# Sticky Prices and the Optimal Return to Money

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## Motivation

- Traditional view on Business Cycles and Money: Money matters!
  - need devices to break Classical Dichotomy: signal extraction problem, menu costs, nominal contracts, segmented markets.
  - Lucas (1972): monetary policy is noisy.
  - Wallace (1997) and Katzman, Keenan and Wallace (2003).
- Our view: correlations between monetary and real variables are not accidental but the result of frictions in the real sector that money alleviates.

## What we do:

- Introduce aggregate uncertainty into a standard search model of money.
- Study optimal allocations (mechanism design problem).
- Show that the return to money (price level) is history dependent in optimal allocations.

**Literature**: Spear and Srivastava (1987) and Green (1987)... **but** the recursive structure for discussing non-stationary allocations in monetary models with heterogeneous agents has not been established. We therefore start simple!

**Environment: Shi-Trejos-Wright with aggregate uncertainty.**

1. Discrete time, discount factor $\beta$.

2. Specialization in production and consumption: $N$ types.

3. Money is indivisible $m \in \{0, 1\}$.

4. Divisible production $y$.

5. Taste-shocks $u_s(y)$, where $s \in \{low, high\}$ with probability $\pi_s$.

## **Definitions**.

- A history is $s^t = (s^{t-1}, s_t)$. Set of all possible histories up to $t$ is $S^t$.

- Allocation is sequence $y_t : S^t \longrightarrow R$ or $y(s^t)$ exchanged for money.

- Welfare Criteria

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t p(s^t) z_{s_t}(y(s^t))$$

  where

$$z_s(y) \equiv m(1-m)\frac{1}{N}(u_s(y) - y).$$

- First best allocation $(y_l^*, y_h^*)$ such that $u'_{s_t}(y(s^t)) = 1$.

## Expectations:

$$
\begin{aligned}
v_1(s^t) &= (1-m)\frac{1}{N}[u_{s_t}(y(s^t)) + \beta(\pi_l v_0(s^t,l) + \pi_h v_0(s^t,h))] + \\
&\quad (1-(1-m)\frac{1}{N})\beta(\pi_l v_1(s^t,l) + \pi_h v_1(s^t,h))
\end{aligned}
$$

$$
\begin{aligned}
v_0(s^t) &= m\frac{1}{N}[-y(s^t) + \beta(\pi_l v_1(s^t,l) + \pi_h v_1(s^t,h))] + \\
&\quad (1-m\frac{1}{N})\beta(\pi_l v_0(s^t,l) + \pi_h v_0(s^t,h)).
\end{aligned}
$$

Denote:

$$\partial v(s^t) \equiv v_1(s^t) - v_0(s^t).$$

## Implementability and Optimality

The producer's participation constraint is

$$y(s^t) \leq \beta(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h))$$

The consumer's participation constraint is

$$u_s(y(s^t)) \geq \beta(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)).$$

**Definitions:**

1. An output allocation $y(s^t)$ is implementable if there exists $v(s^t)$ satisfying participation constraints for all $s^t \in S^t$ and all $t = 0, 1, 2, ....$

2. An allocation is optimal if it maximizes welfare among the set of implementable allocations.

## Promise keeping (rational expectations)

Return on money links $\partial v(s^t)$ to $\partial v(s^t, s_{t+1})$ as follows

$$\partial v(s^t) = f_{s_t}(y(s^t)) + (1 - \frac{1}{N})\beta(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)),$$

where

$$f_s(y) \equiv \frac{1}{N}((1 - m)u_s(y) + my).$$

**The sequential Planner's problem**

$$\max_{y(s^t),\partial v(s^t)} \sum_{t=0}^{\infty} \sum_{s_t \in S^t} p(s^t)\beta^t z_{s_t}(y(s^t))$$

s.t.

$$\begin{aligned}
y(s^t) &\leq \beta(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)) \\
\partial v(s^t) &\leq f_{s_t}(y(s^t)) + \alpha(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)) \\
0 &\leq \partial v(s^t) \leq B \text{ for all } s^t \text{ and all } t.
\end{aligned}$$

**Note:** we ignore consumer's participation constraint

## No Ponzi Games

The constraint $\partial v(s^t) \leq B$ implies that the return to money is bounded above by the discounted-expected utility gain of having one unit of money

$$\partial v(s^t) \leq f_{s_t}(y(s^t)) + \sum_{\tau > t} \sum_{s_\tau \in S^\tau} \alpha^{\tau - t} p(s^\tau) \ f_{s_\tau}(y(s^\tau)).$$

**Proposition 1 (Maximum Sustainable Debt)** *Any sequence* $\{y(s^t), \partial v(s^t)\}$*, satisfying the constraints of the Planner's problem, is such that* $\partial v(s^t) \leq \bar{d}_s$ *for all* $s^t$*, where* $\bar{d}_s$ *solves* $\bar{d}_s = f_s(\beta \bar{d}) + \alpha \bar{d}$ *and* $\bar{d} = \pi_l \bar{d}_l + \pi_h \bar{d}_h$ *for* $s \in \{l, h\}$*.*

If we associate the multipliers $\beta^t p(s^t)\ \mu(s^t)$ and $\beta^t p(s^t)\ \lambda(s^t)$ to the producer's and debt constraints, the FOC with respect to $\partial v(s^t, s')$ yields

$$\lambda(s^t, s_{t+1}) = \mu(s^t) + (1 - \frac{1}{N})\lambda(s^t).$$

- Note that $\lambda(s^t, l) = \lambda(s^t, h)$
- Debt is unrestricted in the initial period: $\lambda(s_0) = 0$.
- History dependence requires $\mu(s^t) > 0$.
- When $\mu(s^t) = 0$, we have $\lambda(s^t, s_{t+1}) = (1 - \frac{1}{N})\lambda(s^t) < \lambda(s^t)$. The rate of decay depends on $\frac{1}{N}$ (matching friction).

The state is $(s, d_l, d_h)$ .... but return $d_s$ is the only relevant promise in realization $s$. We thus write $(s, d)$, where $d$ is a short for $d_s$.

**Bellman's equation**

$$
\begin{aligned}
Tw(s,d) &= \max_{y, d'_l, d'_h} z_s(y) + \beta(\pi_l w(l, d'_l) + \pi_h w(h, d'_h)) \\
&\quad \text{s.t.} \\
y &\leq \beta(\pi_l d'_l + \pi_h d'_h) \\
d &\leq f_s(y) + \alpha(\pi_l d'_l + \pi_h d'_h)
\end{aligned}
$$

**Proposition 2** *Let $w \in W$. Then, $Tw$ is continuous, weakly decreasing in $d$, and concave. The Bellman's equation has a unique solution. Principle of Optimality applies.*

## Economy with no aggregate-uncertainty

**Proposition 3 (No memory)** *In the economy without shocks, the optimal allocation is constant (no dynamics) and the consumer constraint slacks. First best allocation* $y^*$*is only attained when* $\beta$ *is close to* $1$.

Lesson 1: aggregate uncertainty is necessary for history-dependence.

**Proposition 4 (Artificial dynamics.)** *Fixed an initial* $d_0$.
*i) If* $\beta$ *is low, so that* $y^* > \overline{y}$*, no dynamics:* $y(s^t) = \overline{y}$ *and* $d(s^t) = \overline{d}$.
*ii) If* $\beta$ *is high, so that* $y^* < \overline{y}$*, debt and output converge monotonically to* $d^*$ *and* $y^*$ *for all initial* $d_0 \in (d^*, \bar{d})$.

Lesson 2: history-dependence requires that producer's constraint bind ... but not always (so that we can borrow from future states)

## Economy with aggregate-uncertainty

**Main result**: for producer constraint to bind, but not always, discount factor should be not too high *and* not too low.

**Proposition 5** *Assume* $\beta$ *high enough so that* $y_h^* < \beta d^*$, *where* $d^* = \frac{1}{1-\alpha}[\pi_h f_h(y_h^*) + \pi_l f_l(y_l^*)]$. *Then, the optimum is given by First-Best allocation* $y_s^*$ *and is thus not history-dependent.*

**Proposition 6** *There exists* $\beta$ *such that the following holds. The values of* $d^*$ *and* $\bar{d}$ *satisfy* $y_l^* < \beta d^* < y_h^* < \beta \bar{d}$ *and, moreover, the optimum is history-dependent.*

## More on the economy with aggregate-uncertainty

**Proposition 7** *There exists* $\beta_0$ *so that, when* $\beta \leq \beta_0$*, for which output is constant* $y(s,d) = \widehat{y} \leq y_l^*$ *for all* $(s,d)$*. Moreover, output equals* $y_l^*$ *only if* $\beta = \beta_0$.

Key insight: Since participation constraints bind in all states, the Planner can not exploit inter-temporal trade-offs to induce more production when $s$ is high.

## Divisible money: Lagos and Wright.

- LW economy with aggregate-taste shocks at beginning of each period.

- Day: decentralized market with anonymous bilateral matching.

- Night: centralized market where a general good is produced and exchanged.

- Preferences: $u_s(y) - h + U(Y) - H$.

- Growth rate of money $\tau(s^t)$.

## Mechanism design

Trading mechanisms have 2 components:

1. actions sets (include autarkic allocation).

2. outcome functions.

The mechanism we consider has 2 parts:

1. Day-trading mechanism: divide the pie.

2. Night-trading mechanism: spot exchange at competitive price.

## Assume lump sum taxes are available

**Result 1**. For all $\beta > 0$ the first best level of output is implementable with counter-cyclical money-growth rates: $\tau_h < \tau_l$ and $\tau_h < 0$.

## Lump sum taxes are not available

**Result 2**. The first best level of output is implementable if $\beta$ is close to 1. Moreover, optimality requires positive inflation in low state.

**Main Lesson**: price stickiness results from the absence of markets that give fiscal and monetary policy the ability to implement the first best.

## Monitoring: non-monetary mechanisms.

- Any individual deviation can be detected and defectors punished with autarky.

- Full monitoring: whole history of individuals can be recorded.

- Limited monitoring: Planner can only record whether an individual has defected or not in the past

**Main Lesson**: Efficient allocations with limited monitoring are not history-dependent. With full monitoring history dependence can help relax incentive constraints.

## Conclusions: Memory and 2nd Best Efficiency.

- We do no need "special assumptions" such as signal extraction problem, segmented markets, or nominal "rigidities".

- $$\left.\begin{array}{c} anonymity \\ lack\ of\ commitment \\ aggregate\ uncertainty \end{array}\right\} \Rightarrow$$ memory is a “natural” property of money.

- Theory$\Rightarrow$ Money and Business Cycles are intertwined (propagation of shocks).

The optimal allocation is described with the help of threshold debt levels $(\hat{d}_l, \hat{d}_h)$ such that:

1. In state $s = l$, $y(l, d) = y_l^*$ and $(d_l'(l, d), d_h'(l, d)) = (\hat{d}_l, \hat{d}_h)$ for all $d \leq \hat{d}_l$. Output and new debt are increasing functions of $d_l$. Moreover, the policy function for new debt when $s = l$ is such that, for $d_0$ on a right neighborhood of $\hat{d}_l$, the sequence $d^{n+1} = (d_l'(l, d^n), d_h'(l, d^n)$ is a decreasing sequence converging to $(\hat{d}_l, \hat{d}_h)$.

2. In state $s = h$, for $d_h \leq \hat{d}_h$, output is $y_l^* < y_h < y_h^*$ and new debt is $(d_l'(h, d), d_h'(h, d)) > (\hat{d}_l, \hat{d}_h)$. Moreover, output and new debt are increasing functions of $d_h$ for $d_h$ in a right neighborhood of $\hat{d}_h$.