Consumption, wealth, and expected asset returns in the United States: implications of housing wealth and housing consumption

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July 4, 2006

Abstract

Using US quarterly post-war data, this paper documents the existence of a common trend among non-housing non durable consumption, financial wealth, real estate wealth, and labour income (a proxy for human wealth), reflecting the long run stability of the consumption-wealth ratio. The estimated relationship is more consistent with an underlying representative agent's budget constraint than previously found. The paper also shows that deviations from the estimated common trend predict real total stock market returns over horizons ranging from 1 to 24 quarters, also an implication of the underlying theory: high non-housing consumption relative to income and non-human wealth anticipates higher returns. On the other hand, the expenditure ratio between non-housing and total non-durable consumption has no predictive power, which contrasts with recent empirical findings.

1 Introduction[1]

Using cointegration techniques, in this paper I investigate the long run relationship between US household consumption and wealth, using labor income as a proxy for human wealth. The reasons why one should be interested in understanding such relationship are different, and related to two broad fields of the economics literature that deal with the interplay between financial markets and the macroeconomy. One is the long-lived literature on the effects of wealth on consumption. A simple life-cycle model implies that consumption should be approximately a constant fraction of wealth, implying a cointegration between the logs of consumption and wealth. More generally, by estimating a cointegration model one is able to show separately the short- *versus* the long-run properties of the system: to be consistent with the life-cycle model, consumption should respond only to permanent shocks to wealth, but not to transitory ones.

On the other hand, understanding the empirical linkages between the macroeconomy and asset prices has long been one goal of the financial economics literature. The benchmark asset pricing model - the CCAPM - establishes a relationship between expected consumption growth and the risk premium demanded on stocks. However, such model fails to match quantitatively the historical risk premium, which is too high to be consistent with the low volatility of consumption growth, for plausible parametrization of risk aversion. Related to this, another stylized fact in financial markets is the observed cyclical variation and predictability of expected returns. While in principle predictability could be due to a failure of the efficient markets hypothesis, an alternative view is that it may simply reflect the rational response of agents to time varying investment opportunities, possibly driven by cyclical variation in risk aversion or in the joint distribution of consumption and asset returns. That is, although it does not hold unconditionally, the CCAPM might hold if conditioned on some key macroeconomic variables.[2] Lettau and Ludvingson (2001) - henceforth LL - have shown that US stock market returns are predicted by the deviations from a common trend among aggregate consumption, income and non-human wealth. This result is implied by a log-linear approximation of a representative agent's intertemporal budget constraint. In this paper I follow the same approach, but try to fix some problems that arise in their empirical implementation and results. Considering housing and non-housing as separate components of consumption and wealth proves helpful in that respect.

[1]The views expressed in this paper are those of the author, and do not necessarily reflect those of the Bank of Italy. Any errors and omissions are mine.

[2]See Cochrane (2005) for a survey of the relationship between financial markets and the real economy.

Why could it be so? It is important to remark at the outset the very special nature of housing, which is not only one component of household wealth, but also a consumption good affecting individuals' current utility. One implication of the former is that a change in the value of property may affect consumption through a wealth effect. At the same time, real estate provides collateral to liquidity constrained households. In recent years, the United States, have witnessed a sharp increase in home equity withdrawal, whereby homeowners have borrowed against capital gains on their property to finance current consumption or repay other obligations.[3] House price fluctuations are believed to matter for business cycle fluctuations, and to amplify the macroeconomic consequences of monetary policy shocks.[4] As a consumption good, housing also enters the utility function of individuals.[5] Piazzesi et al. (2005) note that if shelter is not a perfect substitute for non-housing consumption, then the CCAPM implies two sources of risk: one is the standard consumption growth risk, the other is a composition risk, arising from the covariation between the share of non-housing in total consumption and the return on assets. An implication of their model is that the expenditure share of non-housing consumption should forecast asset returns. In particular, if the elasticity of intertemporal substitution is lower than the intratemporal, and the latter is not too low, then agents prefer assets whose payoff in terms of non-housing goods provide them with insurance against states in which there is a relative shortfall of housing. It follows that a low share of non-housing consumption should anticipate higher stock market returns, a fact that is confirmed by their empirical analysis.

Why in particular should one wish to consider housing as a separate component in the investigation of the long-run relationship between consumption and wealth? One reason is that consumption may respond differently to shocks to different types of wealth. Although estimates differ by samples and techniques, there is some evidence that the marginal propensity to consume out of real estate wealth is larger than to financial wealth, which could simply result from the different nature of shocks, being financial wealth, especially equity, far more volatile. Another reason is that the expenditure share of non-housing, although stationary, is quite persistent, and could drive results against finding evidence of the supposed common trend between consumption and wealth.

Using quarterly data for US households between 1952 and 2004, I estimate a 4-variable cointegration among the logs of real per capita net housing

[3] Between 2002 and 2004, a period of rapid house appreciation, US mortgage refinancing averaged 1800 billion dollars annually, compared to 440 billions dollars through the earlier decade.

[4] Recent contributions on this subject include Aoki et al. (2004), and Iacoviello (2005).

[5] Note that - although classified as non durable consumption - the service flow of housing is by definition derived from the most durable consumption good, i.e. land and buildings.

wealth, financial wealth, non-housing non-durable consumption, and labour income (which proxies human wealth).[6] I preliminarily show that the log of the expenditure share of non-housing consumption is stationary, which supports the empirical strategy of specifying the above cointegration model in terms of non-housing only. With respect to te work of LL, splitting wealth and consumption into housing and non-housing components is not the only innovation. While LL deflate nominal variables by a index for total consumption, I use the deflator for non-housing non-durable consumption, which is consistent with the underlying intertemporal budget constraint, an issue raised by Rudd and Wehlan (2002). Results from cointegration analysis show that the system is dominated by transitory wealth shocks, and that consumption responds only to permanent shocks, which is consistent with LL's results. In the second part of the analysis I show that the co-integration error works as a predictor of stock market returns over horizons ranging from 1 to 24 quarters, while - contrary to the results obtained by Piazzesi et al. - the expenditure share is insignificant at all horizons.

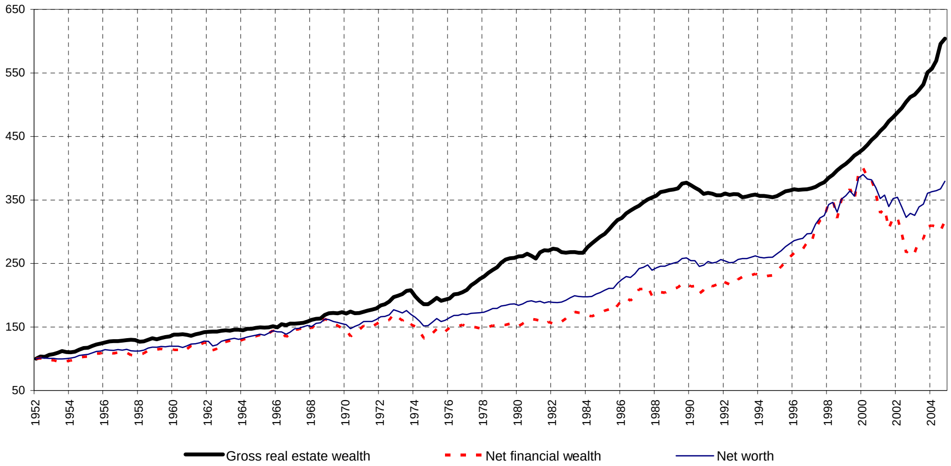
The next section introduces some stylized facts showing the relevance of housing in US household wealth and consumption. Section 3 is a brief overview of the related literature. Section 4 presents the results of the empirical analysis. Section 5 concludes.

2 Housing wealth and consumption: some stylized facts

Between 1952 and 2004, US households net worth increased 280 per cent in real terms, or 2.5 per cent annually on average (figure 1). Over the same period, real net financial wealth, defined as total financial assets minus total liabilities increased by 220 per cent; in contrast, gross real housing wealth increased six-fold. These trends are reflected in the relative shares of the different components of wealth. After remaining stable around 25 per cent through the end of the sixties, the share of real estate wealth in total net worth increased rapidly, to stabilize at above 30 per cent in the eighties (figure 2, solid line). Thereafter, the dramatic cycle of equity prices drove the share first back to historical lows, then up to levels not recorded before (36 per cent in 2004).

[6] Net real estate wealth is defined as gross minus the outstanding value of home mortgages.

Figure 1. Real estate, financial and total household wealth in per capita real terms (1)
(quarterly end of period data; indexes: 1952Q1 = 100)



Source: Bureau of Economic Analysis and Federal reserve. (1) Nominal variables are deflated by the deflator for total personal consumption. - (2) Financial assets minus total financial liabilities.

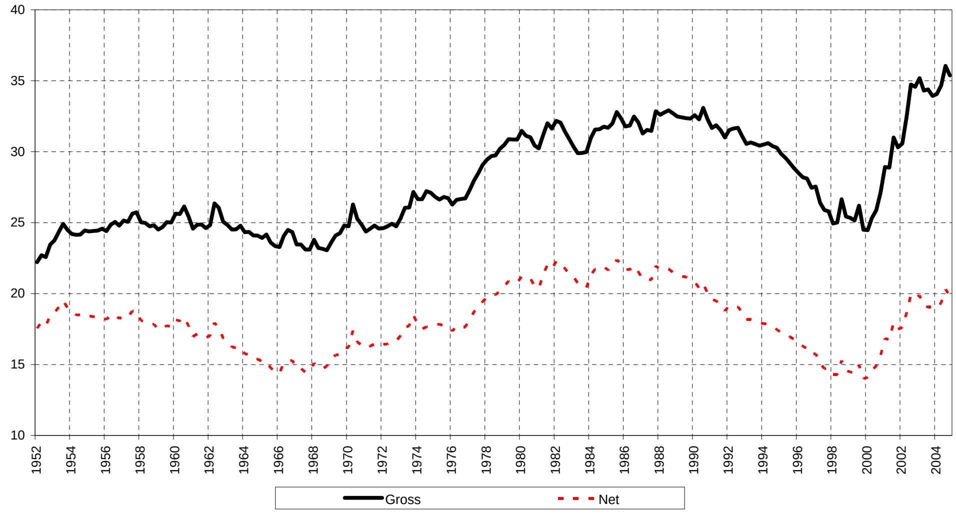
When the outstanding value of home mortgages is subtracted from real estate wealth, the growth rates of **net** real estate and financial wealth appear more balanced over the sample, being equal, on average, to 2.8 per cent and to 2.6, respectively. The share of net real housing wealth (figure 2, dotted line), although highly correlated with its gross counterpart, exhibits less dramatic swings. This is especially evident during the most recent house price boom, due to the concurrent sharp increase in household debt. It is worth noting that at the end of 2004 the share of net real estate wealth was not too far from the historical average.

As is evident from figure 1, net worth is highly correlated with net financial wealth, much less with either gross or net real estate wealth. The reason is that the financial component of wealth is not only quantitatively the most relevant of net worth, but by far also the most volatile. The correlation coefficient between changes in net worth and changes in net real estate is 0.25, while that between the former and changes in financial wealth is almost one and dominated by the equity component. In turn, wealth exhibits a high correlation with asset prices: the correlation coefficient between financial wealth and stock market prices is 0.82, that between net real estate wealth and house prices is 0.57.[7]

Consumption is a second dimension along which housing is an important component. Housing consumption, which essentially consists of monetary

[7] The series for house prices is the OFHEO repeat sales index, which is available starting only in 1975.

Figure 2. Share of gross and net housing wealth out of total household net worth



Source: Federal reserve.

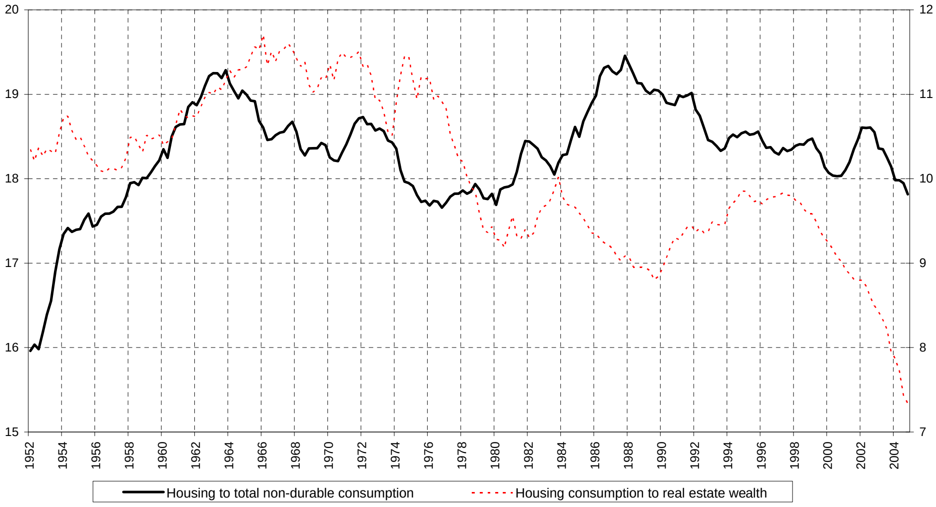
and imputed rents, is on average about 15 per cent of total consumption (figure 3, solid line), and 18 per cent of total non-durable consumption.[8] Variations in the share of housing consumption are quite persistent, but the series seem to exhibit stationarity, which is a maintained hypothesis in Piazzesi et al. (2005). Note that the share of housing consumption is currently not far from its historical average. It is interesting also to compute the ratio of paid and imputed rents to the value of residential property (figure 3, dotted line). While in models it is often assumed that the real service flow of housing is proportional to the quantity of property, their ratio - evaluated at nominal values - exhibits quite persistent swings. In particular, the sharp fall occurred since 1997 - from 9.8 per cent to 7,3 per cent - is largely explained by a decline in the relative price of renting, which followed the recent acceleration in house prices. Taken at face value, such decline could be an indication that real estate prices are above the levels consistent with fundamentals.

3 Related work

In a recent paper, Lettau and Ludvingson (2001) show that US households non-durable consumption (C), non-human wealth (A) and labor income (Y,

[8] In US national accounting, housing consumption is classified as non-durable.

Figure 3. Ratio of housing consumption to total non-durable consumption and to gross real estate wealth



Source: Bureau of Economic Analysis and Federal reserve.

a proxy for human wealth) are log-cointegrated, and that the deviations from their common trend predict subsequent stock returns.[9] Their results are motivated theoretically by a log-linear approximation of a representative agent intertemporal budget constraint like the following:

$$W_{t+1} = (1 + R_{w,t+1})\,(W_t - C_t) \tag{1}$$

Assuming the ratio between consumption and wealth is stationary around $C/W = 1 - \rho_w$, (1) can be log-linearized around around C/W, along the lines of Campbell and Mankiw (1989), resulting in the following expression:

$$cay_t \equiv c_t - \psi a_t - (1-\psi)y_t \simeq E_t \sum_{i=1}^{\infty} \rho_w^i \left(r_{w,t+i} - \Delta c_{t+i}\right) \tag{2}$$

where c and r are the logs of C and of $(1 + R)$, a and y denote the logs of financial wealth and of labour income (which proxies human wealth).[10]

The assumption of stationary consumption growth and asset returns implies the cointegration among c_t, a_t, and y_t. Equation (2) also says that current deviations from their common trend (cay_t) should predict subsequent returns, provided consumption growth is not too volatile. These implications are confirmed by LL's empirical analysis, who show that cay_t indeed anticipates quarterly returns over horizons that range from 1 to 24 quarters.

[9] Unless otherwise specified, I subsequently refer to wealth as to the non-human component of total households net worth (financial plus real estate).

[10] Unimportant constantd are omitted.

LL's empirical implementation of (2) and the related results suffer from a series of problems. First, as noted by Rudd and Whelan (2002), it is inconsistent with the underlying budget constraint, because income and wealth - the latter including the stock of durables - are deflated by a price index for total consumption, while the numeraire in (1) is non-durable consumption excluding shoes and clothings (s&c). Since the ratio between the prices of durable and non-durable consumption trends downwards, the use of different deflators may yield spurious results. In addition, even if the same deflator had been used, the measure of consumption employed by LL would remain inconsistent with the measure of wealth: in fact, expenditure on s&c, excluded from consumption, is not added to wealth. Rudd and Whelan (2002) show that the hypothesis of cointegration is rejected once total consumption expenditure is adopted as the numeraire and the empirical model is specified accordingly.

The exclusion of durable consumption is essentially motivated by the absence of a reliable measure of the service flow from the durables stock, but justified theoretically by the assumption that the log of total consumption is just a multiple of the log of non-durable consumption. LL therefore estimate the following long-run relationship:

$$c_t^{nd} = const + \beta_a a_t + \beta_y y_t \tag{3}$$

assuming $c_t \sim \lambda c_t^{nd}$, for $\lambda > 1$. While Rudd and Whelan note that the log-ratio of real total to real non-durable consumption trends upwards, what really matters for (3) is that their expenditure ratio is stationary, a point related to an argument brought forward by Palumbo, Rudd and Wehlan (2002).[11] In fact, adopting non-durable consumption as the numeraire, one can express real total consumption as the product between real non-durable consumption (the numeraire) and the expenditure ratio between total and non-durable consumption:

$$C_t \frac{P_t}{P_t^{nd}} = C_t^{nd} \frac{C_t P_t}{C_t^{nd} P_t^{nd}} \equiv C_t^{nd} Z_t \tag{4}$$

and

$$c_t = c_t^{nd} + z_t \tag{5}$$

where C_t and C_t^{nd} are quantity indexes (c_t and c_t^{nd} their logs), P_t and P_t^{nd} the corresponding deflators, $Z_t \equiv \frac{C_t P_t}{C_t^{nd} P_t^{nd}}$ and $z_t = \log(Z_t)$. If, as appears in

[11] They observe that in post-war US data the ratio of nominal durable consumption to total consumption is fairly stable, and derive a log-linear approximation of the budget constraint where non-durable consumption, the numeraire, serves as a proxy for total consumption.

the data, Z_t and z_t are stationary, then estimating (3) using the log of real non-durable consumption in place of the log of real consumption should not represent a problem, since it is equivalent to having removed a stationary additive term.[12]

On the other hand, (5) also implies that LL's proportionality coefficient λ is equal to one and that $\beta_a + \beta_y = 1$, which is also consistent with (2). This brings about the third problem with LL's results. They estimate the long-run coefficients in (3) as $\widehat{\beta}_a = 0.31$ and $\widehat{\beta}_y = 0.59$, which implies $\lambda = 1.1$ and that the coefficients do not sum to one. Related to this, it is also worth noting that if long-run coefficients on labor and wealth do not sum to one, then the results from the cointegration analysis may be sensible to the choice of deflator. In fact, while the assumption underlying the log-linear approximation of (1), namely that the consumption-wealth ratio is stationary, does not depend on what numeraire is chosen to express the budget constraint, such 'irrelevance' is inherited by econometric work only if the log-deflator calcels out in (3), which happens only if $\widehat{\beta}_a + \widehat{\beta}_y = 1$.

The above reasoning about the implications of a stationary expenditure ratio between total and non-durable consumption can be extended to the expenditure ratio between total non-durable and non-durable non-housing consumption. Provided the latter is indeed stationary, as is assumed in Piazzesi et al. (2006), one could remove its log from the empirical specification of the long-run relationship between consumption and wealth. One reason for doing so is that such ratio is quite persistent, although stationary, and removing it facilitates detection of the assumed common trend in the data and turns out to yield estimates of the long-run parameters that are more consistent with the underlying theory.

It is worth noting that, under the maintained hypotheses, the long-run elasticities of consumption to wealth and income should identify the shares of capital and labor in GDP. Borrowing from LL, assume that total production (Q_t) is a Cobb-Douglas function of capital (K_t) and labour (L_t):

$$Q_t = K_t^{\alpha} L_t^{1-\alpha} \tag{6}$$

From (6), current payments to capital and labor are given respectively by αQ_t and $(1-\alpha) Q_t$. Since wealth is just the discounted sum of future expected payments, non-human and human wealth can be expressed respectively as

[12] By the same line of reasoning, I will subsequently remove from the empirical specification of the cointegartion relationship the expenditure ratio between total and non-housing consumption, which is also stationary. this eventually leads to results that are more consistent with the underlying theoretical framework.

$$A_t = \alpha E_t \sum_{j=1}^{\infty} \frac{Q_t}{\prod_{i=1}^{j} (R_{t+i})} \tag{7}$$

and

$$H_t = (1 - \alpha) E_t \sum_{j=1}^{\infty} \frac{Q_t}{\prod_{i=1}^{j} (R_{t+i})} \tag{8}$$

Assuming, as in (7) and (8), the same discounting for capital and labor payments, it follows that the shares of the latter in GDP should correspond approximately to the respective shares in wealth. It is therefore natural expect that the analysis of cointegration yields estimates of the elasticities of consumption to wealth and labor close to the respective shares, which the real business cycle literature indicates at approximately 1/3 and 2/3.

4 Cointegration analysis

In this section I provide evidence of a common trend among consumption, labor and wealth with a focus on fixing the problems of previous empirical work, on the basis of the above mentioned consistency issues: (i) variables must be deflated by the same price index (I express them in terms of non-durable non-housing consumption), (ii) the elasticities of consumption to labor and wealth must sum to one, and (iii) they must be close to the relative shares of labor and capital in GDP.

I proceed in steps. I test cointegration first on a 3-variable model as in LL and Rudd and Whelan, including total consumption (c), total wealth (a), and labor income (y) correctly deflated as required in (i): evidence that the three variables cointegrate is mixed, and although the elasticities sum to one, they are far from being close to the theoretical shares. I then split total wealth into housing (a_h) and non-housing (a_{nh}), and estimate the implied 4-variable model: although - assuming a common trend actually exists - there is some improvement in terms of the relative elasticities, evidence of cointegration is still mixed. The third model is 4-variable like the previous one, but includes only the log of non-housing consumption (c_{nh}). In this case, the evidence in favour of the existence of a common trend is less ambiguous, and the estimated parameters are closest to their theoretical counterparts. I argue that the relative success of the third model comes from the removal of the stationary, but quite persistent expenditure ratio between non-housing and housing consumption.

The latter specification can be explicitly derived from the intertemporal constraint (1), as in LL, by additionally assuming that the ratios between total non-durable consumption and its non-housing component, and between total and non-durable consumption are stationary. In the appendix I derive the following approximation of (1), where all variables are expressed in terms of non-housing consumption (the numeraire):

$$c_t^{nh} - \psi a_t - (1-\psi) y_t + \left(c_t^{nd} - c_t^{nh}\right) + \left(c_t - c_t^{nd}\right) \approx \sum_{j=1}^{\infty} \rho^j \left[r_{t+j} - \Delta c_{t+j}^{nh} - \Delta \left(c_{t+j}^{nd} - c_{t+j}^{nh}\right) - \Delta \left(c_{t+j} - c_{t}^{n}\right.\right. \tag{9}$$

where linearization constants have been onmitted and, as in (2), total wealth has been approximated by a weighted average of non-human wealth and labour income.

The assumption of stationary consumption growth and returns, plus the stationarity of the expenditure shares imply the existence of a common trend among non-housing consumption, wealth and income. As was the case for (2), (9) implies that deviations from the underlying common trend among c_t^{nh}, a_t, and l_t must forecast asset returns: a relatively high level of non-housing consumption should be associated with higher subsequent returns. This may be interpreted in terms of a life-cycle model of consumption choice, whereby trransitory deviations of wealth from their long-run equilibrium should not affect consumption. In addition, from (9), it is interesting to investigate the relationship between the expenditure share of non-housing and returns: a relatively high expenditure share could eventually be associated with lower subsequent returns.

In all the above cases I use estimate the cointegration rank by the Johansen approach, and evaluate the evidenge in favour or against cointegration by referring to both the likelihood-ratio test (LR) and the trace test (TR). Model selection analysis indicates that 6 lags are necessary in order to obtain approcimately (multivariate) normal and non-autocorrelated residuals, although simple information criteria would suggest 2 lags are sufficient. I allow for 6 lags in all specifications. In all cases, 3 centered dummies have been added for the periods 1962Q2, 1975Q2, and 1993Q1.

Wealth aggregates are from the Federal reserve Flow of Funds, while consumption and income data are from the Bureau of Economic Analysis NIPA tables. The deflator for non-housing non-durable consumption is constructed according to the BEA chain methodology. Total wealth is defined as households net worth; housing wealth is defined as residential real estate minus home mortgages; the stock of durable goods is included in the stock of financial wealth, but alternative empirical specifications are also analyzed that exclude durables from both consumption and wealth (in this case, financial wealth is denoted by a'_{nh}). Total consumption excludes both durables and expenditure on shoes and clothings (s&c); the stock of s&c is neither added to wealth, since an accurate measure is not available. Labour income is defined as in LL, and is given by the sum of wage and salaries disbursment, plus supplements to wages and salaries (employer contributions for employee pension and insurance funds and for government social insurance) plus personal current transfer receipts), less contributions for government social insurance, less taxes.[13] All variables are expressed in terms of non-durable non-housing consumption, although, as discussed in the previous section, theory implies that the choice of deflator should not matter. Preliminary alanysis indicates that the logs of all variables are I(1), which warrants an empirical investigation of their long-run relationship.

It is worth noting that the long-run elasticities of consumption to wealth estimated in the cointegration relationship can be mapped to the corresponding marginal propensity to consume (MPC) after multiplication by the average consumption-wealth ratio.[14]

4.1 Model 1 (c,a,y)

The first model estimated in this section corresponds to the one estimated by LL, being a 3-variable specification including total consumption, total wealth and labour income. As mentioned in the preceeding discussion, nominal values are obviously deflated by the same index, which is one difference with respect to LL's treatment and could explain some differences in the results obtained. Another point of difference is the time-span, which here covers the period up to 2003Q3, while in LL stops in 1998Q3.

Table 1 reports the L-Max and Trace tests from the Johansen likelihood procedure, together with the 90th percentiles of the corresponding theoretical distributions. The hypothesis under investigation is that the system

[13]Income taxes on the selected aggregate in labour income are estimated, like in LL, assuming overall personal income taxes are proportionally distributed across income subcomponents.

[14]The marginal propensities to consume out of financial and out of net housing wealth are calculated, respectively, as $MPC_{a_{nh}} = \beta_{a'_{nh}} \left(\overline{\frac{C_{nh}+C_h}{A'_{nh}}}\right)$, and $MPC_{a_h} = \beta_{a_h} \left(\overline{\frac{C_{nh}+C_h}{A_h}}\right)$, where the upper bar denotes a sample average.

Table 1

H: r	Eigenv	L-Max	Trace	L-Max-90	Trace-90
0	0.0975	21.04	**25.81**	18.60	26.70
1	0.0229	**4.74**	4.77	12.07	13.31
2	0.0002	0.04	0.04	2.69	2.71

Table 2

H: r	Eigenv	L-Max	Trace	L-Max-90	Trace-90
0	0.1126	**24.48**	44.77	24.73	43.84
1	0.0723	15.40	**20.29**	18.60	26.70
2	0.0233	4.84	4.89	12.07	13.31
3	0.0003	0.06	0.06	2.69	2.71

has rank = 1. While the hypothesis is accepted by the L-Max test, the hypothesis that r = 0 cannot be rejected by the Trace test.

The evidence in favour of the existence of one common trend among the three variables is therefore mixed. In addition, if one imposes r = 3, the estimated cointegration vector is

$$c - 0.509a - 0.552y \tag{10}$$

The coefficients in (10) sum to one, if proportionally rescaled, but their relative values are far from the relative shares of capital and labor (0.35 and 0.65). The data actually reject even the restriction that $\beta_a = 0.4$ and $\beta_y = 0.6$.

4.2 Model 2 (c,a_h,a_{nh},y)

The second model considers housing and non-housing wealth as separate variables. The model has therefore 4-variables, and the hypothesis under investigation is still that the system has rank = 1. The evidence is again mixed (Table 2). In this case, the hypothesis that r = 1 is rejected by the L-Max test, although (marginally) accepted by the Trace test.

If one imposes r = 3, the estimated cointegration vector is

$$c - 0.33a_h - 0.11a_h - 0.61y \tag{11}$$

As before, the coefficients in (11) sum to one. However, once the coefficients of housing and non-housing wealth are summed, the elasticities of

Table 3

H: r	Eigenv	L-Max	Trace	L-Max-90	Trace-90
0	0.1186	25.87	46.75	24.73	43.84
1	0.0671	**14.23**	**20.88**	18.60	26.70
2	0.0316	6.58	6.65	12.07	13.31
3	0.0003	0.06	0.06	2.69	2.71

consumption to wealth and labor are much closer to the theoretical shares. The hypothesis that $\beta_{a_h} = 0.1$, $\beta_{a_{nh}} = 0.3$, and $\beta_y = 0.6$ is accepted, although at the limit (the p-value is of 0.05).

4.3 Model 3 (c_{nh},a_h,a_{nh},y)

The last model considers enters, as model 2, housing and non-housing wealth as separate variables, but considers only the non-housing component of consumption. As before, the model has 4-variables and the hypothesis is that of unit rank. In this latter case both the L-Max and the Trace test lead to accept the hypothesis that r = 1 (Table 3).

Not only is the evidence in favour of the existence on one common trend more neat, but the estimated long-run elasticities exhibit values that are more in line with the theoretical priors. The estimated cointegration vector is

$$c_{nh} - 0.34a_h - 0.11a_h - 0.59y \tag{12}$$

In (12), the elasticities to wealth and labor sum to one and are not statistically distinguishible from the theoretical shares of capital and labor: the hypothesis that $\beta_{a_h} = 0.1$, $\beta_{a_{nh}} = 0.3$, and $\beta_y = 0.6$ is accepted by a large margin (the p-value is 0.25), and the hypothesis that $\beta_{a_h} = 0.08$, $\beta_{a_{nh}} = 0.27$, and $\beta_y = 0.65$ is also accepted (with a p-value of 0.05).

From the cointegration analysis in this section one can conclude that Model 3 has a comparative success in supporting the evidence that there is one common trend among consumption, wealth and labor, and in providing estimates of the coefficients of the assumed long-run relationship that are closest to values that are expected from theory. In order to achieve this, wealth and consumption have been split into their housing and non-hosuing components. How does a separate accounting of housing produce more consistent results is a matter of interpretation. It is fairly intuitive to understand that by considering only the non-housing comoponent of consumption one

Table 4. Loading factors

alpha	value	t-stat
Y	**-0,060**	**-2,48**
C-nh	-0,025	-1,78
A-h	0,058	1,15
A-nh	**0,244**	**3,17**

Notes. Significant coefficients are in bold.

has removed a quite persistent, yet stationary component (the expenditure share of housing) from the stationary linear combination that characterizes the supposed long-run relationship. Less obvious is the mechanism by which splitting wealth leads to improved estimates of the long-run elasticities. A possible interpretation is related to the fact that entering housing and financial wealth separately is equivalent to relaxing restrictions on the long- and short-run responses of consumption to the two types of wealth. Such restrictions, implicitly imposed by Model 1, may appear far too restrictive in light of the abundant existing empirical evidence that the marginal propensity to consume relative to housing is higher than to financial wealth. It must be stressed that the log-linear approximations of the intertemporal budget constraint (2) and (9) have been derived from the assumption that consumption is a constant fraction of **total** wealth, implying identical proportionality of consumption to different types of wealth. It is worth noting that the marginal propensities to consume out of financial and real wealth implied by the estimated long-run elasticities in Model 3 turn out to be very close to each other, which is consistent with the mentioned assumption: the implied MPC out of financial wealth is 4.8%, and the MPC out of real wealth is 5,8%. Such values are also much closer than many other studies have previously found. It turns out that the data are consistent with identical long-run elasticities: for example, an overidentification test does not reject the following values of the cointegration vector: $\beta_{a_h} = 0.07$, $\beta_{a_{nh}} = 0.28$, and $\beta_y = 0.65$ (the p-value is 0.2), which imply theoretically consistent shares of capital and labor and identical long-run MPCs of 5%, a value that is in line with many previous empirical estimates of the MPC to financial wealth.

On the other hand, estimated short run dynamics of the system reveal quite different implications for the relationship between consumption and either real or financial wealth. Table 4 reports the loading factors associated to Model 3.

Only two short-run coefficients turn out significant. The one with the highest t-statistics is the coefficient associated to financial wealth, with positive sign. This is consistent with the deviations from the long-run equilibrium being driven by transitory shocks to financial wealth, while the statistical irrelevance of the coefficient of consumption means that the latter does not respond to such transitory shocks, which is consistent with the implications of a life-cycle model. Note instead that the short-run coefficient associated to housing wealth is not statistically significant, implying that any transitory component in real wealth shocks is far less important. This could be one reason why in many empirical studies of the wealth effect it is found that the marginal propensity to consume is higher relative to real wealth. The performance of Model 3 may be thus explained by the fact that such model relaxes the quite restrictive assumption that both the long- and the short-run joint dynamics of consumption and different types of wealth are identical. While long-run ones are actually not too different, the implications for the short run differ widely. Another point worth a mention is the short-run coefficient associated to labor income, which is significant but has the 'wrong' sign.

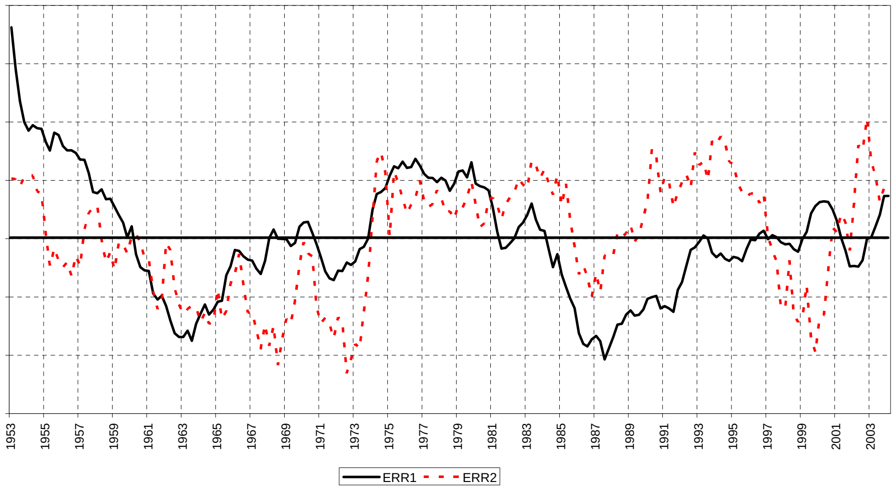
It is interesting to look at the estimated trend deviation from Model 3. Figure 4 plots the normalized cointegration error (ERR2), showing deviations that are significant and quite persistent. Substantial disequilibrium is evident throughout the latest stock market boom, which on the basis of the estimated average short-run responses would be attributed to a transitory increase in wealth, correctly predicting the subsequent fall in share prices. The latter appears to have substantially undershot through the beginning of 2003, to subsequently return in line with closer to equilibrium in 2004. Figure 4 also reports the log expenditure ratio between non-housing and housing consumption (ERR1). ERR1 appears much more persistent than ERR2, which motivates removing it from the long-run empirical model linking consumption to wealth and labor. The next section explores the predictive power of ERR1 and ERR2 for total returns on the US stock market.

4.4 Quarterly forecasting regressions

Return predictability is investigated over horizons that range from one to 24 quarters. The dependent variables are the logs of real total returns on the Standard and Poor's index (the k-period ahead return is denoted by $ret_{t,t+k}$, for $k = 1, ..24$).[15] The main explanatory variables of interest are $ERR1$ and $ERR2$. Some stock market ratios that are typically used in the literature on predictability are also included as controls, including the dividend yield

[15] For any k, $ret_{t,t+k} = ret_{t,t+1} + ret_{t,t+2} + ... + ret_{t+k-1,t+k}$. Data on the S&P are from Robert Shiller's web site.

Figure 4. Deviation from common trend among non-housing, wealth and labor income (ERR2) and log-expenditure ratio between non-housing and housing consumption (ERR1)



(dy), the pay-out ratio (pr) and the price-earnings ratio (pe).[16] A constant term is always added. The most general specification is:

$$ret_{t,t+k} = const + \gamma_1 ERR1_{1,t} + \gamma_2 ERR2_{2,t} + \gamma_3 dy_t + \gamma_4 pr_t + \gamma_5 pe_t + \varepsilon_t$$

Table 5 reports the predictive regression results. Since pr and pe never turned significant, specifications including them are omitted.[17] In all cases, inference is performed by applying the Newey-West correction to account for overlapping observations.

From the first and second specifications it appears that the log-expenditure ratio $(ERR1)$ is never significant, considered either together with $ERR2$ or alone. $ERR2$ is always significant and with the sign implied by the theoretical framework. The explanatory power of regressions including $ERR2$

[16] Lamont (1998) shows that, scaled in different ways, high dividends forecast high future returns, as expected, while high earnings do forecast lower future returns. According to his interpretation, dividends capture the non stationary component of stock prices, possibly because company managers tend to smooth out dividends over medium-long horizons; in contrast, earnings, being positively correlated with the business cycle, show a negative correlation with risk premia, which are high in recessions and low in expansions.

[17] The contrasting results with respect to Lamont are partly explained by different sample coverage Lamont's regressions are estimated over the period 1947-1994, while in this paper the sample runs from 1965 to 2004, and therefore includes the latest stock market cycle. If the above short- and long-run regressions are re-estimated to cover only the period up to 1994, then the coefficient of the payout ratio becomes generally positive and significant.

Table 5

	1	2	3	4	8	12	16	24
const	**0.018**	**0.036**	**0.053**	**0.07**	**0.138**	**0.205**	**0.271**	**0.401**
	(3.26)	**(3.47)**	**(3.70)**	**(3.97)**	**(4.67)**	**(5.51)**	**(6.08)**	**(6.66)**
err1	-0.005	-0.009	-0.014	-0.019	-0.033	-0.049	-0.061	-0.062
	(-0.87)	(-0.87)	(-0.93)	(-0.99)	(-0.96)	(-1.24)	(-1.38)	(-1.11)
err2	**0.021**	**0.043**	**0.063**	**0.083**	**0.152**	**0.205**	**0.241**	**0.335**
	(4.37)	**(4.58)**	**(4.68)**	**(4.90)**	**(5.53)**	**(5.64)**	**(6.24)**	**(8.33)**
R2	0.09	0.15	0.20	0.24	0.37	0.48	0.53	0.62
const	**0.018**	**0.035**	**0.053**	**0.069**	**0.130**	**0.193**	**0.262**	**0.406**
	(3.06)	**(3.15)**	**(3.23)**	**(3.33)**	**(3.31)**	**(3.37)**	**(3.46)**	**(3.44)**
Err1	0.002	0.004	0.005	0.006	0.014	0.014	0.017	0.048
	(0.31)	(0.37)	(0.34)	(0.32)	(0.43)	(0.41)	(0.47)	(0.86)
R2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
const	**0.018**	**0.036**	**0.054**	**0.071**	**0.138**	**0.204**	**0.271**	**0.402**
	(3.26)	**(3.47)**	**(3.69)**	**(3.94)**	**(4.51)**	**(5.15)**	**(5.66)**	**(6.34)**
Err2	**0.019**	**0.040**	**0.059**	**0.077**	**0.141**	**0.188**	**0.220**	**0.311**
	(4.02)	**(4.16)**	**(4.22)**	**(4.38)**	**(4.85)**	**(4.94)**	**(5.14)**	**(7.06)**
R2	0.09	0.15	0.19	0.23	0.35	0.46	0.49	0.60
const	0.039	0.104	0.178	0.249	0.438	0.455	0.431	0.89
	(0.63)	(0.89)	(1.60)	(1.16)	(1.13)	(0.97)	(0.85)	(1.67)
Err2	**0.019**	**0.038**	**0.055**	**0.071**	**0.128**	**0.177**	**0.214**	**0.299**
	(3.61)	**(3.69)**	**(3.67)**	**(3.72)**	**(3.93)**	**(4.08)**	**(4.48)**	**(5.95)**
Dy	0.006	0.020	0.036	0.052	0.088	0.073	0.047	0.148
	(0.34)	(0.58)	(0.73)	(0.82)	(0.76)	(0.53)	(0.32)	(0.94)
R2	0.09	0.15	0.20	0.24	0.37	0.46	0.49	0.60
const	0.078	0.186	**0.305**	**0.433**	**0.917**	**1.246**	**1.422**	**2.228**
	(1.46)	(1.79)	**(2.02)**	**(2.25)**	**(2.48)**	**(2.59)**	**(2.69)**	**(2.77)**
Dy	0.018	0.044	0.074	0.106	**0.228**	**0.306**	**0.340**	**0.542**
	(1.13)	(1.44)	(1.65)	(1.86)	**(2.04)**	**(2.09)**	**(2.09)**	**(2.67)**
R2	0.01	0.02	0.04	0.06	0.13	0.15	0.13	0.13

Notes. The dependent variables are the logs of k-quarter ahead real total returns on the S&P (k in the first row). Const is a constant; err1 is the log-ratio between non-housing and housing consumption; err2 is the k-period lagged trend deviations estimated from the 4-variable model including non-housing consumption, housing wealth, non-housing wealth, and labor inc dy is the k-period lagged log dividend yield. For each specification, the latest column reports the adjusted R^2. T-statistics in parentheses. Significant coefficients are in bold.

Figure 5. Total return and lagged trend deviation: 12-quarter horizon.

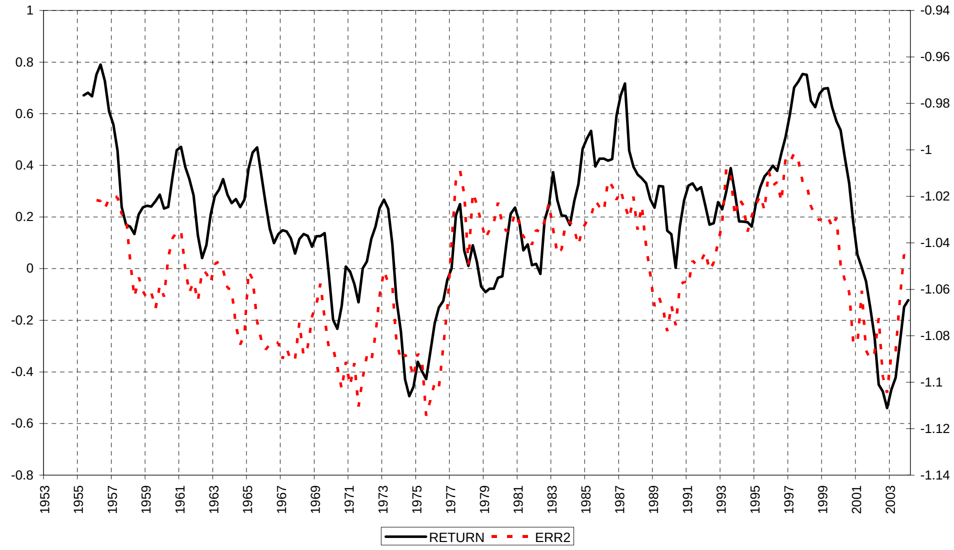
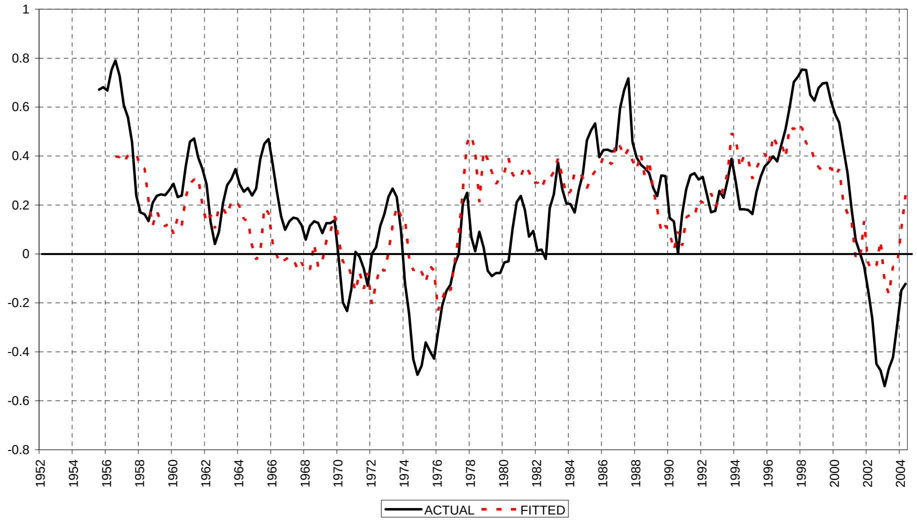


Figure 6. 12-quarter total return: actual *vs* fitted



is substantial and grows with the forecasting hrizon, a feature that is recurrent in the literature and can be rationalized in part by the persistence of the regressors[18]: the adj-R^2 goes from 9 per cent one-quarter ahead to 60 per cent 24-quarters ahead. The fourth specification includes $ERR2$ and dy: the latter variables is never significant, and does not add any explanatory power to the regression including $ERR2$ alone. On the other hand, when dy is the only explanatory variable, its coefficient is significant and has the expected sign for long horizons, from 2 years on, which is in line with previous empirical studies.[19] It can be concluded that the proxy for the consumption-wealth ratio encompasses the information contained in the dividend yield. Figure 5 shows the real total return cumulated over the 12 quarters ending in the reference period and the 12-quarter-lagged trend deviation $ERR2$. The ability of $ERR2$ tyo predict subsequent returns exhibits graphical evidence. Figure 6 shows the actual and fitted value of the 12-quarter total return, confirming the graphical impression that the fit of the model is actually quite high.

Table 6.

[18]The high persistence of $ERR1$ and $ERR2$, as well as that of the control variables is a matter of concern for inference in the subsequent analysis. The problem applies most seriously to $ERR1$, by far the most persistent among the explanatory set. The problem of persistent regressors in this literature is stressed by Campbell et al. (1997), who notice how the explanatory power of predictive regressions may improve as the forecasting horizon is increased simply because regressors exhibit slow mean reversion. Within the context of predictive regressions, Stambaugh (1999) discusses the potential small sample bias related to persistent explanatory variables, and a method to correct for it.

[19]See Campbell (1991), Campbell, Lo and MacKinlay (1997), and Lamont (1998).

	1	2	3	4	8	12	16	24
const	0.015	0.029	0.044	0.057	0.110	0.161	0.212	0.314
	(2.76)	(2.93)	(3.12)	(3.33)	(3.91)	(4.56)	(5.05)	(5.56)
err2	0.018	0.038	0.056	0.073	0.128	0.167	0.189	0.254
	(3.97)	(4.13)	(4.20)	(4.39)	(4.92)	(4.95)	(4.90)	(6.17)
R2	0.09	0.14	0.18	0.21	0.33	0.44	0.47	0.55
const	0.014	0.028	0.041	0.054	0.108	0.158	0.207	0.299
	(2.65)	(2.77)	(2.90)	(3.11)	(3.83)	(4.46)	(4.84)	(4.25)
LL	0.778	2.223	3.650	4.967	9.377	12.622	14.244	17.375
	(2.35)	(3.31)	(3.68)	(3.93)	(5.18)	(5.88)	(6.14)	(5.36)
R2	0.02	0.08	0.14	0.18	0.33	0.43	0.46	0.39

Although the approximated budget constraint implies $ERR2$ has predictive ability for the total return on stocks, for comparison with LL's regressions are re-estimated with the excess return over the three-month T-bill as the dependent variable. Results are broadly unchanged in terms of both the significance of the explanatory variables and of the goodness of fit of the estimated models. Table 6 reports the results for two sets of regressions. The firts, including our $ERR2$ explanatory variable, shows that it is significant over all the considered horizons, and that the adj-R^2 is of a similar magnitude as that for the total return. The second specification regresses the excess return on the trend deviations for LL's model, whose update is available on Martin Lettau's web site. Comparison of the two sets of results indicates that, although LL's explanatory variable is highly significant and provides a good fit too, its performance in terms of adj-R^2 is lower at horizons of one-to-four quarters and at the longest 6-year horizon. Although $ERR2$ does not perform impressively better, and with the caveat that part of the results obtained may be spurious due to the persistence of the regressors, I argue that one may place more confidence in our predictor, since it has been derived from an estimation of a long-run relationship among consumption, income and wealth that is consistent with the underlying theory. Specifications including $ERR1$ are not shown, since this variable never turned significant. This contrasts with the results obtained by Piazzesi et al. (2006), who show that the expenditure ratio on non-housing consumption anticipates higher subsequent returns. One reason for the different results is that they use a sample of yearly observations.

5 Conclusions

TBW

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7 Appendix

In this appendix I derive the approximate expression (9) in the text. The starting point is the law of motion for total wealth, written in terms of the total consumption bundle, including durables and housing consumption. Real aggregates expressed in terms of total consumption are denoted by tildas.

$$\widetilde{W}_{t+1} = \left(1 + \widetilde{R}_{w,t+1}\right)\left(\widetilde{W}_t - \widetilde{C}_t\right) \tag{13}$$

where $\widetilde{C}_t = \widetilde{C}_t^d + \widetilde{C}_t^{nd}$, $\widetilde{C}_t^{nd} = \widetilde{C}_t^h + \widetilde{C}_t^{nh}$, and $\widetilde{C}_t$, $\widetilde{C}_t^d$, $\widetilde{C}_t^{nd}$, $\widetilde{C}_t^h$, $\widetilde{C}_t^{nh}$ denote, respectively, total consumption, durable consumption, non-durable consumption, housing consumption, and non-durable non-housing consumption. The expression (13) can be rewritten in terms of the non-housing component of non-durable consumption, by multiplying for the ratio between the price index for total consumption (P_t) and the price index for non-housing consumption (P_t^{nh}). After rearranging, this yields

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t) \tag{14}$$

where

$W_{t+1} = \widetilde{W}_{t+1}\frac{P_{t+1}}{P_{t+1}^{nh}}$,

$W_t = \widetilde{W}_t \frac{P_t}{P_t^{nh}}$,

$C_t = \widetilde{C}_t \frac{P_t}{P_t^{nh}}$,

and $R_{w,t+1} = \widetilde{R}_{w,t+1} \frac{P_{t+1} P_t^{nh}}{P_t P_{t+1}^{nh}}$.

(14) can be further rewritten by expliciting real non-housing consumption, the expenditure ratio between total and durable consumption, and the expenditure ratio between non-durable and non-housing consumption:

$$W_{t+1} = (1 + R_{w,t+1}) \left(W_t - C_t^{nh} \frac{C_t^{nd}}{C_t^{nh}} \frac{C_t}{C_t^{nd}} \right) \tag{15}$$

Define $\Theta_t = \frac{C_t^{nd}}{C_t^{nh}}$, $\Psi_t = \frac{C_t}{C_t^{nd}}$, and denote by φ, Θ, Ψ the steady state ratios of non-housing consumption to wealth, non-durable to non-housing consumption, and total to non-durable consumption, i.e.:

$\frac{C^{nh}}{W} = \varphi$, $\frac{C^{nd}}{C^{nh}} = \Theta$, and $\frac{C}{C^{nd}} = \Psi$.

Divide both sides of (15) by W_t

$$\frac{W_{t+1}}{W_t} = (1 + R_{w,t+1}) \left(1 - \theta_t \frac{C_t^{nh}}{W_t} \right) \tag{16}$$

Take logs of (16), and define:

$x_t = \log \left(\frac{C_t^{nh}}{W_t} \right) \equiv c_t^{nh} - w_t$,

$y_t = \log \left(\frac{C_t^{nd}}{C_t^{nh}} \right) \equiv c_t^{nd} - c_t^{nh}$,

and $z_t = \log \left(\frac{C_t}{C_t^{nd}} \right) \equiv c_t - c_t^{nd}$.

$$\Delta w_{t+1} = r_{t+1} + \log \left(1 - \exp(x_t) \exp(y_t) \exp(z_t) \right) \tag{17}$$

A Taylor expansion of (17) around the steady state for x_t , y_t, and z_t (x, y, and z) results in

$$\Delta w_{t+1} \approx r_{t+1} + (x_t - x) \left[\frac{-\exp(x_t) \exp(y_t) \exp(z_t)}{1 - \exp(x_t) \exp(y_t) \exp(z_t)} \right] \tag{18}$$

Substituting back for the corresponding expression for x_t, but retaining y_t and z_t, and rearranging (18):

$$\Delta w_{t+1} \approx r_{t+1} + \left[\left(c_t^{nh} - w_t\right) + y_t + z_t\right]\left(1 - \frac{1}{\rho}\right) - (x + y + z)\left(1 - \frac{1}{\rho}\right) \tag{19}$$

where $\rho \equiv 1 - \exp(x_t)\exp(y_t)\exp(z_t) = 1 - \frac{C}{W}$.

Next take the following identity:

$$\Delta w_{t+1} \equiv \Delta c_{t+1}^{nh} + \Delta y_{t+1} + \Delta z_{t+1} + \left(c_t^{nh} - w_t\right) + y_t + z_t - \left[\left(c_{t+1}^{nh} - w_{t+1}\right) + y_{t+1} + z_{t+1}\right] \tag{20}$$

Use (20) in (19) to obtain

$$\left(c_t^{nh} - w_t\right) + y_t + z_t \approx$$

$$\approx \rho\left[\left(r_{t+1} - \Delta c_{t+1}^{nh}\right) - \Delta y_{t+1} - \Delta z_{t+1}\right] + \rho\left[\left(c_{t+1}^{nh} - w_{t+1}\right) + y_{t+1} + z_{t+1}\right] + K \tag{21}$$

where K collects the constant terms. Substituting back the expressions for y_t and z_t into (21), and bringing it forward gives

$$\left(c_t^{nh} - w_t\right) + \left(c_t^{nd} - c_t^{nh}\right) + \left(c_t - c_t^{nd}\right) \approx$$

$$\approx \sum_{j=1}^{\infty} \rho^j \left[r_{t+j} - \Delta c_{t+j}^{nh} - \Delta\left(c_{t+j}^{nd} - c_{t+j}^{nh}\right) - \Delta\left(c_{t+j} - c_{t+j}^{nd}\right)\right] + K' \tag{22}$$

Following LL, total wealth can be approximated by a weighted average of non-human wealth (a) and labour income (y), i.e. $w_t \simeq \psi a_t + (1-\psi) l_t$. (22) becomes

$$c_t^{nh} - \psi a_t - (1-\psi) y_t + \left(c_t^{nd} - c_t^{nh}\right) + \left(c_t - c_t^{nd}\right) \approx$$

$$\approx \sum_{j=1}^{\infty} \rho^j \left[r_{t+j} - \Delta c_{t+j}^{nh} - \Delta\left(c_{t+j}^{nd} - c_{t+j}^{nh}\right) - \Delta\left(c_{t+j} - c_{t+j}^{nd}\right)\right]$$

which is equivalent to (9) in the text.