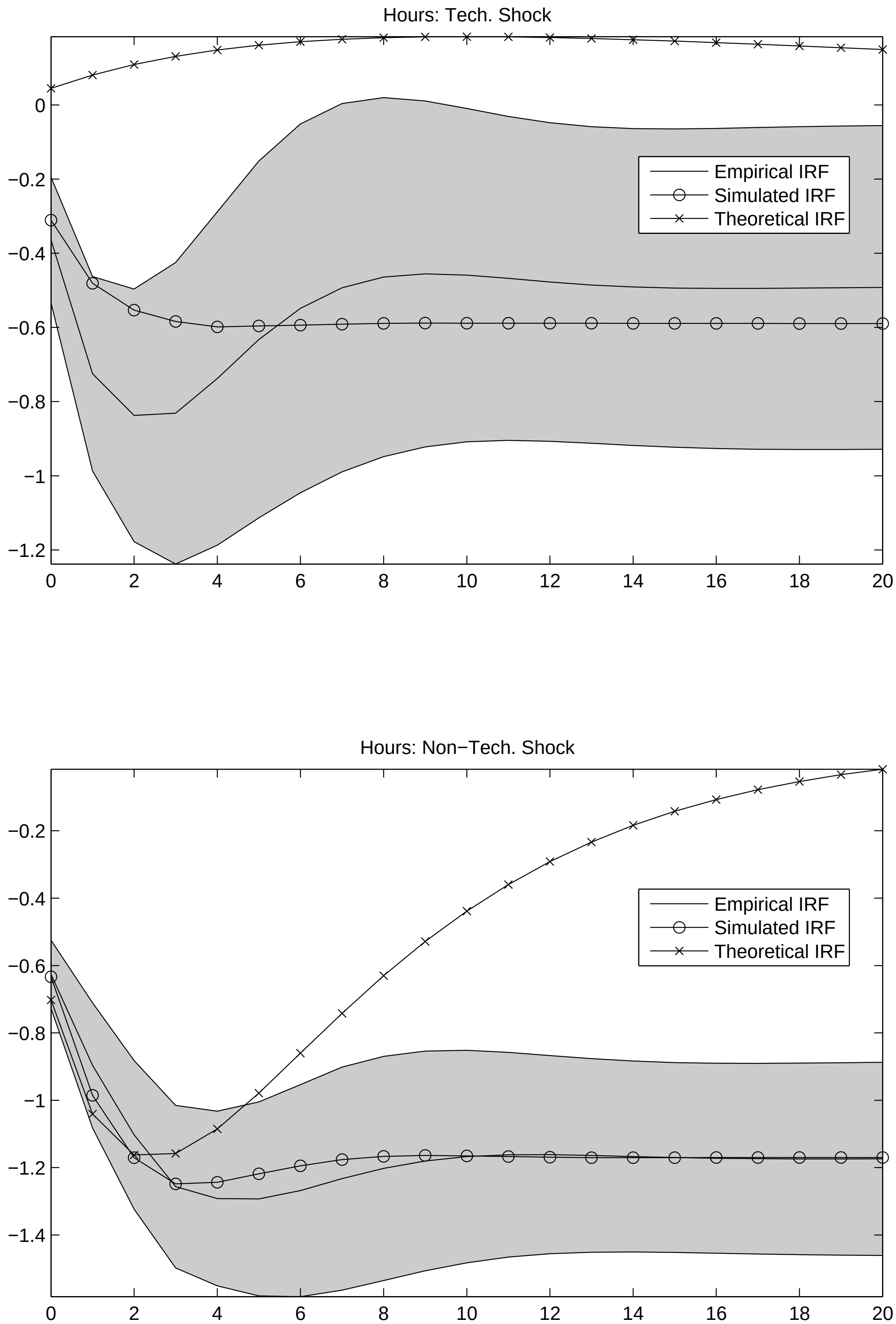


Figure 8: IRF of hours in LSVAR (NFB sector data)

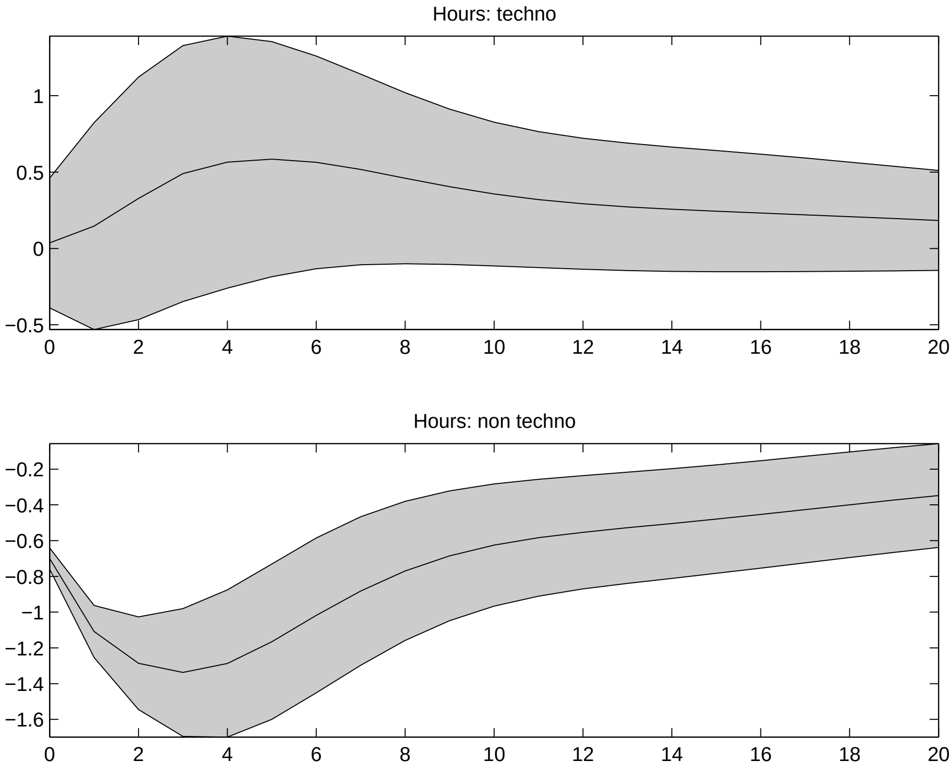


Table 2: Results from SVARs

	DSVAR	LSVAR	DSVAR–LSVAR
b	0.8367	0.9323	0.9469
	(0.2106)	(0.0154)	(0.0125)
σ_z	0.0256	0.0600	0.0376
	(0.0079)	(0.0107)	(0.0043)
ρ_χ	0.6855	0.5597	0.5153
	(0.2981)	(0.0669)	(0.0399)
σ_χ	0.0255	0.0633	0.0760
	(0.0281)	(0.0143)	(0.0184)
$J - stat$	11.82	4.25	32.87
	[100]	[100]	[100]
Q_6	16.26	26.67	–
	[17.9]	[0.9]	–
Q_{11}	23.28	39.10	–
	[38.6]	[1.37]	–
Q_{21}	61.38	80.27	–
	[2.70]	[0.03]	–
$V(\Delta y/\varepsilon_z)$ (in %)	91.2	97.7	95.0
$V(n/\varepsilon_z)$ (in %)	30.7	44.4	21.1

Note: standard–errors in parentheses; P–values in brackets

Figure 9: IRF of hours in DSVAR and LSVAR (estimations from DSVAR)

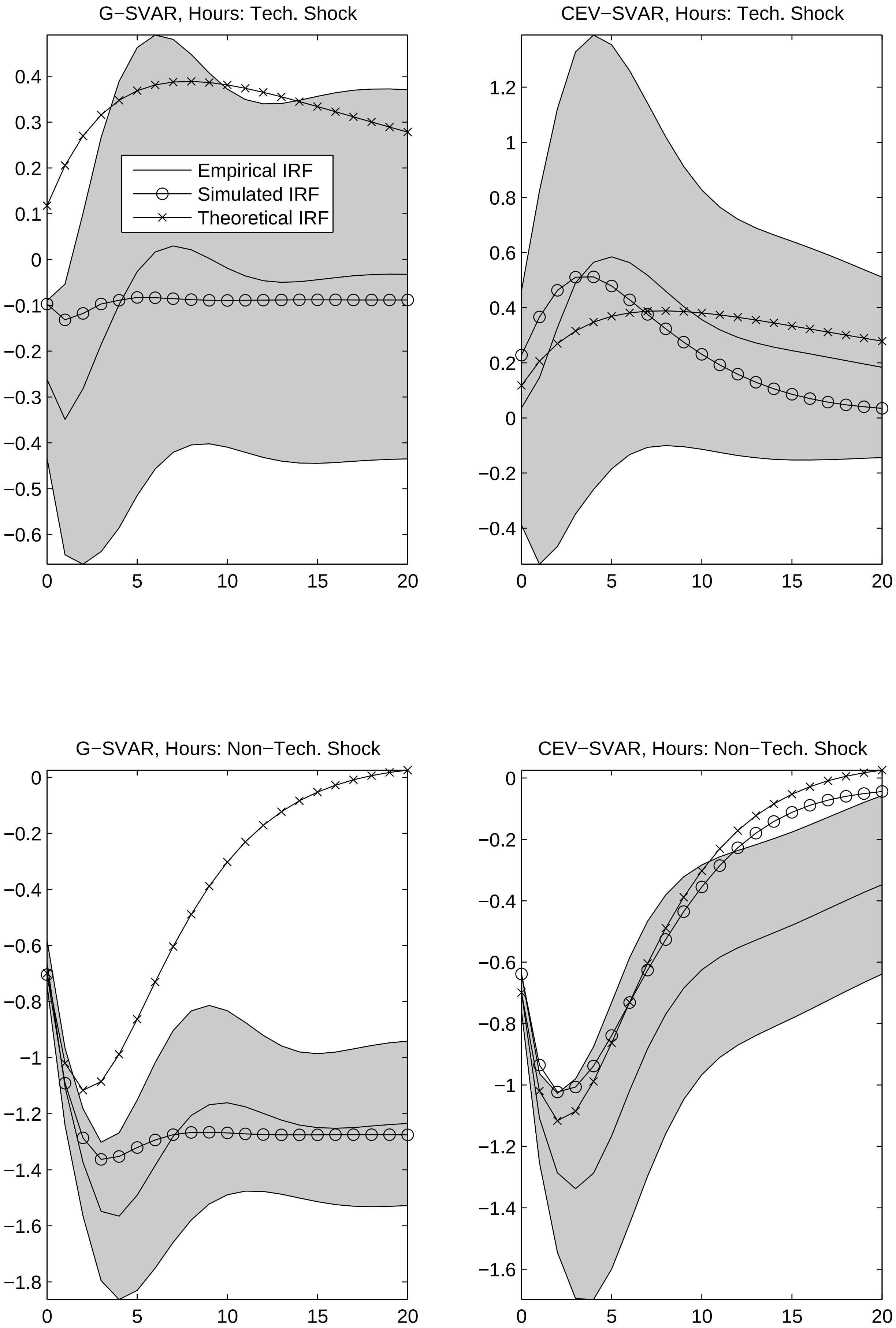


Figure 10: IRF of hours in DSVAR and LSVAR (estimations from LSVAR)

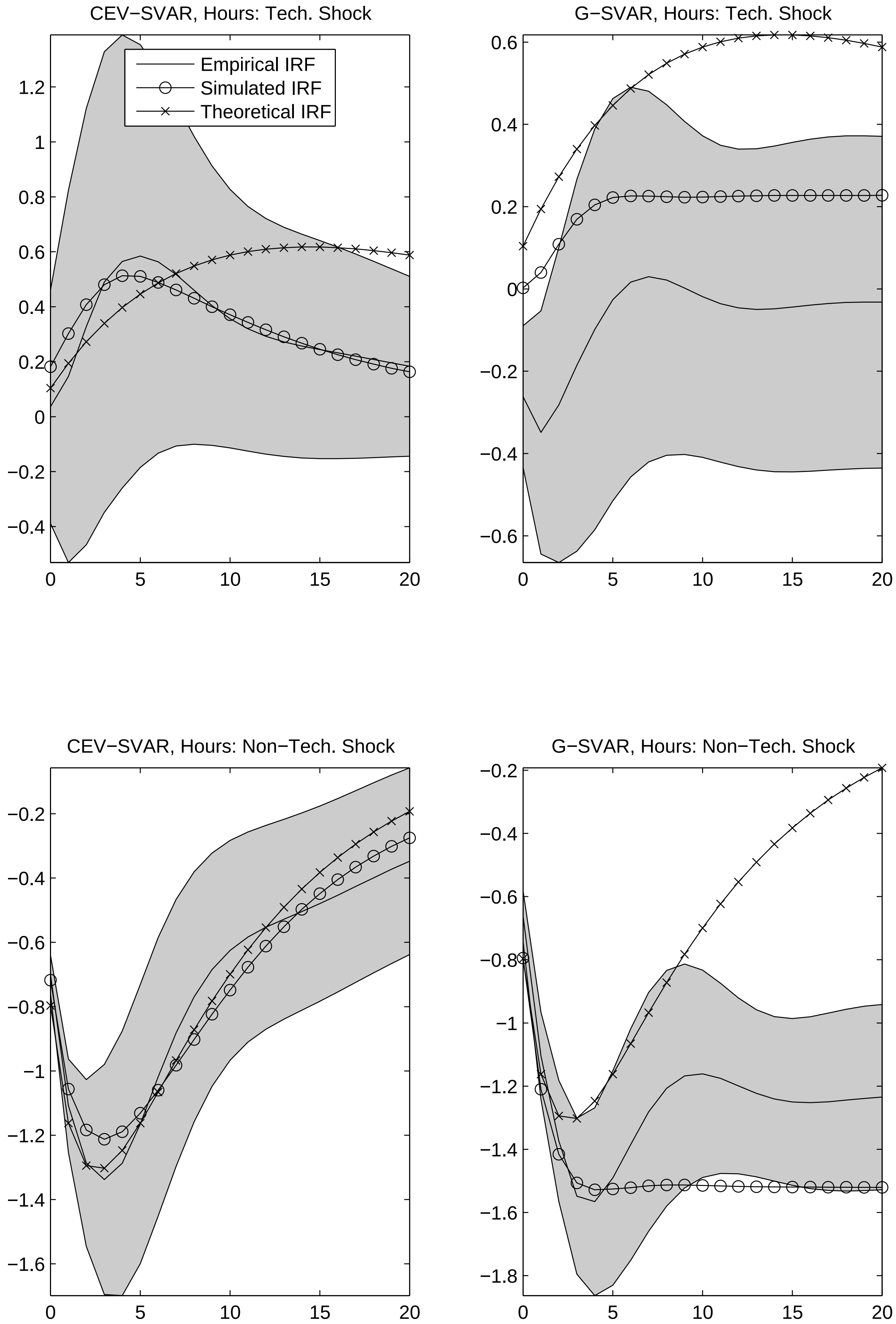


Figure 11: IRF of hours in DSVAR and LSVAR (estimations from DSVAR and LSVAR)