Endogenous Growth through Firm Entry, Exit and Imitation*

Alain Gabler[†] Omar Licandro[‡]

PRELIMINARY - PLEASE DON'T QUOTE

December 13, 2005

Abstract

A simple dynamic general equilibrium model is set up in which firms face idiosyncratic productivity shocks. Firms whose productivity has fallen too low exit, and entrants try to imitate the practice of existing firms, so that the expected productivity of entering firms is a function of current average productivity. Because of the resulting selection and imitation process, aggregate productivity in the economy grows endogenously. When calibrated to U.S. data, the model suggests that around 50 percent of productivity growth may be due to such a selection effect.

1 Introduction

The competitive struggle among heterogeneous firms is among the defining features of a market economy. Not only does this struggle drive the price of goods down to their marginal cost of production; it also ensures that those goods are produced efficiently. Firms which are unable to do that must eventually exit the market, and are replaced by new, more efficient firms. One way to interpret this mechanism is that competition allows for

*The authors acknowledge the financial support of the EC, contract HPRN-CT-2002-00237. Omar Licandro also acknowledges support from the Spanish Ministry of Education under project SEJ 2004-0459/ECON.

[†]Contact: European University Institute, Villa San Paolo, Via della Piazzuola 43, 50133 Florence, Italy; e-mail: alain.gabler@iue.it; tel: +39-348-8881-024; fax: +39-055-4685-902.

[‡]European University Institute and FEDEA.

the selection of good ideas. Productivity growth is driven by the trial of successive ideas and the weeding out of bad ones. Successful ideas will be copied by entering firms, causing productivity to grow endogenously through a continuous process of selection and imitation.

The idea that there can be economic growth through selection dates back at least to Nelson and Winter's (1982) seminal work on evolutionary economics. The strand of literature that has followed it considers the process of growth in analogy to the process of natural selection, in which only the fittest survive, and where 'efficient' behaviour is transmitted to future generations in the form of genes. However, this literature generally focuses on how behavioural rules evolve in a world of bounded rationality. Nevertheless, as this paper shows, there is no inherent contradiction between a mechanism of growth through selection and rational expectations.

One of the few to explicitly model the outcome of selection in terms of growth is Conlisk (1989). He sets up a simple model in which the productivity of new plants is a random draw whose mean depends on current average productivity; labour is then moved from the least productive old plants towards entering plants, causing the former to shut down. As a result, the economy grows at an endogenous rate, which crucially depends on the variance of the random draw of new plants. One of the drawbacks of the model is that only the entry process is stochastic, which is strongly rejected by the data. Furthermore, as is common in the evolutionary economics literature, firms operate in a setting of bounded rationality; in practice, this generally means that the number of entering and exiting firms is set exogenously, which precludes any meaningful statements about the quantitative implications of such models.

This paper proposes to fill that gap by trying to quantitatively link the selection process going on at the firm level to the rate of growth of the aggregate economy. This is done by setting up a dynamic general equilibrium, rational expectations model with idiosyncratic productivity shocks to firms. One way to interpret those shocks is to imagine that each firm represents an idea, or variation of an idea, and that firms try to improve the execution of this idea by progressively making small changes to the production process. The outcome of those changes might be uncertain, although their expected impact on productivity will probably be positive. Also, other existing firms might find it difficult to emulate at least some of these changes, leading to heterogeneity in productivity levels across firms. Entering firms will then try to implement as a whole the production processes of those firms which they think perform best, and after that will focus on making small changes to these processes; some of these changes will be inspired by what other firms in the economy do.

This means that there are two channels through which more efficient production processes spread across the economy. The first is through spillovers between existing firms, and concerns ideas which are relatively easily transferred from one firm to another; the extreme case in which all firms can implement such ideas costlessly is sometimes referred to as 'neutral‘ technical progress. The second is through spillovers from existing towards entering firms, and concerns ideas whose implementation require for example very different organisational structures, and which entering firms might find much easier to implement; this is sometimes called disruptive (or 'non-neutral') technological progress, and should be seen in analogy to the concept of embodied technical change, which stresses that certain technologies can only be implemented by setting up new plants.

As in Arrow's (1962) learning-by-doing model, in which the amount of innovation depends on the economy-wide output, technological progress is a costless externality. However, in this model, the spread of 'best practices' mostly happens through technology spillovers from existing to new firms; this is modeled by assuming that entering firms start with a productivity level which depends on the current average level in the economy, and that the evolution of productivity at a given firm then follows an autoregressive process.

The concept of growth through selection has much in common with the idea of Schumpeterian creative destruction. In Aghion and Howitt's (1992) interpretation of creative destruction, growth is generated by a random sequence of quality-improving, sector-specific innovations; better products or technologies render previous ones obsolete, and this occurs through the replacement of the incumbent sectoral monopolist by a new firm. An analogous mechanism is at work in models of growth through selection, except that it is not the firm of a given sector, but the marginal firm (i.e., the least profitable of all firms) that is rendered obsolete.

Although the idea of selection has its origin in the evolutionary economics literature, this paper, at least from a modelling standpoint, has more in common with models of industrial evolution, which notably includes papers by Jovanovic (1982) and Hopenhayn (1992). While both model idiosyncratic shocks hitting firms each period, the former considers a setup of imperfect information: firms do not directly observe their own productivity level, which leads inefficient plants to delay exit until they have sufficient information. The latter sets up a model with endogenous firm size in order to replicate cross-sectional properties - across size and age cohorts - in the data. However, since the technology of entering firms improves at an exogenous rate, neither of the two models is able to estimate the effect of selection on growth.

A number of recent papers dealing with firm entry and exit, among them

Comin and Mulani (2005) and Luttmer (2005), model firm-level heterogeneity by assuming monopolistic competition. From a quantitative point of view, this approach has the disadvantage of greatly increasing the number of required parameters. In order to keep the complexity of the model to a minimum, we choose to limit ourselves to the case of perfect competition, which greatly facilitates the task of taking the model to the data. Nevertheless, this paper does not, as do Boldrin and Levine (2000), address how innovation can occur under perfect competition (that is, in a world without patents), simply because in our model firms do not have a choice whether to engage in innovation or not.

This paper is also closely related to Campbell (1998), who looks at the business cycle implications of entry and exit. His model is similar to ours except for the fact that he abstracts from imitation, assuming instead that the productivity of entering firms grows at an exogenous rate.

The purpose of this paper is then to set up a simple model of selection and imitation, and to examine its quantitative implications, especially as to how much of economic growth can be attributed to a selection effect, and how much to neutral technological progress.

The remainder of the paper is organised as follows: section 2 describes the model; section 3 deals with its quantitative implications; and section 4 concludes.

2 The Model

In this section we show that a growth model incorporating entry, exit and imitation can be written as a neoclassical growth model with capital-embodied technological change in which the depreciation rate and the relative price of (productivity-adjusted) capital are endogenous.

2.1 Setup

Consider an economy populated by a measure L_t of identical, infinitely lived agents who maximise their lifetime utility from consumption C_t. Time is discrete, and the representative agent solves

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left[\beta^t L_t U\left(C_t/L_t\right)\right] \tag{1}$$

subject to the resource constraints

$$\begin{aligned} Y_t &= C_t + I_t, &(2)\\ \tilde{K}_{t+1} &= \left(1-\tilde{\delta}_t\right)\tilde{K}_t + q_t I_t, &(3)\end{aligned}$$

where Y_t is output, I_t is investment, $\tilde{K}_t$ is the effective (productivity-adjusted) aggregate stock of capital, $\tilde{\delta}_t$ is the depreciation rate of this stock, and q_t is the price of effective capital in terms of the consumption good. Notice that this setup is equivalent to the neoclassical growth model with capital-embodied technological change, except that here, both the depreciation rate $\tilde{\delta}_t$ and the relative price of (effective) capital q_t are endogenous and time-specific: as we will see, while $\tilde{\delta}_t$ depends on the number of firms that exit each period as well as on the technical depreciation rate of capital δ, q_t is endogenous because the productivity of entering firms depends on the productivity of existing firms through imitation. However, we assume that economic agents consider q_t as exogenous when making their decisions.

Output is produced by a continuum of firms which differ in terms of their productivity level z. Defining $K_{z,t}$ and $L_{z,t}$ as the total stock of (non productivity-adjusted) capital and employment of all firms with a given productivity level z, the output of the representative firm at that productivity level is given by

$$Y_{z,t} = A_t\left(zK_{z,t}\right)^{\alpha} L_{z,t}^{1-\alpha}, \tag{4}$$

where A_t is the current state of (firm-neutral) technology. Aggregate output can then be written as:

$$Y_t = A_t \int \left[\left(zK_{z,t}\right)^{\alpha} L_{z,t}^{1-\alpha}\right] dz. \tag{5}$$

While labour $L_{z,t}$ can be costlessly adjusted at any point in time, capital $K_{z,t}$ is assumed to be fixed at the firm level; less formally, the idea is that a firm consists of one plant.

We follow Solow (1957) in defining the "effective" (i.e., productivity-adjusted) capital stock of a representative firm at a given productivity level as $\tilde{K}_{z,t} = zK_{z,t}$. The effective aggregate capital stock is then given by:

$$\tilde{K}_t = \int zK_{z,t}dz.$$

Given that the representative firm at each productivity level optimally chooses its labour force, one can show that the aggregate production technology can be written as:

$$Y_t = A_t\tilde{K}_t^{\alpha}L_t^{1-\alpha}. \tag{6}$$

2.2 Firm Entry and Exit

Stochastic Process While the firm-neutral technology A_t grows at an exogenous rate γ_n, the productivity $z_{i,t}$ of a given firm i follows a random walk:

$$\begin{aligned} z_{i,t+1} &= z_{i,t} + \varepsilon_{i,t}, \\ \varepsilon_{i,t} &\sim N\left(0, \sigma_t^2\right). \end{aligned} \tag{7}$$

The physical depreciation of capital is modelled by assuming that a proportion δ of firms are exogenously destroyed each period.

Exit Firms have the option of costlessly and definitively ceasing production at any point in time, with the restriction that new firms may not exit within the same time period as they have entered. In this case the firm is scrapped, and its capital can be transformed into new capital at a rate $\phi < 1$. The total amount of scrap recovered each period is given by $\phi d_t K_t$, where d_t is the proportion of firms which choose to exit in period t.

Entry Each entering firm draws its productivity $z_{i,t}^e$ from a normal distribution: $z_{i,t}^e \sim N\left(\mu_t^e, \sigma_{e,t}^2\right)$. The number of entering firms is equal to investment I_t plus the amount of scrap recovered from exiting firms, $\phi d_t K_t$. Given that firms operate under constant returns to scale and that agents are risk-averse, entering firms will be atomistic at equilibrium. The number of entering firms with a given productivity z can then be written as:

$$E_{z,t} = \left[I_t + \phi d_t K_t\right] \varphi_t^e\left(z\right), \tag{8}$$

where $\varphi_t^e\left(z\right)$ is the probability density function of z_t^e.

The average productivity of entering firms, μ_t^e, is chosen such that the (output-wheighed) average productivity of entering firms is a constant fraction ψ_e of (output-wheighed) average productivity in the economy:

$$\left[\int z^\alpha Y_{z,t} dz\right] / Y_t = \psi_e. \tag{9}$$

Equation (9) is a simple way of formalising imitation. It states that entering firms' expected productivity depends linearly on the average productivity in the economy.

2.3 Competitive Equilibrium

Households The representative household allocates income from labour L_t and assets B_t between consumption C_t and savings; utility is logarithmic:

$$\max_{\{C_t\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \left[\beta^t \eta^t \ln (C_t)\right]$$
$$\text{s.t. } B_{t+1} = (1 + r_t) B_t + W_t \cdot L_t - C_t \tag{10}$$

where η is the population growth rate, and W_t and r_t are the return rate of assets and the wage rate, respectively. The first order condition for consumption yields the usual Euler equation:

$$C_{t+1}/C_t = \beta (1 + \eta_t) (1 + r_t) . \tag{11}$$

Firms Given that firms differ only with respect of their productivity, there exists one representative firm for each level of productivity z. Let $V_t (z, K_{z,t})$ be the present discounted value of such a firm. If the firm decides to stay, its value is given by its current cash flow plus its expected discounted value in the next period, taking into account the fact that the firm has a probability δ of disappearing; if it decides to exit, its value is simply equal to the scrap value of its capital, which is given by ϕ. The optimal policy then involves choosing a 'reservation' productivity level z_t^* below which the representative firm will find it profitable to exit, so that

$$V_t (z, K_{z,t}) = \begin{cases} \hat{V}_t (z, K_{z,t}) & \text{if } z \geq z_t^*, \\ \phi K_{z,t} & z \leq z_t^*, \end{cases} \tag{12}$$

where

$$\hat{V}_t (z, K_{z,t}) = \Pi_t (z, K_{z,t}) + \frac{1 - \delta}{1 + r_t} \int V_{t+1} (z', K_{z,t}) \varphi (z' - z) dz' \tag{13}$$

is the value of staying in the market, and $\varphi (\cdot)$ is the probability density function of the idiosyncratic shock $\varepsilon_{i,t}$ faced by firms. A firm's current cash flow is given by:

$$\Pi_t (z, K_{z,t}) = A_t (z K_{z,t})^{\alpha} {L_{z,t}}^{1-\alpha} - W_t L_{z,t}. \tag{14}$$

Optimal employment is determined by the first order condition for $L_{z,t}$:

$$L_{z,t} = z K_{z,t} \left[\frac{A_t (1 - \alpha)}{W}\right]^{1/\alpha} . \tag{15}$$

Using the fact that $\int L_{z,t}dz = L_t$ and integrating both sides of equation (15) with respect to z yields the following expression for current cash flow:

$$\Pi_t\left(z, K_{z,t}\right) = \alpha \frac{\tilde{K}_{z,t}}{\tilde{K}_t} Y_t. \tag{16}$$

In other words, a representative firm's share of total revenues depends linearly on its share of the total effective capital stock.

The recursive nature of the problem implies that at each time t the firm chooses a sequence of exit thresholds $\left\{z^*_{t+s}\right\}_{s=0}^{\infty}$ which satisfies

$$\hat{V}_{t+s}\left(z^*_{t+s}, 1\right) = \phi. \tag{17}$$

At equilibrium, the expected profit from entering is driven to zero:

$$\int \hat{V}_t\left[z, \varphi^E_t\left(z\right)\right] dz = 1. \tag{18}$$

The aggregate value of assets held by households is given by

$$B_t = \int V_t\left(z, K_{z,t}\right) dz.$$

To map one period's productivity distribution $K_{z,t}$ into next period's, one has to take into account (i) the idiosyncratic shocks hitting the firms, (ii) the disappearance of those firms which choose to shut down, (iii) the depreciation of existing capital, and (iv) the entrance of new firms. Since there is a continuum of firms in the economy, the evolution of the distribution of firms across productivity levels is deterministic even though each particular firm experiences random shocks. The mapping of the productivity distribution is then given by:

$$K_{z,t+1} = \begin{cases} (1-\delta)\int \varphi\left(z - z'\right) K_{z',t}dz' + E_{z,t} & \text{if } z \geq z^*_t, \\ E_{z,t}, & z < z^*_t. \end{cases} \tag{19}$$

Integrating on both sides of equation (19), one can write the law of motion of aggregate effective capital as:

$$\tilde{K}_{t+1} = \underbrace{(1-\delta)\left[1 - \int d_{z,t}\frac{\tilde{K}_{z,t}}{\tilde{K}_t}dz + q_t\phi d_t K_t\right]}_{1-\tilde{\delta}} \tilde{K}_t + q_t I_t, \tag{20}$$

where $d_{z,t} = \int_{-\infty}^{z^*} \varphi\left(z' - z\right) dz'$ is the percentage of firms at productivity level z which exit in the next period, and $q_t = \int z\varphi^E_t\left(z\right) dz$ is the relative price of effective capital in terms of the consumption good. q_t appears in the term for $1-\tilde{\delta}$ because part of the capital of exiting firms is re-used to create new firms. Equation (20) is the micro-founded equivalent of the law of motion of aggregate effective capital given by equation (3).

Similarly, the law of motion of non productivity-adjusted capital is given by:

$$K_{t+1} = (1-\delta)\left[1-d_t(1-\phi)\right]K_t + I_t. \tag{21}$$

2.4 Balanced Growth

The aim of this section is to transform the model in a way which makes all variables constant at the steady-state. To find the appropriate transformation, notice that the resource constraint (2), households' budget constraint (10) and the transition equation for aggregate effective capital (3) imply that Y, C, I and B all grow at the same rate, say g, along a balanced growth path. Furthermore, we define a balanced growth path as a situation in which the distribution of the productivity-specific variables Y_z, K_z and L_z relative to the average productivity $\bar{z}_t$ remains constant, so that appropriately scaled-down versions of the variables Y_ζ, K_ζ and L_ζ will remain constant, where $\zeta = z - \bar{z}$.

Notice that in the presence of entry and exit, expected productivity growth at individual firms will be less than the aggregate growth rate of the economy. The reason for this is that part of the growth process happens "outside" firms, through the replacement of inefficient firms by new, more efficient entrants. One way to interpret this is that new firms embody more productive capital and more efficient organisational structures, both of which might be more difficult to implement in existing firms or establishments. We then define expected productivity growth at existing firms, γ_n, as (organisation-) neutral technological progress, and growth due to (organisation-) embodied technological progress, γ_e, as growth through selection. Assuming that the state of firm-neutral technology A grows at an exogenous rate of $\gamma_n^{\frac{1}{1-\alpha}}$, total productivity growth is given by $\gamma = \gamma_n \cdot \gamma_e$. Equation (6) then implies that effective capital $\tilde{K}_t$ grows at rate $\gamma_e^{\frac{1-\alpha}{\alpha}} g$.

One can then define transformations which will make all the variables in the model stationary. Specifically, first determine $x_t = X_t/g^t$ for $X_t = Y_t$, C_t, I_t and B_t; second, set $l_t = L_t/\left(L_0\eta^t\right)$; third, set $x_{\zeta,t} = X_{\zeta+\bar{z},t}/g^t$ for $X_t = Y_{z,t}$, $I_{z,t}$ and $K_{z,t}$; fourth, set $\tilde{k}_t = \tilde{K}_t/\left(\gamma_e\eta\right)^t$; finally, set $v_t(\zeta) = V_t\left[\zeta+\bar{z},1\right]/g^t$, and $l_{\zeta,t} = l_{\zeta+\bar{z},t}/\eta^t$. The equilibrium equations of the model can then be rewritten in terms of these transformed variables.

2.5 Equilibrium Conditions

Dropping time subscripts, a stationary equilibrium then consists of a stationary tuple

$$\left\{v\left(\zeta\right), k_\zeta, \tilde{k}, \zeta^*, i, \gamma_e\right\}$$

which solves the following 6 equilibrium conditions:

1. Value of a firm:

$$v\left(\zeta\right) = \begin{cases} \alpha\zeta\tilde{k}^{\alpha-1} + \frac{\beta(1-\delta)}{\gamma}\int v\left(\zeta'\right)\varphi\left(\zeta' - \zeta + \gamma_e^{\frac{1-\alpha}{\alpha}}\right)d\zeta' & \text{if } \zeta \geq \zeta^*, \\ \varphi & \zeta < \zeta^*, \end{cases} \tag{22}$$

2. Transition function of the productivity distribution:

$$k_\zeta = \begin{cases} \begin{array}{c} \frac{1-\delta}{\gamma}\int\varphi\left(\zeta - \zeta' - \gamma_e^{\frac{1-\alpha}{\alpha}}\right)k_{\zeta'}d\zeta' \\ + \left[i + \phi\int_{-\infty}^{\zeta^*}k_{\zeta'}d\zeta'\right]\varphi^E\left(\zeta\right) \end{array} & \text{if } \zeta \geq \zeta^*, \\ \left[i + \phi\int_{-\infty}^{\zeta^*}k_{\zeta'}d\zeta'\right]\varphi^E\left(\zeta\right), & \zeta < \zeta^*, \end{cases} \tag{23}$$

3. Free entry condition:

$$\int\varphi^e\left(\zeta\right)v\left(\zeta\right)d\zeta = 1, \tag{24}$$

4. Exit condition:

$$\alpha\zeta\tilde{k}^{\alpha-1} + \frac{\beta\left(1-\delta\right)}{\gamma}\int v\left(\zeta'\right)\varphi\left(\zeta' - \zeta + \gamma_e^{\frac{1-\alpha}{\alpha}}\right)d\zeta' = \phi, \tag{25}$$

5. Effective capital stock:

$$\tilde{k} = \int\zeta k_\zeta d\zeta, \tag{26}$$

6. Output-wheighed average productivity condition (normalisation):

$$\int\zeta k_\zeta\tilde{k}^{\alpha-1}d\zeta = 1. \tag{27}$$

Equation (22) is obtained by using the first order condition on consumption in equation (11) to substitute for $\frac{1}{1+r}$.

The model distinguishes itself from the evolutionary economics literature through equations (24) and (25), which state that entry and exit follow rational expectations. It distinguishes itself from the industrial evolution literature through equation (23), which states that entering firms' productivity is not exogenous but instead depends on the productivity of existing firms.

The model is solved numerically, following a method which is described in the appendix.

3 Calibration

The aim of this section is to study the behaviour of a parametrised version of the model economy. The aim of the exercise is twofold: first, we would like to check whether our model is consistent with some dimensions of U.S. post-war data. Second, we would like to assess the quantitative impact of entry, exit and imitation on productivity growth in the U.S. over the same time period; this latter part of the exercise is comparable in some ways to work done by Greenwood, Hercowitz, and Krusell (1997), who estimate the contribution of investment-specific technological change on productivity growth.

In order to impose rigour on the quantitative analysis, the procedure advanced by Kydland and Prescott (1982) is followed. The parameters in the model are set either based on a priori information, or so that along the balanced growth path a number of economic variables assume their average values for U.S. data. The length of one period in the model is set to one quarter.

The variables Y, I and K, are matched up with the corresponding nominal variables in NIPA data divided through by a common price deflator. As Greenwood, Hercowitz, and Krusell (1997), we use the consumption deflator for non-durable goods and non-housing services, in order to measure those variables in consumption units as they are in the resource constraints (2) and (3), and to avoid the issue of accounting for quality improvements in consumer durables. Also, the government and housing sectors are netted out of GDP, the former because the selection mechanism which is at work in the model is specific to a competitive economy, and the latter because only capital in the business sector is used to produce output in the model.

The parameters that need to be calibrated are the capital share of income, α; households' discount factor, β; the exogenous rate of destruction of capital, δ; the relative productivity of entrants, ψ_e; the scrap value of establishments, ϕ; the population and productivity growth rates , η and γ; and the variance of shocks to existing and entering establishments, σ^2 and σ_e^2.

The equations characterising balanced growth for the model can be expressed as follows:

$$\beta = \frac{\gamma}{g + (\alpha Y - I)/B}, \tag{28}$$

$$\frac{I}{K} = g - (1-\delta)\left[1 - d(1-\phi)\right], \tag{29}$$

where equation (28) comes from the Euler equation for consumption (11) and households' budget constraint (10). Average quarterly values of NIPA data for the time period 1964-2001 yield the following four equations:

$$\alpha = .332, \tag{30}$$

$$\gamma = 1.0032, \tag{31}$$

$$\eta = 1.0046, \tag{32}$$

$$\frac{I}{K} = .0254. \tag{33}$$

Evidence for the time period 1972-1987 from the Longitudinal Research Database, which tracks between 55'000 and 300'000 establishments in the US manufacturing sector[1], yields four more equations:

$$\psi_e = .99, \tag{34}$$

$$\psi_x = .8727, \tag{35}$$

$$\tilde{d}_e = .0164, \tag{36}$$

$$\tilde{d} = .0083. \tag{37}$$

Foster, Haltiwanger, and Krizan (2001) estimate that the (output-wheighed) average productivity of entering establishments relative to existing ones, ψ_e, is around 99 percent, while the corresponding number for exiting establishments, ψ_x, is .8727. Campbell (1998) reports that the quarterly, employment-wheighed exit rate of establishments, $\tilde{d}$, and that of establishments which are less than one year old, $\tilde{d}_e$, are .83 percent and 1.64 percent respectively. Equations (28) through (37) are then used in order to numerically calibrate the model.

4 Results

The calibrated parameters are $\beta = .991$, $\delta = .0176$, $\phi = .795$, $\sigma^2 = .0368$ and $\sigma_e^2 = .1953$; the rate of establishment-embodied technological progress

[1] For a review of productivity studies on the LRD see Bartelsman and Dhrymes (1998) and Caves (1998).

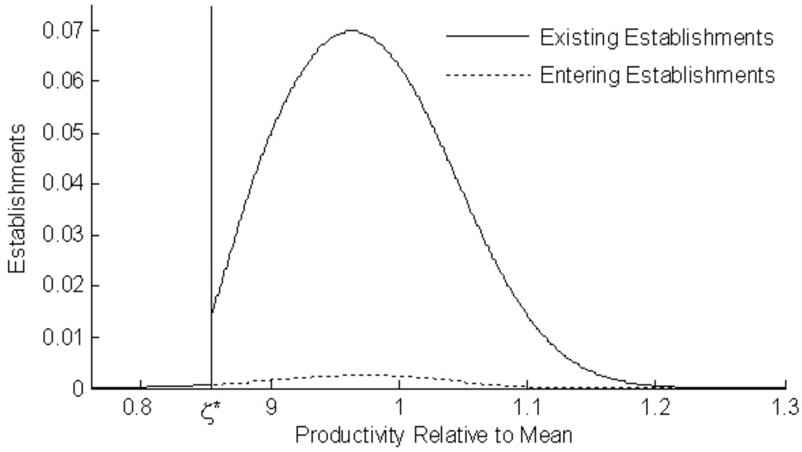

Figure 1: Distribution of establishments across productivity levels

implied by the model is $\gamma_e = 1.0016$. Figure 1 shows the steady-state distribution of capital along productivity levels which corresponds to those parameters.

Given that in the present model, firms have the option to exit and recover part of their initial investment, the value of the fixed factor capital should be somewhat higher than the expected present discounted value of its income share; accordingly, we find an assets to effective capital ratio of $B/\tilde{K} = 1.0334$. This implies that aggregate models which do not incorporate entry and exit are bound to overstate the average real return to assets by that order of magnitude. The latter fact also explains why our calibrated value of the discount factor β is somewhat higher than is usual in the literature. The calibrated value for the scrap value of capital, ϕ, is close to that of Campbell (1998), even though his calibration strategy is quite different.

Finally, the fact that the estimated variance of the productivity shock to establishments which are less than one year old, σ_e^2, is several orders of magnitude larger than the variance of the shock to older establishments, σ^2, is consistent with Bartelsman and Dhrymes' (1998) finding that young plants face substantially more productivity uncertainty than their older counterparts.

One of the key results of the paper concerns the proportion of aggregate productivity growth which is due to establishment-embodied technological progress. The parametrised version of the model economy implies an average annual rate of increase in the productivity of new establishments, q_{t+1}/q_t, of 1.25 percent, which corresponds to 50 percent of total productivity growth.

This number can be compared to microeconomic studies of establishment-level productivity decomposition. Foster, Haltiwanger, and Krizan (2001) estimate that in the U.S. manufacturing sector, between 48 and 65 percent of productivity growth takes place within establishments, with the remainder coming from either the reallocation of inputs from unproductive to more productive establishments, or from entry and exit.

Our results are also closely related to Greenwood, Hercowitz, and Krusell (1997), who find that sixty percent of post-war U.S. productivity growth is due to technical change which is embodied in capital, and to Atkeson and Kehoe (2005), who estimate that over one-third of the payments received by plant owners are due to plant-specific knowledge (i.e., to organisational capital).

5 Conclusion

A model was set up in which firms face idiosyncratic productivity shocks; entry and exit are endogenous, and entering firms start with a productivity level which depends on the average productivity in the economy. This is shown to result in aggregate growth even in the absence of a exogenous positive trend in productivity growth at individual firms, through a process of selection and imitation. The parametrised version of the model economy suggests that around 50 percent of U.S. productivity growth is due to such a selection effect.

The idea of growth through selection does also have some policy implications, although they are not formally investigated here. Chief among them is the fact that since the growth effect of selection turns out to be quite substantial, protecting firms by setting up entry barriers or by not allowing them to fail can have a sizeable effect not only on real income levels through higher prices, but also on long-run growth rates. As an illustration, Levinsohn and Petrin (1999) cite an article by the *Economist*[2] suggesting that Japan's recent poor economic performance has been due at least in part to a Japanese aversion to "outright failure" of firms.

[2]See the June 20, 1998 issue containing the article "Japan's Economic Plight."

Appendix

A Algorithm

The numerical algorithm that is used to solve for the stationary equilibrium of the model is the following:

1. Guess the rate of growth through selection in the economy, γ_e.

2. Guess the aggregate stock of effective capital, $\tilde{k}$. Iterate on the firm's value function $v(z)$ given by equation (22) until convergence is reached. In practice, $v(z)$ is discretised into a matrix of dimension $[2000 \times 1]$. Given that $v(z)$ is decreasing in $\tilde{k}$, use the free entry condition in (24) to update $\tilde{k}$ through a bisection method, and iterate until convergence is reached.

3. Iterate over the productivity transition function given by equation (23), using the definition for the effective capital stock in (26) to determine the number of entering firms before each iteration, until convergence is reached.

4. Given that the right-hand side of the normalisation condition in equation (27) turns out to be a smooth and decreasing function of γ_e, use this equation to update γ_e through a bisection method.

References

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60(2), 323–351.

Arrow, K. J. (1962): "The Economic Implications of Learning-by-Doing.," *Review of Economic Studies*, 29(1), 155–173.

Atkeson, A., and P. J. Kehoe (2005): "Modeling and Measuring Organization Capital," *Journal of Political Economy*, 113(5), 1026–1053.

Baily, M., C. Hulten, and D. Campbell (1992): "Productivity Dynamics in Manufacturing Plants," *Brookings Papers on Economic Activity: Microeconomics*, pp. 187–267.

Bartelsman, E. J., and P. Dhrymes (1998): "Productivity Dynamics: U.S. Manufacturing Plants, 1972-1986," *Journal of Productivity Analysis*, 9(1), 5–34.

Boldrin, M., and D. K. Levine (2000): "Perfectly Competitive Innovation," *UCLA Department of Economics Working Paper*.

Campbell, J. R. (1998): "Entry, Exit, Embodied Technology, and Business Cycles," *Review of Economic Dynamics*, 1(2), 371–408.

Caves, R. E. (1998): "Industrial Organization and New Findings on the Turnover and Mobility of Firms," *Journal of Economic Literature*, 36(4), 1947–1982.

Comin, D., and S. Mulani (2005): "A Theory of Growth and Volatility at the Aggregate and Firm Level," *NBER Working Papers*, 11503.

Conlisk, J. (1989): "An Aggregate Model of Technical Change," *Quarterly Journal of Economics*, 104(4), 787–821.

Davis, S. J., and J. Haltiwanger (1992): "Gross Job Creation, Gross Job Destruction, and Employment Reallocation," *Quarterly Journal of Economics*, 107(3), 819–863.

Foster, L., J. Haltiwanger, and C. Krizan (2001): "Aggregate Productivity Growth: Lessons from Microeconomic Evidence," in *New Developments in Productivity Analysis*, ed. by E. R. Dean, M. J. Harper, and C. R. Hulten, pp. 303–372. University of Chicago Press, Chicago and London.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87(3), 342–362.

Hopenhayn, H. (1992): "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127–1150.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50(3), 649–670.

Kydland, F., and E. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

Levinsohn, J., and A. Petrin (1999): "When Industries Become More Productive, Do Firms?," *NBER Working Paper*, 6893.

Luttmer, E. G. J. (2005): "Growth and the Size Distribution of Firms," *University of Minnesota Working Paper*.

Nelson, R. R., and S. G. Winter (1982): *An Evolutionary Theory of Economic Change*. Belknap Press of Harvard University Press, Cambridge (MA), U.S.A.

Solow, R. M. (1957): "Investment and Technical Progress," *Mathematical Methods in the Social Sciences*, pp. 89–104.