

# Strategic Asset Allocation, Asset Price Dynamics, and the Business Cycle

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## Abstract

The main contribution of this work is to provide a dynamic general equilibrium model of asset allocation, allowing to reconcile economic theory with several puzzling contradictions recently pointed out in the literature: (i) the asset allocation puzzle, (ii) the observed time-variation in aggregate portfolio holdings and (iii) the occurrence of twin peaks in equity and house prices. In this approach, compared to the existing literature, the main difference stems from the fact that, in addition to consumption and dividends, both prices and portfolio decisions are allowed to be endogenously determined within a general equilibrium framework. Secondly, real estate is introduced into the analysis, labor supply is allowed to be endogenously determined and macroeconomic shocks are the main source of riskiness.

- Keywords: Strategic Asset Allocation, House Prices
- JEL: E20, G11, G12

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# 1 Introduction

## 1.1 Issues and Motivation

While linking financial markets to the real economy constitutes one of the most central issue in finance [see Cochrane (2005)], the academic literature has yet failed to provide a convincing model able to reconcile asset pricing facts with the macro economy. Despite some recent improvements, our understanding of the role of macroeconomic risks in explaining fluctuations in financial markets is still limited. From the macroeconomic perspective, the difficulty to account for asset pricing facts using standard models has led many researchers to dismiss asset market data<sup>1</sup>. As regards the finance literature, restrictive assumptions regarding the evolution of consumption or returns that are introduced in most finance models are to be blamed. For instance, following the endowment economy approach of Lucas (1978), exogenous processes are often introduced in models to describe the evolution of key economic variables such as consumption.

However, as emphasized by the literature on asset pricing in production economies, ignoring the saving and investment decision, which is the building block of dynamic macroeconomic theory, can be seriously misleading. Once consumption is allowed to be endogenously determined, as shown by Jermann (1998)<sup>2</sup>, the key finding that habit formation is a solution to the equity premium puzzle collapses. In a general equilibrium model with production, when consumption and dividends are both endogenously determined, habit formation alone has hardly no impact on the equity premium. The fact that such a major implication of endowment economy models do not survive in a more general setting raises some methodological concerns. And as suggested by Campbell (2002), it seems that adopting an approach where dividends and consumption could be endogenously derived within a general equilibrium model would constitute a major improvement.

When it comes to finding a successor the endowment economy model, as stated by Cochrane (2005), the recent developments in the macroeconomic literature on asset pricing offer some promising new directions of research. Following the literature on real business cycles [see King and Rebelo (2000)], several authors have shown that dynamic general equilibrium models were able to generate implications that could reconcile asset pricing and business

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<sup>1</sup>See Cochrane (2005)

<sup>2</sup>See also Christiano, Boldrin and Fisher (2001)

cycle facts. Successful contributions, including the work of Jermann (1998); Boldrin, Christiano and Fisher (2001); and Jermann (2005), have shown that third-generation real business cycle models are able to explain business cycle facts while generating predictions for the equity premium and the mean risk free rate that are in line with the puzzling empirical evidence. As shown by these studies, in a model where both dividends and consumption are endogenously determined, an additional friction such as capital adjustment cost or limited labor mobility is needed in addition to habit formation, to explain the equity premium puzzle.

However, despite these recent improvements, when it comes to the study of another central issue in financial economics, asset allocation decisions, the dynamic general equilibrium approach appears to remain widely unexplored. The widening gap between financial planners, offering portfolio advice to long-term investors, and finance theory, illustrates the failure of current models to deliver plausible predictions. The fact that economic models have been disregarded by financial advisors is probably due to the technical complications that have to be overcome in order to build realistic frameworks. As explained by Campbell, Chan and Viceira (2003), until recently, the difficulties to solve multi-period choice portfolio settings were a major obstacle to building models able to account for the dynamic dimension of the problem. However, despite the recent developments in numerical methods, very few studies have attempted to provide a framework that could be used to study the impact of business cycle fluctuations on asset allocation decisions. In addition, although in America, for instance, the typical household on an average income holds six times as much wealth in residential property as in shares<sup>3</sup>, existing asset allocation models ignore real estate [see Campbell and Viceira (2001); Brennan, Schwartz and Lagnado (1997)]. This illustrates, as stated by Campbell and Viceira (2001), the current failure of the academic literature to provide a unifying framework able to account for the complete portfolio problem.

The main objective of this work is to contribute to fill this gap by constructing a dynamic general equilibrium model of asset allocation. Compared to the existing literature, the main difference stems from the fact that, firstly, in this economy, in addition to consumption and dividends, both prices and portfolio decisions are allowed to be endogenously determined. Secondly, real estate is introduced into the analysis, labor supply is allowed to be endoge-

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<sup>3</sup>Source: The Economist, Survey Property: House of Cards, May 29th 2003

nously determined and the main source of macroeconomic risks stems from aggregate technology shocks. Compared to the standard literature on the subject, in this economy, the representative agent faces the following allocation problem. Each period, households have to choose the number of hours to work and how much of their revenue to share between consumption, investment and saving. The investment decision consists of deciding how much to invest in the representative firm and to allocate to the accumulation of residential real estate. The saving decision is modelled as in Coreia, Neves and Rebelo (1995), the only difference being that a demand for domestic saving is introduced, which allows for an endogenous determination of the interest rate on saving. Domestic savings are assumed to be absorbed by a foreign firm which issues bonds to finance its activity. The model is presented in section 2.

The second objective is to investigate whether the framework that has been constructed is able to bring some new implications in terms of asset allocation and asset pricing facts. We start by asking whether the model that has been constructed could reconcile economic theory with several puzzling contradictions recently pointed out in the macroeconomic and finance literature. Firstly, as documented by Canner, Mankiw and Weil (1997), the standard mean-variance approach leads to the striking conclusion that investors should hold risky assets in the same proportion, regardless of their risk preference. Not surprisingly, this conclusion is contradicted by the facts. In practice financial advisors recommend customers with different risk aversion to hold bonds and stocks in different proportions. In section 3, compared to what is usually done in the finance literature, we propose to reexamine these facts taking a different perspective. In particular, we propose to investigate how changes in investors' attitude toward intertemporal substitution affect the allocation of bonds and stocks at the aggregate level. We start with the case of a conservative representative investor, which in our framework translates into considering agents that are reluctant to bear important variations in consumption over time, and investigate how changes in attitudes affect the optimal portfolio structure. The model predictions are then compared with the facts reported by Canner, Mankiw and Weil (1997), and the section is concluded by discussing whether an "asset allocation puzzle" arise in our framework.

Next, we take a closer look at the evidence recently reported by Normandin and St-Amour (2005), where it is shown that the inability of a vast majority of models to generate time-variation in portfolio shares is in con-

tradiction with the empirical facts. While in theory there exists cases where the dynamic problem reduces to the familiar static problem [see Campbell and Viceira (2001), ch. 1], the pronounced fluctuations through time of aggregate portfolio shares is not consistent with the myopic view. Moreover, as shown by Normandin and St-Amour (2005), where a framework nesting the two alternatives is provided, while purely tactical and myopic strategies are unambiguously rejected, the data strongly supports strategic investment behaviors. In section 4, we ask whether the model that has been developed is able to generate the observed time-variation in aggregate portfolio shares.

Finally, section 5 investigates the model's implications regarding asset pricing facts, by asking whether the striking empirical regularities recently documented by Borio and McGuire (2004) can be explained. As shown by their study which investigates the cycle in equity and housing prices since the early 1970s, equity price peaks tend to be followed by housing price peaks. The average lag between the two peaks which is found to be of about two years, or the occurrence of what the authors have labelled as "twin peaks in housing and equity prices", constitutes a challenging empirical fact to explain. This section is concluded by confronting the theoretical implications of the model with these empirical facts in order to investigate whether twin peaks as well as boom and bust phases in asset prices can arise in our framework.

## 1.2 Methodology and Findings

As regards the methodological approach, this study proposes to apply the numerical methods developed by the dynamic general equilibrium literature on macroeconomics [see King, Plosser and Rebelo (2002)] to the portfolio choice problem. The steady state implications of the model are firstly derived in order to characterize the optimal asset allocation that an agent would choose if the economy was not subject to shocks. The static allocation is then studied in order to assess how changes in parameters affect the portfolio structure. As regards the asset allocation puzzle, the main finding of this paper is that the predictions of the framework that has been constructed can be reconciled with the facts reported by Canner, Mankiw and Weil (1997). A general equilibrium model where changes in behaviors are captured by differences in attitudes toward intertemporal substitution seems therefore to constitute a promising starting point. As shown by the results of our simulations, in the steady state, with preferences that are consistent with

balanced growth, changes in the elasticity of intertemporal substitution lead to variations in optimal portfolio holdings. For plausible parameter values, the rise in the bond to stock ratio predicted by the model, as the elasticity of intertemporal substitution decreases, is compatible with the facts reported by Canner, Mankiw and Weil (1997).

As regards time-variation in aggregate portfolio shares, the contribution of this work is to show that in a dynamic general equilibrium framework, agents find optimal to strategically adjust their portfolio holdings over the business cycle, in response to macroeconomic shocks. An additional implication of the model stems from the fact that changes in the dynamic structure of the portfolio are found to be closely linked to the coefficient of intertemporal substitution. In periods of boom, while aggressive investors find optimal to hold a larger proportion of their financial wealth in stocks, as investors become more reluctant to see their consumption profile vary over time, the rise in the weight associated with equity holding is considerably reduced. Another implication of this model is that in periods of economic downturn, agents find optimal to hold a larger fraction of their financial wealth in real estate and in bonds.

Finally, when it comes to explaining the occurrence of twin peaks and boom and bust phases in asset prices, the model predictions are found to be consistent with the empirical facts reported by Borio and McGuire (2004). The key ingredient that allows the model to correctly predict that equity price peaks tend to be followed by housing price peaks with an average lag of about two years, is the introduction of real estate in the utility function. Real estate investment dynamics is influenced by the desire of agents to smooth their holding of residential real estate. Real estate smoothing is facilitated by the fact that, in this economy, wealth can easily be transferred from current to future periods by investing in capital or by increasing savings. As a result, residential investment responds gradually to shocks, leading house prices to respond in a hump-shaped manner.

## 2 The Model

This economy is composed of a representative agent, a firm producing a numeraire good, a firm producing a real estate composite good and a foreign firm. Each period agents purchase a numeraire consumption good and a composite real estate good from the firms in the domestic sector. The asset

allocation problem of the agents consists of deciding how much to invest in business capital and in residential real estate, and to save. The domestic firm uses both domestic labor and capital as production factors. The firm rents these two factors from the household. The real estate good is produced using residential capital. Compared to the standard literature, in this study we move away from the standard small economy assumption by allowing the interest rate on saving to be endogenously determined. The model is closed by introducing a demand for domestic savings emanating from a foreign country. Domestic savings are absorbed by the foreign firm which uses foreign workers in addition to domestic savings as inputs.

## 2.1 The Competitive Equilibrium

### 2.1.1 The Consumer

Consumer are assumed to derive utility from the consumption of a standard numeraire consumption good as well as from the stock of housing that they have accumulated over time.

$$u(c_t, x_t^c, h_t, x_t^h) = \left\{ \frac{(c_t + m^c x_t^c)^\kappa (h_t + m^h x_t^h)^{1-\kappa}}{1 - \sigma} \right\}^{1-\sigma} \quad (1)$$

$c_t$  and  $h_t$  denote respectively numeraire consumption and real estate. As it is common in the asset pricing literature, we assume habit formation in consumption.  $x_t^c$  denotes the stock of consumption habit which, following Fuhrer (2000), evolves according to the following law of motion:

$$\gamma x_{t+1}^c = a^c x_t^c + b^c c_t$$

where  $\gamma$  is the growth rate of the deterministic component of productivity [see King and Rebelo (2000)]. The parameter  $a^c$ , where  $0 \leq a^c \leq 1$ , can be interpreted as the rate at which the stock of habit depreciates. As regards  $b^c$ , where  $0 \leq b^c \leq 1$ , this parameter captures the sensitivity of the actual stock of habit on past levels of consumption. As for  $m^c$ , where  $m^c = \{-1; 0; 1\}$ , this coefficient will determine whether we are in the case of habit formation,  $m^c = -1$  or durability,  $m^c = 1$ . The case  $m^c = 0$  corresponds to the standard case without habit formation or durability in consumption.

Similarly, habit formation in real estate is introduced, where  $x_t^h$  denotes the stock of housing habit. The law of motion for the stock of housing habit

is given by:

$$\gamma x_{t+1}^h = a^h x_t^h + b^h h_t$$

where the coefficients governing the accumulation of housing habit can be interpreted as in the case of consumption habit. Regarding the form of the utility function, the standard Cobb-Douglas specification is chosen where  $\kappa$  and  $1 - \kappa$  are the weights attached to consumption and real estate and  $\kappa$  is such that  $0 < \kappa < 1$ . The curvature coefficient is denoted by  $\sigma$ .

The representative household faces the following budget constraint:

$$c_t + z_t h_t + i_t^e + i_t^h + i_t^b = W_t N_t + r_t^e k_t^e + z_t k_t^h + r_t^b b_t \quad (2)$$

Each period agents purchase a real estate composite good,  $h_t$ , from the firms in the real estate sector and a standard consumption good,  $c_t$ , from the firms in the numeraire sector. The expenditure related to housing is denoted by  $z_t h_t$ ,  $z_t$  being the relative price of one unit of real estate. The price of the consumption good is normalized to 1. Investment in the foreign firm is denoted by  $i_t^b$ .  $i_t^e$  and  $i_t^h$  denote respectively the amount that the agents invest in capital and in residential real estate. Agents receive a labor income,  $W_t N_t$ , as well as a revenue from owning the three assets, foreign bonds, capital and real estate.  $r_t^e k_t^e$  is the capital income,  $r_t^e$  denoting the rental rate of capital. As in the open economy model [see Coreia, Neves and Rebelo (1995)],  $r_t^b b_t$  is the income generated by the agent savings, with  $r_t^b$  representing the interest rate on savings.  $b_t$  therefore denotes the amount of foreign asset that has been accumulated over time and will thus be assimilated to the net holding of foreign bonds. Compared to the closed economy case, the opportunity to invest abroad using the international capital market provides an additional mean to smooth consumption. The payoff associated with investing in the foreign firm, which is given by the interest rate on savings,  $r_t^b$ , is endogenously determined by the supply of saving from the domestic economy and the demand for funds emanating from a foreign firm. The interest rate on saving is thus going to be affected by shifts in the supply and demand for domestic savings. The foreign firm is foreign-owned.

**Accumulation Equations:** As in Baxter and Crucini (1993), business capital accumulates over time according to:

$$\gamma k_{t+1}^e = (1 - \delta^e) k_t^e + \phi^e \left( \frac{i_t^e}{k_t^e} \right) k_t^e \quad (3)$$



where the parameters of the capital adjustment costs function  $\phi^e\left(\frac{i_t^e}{k_t^e}\right)$  are set so that the model with adjustments costs has the same steady state as the model without adjustment costs and it is assumed that near the steady state point:  $\phi^e > 0$ ,  $\phi^{e'} > 0$  and  $\phi^{e''} < 0$ . This captures the idea that increasing the capital stock rapidly is more costly than changing it slowly.

Similarly, the law of motion for residential capital is given by:

$$\gamma k_{t+1}^h = (1 - \delta^h)k_t^h + \phi^h\left(\frac{i_t^h}{k_t^h}\right)k_t^h \quad (4)$$

investment in the foreign firm is also subject to adjustment costs:

$$\gamma b_{t+1} = (1 - \delta^b)b_t + \phi^b\left(\frac{i_t^b}{b_t}\right)b_t \quad (5)$$

**Pricing Equations:** Defining respectively  $\lambda_t$ ,  $\mu_t$ ,  $\psi_t$  and  $\theta_t$  as the Lagrange multipliers associated with the budget constraint (2), the business capital accumulation equation (3), the residential capital accumulation equation (4) and the foreign capital accumulation equation (5), the dynamics of prices can be characterized by the following system of first-order conditions:

$$P_t^e = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} [P_{t+1}^e \Gamma_{t+1}^e + r_{t+1}^e] \quad (6)$$

$$P_t^h = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} [P_{t+1}^h \Gamma_{t+1}^h + z_{t+1}] \quad (7)$$

$$P_t^b = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} [P_{t+1}^b \Gamma_{t+1}^b + r_{t+1}^b] \quad (8)$$

where equations<sup>4</sup> (6), (7) and (8) are the three Euler equations associated with the first-order condition with respect to  $k_{t+1}^e$ ,  $b_{t+1}$  and  $k_{t+1}^h$ . Following the resolution methods of King, Plosser and Rebelo (2002), the fact that

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<sup>4</sup>where we have defined for instance in the case of business capital:

$$\Gamma_{t+1}^e = \left[ \phi^e\left(\frac{i_{t+1}^e}{k_{t+1}^e}\right) - \phi^{e'}\left(\frac{i_{t+1}^e}{k_{t+1}^e}\right) \frac{i_{t+1}^e}{k_{t+1}^e} + (1 - \delta^e) \right],$$

and

$$P_t^e = \frac{\mu_t}{\lambda_t}, P_t^h = \frac{\psi_t}{\lambda_t}, P_t^b = \frac{\theta_t}{\lambda_t}$$

steady state growth is taken into consideration implies that the discount factor attached to the Euler equations differs from the subjective rate  $\beta$ . The modified discount factor used in the pricing formulae is given by:

$$\tilde{\beta}^* = \beta\gamma^{-\sigma}$$

where  $\gamma$  denotes the growth rate of the economy. In the case of equity, compared to the standard case where quantities are fixed [see Lucas (1978)], the main difference is that an additional term,  $\Gamma_{t+1}^e$ , which is linked to the capital gain component of the valuation, shows up in the asset pricing formula.

As shown in the technical appendix, the price of capital, real estate, and the foreign asset can equivalently be stated as:

$$P_t^e = \frac{\mu_t}{\lambda_t} = 1/\phi^{e'} \left( \frac{i_t^e}{k_t^e} \right)$$

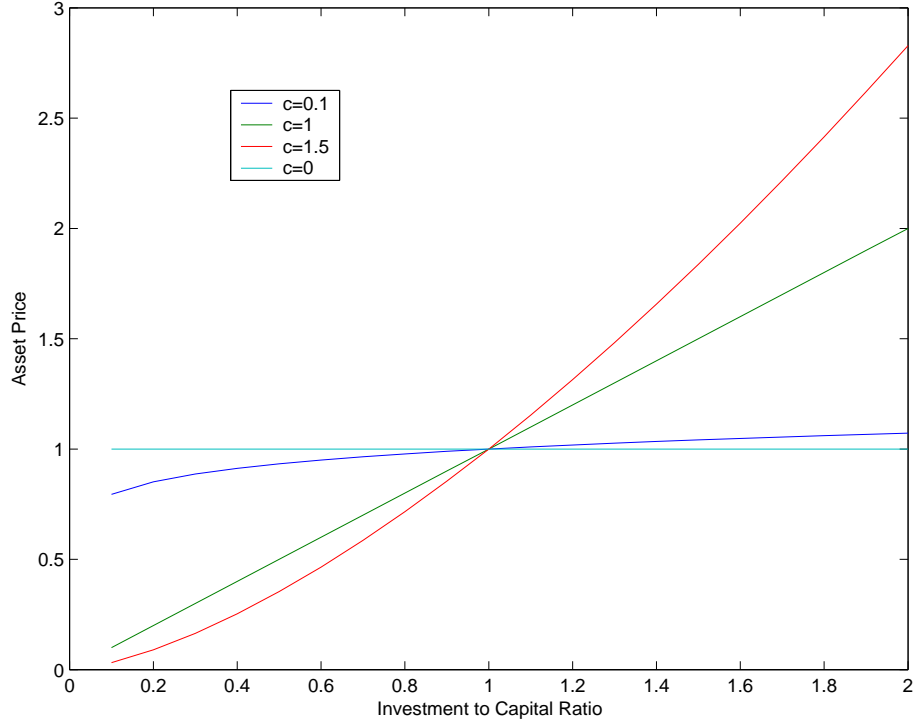
$$P_t^h = \frac{\psi_t}{\lambda_t} = 1/\phi^{h'} \left( \frac{i_t^h}{k_t^h} \right)$$

$$P_t^b = \frac{\theta_t}{\lambda_t} = 1/\phi^{b'} \left( \frac{i_t^b}{b_t} \right)$$

Since  $\phi^{e''}$ ,  $\phi^{h''}$  and  $\phi^{b''} < 0$ , asset prices are increasing functions of the investment to stock ratios  $\frac{i_t^e}{k_t^e}$ ,  $\frac{i_t^h}{k_t^h}$  and  $\frac{i_t^b}{b_t}$ . In the case of real estate, for instance, this relationship captures the idea that when investment is high relative to the existing stock of housing, the price of residential capital,  $P_t^h$ , increases to signal that agents are willing to increase the existing stock of housing. In contrast, a low value for  $P_t^h$  reflects that, according to agents, the stock of existing houses is sufficiently high and that little investment in residential capital is needed.

As illustrated by figure 1, the sensitivity of asset prices to changes in the investment to stock ratio will depend on the curvature of the adjustment cost function. In the case without adjustment costs ( $c = 0$ ), asset prices are always equal to 1 and variations in the investment to stock ratio have no impact on the valuation. The introduction of adjustment costs therefore allows variations in both prices and quantities. When adjustment costs are low (case  $c = 0.1$ ), prices will be moderately affected by changes in investment, while in contrast, when adjustment costs are high (case  $c = 1.5$ ), small variations in the investment to stock ratio generate important fluctuations in prices.

Figure 1: Sensitivity of Asset Prices to Investment/Stock Ratio



### 2.1.2 The Numeraire Good Producer

From the structure of the model, all laws of motion are taken into account in the consumer's problem. The problems of the firms are static and, as regards the numeraire good producer, consists in choosing how much business capital,  $k_t^e$ , to rent from the household at the rental price  $r_t^e$ , and how much labor to hire,  $N_t$ . The wage rate is denoted by  $W_t$ , and  $A_t$  is the traditional random productivity shock.

$$\text{Max } \pi_t^e = Ak_t^e{}^\alpha N_t^{1-\alpha} - r_t^e k_t^e - W_t N_t$$

The optimal demand of labor and capital can be described by the usual first-order conditions, relating the cost of labor and capital to their marginal productivity:

$$r_t^e = \alpha A_t k_t^e{}^{\alpha-1} N_t^{1-\alpha}$$

$$W_t = (1 - \alpha)A_t k_t^{\alpha} N_t^{-\alpha}$$

### 2.1.3 The Real Estate Sector

The firm in the real estate sector produces the final composite real estate good using residential capital. Profits in the real estate sector are given by:

$$\pi_t^h = z_t y_t^h - z_t k_t^h$$

where  $z_t$  is the relative price of the real estate good and  $y_t^h$  denotes the output of the real estate firm. As a result of perfect competition, firms in the real estate sector make zero profit implying that:

$$y_t^h = k_t^h$$

### 2.1.4 Demand for Domestic Savings

As in the standard open economy case, savings are absorbed by the rest of the world and are used to accumulate a composite foreign asset. While the small open economy assumption is often used to avoid modelling the demand for savings, this assumption is relaxed in what follows. The interest rate is endogenously determined by the equalization of the supply of savings, determined in the domestic economy, and the demand for domestic capital emanating from the foreign firm. It will be assumed that the foreign firm uses domestic savings exclusively to finance its activity. The technology of the foreign firm is Cobb-Douglas and is given by:

$$Y_t^* = F_t k_t^{*\xi} N_t^{*1-\xi}$$

where  $k_t^*$  is the quantity of capital that is used to produce and where  $N_t^*$  denotes the quantity of foreign labor hired by the foreign firm.  $F_t$  denotes total factor productivity.  $F_t$  and  $N_t^*$  will be treated as exogenous variables.

The foreign firm maximizes profit and chooses each period the amount of factors to rent:

$$Max \pi_t^* = F_t k_t^{*\xi} N_t^{*1-\xi} - r_t^b k_t^* - W_t^* N_t^*$$

The demand for domestic saving can thus be characterized by the following optimality condition:

$$r_t^b = \xi F_t k_t^{*\xi-1} N_t^{*1-\xi}$$

where  $r_t^b$  is the saving rate that the foreign firm has to pay to the representative agent.

### 2.1.5 Market Clearing

Finally the characterization of the competitive equilibrium requires that all markets clear. Firstly, equilibrium in the domestic factor markets implies that the quantity of labor and business capital supplied by the household equals the demand from the domestic firm.

$$k_t^e S = k_t^e D$$

$$N_t^S = N_t^D$$

Equilibrium on the real estate market implies that the quantity of the real estate good demanded by the consumers equals the quantity produced:

$$h_t = y_t^h$$

Second, equilibrium on the international capital market requires the supply of domestic saving to be equal to the demand of capital by the foreign firms:

$$b_t = k_t^*$$

and finally the aggregate resource constraint is given by:

$$y_t + r_t^b b_t = c_t + i_t^e + i_t^h + i_t^b$$

## 2.2 Calibration

### 2.2.1 Steady State Ratio and Preference Parameters

As for the curvature coefficient, according to Mehra and Prescott (1985), acceptable choices for this parameter include values ranging from 1 to 10. For the baseline calibration, we use the value suggested by Kocherlakota (1996) and set  $\sigma$  to 3. Regarding the value for  $\kappa$ , the weight attached to

consumption in the utility function, it set so as to imply a share of real estate investment to output,  $\frac{i^h}{y}$ , of 11%, which corresponds the observed value added as a percentage of gross domestic product in the United States<sup>5</sup>.

### 2.2.2 Production, Growth Rate and Saving to Output Ratio

Following the literature, we set the capital share in the production function of the domestic firm,  $\alpha$ , to 1/3. The capital share in the production function of the foreign firm,  $\xi$ , is also set to 1/3. The quarterly growth rate,  $\gamma$ , is 1.004. These are the standard values used in the literature. In order to facilitate comparisons with Canner, Mankiw and Weil (1997), the saving to output ratio,  $b/y$ , is chosen such that an investor with a moderate elasticity of intertemporal substitution ( $1/\sigma = 1/3$ ), chooses a ratio of bond to stock equal to 1. It is set to 11.5.

### 2.2.3 Habit Formation

Firstly, following the asset pricing literature, we assume habit formation in consumption and set  $m^c = -1$ . As regards the accumulation of the stock of consumption habit, we follow Jermann (1998) in setting  $b^c = 0.82$  and  $a^c = 0$ . Second, given that the introduction of habit formation in real estate does not affect significantly the dynamics of the model, we set  $m^h = 0$ <sup>6</sup>.

### 2.2.4 Adjustment Costs

The introduction of adjustment costs allows to generate variations in both prices and quantities. As shown in figure 1, the case with no adjustment costs corresponds to the case where prices are fixed to 1. The parameters governing the curvature of the adjustment cost functions,  $c^e$ ,  $c^h$  and  $c^b$  determine how quantity and prices will interact. In the baseline calibration, given the lack of a priori knowledge regarding adjustment costs, these parameters are set such as to maximize the ability of the model to account for the set of stylized facts described in section 5. We set  $c^e = 0.016$ ,  $c^h = 0.05$  and  $c^b = 0.03$ , to

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<sup>5</sup>Source: Bureau of economic analysis, Gross-Domestic Product (GDP) by Industry Data

<sup>6</sup>This is due to the fact that in equilibrium  $k_t^h = h_t$  and thus that real estate inherits the dynamic properties of the stock of residential capital. Since  $k_t^h$  is predetermined,  $h_t$  respond gradually to shocks. Introducing habit in real estate imply a profile for  $h_t$  which is even smoother.

capture the idea that it is more costly to invest abroad than in the domestic firm and that increasing the stock of residential capital is more costly than increasing the stock of business capital ( $c^h > c^b > c^e$ ). Adjustment costs have no impact on the steady state allocation and only affect the around steady state dynamics.

### 2.2.5 Baseline Calibration

The following tables summarize the baseline calibration that has been used to generate all the results presented in section 3,4 and 5.  $\sigma, \beta$  and  $\kappa$  denote respectively risk aversion, the weight associated with consumption in the utility function and the non-modified discount factor.

#### Preference Parameters

$\sigma$	$\kappa$	$\beta$
3.0	0.75	0.99814

$\delta^e, \delta^b$  and  $\delta^h$  are the depreciation rates of business capital, the foreign asset and residential capital:

#### Depreciation Rates

$\delta^e$	$\delta^b$	$\delta^h$
0.01508	0.00	0.00439

$c^e, c^b$  and  $c^h$  are the adjustment cost parameters of capital, the foreign asset and residential capital:

#### Adjustment Costs Parameters

$c^e$	$c^b$	$c^h$
0.016	0.03	0.05

$\alpha$  and  $\xi$  denote the share of capital in the production function of the domestic and of the foreign firm.

#### Capital Share

$\alpha$	$\xi$
1/3	1/3

## 2.3 Solution Method

Following the resolution method of King, Plosser and Rebelo (2002), the dynamic system is composed of two control variables  $c_t$  and  $N_t$ , 5 control state variables  $k_t^e, k_t^h, x_t^c, x_t^h$  and  $b_t$ , 5 co-state variables or shadow prices associated to each of the four control state variables  $\mu_t, \psi_t, \varphi_t^c, \varphi_t^h$  and  $\theta_t$  and one exogenous state variable given by the random productivity shock  $A_t$ <sup>7</sup>.

The solution of the system implies the computation of 10 eigenvalues. Uniqueness of the solution and stability of the system require that the eigenvalues associated with the predetermined variables  $k_t^e, k_t^h, x_t^c, x_t^h$  and  $b_t$  are less than unity and that the eigenvalues associated with the non-predetermined variables  $\mu_t, \psi_t, \varphi_t^c, \varphi_t^h$  and  $\theta_t$  are outside the unit circle.

## 3 Asset Allocation in the Long Run

### 3.1 Defining Financial Wealth

Following the real business cycle literature, we can, firstly, derive an expression for  $\frac{c}{h}$ , the consumption to housing ratio by evaluating equation (7) in the steady state. For a given level of foreign asset to output ratio,  $\frac{b}{y}$ , the consumption share,  $\frac{c}{y}$ , as well as all the other remaining steady state ratios including the housing to output ratio,  $\frac{h}{y}$ , and the capital to output ratio,  $\frac{k^e}{y}$  can be computed. Then, defining financial wealth as:

$$W^F = P^e k^e + P^b b + P^h h$$

the steady state ratio of capital, housing and bond relative to wealth<sup>8</sup> :

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<sup>7</sup>Shocks affecting total factor productivity or the number of workers in the foreign firm could also be studied.

<sup>8</sup>where  $\frac{W^F}{y} = \frac{k^e}{y} + \frac{b}{y} + \frac{h}{y}$  and using the fact that:

$$\frac{k^e}{y} \frac{y}{W^F} = \frac{k^e}{W^F}$$

$$\frac{h}{y} \frac{y}{W^F} = \frac{h}{W^F}$$

$$\frac{b}{y} \frac{y}{W^F} = \frac{b}{W^F}$$

and using the fact that in the steady state  $P^h = P^b = P^e = 1$



$$\frac{P^e k^e}{W^F}, \frac{P^h h}{W^F} \text{ and } \frac{P^b b}{W^F}$$

can be derived analytically and expressed in terms of the structural parameters of the model.

For the baseline calibration described above, the steady state allocation chosen by the agent implies the following portfolio weights:

$$\frac{P^h h}{W^F} = 0.37, \frac{P^e k^e}{W^F} = 0.315 \text{ and } \frac{P^b b}{W^F} = 0.315$$

so the ratio of bond to stock is:

$$\frac{P^b b}{P^e k^e} = 1$$

### 3.2 The Asset Allocation Puzzle

To compare our results with the numbers reported in Canner and al. (1997), the model is calibrated such that an agent that is moderately risk averse chooses a portfolio with a ratio of bond to stock equal to 1. We then look at the implications of varying the curvature coefficient within the range suggested by Mehra and Prescott (1985) and consider the case of an aggressive investor, ( $\sigma = 1$ ), a moderate investor, ( $\sigma = 3$ ), and of a conservative investors ( $\sigma = 5$  and  $\sigma = 10$ ).

As illustrated in table 2, the evolution of the bond to stock ratio is consistent with the evidence reported by Canner and al. (1997). According to our framework, as the curvature coefficient increases, investors becomes more conservative, and they increase their holding of bonds relative to stocks. This ratio increases from 1 to 1.28, when  $\sigma$  increases from 3 to 5. When the curvature coefficient increases from 5 to 10, the bond to stock ratio increases from 1.28 to 2.

**Table 2: Impact of attitude toward intertemporal substitution on the bond to stock ratio**

$\sigma$	1	3	5	10
$\frac{P^b b}{P^e k^e}$	0.72	1.0	1.28	2

The influence of the elasticity of intertemporal substitution works through the impact of  $\sigma$  on the modified discount factor. In a model that is consistent with balanced growth, the usual transformation implies that:

$$\tilde{\beta}^* = \beta\gamma^{-\sigma}$$

where  $\gamma$  can be interpreted as the growth rate of population and productivity.

Intuitively, the positive influence of the curvature coefficient on the bond to stock ratio stems from the fact that a more conservative investor is willing to consume a larger fraction of its income. This increase in steady state consumption comes at the cost of decreasing steady state investment. Since the capital share is determined by:

$$\frac{k^e}{y} = \frac{\alpha\tilde{\beta}^*}{1 - \tilde{\beta}^*(1 - \delta^e)}$$

with  $\frac{b}{y}$  given, a rise in  $\sigma$  unambiguously implies a decrease in  $k^e/y$  and therefore a rise in  $P^b b / P^e k^e$ <sup>9</sup>.

As regards the real estate to stock ratio, as illustrated in table 3, for this set of parameters, a rise in  $\sigma$  leads agents to increase the relative weight of housing, as they become less willing to bear consumption risk.

**Table 3: Impact of attitude toward intertemporal substitution on the real estate to stock ratio**

$\sigma$	1	3	5	10
$\frac{P^b h}{P^e k^e}$	1.14	1.17	1.21	1.30

Finally, as regards the bond to real estate ratio, as shown in table 4, the agent chooses to increase the relative weight of bonds when they become less willing to substitute consumption intertemporally.

**Table 4: Impact of attitude toward intertemporal substitution on the bond to real estate ratio**

$\sigma$	1	3	5	10
$\frac{P^b b}{P^e k^e h}$	0.63	0.85	1.05	1.53

Table 5 shows the optimal portfolio structure predicted by the model and how it is influenced by differences in attitudes toward intertemporal substitution. In this economy a conservative investor, ( $\sigma = 10$ ), chooses to invest 47% of its financial wealth in bonds, 30% in real estate and 23% in stocks. An aggressive investors, ( $\sigma = 1$ ), in contrast, finds optimal to invest 25% in bonds, 40% in real estate and 35% in stocks.

<sup>9</sup>Since  $\frac{b}{k^e} = \frac{b}{y} \frac{1}{k^e/y}$  and  $P^b = P^e = 1$

**Table 5: Asset Allocation and Intertemporal Substitution**

$\sigma$	1	3	5	10
$\frac{P^b b}{W^F}$	0.25	0.315	0.37	0.47
$\frac{P^h h}{W^F}$	0.40	0.370	0.34	0.30
$\frac{P^k k}{W^F}$	0.35	0.315	0.29	0.23
	1	1	1	1

## 4 Dynamic Asset Allocation

To investigate the dynamic properties of the model, we now introduce labor in order to consider an economy where, each period, agents decide how much to invest in bonds, shares, real estate and decide how many hours to supply.

We assume separability between consumption goods and leisure:

$$u(c_t, x_t^c, h_t, x_t^h, l_t) = \left\{ \frac{(c_t + m^c x_t^c)^\kappa (h_t + m^h x_t^h)^{1-\kappa}}{1-\sigma} \right\}^{1-\sigma} + v(l_t)$$

where  $l_t$  denotes leisure and where we have that  $v'(l) > 0$  and  $v''(l) < 0$ . To focus on the dynamic implications of varying the elasticity of intertemporal substitution, in this section, we abstract from balanced growth in order to have a steady state that is independent from the curvature coefficient.

### 4.1 Asset Characteristics

The aim of this section is to present the main characteristics of each of the three assets in order to assess how their behavior will be affected by macroeconomic shocks. As illustrated by the asset pricing formulae describing the dynamics of equity, bond and housing prices, the expected payoff is the main component of the valuation that differentiates the three assets. Total expected returns, which in the case of capital, are given by:

$$\frac{P_{t+1}^e \Gamma_{t+1}^e + r_{t+1}^e}{P_t^e}$$

are determined by three distinct components: the payoff associated with the asset<sup>10</sup>, the capital gain associated with the change in prices, and an

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<sup>10</sup>Rents,  $z_t$ , in the case of real estate; the rental cost of capital,  $r_t^e$ , in the case of equity and the saving rate,  $r_t^b$ , in the case of the foreign asset.

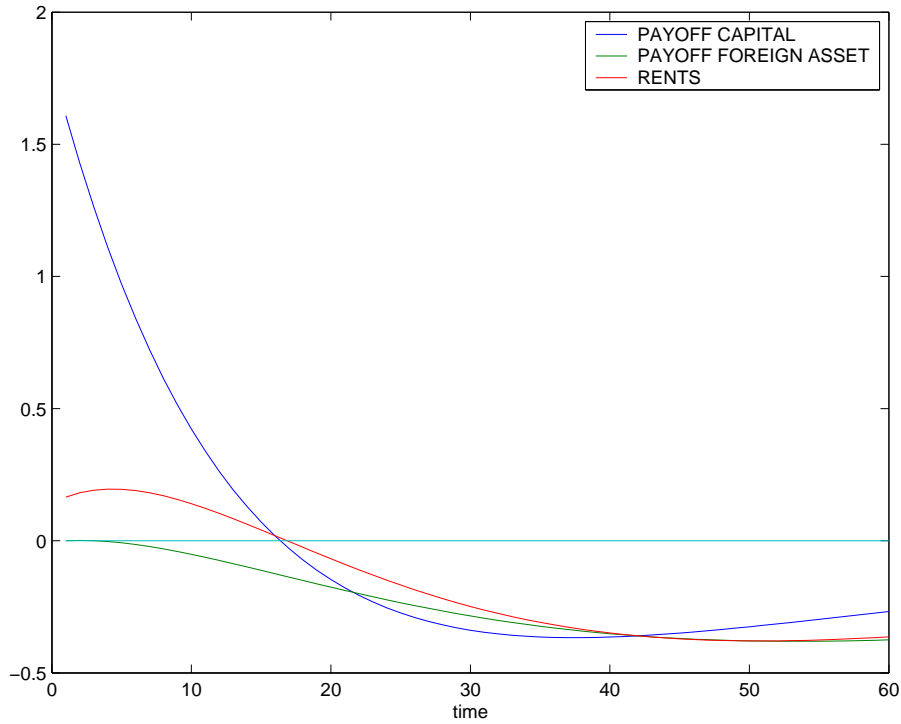
additional term related to variations in quantities, that captures the impact of adjustment costs on the valuation as well as total depreciation<sup>11</sup>.

In the case of equity prices, the payoff associated with holding one unit of capital is determined by the rental cost of capital, which from the problem of the firm is given by:

$$r_t^e = \alpha A_t k_t^{\alpha-1} N_t^{1-\alpha} \quad (9)$$

As illustrated in figure 6, which shows the response of the payoffs associated to each asset to a technology shock, equation (9) implies that  $r_t^e$  will jump in response to a productivity shock (blue line) since the rental cost of capital is directly affected by  $A_t$ . With moderate capital adjustment cost<sup>12</sup>, this effect is reinforced by the instantaneous rise in labor.

**Figure 6: Rental cost of capital, interest rate on saving and rents**



<sup>11</sup>In the case of capital, it is given by:  $\Gamma_{t+1}^e = \left[ \phi^e \left( \frac{i_{t+1}^e}{k_{t+1}^e} \right) - \phi^{e'} \left( \frac{i_{t+1}^e}{k_{t+1}^e} \right) \frac{i_{t+1}^e}{k_{t+1}^e} + (1 - \delta^e) \right]$

<sup>12</sup>In the case of high adjustment cost, hours worked decline in response to a productivity shock [see Christiano, Boldrin and Fisher (2001)].

The capital stock being predetermined, an instantaneous rise in labor implies an additional rise in the productivity of capital since for a given stock of capital there is now more labor involved in the production process. This indirect effect due to labor combined with the direct effect of the shock leads  $r_t^e$  to rise significantly in response to a shock. The response of  $r_t^b$ , the interest rate on savings (green line), is, in contrast, considerably more sluggish and gradual. This can be easily explained by looking at the expression for  $r_t^b$  :

$$r_t^b = \xi F_t k_t^{*\xi-1} N_t^{*1-\xi}$$

Since a technology shock affecting the domestic economy has no direct effect on the saving rate, with  $N_t^*$  and  $F_t$  being exogenous, the dynamics of  $r_t^b$  is entirely determined by the evolution of savings.  $k_t^*$  being predetermined, this explains the gradual response of the interest rate. In periods of boom, consumption smoothing of both the numeraire good and real estate, implies a rise in savings. The demand for domestic savings being not directly affected by the shock, this rise in the supply of fund induces the interest rate to decline progressively. The saving rate is thus countercyclical.

As regards the payoff associated with investing in real estate, rents (red line),  $z_t$ , are determined by the first-order condition in the consumer's problem with respect to  $h_t$ . In the case without habit formation, rents can be expressed as:

$$z_t = \frac{1 - \kappa c_t}{\kappa h_t}$$

where  $\kappa$  denotes the weight attached to consumption in the utility function. Rents are thus essentially determined by the ratio of numeraire consumption to housing. Housing being predetermined, since productivity has a positive indirect impact on consumption, this ratio will rise which implies a procyclical behavior for rents. Consumption and housing smoothing will thus lead the response of rents to be moderate.

#### 4.1.1 Strategic Allocation

As shown in the previous section, the long run allocation can be characterized by the steady state of the economy. This static allocation is reached when the economy has converged to its long-run equilibrium. In reality, as documented by Normandin and St-Amour (2005), agents find optimal to strategically adjust their holdings of asset and there is a considerable amount of time-variation in aggregate portfolio shares. The objective of this section is to

assess whether the framework that has been constructed is able to generate implications that are consistent with these empirical observations.

In the economy described in the previous section, the agent accumulates three assets, savings ( $b_t$ ), housing ( $h_t$ ) and business capital ( $k_t^e$ ). Using equations (6) to (8), the market value of the agent financial wealth ( $W_t^F$ ) can be defined as:

$$W_t^F = P_t^e k_t^e + P_t^h h_t + P_t^b b_t \quad (10)$$

The evolution of portfolio shares can also be analyzed and are given by:

$$1 = \frac{P_t^e k_t^e}{W_t^F} + \frac{P_t^h h_t}{W_t^F} + \frac{P_t^b b_t}{W_t^F} \quad (11)$$

This framework can now be used to study the evolution of the portfolio weights chosen by agents in response to macroeconomic shocks.

## 4.2 Impulse Response Analysis

**Figure 3: Moderate Investor**

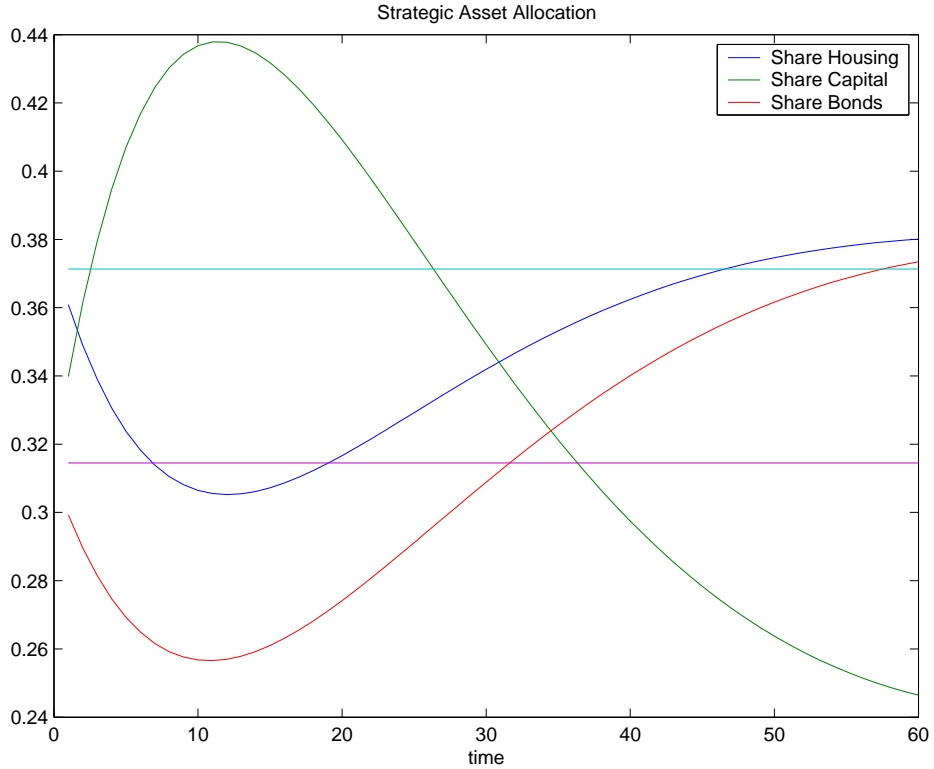
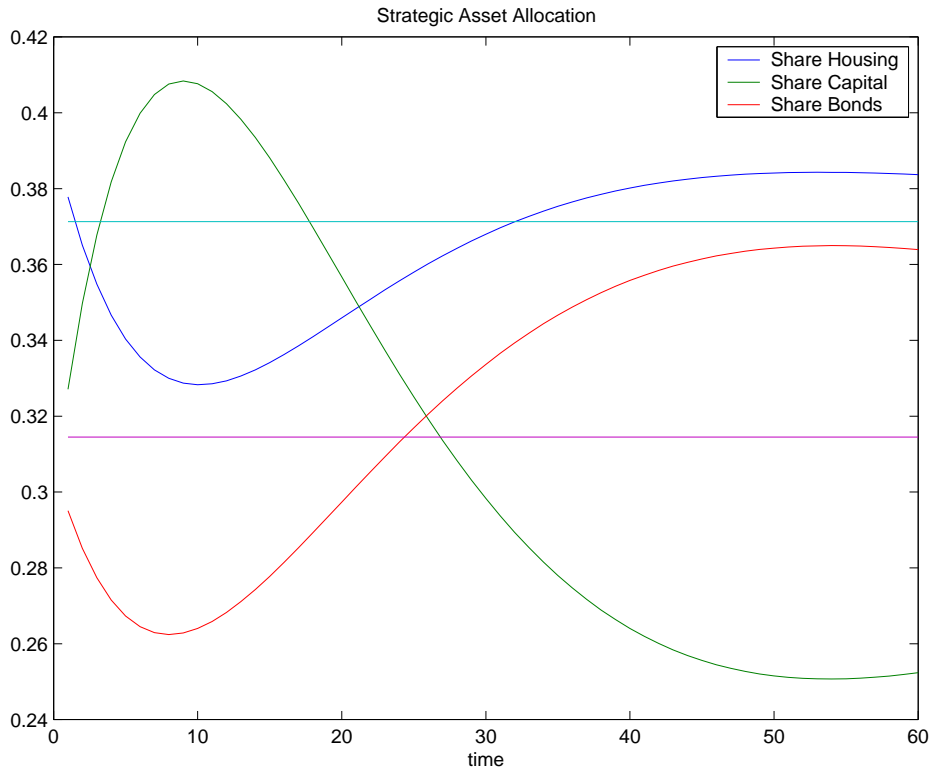


Figure 3 shows the responses of the portfolio weights associated to the position on housing ( $h_t$ ), capital ( $k_t^e$ ) and savings ( $b_t$ ) of the investor, following a standard one percent technology shock.

The purple and the blue horizontal lines denote the steady state positions of bonds and stocks (0.315), and real estate (0.370). The red, the blue and the green lines denote respectively the evolution of the saving share, the housing share and the capital share. As shown in figure 3, for the baseline calibration described above ( $\sigma = 3$ ), in the case of a moderate representative investor, on impact, the weight associated with the position on equity holding jumps from 31.5% to 34% and then progressively increases to reach a peak about 10 quarters later at 44%.

**Figure 4: Conservative Investor**



As shown in figure 4, an increase in the curvature coefficient from 3 to 10, holding the steady state level of the portfolio shares constant, has a significant impact on the dynamic allocation. The rise in the weight associated

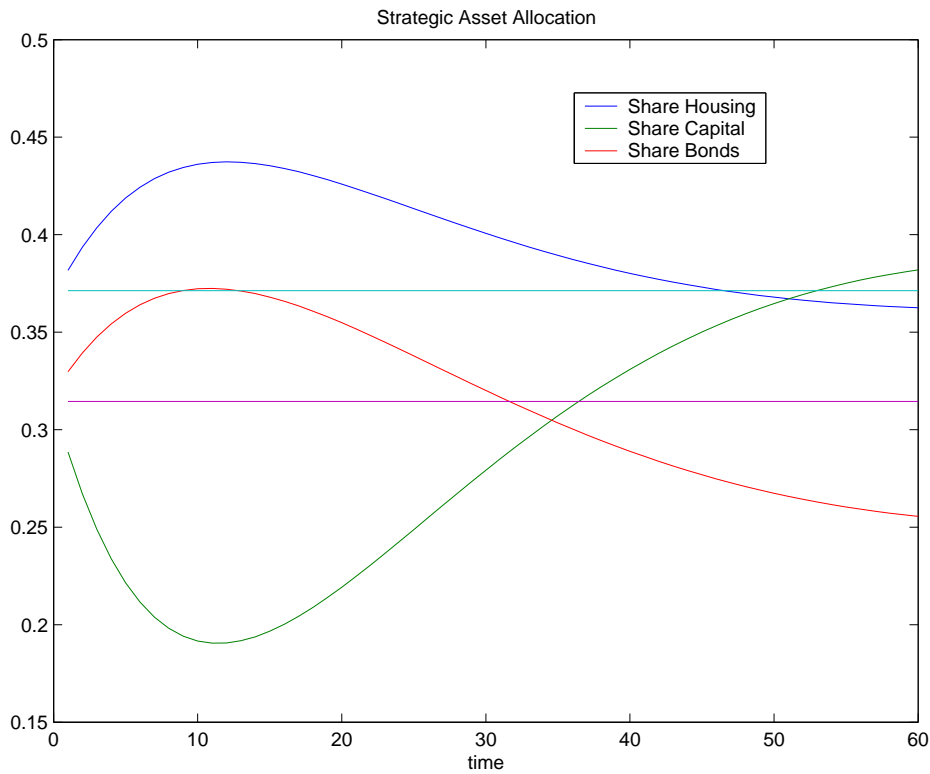
with equity holding is considerably reduced and compared to the case of a moderate investor, the decrease in the real estate position is less pronounced. The decrease in equity is financed by a reduction in both the housing and the saving positions. It is interesting to note that the model is able to generate a substantial amount of time-variation in aggregate portfolio weights in response to a macroeconomic shock. The model prediction is thus consistent with the empirical evidence reported in Normandin and St-Amour (2005).

To facilitate comparisons, the evolution of the portfolio shares associated with capital, bonds and housing in the case of a moderate and of a conservative investor are shown in the appendix (see figure 8, sect. 8.1).

#### 4.2.1 Real Estate and the Business Cycle

We now turn to the analysis of the optimal dynamic allocation implied by the model in periods of recession. Figure 5 shows the optimal reallocation that agents choose following a negative technology shock.

**Figure 5: Negative Productivity Shock**





As illustrated in figure 3 and 4, in periods of economic boom, agents find optimal to increase their holding of capital. It is interesting to note that most of the rebalancing is done by reducing the portfolio share of real estate and bonds. When agents do not mind substituting consumption over time, the reduction of the weight associated with real estate is even more pronounced.

As shown in figure 5, in periods of economic downturn, in contrast, agents choose to increase the real estate share and most of the rebalancing implies a reduction in the capital share. In bad times, the model therefore predicts that agents will be willing to hold a larger fraction of their financial wealth in real estate. The main reason that explains why real estate is favoured in periods of recession is linked to the fact that housing enters the utility function. In periods of economic downturn, agents have to reduce investment. The question is what asset to sacrifice first. Since real estate enters the utility function, agents are willing to smooth the level of housing stock that they own. As a result, they will choose to disinvest in real estate only gradually. The sharp decline in the position associated with capital can be explained by the fact that agents are willing to smooth consumption. As a result, since in bad times consumption is low, agents have little incentive to invest in an asset that allows them to transfer wealth from the current to the next period.

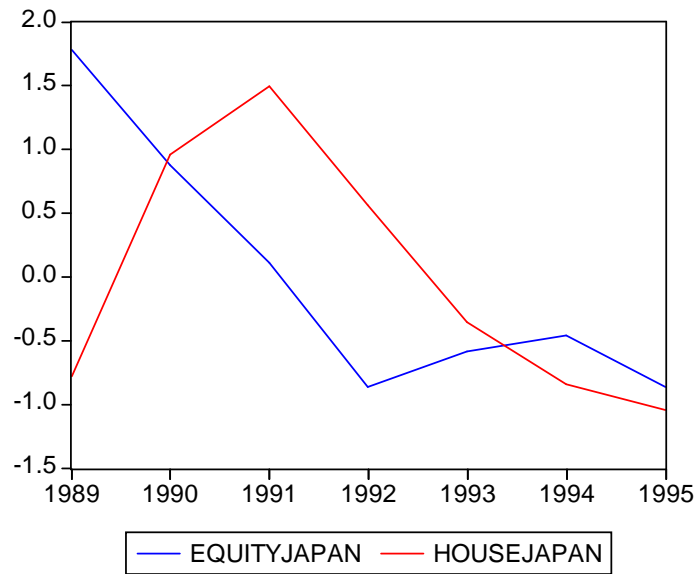
In addition, the productivity shock has a negative impact on the rental cost of capital. This additional effect implies that capital is not an attractive investment in periods of recession. The agent will thus consider alternative investment opportunities. The fact that the bond share increases is due to the fact that the interest rate on savings is countercyclical. When a bad shock hits the economy, agents are less willing to save. As a result, this decrease in the supply of funds on the international capital market forces the foreign firm to pay a higher interest rate in order to attract savings. The resulting rise in interest rate implies that investing in bonds is more attractive than investing in capital, in periods of recession.

## **5 Twin Peaks in Equity and Housing Prices**

As documented by Borio and McGuire (2004), the joint dynamics of equity and housing prices is characterized by some interesting empirical regularities. In their study, which investigates the cycle in equity and housing prices since the early 1970s, the authors have shown that equity price peaks tend to be followed by housing price peaks.

The average lag between the two peaks, which is found to be of about two years, constitutes a challenging empirical fact to explain and is closely linked to the debate regarding the sustainability of the recent worldwide surge in housing prices. As shown in figure 6, Japan provides a good illustration of what is documented in Borio and McGuire (2004).

**Figure 6: Housing and Equity Prices Japan**



Annual data. Source: BIS (data collected from national sources).

The red and the blue line denote the observed series of housing and capital prices in Japan corresponding to the 1989-1995 period. The increase in equity, which peaks in 1989, is followed by a peak in housing prices about two years later. The hump shaped evolution of house prices characterized by an initial gradual increase, followed by a sharp decline, has led many economists to investigate the potential reasons that could explain the occurrence of what is often described as "house prices bubbles".

## 5.1 Discussion

As illustrated in figure 7, which shows the response of both equity and housing prices to a productivity shock, the model that has been developed can

account for the observed dynamics of equity and housing prices. In a neo-classical multi-asset framework, the lagged response of housing prices documented by Borio and McGuire (2004) can thus be generated by technology shocks. It is also interesting to note that a market clearing model with rational expectations is able to generate a hump-shaped response for housing prices.

**Figure 7: Twin Peaks**

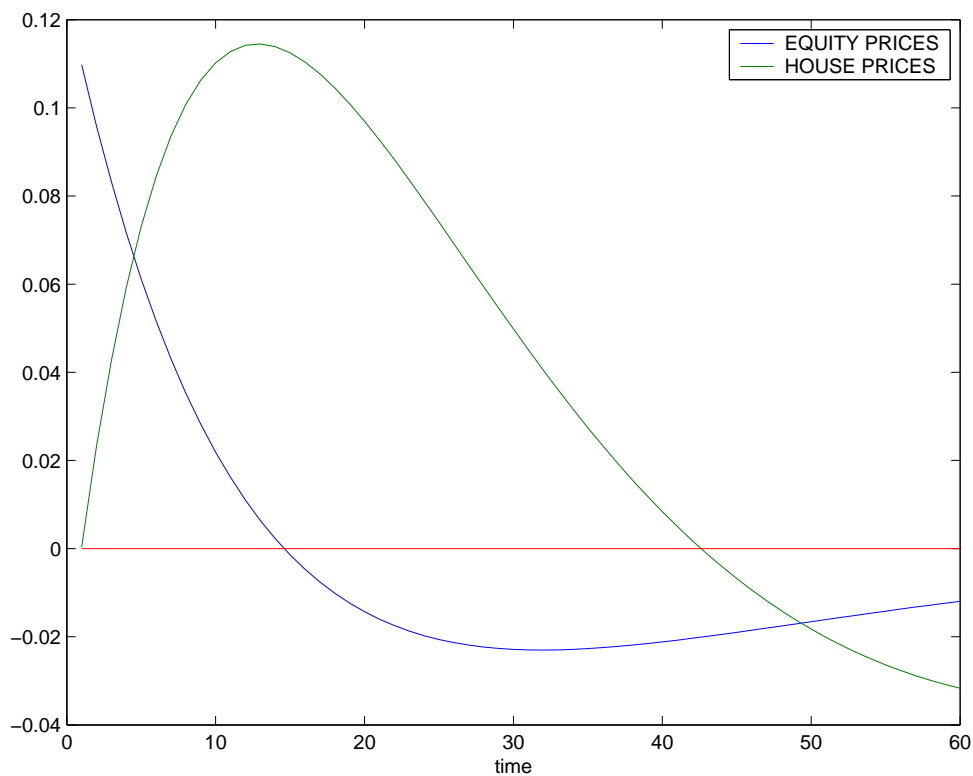


Figure 7 shows the response of equity and house prices to a standard percent deviation productivity shock. The blue line denotes the response of equity prices and the green line of house prices. The horizontal axis denotes time, measured in quarters.

The key ingredients that allow the model to correctly predict that, following a productivity shock, a peak in equity prices will be followed by a peak in housing prices about 2 years later, are the introduction of differences

in depreciation of both business and residential capital, and the fact that the housing stock enters the utility function. From the first-order conditions, housing prices can be equivalently expressed as:

$$P_t^h = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [P_{t+1}^h x_{t+1}^h + z_{t+1}] \quad (12)$$

or:

$$P_t^h = 1/\phi^{th} \left( \frac{i_t^h}{k_t^h} \right) \quad (13)$$

From equation (13), it can be seen that the capital stock being predetermined, the dynamics of housing prices will be mostly influenced by the behavior of investment. As a result, if residential capital depreciates slowly over time, the consumer understands that, following a productivity shock, less investment is required to maintain a given level of housing stock. The dynamics of housing prices is then influenced by the moderate response of investment. Since the increase in the investment to stock ratio,  $\frac{i_t^h}{k_t^h}$ , is moderate, prices will increase gradually to signal that agents do not find that investing in real estate is a priority. Secondly, the fact that the housing stock enters the utility function also has a significant impact on the dynamics of real estate prices. In this framework, where the consumer faces an investment choice involving many assets, the dynamics of residential investment is influenced by the agent's willingness to avoid fluctuations in its stock of real estate. In good times, the problem the agent is facing consists of deciding in which of the 3 assets to invest. Since in this framework investing in capital or savings allows agents to transfer wealth from the current to the next periods, it is possible to postpone the investment in residential capital as long as needed. Real estate smoothing is thus facilitated by the fact that in this economy many opportunities to transfer wealth are offered to investors.

## 6 Conclusion

The aim of this study is to construct a model linking financial markets to the real economy [see Cochrane (2005)] in order to assess whether a multi-asset neoclassical framework is able to account for: (i) the asset allocation puzzle, (ii) the time-variation in aggregate portfolio shares and (iii) the occurrence

of twin peaks in equity and housing prices. In contrast to the standard literature on asset pricing, in this model asset prices and portfolio shares are endogenously determined within a unified general equilibrium framework. Fluctuations in quantity and prices are entirely driven by aggregate technology shocks.

When it comes to the asset allocation puzzle, the model prediction regarding the evolution of the bond to stock ratio is consistent with the evidence presented by Canner, Mankiw and Weil (1997). The model correctly predicts that the bond to stock ratio should decrease as investors become more willing to substitute consumption over time. Adopting preferences that are consistent with balanced growth path is key in order to study the impact of changes in the elasticity of intertemporal substitution on the steady state allocation.

In addition, in this economy, the representative agent finds optimal to rebalance its portfolio in response to a macroeconomic shock. The implication regarding the evolution of the aggregate portfolio shares are thus consistent with the evidence reported by Normandin and St-Amour (2005). In contrast to what is predicted by the myopic setting, this framework is able to account for the observed time-variation in aggregate portfolio shares.

Regarding the joint dynamics of equity and housing prices, the model is able to correctly predict that following a productivity shock, a peak in equity prices will be followed by a peak in housing prices about two years later [see Borio and McGuire (2004)]. It is also possible to account for the sluggish and hump-shaped response of housing prices. In this economy, housing price bubbles are rational and can be generated by productivity shocks.

In sum, the main contribution of this paper is to provide a framework able to generate implications in terms of both asset price dynamics and asset allocation facts that are consistent with a series of empirical observations that have been reported in the literature. Moreover, it has been shown that for a unique set of parameters, the model is able to generate both steady state and dynamic implications that are in line with the empirical facts. Regarding future research, a natural extension would consist of conducting additional empirical tests of the model predictions. For instance, the ability of the model to account for the cyclical properties of aggregate portfolio shares could be assessed. Moreover, in this economy, the model predicts that, in periods of recession, agents find optimal to hold a larger proportion of their wealth in real estate and in bonds. This additional implication could also be tested.

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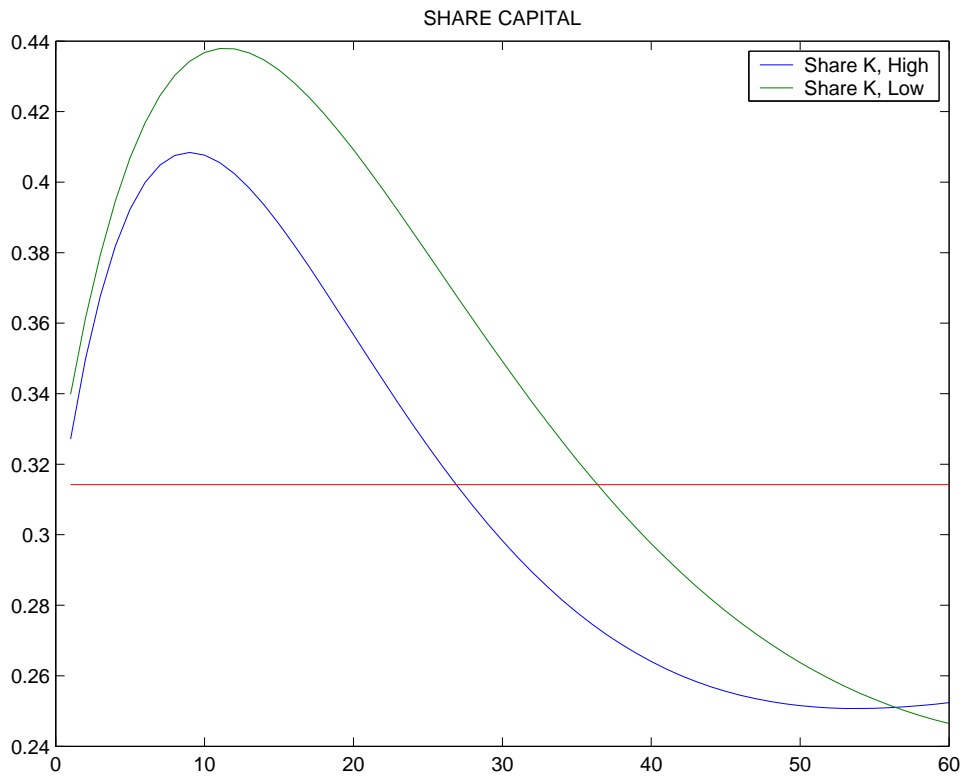


## 8 Appendix

### 8.1 Strategic Allocation and Intertemporal Substitution

To assess the impact of changing the curvature coefficient,  $\sigma$ , we now present the dynamic response of the positions on each asset, in the case of low curvature, ( $\sigma = 3$ ), and high curvature, ( $\sigma = 10$ ). The case  $\sigma = 3$  corresponds to the case of a moderate investor and the case  $\sigma = 10$  correspond to the case of a conservative investor.

**Figure 8: Impact of changing the elasticity of intertemporal substitution**



On each picture, the green line represents the low curvature case (moderate investor). The horizontal red line denotes the steady state share of the portfolio.

