Subsidies for FDI: Implications from a Model with Heterogenous Firms*

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Comments are welcome

Abstract

This paper develops a two-country version of the Helpman, Melitz and Yeaple (2004) model with heterogenous firms to analyze the welfare effects of subsidy schemes to attract multinationals. Considering policies financed by a tax on labor income, I show formally that the use of a small cost subsidy by the host country to multinational firms raises welfare in that country. This welfare improvement stems from a selection effect: The subsidy attracts the most productive home country exporters to switch to servicing the foreign market via FDI, allowing foreign consumers to access these firms' products at a lower price by saving on cross-border transport costs. This consumption gain to the foreign country outweighs the direct costs of funding the subsidy precisely because it is the most productive home country exporters that respond to the FDI subsidy. Some benchmark calibrations show that the magnitude of the welfare gains from a subsidy to variable costs is substantially larger than from a subsidy to the fixed cost of conducting FDI. Intuitively, a variable cost subsidy also helps to raise the inefficiently low output levels of each firm stemming from their mark-up pricing power.

Keywords: FDI subsidies, import subsidies, heterogenous firms.

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1 Introduction

This paper presents an application of the trade models with heterogenous firms advanced by Melitz (2003) and Helpman, Melitz and Yeaple (2004) to an analysis of policy interventions related to foreign direct investment (FDI). It examines specifically the use of cost subsidies to attract multinational corporations (MNCs), which have become an increasingly popular practice among prospective host countries. Importantly, the modelling framework allows us to undertake a positive analysis of the welfare implications for countries that extend such incentives to attract MNCs, to determine in particular the scope for a welfare improvement from subsidizing FDI.

As an economic phenomenon, FDI has expanded considerably over the past two decades.[1] Many countries, including many in the developing world, are in fact now keen to attract FDI to their shores for a variety of reasons. The potential consumption gains are perhaps the most direct effect, since the relocation of production lowers the prices that multinationals charge in their host country's market, due to the savings on cross-border transport costs and possibly also labor costs (if the host is a developing country where labor is less expensive). In addition, host countries often value the increased demand for labor, which bolsters local employment and real wages, as well as the injection of foreign capital.[2] The policy arguments in favor of FDI have also stressed other perceived long-term benefits for economic growth, such as industrial spillovers and transfers of technological expertise, although such effects have been more difficult to identify empirically.[3]

Not surprisingly, countries that adopt these positive views towards FDI have used an array of incentive measures to attract a larger share of the FDI pie, ranging from tax breaks to the subsidization of construction and rent for multinationals. A recent edition of the *World Investment Report* (UNCTAD 2003) surmised that "[t]he use of locational incentives to attract FDI has considerably expanded [sic.] in frequency and value" (p. 124), resulting in an intensifying competition among countries for FDI projects. The *Report* cited the example

[1] Of note, inward stocks of FDI into developing countries increased almost threefold as a percentage of GDP, from 12.6% in 1980 to 36.0% in 2002 (UNCTAD 2003).

[2] On the positive labor market effects of FDI, Rama (2001) reports some suggestive evidence that PPP-adjusted real wages are positively correlated with the FDI-to-GDP ratio across countries.

[3] For example, Aitken and Harrison (1999) find relatively small net effects of foreign investment on domestic firms in Venezuela. On the other hand, a recent piece by Javorcik (2004) does find evidence among Lithuanian firms of positive spillovers stemming from backward linkages improving the productivity levels of local suppliers. As for the cross-country literature on the growth effects of FDI, Nunnenkamp's (2004) overview of the current evidence concludes that whether these benefits materialize depends crucially on host-country conditions such as the quality of the workforce and the strength of local institutions.

of how BMW was reportedly wooed by up to 250 European sites before finally locating a plant in Leipzig in 2001.[4] There has also been brewing unease among some Western European countries over the aggressive use of corporate tax cuts by several Central and Eastern European countries, such as Poland and Slovakia, to attract FDI, prompting France and Germany to propose harmonizing the basic tax rate within the European Union (*The Economist*, July 24th, 2004).[5]

Although FDI subsidies have become a common feature of the international economic landscape, it is not clear *a priori* that such policies are necessarily welfare-improving for the host country even in the absence of strategic competition for FDI. On net, the direct fiscal costs of financing the subsidy schemes have to be weighed against the benefits of an increased multinational presence.

To assess this tradeoff formally, I develop a two-country model that considers the interaction between a Home country where multinationals are headquartered and a Foreign country seeking to attract FDI. Firms within an industry differ in their innate productivity levels, as determined by a draw from a pre-existing distribution of technological possibilities. The initial industry equilibrium sees only the most productive Home firms conducting FDI in Foreign to service that market, since firms need to be sufficiently productive to compensate for the higher fixed costs of operating an overseas plant. Crucially, the model I formulate admits a closed-form expression for consumer welfare, which facilitates analysis of the impact of various policy interventions. The key exercise I conduct examines whether a subsidy to attract even more Home firms to conduct FDI, financed by revenues from an income tax on Foreign's workers, can in fact be welfare-improving for the host country. I focus in this analysis solely on the consumption gains accruing to the Foreign country from attracting more MNCs, as consumers there pay a lower price for MNC's products so long as wage costs in Foreign are not too high. Although this puts aside other potential gains such as technological spillovers or agglomeration effects, it nevertheless serves as an important benchmark, since these additional dynamic effects would further reinforce the benefits of attracting FDI. A more salient caveat is that this leaves out feedback effects that MNCs could have on domestic labor mar-

[4]See also Table 1 in Davies (2005) for a list of publicly-announced incentive packages extended in North America to automobile producers between the late 1970s and the late 1990s.

[5]The article in *The Economist* reports that "Poland reduced its basic rate this year (2004) from 27% to 19%, and Slovakia from 25% to 19%. Hungary has a 16% rate, while Estonia does not even levy corporate tax on reinvested earnings. By contrast, Germany levies a 38.3% rate ... and France 34.3%." See Hines (1996) and Devereux and Griffith (1998) for empirical evidence on the importance of differences in corporate tax rates in explaining cross-state or cross-country variation in volumes of multinational activity received.

kets. Notwithstanding this, the results that I derive will still hold so long as the labor supply in the host country is sufficiently elastic, so that wages do not rise so much in response to the increased demand for labor that it chokes off the original incentive for MNCs to locate production in that country.

Previewing the main results in Section 3, I establish that a small subsidy for FDI does indeed improve welfare in the host country. This result holds both for a subsidy that reduces MNCs' fixed costs of operation (such as through the construction of industrial parks and infrastructure) and when the subsidy is applied to their variable costs of production (such as via job-creation subsidies or corporate tax cuts). Importantly, this welfare improvement is driven by a selection effect that arises when firms are heterogenous in their productivity levels: The subsidy allows the host country to attract the most productive Home firms that were previously servicing Foreign via exports rather than via horizontal FDI. When the distribution of firm productivities is sufficiently thick-tailed – a condition found to be satisfied empirically by Helpman, Melitz and Yeaple (2004) – this selection effect allows the host country to generate consumption gains that are larger than the direct costs of financing the subsidy. The key role played by firm heterogeneity for this result is made clear in Section 3.3, which contrasts how the scope for a welfare improvement from a subsidy is theoretically ambiguous when all firms have identical productivity levels as in the antecedent model of Krugman (1980).

Apart from the selection effect highlighted above, there is an additional varieties effect that emerges when we take into account how the subsidy scheme raises the *ex ante* profitability of potential entrant firms to the Home industry. Section 3.4 shows how this increases the measure of Home firms in the full industry equilibrium, and amplifies the consumption gains to Foreign.

Are there any substantive differences then between the use of fixed versus variable cost subsidies? A simple calibration exercise (Section 3.5) shows that variable cost subsidies have a quantitatively much larger impact on aggregate Foreign welfare than fixed cost subsidies. Intuitively, a variable cost subsidy alters both the entry and production margins for Home MNCs, whereas a fixed cost subsidy affects only the former. The decline in variable costs raises output levels at each firm, which delivers an additional kick to consumption and counteracts some of the inefficiency arising from each firms' monopoly pricing power. However, this favorable comparison of variable cost over fixed cost subsidies warrants some qualification. If the production facility in Foreign also serves as a platform to service third-country markets (as in Grossman et al. (2006)), then not all the consumption gains from the FDI subsidy accrue

to the domestic economy. When this re-export motive for FDI is large, a variable cost scheme that commits to subsidizing each unit of production can raise the subsidy bill substantially, potentially even lowering Foreign's welfare, unless the subsidy takes the form of a sales credit or rebate to domestic consumption as opposed to a direct subsidy to production costs. A fixed cost subsidy, on the other hand, would be immune to this criticism, since it affects only the entry decision of MNCs.

This paper contributes to an extensive literature on the effects of subsidies and competition for FDI, but several features set it apart from the existing work. In particular, it presents a first attempt to the best of my knowledge to apply a framework with heterogenous firms to these policy issues. The paper also falls within a growing body of research (exemplified by Baldwin et al. (2003)), that takes an explicit industry equilibrium approach towards analyzing the effects of trade policy interventions.[6] In this paper, the incorporation of firm heterogeneity enables us to be very precise in describing the industry equilibrium, specifically how each firm's productivity level and the size of the FDI subsidy pins down its location decision and mode for servicing the Foreign market (via exports or FDI).

Separately, this paper also speaks to a broader literature on optimal policy towards foreign investment. The early theoretical contributions on this topic, by MacDougall (1960), Kemp (1966) and Jones (1967), found that with cross-country specialization in production, the optimal policy for a country acting unilaterally requires taxing both imports and capital inflows. Intuitively, without a tax on capital flows, the use of an optimal tariff could prompt more tariff-jumping foreign investment, which could ultimately erode the gains from the tariff on imports. However, this strand of work views foreign investment as international flows of capital, in contrast to the more recent literature on MNCs which treats FDI more concretely as the production activities of overseas affiliates. Along these lines, there has been work exploring various economic settings under which FDI subsidies might generate welfare improvements. For example, Haaland and Wooten (1999) examine how FDI subsidies can foster agglomeration effects, while Pennings (2005) shows that a positive subsidy is indeed optimal when foreign investors face some uncertainty over demand conditions in the host economy.[7] Other authors

[6]For example, Haufler and Wooten (1999) and Ottaviano and van Ypersele (2005) consider the advantage that market size can confer on countries in the competition to attract firms or mobile capital in a general equilibrium model.

[7]Janeba (2002) models this uncertainty as arising from an inability by the host government to make credible long-term commitments to maintain their announced tax or subsidy policies. His analysis also takes into full account that there are two dimensions to this credibility issue, namely that MNCs themselves may not be able to credibly commit to invest in only one country.

have argued that FDI subsidies can serve to counteract either a pre-existing distortion in the economy (Black and Hoyt 1989) or improve the allocation of firms' production facilities to countries from the standpoint of aggregate efficiency (Fumagalli 2003).[8] In this paper, the welfare improvement stems instead from the reduction of barriers to entry (the high fixed costs of conducting FDI) for the most productive Home firms that would otherwise service the Foreign market by exporting without the FDI subsidy.

The rest of the paper proceeds as follows. Section 2 lays out the building blocks of the model. Section 3 establishes the main propositions concerning the scope for a welfare improvement from either a fixed or variable cost subsidy to FDI. Section 4 briefly explores some extensions. A parallel analysis shows that there is a similar scope for improving welfare in the foreign country through an import subsidy (Section 4.1). I also show that my results are robust to the use of an alternative functional form for utility that allows for income effects, wherein the reduction in disposable income in Foreign caused by the labor tax dampens consumer demand and thus potentially reduces utility levels (Section 4.2). Section 5 concludes. Detailed proofs are relegated to the Appendix.

2 The Model Set-up

We proceed in steps to introduce the various building blocks of the model. There are two countries in the baseline model, called Home and Foreign, indexed by H and F respectively. Each economy is made up of two sectors: (i) a homogenous good sector, and (ii) a (country-specific) differentiated goods sector. Labor is the sole factor of production.

Utility: The utility of the representative consumer in country i is given by:

$$U_i = x_i^0 + \sum_{c=H,F} \frac{1}{\mu}(X_i^c)^\mu \tag{2.1}$$

Here, x_i^0 denotes consumption of homogenous goods, which we treat as our numeraire. X_i^c is the familiar Dixit-Stiglitz aggregator of consumption over products, x_i^c, from country c's differentiated goods sector:

$$X_i^c = \left[\int_{\Omega_i^c} x_i^c(a)^\alpha dG^c(a)\right]^{\frac{1}{\alpha}} \tag{2.2}$$

[8] Specifically, Black and Hoyt (1989) consider how a subsidy to firms may reduce the distortion caused by the average cost pricing of public services, when the marginal cost of providing these services is less than the tax revenue that they generate. In the case of Fumagalli (2003), firms prefer to be located in a region that is a more developed market, but locating the MNC in a less developed region may yield greater gains from technological spillovers.

where Ω_i^c is the set of products from country c available to consumers in country i; for example, when $c = H$ and $i = F$, this set is the union of goods imported from Home and those produced in Foreign by Home MNCs. I shall assume that $0 < \mu < \alpha < 1$. This means that the differentiated goods are pair-wise substitutes, and moreover, that products from the same country are closer substitutes than products from different countries.

Differentiated products are indexed by a, which is the amount of labor required to produce one unit of the good. $1/a$ is thus a measure of a firm's labor productivity. Each firm draws its a upon paying the fixed cost of entry into the industry from an exogenous technological distribution $G^c(a)$, and so the resulting productivity differences are the key dimension along which firms in the differentiated goods sector are heterogeneous.

The utility function in (2.1) is maximized with respect to the budget constraint:

$$x_i^0 + \sum_{c=H,F} \int_{\Omega_i^c} p_i^c(a) x_i^c(a) dG^c(a) = w_i \tag{2.3}$$

where w_i is the wage income of a representative consumer in country i, and p_i^c is the unit price of good x_i^c. (The homogenous good price is normalized to 1.) This utility specification generates a demand function for each product with constant elasticity, $\varepsilon = \frac{1}{1-\alpha} > 1$, given by: $x_i^c(a) = (X_i^c)^{\frac{\mu-\alpha}{1-\alpha}} p_i^c(a)^{-\varepsilon}$. Substituting this into the definition in (2.2) yields the following expression for X_i^c in terms of goods prices that is useful for future computations:

$$X_i^c = \left[\int_{\Omega_i^c} p_i^c(a)^{1-\varepsilon} dG^c(a) \right]^{\frac{1-\alpha}{\alpha} \frac{1}{1-\mu}} \tag{2.4}$$

Intuitively, the CES aggregator for consumption decreases as individual goods prices rise.[9]

Welfare measure: As a measure of welfare for the subsequent analysis, I also derive the indirect utility function, W_i, for a representative consumer. The demand function for differentiated products, $x_i^c(a)$, and the budget constraint (2.3) together imply a level of demand for the homogenous good, x_i^0. Substituting this expression for x_i^0 into the utility function (2.1) and simplifying, one obtains:

$$W_i = w_i + \left(\frac{1-\mu}{\mu} \right) \sum_{c=H,F} (X_i^c)^\mu \tag{2.5}$$

The analysis which follows focuses on the industry equilibrium for the Home differentiated goods sector, namely $c = H$, and the effects of a Foreign subsidy on FDI from this sector. For brevity, I suppress the c superscript throughout unless there is cause for ambiguity.

[9] An alternative way to see this is to realize that X_i^c is equal to the ideal price index for Home goods raised to the power of $-1/(1-\mu)$.

Nominal wages: The homogenous good is produced under a constant returns to scale technology. I assume that the labor force in each country is sufficiently large, so that output in this numeraire sector is strictly positive in equilibrium. The nominal wage in each economy is then pinned down by the marginal product of labor in this sector, which facilitates a closed-form solution. A more general model would allow for the wage in the host country to rise in response to the increased demand for Foreign labor, but this would intuitively deliver a positive income effect and raise welfare in Foreign. The results derived below would continue to hold, so long as the Foreign labor market is sufficiently flexible (labor supply there is sufficiently elastic), so that the rise in w_F is not so large that it erodes the incentives for MNCs to locate production in Foreign.[10]

Production and profits: The structure of production in the differentiated goods sector follows closely that in Melitz, Helpman and Yeaple (2004). Upon entering the industry, each Home firm takes a productivity draw, a, from the distribution $G^H(a)$. Production for the Home domestic economy requires a fixed cost of f_D units of labor in each period, while the marginal cost of each unit of output is aw_H. Since firms are profit maximizing, they set prices equal to a constant mark-up, $\frac{1}{\alpha}$, over marginal costs.

Home firms may service the Foreign market through one of two means, namely exporting or horizontal FDI. Firms that export incur two additional costs: (i) a per-period fixed cost of exporting, equal to f_X units of Home labor; and (ii) the conventional iceberg transport costs, which raise unit production costs by a factor $\tau > 1$. (For simplicity, the homogenous good is assumed to be costlessly transportable.) On the other hand, Home firms which are sufficiently productive may opt to start up an additional manufacturing plant in Foreign, in order to save on transport costs. However, FDI requires a higher per-period fixed cost, $f_I > f_X$, than exporting.

Denote the number of workers in country i by M_i ($i = H, F$). For the Home firm with productivity level $1/a$, the per-period profits from production for the domestic economy, from exporting, and from FDI in Foreign are given respectively by $\pi_D(a)$, $\pi_X(a)$ and $\pi_I(a)$:

[10]More generally, one can endogenize the nominal wage by removing this outside sector from the model, but the resulting expression for w_F is the root of a non-linear polynomial. A full solution would require the use of computational methods, sacrificing analytical tractability.

$$\pi_D(a) = A_H\left(\frac{aw_H}{\alpha}\right)x_H(a) - aw_Hx_H(a) - f_Dw_H = (1-\alpha)A_H\left(\frac{aw_H}{\alpha}\right)^{1-\varepsilon} - f_Dw_H \quad (2.6)$$

$$\pi_X(a) = A_F\left(\frac{\tau aw_H}{\alpha}\right)x_F(a) - \tau aw_Hx_F(a) - f_Xw_H = (1-\alpha)A_F\left(\frac{\tau aw_H}{\alpha}\right)^{1-\varepsilon} - f_Xw_H \quad (2.7)$$

$$\pi_I(a) = A_F\left(\frac{aw_F}{\alpha}\right)x_F(a) - aw_Fx_F(a) - f_Iw_H = (1-\alpha)A_F\left(\frac{aw_F}{\alpha}\right)^{1-\varepsilon} - f_Iw_H \quad (2.8)$$

where $A_i = M_i(X_i)^{\frac{\mu-\alpha}{1-\alpha}}$ $(i = H, F)$ is the level of demand in country i. Since there is a continuum of firms, individual firms take these levels of demand as given.[11]

Productivity cut-offs: Firms engage in production for the domestic market if profits from (2.6) are positive. Solving $\pi_D(a) = 0$, this establishes a cut-off value, a_D, which is the maximum labor input coefficient at which production for the Home market is profitable. In addition, firms for which $\pi_X(a) \geq 0$ export to Foreign. This implies a cut-off value, a_X, such that exporting is profitable for all firms with $a < a_X$. However, Home firms service the Foreign market via FDI instead if $\pi_I(a) > \pi_X(a)$; solving for the value of a that equates (2.7) and (2.8) yields a third cut-off, a_I, such that the Home firm opts for FDI over exporting if $a < a_I$. The explicit expressions for these three productivity cut-offs are:

$$(a_D)^{1-\varepsilon} = \frac{f_Dw_H}{(1-\alpha)A_H(w_H/\alpha)^{1-\varepsilon}} \quad (2.9)$$

$$(a_X)^{1-\varepsilon} = \frac{f_Xw_H}{(1-\alpha)A_F(\tau w_H/\alpha)^{1-\varepsilon}} \quad (2.10)$$

$$(a_I)^{1-\varepsilon} = \frac{(f_I - f_X)w_H}{(1-\alpha)A_F[(w_F/\alpha)^{1-\varepsilon} - (\tau w_H/\alpha)^{1-\varepsilon}]} \quad (2.11)$$

Following Helpman, Melitz and Yeaple (2004), I introduce several restrictions on parameter values that ensure a natural sorting pattern of firms to the various modes of servicing the two markets. In particular, I require $a_D > a_X > a_I$, so that only the most productive firms $(a > a_I)$ are able to conduct FDI, while firms with an intermediate level of productivity $(a_X > a > a_I)$ export to Foreign. Firms with $a_D > a > a_X$ serve only the Home market, while firms that draw an a larger than a_D have labor input requirements that are too high and thus exit the industry immediately. Formally, this boils down to: $a_D > a_X \Leftrightarrow \tau^{\varepsilon-1}(\frac{f_X}{A_F}) > \frac{f_D}{A_H}$, and $a_X > a_I \Leftrightarrow f_I > (\frac{\tau w_H}{w_F})^{\varepsilon-1}f_X$. The fixed cost of exporting (normalized by the level of Foreign

[11] Due to the additive separability of the utility derived from Home and Foreign goods in (2.1), actions taken by firms in the Foreign differentiated goods sector do not affect the demand functions and hence profit levels of Home firms. See Levy and Nolan (1992) for an analysis of FDI policy when domestic firms and MNCs are oligopolistic competitors in the same market. In their context, the net effect on welfare depends on how the gains from having the potentially more productive MNC supply the domestic market are weighed against the negative impact on the production levels of domestic firms.

demand) must be sufficiently larger than the fixed cost of domestic production, so that only sufficiently productive firms are able to overcome the higher fixed cost barrier to exporting.[12] Similarly, the fixed cost of FDI must be large enough relative to that for exporting, so that FDI is more profitable only for the most productive Home firms. Figure 1 illustrates this sorting pattern of firms according to $a^{1-\varepsilon}$, which proxies for firms' productivity levels (since $1-\varepsilon < 0$). Finally, (2.11) requires that $w_F < \tau w_H$, in order that $a_I > 0$. Thus, Foreign wages are lower than the marginal cost of the exporting option, in order to make horizontal FDI a profitable operation for some positive productivity levels.

Technology: The distribution, $G^H(a)$, of productivity draws, $1/a$, is parameterized by a Pareto distribution with shape parameter k, as is common in the industrial organization literature. Here, a higher k corresponds to a thicker right-tail in the distribution of productivity levels. It is convenient to define $V^H(a) = \int_0^a \tilde{a}^{1-\varepsilon} dG^H(\tilde{a})$, as this expression will show up repeatedly. The Pareto distribution facilitates an analytical solution, since G^H and V^H are both polynomials in a:

$$G^H(a) = \left(\frac{a}{a_H}\right)^k \tag{2.12}$$

$$V^H(a) = \frac{k}{k-\varepsilon+1}\left(\frac{a}{a_H}\right)^k a^{1-\varepsilon} \tag{2.13}$$

Note that $a < a_H$, with $1/a_H$ being a lower bound on Home productivity levels.

Helpman, Melitz and Yeaple (2004) show that if the underlying productivity distribution is Pareto with shape parameter k, then the distribution of observed firm sales will be Pareto with shape $k - \varepsilon + 1$. In fact, this distribution is equal to $V^H(a)$ up to a multiplicative constant. Their estimation based on European firm-level data establishes the goodness of fit of this parametric distribution for firm sales, while yielding estimates for $k - \varepsilon + 1$ that are always significantly greater than 0 across manufacturing industries. This empirical evidence motivates a key assumption: $k > \varepsilon - 1$. In essence, this is a condition on the extent of firm heterogeneity in the Home sector, namely that the distribution of Home firm productivities is sufficiently thick-tailed, placing a sufficiently large mass on high productivity levels.

Equilibrium consumption of differentiated goods: We can now solve for the equilibrium level of X_H and X_F, namely the CES consumption aggregates for Home's differentiated

[12] To be fully precise, one needs to take the expressions for A_H and A_F that are solved for in general equilibrium and substitute them into this inequality for the full condition based only on the structural parameters of the model.

goods sector in the Home and Foreign markets respectively. In particular, these will be key expressions for evaluating welfare based on the indirect utility function, (2.5). From (2.4) and the sorting pattern within the Home industry, these CES aggregates can be re-written as:

$$X_H = \left[N\int_0^{a_D}\left(\frac{aw_H}{\alpha}\right)^{1-\varepsilon}dG^H(a)\right]^{\frac{1-\alpha}{\alpha}\frac{1}{1-\mu}} = \left[N\frac{k}{\alpha}\Lambda_H(A_H)^{\frac{k-\varepsilon+1}{\varepsilon-1}}\right]^{\frac{1-\alpha}{\alpha}\frac{1}{1-\mu}} \tag{2.14}$$

$$\begin{aligned} X_F &= \left[N\left(\int_{a_I}^{a_X}\left(\frac{\tau aw_H}{\alpha}\right)^{1-\varepsilon}dG^H(a)+\int_0^{a_I}\left(\frac{aw_F}{\alpha}\right)^{1-\varepsilon}dG^H(a)\right)\right]^{\frac{1-\alpha}{\alpha}\frac{1}{1-\mu}} \\ &= \left[N\frac{k}{\alpha}\Lambda_F(A_F)^{\frac{k-\varepsilon+1}{\varepsilon-1}}\right]^{\frac{1-\alpha}{\alpha}\frac{1}{1-\mu}} \end{aligned} \tag{2.15}$$

where N is the measure of Home firms. Note that I have substituted the expressions for the productivity cut-offs from (2.9), (2.10) and (2.11), as well as for the Pareto distribution from (2.12) to evaluate the integrals above. The terms Λ_H and Λ_F are given explicitly by:

$$\Lambda_H = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\frac{(f_D)^{\frac{k}{1-\varepsilon}+1}w_H}{(w_H)^k} \tag{2.16}$$

$$\Lambda_F = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k}+\frac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right] \tag{2.17}$$

Note that Λ_H and Λ_F are equal to the *ex ante* profit levels that a prospective entrant firm in the Home sector would obtain in expectation (ie before observing its productivity draw a) from sales in Home and Foreign respectively, if the market demand levels A_H and A_F were normalized to 1. I therefore refer intuitively to Λ_H and Λ_F as the normalized profit levels in these respective markets.

Recall though that $A_i = M_i(X_i)^{\frac{\mu-\alpha}{1-\alpha}}$ (for $i = H, F$), which is a definition I now substitute into (2.14) and (2.15). Some algebraic simplification then yields an expression for the CES consumption aggregates:

$$(X_i)^\mu = \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}\frac{\tilde{M}_i}{M_i}\tilde{\Lambda}_i, \quad i = H, F \tag{2.18}$$

where $\tilde{\Lambda}_i = (\Lambda_i)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}$ and $\tilde{M}_i = (M_i)^{\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}}$. (2.18) is a very useful expression for computing country welfare levels; in particular, observe from (2.5) that $(X_F)^\mu$ is proportional to the quantum of utility derived by Foreign from the consumption of Home differentiated goods. Note that $(X_F)^\mu$ and hence Foreign welfare is increasing in N, the measure of Home firms, so that there is the usual "love of varieties" effect. In addition, $(X_F)^\mu$ rises with Λ_F (since $\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)} > 0$ as $\mu < \alpha$); a higher normalized profit level of Home firms from sales

in Foreign implies that Foreign consumers have access to goods from more Home firms, thus raising the level of welfare in Foreign. Finally, we have a market size effect, whereby $(X_F)^\mu$ increases with M_F (since $\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)} - 1 = \frac{\mu(k-k\alpha-\alpha)}{\mu\alpha-k(\mu-\alpha)} > 0 \Leftrightarrow k > \frac{\alpha}{1-\alpha} = \varepsilon - 1$, which holds by the assumption on the extent of firm heterogeneity).[13]

To close the model fully, one needs to pin down the measure of Home firms, N, with a free-entry condition for the Home sector. However, I defer this discussion to Section 3.4, since (2.18) already facilitates a closed-form expression for Foreign welfare that corresponds loosely to the case of a "small" Foreign country, namely when the foreign country is too small to affect the entry decisions of prospective entrant firms in Home. Analyzing this case where N is exogenous helps to isolate and highlight the selection effect of FDI subsidies, inducing existing Home firms to switch their mode of servicing Foreign from exporting to FDI. It will turn out later in the endogenous N case that the additional entry or varieties effect works to reinforce this selection effect, so that the welfare implications are qualitatively identical. I therefore turn first to the policy analysis of the exogenous N case.

3 The Welfare Implications of FDI Subsidies

We now consider the effects of FDI subsidies to attract more Home firms to locate in Foreign, focusing on firm location decisions and the welfare implications for the host economy. I first establish that the use of a small subsidy increases welfare levels both when the subsidy is applied to the fixed costs of FDI (Section 3.1) or to the variable component of MNCs' production costs (Section 3.2). Section 3.3 highlights the key role played by firm heterogeneity in the analysis, by contrasting the results against a model where all firms have identical productivity levels. As promised, Section 3.4 shows how to endogenize N for the full industry equilibrium, a step that leaves the welfare implications unchanged. Section 3.5 provides a discussion on the relative efficacy of fixed versus variable cost subsidies.

3.1 FDI subsidy to fixed costs

Consider first the use of a subsidy by the Foreign government that reduces the per-period fixed costs of FDI for Home multinationals by the amount: $s_f(f_I - f_X)w_H$, where $s_f < 1$. For

[13] It is important for the derivation of the neat closed-forms in (2.14), (2.15) and (2.18) that f_D and f_X both be strictly larger than zero. For instance, if $f_X = 0$, then the expression for the cut-off a_D would involve both A_H and A_F, since all firms that engage in domestic production also export when the fixed cost of exporting is 0. A_H (in addition to A_F) would then enter on the right-hand side of (2.15) when computing that CES consumption aggregate, and it would in general not be possible to isolate a closed-form for X_F or A_F.

example, this subsidy could come in the form of the provision of basic infrastructure such as roads or communications networks, which the MNC would otherwise have to pay for on its own. Alternatively, the subsidies could remove certain lump-sum regulatory fees that need to be paid on a recurrent basis. Note that the subsidy is applied to the difference between the fixed costs of investment and exporting, to capture how it closes the gap between the upfront costs of these two modes of servicing the Foreign market. (The proposition below on the scope for a welfare improvement still holds if the subsidy is applied to f_I instead of $f_I - f_X$, with the proofs being entirely analogous.)

Throughout the analysis, I restrict myself to subsidy policies that are "budget-neutral", in that the subsidy bill is exactly paid for by revenues raised from a tax on Foreign labor income. In the case of a fixed cost subsidy, the income tax rate, t_f, must therefore satisfy the following equation to balance Foreign's state budget:

$$t_f w_F M_F = s_f(f_I - f_X) w_H N G^H(a_I) \tag{3.1}$$

The expression on the right-hand-side of this balanced-budget constraint is the total fiscal bill from subsidizing each Home firm with $a < a_I$ by the amount $s_f(f_I - f_X)w_H$. Firms continue to pay w_F for each unit of Foreign labor they employ, but Foreign workers now maximize utility subject to the budget constraint (2.3) with w_F replaced by $(1 - t_f)w_F$. Note that t_f is thus the minimum tax rate that needs to be levied on consumers in order to cover the costs of funding the subsidy to MNCs.

To evaluate the net impact on welfare, it suffices to examine what happens to the terms, $W_{Ff} \equiv (1 - t_f)w_F + \frac{1-\mu}{\mu}(X_F)^\mu$, in the formula for indirect utility in (2.5).[14] This expression for W_{Ff} illustrates the nature of the tradeoff facing Foreign: The FDI subsidy lowers the productivity cut-off, $a_I^{1-\varepsilon}$, allowing some Home firms that were previously exporting to turn to FDI as their mode of servicing Foreign. There is thus a margin of goods that were previously exported to Foreign at price $\frac{\tau a w_H}{\alpha}$ that are now priced more cheaply at $\frac{a w_F}{\alpha}$ by the MNC's Foreign facility (by assumption, $\tau w_H > w_F$). While this boosts the CES consumption aggregate X_F^μ, the consumption gain must be weighed against the direct cost of the income tax, $-t_f w_F$.

The following proposition formally establishes that the net effect is a welfare improvement from a "small" positive subsidy to the fixed costs of FDI:

[14] This follows from the additively separable nature of utility obtained from consuming differentiated products from each country, $c = H, F$.

Proposition 1 [Fixed Cost Subsidy]: *Consider the family of fixed cost FDI subsidies characterized by s_f that satisfy the balanced-budget constraint (3.1). Then the optimal policy, s_f^*, that maximizes welfare in Foreign is a strictly positive subsidy level, namely $s_f^* \in (0,1)$.*

Proof: Using (2.11), (2.12) and (3.1), one can express t_f in terms of s_f and the underlying model parameters. Together with the expression for $(X_F)^\mu$ from (2.18), this yields:

$$W_{Ff} = w_F \; + \; \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\left[\frac{1-\mu}{\mu}\tilde{\Lambda}_{Ff} - s_f\left(\frac{\alpha}{k}\right)\frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}\right] \tag{3.2}$$

where Λ_{Ff} and $\tilde{\Lambda}_{Ff}$ are Λ_F and $\tilde{\Lambda}_F$ respectively with $f_I - f_X$ replaced by $(1-s_f)(f_I - f_X)$, while:

$$\frac{\partial \Lambda_{Ff}}{\partial s_f} = \left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{\left((w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}\right)^{\frac{k}{1-\varepsilon}}}(1-s_f)^{\frac{k}{1-\varepsilon}}$$

(Note that the expression $\frac{\partial \Lambda_{Ff}}{\partial s_f}$ appears in (3.2) via an algebraic substitution; no first-order conditions have been taken yet.)

Now, differentiating (3.2) with respect to s_f and simplifying gives:

$$\begin{aligned}\frac{\partial W_{Ff}}{\partial s_f} \;=\; & \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\frac{\tilde{\Lambda}_F}{\Lambda_F}\left[\left(\frac{1-\mu}{\mu}\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)} - \frac{\alpha}{k}\right)\frac{\partial \Lambda_{Ff}}{\partial s_f}\ldots\right. \\ & \left.\ldots - s_f\left(\frac{\alpha}{k}\right)\frac{k(\mu-\alpha)}{\mu\alpha - k(\mu-\alpha)}\frac{1}{\Lambda_{Ff}}\left(\frac{\partial \Lambda_{Ff}}{\partial s_f}\right)^2 - s_f\left(\frac{\alpha}{k}\right)\left(\frac{\partial^2 \Lambda_{Ff}}{\partial s_f^2}\right)\right]\end{aligned} \tag{3.3}$$

The terms outside the square brackets in (3.3) are clearly positive (in particular, $\tilde{\Lambda}_F > 0$ whenever $s_f < 1$). It suffices therefore to examine the terms in square brackets to deduce how Foreign welfare varies with the level of the fixed cost subsidy.

To examine the scope for a welfare improvement from a small subsidy, set $s_f = 0$ in (3.3). Since $\frac{\partial \Lambda_{Ff}}{\partial s_f} > 0$ for $s_f < 1$, a sufficient condition for $\frac{\partial W_{Ff}}{\partial s_f}$ to be positive at $s_f = 0$ is:

$$\begin{aligned}\frac{1-\mu}{\mu}\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)} - \frac{\alpha}{k} > 0 \;&\Leftrightarrow\; k - \mu\alpha - k\alpha > 0 \\ &\Leftrightarrow\; \frac{k}{\varepsilon - 1} > \mu\end{aligned} \tag{3.4}$$

But this last inequality holds because $\frac{k}{\varepsilon-1} > 1 > \mu$. Thus, $\frac{\partial W_{Ff}}{\partial s_f} > 0$ in a neighborhood of $s_f = 0$. One can show moreover that $\frac{\partial W_{Ff}}{\partial s_f} > 0$ for all $s_f < 0$ (when the policy is in fact a tax on FDI), while $W_{Ff} \to -\infty$ as $s_f \to 1^-$. (The details of these proofs are not particularly illuminating, and are relegated to Appendix 7.1.)

Welfare in Foreign therefore increases with s_f for small values of the subsidy, but has at least one turning point in the interval $(0, 1)$. There thus exists an optimal positive subsidy level that maximizes welfare for the host country. In particular, a small fixed cost subsidy unambiguously improves welfare for Foreign's workers. ■

It is useful at this point to understand what drives the scope for welfare improvement. The presence of a fixed cost for investment prevents a segment of Home firms from servicing the Foreign market directly through FDI. The subsidy thus helps to alleviate some of the inefficiency caused by this fixed cost barrier. As illustrated in Figure 1, the Home exporters sitting just to the left of the productivity cut-off, $a_I^{1-\varepsilon}$, switch to servicing the Foreign market via FDI as a result of the subsidy. For the consumption gains to Foreign from this switch to be large enough to outweigh the direct costs of the subsidy, it must intuitively be the case that these Home firms drawn into FDI need to be sufficiently productive. (The more productive a firm is, the lower the unit price of its output, and hence the larger the increase in volume of consumption when the price of this good falls from $\frac{\tau a w_H}{\alpha}$ (under exporting) to $\frac{a w_F}{\alpha}$ (under FDI).) The distribution of firm productivities must therefore be relatively thick-tailed, in order to ensure that the FDI subsidy indeed attracts a margin of sufficiently productive firms. This is why the proof above requires that k be sufficiently large in the inequality (3.4) in order to sign the slope of the welfare function at $s_f = 0$, which is precisely the role that firm heterogeneity plays in generating this selection effect. Reassuringly, the extent of firm heterogeneity required for the result to hold is also empirically relevant, consistent with the earlier estimates showing that $k - \varepsilon + 1 > 0$ from Helpman, Melitz and Yeaple (2004).[15]

It is also worth emphasizing that the welfare improvement in this result arises simply from consumption gains to the host economy, putting aside other considerations that have been highlighted in the literature as potential sources of gains from attracting FDI. Such effects as technological spillovers (Fumagalli 2003, Javorcik 2004), agglomeration economies (Haaland and Wooten 1999), and an increase in the demand for Foreign labor would intuitively reinforce and amplify the welfare improvement from a subsidy to FDI (so long as wages do not rise too much to undermine MNCs' incentives to undertake production in the Foreign country).

A natural question to ask at this point is whether the optimal fixed cost subsidy is unique.

[15]This optimal subsidy discussion puts aside normative issues with regards to whether the subsidy can indeed be implemented. A more thorough treatment of the determinants of FDI subsidy levels would almost certainly need to incorporate political economy considerations. See for example Janeba (2004) in which FDI subsidies generate a re-distributive effect from workers to firms, which inherently limits the scope for large subsidies if workers have sufficient political clout.

Note first that there does not exist a closed-form expression for s_f^*. Moreover, the welfare function is not globally concave, and there are parameter values, albeit extreme ones, for which W_{Ff} exhibits more than one turning point in the interval $(0, 1)$.[16] For all practical purposes though, the calibration exercise in Section 3.5 will confirm that for the empirically relevant values of the parameter space, the welfare function does exhibit a unique turning point. For those that desire a more formal treatment of this issue, Appendix 7.1 derives a sufficient condition for the uniqueness of the optimal subsidy: $2(\varepsilon - 1) > k$. This restriction is typically satisfied by the parameter choices of similar calibrations in the literature, in particular Ghironi and Melitz (2005) where the baseline values are $k = 3.4$ and $\varepsilon = 3.8$.

3.2 FDI subsidy to variable costs

What happens if the financial incentives to Home multinationals are targeted towards their variable costs of production instead? Many of the incentive schemes offered in practice, such as job-creation subsidies or corporate tax rate cuts, fall into this category. The analysis in this subsection shows that in the baseline two-country model, the welfare implications of a variable cost subsidy turn out to be qualitatively identical to what we have seen for a fixed cost subsidy.

Consider then a subsidy to the variable costs of Home MNCs' production in Foreign that reduces their effective unit wage costs from w_F to $(1 - s_v)w_F$, where $s_v < 1$. As before, suppose that the state pays for these subsidies by raising an income tax on its citizens, t_v. If this incentive scheme is to be budget-neutral, then:

$$t_v w_F M_F = s_v w_F N A_F \left(\frac{(1 - s_v) w_F}{\alpha} \right)^{-\varepsilon} V^H(a_I) \tag{3.5}$$

where the right-hand side of (3.5) is the total amount paid out as production subsidies to the multinationals. Note that a higher demand for Home final goods, A_F, will raise the total subsidy bill directly under a variable cost subsidy scheme.

We now have a parallel result concerning the welfare improvements from a subsidy to MNCs' variable costs of production:

Proposition 2 [Variable Cost Subsidy]: *Consider the family of variable cost FDI subsidies characterized by s_v that satisfy the balanced-budget constraint (3.5). Then the opti-*

[16] Some experimentation with the calibration parameters in Section 3.5 shows that when both k is increased (to say $k = 50$) and μ is raised closer to the value of α (to say $\mu = 0.6$), then W_{Ff} actually has 3 turning points in $(0, 1)$.

mal policy, s_v^*, that maximizes welfare in Foreign is a strictly positive subsidy level, namely $s_v^* \in (0,1)$.

Proof: Once again, it suffices to examine the behavior of the terms $W_{Fv} \equiv (1-t_v)w_F + \frac{1-\mu}{\mu}(X_F)^\mu$ in the indirect utility function. The state's budget constraint (3.5) allows us to re-write t_v in terms of s_v; making this substitution and using the expression for $(X_F)^\mu$ from (2.18), one obtains after some simplification:

$$W_{Fv} = w_F \; + \; \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\left[\frac{1-\mu}{\mu}\tilde{\Lambda}_{Fv} - s_v\left(\frac{\alpha}{k}\right)\frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}}\frac{\partial\Lambda_{Fv}}{\partial s_v}\right] \tag{3.6}$$

where Λ_{Fv} and $\tilde{\Lambda}_{Fv}$ are given by Λ_F and $\tilde{\Lambda}_F$ respectively except with $(w_F)^{1-\varepsilon}$ replaced by $((1-s_v)w_F)^{1-\varepsilon}$ in the denominator, while

$$\frac{\partial\Lambda_{Fv}}{\partial s_v} = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \frac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}+1}} k w_F^{1-\varepsilon}(1-s_v)^{-\varepsilon} \tag{3.7}$$

With this formulation, the welfare function to be maximized in (3.6) is completely analogous to that from the fixed cost subsidy case in (3.2). It is straightforward to check that the proof that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ at $s_v = 0$ follows the same steps in Proposition 1 with the relevant terms involving Λ_{Ff} now replaced by terms in Λ_{Fv}. We will once again require that the distribution of firm productivities be sufficiently thick-tailed (with k large enough to satisfy the condition (3.4)) in order to sign this derivative in a neighborhood of $s_v = 0$. In addition, one can show that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ for all $s_v < 0$, while $W_{Fv} \to -\infty$ when s_v approaches 1 (see Appendix 7.2).

In short, the welfare function in the variable cost subsidy case has a positive slope at $s_v = 0$, a turning point in the interior of $(0,1)$ and an asymptote to $-\infty$ at $s_v \to 1^-$. The optimal policy is therefore a strictly positive subsidy level, $s_v^* \in (0,1)$. Once again, while the equation $\frac{\partial W_{Fv}}{\partial s_v} = 0$ may in principle have more than one zero in $(0,1)$, the optimal subsidy is nevertheless unique for the range of parameters that is empirically relevant. ■

As with a fixed cost subsidy, the variable cost subsidy exhibits the same selection effect of drawing in the most productive Home exporters who were just shy of the $a_I^{1-\varepsilon}$ cut-off for FDI. There is however also a production effect at play here, in that the variable cost subsidy raises output levels at all Foreign assembly plants, even those belonging to MNCs that would have conducted FDI without the subsidy. This additional effect raises further the consumption gains from a variable cost subsidy. Put otherwise, the variable cost subsidy helps to counteract

the inefficiency stemming from the firms' monopoly pricing power, by reducing the effective mark-up of MNCs from $\frac{1}{\alpha}$ to $\frac{1-s_v}{\alpha}$.

Section 3.5 shall turn to a more careful comparison of the relative efficacy of fixed versus variable cost subsidies. But first, it is useful to isolate the key role played by firm heterogeneity for the welfare results, by explicitly contrasting what happens with the use of FDI subsidies when all Home firms have identical productivity levels.

3.3 Comparison with a model with homogenous firms

Consider now the case where all firms in Home's differentiated goods sector share the same unit labor input coefficient, $\bar{a}$, as in Krugman (1980). Within the framework set up in Section 2, this corresponds to a situation where $G^H(a)$ has its entire density concentrated at a single point. For simplicity, I shall continue to treat N as exogenous.

Suppose to begin with that all the Home firms initially service Foreign via exports instead of via FDI, namely $\bar{a}$ satisfies $\pi_D(\bar{a}), \pi_X(\bar{a}) > 0$, but $\pi_X(\bar{a}) < \pi_I(\bar{a})$, where the profit functions come from (2.6), (2.7) and (2.8). The question of interest would then be whether a subsidy to the Home firms inducing a switch from exporting to FDI improves welfare in Foreign. It turns out that in this setting, the scope for welfare improvements from a subsidy to FDI is **not** guaranteed when firms have identical productivity levels.

For the purpose of illustration, let us focus on the case of a fixed cost subsidy. Consider a subsidy, s_f, that would make the Home firms indifferent between exporting and FDI as their preferred mode of servicing the Foreign market. This is the smallest subsidy level that would make the policy effective in terms of inducing Home firms to switch to FDI. s_f is therefore given by:

$$(1-\alpha)A_{F,X}\left(\frac{\bar{a}w_F}{\alpha}\right)^{1-\varepsilon} - (1-\alpha)A_{F,I}\left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{1-\varepsilon} = (1-s_f)(f_I - f_X)w_H \tag{3.8}$$

where $A_{F,X}$ and $A_{F,I}$ are respectively the levels of Foreign market demand in the old equilibrium where all Home firms export and in the new equilibrium where all Home firms conduct FDI. Now, the definition of X_i^c in (2.2) implies that $X_i^c = N^{\frac{1}{\alpha}} x_i^c(\bar{a})$. The demand for individual products, $x_i^c(\bar{a})$, is given in turn by $(X_i^c)^{\frac{\mu-\alpha}{1-\alpha}}\left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{-\varepsilon}$ when Home firms export and by $(X_i^c)^{\frac{\mu-\alpha}{1-\alpha}}\left(\frac{\bar{a}w_F}{\alpha}\right)^{-\varepsilon}$ when Home firms conduct FDI. A quick substitution back into (2.2) then delivers expressions for the respective CES consumption aggregates in Foreign. In particular, $X_{F,X}^{\mu} = N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{-\frac{\mu}{1-\mu}}$ and $X_{F,I}^{\mu} = N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{\bar{a}w_F}{\alpha}\right)^{-\frac{\mu}{1-\mu}}$, where the X and I subscripts refer to whether Home's sales to Foreign are conducted via exporting or FDI.

From this, one can compute the market demand levels, $A_{F,X}$ and $A_{F,I}$, using the definition: $A_F = M_F(X_F)^{\frac{\mu-\alpha}{1-\alpha}}$.

As before, let the fixed cost subsidy be paid for by revenues from a tax, t_f, on citizens:

$$t_f w_F M_F = s_f(f_I - f_X) w_H N \tag{3.9}$$

Recall that the relevant welfare measure is: $(1-t_f)w_F + \frac{1-\mu}{\mu}(X_F)^{\mu}$. The minimum effective FDI subsidy level given by (3.8) can be substituted into the balanced-budget constraint (3.9) to obtain an expression for t_f. One can now derive an expression for the change in welfare for Foreign from the use of this fixed cost subsidy to Home firms to induce a switch to FDI:

$$\Delta W_F = -\frac{(f_I - f_X)w_H N}{M_F} + N^{\frac{1-\alpha}{\alpha}\frac{\mu}{1-\mu}}\left(\frac{1}{\mu} - \alpha\right)\left[\left(\frac{\bar{a}w_F}{\alpha}\right)^{-\frac{\mu}{1-\mu}} - \left(\frac{\tau\bar{a}w_H}{\alpha}\right)^{-\frac{\mu}{1-\mu}}\right] \tag{3.10}$$

The second summand in (3.10) is positive given the parameter assumptions $\mu, \alpha < 1$ and $w_F < \tau w_H$. This term represents the consumption gains to Foreign from a lower price of Home final goods. Thus, for the net change to Foreign welfare to be positive, one needs the first term in (3.10) to be sufficiently small. In particular, the higher the fixed cost of FDI (the larger is $(f_I - f_X)$), the higher the total subsidy bill required to attract multinationals, and this potentially overwhelms the consumption gains from this policy action. (Note that the condition $\pi_X(\bar{a}) < \pi_I(\bar{a})$ delivers a positive lower bound on the magnitude of $(f_I - f_X)$, and not an upper bound, as would be needed to limit the size of the subsidy bill represented by the term $-\frac{(f_I-f_X)w_H N}{M_F}$. Thus, the set-up of our model does not constrain how large $f_I - f_X$ can be.)

This simple exercise highlights the key role played by firm heterogeneity and the selection effect for the results of Sections 3.1 and 3.2. When firms are instead homogenous with respect to their productivity levels, any subsidy that is effective in attracting a given Home multinational necessarily also induces the full measure of Home exporters to switch to FDI. This can imply a large subsidy burden if the fixed cost of conducting FDI is very high. In essence then, the industry equilibrium with heterogenous firms moderates the amount of FDI induced by the subsidy by sieving out the most productive Home exporters, a selection mechanism that is critical for delivering a welfare improvement.

3.4 N endogenous

Up till now, the measure of Home firms, N, has been treated as exogenous in order to highlight the shifting of the FDI cut-off, $a_I^{1-\varepsilon}$, that an FDI subsidy induces. However, when the host

country is a large market for Home, this policy action by Foreign also increases the *ex ante* profitability of potential entrant firms to the Home differentiated goods sector. I show briefly in this subsection how to incorporate this additional entry effect by endogenizing N in the full industry equilibrium. The model remains highly tractable, with the subsequent increase in the measure of Home firms reinforcing the gains that accrue to Foreign consumers.

Free-entry: N is pinned down by a free-entry condition for the Home sector, which closes the industry equilibrium in Section 2. Potential entrants do not observe their productivity draw $1/a$ until after they have started paying a per-period fixed cost of entry, f_E, expressed in units of labor. These firms therefore weigh their expected profits after entry against this fixed cost, with zero *ex ante* profits prevailing in equilibrium. This free-entry condition for the Home sector is:

$$\begin{aligned} f_E w_H &= (1-\alpha)A_H \left(\tfrac{w_H}{\alpha}\right)^{1-\varepsilon} V_H(a_D) + (1-\alpha)A_F \left(\tfrac{\tau w_H}{\alpha}\right)^{1-\varepsilon} (V_H(a_X) - V_H(a_I)) \ldots \\ &\quad \ldots + (1-\alpha)A_F \left(\tfrac{w_F}{\alpha}\right)^{1-\varepsilon} V_H(a_I) - f_D w_H G_H(a_D) \ldots \\ &\quad \ldots - f_X w_H (G_H(a_X) - G_H(a_I)) - f_I w_H G_H(a_I) \end{aligned} \tag{3.11}$$

By substituting the productivity cut-offs in (2.9)-(2.11) and the distributions in (2.12)-(2.13) into (3.11), this free-entry condition can be simplified as:

$$f_E w_H = \Lambda_H (A_H)^{\frac{k}{\varepsilon-1}} + \Lambda_F (A_F)^{\frac{k}{\varepsilon-1}} \tag{3.12}$$

This last equation has an intuitive interpretation: $\Lambda_H (A_H)^{\frac{k}{\varepsilon-1}}$ captures the normalized profits from sales in Home weighted by a measure of the level of demand in the Home market (similarly for $\Lambda_F (A_F)^{\frac{k}{\varepsilon-1}}$). The free-entry condition (3.12) thus equates the fixed cost of entry with the total expected profits for the firm from both markets.[17]

Substituting now from (2.14) and (2.15) into the definition of A_i yields:

$$(A_i)^{\frac{k}{\varepsilon-1}} = \left[(M_i)^{\frac{\alpha(1-\mu)}{\mu-\alpha}} N \frac{k}{\alpha} \Lambda_i \right]^{\frac{k(\mu-\alpha)}{\mu\alpha - k(\mu-\alpha)}}, \quad i = H, F \tag{3.13}$$

The free-entry condition (3.12) and the two equations in (3.13) comprise a system of three equations in three unknowns, N, A_H and A_F, that can be solved for the equilibrium in the Home sector. Specifically, substituting from (3.13) into (3.12) and re-arranging delivers the

[17] The key advantage of equation (3.12) is that we can collect the terms involving A_H and A_F to get a linear equation linking $(A_H)^{k/(\varepsilon-1)}$ and $(A_F)^{k/(\varepsilon-1)}$. This would not be possible if one attempted to reduce the number of model parameters by setting f_D, f_X or f_I to 0.

measure of firms N as a function of the model parameters:

$$\begin{aligned} N &= \frac{\alpha}{k}\left[\left((M_H)^{\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}}(\Lambda_H)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}+(M_F)^{\frac{k\alpha(1-\mu)}{\mu\alpha-k(\mu-\alpha)}}(\Lambda_F)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}\right)/f_E\right]^{\frac{\mu\alpha-k(\mu-\alpha)}{k(\alpha-\mu)}} \\ &= \frac{\alpha}{k}\left[\left(\tilde{M}_H\tilde{\Lambda}_H+\tilde{M}_F\tilde{\Lambda}_F\right)/f_E\right]^{\frac{\mu\alpha-k(\mu-\alpha)}{k(\alpha-\mu)}} \end{aligned} \tag{3.14}$$

Since $\mu < \alpha$, (3.14) implies that the measure of varieties decreases when fixed entry costs, f_E, or Home wages, w_H, increase. This expression for N can then be used in (2.18) to evaluate the CES consumption aggregate $(X_F)^\mu$ used in computing Foreign welfare levels. It is easy to observe that both N and $(X_F)^\mu$ exhibit similar comparative statics with respect to most of the structural parameters of the model; in particular, both variables rise as the market size, M_i, or normalized profit levels, Λ_i, of either country increase.

Allowing N to be endogenous introduces an additional varieties effect from the use of an FDI subsidy. Denoting N_{Ff} (respectively N_{Fv}) to be the measure of Home firms in the equilibrium with a fixed cost subsidy s_f (respectively, a variable cost subsidy s_v), we have:

Lemma 1: $\frac{\partial N_{Ff}}{\partial s_f}, \frac{\partial N_{Fv}}{\partial s_v} > 0$ for all $s_f, s_v < 1$.

Intuitively, the subsidy to FDI tends to raise the profitability of potential Home entrants. In equilibrium, the thickness of the supply side of this sector has to increase in response to ensure that firms continue to earn zero *ex ante* profits. Due to the "love of varieties" exhibited by the utility function, this increase in N amplifies the consumption gains arising from the use of a subsidy. For the net effect on Foreign welfare, one must weigh this against the higher subsidy bill to be paid to the large mass of Home firms. It turns out nevertheless that when N is endogenous, the welfare functions W_{Ff} or W_{Fv} continue to inherit the same shape as in the baseline case where N is fixed: Welfare is an increasing function of s_f (or s_v) when the subsidy level is negative; exhibits a positive slope when the subsidy level is zero; but hits a negative asymptote as the subsidy level approaches 1. I summarize this result in the following proposition (see Appendix 7.3 for a sketch of the proof):

Proposition 3 [N **endogenous**]: *For either fixed or variable cost subsidy schemes, the optimal subsidy policy that maximizes welfare in Foreign when N is endogenous continues to be a strictly positive subsidy level that lies in the interior of the interval* $(0, 1)$.

In sum, the varieties effect introduced when N is endogenous is an additional effect that does not alter the welfare implications from a subsidy to FDI. In fact, Appendix 7.3 shows that

the slope of the welfare function when the subsidy level is 0 is larger when N is endogenous compared to the baseline case where N is fixed (for both the fixed and variable cost cases). Thus, for small subsidy levels, the increase induced in the measure of Home firms amplifies the welfare gains accruing to Foreign. Note also that since Home consumers also benefit from the expansion of varieties, the FDI subsidy in fact generates a Pareto improvement for both countries.

3.5 Fixed versus Variable Cost Subsidies

As a final exercise for this two-country baseline model, I compare the impact and efficacy of the two types of subsidy schemes considered. In particular, how does the welfare level at the optimal fixed cost subsidy, s_f^*, compare to that at the optimal variable cost subsidy, s_v^*?

The answer to this question is best illustrated graphically. Figure 2 plots the welfare functions W_{Ff} and W_{Fv} from a simple calibration of the model. Based on Ghironi and Melitz (2005), I set the elasticity of consumer demand to $\varepsilon = 3.8$ (which implies $\alpha = 0.74$), and the key heterogeneity parameter to $k = 3.4$.[18] (The qualitative nature of Figure 2 is unchanged if the higher value of $\varepsilon = 6$ more commonly seen in the macro literature is used instead to imply a smaller price mark-up.) Following their lead, I also fix $f_X = 0.23$, $f_E = 1$, and $\tau = 1.3$. There is less precedent in the empirical literature for the remaining model parameters, although the conditions $\mu < \alpha$, $\tau w_H > w_F$, and $a_D > a_X > a_I$ impose some discipline on the values that can be chosen. While the baseline calibration in Figure 2 adopts the values: $\mu = 0.3$, $f_D = 0.1$, $f_I = 2$, $w_H = w_F = 1$, $a_H = 1$, and $M_H = M_F = 1$, the general shape of the welfare functions is nevertheless robust to alternative calibrations that continue to respect the structural assumptions of the model, including the ordering of the respective productivity cut-offs.

Several observations emerge from Figure 2. First, the shape of the welfare functions confirms the existence of a unique optimal subsidy for this parametrization, both in the fixed and variable cost cases.[19] Second, the variable cost subsidy appears to generate much higher levels of welfare than the fixed cost subsidy for subsidy rates in the interval $(0, 1)$. This confirms the earlier intuition articulated in Section 3.2 that a variable cost subsidy has the potential to generate a greater kick to welfare by reducing the distortion arising from firms'

[18] Ghironi and Melitz (2005) adopt the value $\varepsilon = 3.8$ from the empirical estimation of Bernard et al. (2003) based on US plant and trade data. Bernard et al. (2003) also estimate the log standard deviation of plant sales to be 1.67; since this moment is equal to $1/(k - \varepsilon + 1)$ in our model, this implies a value of $k = 3.4$.

[19] This is not surprising for the case of the fixed cost subsidy: As noted before, the calibration parameters satisfy the sufficient condition $2(\varepsilon - 1) > k$ for a unique turning point.

monopoly pricing power: The effective mark-ups that consumers pay is lowered, while output at each firm is raised from their inefficiently low levels. The resulting increase in equilibrium consumption potentially generates greater utility gains for Foreign workers. This production effect is absent with a fixed cost subsidy, which only affects a Home firms' decision on exporting versus FDI, but not its output levels. Third, allowing the measure of firms N to respond to the introduction of the subsidy accentuates the welfare functions without altering their general shape, as was asserted in Proposition 3. For the small to moderate positive subsidy levels graphed in Figure 2, the case with endogenous N (dotted-line graphs) has welfare levels raised above that when the measure of Home firms is fixed (solid-line graphs).[20]

Figure 2 suggests that there is a *prima facie* case in favor of variable cost subsidies, such as a reduction in the corporate tax rate or job-creation grants, from the perspective of Foreign welfare levels. We can in fact state the following result for the case where N is exogenous (see Appendix 7.4 for a proof):

Proposition 4 [Fixed versus variable cost subsidy]: *Suppose that $\varepsilon > 2$ and that the measure of Home firms is fixed. Then, a variable cost subsidy that incurs the same total subsidy bill as a fixed cost subsidy always delivers greater consumption gains to Foreign.*

In words, a variable cost subsidy has more bang for the buck, delivering a greater increase in utility from consumption than a corresponding fixed cost subsidy that incurs the same amount of public spending. It follows immediately that the welfare level achieved by the optimal variable cost subsidy, s_v^*, is higher than that reached by the optimal fixed cost subsidy, s_f^*. The requirement that $\varepsilon > 2$ also has an intuitive interpretation: Consumer demand needs to be sufficiently elastic, so that a given price decrease will generate a large increase in consumption.[21]

It is important though to identify a key caveat that will qualify the above result putting variable cost subsidies in a more favorable light. As it stands, the only motive in the model for a Home firm to open a plant in Foreign is to take advantage of the proximity-concentration tradeoff in servicing the Foreign market, so that FDI is of a purely horizontal nature. However, much of the foreign affiliate activity that takes place in the real world is intended to service more than just the local market, with some output from the foreign assembly plant being

[20]For the exogenous N graphs, the value of N used is given by (3.14) with the subsidy level set to 0.

[21]The proof of this proposition in Appendix 7.4 makes it evident that this lower bound ε can be made tighter if one is willing to take a position on the value of k. A more precise lower bound is $\varepsilon > \frac{2k+1}{k+1}$, which clearly implies $\varepsilon > 2$.

shipped back to the home or third-country markets. It turns out that the scope for welfare improvement from a variable cost subsidy to production is potentially fragile to incorporating such a re-exporting motive, since re-exports represent subsidized output for which the consumption gains accrue purely to foreigners. If the third-country market that the affiliate is servicing is large, re-exports would raise the total subsidy bill significantly without generating corresponding gains to domestic consumers, potentially negating the scope for a welfare improvement (the slope of the welfare function at $s_v = 0$ could even be negative).[22] To avoid this outcome, it would thus be important for the subsidy to be administered as a domestic sales or retail credit, instead of as a rebate to production. Note that a fixed cost subsidy, on the other hand, is robust to this criticism, since it alters only firms' mode of servicing the Foreign market, but does not affect firms' choice of output levels.

4 Some extensions

I discuss briefly now two extensions that illustrate the usefulness of the baseline framework, as well as the robustness of the welfare results on the effects of FDI subsidies.

4.1 Import subsidies

The model presented above lends itself naturally to an analysis of trade policy. It is well-established that the optimal policy for a country with a downward-sloping demand curve is to levy a positive import tariff, since the gains in tariff revenue outstrip the loss in consumer surplus when the tariff is small (see Helpman and Krugman (1989)). In view of this result, previous arguments advanced in favor of import subsidies have relied on the existence of dynamic gains: Dasgupta and Stiglitz (1988) for example posit that when learning-by-doing effects are large, it may be optimal to subsidize imports to promote learning by foreign producers, thus lowering prices in the long-run.

The framework with heterogenous firms suggests one additional mechanism through which import subsidies might be beneficial, by increasing the variety of goods that is exported to the Foreign market. In this case, however, the intuition is a little less neat: A subsidy extended by Foreign to exporters will induce some Home firms to start exporting to Foreign, delivering a positive varieties effect, but it will also prompt some Home MNCs to switch from horizontal

[22]This can be shown more formally using a three country set-up, by introducing an additional cut-off in the industry equilibrium, a_{IX}, such that Home firms with $a < a_{IX}$ will find it more profitable to service the third-country market via this re-exporting option than by direct exports from Home.

FDI back to exporting, raising prices for Foreign consumers for this margin of goods. There is thus a positive selection effect from the leftward shift of the $a_X^{1-\varepsilon}$ cut-off in Figure 1, but a detrimental effect as the $a_I^{1-\varepsilon}$ cut-off moves to the right. *A priori* at least, it is not clear if the net effect will be a positive welfare gain accruing to Foreign.

It is nevertheless straightforward to compute the welfare impact of either a fixed or variable cost subsidy to impacts, following the methodology in Sections 3.1 and 3.2. It turns out that for this alternative policy intervention, a small subsidy is indeed welfare-improving and we have the following parallel result:

Proposition 5 [Import Subsidies]: *Consider the family of fixed cost (respectively variable cost) import subsidies that satisfy a balanced-budget constraint. Then the optimal policy that maximizes welfare in Foreign is a strictly positive subsidy level.*

The proof of this proposition when N is exogenous is essentially identical to that for Propositions 1 and 2, hinging on the fact that the welfare function has a positive slope in the neighborhood where the subsidy level is zero. (See Appendix 7.5 for details.) Once again, the result relies on inequality (3.4) holding ($\frac{k}{\varepsilon-1} > \mu$, or equivalently that the distribution $G^H(a)$ is sufficiently thick-tailed), so that the firms newly drawn into exporting to Foreign are sufficiently productive firms. When N is allowed to be endogenous, the entry of more firms in the Home differentiated sector generates a positive varieties effect that reinforces the welfare gains from a small subsidy, akin to Proposition 3. Therefore, it turns out that the consumption gains from drawing in more firms at the $a_X^{1-\varepsilon}$ cut-off outweigh the loss of MNCs at the $a_I^{1-\varepsilon}$ margin. Intuitively, this is because there is a greater density of firms at the cut-off for exporting than at the cut-off for FDI.

4.2 Income effects

To this point, the quasilinear utility function (2.1) has been convenient for the analysis because the income effect from the imposition of a tax on labor income affects only the level of consumption of the homogenous good. If however demand for differentiated goods is also subject to income effects, then this decrease in disposable income could dampen the consumption gains from an FDI subsidy.

To examine the robustness of the welfare results to incorporating such income effects, I consider the other standard utility specification used in the literature in such models of heterogenous firms, in which utility is a Cobb-Douglas aggregate over consumption of homogenous

and differentiated goods. For country i, this utility function is given by:

$$U_i^{CD} = (1 - \sum_{c=H,F} \eta_c) \ln x_i^0 + \sum_{c=H,F} \eta_c \ln X_i^c \tag{4.1}$$

Here, $\eta_c \in (0,1)$ is the share of income spent on country c's differentiated goods sector; I assume that $1 - \eta_H - \eta_F > 0$, so that the income share spent on the outside good is positive. With this choice of utility function, a labor tax on Foreign workers will lower the level of demand in that market for differentiated products, but consumers can potentially compensate for this by substituting towards homogenous goods.

It turns out that even with this alternative utility function, the optimal policy for Foreign continues to be a small subsidy to multinationals, namely that the consumption gains from attracting more FDI outweigh the direct costs of funding the policy:

Proposition 6 [**Income effects**]: *The welfare results pertaining to the impact of fixed cost and variable cost subsidies in Propositions 1 and 2 continue to hold with the Cobb-Douglas utility function in (4.1). In particular, the optimal policy that maximizes welfare in Foreign is a strictly positive subsidy level in the interior of* $(0,1)$.

The proof of this proposition for the case of a fixed cost subsidy is sketched out in Appendix 7.6. It is therefore reassuring that the main welfare implications of FDI subsidies carry through even when the demand for differentiated goods is subject to income effects with a Cobb-Douglas utility function.

5 Conclusion

There has been much recent work in international trade on models of firm heterogeneity aimed at understanding the interaction between global forces and industry structure. This paper builds upon this work by extending it to a policy analysis of FDI subsidies, which have been used with increasing frequency by countries that perceive potential economic gains from attracting multinationals to their shores.

To this end, this paper has developed a two-country version of the Helpman, Melitz and Yeaple (2004) model that admits a tractable closed-form expression for consumer welfare in each country, a crucial prerequisite for a positive welfare analysis of subsidies to FDI. Although the framework in Section 2 admittedly focuses only on the consumption gains to attracting horizontal FDI (from the lowered prices of MNCs' products in the host country), it

nevertheless delivers a sharp benchmark result, namely that an FDI subsidy to either MNCs' fixed or variable costs of operation leads unambiguously to a rise in consumer welfare in the host country, after netting out the cost of financing this policy through a tax on workers' income. Of note, this result does not require us to appeal to other potential benefits of FDI, such as technological spillovers or agglomeration economies, that have received a fair amount of attention in the related policy debates.

This scope for a welfare improvement from subsidizing FDI is driven by a selection effect, highlighting the key role played by firm heterogeneity in these theoretical results. The FDI subsidy enables the host country to attract the most productive Home country exporters to switch to servicing the Foreign market via FDI. Intuitively, this result requires that the distribution of firm productivities be sufficiently thick-tailed, so that the margin of firms drawn in consists of sufficiently productive firms. What is particularly nice is that the analytic condition on the heterogeneity parameter of firm productivities is one that prior empirical research (by Helpman, Melitz and Yeaple (2004)) has shown is satisfied in practice from observed industry distributions. From a quantitative point of view, the model also suggests that a variable cost subsidy generates much greater gains to the host country than a fixed cost subsidy, given that a reduction in MNCs' variable costs of operation has an additional production effect that helps to partially correct the inefficiently low output levels stemming from individual firms' monopoly pricing power.

The framework developed in this paper lends itself readily to future work. The most natural extension to be explored is how to apply the set-up to an analysis of FDI competition among two prospective host countries, seeking to draw in multinationals from the Home country. My preliminary work on this topic has shown that it is relatively easy to derive a closed-form expression for welfare in each host country in a symmetric equilibrium in which both countries offer the same subsidy level and attract precisely half the measure N of Home MNCs to start an affiliate within their respective borders. However, it is not possible to get similarly neat closed-forms when the country subsidy offers differ and the corresponding industry equilibrium is asymmetric. A more rigorous analysis of this problem will therefore likely require the use of computational methods to understand the shape of each country's reactions function in response to the other country's choice of subsidy level. An analysis along these lines to understand the properties of the competitive subsidy equilibrium is on my research plans for the near future.

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7 Appendix

7.1 Details of Proof of Proposition 1

Proof that $W_{Ff} \to -\infty$ when $s_f \to 1^-$: Recall from (3.2) that:

$$W_{Ff} = w_F \; + \; \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\tilde{\Lambda}_{Ff}\left[\frac{1-\mu}{\mu} - s_f\left(\frac{\alpha}{k}\right)\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\right] \tag{7.1}$$

Now, from the definition of Λ_F in (2.17):

$$\Lambda_{Ff} = \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k} + \frac{((f_I-f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}w_H}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]$$

which implies that $\lim_{s_f\to 1^-}\Lambda_{Ff} = +\infty$, since $1-\varepsilon < 0$. It follows that $\lim_{s_f\to 1^-}\tilde{\Lambda}_{Ff} = +\infty$, since $\tilde{\Lambda}_{Ff}$ is Λ_{Ff} raised to a positive power ($\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)} > 0$ as $\mu < \alpha$).

Moreover, some algebraic manipulation shows that $\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} = \frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{1-s_f}g(s_f)$, where $g(s_f)$ is given by:

$$g(s_f) = \frac{\frac{((f_I-f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}{\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}}{(\tau w_H)^k} + \frac{((f_I-f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}$$

Clearly, $\lim_{s_f\to 1^-} g(s_f) = 1$, which implies that $\lim_{s_f\to 1^-}\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} = +\infty$. Hence, the limit of the term in the square brackets in (7.1) as s_f approaches 1 is $-\infty$. Together with the fact that $\lim_{s_f\to 1^-}\tilde{\Lambda}_{Ff} = +\infty$, we have that $W_{Ff} \to -\infty$ when $s_f \to 1^-$ as claimed. ■

Proof that $\frac{\partial W_F}{\partial s_f} > 0$ for all $s_f < 0$: With some algebraic manipulation, one can re-write the derivative in (3.3) as follows:

$$\begin{aligned}\frac{\partial W_{Ff}}{\partial s_f} \;=\; & \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}\frac{\tilde{M}_F}{M_F}\frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\frac{\alpha}{k}\left[\frac{k(1-\alpha)-\mu\alpha}{\mu\alpha-k(\mu-\alpha)}\cdots\right.\\ & \left.\cdots + s_f\frac{k(\alpha-\mu)}{\mu\alpha-k(\mu-\alpha)}\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} - \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1}\right]\end{aligned} \tag{7.2}$$

The first summand on the right-hand side is positive, since $k(1-\alpha)-\mu\alpha > 0$ follows from $\frac{k}{\varepsilon-1} > \mu$. Now observe that for $s_f < 0$, the last two summands are:

$$\begin{aligned} s_f\frac{k(\alpha-\mu)}{\mu\alpha-k(\mu-\alpha)}\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} - \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1} &= \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1}\left[\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha-k(\mu-\alpha)}g(s_f)-1\right]\\ &> \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1}\left[\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha-k(\mu-\alpha)}-1\right]\\ &= \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1}\frac{(\alpha-\mu)(1-\varepsilon)-\mu\alpha}{\mu\alpha-k(\mu-\alpha)}\\ &> 0 \end{aligned} \tag{7.3}$$

where we have relied on the fact that $\mu < \alpha$ and $\varepsilon > 1$.

Hence, the expression in the square brackets in (7.2) is positive, from which it follows that $\frac{\partial W_{Ff}}{\partial s_f} > 0$ whenever $s_f < 0$. ■

Proof that $2(\varepsilon - 1) > k$ is a sufficient condition for W_{Ff} to have a unique turning point: Setting the derivative in (7.2) equal to zero, we have that any turning point of the welfare function must satisfy:

$$\tilde{g}(s_f) \equiv \frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu - \alpha)} \frac{1 - s_f}{s_f} + \frac{k}{\varepsilon - 1} \left(\frac{(k - \varepsilon + 1)(\alpha - \mu)}{\mu\alpha - k(\mu - \alpha)} g(s_f) - 1 \right) = 0 \tag{7.4}$$

Note first that $\frac{1-s_f}{s_f}$ is a decreasing function of s_f, while $g(s_f)$ is increasing in s_f, so it is not possible to conclude in general that $\tilde{g}(s_f)$ is a monotonic function. Nevertheless, $\lim_{s_f \to 0^+} \tilde{g}(s_f) = +\infty$, while $\lim_{s_f \to -} \tilde{g}(s_f) = \frac{k}{\varepsilon - 1} \left(\frac{(k-\varepsilon+1)(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - 1 \right) < 0$. For $\tilde{g}(s_f)$ to have a unique zero in $(0, 1)$, it suffices that $\tilde{g}(s_f)$ be a strictly convex function in this interval. It is easy to check that $\frac{1-s_f}{s_f}$ is indeed strictly convex. Since the sum of two convex functions is also convex, I explore a sufficient condition for $g(s_f)$ to be convex. Some differentiation and algebraic substitution yields:

$$g''(s_f) = \frac{k - \varepsilon + 1}{\varepsilon - 1} \frac{g(s_f)(1 - g(s_f))}{(1 - s_f)^2} \left[\frac{k}{\varepsilon - 1} - 2\frac{k - \varepsilon + 1}{\varepsilon - 1} g(s) \right]$$

Since $g(s_f) \in (0, 1)$ for $s_f \in (0, 1)$, we have strict convexity if and only if $g(s_f) < \frac{k}{2(k-\varepsilon+1)}$. A sufficient condition is therefore: $1 < \frac{k}{2(k-\varepsilon+1)}$, or equivalently $2(\varepsilon - 1) > k$. ■

7.2 Details of Proof of Proposition 2

Proof that $W_{Fv} \to -\infty$ when $s_v \to 1^-$: Recall from (3.6) that:

$$W_{Fv} = w_F \; + \; \left(N \frac{k}{\alpha} \right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \tilde{\Lambda}_{Fv} \left[\frac{1 - \mu}{\mu} - s_v \left(\frac{\alpha}{k} \right) \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \right] \tag{7.5}$$

where:

$$\Lambda_{Fv} = \frac{\varepsilon - 1}{k - \varepsilon + 1} \left(\frac{\alpha}{a_H} \right)^k \left(\frac{1 - \alpha}{w_H} \right)^{\frac{k}{\varepsilon - 1}} \left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(\tau w_H)^k} + \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{(((1 - s_v) w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}} \right]$$

Analogous to the proof in Appendix 7.1, we have $\lim_{s_v \to 1^-} \Lambda_{Fv} = \lim_{s_f \to 1^-} \tilde{\Lambda}_{Ff} = +\infty$.

Some algebraic work shows that $\frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} = \frac{k}{1-s_v} \frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} h(s_v)$, where $h(s_v)$ is given by:

$$h(s_v) = \frac{\frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}{\frac{(f_X)^{\frac{k}{1-\varepsilon}+1}}{(\tau w_H)^k} + \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}}$$

Now, $\lim_{s_v \to 1^-} h(s_v) = 1$, while $\lim_{s_v \to 1^-} \frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} = \left[1 - \left(\frac{\tau w_H}{w_F}\right)^{1-\varepsilon}\right]^{-1} > 0$. Together, these imply that $\lim_{s_v \to 1^-} \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} = +\infty$. The limit of the term in the square brackets in (7.5) as $s_v \to 1^-$ is therefore $-\infty$, so that we have $\lim_{s_v \to 1^-} W_{Ff} = -\infty$ as desired. ■

Proof that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ for all $s_v < 0$: With some algebraic manipulation, one can re-write the derivative in (3.7) as:

$$\begin{aligned} \frac{\partial W_{Fv}}{\partial s_v} &= \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F} \frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \frac{\alpha}{k} \left[\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} + s_v \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} \cdots \right. \\ &\qquad \left. \cdots - \frac{s_v}{1-s_v} \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}\right] \end{aligned} \tag{7.6}$$

Since we know that $\frac{k(1-\alpha)-\mu\alpha}{\mu\alpha - k(\mu-\alpha)} > 0$, it suffices to show that the last two summands on the right-hand side add up to a positive quantity. Observe first that $(a_X)^{1-\varepsilon} < (a_I)^{1-\varepsilon}$ implies that $\frac{f_X}{(\tau w_H)^{1-\varepsilon}} < \frac{f_I - f_X}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}$. Using this inequality to replace f_X in the denominator of $h(s_v)$ and simplifying, one obtains: $h(s_v) < \frac{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}}$. For $s_v < 0$, we then have:

$$\begin{aligned} & s_v \frac{k(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{1}{\Lambda_{Fv}} \frac{\partial \Lambda_{Fv}}{\partial s_v} - \frac{s_v}{1-s_v} \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} \\ = \; & \frac{s_v}{1-s_v} \left[\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} \frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} h(s_v) - \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}\right] \\ > \; & \frac{s_v}{1-s_v} \left[\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - \frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}}\right] \end{aligned}$$

For this last expression to be positive whenever $s_v < 0$, it suffices to show that:

$$\frac{(k+1)((1-s_v)w_F)^{1-\varepsilon} - \varepsilon(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon}} > k \tag{7.7}$$

since $\frac{k^2(\alpha-\mu)}{\mu\alpha - k(\mu-\alpha)} - k = \frac{-k\mu\alpha}{\mu\alpha - k(\mu-\alpha)} < 0$. This is sufficient to ensure that the expression in the square brackets in (7.6) is positive, so that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ whenever $s_v < 0$.

Bearing in mind that $((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon} > 0$, a re-arrangement of (7.7) yields:

$$((1-s_v)w_F)^{1-\varepsilon} - (\tau w_H)^{1-\varepsilon} + (k - \varepsilon + 1)(\tau w_H)^{1-\varepsilon} > 0$$

which clearly holds, since $k - \varepsilon + 1 > 0$. ■

7.3 Proofs from Section 3.4 (N endogenous)

Proof of Lemma 1: Log-differentiating (3.14), one has:

$$\frac{\partial N_{Ff}}{\partial s_f} = N_{Ff}\frac{\mu\alpha}{k(\alpha-\mu)}\frac{\tilde{M}_F\tilde{\Lambda_{Ff}}}{\tilde{M}_H\tilde{\Lambda_H}+\tilde{M}_F\tilde{\Lambda_{Ff}}}\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} > 0$$

for all $s_f < 1$. An analogous expression holds for $\frac{\partial N_{Fv}}{\partial s_v}$ with s_f replaced by s_v and the subscript sf replaced by sv. ■

Sketch of proof of Proposition 3: I illustrate this proof for the case of a fixed cost subsidy, since the argument for the case of a variable cost subsidy is virtually identical. Using the expression for W_{Ff} from (7.1), we have shown in Appendix 7.1 that $\tilde{\Lambda}_{Ff}\left[\frac{1-\mu}{\mu} - s_f\left(\frac{\alpha}{k}\right)\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\right]$ is a positive increasing function of s_f when $s_f < 0$. Since N_{Ff} is also a positive increasing function of s_f for all $s_f < 1$, this implies that W_{Ff} must be increasing in s_f when s_f is negative.

Also, observe that $\lim_{s_f\to 1^-} N_{Ff} = +\infty$. Since $\tilde{\Lambda}_{Ff}\left[\frac{1-\mu}{\mu} - s_f\left(\frac{\alpha}{k}\right)\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\right]$ tends to $-\infty$ when s_f approaches 1, this implies that $\lim_{s_f\to 1^-} W_{Ff} = -\infty$.

Finally, the expression for $\frac{\partial W_{Ff}}{\partial s_f}$ when N is endogenous is given by (7.2) plus an extra term (from the product rule) to reflect the effect of s_f on N:

$$\left(\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}(N_{Ff})^{\frac{k(\mu-\alpha)}{\mu\alpha-k(\mu-\alpha)}}\frac{\partial N_{Ff}}{\partial s_f}\frac{\tilde{M}_F}{M_F}\tilde{\Lambda}_{Ff}\left[\frac{1-\mu}{\mu} - s_f\left(\frac{\alpha}{k}\right)\frac{1}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f}\right]$$

This term is clearly positive when evaluated at $s_f = 0$, and hence the slope of W_{Ff} at $s_f = 0$ is larger when N is endogenous when compared to the baseline case when N is fixed. ■

7.4 Proof of Proposition 4

Proof: The proof proceeds via contradiction. Suppose that the total subsidy bills from s_f and s_v are equal. From (7.1) and (7.5), this implies that: $s_f\frac{\tilde{\Lambda}_{Ff}}{\Lambda_{Ff}}\frac{\partial\Lambda_{Ff}}{\partial s_f} = s_v\frac{\tilde{\Lambda}_{Fv}}{\Lambda_{Fv}}\frac{\partial\Lambda_{Fv}}{\partial s_v}$, or equivalently that:

$$\frac{s_f}{1-s_f}\tilde{\Lambda}_{Ff}\frac{k-\varepsilon+1}{\varepsilon-1}g(s_f) = \frac{s_v}{1-s_v}\tilde{\Lambda}_{Fv}\frac{k((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}h(s_v) \tag{7.8}$$

However, suppose to the contrary that the consumption gains from the fixed cost subsidy are larger; pulling out the relevant terms from the welfare functions, this means that $\tilde{\Lambda}_{Ff} \geq \tilde{\Lambda}_{Fv}$. From the definition in (2.17), this assumption simplifies to:

$$\frac{((f_I-f_X)(1-s_f))^{\frac{k}{1-\varepsilon}+1}}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}} \geq \frac{(f_I-f_X)^{\frac{k}{1-\varepsilon}+1}}{(((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}} \tag{7.9}$$

Observe though that (7.9) implies that $g(s_f) \geq h(s_v)$, and hence that $\tilde{\Lambda}_{Ff} g(s_f) \geq \tilde{\Lambda}_{Fv} h(s_v)$. Looking back at (7.8), we must therefore have:

$$\frac{s_f}{1-s_f}\frac{k-\varepsilon+1}{\varepsilon-1} < \frac{s_v}{1-s_v}\frac{k((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}} \tag{7.10}$$

Now (7.9) simplifies directly to $s_f \geq 1 - \left[\frac{(w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}\right]^{\frac{k}{k-\varepsilon+1}}$, so that:

$$\frac{s_f}{1-s_f} \geq \left[\frac{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}\right]^{\frac{k}{k-\varepsilon+1}} - 1 \tag{7.11}$$

Combining (7.10) and (7.11), and eliminating $\frac{s_f}{1-s_f}$, the following inequality needs to be satisfied:

$$\left[\frac{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}\right]^{\frac{k}{k-\varepsilon+1}} - 1 - \frac{s_v}{1-s_v}\frac{k(\varepsilon-1)}{k-\varepsilon+1}\frac{((1-s_v)w_F)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}} < 0 \tag{7.12}$$

Let us define the function in s_v on the left-hand side of this last inequality as $\psi(s_v)$. Observe that $\psi(0) = 0$. I shall now show that if $\varepsilon > 2$, then $\psi'(s_v) > 0$ for all $s_v \in (0,1)$, so that in fact $\psi(s_v) > 0$ for all positive subsidy levels. This will yield the desired contradiction to (7.12). Some algebra shows that $\psi'(s_v)$ is equal up to a positive multiplicative constant to:

$$\begin{aligned}
& \left[\frac{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}{(w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}\right]^{\frac{k}{k-\varepsilon+1}} - \frac{1}{1-s_v} + \frac{s_v}{1-s_v}\frac{(\varepsilon-1)(\tau w_H)^{1-\varepsilon}}{((1-s_v)w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}} \\
> & \left[1 + \frac{((1-s_v)^{1-\varepsilon}-1)(w_F)^{1-\varepsilon}}{(w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon}}\right]^{\frac{k}{k-\varepsilon+1}} - \frac{1}{1-s_v} \\
> & (1+((1-s_v)^{1-\varepsilon}-1))^{\frac{k(1-\varepsilon)}{k-\varepsilon+1}} - \frac{1}{1-s_v} \\
= & \frac{1}{1-s_v}\left[(1-s_v)^{\frac{k(1-\varepsilon)}{k-\varepsilon+1}+1} - 1\right]
\end{aligned}$$

Since $(1-s_v) \in (0,1)$, this last expression is positive if and only if: $\frac{k(1-\varepsilon)}{k-\varepsilon+1} + 1 < 0$. This holds when $\varepsilon > \frac{2k+1}{k+1}$, and in particular when $\varepsilon > 2$. ■

7.5 Sketch of proof of Proposition 5

The proofs concerning the welfare implications of an import subsidy mirror closely those for an FDI subsidy in Sections 3.1 and 3.2, as well as Appendix 7.1 and 7.2. The exposition below is therefore brief, showing once again that the welfare function under an import subsidy has a positive slope when the subsidy level is less than or equal to zero, but asymptotes towards

$-\infty$ as the subsidy level tends towards its maximum value. The optimal policy is therefore a subsidy level that is strictly positive.

Fixed cost subsidy to Home exporters: Consider first a subsidy that reduces the fixed cost of exporting for each Home firm by the amount $s_f f_X w_H$, with $s_f < 1$. Suppose as before that this is financed by a tax on labor income equal to $t_f w_H$. The relevant balanced budget constraint for Foreign is now:

$$t_f w_F M_F = s_f f_X w_H N(G^H(a_X) - G^H(a_I)) \tag{7.13}$$

Substituting the implied value of t_f from (7.13) into the definition of $W_{Ff} = (1-t_f)w_F + \frac{1-\mu}{\mu}(X_F)^\mu$, one obtains the following expression after some work for welfare in Foreign:

$$W_{Ff} = w_F \; + \; \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\left[\frac{1-\mu}{\mu}\tilde{\Phi}_{Ff} - s_f\left(\frac{\alpha}{k}\right)\frac{\tilde{\Phi}_{Ff}}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f}\right] \tag{7.14}$$

where:

$$\begin{aligned}
\Phi_{Ff} &= \frac{\varepsilon-1}{k-\varepsilon+1}\left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{((1-s_f)f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k} + \frac{(f_I-(1-s_f)f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right] \\
\frac{\partial \Phi_{Ff}}{\partial s_f} &= \left(\frac{\alpha}{a_H}\right)^k\left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}}\left[\frac{(1-s_f)^{\frac{k}{1-\varepsilon}}(f_X)^{\frac{k}{1-\varepsilon}+1}w_H}{(\tau w_H)^k} - \frac{(f_I-(1-s_f)f_X)^{\frac{k}{1-\varepsilon}}f_X w_H}{((w_F)^{1-\varepsilon}-(\tau w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right]
\end{aligned}$$

and $\tilde{\Phi}_{Ff} = (\Phi_{Ff})^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}$. Note that Φ_{Ff} is precisely equal to Λ_F with f_X replaced by $(1-s_f)f_X$. (The switch of notation to Φ is intended to avoid a clash with Λ, which has been used for the analysis of FDI subsidies.) The welfare function in (7.14) clearly parallels that in (7.1) for the case of a fixed cost subsidy to FDI, except that the *ex ante* profits from sales in Foreign are now given by Φ_{Ff}. The expression for Φ_{Ff} also makes apparent the two opposing effects that an import subsidy has: The first summand in the square brackets captures how s_f lowers the $a_X^{1-\varepsilon}$ threshold for exporting, which tends to increase the consumption gains for Foreign, but the second summand captures how s_f raises the $a_I^{1-\varepsilon}$ cut-off for FDI, which acts to lower these consumption gains instead.

Differentiating (7.14) with respect to s_f yields:

$$\begin{aligned}
\frac{\partial W_{Ff}}{\partial s_f} &= \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha-k(\mu-\alpha)}}\frac{\tilde{M}_F}{M_F}\frac{\tilde{\Phi}_{Ff}}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f}\frac{\alpha}{k}\left[\frac{k(1-\alpha)-\mu\alpha}{\mu\alpha-k(\mu-\alpha)}\cdots\right. \\
&\qquad \left.\cdots + s_f\frac{k(\alpha-\mu)}{\mu\alpha-k(\mu-\alpha)}\frac{1}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f} - s_f\left(\frac{\partial^2\Phi_{Ff}}{\partial s_f^2}\Big/\frac{\partial \Phi_{Ff}}{\partial s_f}\right)\right]
\end{aligned} \tag{7.15}$$

Evaluating s_f at 0, it is straightforward to check once again that $\frac{\partial W_{Ff}}{\partial s_f} > 0$, given that $\frac{k}{\varepsilon-1} > \mu$. Thus, a small subsidy to exporting firms from Home raises indirect utility in Foreign.

Moreover, as $s_f \longrightarrow 1^-$, we have $\Phi_{Ff}, \tilde{\Phi}_{Ff}, \frac{1}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f} \longrightarrow +\infty$. This implies from (7.14) that W_{Ff} asymptotes to $-\infty$ as s_f tends towards its maximum value of 1.

Last but not least, $\frac{\partial \Phi_{Ff}}{\partial s_f} > 0$ so long as $a_X^{1-\varepsilon} < a_I^{1-\varepsilon}$. Also, one can verify (albeit tediously) that $\frac{1}{\Phi_{Ff}}\frac{\partial \Phi_{Ff}}{\partial s_f} < \frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{1-s_f}$ and $\frac{\partial^2 \Phi_{Ff}}{\partial s_f^2} / \frac{\partial \Phi_{Ff}}{\partial s_f} > \frac{k}{\varepsilon-1}\frac{1}{1-s_f}$. Substituting these two inequalities into the expression in square brackets in (7.15), one can then show that when $s_f < 0$, we have $\frac{\partial W_{Ff}}{\partial s_f} > 0$. ■

Variable cost subsidy to Home exporters: I examine now a subsidy that reduces the unit cost of exporting for a Home firm with productivity parameter a from $\frac{\tau a w_H}{\alpha}$ to $\frac{(\tau - s_v) a w_H}{\alpha}$. This subsidy scheme is financed by a tax on labor income in Foreign at tax rate t_v satisfying the balanced-budget constraint:

$$t_v w_F M_F = s_v w_H N A_F \left(\frac{(\tau - s_v) w_H}{\alpha}\right)^{-\varepsilon} (V^H(a_X) - V^H(a_I)) \tag{7.16}$$

Substituting the implied value of t_v from (7.16) into $W_{Fv} = (1 - t_v) w_F + \frac{1-\mu}{\mu}(X_F)^\mu$, one obtains:

$$W_{Fv} = w_F \; + \; \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\left[\frac{1-\mu}{\mu}\tilde{\Phi}_{Fv} - s_v\left(\frac{\alpha}{k}\right)\frac{\tilde{\Phi}_{Fv}}{\Phi_{Fv}}\frac{\partial \Phi_{Fv}}{\partial s_v}\right] \tag{7.17}$$

where:

$$\begin{aligned}
\Phi_{Fv} &= \frac{\varepsilon - 1}{k - \varepsilon + 1}\left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} \left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((\tau - s_v) w_H)^k} + \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((w_F)^{1-\varepsilon} - ((\tau - s_v) w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}}}\right] \\
\frac{\partial \Phi_{Fv}}{\partial s_v} &= \frac{k(\varepsilon - 1)}{k - \varepsilon + 1}\left(\frac{\alpha}{a_H}\right)^k \left(\frac{1-\alpha}{w_H}\right)^{\frac{k}{\varepsilon-1}} (\tau - s_v)^{-\varepsilon} w_H^{1-\varepsilon} \ldots \\
&\qquad \ldots \times \left[\frac{(f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((\tau - s_v) w_H^{1-\varepsilon})^{\frac{k}{1-\varepsilon}+1}} - \frac{(f_I - f_X)^{\frac{k}{1-\varepsilon}+1} w_H}{((w_F)^{1-\varepsilon} - ((\tau - s_v) w_H)^{1-\varepsilon})^{\frac{k}{1-\varepsilon}+1}}\right]
\end{aligned}$$

and $\tilde{\Phi}_{Fv} = (\Phi_{Fv})^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}}$.

Differentiating (7.17) with respect to s_v yields:

$$\begin{aligned}
\frac{\partial W_{Fv}}{\partial s_v} &= \left(N\frac{k}{\alpha}\right)^{\frac{\mu\alpha}{\mu\alpha - k(\mu-\alpha)}} \frac{\tilde{M}_F}{M_F}\frac{\tilde{\Phi}_{Fv}}{\Phi_{Fv}}\frac{\partial \Phi_{Fv}}{\partial s_v}\frac{\alpha}{k}\left[\frac{k(1-\alpha) - \mu\alpha}{\mu\alpha - k(\mu-\alpha)} \ldots\right. \\
&\qquad \left. \ldots + s_f \frac{k(\alpha - \mu)}{\mu\alpha - k(\mu-\alpha)}\frac{1}{\Phi_{Fv}}\frac{\partial \Phi_{Fv}}{\partial s_v} - s_v\left(\frac{\partial^2 \Phi_{Fv}}{\partial s_v^2} / \frac{\partial \Phi_{Fv}}{\partial s_v}\right)\right]
\end{aligned} \tag{7.18}$$

which parallels (7.18) closely with s_f replaced by s_v and Φ_{Ff} replaced by Φ_{Fv}. It follows immediately that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ at $s_v = 0$ as before.

In addition, it is straightforward to check that as $s_v \longrightarrow \tau^-$, we have $\Phi_{sv}, \tilde{\Phi}_{sv}, \frac{1}{\Phi_{sv}}\frac{\partial \Phi_{Fv}}{\partial s_v} \longrightarrow +\infty$. Thus, $W_{sv} \longrightarrow -\infty$ as s_v approaches its maximum value of τ.

Finally, one can show (with some work) that $\frac{1}{\Phi_{sv}}\frac{\partial \Phi_{Fv}}{\partial s_v} < \frac{k}{\tau - s_v}$ and $\frac{\partial^2 \Phi_{Fv}}{\partial s_v^2} / \frac{\partial \Phi_{Fv}}{\partial s_v} > \frac{k+1}{\tau - s_v}$. In addition, $\frac{\partial \Phi_{Fv}}{\partial s_v} > 0$ so long as $a_X^{1-\varepsilon} < a_I^{1-\varepsilon}$. Together, these observations imply that $\frac{\partial W_{Fv}}{\partial s_v} > 0$ whenever $s_v < 0$. ■

7.6 Sketch of proof of Proposition 6

Proof: I illustrate the robustness of the welfare results to the alternative utility specification in (4.1) for the case of a subsidy to the fixed costs of FDI. The proof for a variable cost subsidy is very similar. The derivations below follow closely that from Sections 2 and 3.1.

It is well-known that maximizing (4.1) subject to the budget constraint (2.3) delivers the following individual demand functions for homogenous goods and differentiated products respectively in Foreign: $(x_F^0)^{CD} = (1 - \eta_H - \eta_F)w_F$ and $x_F^H(a) = \frac{(A_F^{CD})}{M_F} p_F^H(a)^{-\varepsilon}$, where A_F^{CD} is the market demand level in Foreign for Home's differentiated products, given explicitly by:

$$A_F^{CD} = \frac{\eta_F M_F w_F}{\int_{\Omega_F^H} p_F^H(a)^{1-\varepsilon} dG^H(a)} \tag{7.19}$$

Note that the superscript "*CD*" is used to refer to variables for the solution under this Cobb-Douglas utility specification.

The industry equilibrium for the Home differentiated goods sector is identical to that in the baseline model with quasilinear utility in Section 2. Therefore, following from (2.15), the ideal price index in the denominator of (7.19) is still equal to:

$$\int_{\Omega_F^H} p_F^H(a)^{1-\varepsilon} dG^H(a) = N\frac{k}{\alpha}\Lambda_F (A_F^{CD})^{\frac{k-\varepsilon+1}{\varepsilon-1}} \tag{7.20}$$

Substituting from (7.20) into (7.19), one obtains:

$$(A_F^{CD})^{\frac{k}{\varepsilon-1}} = \frac{\eta_F M_F w_F}{N\frac{k}{\alpha}\Lambda_F} \tag{7.21}$$

Meanwhile, substituting for $x_F^H(a)$ in the definition of X_F^H implies that:

$$(X_F^H)^{CD} = \eta_F w_F \left(\int_{\Omega_F^H} p_F^H(a)^{1-\varepsilon} dG^H(a) \right)^{\frac{1}{\alpha}} = \eta_F w_F \left[N\frac{k}{\alpha}\Lambda_F (A_F^{CD})^{\frac{k-\varepsilon+1}{\varepsilon-1}} \right]^{\frac{1}{\varepsilon-1}} \tag{7.22}$$

We can now compute the indirect utility function by substituting the consumer demand functions into (4.1). Making the relevant substitutions using (7.21) and (7.22), this is equal up to an additive constant to the following which I will use as our relevant measure of welfare:

$$W_F^{CD} = \left[1 + \frac{k - \varepsilon + 1}{k(\varepsilon - 1)}(\eta_H + \eta_F)\right] \ln w_F + \frac{\eta_F}{k} \ln(\Lambda_F) \tag{7.23}$$

Let us examine now a subsidy by Foreign that reduces the fixed cost of FDI for Home firms by $s_f f_I w_H$. The balanced budget constraint (3.1) still applies. Substituting from (7.21) into this budget constraint and simplifying, one obtains an implied income tax rate of $t_f = s_f \frac{\eta_F \alpha}{k} \frac{1}{\Lambda_{Ff}} \frac{\partial \Lambda_{Ff}}{\partial s_f}$ in Foreign. Replacing w_F by $(1 - t_f)w_F$ in (7.23) and differentiating with respect to s_f yields:

$$\frac{\partial W_F^{CD}}{\partial s_f} = -\frac{\eta_F \alpha}{k}\left[1 + \frac{k - \varepsilon + 1}{k(\varepsilon - 1)}(\eta_H + \eta_F)\right]\left[\frac{\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{sf}}{\partial s_f} - s_f\left(\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}\right)^2 + s_f \frac{1}{\Lambda_{Ff}}\frac{\partial^2 \Lambda_{Ff}}{\partial s_f^2}}{1 - s_f \frac{\eta_F \alpha}{k}\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}}\right] + \frac{\eta_F}{k}\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f} \tag{7.24}$$

When $s_f = 0$, the sign of this derivative is given by $sign\left\{-\frac{\eta_F \alpha}{k}\left[1 + \frac{k-\varepsilon+1}{k(\varepsilon-1)}(\eta_H + \eta_F)\right]\right\} = sign\left\{k(1 - \eta_H - \eta_F) + (\varepsilon - 1)(\eta_H + \eta_F)\right\} > 0$. Hence, a small subsidy does indeed improve welfare with Cobb-Douglas preferences.

Note that $\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}$ is an increasing function in s_f which tends to $+\infty$ as s_f tends to 1. However, the maximum value that t_f can take is 1, which implies a maximum feasible value for the subsidy s_f as $s_f \frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f} \to 1$. But as $t_f \to 1^-$, W_F^{CD} asymptotes towards $-\infty$.

Last but not least, for $s_f < 0$, it suffices to check that $s_f\left(\frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}\right)^2 - s_f \frac{1}{\Lambda_{Ff}}\frac{\partial^2 \Lambda_{Ff}}{\partial s_f^2} = \frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f}\left(s_f \frac{1}{\Lambda_{Ff}}\frac{\partial \Lambda_{Ff}}{\partial s_f} - \frac{s_f}{1-s_f}\frac{k}{\varepsilon-1}\right) > 0$ to ensure that the derivative in (7.24) is positive. But this last inequality can be verified in a manner analogous to (7.3) in Appendix 7.1. ■

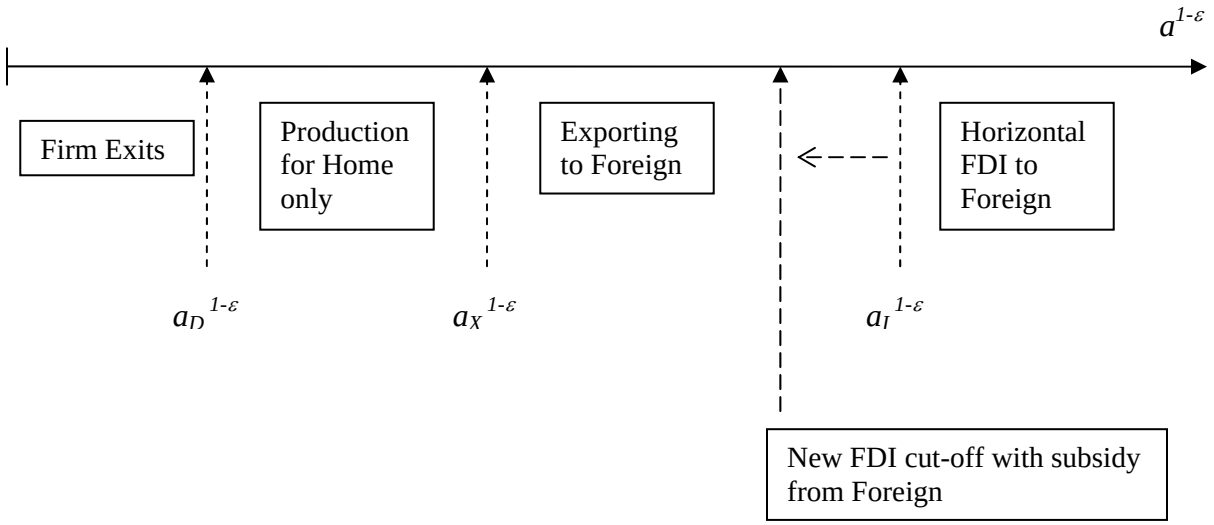

Figure 1: The Sorting Pattern within Home's Differentiated Goods Sector

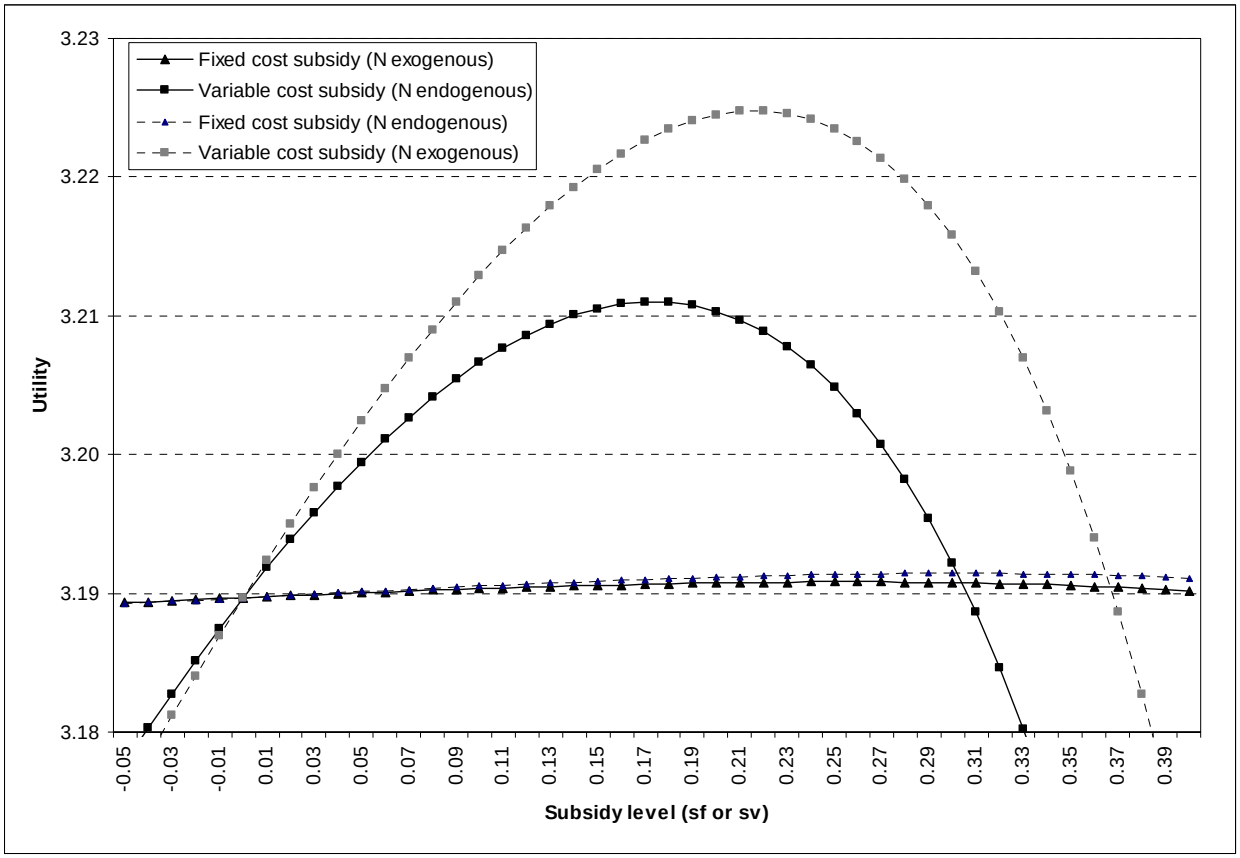

Figure 2: Some Sample Calibrated Welfare Functions

Notes: Calibration parameters are: $k = 3.4$, $\varepsilon = 3.8$ (which implies $\alpha = 0.74$), $\mu = 0.3$, $f_D = 0.1$, $f_X = 0.23$, $f_I = 2$, $w_H = w_F = 1$, $\tau = 1.3$, $a_H = 1$, $f_E = 1$, $M_H = M_F = 1$. These parameter choices imply the order of productivity cut-offs imposed in the model, namely $a_D > a_X > a_I$. For the cases where the measure of Home firms is exogenous, the value of N used is that obtained when setting the subsidy level to 0 in equation (3.14).