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Financial Integration and International Risk Sharing*

Yan Bai**

Arizona State University

Jing Zhang

University of Michigan

ABSTRACT

In the last two decades, financial integration has increased dramatically across the world. At the same time, the fraction of countries in default has more than doubled. Contrary to theory, however, there appears to have been no substantial improvement in the degree of international risk sharing. To account for this puzzle, we construct a general equilibrium model that features a continuum of countries and default choices on state-uncontingent bonds. We model increased financial integration as a decrease in the cost of borrowing.

Our main finding is that as the cost of borrowing is lowered, financial integration and sovereign default increases substantially, but the degree of risk sharing as measured by cross section and panel regressions increases hardly at all. The explanation, we propose, is that international risk sharing is not sensitive to the increase in financial integration given the current magnitude of capital flows because countries can insure themselves through accumulation of domestic assets. To get better risk sharing, capital flows among countries need to be extremely large. In addition, although the ability to default on loans provides state contingency, it restricts international risk sharing in two ways: higher borrowing rates and future exclusion from international credit markets.

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** Corresponding author, Yan Bai, Department of Economics, W.P. Carey School of Business, Arizona State University, P.O.Box 873806, Tempe, AZ, 85287. E-mail: yan.bai@asu.edu.

1. Introduction

Over the last two decades, as a result of widespread deregulation of capital flows, financial integration among countries has grown dramatically. Economists have studied the phenomenon from several angles, all of which confirm this remarkable increase. WEO (2001), Prasad, Rogoff, Wei and Kose (2003), and Lane and Milesi-Ferretti (2003) examine current controls on capital flows as a measure of financial integration. They all find that there has been less official restriction of capital flows over time and a significant increase in actual capital inflows and outflows among countries over the same period. Obstfeld and Taylor (2002) measure financial integration by looking at differences in interest rates among the United States, United Kingdom, France and Germany; they find smaller interest rate differentials over time between the U.S. and the other three countries.

Despite the extensive research on the effect of financial integration on economic growth, relatively little work has been done to study the impact of financial integration on international risk sharing. In their empirical work, Prasad, Rogoff, Wei and Kose (2003) examine the impact of financial integration on macroeconomic volatility, one way to measure the international risk sharing. They find that for more financial integrated developing countries, the consumption volatility relative to GDP volatility has actually risen, implying that no improvement in international risk sharing, after financial integration. Using a more formal econometric method. Kose, Prasad, and Terrones (2003a) also stress that consumption volatility increases after financial integration.

In this chapter, we first confirm that there is no substantial improvement in international risk sharing in the last two decades when financial integration increases dramatically. To measure changes in international risk sharing, instead of looking at consumption volatility, we follow standard consumption theory literature and run a regression of average real consumption growth rates against average real GDP growth rates over two 13-year periods, 1973-85 and 1986-98. As in Cochrane (1991), "*With full insurance, consumption growth rate should be cross-sectionally independent of idiosyncratic variables.*" In each period, the regression coefficient is significantly different from zero, meaning that international risk sharing is not perfect. More importantly, the difference between the two coefficients is not statistically significant, indicating that there was no substantial improvement in international risk sharing from one period to the next. This result is robust to the panel regression.

These two observations, increase in financial integration but no substantial improvement in international risk sharing, seem puzzling. Most theories predict that as financial integration increases, countries should share their risk and smooth their consumption better. As in Prasad et al.,

"*In theory, financial globalization can help developing countries to better manage output and consumption volatility. Indeed, a variety of theories implies that the volatility of consumption relative to that of output should go down as the degree of financial integration increases; the essence of global financial diversification is that a country is able to offload some of its income risk in world markets*".

In addition, during same period as financial integration increases, sovereign default has also increased. Data provided by Standard & Poor's indicate that the average fraction of countries in default more than doubled from 1973-85 to 1986-98, increasing from 5% to 11%.

This is also puzzling. Grossman and Huyck (1988) argue that sovereign default is a way of risk lifting in that it turns state uncontingent debt into state contingent debt. That is, if a country defaults while in bad states, that country shares risk by paying back less than it does in good states, when it repays the face amount of the debt. Theory thus suggests that risk sharing should increase both when financial integration increases and as sovereign default rises. But our third finding indicates that, in fact, international risk sharing has not greatly improved.

To account for these observations, increase in financial integration, increase in sovereign default but no better risk sharing, we study a heterogeneous countries general equilibrium model. There are two types of agents in the economy: a continuum of small open economies and a finite number of international financial integrations. Countries trade bonds with international financial intermediaries. The model has several frictions in the credit market. First, markets are incomplete in that the only assets that countries trade with international financial intermediaries are uncontingent bonds. Second, enforcement of contracts is limited; if countries default, they are subject only to limited sanctions consisting of a random period during which they are excluded from credit markets and their productivity is reduced. The international financial intermediaries internalize countries' default choices and design a country-specific bond price schedule incorporating the default premium The bond price depends on each country's state and loan demand. Third, borrowing is costly in that it uses up real resources. We model increased financial integration as a decrease in the cost of borrowing.

These modeling choices are made for the following reasons. First, to account for the cross-section regularities involving a large number of countries in a tractable way, we use a model with a continuum of small open economies in that the world interest rate is determined endogenously[1]. This is contrast to the standard international literature, which either use a small open economy model or a two-country model[2]. And neither of these two models is suitable to account for the cross-section regularities. Second, sovereign defaults are observed in the data. Moreover countries' borrowing rates incorporating the default premium vary with countries characteristics. This leads us to model default and hence the country-specific interest rate explicitly. Third, motivated by the widespread deregulation of capital flows, we model the increased financial integration as a result of lowering borrowing costs.

We calibrate the model twice, one corresponding to the first sub-period 1973-85 of less financial integrated, and the other corresponding to second sub-period 1986-98 of more financial integrated. To solve the model, the world productivity process is calibrated. The productivity process has three key features: wide range, higher volatility in poor countries, and countries that change properties over time. To capture the richness of the data, we use regime-switching process to estimate the world productivity process.

Our main finding is that as the cost of borrowing is lowered, financial integration increases substantially, in that the volume of loans increases greatly, but the degree of risk sharing as measured by the cross-section and panel regressions increases very little. The explanation we propose is that sovereign default increases since countries tend to default more when they borrow

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[1] Castro (2004) also adopts a continuum of countries model at the same time we wrote the first draft of this chapter.

[2] See Backus, Kehoe and Kydland (1992) for a standard two-country model, and Mendoza (1991) for a standard small open economy model.

more. Also, the smoothness of consumption is not sensitive to changes in financial integration when capital flows are at levels well below current magnitudes since countries can insure themselves through accumulation of domestic assets. To get better risk sharing, capital flows among countries need to be extremely large. To understand these results, several comparative analyses are made; all of them suggest that international risk sharing is not sensitive to changes in financial integration under current levels. Default also plays an important role in restricting international risk sharing in that the country specific risk premium generates a time-varying impediment to international risk sharing. In addition, defaulting countries will lose access to international credit market for some random period of time during which no borrowing and lending is done to smooth the consumption. Without default, international risk sharing will be improved in both periods.

In this chapter, we also address a related question: can we get full risk sharing under this incomplete market model with default choices? We find that as the discount factor close to 1, near to perfect risk sharing can be reached even under this incomplete market model. This is consistent with the theoretical finding by Levine and Zame (2001). They show in a pure exchange economy incomplete market result is close to that of complete market as long as agents are patient enough. We confirm that this is still true under a production economy. However, to get the near perfect risk sharing, the financial integration needs to be wildly large in that the net foreign capital stock is more than 20 times GDP.

Our model is related to Chatterjee, Corbae, Nakajima and Rios-Rull (2002), Arellano (2005), Aguiar and Gopinath (2004), and Yue (2005). They focus on partial equilibrium under a pure exchange economy in which the world interest rate is taken as given. Our model differs in that we use a general equilibrium model with a continuum of small, open economies. The world interest rate is endogenously determined. Instead of using a pure exchange economy, we add production to our economy. Adding capital into the model introduces an important subtlety: to optimally design their bond price schedule, international intermediaries have to think through each country's off-equilibrium allocations. These features confer several advantages. A model with production helps generate a higher default rate, one closer to that seen in actual data[3]. With

[3] Arellano (2003) mentions that it is hard to generate a default rate in a pure exchange economy with one good since when international intermediaries predict that countries will default, they will charge a higher interest rate to keep

capital, a country in financial autarky can self insure against bad shocks. The utility of financial autarky in a production economy is therefore higher than in a pure exchange economy. Hence, countries have a larger incentive to default.

Our work is also related to Perri and Heathcote (2004). They account for the relationship between financial integration and international risk sharing measure by the consumption volatility to output volatility, quantitatively. The distinction between our work and theirs is that they focus only on the developed countries: U.S., Europe, Canada and Japan, while in this paper, we study the statistics of a larger sample of countries: both developed countries and more financial integrated developing countries.

This chapter is organized as follows. Section 2 shows in detail the apparent inconsistency: financial markets that became more highly integrated from 1973 to 1998, but without simultaneous improvement in international risk sharing. Additionally, some facts on sovereign default rates are presented. Section 3 provides a continuum country model which features a production economy and default choices on country's uncontingent bond. We calibrate parameters and compute the model in section 4. Section 5 presents some sensitivity tests of our model. Section 6 concludes.

2. Empirical Facts

This section presents the empirical facts on financial integration, international risk sharing and sovereign default. Section 2.1 reviews the two most commonly used measurement of financial integration: restriction measure and openness measure. They all point to the same fact: increase in financial integration. This fact is also robust if we include debt in the second measurement. Using the data from Standard & Poor's, section 2.2 shows that the fraction of countries in the state of default is more than doubled at the same period. In section 2.3, we explore the change in international risk sharing, which is uncovered through the cross-section and panel regression. Both of the two regressions reveal that there is little change in international risk sharing over periods.

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countries from borrowing. Therefore, fewer defaults occur in equilibrium. With stochastic trend and/or endogenous default penalty, Aguiar et al. (2004) and Yue (2005) generate higher default rates but still smaller than that in data.

2.1 Measure of Financial Integration

There is a wide consensus that financial integration has increased remarkably since the 1970s. This has been shown using several different measures, including official restrictions on capital flows and openness to actual capital flows. In this section, we first review the literature on these two commonly used measures, restriction measure and openness measure. Second, we show that the increase in financial integration is also robust if we include debt in the second measurement, the openness measure.

2.1.1 Restriction Measures Review

Since financial integration is treated as a natural result of the reduction of official restrictions on international capital flows, one strand of literature looks at the change in official restrictions on cross-country capital flows. This type of measurement is called a restriction measure. Edison, Klein, Ricci and Sloek (2002) give a thorough review of studies on restriction measures of financial integration. There are two types of restriction measures: simple restriction measure and intensive restriction measure. A simple restriction measure is usually a binary indicator, under which a country is either counted as open or closed in terms of control on capital flows. In contrast, an intensive restriction measure considers the intensity of control of capital flows.

Most simple restriction measures are constructed based on the IMF publication *Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER)*, which describes the current controls on cross-border capital flows for 117 countries[4]. It's a binary indicator with 1 indicating a closed economy and 0 indicating a fully open economy. For example, based on ARERER, WEO (2001) calculates the average on/off indicator for different country groups[5] during the period 1970-98. The solid line in Figure 2.1 from WEO plots the average on/off indictor for 20 developed countries. It shows that since capital account liberalization in the 1970s, financial integration has increased greatly for developed countries as the restriction indicator decreases from 0.78 to 0.05. Similarly, the solid line in Figure 2.2 plots the restriction indicator for 55 developing countries. For developing countries, capital account liberalization reversed during the

[4] After 1986, IMF increases the number of countries to 137.

[5] Two country groups are considered in WEO (2001), 20 developed countries and 55 developing countries.

1980s but resumed in the 1990s, for an overall period trend toward greater liberalization, indicating increased financial integration also.

Some other simple restriction measures, for example Klein and Olivei (2000), are based on *OECD Code of Liberalization of Capital Movements,* which provides data on the restrictions of international transactions of OECD countries. It reports the binary indicators of 11 categories of international transactions as direct investment, liquidation of direct investment, and admission of securities to capital markets etc.

Since a simple restriction measure provides no information about how strong a country's capital controls, Quinn (1997) first attempts to construct the intensity restriction measure by assigning scores separately to the intensity of controls for capital account receipts and capital account payments. He scores each of these two categories according to the description in IMF AREAER as: 0 indicating that payments are forbidden; 0.5 indicating existence of quantitative or other regulatory restrictions; 1 indicating that transactions are subject to heavy taxes; 1.5 indicating less severe taxes; and 2.0 indicating transactions are free of restrictions or taxes. Ranging between 0 and 4, the intensive restriction indicator is the sum of the scores of these two categories. Using the similar method, Montiel (1996), and Montiel and Reinhart (1999) also construct intensive restriction measure according to IMF AREAER.

Except for the simple restriction measures based on IMF AREAER, other studies on restriction measures only have limited time samples and country samples mainly OECD countries included. It is therefore difficult to study trends in financial integration, especially for developing countries, through the restriction measures.

2.1.2 Openness Measures Review

An alternative way to measure financial integration, openness measure, is to look at the actual capital inflows and outflows among countries. WEO, Prasad et al., Lane and Milesi-Ferretti construct an openness indicator as the gross stock of foreign assets and liabilities as a share of GDP. The gross stock of foreign assets and liabilities is defined as the sum of foreign direct investment (FDI) asset and liabilities, portfolio equity asset and liabilities and portfolio asset and liabilities, all in stock. However portfolio debt asset and liabilities are usually dropped from the construction. Though each uses different sets of countries, they all show that there has been a remarkable increase in openness for both developed countries and developing countries. The dotted line in Figure 2.1 and 2.2, provided by WEO, show that the average gross foreign capital stock over GDP is increasing for both developed countries and developing countries.

Lane and Milesi-Ferretti examine the effects of the 1990s stock market boom and size of capital flow on gross stock of foreign assets and liabilities. They show that the large size of capital flow helps increase gross stock. Similarly, Heathcote and Perri find that the sum of US foreign direct investment and portfolio equity, relative to US capital stock, grows dramatically from 1972 to 1999. The analogous measure of liabilities also grows dramatically at the same period of time. This also indicates that the size of capital flows increases dramatically.

Due to the volatility of foreign debt data, WEO, Prasad et al. and Lane et al. include only foreign direct investment and portfolio equity in their definition of gross stock of foreign assets and liabilities. For developing countries, however, this may not be a good indicator for measuring foreign capital flows since debt represents an important part of their foreign assets and liabilities. Also it is a key instrument for developing countries to do risk sharing. We therefore include debt in our measurement of financial integration.

2.1.3 Our Measurement

The financial integration in this paper is measured following the strand of openness measurement literature. Our work differs from the literature in two dimensions. First, we include debt asset and liabilities into our measurement of financial integration. Second, we use the net foreign capital stock over GDP ratio instead of gross foreign capital stock over GDP as in the literature[6].

The degree of financial integration in period *t* is measured according to the following equation:

[6] We also look at the gross foreign capital stock over GDP ratio for the sample countries. The conclusion of increase in financial integration still holds. We find that the weighted average gross foreign capital stock over GDP ratio increase from 11% to 30% for all the 40 countries; from 9% to 28% for developed countries; and from 20% to 32% for developing countries.

$$indicator_t = \frac{\sum_{i=1}^{N} \left| Net\ Foreign\ Stock_{i,t} \right|}{\sum_{i=1}^{N} \left| GDP_{i,t} \right|} \tag{2.1}$$

and

$$Net\ Foreign\ Stock = FDIA - FDIL + EQA - EQL + DEBTA - DEBTL \tag{2.2}$$

where, *FDIA (FDIL)* is stock of foreign direct investment assets (liabilities), *EQA (EQL)* is stock of portfolio equity assets (liabilities), and *DEBTA (DEBTL)* is stock of portfolio debt assets (liabilities)[7].

Two points should be emphasized here. First, to avoid some high debt country driving up the ratio of foreign stock over GDP, weighted average of foreign stock over GDP ratio among countries is calculated in each period. Second, we use stock data rather than flow data from balance of payments is that stock data are less volatile and less prone to measurement error[8]. Unfortunately, data on foreign assets and liabilities stock are incomplete and not comparable among countries over time. The IMF provides International Investment Position data for just 15 OECD countries after 1980. To better understand the capital flows among countries, Lane and Milesi-Ferretti construct foreign assets and liabilities stock data for 67 countries over the period 1970 to 1998 by flow data. They take into consideration both exchange rate changes and price changes when constructing the foreign capital stock.

We choose 21 industrial countries[9] and 19 developing countries[10]. Our choice of industrial countries is the same as Lane and Milesi-Ferretti (2003) except that we count Belgium as industrial country and Portugal as a developing country. We based our selection of developing countries on Prasad, Rogoff, Wei and Kose (2003), who divide 55 developing countries into 22 that are more financially integrated and 33 that are less well integrated. Since we are trying to understand the link between international financial integration and risk sharing, we select only the

[7] All the data are from Lane and Milesi-Ferretti (2001). See Appendix 2.7.1 for details.

[8] See Lane and Milesi-Ferretti (2001)

[9] Industrial countries: Australia, Austria, Belgium, Canada, Switzerland, Denmark, Finland, Germany, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Spain, Sweden, UK, USA

[10] Developing countries: Argentina, Brazil, Chile, Colombia, Egypt, Indian, Indonesia, Israel, Korea, Malaysia, Mexico, Pakistan, Peru, Philippine, Portugal, Singapore, South Africa, Thailand, Venezuela

more financially integrated nations. China, Morocco and Turkey are dropped because of inadequate quality and availability of data.

Table 2.1 Average Net Foreign Capital Stock as a Share of GDP

	1973 – 85	1986 – 98
All countries (40)	0.09	0.17
Industrial countries (21)	0.06	0.14
Developing countries (19)	0.27	0.37

Figures 2.3 plots the international financial integration for different groups of countries over time. There is a clear increasing pattern in all three figures. The hump in developing countries' graph is due to the 1982 debt crisis. Table 2.1 summarizes the ratio of average net foreign stock to GDP in the two 13-year periods: 1973-85 and 1985-98. For all the 40 countries, the degree of financial integration increases from 9% to 17%, nearly doubled from one period to the next. Financial integration increases dramatically in developed countries; the increase in developing countries is less dramatic, but still quite substantial.

Table 2.2 Average Net Debt Stock as a Share of GDP

	1973 – 85	1986 – 98
All countries (40)	0.05	0.14
Developed countries (21)	0.02	0.11
Developing countries (19)	0.23	0.28

Debt is important for developing countries as a means of obtaining resources to smooth consumption. Figures 2.4 plots the average debt assets and liabilities as a ratio of GDP for all 40 countries, for just the 21 industrial countries and for just 19 developing countries. As was the case for foreign capital stock, the weighted average net debt stock increases over time. Table 2.2 shows the period average of debt stock as a share of GDP. The number more than doubles if all the 40 courtiers are considered.

2.2 Sovereign Default Rate

Standard & Poor's defines sovereign default as "*the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of* *the debt issue*[11]." Standard and Poor's surveys two types of sovereign default: local currency debt and foreign currency debt. The latter includes foreign currency bond and foreign bank debt. Default on local currency debt is rare since governments can use fiscal policy (tax increases) or monetary policy (issuing money) to repay local currency debt. We therefore only look at foreign currency debt default. However, including local currency default does not affect our conclusion about sovereign default rates.

Because few countries with low ratings issue foreign currency bonds and they depend primarily on foreign bank debt to finance domestic demand, defaults on foreign currency bonds were less frequent prior to the 1990s. After the 1980s' debt crisis, the share of foreign bank debt decreased, due to a significant extent to Brady bond exchanges and buybacks. Foreign currency bond defaults have tended to increase in the last ten years though it still does not exceed the default rate of foreign currency debt.

Most of the literature on sovereign default focuses on the new default rate which is the ratio of the number of new defaults to the number of countries with debt in a given period[12]. From Reinhart (2002), the average new default rate for 59 countries is around 3% per year over 1970-99.

Since we are interested in the factors that drive low international risk sharing, a statistic on the fraction of countries in the state of default is more relevant. A delinquent country may be excluded from financial markets for some years[13]. This will definitely restrict its ability to do. We therefore use a definition of default rate as defined by Standard & Poors. The fraction of countries in the state of default, shorted as Fraction in Default, is formally defined as

$$\text{Fraction in Default at period } t = \frac{\text{number of countries in the state of defaults in period t}}{\text{total number of countries}}$$

Beers and Chambers report the 202 sovereign countries' years in default by Standard & Poors. Over the 202 sovereign countries, the fraction in default in period 1973-85 is 10% and 26% in 1986-98. The years in default for our sample countries are summarized in Table 2.3. All are

[11] See Beers and Chambers (2003)

[12] See Reinhart (2002),Beim and Calomiris (1999). Reinhart gives the example of Sierra Leone which was in default during the periods 1983-84 and 1986-95, but treated as a single default episode beginning in 1983.)

[13] Examples see Gelos and Sahay (2003)

defaults on foreign currency bank debt except for Argentina 1989 and Venezuela 1995-97, which are defaults on foreign currency bond debt. Through this table, we can construct the fraction in default from period 1973 to 1998. The average number of countries in default in period 1973-85 is 2.15, which is of around 5% of the 40 countries. Similarly, the average number of countries in default in period 1986-98 is 4.25, or 11% of the 40 countries.

Table 2.3 Year in Default, 1973-98

Country	Year in Default
Argentina	1989,1982-93
Brazil	1983-94
Chile	1983-90
Egypt	1984
Indonesia	1998
Mexico	1982-90
Pakistan	1998
Peru	1976,78,80,83-97
Philippine	1983-92
South Africa	1985-87,89,93
Venezuela	1983-88,90,95-97

2.3 Measure of International Risk Sharing

Most models predict that as financial integration increases, countries can pool their risk and smooth consumption better. Also default is a way of turning state uncontingent debt into state contingent debt and so provides risk sharing. In the previous section, we showed that both financial integration and sovereign default have increased over the last two decades. But the question remains: Has international risk sharing increased as well?

Nearly all of the international risk sharing literature focuses on time series or high frequency correlation among countries' consumption and output. Backus, Kehoe and Kydland (1992) first study the cross-country consumption correlations among 16 OECD countries. They found the so-called Backus-Kehoe-Kydland consumption correlation puzzle: Cross-country *consumption* correlations are smaller than cross-country *output* correlations. The literature following them that seeks to explain this puzzle all relies on detrended consumption and output series. Most of these papers have a two-country model or a small open economy. In a small open economy, the world interest rate is exogenous and taken as given. In a two-country economy, although the world interest rate is endogenously determined, the limited number of countries restricts the assets available and therefore restricts international risk sharing.

In this paper, we follow the consumption theory literature and put our focus on cross-section or high-frequency behavior. The direct implication of the standard complete market model is that consumption growth rates should not respond to individual income growth rates, independent of high or low frequency. In the following, we first run a cross-section regression to test international risk sharing. To show the robustness of the result, we also run a panel regression designed to test international risk sharing.

2.3.1 Cross-section Regression

The key idea of using cross-section regression to test international risk sharing is that "With full consumption insurance, consumption growth should be cross-sectionally independent of idiosyncratic variables." (From Cochrane (1991)). The specification for the regression is as follows,

$$\triangle \log c_i = \alpha_0 + \alpha_1 \triangle \log y_i + \varepsilon_i, \quad for\ i = 1, 2, .., N \tag{2.3}$$

where $\triangle \log c_i$ is the average growth rate in real final consumption of country i over time, $\triangle \log y_i$ is the average growth rate of real GDP of country i over time. For x to be real consumption or real GDP, $\Delta \log x$ is defined as

$$\triangle \log x_i = \frac{\log x_{i,T} - \log x_{i,1}}{T-1} \tag{2.4}$$

It is assumed that ε_i is i.i.d. and normally distributed, $\varepsilon_i \sim N(0,\sigma)$. Both consumption and GDP are in per capita. We use final consumption which is the sum of government expenditure and private consumption. According to Cochrane (1991), this specification is simple and independent of nonseparability of preference.

We designate two sub-periods: 1973-1985 and 1986-1998. $\triangle \log c_i$ and $\triangle \log y_i$ are then the average 13-year consumption and GDP growth rates for each country. In each sub-period, we run three separate regressions: for all 40 countries, for the 21 developed countries and for the 19 more financially integrated developing countries. With full international risk sharing, the consumption growth rate should be independent of the GDP growth rate. The null hypothesis is $\alpha_1 = 0$.

Table 2.4 Cross-Section Regression

	α_1 : 1973 – 1985	α_1 : 1986 – 1998
All countries (40)	0.78 (0.08)	0.76 (0.05)
Industrial countries (21)	0.73 (0.13)	0.77 (0.12)
Developing countries (19)	0.80 (0.12)	0.81 (0.07)

The regression results are reported in Table 2.4. Number in bracket is the standard errors. For all 40 countries, the regression coefficient is 0.78 in the first sub-period and 0.76 in the second sub-period. Both coefficients are significantly different from zero, thus rejecting the null hypothesis of $\alpha_1 = 0$ in both sub-periods. Moreover, a t-test shows that we cannot reject the possibility that the regression coefficients in the two sub-periods are the same. This implies that there is no improvement in international risk sharing from one period to the next. Similarly, for 21 developed countries, the regression coefficients in both sub-periods are significantly different from zero and we cannot reject they are the same. These results hold for the 19 developing countries as well.

Thus, the cross-section regression analysis does not show that there is better risk sharing among countries despite a substantial increase in international capital flows.

2.3.2 Panel Regression

In the same issue of the *Journal of Political Economy* where Cochrane (1991) proposes his test of consumption insurance, Mace (1991) uses a different specification to the same end. The idea is similar to Cochrane's. "*Key feature of risk sharing: individual consumption responds to aggregate risk but not to idiosyncratic risk.*" (Mace (1991)). She runs a panel regression with individual consumption growth rate as the explained variable, and individual output growth rate and aggregate consumption growth rate as the independent variables. The panel regression is related to the cross-section regression in that the constant term in the cross-section regression corresponds to the aggregate consumption growth rate.

Formally, under the assumption that the utility function is CES and separable on consumption and labor, the panel regression is specified as follows,

$$\triangle \log c_{it} = \beta_0 + \beta_1 \triangle \log c_t^a + \beta_2 \triangle \log y_{it} + \varepsilon_{it} \tag{2.5}$$

where $\triangle \log c_{it}$ is the period t real final consumption growth rate of country i; $\triangle \log y_{it}$ is period t country $i's$ growth rate of real GDP; $\triangle \log c_t^a$ is the average consumption growth rate across all countries. $\triangle \log c_t^a$ is defined as follows,

$$\triangle \log c_t^a = \frac{1}{N} \sum_{i=1}^{N} \triangle \log c_{it} \tag{2.6}$$

For x to be real consumption and real GDP, the growth rate of x is defined as

$$\triangle \log x_{it} = \log x_{i,t} - \log x_{i,t-1} \tag{2.7}$$

$\varepsilon_{it} \sim N(0, \sigma)$ contains the time-varying components of individual and aggregate preference shocks and might also include measurement errors from the data (From Mace). With full risk sharing, the individual consumption growth rate is independent of the individual output growth rate and only responds to the world average consumption growth rate. The null hypothesis is therefore $\beta_0 = 1$ and $\beta_1 = 0$.

Table 2.5 Panel Regression

	1973 – 85		1986 – 98	
	β_1	β_2	β_1	β_2
All countries (40)	0.21 (0.13)	0.72 (0.04)	-0.10 (0.14)	0.84 (0.02)
Industrial countries (21)	0.19 (0.10)	0.56 (0.04)	0.05 (0.14)	0.67 (0.03)
Developing countries (19)	0.32 (0.25)	0.77 (0.06)	-0.33 (0.26)	0.90 (0.03)

Table 2.5 shows the panel regression results. In each regression, $\triangle \log c_t^a$ is the average consumption growth rate of 40 countries. The null hypothesis of $\beta_0 = 1$ and $\beta_1 = 0$ is rejected for all six regressions. Most importantly, we cannot reject that the regression coefficient before

individual output growth rate in the first sub-period is larger than that in the second sub-period. Once more, we show that there is no substantial improvement in international risk sharing over time.

The conclusion of no improvement in international risk sharing is robust to using private consumption or final consumption, and the sum of private and final consumption. The results still hold if we use aggregate final consumption and total GDP instead of per capita data. Additionally, if we change the cut-off dates for the sub-periods, there is still no substantial improvement in international risk sharing.

In summary, two puzzling facts are observed in the data: the degree of financial integration nearly doubles from the first period to the second, but there is no substantial improvement in international risk sharing. In addition, we have found a third interesting fact: the fraction of nations in default also increases greatly over this time span. See Table 2.6 for summary of the statistics for all the 40 countries with risk sharing measured by the cross-section regression coefficients. In the next section, we construct a model with continuum countries and default choices to account for these three facts

Table 2.6 Summary of Empirical Results

	1973 – 85	1986 – 98
Degree of financial integration	0.09	0.17
Fraction in default	0.05	0.11
Degree of risk sharing	0.78	0.76

3. The Model

In this economy, there is only one good, which can be either consumed or invested as capital. There are two types of agents in the world: a continuum of measure 1 countries[14] and finite

[14] Here we are considering each country's planner's problem. It can be decentralized similarly as in Kehoe and Perri (2004) by a problem in which consumers decide how much to borrow abroad, governments make default choices and levy capital income tax.

number of international financial intermediaries. Countries trade state uncontingent bonds with financial intermediaries. International contracts have limited enforcement in that countries have the option to default on their debts if it is optimal. After defaulting a country's productivity drops in the following periods. The defaulting country is also excluded from international financial markets meaning that it can only self insure through capital accumulation but cannot save or borrow abroad. With some exogenous probability, its default behavior is forgiven and it reenters international financial markets. The financial intermediaries internalize countries' default choices and design a country-specific bond price schedule incorporating the default premium.

In each period, the world economy experiences a finite number of events s_t. We use $s^t = (s_0, s_1, ..., s_t)$ to denote the history of events. Let $\pi(s^t)$ be the probability that event s^t occurs. There is no aggregate uncertainty in this world[15]. A single good is produced in country i using input capital $k_i(s^{t-1})$ and labor $l_i(s^t)$, which is assumed to be inelastically supplied and normalized as one for each country[16]. The production function of country i is given by $a_i(s^t) f(k_i(s^{t-1}))$. where $a_i(s^t)$ is a country-specific productivity shock following an exogenous process.

Each country i maximizes lifetime-discounted utility of the form:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_i(s^t)) \tag{3.1}$$

β denotes the discount factor and $c_i(s^t)$ denotes country i 's consumption under history s^t. The period utility function u takes the form of CES as $u(c) = \dfrac{c^{1-\sigma} - 1}{1-\sigma}$, σ is the degree of risk aversion.

A country is either in Normal Phase or in Penalty Phase. Only countries in Normal Phase can borrow from or save in the international financial market. Countries in Normal Phase can also choose to renege on their international debts. If a country in Normal Phase defaults, its debt will

[15] Since international risk sharing among countries can only smooth out idiosyncratic shocks, here we assume there is no aggregate uncertainty.

[16] This paper focuses on low frequency cross-section regularities. Labor supply is less volatile in low frequency. We therefore assume that it is supplied inelastically without loss of generality.

be written off but with some penalties. First, from then on it will stay in Penalty Phase – financial autarky in which no saving and borrowing abroad are allowed – until it returns to Normal Phase. Second its productivity will drop γ percent as long as it stays in Penalty Phase[17]. To model that countries only stay in Penalty Phase temporarily, it is assumed that there is an exogenous probability λ defaulting country will return to Normal Phase.

The state of each country is denoted by $x=(s,h)\in X$, where $s=(a,k,b)$ with productivity shock a, domestic capital stock k and foreign bond holding b. And h indicates which phase it's in: P for Penalty Phase and N for Normal Phase. Let $X=S\times H$ be the state space with $S=A\times\mathbb{R}_{+}\times\mathbb{R}$, $H=\{N,P\}$. In the remaining of this paper, variables with prime are realized in next period.

There are a finite number of risk-neutral financial intermediaries[18] in this economy. They are in Bertrand competition. They offer contracts $\{q(s,b'),b'\}$ for any $b'\in\mathbb{R}$ to countries in Normal Phase. A country-specific bond price schedule incorporates default risk of a country and depends on the state and borrowing of each country in Normal Phase: $q(s,b')$. If $b'\geq 0$, country s deposits $q(s,b')b'$ amount of goods in a financial intermediary in this period and is promised b' unit of the good in the next period. Financial intermediaries are not allowed to default[19]. If $b'<0$, country s will get $q(s,b')b'$ amount of loan, and repay b' unit of good to the financial intermediary in the next period. Countries with debt may default.

International borrowing and lending is costly in that it used up real resources. Country will incur bond transaction cost[20] $\tau(b')$ [21] for its foreign bonds holding b' .

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[17] According to Rose (2002), there is about average 8% of trade loss after country defaults. One reason defaulting countries incur loss in trade is that they may lose trade credit – intermediate goods – as a punishment from creditors. And decrease in intermediate goods can be simplified as a loss in total factor productivity.

[18] The number of financial intermediaries is larger than 2.

[19] The financial intermediaries' problem is actually deterministic given no aggregate uncertainty in this economy. They therefore will not default on their deposits.

[20] International financial integration is modeled as a decrease in bond transaction cost. This is motivated by the fact that there is large worldwide capital account liberalization from 70s to 80s. And financial integration is treated a natural result of reduction in official restriction of international capital flows. See reviews by Edison, Klein, Ricci and Sloek (2002).

The timing is as follows. At each period, first, a productivity shock a is realized for each country. This is public information. Second, international financial intermediaries post bond price schedule $q(s,b')$, which depends on a country's state s and loan demand b'. Finally, given the bond price schedule, a country in Normal Phase decides whether or not to default and also decides its consumption, investment and bond holding. A country in Penalty Phase cannot borrow or save abroad and so only decides its consumption and domestic investment.

The remaining of this section presents details of each country and financial intermediaries' problem. Subsection 3.1 shows each country's maximization problem. Subsection 3.2 presents how financial intermediaries design their optimal bond price schedule. A stationary recursive equilibrium is defined in subsection 3.3.

3.1 Individual Country

There are two phases countries can be in: Normal Phase and Penalty Phase. For a country in Normal Phase, given the bond price schedule provided by the international financial intermediaries, it can choose either to default on its debt or to renew its old debt. For a country in Penalty Phase, it can only self insure through domestic capital accumulation.

3.1.1 Country in Normal Phase

A county in Normal Phase will first decide whether to default according to which welfare is larger: default or repay. Let $V^N(s)$ denote the value function of country s in Normal Phase. Then

$$V^N(s) = \max\,\{W^R(s), W^D(a,k)\} \tag{3.1}$$

where $W^R(s)$ is the value function if it repays, $W^D(a,k)$ is the value function if it defaults. The default value function $W^D(a,k)$ only depends on country's shock realization a and domestic capital stock but not on foreign bonds holding, since after default country's debt is written off.

[21] Here we follow the literature and assume that the transaction cost depends on the bond holding. See Perri and Heathcote (2004). It can be also assumed that the transaction cost depends on current and next period's bond holding $\tau(b,b')$.

Let d denote the default decision. Then if country s chooses to repay, $d(s)=0$; if otherwise it defaults, $d(s)=1$.

If choose to repay $d(s)=0$, country s still has access to international financial market. Given its income from production ak^{α} and bonds holding b, it chooses consumption, next period's domestic capital stock k' and bonds holding b' with transaction cost $\tau(b')$. The budge set is defined as $\Gamma^N : S \to\to \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}$ such that for any $s \in S$

$$\Gamma^N(s) \equiv \{(c,k',b') \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \mid c + k' + q(s,b')b' + \tau(b') \le ak^{\alpha} + (1-\delta)k + b\} \quad (3.10)$$

For a country with low production, large amount of debt and expensive new debt, its budget set Γ^N may be empty and then it has to default. This type of default is called involuntary default.

In addition, the non-defaulting country will still stay in Normal Phase in next period. For country s in Normal phase with nonempty budget set $\Gamma^N(s)$, its value function of repaying is defined as,

$$\begin{aligned} W^R(s) = \max_{c,k',b'} u(c) + \beta \textstyle\sum_{a'} \pi(a' \mid a) V^N(s') \\ s.t.\ c + k' + q(s,b')b' + \tau(b') \le ak^{\alpha} + (1-\delta)k + b \end{aligned} \quad (3.11)$$

If default, country s will enter into Penalty Phase with some penalties. First, its productivity drops γ percent from this period on until it is back to Normal Phase. And the international financial market shuts off to it temporarily, meaning that it has no access to international financial market and can only insure through domestic capital accumulation. It will be in Penalty Phase from next period on until it returns to Normal Phase. The value function of default corresponding to lifetime-expected utility is

$$\begin{aligned} W^D(a,k) = \max_{c,k' \in \mathbb{R}_+ \times \mathbb{R}_+} u(c) + \beta \textstyle\sum_{a'|a} \pi(a' \mid a) V^P(a',k') \\ s.t.\ c + k' \le (1-\gamma)ak^{\alpha} + (1-\delta)k \end{aligned} \quad (3.12)$$

where $V^P(a,k)$ is the value function for a country in Penalty Phase with productivity shock a and capital stock k.

3.1.2 Country in Penalty Phase

For a country in Penalty Phase, its productivity drops γ percent each period and so its production becomes $(1-\gamma)ak^{\alpha}$, and it only chooses consumption and domestic capital holding; no international borrowing and saving are allowed. In next period it has probability λ of returning to financial markets (Normal Phase) with 0 debt, with probability $1-\lambda$ it still stays in Penalty Phase. The value function of country (a,k) [22] in Penalty Phase is defined as:

$$\begin{aligned} V^{P}(a,k) = \max_{c,k'} \; & u(c) + \beta \sum\nolimits_{a'} \pi(a' \mid a)[(1-\lambda)V^{P}(a',k') + \lambda V^{N}(a',k',0)] \\ s.t. \; & c + k' \leq (1-\gamma)ak^{\alpha} + (1-\delta)k \end{aligned} \tag{3.13}$$

To better understand the model, we explore the first order conditions of a country that is in Normal Phase and chooses not to default[23]. There are two Euler equations: one for capital and one for bonds.

Equation (3.14) is the Euler equation for capital, which equalizes the marginal utility of consumption in the current period to the expected value of future marginal utility of consumption.

$$\begin{aligned} u'(c_R) = \beta \sum\nolimits_{a'} \pi(a' \mid a) \Bigg[& (1-d')u'(c_R') \left(\alpha a'(k_R')^{\alpha-1} + (1-\delta) - \frac{\partial q(s',b'')}{\partial k_R'} b'' \right) \\ & + d'u'(c_D') \Big((1-\gamma)\alpha a'(k_R')^{\alpha-1} + (1-\delta) \Big) \Bigg] \end{aligned} \tag{3.14}$$

Where state space $s' = (a', k_R', b_R')$ with k_R', b_R' denoting capital stock choice and bonds holding in current period if a country repays. And $d' = d'(s')$ denotes next period's default choice given next period's state s'. c_R' denotes the consumption if repay in next period, and c_D' denotes the consumption if default in next period. Countries will consider their future default decisions when they make decisions on domestic investment. If a country does not default at some future state

[22] Since a country in Penalty Phase has no foreign bond holdings, we omit $b=0$ here and write the value function of Penalty Phase only as a function of (a,k).

[23] We will not give the conditions under which the value functions are differentiable, but simply assumed they are differentiable almost everywhere.

a', then the marginal benefit of investing one extra unit of consumption is the marginal return on investment plus the benefit of a higher bond price for next period's borrowing, if the bond price schedule is increasing in its second argument *k*. However, if bond price is decreasing in *k*, the cost of a lower borrowing price in the future is subtracted from the benefit of marginal return.

If a country defaults at some future state, the marginal benefit of investing one extra unit of consumption is the marginal return on investment with productivity decreased by γ percent.

Similarly, the Euler equation for bond holding,

$$u'(c_R)(q(s,b'_R)+\frac{\partial q(s,b'_R)}{\partial b'_R}b'_R)=\beta\sum_{a'}\pi(a'\mid a)\left[(1-d')u'(c'_R)\left(1-\frac{\partial q(s',b'')}{\partial b'_R}b''\right)\right] \quad (3.15)$$

Similarly, the benefit of borrowing for additional consumption is $u'(c_R)$. Since bond price is country specific, the agent will consider the change in bond price with additional borrowing. At the margin, today's borrowing affects future consumption only when the agent chooses not to default, since the debt of default countries is written off. As in the capital Euler equation, the marginal cost of additional borrowing includes a future low bond price if q is an increasing function of debt holding.

If a country chooses to save, then the bond price is fixed at the reciprocal of world interest rate and so extra saving will not affect the bond price. In addition, in next period this country will not default $d'=0$. The Euler equation for bonds becomes

$$u'(c_R)q(s,b'_R)=\beta\sum_{a'}\pi(a'\mid a)\left[u'(c'_R)(1-\frac{\partial q(s',b'')}{\partial b'_R}b'')\right] \quad (3.16)$$

Therefore, when a country in Normal Phase makes decision in investment under repayment, it not only considers about future expected return on investment, but also takes into account that current decision on capital stock and bonds holding will also affect its future borrowing rate from financial intermediaries. The international financial intermediaries will internalize this when they design the bond price schedule. The default model under production economy is therefore subtler than that under endowment economy. In next subsection, financial intermediary' problem is presented.

3.2 International Financial Intermediaries

In this economy, there are $N \geq 1$ financial intermediaries that are in Bertrand competition. They are risk neutral[24]. And the financial intermediaries are assumed to be large enough that any amount of loan demand from a single country can be satisfied. This assumption ensures that coordination problems will not arise[25]. At the beginning of each period, given a world interest rate and sovereign countries' response choices, international financial intermediaries design their borrowing and lending contracts as $(q(s,b'),b')$. It is assumed that countries in Penalty Phase have no access to international financial market. We therefore omit the state $N = \textit{Normal Phase}$ for each country below.

The international financial intermediaries chooses bond price $q(s,b')$ to solve the following problem

$$\max_{q} \quad qb' - \frac{1-p(s,b';q)}{R}b' \tag{3.17}$$

where, $p(s,b';q)$ is the expected default probability of a country in state s which would like to borrow b' amount given optimal contracts $(q(s,b'),b')$. For $b'<0$, the financial intermediary lends qb' to country s in Normal Phase with loan request b' today and with probability $1-p(s,b';q)$ that the loan will be repaid. But it will also forgo the world interest rate.

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[24] The ssumption of risk neutrality of international financial intermediaries does not affect the setup of the bond price schedule. This is because in this economy, there is no aggregate uncertainty. International financial intermediaries can therefore fully predict the measure of agents in each state. For them, the entire problem is stationary and they will therefore choose a stationary consumption path.

[25] Coordination problem: a country may have to choose to default if everyone thinks that it will default and refuses to roll over its old debt. If otherwise, everyone agrees to lend to it, this country will not choose to default. But if there are finite number of large financial intermediaries such that they can satisfy any country's loan demand, then this coordination problem will not come up since it is still beneficiary to lend to that country.

Countries who save will not default and their default probability is zero. They will get $1/R$ for any $b' \geq 0$. The optimal borrowing contract is $(q(s,b'),b')$ which solves the above problem.

Zero profit from Bertrand competition implies that for any (s,b')

$$q(s,b') = \frac{1 - p(s,b';q)}{R} \tag{3.18}$$

Note that the bond price schedule is defined on any possible borrowing not just the optimal borrowing.

To get the bond price schedule, the financial intermediary needs to think through each country's problem to get default probabilities.

Let $\Gamma(s,b')$ be the feasible set of country s that intends to borrow b' given contracts $q(.)$,

$$\Gamma(s,b') = \{(c,k') \in \mathbb{R}_+ \times \mathbb{R}_+ \mid c + k' \leq ak^\alpha + (1-\delta)k + b - q(s,b')b' - \tau(b')\} \tag{3.19}$$

If country s in Normal Phase chooses to default after the financial intermediaries announce the bond price schedules, the best choice for the international intermediary is to lend it nothing. And so it will announce that the bond price to this country is zero whatever amount of debt it borrows.

Given country s does not default on its current debt, optimal capital stock choice can be inferred from the country's optimization problem.

If b' is not feasible for country s in the sense that $\Gamma(s,b')$ is empty, the financial intermediaries will not issue b' to country s. The bond price $q(s,b')$ will be set to zero.

From above, only countries that are not defaulting in the current period and the amount of loans is feasible under s may be issued a bond with positive price.

Let $B(s) = \{b' \mid d(s) = 0$ and $\Gamma(s,b')$ is non-empty$\}$

Then the default probability is defined as

$$p(s,b';q)=\begin{cases}\sum_{a'}\pi(a'\mid a)d(a',k'(s,b';q),b') & if\ b'\in B(s;q)\\ 1 & otherwise\end{cases} \tag{3.20}$$

The optimal capital stock given (s,b') is given by

$$\begin{aligned} k'(s,b';q)\in \arg\max_{(c,k')} & \ u(c)+\beta\sum_{a'}\pi(a'\mid a)V^N(s') \\ s.t.\quad & c+k'\le ak^{\alpha}+(1-\delta)k+b-q(s,b')b'-\tau(b') \end{aligned} \tag{3.21}$$

Since country s does not default in current period, it will still stay in Normal Phase in next period and so its expected future discounted welfare is given by $\beta\sum_{a'}\pi(a'\mid a)V^N(s')$.

3.3 Stationary Recursive Equilibrium

Since there is no aggregate uncertainty in our model, the distribution will stay constant across periods. Before we define the stationary recursive equilibrium, let's first define the distribution. Let μ be the probability measure on $(X,\aleph)$, where X is the state space defined in section 3.1 and $\aleph$ is the Borel σ-algebra. For any $M\in\aleph$, $\mu(M)$ indicates the mass of agents whose individual state vectors lie in *M*. Define the transition matrix of the state as follows: $Q:X\times\aleph\to[0,1]$. $Q(x,M)$ is the probability of a country with state *x* having next period's state falling into set *M*.

Definition. The probability measure μ is said to be stationary if

$$\mu(M)=\int_X Q(x,M)d\mu \qquad \text{for any } M\in\aleph \tag{3.22}$$

Definition: A stationary recursive equilibrium is a set of world interest rate *R,* bond price $q(s,b')$, allocations $\{c(x),k'(x),b'(x),d(s)\}$, value function $V^N(s)$, $V^P(a,k)$, and stationary distribution μ, such that

(1) Given R and sovereign countries' response functions, Bertrand competitive financial intermediaries choose borrowing and lending contracts as $(1/R, b')$ for any $b' \geq 0$ and $(q(s,b'), b')$ for any $b' < 0$.

(2) Given R and $q(s,b')$, $\{c(x), k'(x), b'(x), d(s)\}$ and $V^N(s)$, $V^P(a,k)$ solve each country's problem.

(3) Bond market clearing: $\int_X q(x,b')b'(x)d\mu = 0$

(4) μ is stationary in that $\mu(M) = \int_X Q(x,M)d\mu$ for any $M \in \aleph$.

Condition (3) shows the resource constraint. Aggregate loans and aggregate deposits should be equalized. Given bond price schedule *q*, the recursive formation of each country is well-defined, following from Contraction Mapping Theorem[26].

Proposition 1. If a country with debt b_2 defaults, it will also default on b_1 with $b_1 < b_2$ given the same (a,k).

See appendix for proof.

Proposition 1 simply states that default choice $d(s)$ is weakly decreasing in b. When a country defaults on a small amount of debt, it will also default on a large amount of debt given the same level of capital stock and productivity shock. This is intuitive. From proposition 1, there exists a cutoff point for debt $\tilde{b}(a,k)$ such that for any $b \leq \tilde{b}(a,k)$, $d(a,k,b) = 1$.

Proposition 2. A country with debt to GDP ratio smaller than γ will not default.

See appendix for proof.

When its debt-to-GDP ratio is low, repaying debt is not painful for a country. The benefit of risk sharing is therefore larger than the cost of repaying the debt. A country will simply not choose to default. This is only a sufficient condition. Countries with larger debt-to-GDP ratios may also choose not to default. The real endogenous borrowing limit may be larger than this.

[26] See Stokey and Lucas, chapter 4

4. Calibration and Computation

In this section, we first calibrate world productivity process and the other parameters. A dynamic non-linear method is then used to compute the model. We experiment the model with different values of portfolio transaction cost parameter and show that as the transaction cost is lowered, financial integration increases. In addition, the fraction of countries in default increases endogenously after lowering transaction cost. However there is no substantial improvement in international risk sharing.

4.1 World Productivity Process

To estimate the world productivity process, total factor productivities are calibrated for sample countries. The calibrated productivity processes have three key features that motivate us to use regime-switching process to estimate the world productivity process.

4.1.1 Calibration of Productivity Processes

The total factor productivity is calibrated from the Cobb-Douglas production function:

$$\log A_{it} = \log Y_{it} - \alpha \log K_{it} - (1-\alpha) \log L_{it}$$

where, $A_{it}, Y_{it}, K_{it}, L_{it}$ are country i's total factor productivity, real GDP, capital stock and employment at period *t,* respectively. Here capital share $\alpha = 0.33$ [27]. Capital stock is constructed from gross capital formation for each country at each period. Both real GDP and capital stock are normalized by 1995's nominal exchange rate relative to US to make them comparable. Data source and construction of capital stock are presented in Appendix A.2.

It is assumed that all countries' realization of TFPs are drawn from one common world productivity process. This assumption can be justified as follows. First, due to the computation consideration, it is tractable to have one common shock process instead that each country has its own shock process. Second, the length of available data is short. For most countries especially developing countries, data statistics before 1960 is totally not available. The small sample period

[27] See below calibration of parameters for details.

will make bias the estimation of each country's shock process. We therefore choose a larger sample of countries and longer period 1960-2000 to estimate this common shock process for all the countries.

There are 49 sample countries in our sample of world productivity process estimation, with 29 countries from previous sample in measure of financial integration and 20 other countries, which are chosen as long as the employment data are available[28]. See Appendix A.2 for country selections. Countries with default history are not included in this sample since according to Rose (2002) there is about 8% of trade loss after default. And the trade loss will be reflected in the loss in total factor productivity. The TFP of defaulting countries therefore has two components: realization drawn from the common process and the loss of productivity as penalty from international creditors. Since it is complicated to separate those two components, here we simply drop the 11 defaulting countries[29] from our productivity estimation country sample.

Each country's productivity process is detrended with the world average productivity growth rate, $g_a = 1.3\%$ [30]. The detrened TFP is:

$$\log a_{it} = \log A_{it} - t\log(1+g_a)$$

Figure 4.1 plots the detrended productivity processes $\log a_{it}$ for 49 countries from 1960 to 2000.

The calibrated productivity processes have three key features:

(1) The range of the processes is wide relative to the standard deviation

(2) The poor countries are more volatile, on average

(3) Some countries switch properties at some point

The country with the highest productivity process is the U.S. with TFP around 3.75. Senegal has nearly the lowest productivity process which is around 1.75. The range is therefore about 2.0, while the mean of the time series standard deviation is around 0.12. This means that the range of the processes is more than 17 times the standard deviation. The first graph in Figure 4.2 presents productivity processes for some rich and poor countries. It shows features (2), poor countries are

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[28] Countries with real GDP per capita less than $1050 are not included.

[29] See section 2.2.2 for names of the defaulting countries.

[30] The average world real GDP per capita growth rate is 2%.

more volatile. The productivity processes of El Salvador, Iran, Jamaica, and Uruguay are also shown in the second graph of Figure 4.2. The persistence, volatility or mean of those countries' processes changed over time.

4.1.2 Estimation of World Productivity Process

The above features of the TFP processes motivate our choice of regime-switching process. There are three regimes.[31] Conditional on being in regime $\Re_t^i = 1,2,3$, we assume that TFP shocks follow an autoregressive process given by

$$a_t^i = \mu_{\Re_t^i}(1-\rho_{\Re_t^i}) + \rho_{\Re_t^i} a_{t-1}^i + \sigma_{\Re_t^i} \varepsilon_t^i, \tag{4.23}$$

where, a_t^i denotes the idiosyncratic TFP shock of country i at period t; ε_t^i is i.i.d. and drawn from a standard normal distribution $N(0,1)$; and $\Re_t^i$ denotes the regime that country i is in at period t. The probability of switching from one regime to another is given by the transition matrix Π.

We use maximum likelihood method to estimate the unknown parameters: $\Theta = \{\{\mu_{\Re}, \rho_{\Re}, \sigma_{\Re}\}_{\Re=1,2,3}, \Pi\}$. The method we use is an extension of that in Hamilton (1991) for one time series to panel data series. The Expectation-Maximization (EM)[32] principle is used to compute the solutions. The detailed algorithm is described in the Technical Appendix A.4.

The parameter values are shown in Table 4.1; the numbers in parentheses are the standard errors. We order the processes according to their volatility and label them as low, middle and high regime. The high regime is most volatile and has the lowest mean. This coincides with the well-known fact that emerging countries are very volatile and poor. The low regime is least volatile. The rich countries are at the top of Figure 4.1 and less volatile, they therefore have large probability in low regime.

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[31] The justification for the three-regime specification is as follows. The three-regime specification significantly improves the goodness of fit from the two-regime, while the four-regime specification barely improves the goodness of fit from the three-regime specification,

[32] Dempster, Laird, and Rubin (1977)

Table 4.1 Estimated Parameters for Regime-Switching Process

		Low regime	Middle regime	High regime
μ		4.02 (.0010)	6.20 (.0143)	1.48 (.0065)
ρ		0.997 (.0012)	0.992 (.0018)	0.988 (.0066)
σ		0.017 (.0001)	0.025 (.0002)	0.061 (.0001)
P	L	0.95 (0.06)	0.001 (0.01)	0.049 (0.01)
	M	0.06 (0.03)	0.88 (0.04)	0.06 (0.02)
	H	0.08 (0.02)	0.07 (0.03)	0.85 (0.06)

4.2 Parameter Calibration

To compute the model, we still need to calibrate the other parameters. Risk aversion σ in the utility function is chosen to be 2.0, the standard value in the literature[33]. Capital share $\alpha = 0.33$ is calibrated to match the average U.S. labor compensation share for the period 1973-98[34]. We follow Stokey and Rebelo (1990) to calibrate a capital depreciation rate δ from U.S. average capital consumption allowance and capital-GDP ratio. Average capital consumption allowance to GDP is 0.107 in the period 1973-98. The average capital-GDP ratio is around 1.73 (Christiano (1988). δ is hence calibrated to be 0.06.

The probability of reentry to market after default λ is calibrated to be 0.20. From Gelos and Sahay (2003), countries are denied access to markets for 4.5 years on average from 1980 to 1998 after default. Access is defined as public or publicly guaranteed bond issuances or public or publicly guaranteed borrowing through a private syndicated bank loan. The discount factor β is

[33] Degree of risk aversion is an important parameter. Different values of risk aversion are experimented in section 2.5 sensitivity analysis.

[34] Gollin (2002) recalibrates α for some industrial countries as well as some developing countries. He found that by adding back the unrecorded self-employed labor, developing countries also have a capital share around 0.33.

calibrated to be 0.90 to match the first sub-period world interest rate as 4%, which is the average capital return in U.S as from McGrattan and Prescott (2003).

We follow Neumeyer and Perri (2001) in using $\tau(b') = \tau b'^2$ as the quadratic portfolio transaction cost. We first pick the transaction cost and output drop (τ, γ) to match both the net foreign capital stock over GDP ratio 9% and the fraction in default 5% as in the first period. And then we rerun the model by fixing all the parameters except τ, which is chosen to match the second period's net capital stock over GDP ratio.

Table 4.2 summarizes the calibrated parameters.

Table 4.2 Parameter Values

Risk aversion	$\sigma = 2.0$
Discount factor	$\beta = 0.90$
Capital share	$\alpha = 0.33$
Depreciation	$\delta = 0.06$
Re-entry probability	$\lambda = 0.20$
Output drop	$\gamma = 0.03$

4.3 Computation Algorithm

In order to solve the model numerically, we follow Tauchen (1991) and discretize the continuous regime-switching process into 9 points with 3 points in each regime. The computation algorithm is described below.

(1) First, guess initial world interest rate R and loan price function $q^n(s,b')$, which is initialized as the reciprocal of the world interest rate. Here $s = (a,k,b)$.

(2) Given bond price function, value functions and decision rules can be solved through value function iteration, $V^N(s)$, $V^P(a,k)$ and $\{c(s,h), k'(s,h), b'(s,h), d(s)\}$.

(3) Given the decision rules and value function $V^N(s)$, we can compute the decision rule of $k'(s,b')$, the optimal capital stock for a country s with intention of borrowing b', according to (3.21).

(4) The default probability $p^n(s,b')$ of a country in Normal Phase with state s can be updated using default decisions $d(s)$ and $k'(s,b')$ by equation (3.20). The bond price schedule is then computed by $q^{n+1}(s,b') = (1-p^n(s,b'))/R$.

(5) Iterate (2)-(4) until q converges, $\| q^{n+1}(s,b') - q^n(s,b') \| < \varepsilon$.

(6) Calculate the invariant distribution μ according to (3.22). The transition matrix of states can be obtained through the following calculation, given the decision rules. Let I denotes indicator function. The transition matrix of the state defined in Section 3.3 $Q(x,M)$, conditional probability for a country with state $x \in X$ having next period's state falling into set M, is updated as:

$$Q(s,h=0,s',h'=1) = d(s)\pi(a' \mid a) I_{k'(s,h)=k'}$$

$$Q(s,h=0,s',h=0) = (1-d(s))\pi(a' \mid a) I_{k'(s,h)=k' \,\&\, b'(s,h)=b'}$$

$$Q(s,h=1,s',h=0) = \lambda\, \pi(a' \mid a) I_{k'(s,h)=k'}, \text{ for } b=0 \textit{ and } b'=0$$

$$Q(s,h=1,s',h=0) = (1-\lambda)\, \pi(a' \mid a) I_{k'(s,h)=k'}, \text{ for } b=0 \textit{ and } b'=0$$

(7) Check if excess demand is close to zero: $\left| \sum_s \mu(s,0) b'(s,0) \right| < \varepsilon$. If it is close to zero, we are done; otherwise, guess a new R and return to step (1).

4.4 Simulation Results

In this section, we report the simulation results. The borrowing costs τ are chosen to be 0.12 and 0.01 to match the average net foreign capital stock-to-GDP ratio ($|B|/GDP$) in 1973-85 and 1986-98, respectively. We run 1,000 simulations, with 13 periods and 40 countries for each run. The simulation is started, for each run, from the invariant stationary distribution. The main findings are shown in Table 4.3. As τ decreases from 0.12 to 0.01, the average $|B|/GDP$ increases from 0.09 to 0.17, and the fraction of countries in default increases from 0.04 to 0.09, endogenously. But there is no substantial improvement in international risk sharing. The change in the cross-section regression coefficient measuring risk sharing among countries, a drop from 0.66 to 0.64, is statistically insignificant. Though the degree of international risk sharing generated in the model is better than that in the data, the hypothesis of perfect—or even substantially improved—risk sharing, can be rejected.

The increase in financial integration is a natural result of the decreased cost of borrowing. With only one state uncontingent bond, a country tends to default more when it borrows more. The rate of sovereign default thus increases. However, the smoothness of consumption is not sensitive to increases in financial integration under the current magnitude since countries can smooth consumption by changing domestic investment. In particular, when there is a bad shock, countries can turn their investment into consumption instead of borrowing. Similarly, when there is a good shock, countries can save into domestic investment instead of saving in international credit market. To get a substantial improvement in risk sharing, the demand for foreign capital stock must be quite large—higher than the levels currently seen. When domestic capital stock is sufficient for consumption smoothing, there will not be substantial improvements in risk sharing.

Table 4.3 Simulation Results

	Data		Model	
	1973 –85	1986 - 98	$\tau = 0.12$	$\tau = 0.01$
|B|/GDP	0.09	0.17	0.09	0.17
Fraction in default	0.05	0.11	0.04	0.09
Cross-section reg.	0.78	0.76	0.66	0.65

In addition, though it provides state contingency, default also hinders risk sharing in that it comes with high risk premiums and exclusion from financial markets. And yet, a country most needs risk sharing when it hits the bad shocks that are likely to cause default. In this situation, there are two possibilities. First, it can choose to default. That helps lower payments in the current period. But from then on, it will be excluded from financial markets, and can only use domestic assets to smooth consumption. Second, it can repay and then borrow more. But when a country is in a bad state, financial intermediaries charge a high risk premium and may actually refuse to lend more than a certain amount since the probability of default for countries in bad states is quite high. Moreover, since shocks are serially correlated, there is a high probability that next period it will be in the same situation as in the current period. In short, a country in a bad state has two options: It can borrow only a small amount at a high interest rate, or it can default and be excluded from financial markets in the future. Either way, countries in bad states cannot smooth consumption well.

In the following, we show in detail that smoothness of consumption is not sensitive to increases in financial integration under the current magnitude and how default hurts risk sharing.

4.4.1 Insensitivity of Risk Sharing

This subsection will show that risk sharing is not sensitive to increases in foreign capital flow under magnitudes seen in the data. Since default comes with the endogenous borrowing limits, even if when the portfolio transaction cost is set as 0, the capital flows among countries are too low to generate substantial improvement in risk sharing. Table 4.4 shows that under default, even at $\tau = 0$, $|B|/GDP = 0.22$ and cross-section regression coefficient is 0.64. To separate the effect of default on risk sharing, we rerun the model and set the default penalty large enough as $\lambda = 0.001$ and $\gamma = 0.60$ so that there is essentially no default in equilibrium. Table 4.4 shows that under no default, the risk sharing improves from 0.58 to 0.36 substantially as the transaction cost decreases from 0.30 to 0. At the same time, the financial integration, measured as average net foreign capital stock over GDP ratio, increases dramatically from 0.09 to 5.2. However if the borrowing costs are chosen to match the data on foreign capital stock-to-GDP ratios, the degree of risk sharing is not sensitive to moderate increases in financial integration, changing from 0.58 to 0.56. Therefore to get a substantial improvement in international risk sharing, the financial integration needs to increase to at least 30 times of current level.

Table 4.4 Default vs. Non-Default

	With Default			Without Default				
τ	0.12	0.01	0	0.30	0.18	0.12	0.01	0
|B|/GDP	0.09	0.17	0.22	0.09	0.17	0.39	4.39	5.20
Fraction in default	0.04	0.09	0.10	0	0	0	0	0
Risk Sharing	0.66	0.65	0.64	0.58	0.56	0.52	0.42	0.36

Table 4.4 also shows that even under zero portfolio transaction cost, there is no perfect risk sharing with or without default happened in equilibrium. One interesting question is whether this model can generate perfect risk sharing. To answer this question, we experiment the model with different discount factors. Levine and Zame (2001) showed in theory that even with one state uncontingent debt, if agents are patient enough, the equilibrium of an incomplete market under no aggregate uncertainty is close to that of a complete market. Two limitations of their paper are no

production in the economy and no numerical estimates[35]. The following sensitivity analysis shows quantitatively that even with a production economy, when the discount factor is close to 1, the incomplete market results are close to those of complete markets. Table 4.5 gives the foreign capital stock-to-GDP ratios and risk-sharing coefficients under different discount factors. As the discount factor becomes close to 1, risk sharing improves. When the discount factor arrives 0.999, the risk sharing is about 0.005 that is close to perfect risk sharing. However to get close to full risk sharing, financial integration needs to be wildly large, with foreign capital stock more than 20 times GDP.

Table 4.5 Sensitivity Analysis – Discount Factor

β	|B|/GDP	Cross-section Reg.
0.90	3.39	0.51
0.95	5.87	0.35
0.99	7.37	0.20
0.995	13.13	0.12
0.998	17.37	0.07
0.999	20.25	0.005

Note the above results shown in Table 4.5 are under high default penalties $\lambda = 0.001$ and $\gamma = 0.60$ so that there is essentially no default in equilibrium. Even under low default penalty, the conclusion of near perfect risk sharing as discount factor close to 1 will still hold, since as $\beta \to 1$, countries extremely value future and so will not choose to default. Therefore as $\beta \to 1$, the results are independent of default penalty parameters.

The above tests show that smoothness of consumption, and international risk sharing, are not sensitive to moderate increases in financial integration when capital flow is at the magnitudes currently seen. In the following subsection, we will show how default hinders risk sharing.

4.4.2 Default and Risk Sharing

Financial intermediaries incorporate a default premium into the contracts they sign with countries. The country-specific bond price is therefore the product of the reciprocal of risk-free rate and the default probability. To better understand the properties of bond prices, Figure 4.3

[35] See Levine and Zame (2001)

plots the default zones under $\tau = 0$. Each line is the default cutoff for a given productivity shock realization with different capital stock. Given each (a,k), the area above each line is the set of debts for which default is the optimal choice. The larger the area above the default cutoff line, the larger the default incentive. The x-axis is capital and y-axis is debt ($-b$).

The default zone has the following properties:

(1) The high regime has larger default incentive under low capital stock
(2) The low regime has larger default incentive under high capital stock
(3) As capital stock increases, default incentive decreases

Given low capital stock, the default incentive depends on countries' income since involuntary default tends to occur to low-income country. Therefore, under low capital stock, the high regime countries have lowest mean and so has the largest incentive to default. However, as capital stock increases to some larger level, the effect of regime volatility dominates the effect of regime level. The reason is because as capital becomes larger, involuntary default is not an issue any more, but the value of staying in the market becomes more important. As the low regime is least volatile, the benefit of staying in the market and doing risk sharing is smallest. Countries in the low regime therefore have largest incentive to default. Given same level of shock, as capital increases, countries become richer and therefore repaying debts is less painful for them, the default incentives therefore become smaller.

Given default choices, the bond price schedule can be computed and is shown in Figure 4.4. The bond price is increasing in (k,b) but not monotone in increases to productivity shocks. Countries with bad shocks and low capital stock have a larger incentive to default and so face a higher borrowing rate. In fact, there is an endogenous borrowing limit for each country. Moreover, the endogenous borrowing limit is especially tight when countries are in bad states though they have a larger need to borrow to smooth consumption.

To further show that the cost of default is larger than the benefit of default in terms of risk sharing, Table 4.4 compares results under default to those under no default. It is shown that given the same borrowing cost, the equilibrium with default can generate a degree of risk sharing and a default rate close to the data. In particular, if the borrowing costs are chosen to match the data on foreign capital stock-to-GDP ratios, risk sharing under default is worse. The reason is because the bond price is shock specific with default happened in equilibrium, comparing to fixed bond price under no default. Moreover the bond price is especially expensive when countries face a bad shock and need risk sharing most. Default therefore hinders international risk sharing.

4.4.3 Other Predictions

One striking fact in the data is that OECD countries seldom default after the 1970s. Defaults are seen only in developing countries, and especially among emerging countries. This regularity also occurs in our model. In the data, OECD countries' productivity processes are least volatile. They therefore have high probability in the low regime. The productivity processes of emerging countries are most volatile and have lowest realizations. Hence there is quite high probability for emerging countries staying in the high regime. In the following, it is shown that most defaults happen in the high regime in our model.

The conditional probabilities of regime switches given the occurrence of default in our simulation are given in Table 4.6. Five percent of defaults happen when countries stay at high regime, 62% occur when countries jump from low regime to high regime, and 25% when they jump from middle regime to high regime. In sum, 92% of default happens to countries in or shifting to the high regime (emerging countries). Only 0.2% of defaults occur when countries stay at the low regime. And countries in the middle regime do not default. Also 92% of this simulated default occurs in the countries with income below 50% of world income. In our model, therefore, as in the data, almost all the defaults happen in the high regime countries or emerging countries, and the low regime countries or rich countries seldom default. The reason is because emerging countries usually have low shock realization and low capital stock. From Figure 4.3, the high regime countries with low capital stock have the largest incentive to default.

Debt thresholds have been extensively discussed in the literature. Kehoe et al. (2000) discussed the amount of debt that is sustainable for Mexico not to default. Reinhart et al. (2003) found that the average external debt-to-GNP ratio in the initial year of default is 70.6% on average. This number is somewhat upwardly biased since a debt crisis is usually accompanied by depreciation in local currency. Some sovereign default occurs with a low debt-to-GNP ratio. For example, the ratio of debt-to-GNP was only 47% in Mexico's 1982 debt crisis. Also, they show that non-default countries have lower ratios of debt-to-GNP than default countries. The mean ratio of debt-to-GNP for non-default countries is around 34% while that of default countries is 41%. Therefore, credit is more risky as the debt-to-GNP ratio exceeds 35%.

Table 4.6 Probability of regime switching when there is a default
(Default occurs at period t)

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Period t</td></tr>
<tr><td>Low regime</td><td>Middle regime</td><td>High regime</td></tr>
<tr><td rowspan="3">$t-1$</td><td>Low regime</td><td>0.002
(0.007)</td><td>0.00
(0.00)</td><td>0.62
(0.08)</td></tr>
<tr><td>Middle regime</td><td>0.078
(0.05)</td><td>0.00
(0.00)</td><td>0.25
(0.08)</td></tr>
<tr><td>High regime</td><td>0.00
(0.00)</td><td>0.00
(0.00)</td><td>0.05
(0.04)</td></tr>
</table>

In our model, the mean debt-to-GDP ratio for defaulters is 35.3%, and 13% for non-defaulters. Our debt-to-GDP ratio is smaller than the results from Reinhart et al. Still, Figure 4.5 shows that defaulters have larger debt-to-GDP ratios than non-defaulters. And when borrowing costs are lower, the average debt-to-GDP ratio is higher.

5. Sensitivity Analysis

In this section, first we experiment different values of parameters, output drop γ, reentry probability λ and discount factor β, which will affect our main results. Second, we examine factors that generate high fraction in default in our model: regime-switching process and a production economy model framework. It is shown that model with capital helps in generating high default rate since under production economy countries have more incentive to default with extra ways to do risk sharing.

5.1 Parameter Change

Table 5.1 shows that as output drop after default increases from 0.00 to 0.05, the fraction in default increases from 3% to 11%, debt-over-GDP ratio increases from 12% to 48% and there is no significant change in the cross-section regression coefficient in that the cross-section regression coefficients change from 0.65 to 0.59. The intuition is that as the output drop after default increases, countries have a smaller incentive to default. The financial intermediaries internalize this reasoning and charge a lower interest rate on loans. The low interest rate stimulates borrowing and generates high foreign capital stock over GDP ratio, in equilibrium. However, the fraction in default also grows as borrowing increases. Countries that borrow more also tend to default more. Little change is observed in the cross-section regression coefficient since the financial market is still incomplete.

Similarly borrowing limits become looser as the probability of reentry decreases from 0.20 to 0.10, with higher default penalty. Using similar logic, the fraction in default increases with the higher default penalty. Also there is little change in international risk sharing.

Changing the discount factor has little effect on the foreign capital stock-over-GDP ratio and the fraction in default. But we still can see that as the discount factor decreases from 0.90 to 0.88 (as countries become more impatient), the fraction in default increases. But debt-to-GDP also decreases since as β decreases, countries tend to borrow more and save less. See Table 5.1 below.

Table 5.1 Sensitivity Analysis

	$\lvert B \rvert / GDP$	Fraction in default	Cross-section reg.
Output drop γ			
0	0.12	0.03	0.65
0.03	0.22	0.10	0.64
0.05	0.48	0.11	0.59
Reenter prob. λ			
0.10	0.34	0.17	0.62
0.20	0.22	0.10	0.64
0.30	0.19	0.08	0.64
Discount fact β			
0.88	0.22	0.11	0.67
0.90	0.22	0.10	0.64

5.2 High Fraction in Default: Productivity Process or Production Economy?

One problem in models with a pure exchange economy is that they find it difficult to generate default rates close to the data when using calibrated parameters. In the literature, the default rate is defined as the ratio of number of newly defaulting countries over the total number of sample countries. Reinhart (2003) estimated the average default rate to be 6% annually[36]. Arellano (2003) found that under a pure exchange economy with one good, the default rate is zero. Aguiar et al. (2004) found that the default rate is about 0.04% yearly with a stationary endowment process under a pure exchange economy. This chapter generates a higher default rate , closer to the rate seen in the literature.

The high default rate generated in our model is the result of two factors: productivity process and production economy. To test the importance of these factors, two experiments are undertaken. In each experiment, we fix one factor and test the effect of the other. The parameter values remain the same as in the previous section except when stated otherwise. More concretely, to test the impact of production economy on the default rate, the productivity process is fixed and treated as an endowment process. The setup of the model with pure exchange is the similar to that in Aguiar et al. The simulation results are reported in the first row of Table 5.2.

Table 5.2 Pure exchange economy Simulation Results

β	B/GDP	Fraction in default	New default rate	Cross-section Reg.
0.90	0.51	0.000	0.000	0.88
0.80	0.23	0.034	0.005	0.95

Both the fraction in default and the default rate are zero. The model framework of production economy does help us in generating a high default rate. The idea behind this is that saving is not permitted in the literature with a pure exchange economy. On the other hand, with capital, countries are able to insure against bad shocks without having to borrow from international markets. The utility of financial autarky is hence higher than it is in a pure exchange economy. Countries have a larger incentive to default, which results in a higher default rate and therefore a higher fraction in default in equilibrium in our model.

[36] There were 113 defaults over the period 1970-1999 for 58 countries.

The cross-section regression coefficient under the pure exchange economy is also higher than our model prediction. This is intuitive as a larger fraction of countries in default implies a larger fraction of countries in financial autarky, which causes worse risk sharing than that seen under a production economy.

The calibrated world productivity process is unique in this chapter. It is different from a simple AR(1) process in that it has wide range, and countries may switch properties at some period of time. Countries have a large incentive to default when they jump from High regime to Low regime with a large amount of debt. Hence, the productivity process under AR(1) with regime change should generate a higher default rate. The second experiment tests this argument by computing the model under a pure exchange economy but with the discount factor decreased to $\beta = 0.80$. The endowment process is the same as our productivity process. The simulation result has 3% fraction in default, and a 0.5% newly default rate. See the second row in Table 5.2. As shown before, Arellano, and Aguiar et al. obtained default rates far below this result, using a simple AR(1) process. This experiment shows that the productivity process of regime-switching process also helps in generating a high default rate. The international risk sharing under $\beta = 0.80$ in a pure exchange economy is worse than that under a higher discount factor of 0.90 since the fraction in default increases.

6. Conclusion

This chapter constructs a general equilibrium model with a continuum of small open economies and default choices to account for a puzzling fact observed in the data: an increase in financial integration without a substantial improvement in international risk sharing. In the model, only state uncontingent bonds are available to countries. Also international contracts have limited enforcement in that countries can default on their state uncontingent debt if it is optimal to do so.

This chapter makes three main contributions. First, the model developed here explains why risk sharing has not improved despite increased financial integration. As borrowing costs are lowered, international financial integration increases and the fraction of countries in default also increases endogenously, which hinders international risk sharing. This emphasizes the importance of incompleteness in financial markets. Second, this chapter extends current models on international lending by adding production into the model economy. In contrast to models with pure exchange economies, under a production economy the determination of the country-specific bond price is subtle because financial intermediaries have to think through each country's optimal problem and guess the optimal capital stock for a specific country with any loan request even the loan request is not optimal for the country. In addition, the equilibrium default rate is closer to the data than that generated in the literature. In the production economy, countries can self insure from bad shock through accumulating capital stock. The utility of financial autarky is therefore higher in a production economy than in pure exchange economy. Countries have a larger incentive to default. Finally, we estimate the world productivity process with AR(1) with regime change which also helps us in generating the high fraction in default seen in the data. Most importantly, this can be useful in the future studies.

In real life, defaulting countries seldom go to complete financial autarky. There is always negotiation between lenders and defaulters. And defaulters may choose not return to the financial markets if they have continuous bad shocks and there is a request for repayment of previously defaulted debts. Future work will extend the model to incorporate these more realistic features.

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World Economic Outlook, September 2002, October 2001

Appendix

A.1 Measure of Financial Integration

The degree of financial integration at each period is measured as follows:

$$indicator_t = \frac{\sum_{i=1}^{N} \left| FDIA_{it} - FDIL_{it} + EQA_{it} - EQL_{it} + DEBTA_{it} - DEBTL_{it} \right|}{\sum_{i=1}^{N} \left| GDP_{i,t} \right|}$$

N: number of countries

FDIA (FDIL): stock of foreign direct investment asset (liabilities)

EQA (EQL): stock of portfolio equity asset (liabilities)

DEBTA (DEBTL): stock of portfolio debt asset (liabilities)

All the foreign capital stock data are from Lane and Milesi-Ferretti (2001). The followings are the variable names we select from their data set. All variables are in US dollar.

This paper	Notation from Lane	Explanation from Lane
FDIA	CFDIAH	Estimate stock of direct investment assets
FDIL	CFDILH	Estimate stock of direct investment liabilities
EQA	CEQAR	Estimate of stock of portfolio equity assets
EQL	CEQLR	Estimate of stock of portfolio equity liabilities
DEBTA (industrial country)	IPPDA	Stock of portfolio debt assets (Source: BOPS and IFS, International Investment Position)
DEBTL (industrial country)	IPPDL	Stock of portfolio debt liabilities (Source: BOPS and IFS, International Investment Position)
DEBTA (developing country)	CUMLOAN	Sum of cumulative flows of portfolio debt assets, other assets and net errors and omissions
DEBTL (developing country)	DEBTWB	Gross external debt (developing countries) (Source: World Bank, Global Development Finance)
GDP	DGDP	Nominal GDP in current US dollar

A.2 Calibration of Total Factor Productivities

Our data comes from IFS, World Development Indicator (WDI) and Penn World 6.1. Countries with available employment data are included in the sample. In particular, there are 49 countries in total. Sample period is from 1960 to 2000. 8 countries with 1995 real GDP per capita less than \$1050 are dropped from sample: Lesotho, Rwanda, Malawi, Burundi, Malta, Niger, Togo, and Zambia.

Productivity for each country is computed as $\log A_t = \log Y_t - \alpha \log K_t - (1-\alpha)\log L_t$, where Y_t *is* real GDP from WDI.

Capital stock K is constructed from investment data from WDI. If the initial year's capital stock k_0 is available, each year's capital stock can be backed up through $k_{t+1} = (1-\delta)k_t + I_t$ accumulatively (I_t: investment; δ depreciation rate, equal to 0.06). The issue here is how to estimate the capital stock in initial year. Chari, Kehoe and McGrattan (1997) argue that different ways of estimating initial capital stock generate similar result of final capital stock. Here we follow Chari et al. and adopt one of the estimations as follows. Assume that countries are in balanced growth path since 1960s, then $E(k/y) = \dfrac{E(I/Y)}{g_y + \delta}$ with g_y denoting output growth rate. The initial capital stock k_0 can therefore be estimated as the multiplication of capital-output ratio and initial year's output.

Employment data is hard to get. From OECD database we can get employment data for 24 OECD countries. We also get employment data for 13 countries from National Statistics. The employment data for the other 12 countries are supplemented by Penn World 6.1. Penn World 6.1 reports population, real GDP per capita and real GDP per worker. We can therefore calculate the employment through

$$real\ GDP\ per\ capita * population / real\ GDP\ per\ worker$$

However, since Penn World 6.1 interpolates many poor countries' employment data, this is not a good measurement of employment.

Countries included

37 countries with good labor statistics:

Australia, Austria, Belgium, Cameroon, Canada, Costa Rica, Columbia, Cyprus, El Salvador, Denmark, Fiji, Finland, France, Germany, Greece, Iceland, Israel, India, Ireland, Italy, Jamaica, Japan, Korea, Malaysia, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, Thailand, Trinidad and Tobago, UK, USA, Uruguay

12 countries with labor statistics constructed from Penn World 6.1:

Cote d'Ivoire, Guatemala, Honduras, Iran, Morocco, Panama, Papua New Guinea, Paraguay, Senegal, Sri Lanka, Tunisia, Zimbabwe

A.3 Characteristics of Equilibrium

Proof of Proposition 1: From the definition of V^N, we know that

$$V^N(a,k,b_1) = \max\{W^R(a,k,b_1), W^D(a,k)\}$$

Also since the welfare of repaying W^R is increasing in *b*, we have

$$W^R(a,k,b_1) \le W^R(a,k,b_2)$$

This implies $V^N(a,k,b_1) \le \max\{W^R(a,k,b_2), W^D(a,k)\} \le W^D(a,k)$

Since $V^N(a,k,b_1) \ge W^D(a,k)$ by definition, we have $V^N(a,k,b_1) = W^D(a,k)$ Q.E.D.

Proof of Proposition 2: It is obvious a country will not default on its positive asset holdings. Given a country with state $s=(a,k,b)$, define $\tilde{k}$ as the solution to $\gamma a\tilde{k}^\alpha = -b$. If $k > \tilde{k}$, then $-\gamma ak^\alpha < b$. This implies that as $k > \tilde{k}$, the budget set of non-default is larger than the budget set of default. The optimal allocation under default $(c^d, k^{d\prime}, 0)$ is feasible under a non-default budget constraint. If the optimal choice of non-default is $(c^r, k^{r\prime}, b^{r\prime})$, we must have

$$u(c^r) + \beta\sum \pi(a' \mid a)V^N(a', k^{r\prime}, b^{r\prime}) \ge u(c^d) + \beta\sum \pi(a' \mid a)V^N(a', k^{d\prime}, 0)$$

$$\ge u(c^d) + \beta\sum \pi(a' \mid a)V^P(a', k^{d\prime})$$

i.e. $W^R(a,k,b) \ge W^D(a,k)$

Therefore for any $k \geq \tilde{k}$, the agent (a,k,b) will choose not to default.

Q.E.D.

A.4 Estimation of the World Productivity Process

We use the maximum likelihood method to estimate the following parameters in the regime-switching process specified in **(4.23)**. The unknown parameters are

$$\Theta = \{\{\mu_{\Re}, \rho_{\Re}, \sigma_{\Re}\}_{\Re=1,2,3}, \Pi\}.$$

Define the following variables to simplify notations: $\Psi = (\Psi^i)_{i=1}^N$, $\Psi^i = \{a_T^i, a_{T-1}^i, ..., a_1^i,\}$, $\Psi_t^i = (a_t^i, a_{t-1}^i, ..., a_1^i)$, $\Re = (\Re^i)_{i=1}^N$ and $\Re^i = (\Re_T^i, \Re_{T-1}^i, ..., \Re_1^i)$ with $\Re$ denoting regime, N number of countries, T number of periods.

The Maximum Log-likelihood function is given by

$$\max_{\Theta} f(\Psi;\Theta) = \sum_{i=1}^{N} \log(f(\Psi^i;\Theta))$$

where, $f(\Psi^i;\Theta) = \sum_{\Re^i} f(\Psi^i, \Re^i;\Theta)$,

$$f(\Psi^i, \Re^i;\Theta) = f(a_T^i \mid \Re_T^i, a_{T-1}^i;\Theta) .... f(a_2^i \mid \Re_2^i, a_1^i;\Theta) p(\Re_T^i \mid \Re_{T-1}^i) ... p(\Re_2^i \mid \Re_1^i) p(\Re_1^i)$$

Due to nonlinearity of the problem, we cannot solve the parameters analytically. We use the EM algorithm to iteratively solve the maximum likelihood estimates. The following theorem shows the validity of the EM method.

Theorem: If $\frac{\partial Q(\Theta_{m+1};\tilde{\Theta}_m, \Psi)}{\partial \Theta_{m+1}} |_{\Theta_{m+1}=\tilde{\Theta}_m} = 0$, then $\frac{\partial f(\Psi;\Theta)}{\partial \Theta} |_{\Theta=\tilde{\Theta}_m} = 0$, where,

$$Q(\Theta_{m+1}, \Theta_m;\Psi) \equiv \sum_{i=1}^{N} \frac{1}{f(\Psi^i, \Theta_m)} \sum_{\Re^i} \log(f(\Psi^i, \Re^i;\Theta_{m+1})) \cdot f(\Psi^i, \Re^i;\Theta_m).$$

Proof: We have

$$\frac{\partial Q(\Theta_{m+1};\tilde{\Theta}_m, \Psi)}{\partial \Theta_{m+1}} |_{\Theta_{m+1}=\tilde{\Theta}_m} = \sum_{i=1}^{N} \frac{1}{f(\Psi^i, \tilde{\Theta}_m)} \sum_{\Re^i} \frac{\partial f(\Psi^i, \Re^i, \Theta_{m+1})}{\partial \Theta_{m+1}} |_{\Theta_{m+1}=\tilde{\Theta}_m},$$

which satisfies the first order condition of the maximum likelihood problem. Thus,

$$\frac{\partial f(\Psi,\Theta)}{\partial\Theta}|_{\tilde{\Theta}_m}=\sum_{i=1}^{N}\frac{1}{f(\Psi^i,\Theta)}\frac{\partial f(\Psi^i,\Theta)}{\partial\Theta}|_{\tilde{\Theta}_m}=\sum_{i=1}^{N}\frac{1}{f(\Psi^i,\tilde{\Theta}_m)}\sum_{\Re^i}\frac{\partial f(\Psi^i,\Re^i,\Theta)}{\partial\Theta}|_{\tilde{\Theta}_m}.$$

Q.E.D.

Parameter Estimation

Given the above theorem, the EM algorithm is described as below:

Let M denotes the number of regimes.

(1) Start with an initial guess of Θ_{n-1}

(2) Calculate the conditional probabilities for each country $i=1,...,N$:

The conditional probabilities are updated by Bayesian rule. The superscription i indicating country is omitted for simplicity.

(2.1) Calculate $f(a_t\,|\,\Psi_{t-1}),\, f(\Re_t,\Re_{t-1}\,|\,\Psi_t),\, f(\Re_t\,|\,\Psi_t)$

Starting from $p(\Re\,|\,a_1)=[\pi_1,...,\pi_M]$

Iterating over $t=2,...,T$, compute

$$f(a_t\,|\,\Psi_{t-1})=\sum_{\Re_t=1}^{M}\sum_{\Re_{t-1}=1}^{M}f(a_t\,|\,\Re_t,\Psi_{t-1})p(\Re_t\,|\,\Re_{t-1})f(\Re_{t-1}\,|\,\Psi_{t-1})$$

$$f(\Re_t,\Re_{t-1}\,|\,\Psi_t)=\frac{f(a_t\,|\,\Re_t,\Psi_{t-1})p(\Re_t\,|\,\Re_{t-1})f(\Re_{t-1}\,|\,\Psi_{t-1})}{f(a_t\,|\,\Psi_{t-1})}$$

$$f(\Re_t\,|\,\Psi_t)=\sum_{\Re_{t-1}=1}^{M}f(\Re_t,\Re_{t-1}\,|\,\Psi_t)$$

(2.2) Calculate $f(\Re_t,\Re_{t-1}\,|\,\Psi_T),\, f(\Re_t\,|\,\Psi_T)$

(2.2.1) $\forall\, t=2,...,T-1$

(i) Starting from $\tau=t+1$

$$f(\Re_{t+1},\Re_t,\Re_{t-1}\,|\,\Psi_{t+1})=\frac{p(\Re_{t+1}\,|\,\Re_t)f(a_{t+1}\,|\,\Re_{t+1},\Psi_t)f(\Re_t,\Re_{t-1}\,|\,\Psi_t)}{f(a_{t+1}\,|\,\Psi_t)}$$

(ii) For $\tau=t+2,...,T$

$$f(\Re_\tau,\Re_t,\Re_{t-1}\,|\,\Psi_\tau)=\frac{\sum_{\Re_{\tau-1}=1}^{M}p(\Re_\tau\,|\,\Re_{\tau-1})f(a_\tau\,|\,\Re_\tau,\Psi_{\tau-1})f(\Re_{\tau-1},\Re_t,\Re_{t-1}\,|\,\Psi_{\tau-1})}{f(a_\tau\,|\,\Psi_{\tau-1})}$$

$$f(\Re_\tau, \Re_t \mid \Psi_\tau) = \frac{\sum_{\Re_{\tau-1}=1}^{M} p(\Re_\tau \mid \Re_{\tau-1}) f(a_\tau \mid \Re_\tau, \Psi_{\tau-1}) f(\Re_{\tau-1}, \Re_t \mid \Psi_{\tau-1})}{f(a_\tau \mid \Psi_{\tau-1})}$$

(iii) Summing over

$$f(\Re_t, \Re_{t-1} \mid \Psi_T) = \sum_{\Re_T=1}^{M} f(\Re_T, \Re_t, \Re_{t-1} \mid \Psi_T)$$

$$f(\Re_t \mid \Psi_T) = \sum_{\Re_T=1}^{M} f(\Re_T, \Re_t \mid \Psi_T)$$

(2.2.2) For $t = T$

$$f(\Re_T, \Re_{T-1} \mid \Psi_T) = f(\Re_T, \Re_{T-1} \mid \Psi_T)$$

$$f(\Re_T \mid \Psi_T) = f(\Re_T \mid \Psi_T)$$

(3) Given the conditional probability, compute Θ_n as following:

$$\mu_m = \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} (a_t^i - \rho_m a_{t-1}^i) f(\Re_t^i = m \mid \Psi^i)}{(1-\rho_m) \sum_{i=1}^{N} \sum_{t=2}^{T} f(\Re_t^i = m \mid \Psi^i)}, \ m = 1, 2, ..., M$$

$$\rho_m = \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} (a_t^i - \mu_m)(a_{t-1}^i - \mu_m) f(\Re_t^i = m \mid \Psi^i)}{\sum_{i=1}^{N} \sum_{t=2}^{T} (a_{t-1}^i - \mu_m)^2 f(\Re_t^i = m \mid \Psi^i)}, m = 1, 2, ..., M$$

$$\sigma_m^2 = \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} f(\Re_t^i = m \mid \Psi^i)(a_t^i - (1-\rho_m)\mu_m - \rho_m a_{t-1}^i)^2}{\sum_{i=1}^{N} \sum_{t=2}^{T} f(\Re_t^i = m \mid \Psi^i)}, m = 1, 2, ..., M$$

$$p_{j,k} = \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} f(\Re_t^i = k, \Re_{t-1}^i = j \mid \Psi^i)}{\sum_{i=1}^{N} \sum_{t=2}^{T} f(\Re_{t-1}^i = j \mid \Psi^i)} \ \text{ for any } \ j, k \in \{1, 2, ..., M\}$$

(4) Iterate (2)-(3) until Θ converges.

Standard Error Estimation

The standard errors of the estimated parameters can be obtained by calculating the score of log likelihood function[37].

[37] See Hamilton (1996) for details

A. Distribution of Estimation

Let Θ_0 be the set of true parameters, $\hat{\Theta}$ be the estimation. The score $h_t^i(\Theta_0)$ for country i at period t is defined as

$$h_t^i(\Theta_0) \equiv \frac{\partial \log f(a_t^i \mid \Psi_t; \Theta)}{\partial \Theta}\Big|_{\Theta=\Theta_0}$$

Let

$$\phi_{OP} = \frac{1}{T \times N} \sum_{i=1}^{N} \sum_{t=1}^{T} \left[h_t^i(\hat{\Theta}) \bullet h_t^i(\hat{\Theta})' \right]$$

Then the distribution of estimation is given by

$$(\hat{\Theta} - \Theta_0) \approx N(0,\ \frac{1}{T \times N} \phi_{OP}^{-1})$$

B. Score Calculation

(1) The score with respect to $\theta = (\mu_{\Re}, \rho_{\Re}, \sigma_{\Re})_{\Re=1,2,..M}$, for $t = 1,2,...,T$

$$\begin{aligned} h_t^i(\theta) &= \sum_{j=1}^{M} \psi_{t,j}^i f(\Re_t = j \mid \Psi_t) \\ &+ \sum_{\tau=1}^{t-1} \sum_{j=1}^{M} \psi_{\tau,j}^i [f(\Re_\tau = j \mid \Psi_t) - f(\Re_\tau = j \mid \Psi_{t-1})] \end{aligned}$$

Where

$$\psi_{t,j}^i = \frac{\partial \log f(a_t^i \mid a_t^i, \Re_t = j; \theta)}{\partial \theta}$$

(2) The score with respect to Π

$\Pi = (p_{mj})_{m,j \in \{1,2,..,M\}}$, where p_{mj} denotes the transition probability from regime m to regime j.

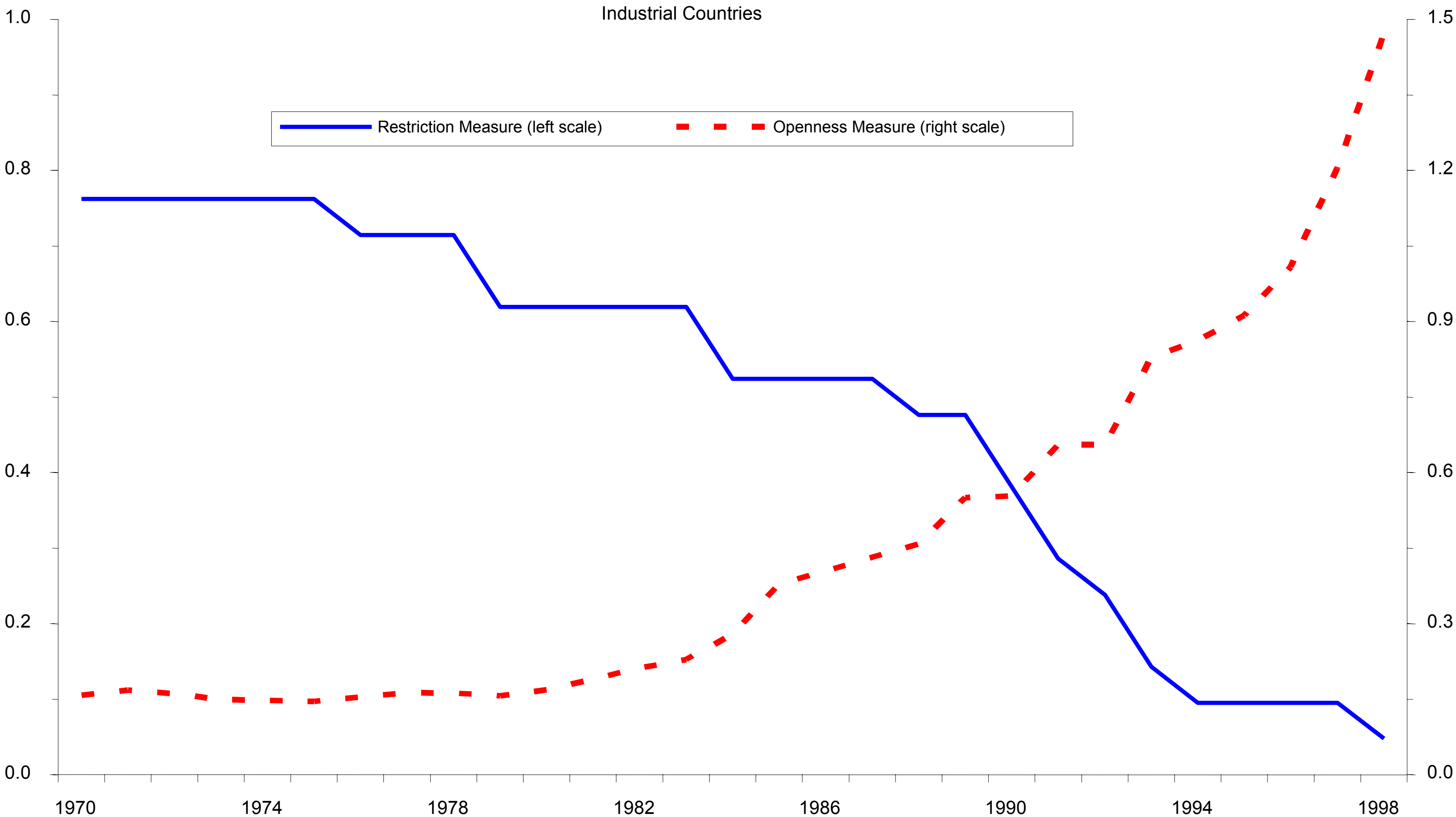
For $t = 2,...,T,\ m = 1,...,M,\ j = 1,...,M-1$

$$
\begin{aligned}
h_t^i(p_{mj}) = {} & p_{mj}^{-1} f(\Re_t = j, \Re_{t-1} = m \mid \Psi_t) - p_{mM}^{-1} f(\Re_t = M, \Re_{t-1} = m \mid \Psi_t) \\
& + p_{mj}^{-1} \left\{ \sum_{\tau=2}^{t-1} [f(\Re_\tau = j, \Re_{\tau-1} = m \mid \Psi_t) - f(\Re_\tau = j, \Re_{\tau-1} = m \mid \Psi_{t-1})] \right\} \\
& - p_{mM}^{-1} \left\{ \sum_{\tau=2}^{t-1} [f(\Re_\tau = M, \Re_{\tau-1} = m \mid \Psi_t) - f(\Re_\tau = M, \Re_{\tau-1} = m \mid \Psi_{t-1})] \right\} \\
& + \sum_{\Re_1=1}^{M} \frac{\partial \log f(\Re_1; \Pi)}{\partial p_{mj}} [f(\Re_1 \mid \Psi_t) - f(\Re_1 \mid \Psi_{t-1})]
\end{aligned}
$$

For period 1, $t = 1$, $m = 1, \ldots, M$, $j = 1, \ldots, M-1$

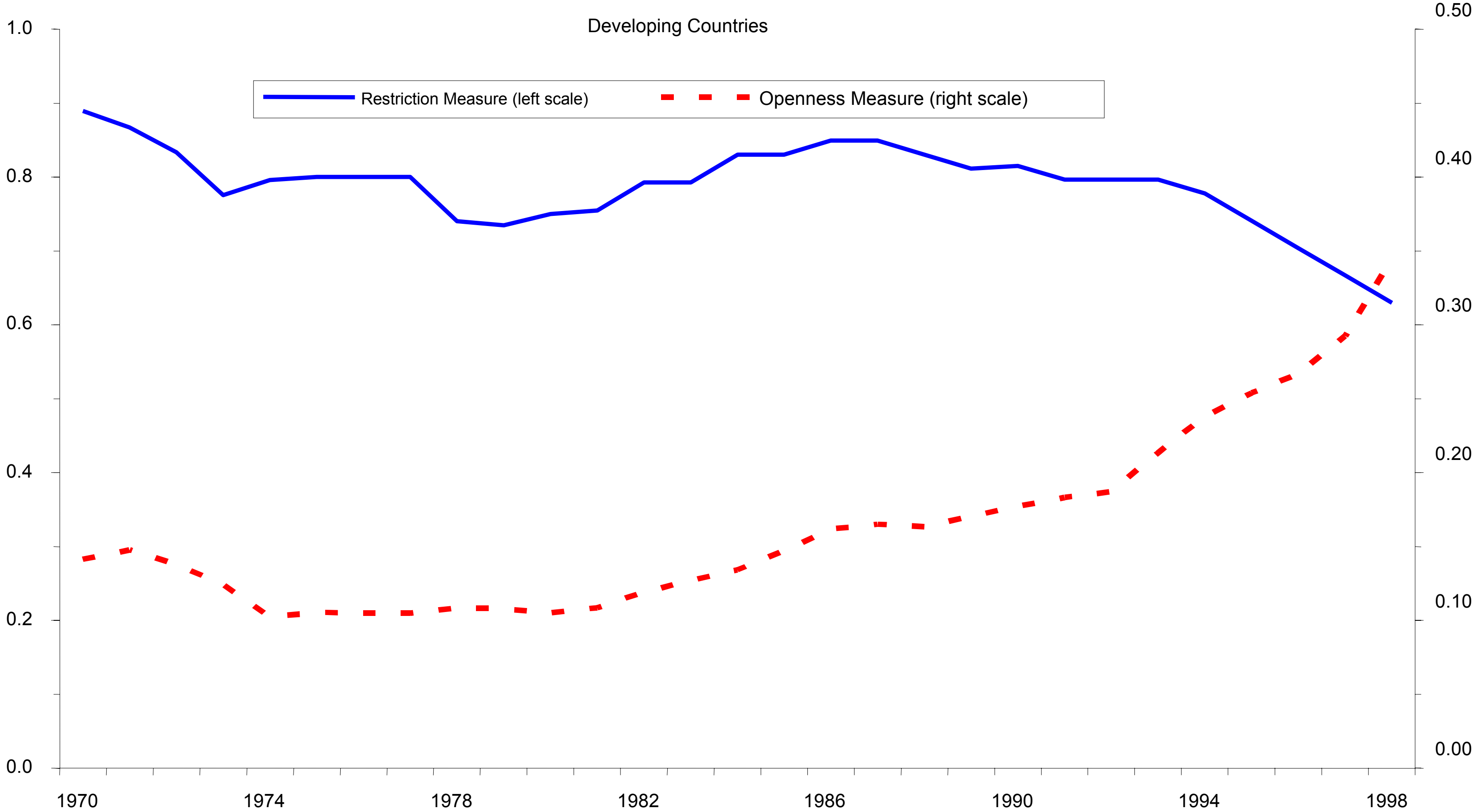
$$
h_1^i(p_{mj}) = \sum_{\Re_1=1}^{M} \frac{\partial \log f(\Re_1; \Pi)}{\partial p_{mj}} p(\Re_1 \mid \Psi_1)
$$

Figure 2.1 Measures of Financial Integration



Source: WEO, Lane and Milesi-Ferreti (2003)

Figure 2.2 Measures of Financial Integration



Source: WEO, Lane and Milesi-Ferreti (2003)

Figure 2.3 Net Foreign Capital Stock over GDP Ratio

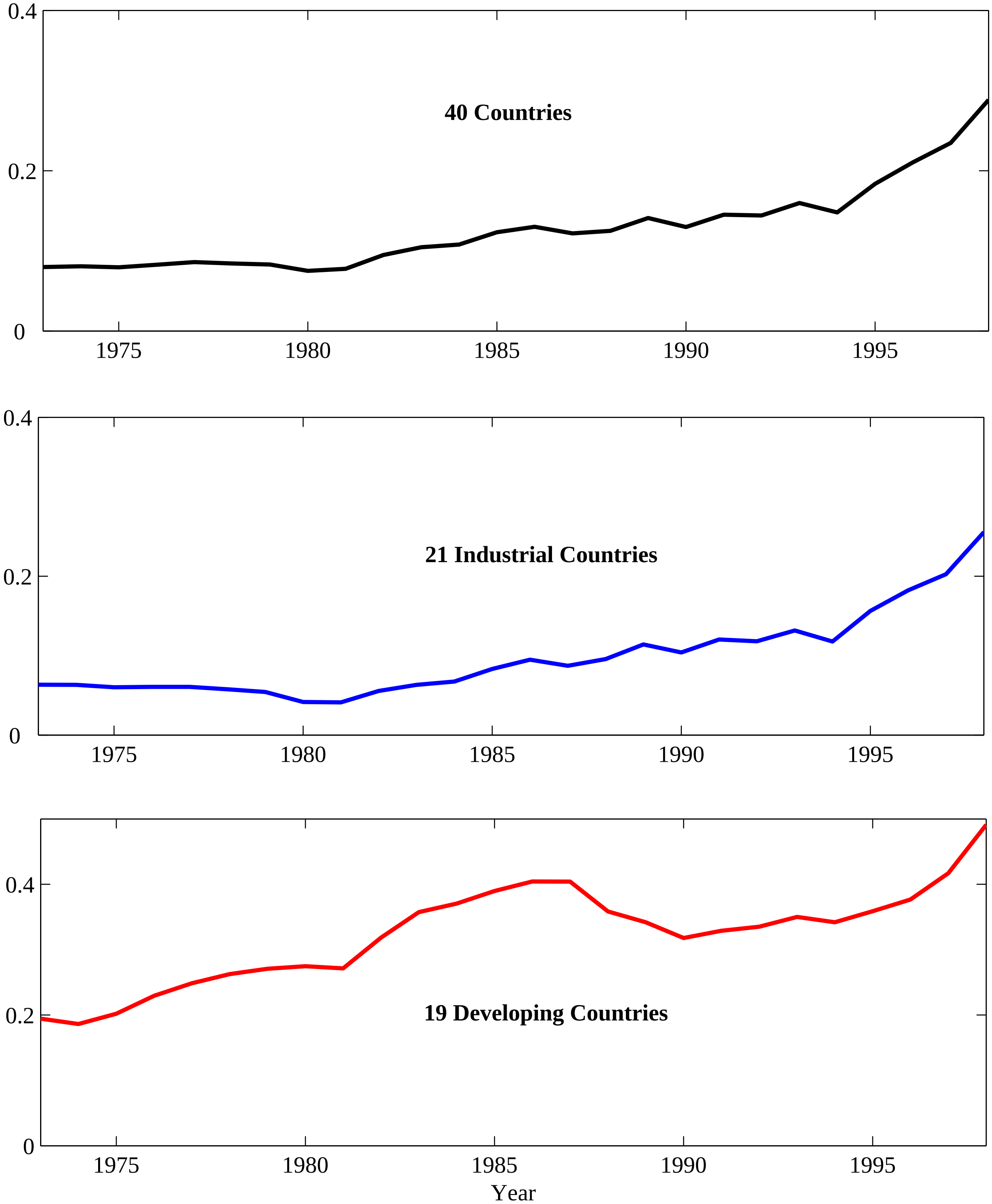


Figure 2.3 Each line is the weighted average of net foreign capital stock over GDP ratio of sample countries. Data is from Lane and Milesi-Ferretti (2001).

Net foreign capital stock = net FDI + net Equity + net Debt

Net FDI = FDI asset -FDI liabilities. Similarly for net equity and net debt.

Figure 2.4 Net Debt Stock over GDP Ratio

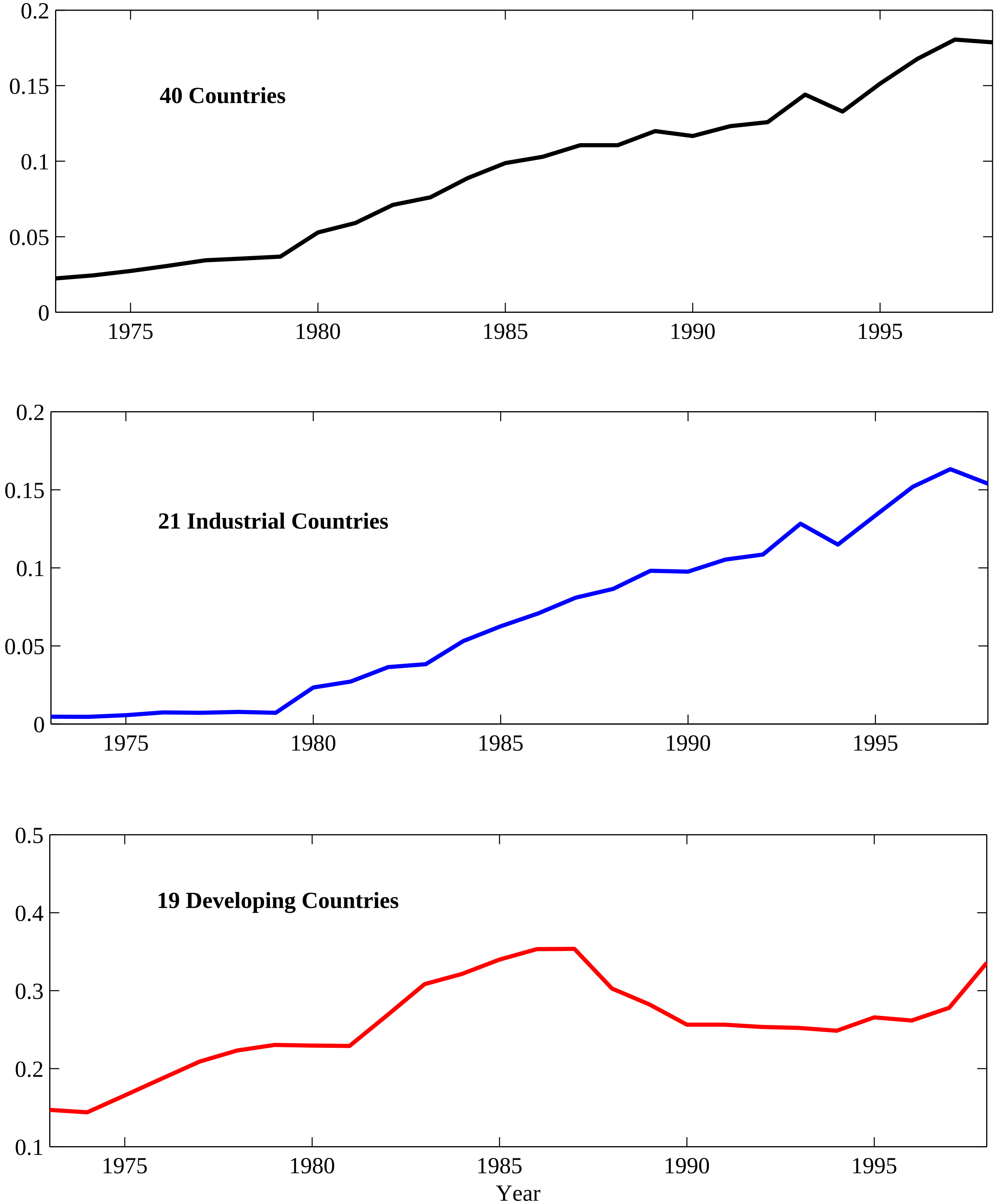


Figure 2.4 Each line is the weighted average of net debt stock over GDP ratio of sample countries. Data is from Lane and Milesi-Ferretti (2001).

Net Debt = Debt asset - Debt liabilities

Figure 4.1 Total Factor Productivities (49 Countries)

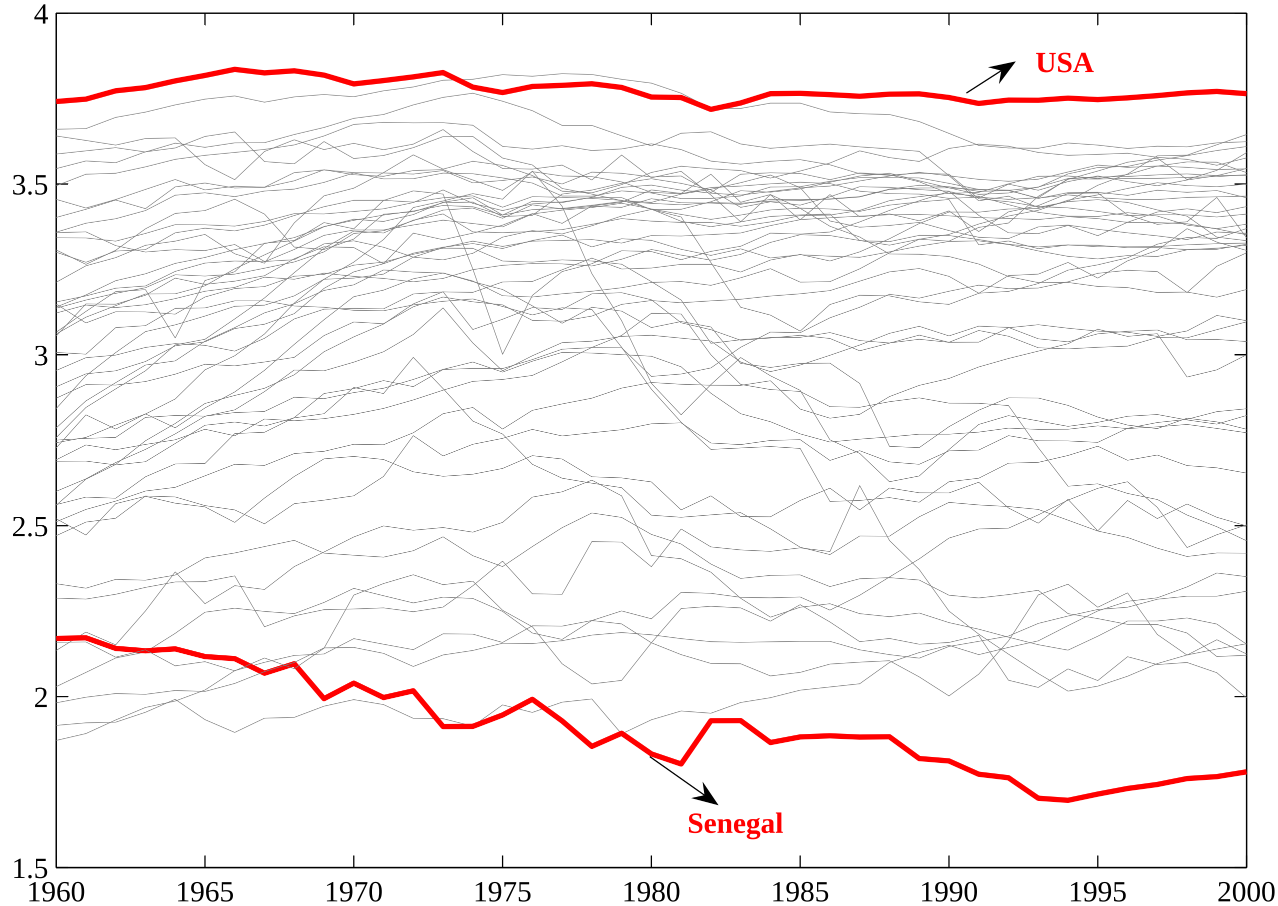
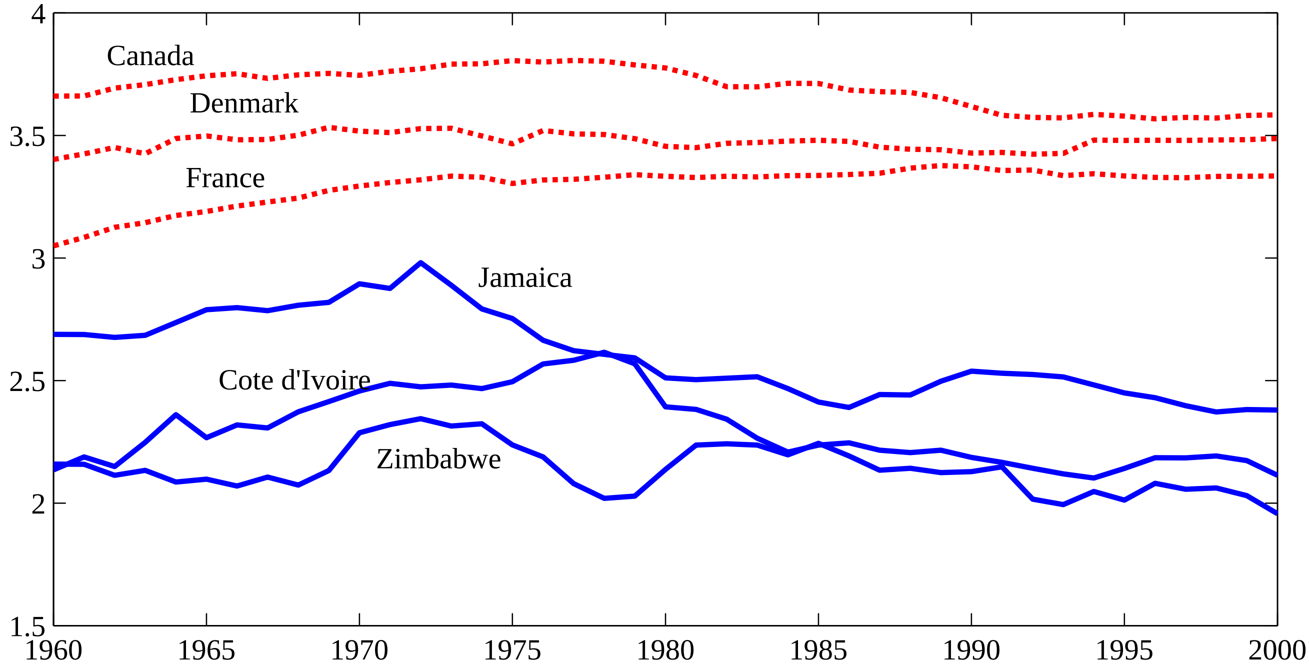


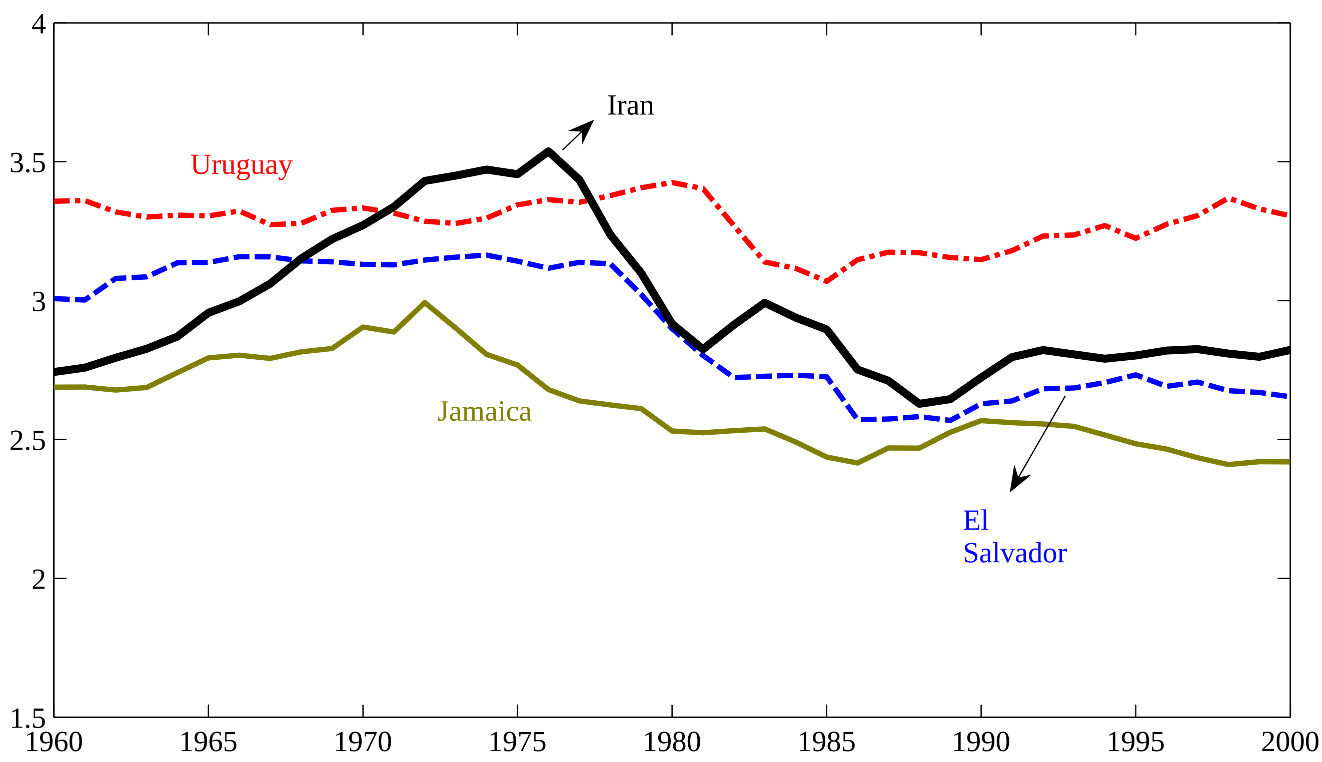
Figure 4.1 plots the total factor productivities for 49 countries with 20 industrial countries and 29 developing countries. The countries with default histories are not included. The total factor productivity for each country at each period is first calibrated from the Cobb-Douglas prodcution function with capital share equal to 0.33. It is then detrended by the world average productivity growth rate 1.3%.
Data source: World Bank, Penn World Table 6.1

Note: the range of processes is more than 17 times of average time series standard deviation.

Figure 4.2. Features of Total Factor Productivities



Poor countries are relatively more volatile than rich countries



Some countries have properties switched over period of time

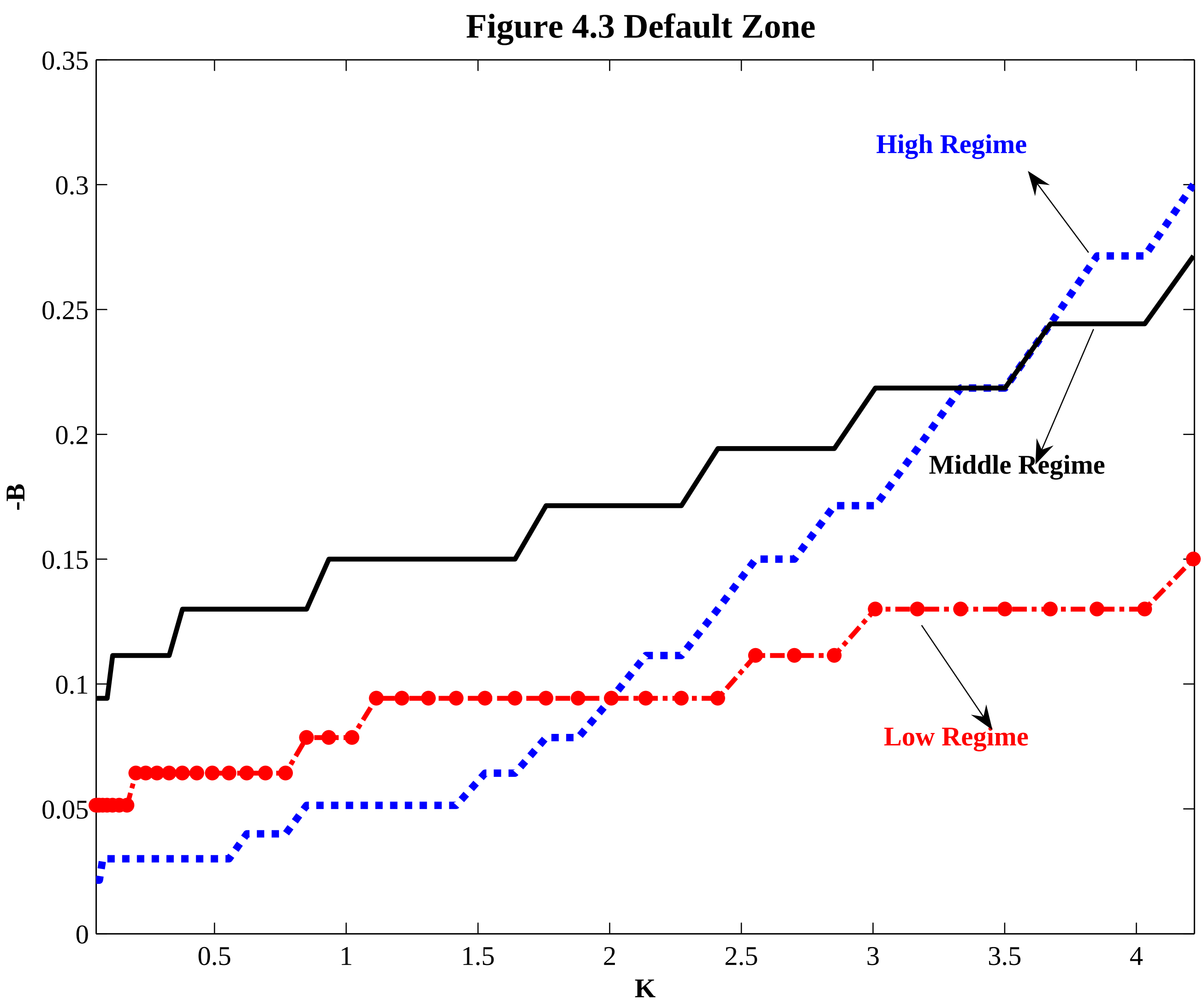


Figure 4.3 plots the default decisions under zero portfolio transaction cost. Each line is the default cutoff for a given productivity shock with different capital shock. The area above each line is the set of debts for which default is the optinal choice. Note the y-axis is -b, negative asset.

Figure 4.4 Bond Price Schedule ($\tau = 0$)

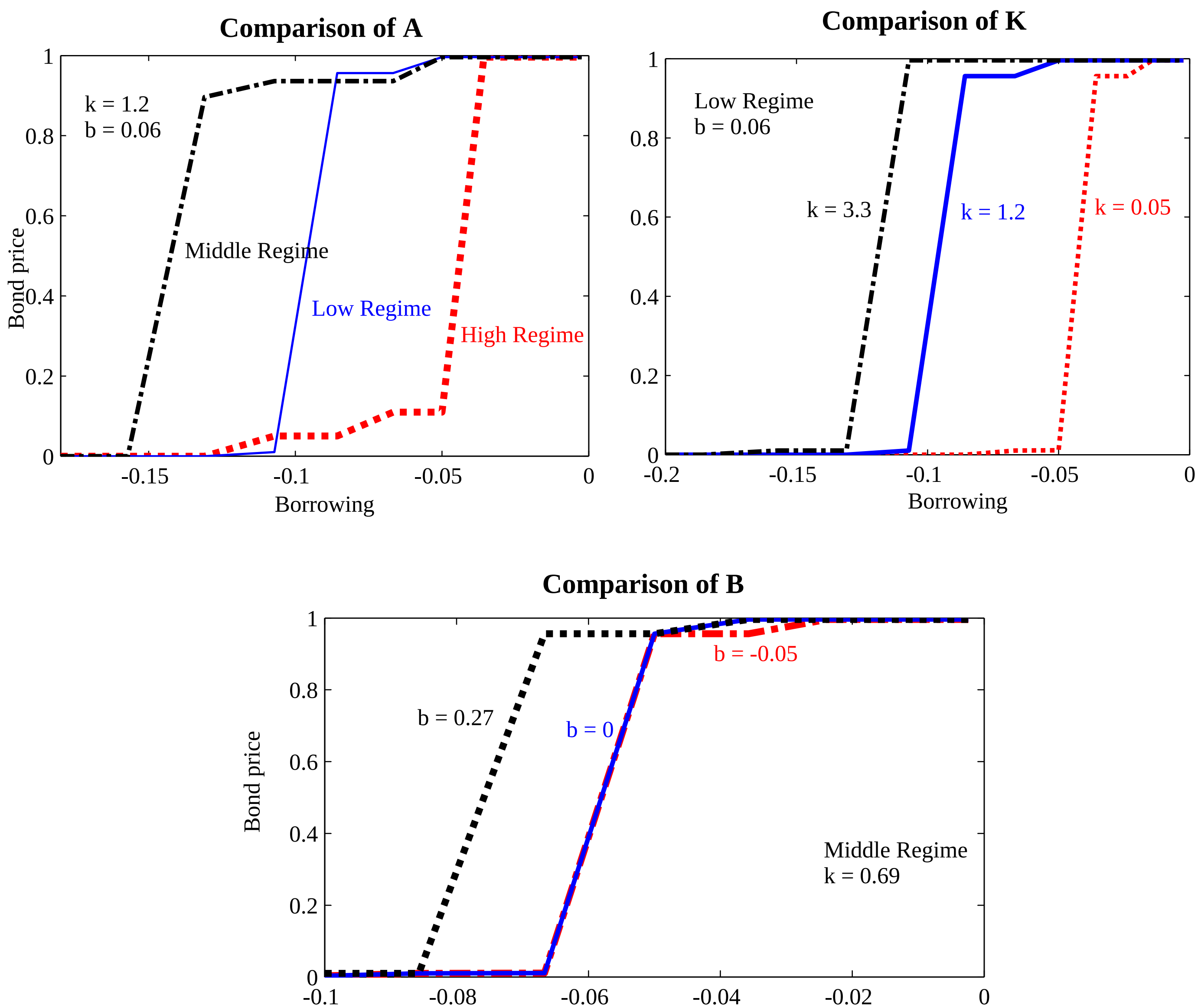


Figure 4.4 plots the bond price schedule under zero portfolio transaction cost.

Figure 4.5 Cumulative B/GDP Distribution

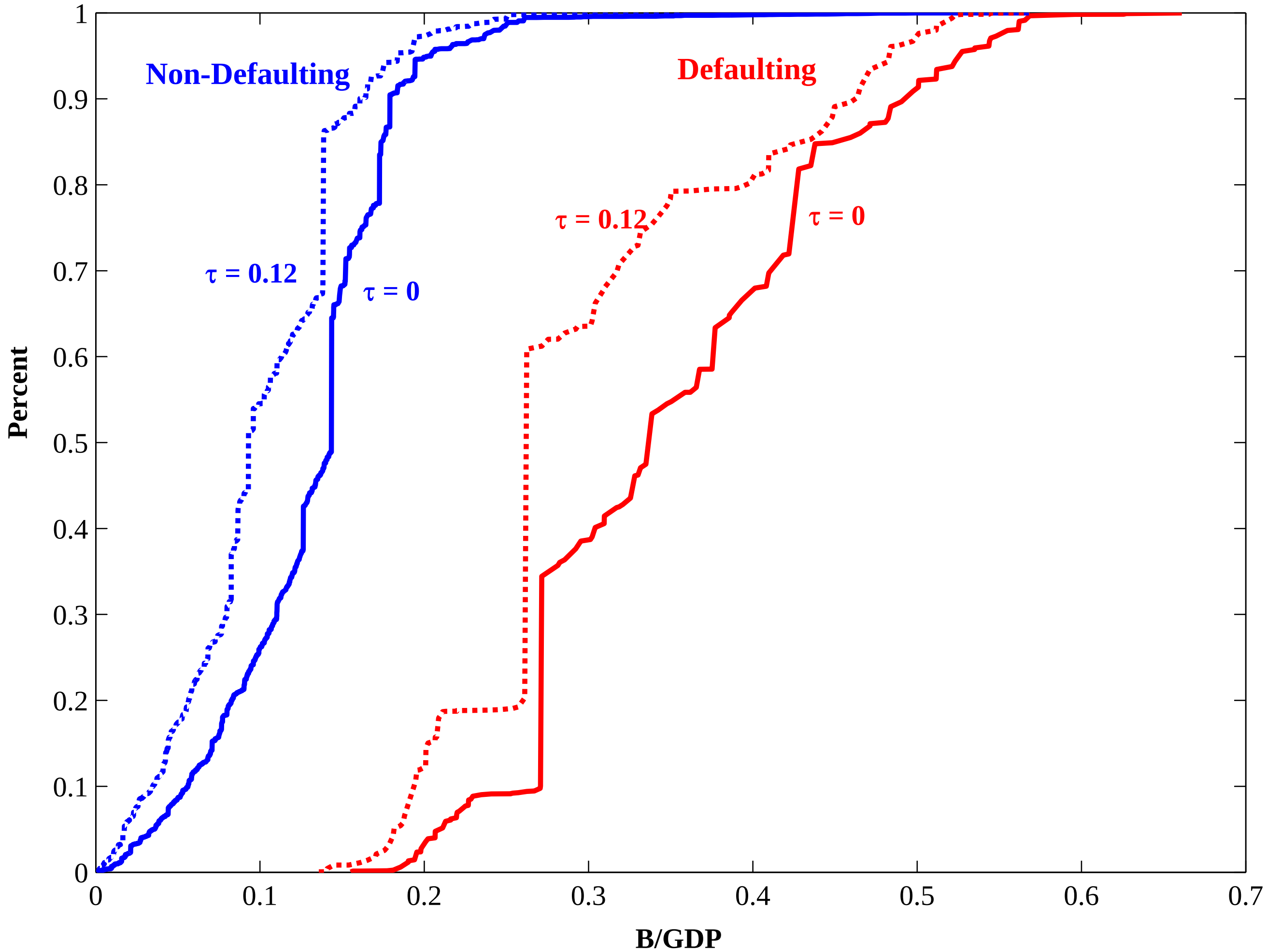


Figure 4.5 plots cumulative B/GDP distribution. Blue lines are the B/GDP distribution on the condition that countries will not default. Red lines are the B/GDP distribution on the condition that countries will default. Solid lines and dotted lines are under zero portfolio transaction cost and 0.12 cost, respectively.