Financial Frictions, Investment and Tobin's q^*

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Abstract

We develop a model of investment with financial constraints and use it to investigate the relation between investment and Tobin's q. A firm is financed partly by insiders, who control its assets, and partly by outside investors. When insiders' wealth is scarce, they earn a rate of return higher than the market rate of return, i.e. insiders earn a quasi-rent on invested capital. This rent is priced into the value of the firm, so Tobin's q is driven by two forces: changes in the value of invested capital, and changes in the value of the insiders' future rents. The second effect weakens the correlation between q and investment. We calibrate the model and show that, thanks to this effect, it can generate realistic correlations between investment, q and cash flow.

Keywords: Financial constraints, Tobin's q, limited enforcement, optimal capital structure.

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1 Introduction

The standard model of investment with convex adjustment costs predicts that movements in the investment rate should be entirely explained by changes in Tobin's q. This prediction has generally been rejected in empirical studies. Furthermore, several studies have shown that cash flow and other measures of current profitability have a strong predictive power for investment. This has been taken by many authors as *prima facie* evidence of the presence of financial constraints at the firm level.

Some recent papers — in particular Gomes (2001) and Cooper and Ejarque (2003) — have challenged the above interpretation. These papers compute dynamic general equilibrium models with financial frictions, calibrate them, and look at the relation between Tobin's q and investment in the simulated series. They show that, in presence of financial frictions alone, Tobin's q still explains most of the variability in investment, and cash flow does not provide any additional explanatory power. These results seem to echo a concern raised by Chirinko (1993):

> "Even though financial market frictions impinge on the firm, q is a forward looking variable capturing the ramifications of these constraints on all the firm's decisions. Not only does q reflect profitable opportunities in physical investment but, depending on circumstances, q capitalizes the impact of some or all financial constraints as well."[1]

In this paper we analyze this issue using a model of investment with a financial friction due to limited enforcement of financial contracts. We allow firms to use a rich set of state contingent liabilities, which can include debt and equity claims. For each firm there is an "insider," which can be interpreted as the entrepreneur, the manager or the controlling shareholder. The financial constraint imposes a lower bound on the fraction of the firm's value held by the insider at each point in time. In this framework, we explicitly derive the market value of the total outstanding claims of the firm and use it to compute average q.

The contribution of this paper is twofold. First, we show that the presence of the financial constraint introduces a positive *wedge* between average q and marginal q. This wedge reflects the tension between the future profitability of investment and the availability of internal funds in the short run. Second, we show that this wedge varies over time, and this weakens the observed correlation between q and investment. Using a calibrated version of our model, we show that our model can generate realistic correlations between investment, q and cash-flow.

The paper provides a tractable model of optimal long-term financial contracts with financial frictions, that can be used to explore the relation between investment and asset prices. The model has two main ingredients: convex adjustment costs with constant returns to scale, as in the classic Hayashi (1982) model, and a specific assumption of limited enforcement in financial contracts that delivers a linear financial constraint for the single entrepreneur. Thanks to the second assumption, the problem of the individual entrepreneur retains the linearity of the original Hayashi (1982) model. This makes it easier to compare the model with financial frictions with the frictionless benchmark. An additional advantage of these assumptions is that aggregation is straightforward. In this sense, the model retains the simplicity of a representative agent model, while allowing for rich dynamics of the financial constraint.

[1] Chirinko (1993) p. 1903.

The main difference between our paper and the papers by Gomes (2001) and Cooper and Ejarque (2003), is in the way we treat the financial friction. Namely, they adopt a relatively "reduced form" specification, assuming a convex cost of outside finance, while we derive explicitly the optimal financial contract in an environment with limited enforcement.

Our paper is related to the large theoretical literature on the macroeconomic implications of financial frictions, e.g. Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Cooley, Marimon and Quadrini (2004). The form of financial imperfection presented in this paper is close in spirit to Kiyotaki and Moore (1997). The main difference with that paper, apart from the introduction of adjustment costs, is that we allow for fully state contingent securities and that we introduce aggregate shocks explicitly.

A model that combines convex adjustment costs and financial frictions is Bernanke et al. (2000). There are two crucial differences between our approach and theirs. They concentrate on short term financial contracts, and on debt instruments. In the present paper, instead we consider long term financial arrangements, and we allow for fully state contingent contracts. An advantage of our approach, is that we can define q looking at the value of total financial claims issued by the entrepreneurs, and therefore we can map the measure of q in the model, with the q observed on financial markets.

Our model is also related to the model of financial contracting with limited enforcement in Albuquerque and Hopenhayn (2004). The main differences between our setup and theirs are the assumption of constant returns to scale and the way in which we model the outside option of the entrepreneur. They assume that after default an entrepreneur goes into autarky, while we assume that he loses a fraction of his wealth but retains access to financial markets. The assumption of constant returns to scale implies that the optimal financial contract is linear in the entrepreneur's initial wealth. This allows us to give a simple characterization for the risk-management problem of the single entrepreneur. Moreover, it greatly simplify aggregation and provides a tractable way of introducing financial frictions in a general equilibrium setting.

Following Fazzari *et al.* (1988) there has been a large empirical literature exploring the relation between investment and asset prices in panel data. The great majority of these papers have found small coefficients on average q and positive and significant coefficients on cash flow, or other variables describing the current financial condition of a firm.[2] An early critical interpretation of these results was that cash flow contained information regarding future profits that, for some reason, (measurement error or non-fundamental stock market movements) was not captured by the empirically observed q. This interpretation was rejected by Gilchrist and Himmelberg (1995), who show that cash flow has significant additional predictive power on investment even after controlling for the information value in current cash flow.

The idea of looking at the statistical implications of a simulated model to understand the empirical correlation between investment and q goes back to Sargent (1980). Recently Gomes (2001), Cooper and Ejarque (2001, 2003) and Abel and Eberly (2005) have followed this route, introducing either financial frictions or decreasing returns and market power to try to match the existing empirical evidence. The conclusion one reaches from this literature is that decreasing returns and market power can generate realistic correlations, while financial frictions do not help in matching the observed correlations. In this paper we show that the second conclusion is unwarranted, and depends crucially on the way

[2] E.g. Gilchrist and Himmelberg (1995), Gilchrist and Himmelberg (1998). See Hubbard (1998) for a survey.

one models the financial constraint. On the other hand, there are some parallels between our approach and these papers, in particular with the "growth options" approach of Abel and Eberly (2005). Both approaches imply that movements in q may reflect changes in future rents that are unrelated with current investment. In the current paper these rents are not due to market power, but to the scarcity of entrepreneurial wealth, which evolves endogenously.

The organization of the paper is as follows. Section 2 introduces the model. In Section 3, we look at the optimal financial contract from the point of view of a single entrepreneur. Section 4 characterizes a competitive equilibrium and characterizes the equilibrium relation between investment and asset prices. Section 5 describes the calibration and simulation results. In Section 6 we discuss some extensions. Section 7 concludes.

2 The model

Preferences and technology. There are two groups of agents: consumers and entrepreneurs. There are two goods, a perishable consumption good and physical capital. Each group of agents is a continuum of mass 1. Consumers are infinitely lived and have linear preferences represented by the utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t c_t\right].$$

They have a constant endowment of labor l_C which they supply on the labor market each period.

Entrepreneurs have random, finite lives. Each period a random fraction γ of entrepreneurs dies and is replaced by an equal mass of young entrepreneurs. Young entrepreneurs are endowed with l_E units of labor in the first period of their life. We normalize total labor supply to one, so that $l_C + \gamma l_E = 1$.

The preferences of entrepreneur i, born at date t, are described by the utility function

$$\mathbb{E}_t\left[\sum_{j=0}^{J_i}\beta_E^j c_{i,t+j}^E\right],$$

where J_i is the random duration of the entrepreneur's life. We allow for the discount factors of consumers and entrepreneurs, β and β_E, to be different. We assume that $\beta_E < \beta$. This assumption, together with the assumption of a finite life for entrepreneurs, guarantees the existence of a steady state where the borrowing constraint is always binding. We will further discuss this assumption below.

Each period t entrepreneurs have access to a constant returns to scale technology described by the concave production function $A_t F(k_{i,t}, l_{i,t})$, where $k_{i,t}$ is capital installed in period $t-1$. The productivity parameter A_t is the same for all entrepreneurs. Entrepreneurs face convex adjustment costs. By employing $k_{i,t}^o$ units of used capital, or "old capital," and $G\left(k_{i,t+1}, k_{i,t}^o\right)$ units of the consumption good in period t an entrepreneur installs $k_{i,t+1}$ units of new capital, ready for production in period $t+1$. The function G is convex in $k_{i,t+1}$, homogeneous of degree 1 and satisfies $G_1(k,k) = 1$.

The timing of events is as follows. At the beginning of period t, production is realized and entrepreneur i learns if period t is his last period of activity. Then, entrepreneurs exchange used capital. An entrepreneur can set $k^o_{i,t} \neq k_{i,t}$ by trading the difference on the used capital market. Finally, new capital is installed using old capital and consumption goods as inputs. With this timing assumption entrepreneurs are able to liquidate all old capital on their last period of activity, while continuing entrepreneurs acquire it from them. The assumption that used capital is homogeneous and can be traded across firms is useful to simplify the entrepreneurs' problem in the last period of their life. It also helps in modelling the liquidation proceedings in the event an entrepreneur defaults.

Aggregate uncertainty is described by the Markov process s_t in the finite state space $\mathcal{S}$, with transition probability $\pi(s_{t+1}|s_t)$. The state s_t determines current productivity according to $A_t = A(s_t)$. This general formulation will allow us to introduce both persistent and temporary productivity shocks. The theory will be developed for the case of a finite state space $\mathcal{S}$, for ease of exposition. However, when we turn to the simulations we will use continuous random variables. Individual uncertainty is described by the random variable $\chi_{i,t}$, which is equal to 1 in all the periods when entrepreneur i is active, except in the last period, when $\chi_{i,t} = 0$.

Financial contracts. Consider an entrepreneur born at time t. The entrepreneur finances his current and future investment by selling a long-term financial contract $\mathcal{C}_{i,t}$. The contract specifies: a sequence of state-contingent transfers $\{d_{i,t+s}\}_{s=0}^{\infty}$,[3] a sequence of state-contingent labor inputs, old capital inputs, and capital stocks $\left\{l_{i,\tau}, k^o_{i,\tau}, k_{i,\tau+1}\right\}_{\tau=t}^{\infty}$ for all the periods in which the entrepreneur is alive. The transfers and input levels are contingent both on the history of aggregate shocks, $\{s_0, s_1, ..., s_t\}$, and on the idiosyncratic termination shock of entrepreneur i. The choice variables $k^o_{i,\tau}$ and $k_{i,\tau+1}$, and the transfer $d_{i,\tau}$, are set after the idiosyncratic termination shock is realized. Let q^o_t denote the price of old capital in period t. Feasibility requires that the transfers $\{d_{i,\tau}\}$ satisfy:

$$c^E_{i,\tau} + d_{i,\tau} + G\left(k_{i,\tau+1}, k^o_{i,\tau}\right) \leq A_\tau F\left(k_{i,\tau}, l_{i,\tau}\right) - w_\tau l_{i,\tau} - q^o_\tau\left(k^o_{i,\tau} - k_{i,\tau}\right), \tag{1}$$

for all the periods where the entrepreneur is active.[4] All input levels and transfers are set to zero afterwards.

Limited enforcement. Financial contracts are subject to limited enforcement. The entrepreneur has full control over the firm's assets. In each period, after production takes place, the entrepreneur can choose to divert part or all of the current profits and the capital stock. In this way he can capture up to a fraction $(1-\theta)$ of the firm's *liquidation value*, $v_{i,t}$, which is equal to current profits plus the resale value of the capital stock:

$$v_{i,t} = A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q^o_t k_{i,t}.$$

The only recourse outside investors have against such behavior is the liquidation of the firm. Upon liquidation, the investors can recover the remaining fraction θ of the firm's liquidation value. After liquidation the entrepreneur can start anew with initial wealth $(1-\theta)\, v_{i,t}$. That is, the only punishment for a defaulting entrepreneur is the loss of a fraction θ of the firm's

[3] The transfer will typically be negative in the first period (initial investment) and can be positive or negative in the following periods, corresponding to dividend payments minus new investment in the firm.

[4] In the first period of activity the constraint is:

$$c^E_{i,t} + d_{i,t} + G\left(k_{i,t+1}, k^o_{i,t}\right) \leq A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} - q^o_\tau\left(k^o_{i,t} - k_{i,t}\right) + w_t l_E,$$

with $k_{i,t} = 0$.

liquidation value. Assuming that the entrepreneur is only allowed to re-enter the financial market after a certain number of periods would not alter the qualitative features of the model. We discuss below the relation between this form of limited enforcement and other contractual imperfections used in the literature.

3 Optimal financial contracts

Before turning to the competitive equilibrium, we concentrate on the decision problem of a single entrepreneur. We begin by introducing some preliminary definitions that will simplify the analysis. Then we give a recursive characterization of the optimal financial contract and show that, under constant returns to scale and given the notion of limited enforcement introduced above, the optimal financial contract is linear.

3.1 Preliminaries

We will study equilibria where consumers always have positive consumption, $c_t > 0$. Therefore, the price of a sequence of state-contingent transfers $\{d_{i,t+s}\}_{s=0}^{\infty}$ is equal to its expected present value, discounted at the rate β. An entrepreneur born at date t will choose the financial contract $\mathcal{C}_{i,t}$ to maximize his expected utility subject to feasibility, (1), to the intertemporal budget constraint:

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t \left[d_{i,t+s} \right] \geq 0,$$

and to the condition that future promised transfers be credible. The last condition is satisfied if, at each date, the entrepreneur prefers repayment to diversion and default. This condition is stated formally below. For a recursive formulation of the problem it is useful to define the net present value of the firm's liabilities at date τ:

$$b_{i,\tau} = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_\tau \left[d_{i\tau+s} \right].$$

The entrepreneur's problem can be simplified by exploiting the assumption of constant returns to scale. Under constant returns to scale the liquidation value of the firm can be written as:

$$v_{i,t} = R_t k_{i,t} = \max_{l_{i,t}} \left\{ A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q_t^o k_{i,t} \right\},$$

where R_t, the gross return on capital, is taken as given by the single entrepreneur and is a function of the prices w_t and q_t^o. Also, constant returns to scale for G, and the presence of a competitive market for old capital, imply that there exists a shadow price of new capital, q_t^m, such that:

$$q_t^m k_{i,t+1} = \min_{k_{i,t}^o} \left\{ q_t^o k_{i,t}^o + G(k_{i,t+1}, k_{i,t}^o) \right\}. \tag{2}$$

Thanks to constant returns to scale, the value of q_t^m is a function of q_t^o and, thus, is taken as given by the single entrepreneur.[5]

[5] The first order condition for problem (2) gives:

$$q_t^o = G_2\left(k_{i,t+1}, k_{i,t}^o\right) = G_2\left(\frac{k_{i,t+1}}{k_{i,t}^o}, 1\right)$$

Note that in this model there is a one-to-one relation between the investment rate and the shadow price of new capital:

$$q_t^m = G_1\left(\frac{k_{i,t+1}}{k_{i,t}}, 1\right) \tag{3}$$

That is, this shadow price corresponds to the usual definition of marginal q and is a sufficient statistic for the firm's investment rate. The open question is whether the q observed in financial markets corresponds to marginal q in a model with financial frictions. This is the issue we address in 4.1.

Putting together the definitions above, the feasibility constraint (1) can be written as:[6]

$$c_{i,\tau}^E + d_{i,\tau} + q_\tau^m k_{i,\tau+1} \le v_{i,\tau}. \tag{4}$$

3.2 Recursive characterization

We study recursive competitive equilibria, where the state of the economy is captured by a vector of aggregate state variables $X_t \in \mathcal{X}$, including the exogenous state s_t, with transition probability $H(X_{t+1}|X_t)$. The vector X_t will be defined and discussed in section 4. For now, consider a single entrepreneur, who takes as given the law of motion for X_t. The state X_t determines the wage rate, w_t, and the price of used capital, q_t^o. Therefore, it also determines the gross rate of return, R_t, and the shadow price of new capital, q_t^m. Let this dependence be captured by the functions $R(X_t)$ and $q^m(X_t)$.

Now we can use a recursive approach to characterize the optimal financial contract. The individual state variables for the entrepreneur are given by $v_{i,t}$, $b_{i,t}$, and $\chi_{i,t}$. Define $W(v, b; \chi, X)$ as the expected utility, in state X, of an entrepreneur who controls a firm with liquidation value v and outstanding liabilities b.[7] The expected utility W is defined at the time when production has already taken place and the idiosyncratic termination shock has been observed. Also, W is defined after the default decision has taken place, assuming that the entrepreneur does not default in the current period. For now, we will assume that the entrepreneur's problem has a solution in each state $X \in \mathcal{X}$, and the expected utility W is finite. This will be the case in the recursive equilibria we study below (see Proposition (4)).

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which determines the ratio $k_{i,t+1}/k_{i,t}^o$. Moreover, the envelope theorem gives:

$$q_t^m = G_1\left(k_{i,t+1}, k_{i,t}^o\right) = G_1\left(\frac{k_{i,t+1}}{k_{i,t}^o}, 1\right).$$

Therefore q_t^o determines the ratio $k_{i,t+1}/k_{i,t}^o$, which determines q_t^m.

[6] The first order condition for problem (2) (see footnote 5) and constant returns to scale give:

$$\begin{aligned} G\left(k_{i,t+1}, k_{i,t}^o\right) &= G_1 k_{i,t+1} + G_2 k_{i,t}^o \\ &= q_t^m k_{i,t+1} - q_t^o k_{i,t}^o. \end{aligned}$$

Substituting in the feasibility constraint and rearranging we obtain (4).

For a newborn entrepreneur the constraint is:

$$c_{i,t}^E + d_{i,t} + q_t^m k_{i,t+1} \le w_t l_E.$$

[7] For a newborn entrepreneur, v is the entrepreneur's initial labor income, and b is zero.

In all periods prior to the last period of activity, i.e. for $\chi = 1$, W satisfies the Bellman equation:

$$W(v, b; 1, X) = \max_{\substack{c^E, d \\ k', v'(.), b'(.)}} c^E + \beta_E \mathbb{E}[W(v', b'; \chi', X') | X] \tag{P}$$

$s.t.$

$$c^E + d + q^m(X) k' \leq v, \tag{5}$$

$$b = d + \beta \mathbb{E}[b'(\chi', X') | X], \tag{6}$$

$$v'(X') = R(X') k' \quad \forall X', \tag{7}$$

$$W(v'(X'), b'(\chi', s'); \chi', X') \geq W((1-\theta) v'(X'), 0; \chi', X') \quad \forall \chi', X', \tag{8}$$

where the conditional expectation $\mathbb{E}[.|X]$ is computed according to the transition $H(X'|X)$, with χ' independent of X'.

Problem (P) can be interpreted as follows. At each date, an entrepreneur who does not default has to decide how to allocate the current firm's resources, v, to its potential uses: payments to insiders, c^E, payment to outsiders, d, and investment in physical capital, $q^m k'$. This is captured by the feasibility constraint (5). Moreover, the entrepreneur has to satisfy the "promise keeping" constraint (6): current and future payments to outsiders have to cover the current liabilities of the firm, b. The current payments are d, the future payments are captured by the net present value of the firm's liabilities in the following period, $b'(\chi', X')$. These liabilities are allowed to be contingent on the realization of the idiosyncratic termination shock χ' and of the aggregate state X'.[8] Constraint (7) simply says that liquidation value of the firm next period will be given by the total returns on the firm's installed capital k'. Finally, the no-default constraint (8) ensures that, in all future states of the world, the future liabilities b' are credible. The no-default constraint take this form, given that the entrepreneur has the option to default and start anew with a fraction $(1-\theta) v'$ of the firm's liquidation value and zero liabilities.

An entrepreneur in his last period of activity will simply liquidate all capital and pay existing liabilities. Therefore, for $\chi = 0$ we have:

$$W(v, b; 0, X) = v - b.$$

Lemma 1 *The value function satisfies*

$$W(v, b; \chi, X) = W(v - b, 0; \chi, X)$$

and the no-default condition can be written as

$$b \leq \theta v. \tag{9}$$

Proof. The first result follows by simply substituting d in problem (P). To prove the second result it is sufficient to show that W is monotone increasing in its first argument. ■

Lemma 1 allows us to replace constraint (8) with constraint (9). The latter can be interpreted as a "collateral constraint," where the total value of the entrepreneur liabilities are bounded from above by a fraction θ of the liquidation value of the firm.

[8] In equilibrium the distribution of all the elements of X', conditional on the exogenous state s', will be degenerate. Therefore, we could restrict b' to be contingent only on χ' and s'. We allow b' to be contingent on all the elements of X' only for notational convenience.

An alternative set of assumptions that would deliver the same "collateral constraint" is the following: (1) the entrepreneur loses access to the technology after default, (2) before liquidation takes place there is a round of renegotiation, and (3) in the renegotiation stage the entrepreneur can make a take-it-or-leave it offer to the outside investors. With these assumptions, the entrepreneur has all the bargaining power, and whenever the net present value of the entrepreneur's outstanding liabilities exceeds θv he will renegotiate them down to θv. Apart from the presence of state-contingent dividends, this is the set of assumptions used in Kiyotaki and Moore (1997).

If we replace constraint (8) with constraint (9), problem (P) is linear and we obtain the following proposition.

Proposition 2 *The value function $W(.,.;\chi,X)$ is linear in its first two arguments and takes the form:*

$$\begin{aligned} W(v,b;1,X) &= \phi(X)(v-b), \\ W(v,b;0,X) &= v-b. \end{aligned}$$

There is an optimal policy for k', c^E, d and b' which is linear in $v-b$.

Define the *net worth* of entrepreneur i:

$$n_{i,t} = v_{i,t} - b_{i,t}.$$

This variable represents the difference between the market value of the firm's capital (including current profits) and the value of the claims issued to outsiders. Proposition 2 shows that the expected utility of the entrepreneur is a linear function of the entrepreneur net worth. The factor ϕ, which determines the marginal value of the entrepreneur net worth, depends on current and future prices, and hence it is, in general, dependent on X.

The following proposition gives a further characterization of the optimal solution.

Proposition 3 *For a given law of motion $H(X'|X)$, let $\phi(X)$ be defined by the recursion:*

$$\phi(X) = \max\left\{\frac{\beta_E(1-\theta)\mathbb{E}[(\gamma+(1-\gamma)\phi(X'))R(X')|X]}{q^m(X)-\beta\theta\mathbb{E}[R(X')|X]}, 1\right\}. \tag{10}$$

Suppose that

$$\beta\phi(X) \geq \beta_E\phi(X') \tag{11}$$

for all pairs X, X' such that $H(X'|X) > 0$. Then, the optimal policy for the individual entrepreneur involves: (i) $k' > 0$, (ii) $c^E = 0$ if $\phi(X) > 1$, and (iii) $b(1,X') = \theta v(X')$ if $\beta\phi(X) > \beta_E\phi(X')$.

A central result of this proposition is point (iii), which characterizes the cases where the state contingent liabilities are set to their maximum level. Consider an entrepreneur in state X choosing his financial liabilities next period, in state X'. The entrepreneur compares the marginal value of a dollar today, $\phi(X)$, to the marginal value of a dollar tomorrow, given by $\beta_E\phi(X')H(X'|X)$. On the financial market, the price of a dollar in state X' is $\beta H(X'|X)$. Therefore, if (11) holds as a strict inequality, then it is optimal to borrow as much as possible against the revenue realized in state X' and use the proceeds to invest today.

3.3 An asset pricing interpretation

Consider the ratio

$$m\left(X', X\right) = \beta_E \frac{\gamma + (1-\gamma)\,\phi\left(X'\right)}{\phi\left(X\right)},$$

this ratio represents the shadow discount factor for the entrepreneur, i.e. it represents the ratio of the marginal value of inside wealth tomorrow, in state X', to the marginal value of inside wealth today.

Let $\tilde{R}$ represent the leveraged rate of return on entrepreneurial net worth:

$$\tilde{R}\left(X', X\right) \equiv \frac{(1-\theta)\,R\left(X'\right)}{q^m\left(X\right) - \beta\theta\mathbb{E}\left[R\left(X'\right)|X\right]}.$$

For each dollar of net worth the entrepreneur can borrow up to $q^m / \left(q^m - \beta\theta\mathbb{E}\left[R'|X\right]\right)$ dollars today, because he can pledge a fraction θ of the gross return $\mathbb{E}\left[R'|X\right]$ and sell it to the consumers. This investment yields a return R_{t+1}/q_t^m, of which the entrepreneur will retain a fraction $(1-\theta)$.

Then, condition (10) can be rewritten as the familiar asset pricing expression

$$\mathbb{E}\left[m\left(X', X\right)\tilde{R}\left(X', X\right)|X\right] = 1.$$

Notice that consumers do not have access to direct investment in entrepreneurial capital, so this condition does not hold using the consumers' discount factor. Using condition (11) one can show that:

$$\beta\mathbb{E}\left[\tilde{R}|X\right] \geq \mathbb{E}\left[m\tilde{R}|X\right] = 1.$$

This inequality also implies that the market return on non-leveraged entrepreneurial capital also is larger than 1, that is,

$$\beta\mathbb{E}\left[\frac{R\left(X'\right)}{q^m\left(X\right)}|X\right] \geq 1$$

The difference $\beta\mathbb{E}\left[R'\right]/q^m - 1$ is sometimes called the "external finance premium," as it reflects the premium that outsiders would be willing to pay to invest directly in the firms' capital. This premium is closely related to the wedge between average q and marginal q, that will be analyzed below.

4 Equilibrium and asset prices

We are now in a position to define a recursive competitive equilibrium. The aggregate state is given by

$$X = \left(K, B, s\right),$$

where K is the aggregate capital stock and B represents the aggregate liabilities of the entrepreneurs who are not in their last period of activity.

A recursive competitive equilibrium is given by a transition probability, $H\left(X'|X\right)$, such that the optimal behavior of entrepreneurs is consistent with this transition probability, and the goods market, labor market, and capital market clear. The formal definition is given in the Appendix.

A crucial property of our model is that the entrepreneur's problem is linear, and we obtain optimal policies that are linear in entrepreneurial net worth, $v_{i,t} - b_{i,t}$. Given the linearity of the optimal policies it is straightforward to aggregate the behavior of the entrepreneurial sector. We illustrate the aggregation properties of the model in the case where the collateral constraint is always binding. This is the case where the condition

$$\beta\phi(X) > \beta_E\phi(X') \tag{12}$$

holds for every pair X, X' such that $H(X'|X) > 0$. Proposition 4 shows that, in economies with "small" productivity shocks, such an equilibrium exists. This case will be the basis for the numerical analysis in the next section. In section 6 we discuss the more general case where the financial constraint is occasionally binding.

Condition 12 implies that, in each state X, the state-contingent liabilities are set to their maximum level for each future value of X', i.e. $b'(\chi', X') = \theta v'(X')$. Therefore, the optimal level of investment is given by:

$$k' = \frac{1}{q^m(X) - \beta\theta\mathbb{E}[R(X')|X]}(v-b). \tag{13}$$

Consider an economy that enters period t with an aggregate stock of capital K_t, in the hands of old entrepreneurs. The agents who invest in period t are: a mass $(1-\gamma)$ of the old entrepreneurs, who have $v_{i,t} = R_t k_{i,t}$ and $b_{i,t} = \theta R_t k_{i,t}$, and a mass γ of newborn entrepreneurs with $v_{i,t} = w_t l_E$. Therefore, the aggregate entrepreneurial net worth of investing entrepreneurs is:[9]

$$N_t = (1-\gamma)(1-\theta)R_t K_t + \gamma w_t l_E,$$

Using the optimal policy (13) and aggregating we obtain:

$$K_{t+1} = \frac{1}{q_t^m - \beta\theta\mathbb{E}_t[R_{t+1}]}N_t.$$

From these two equations we get the following law of motion for the aggregate capital stock

$$K_{t+1} = \frac{(1-\gamma)(1-\theta)R_t K_t + \gamma w_t l_E}{q_t^m - \beta\theta\mathbb{E}_t[R_{t+1}]}. \tag{14}$$

The next proposition shows that for a Cobb-Douglas economy with quadratic adjustment costs and bounded productivity shocks, we can construct a recursive equilibrium of this type.

Let the production function and the adjustment cost function be:

$$A_t F(k_t, l_t) = A_t k_t^\alpha l_t^{1-\alpha}, \tag{15}$$

$$G(k_{t+1}, k_t) = k_{t+1} - (1-\delta)k_t + \frac{\xi}{2}\frac{(k_{t+1}-k_t)^2}{k_t}. \tag{16}$$

Let the unconditional mean of A_t be $\hat{A}$, and let the support of A_t be $[\underline{A}, \overline{A}]$. To show that a recursive equilibrium with binding constraint exists we first check that there is a deterministic steady state with binding constraints. This requires that θ is not too large, inequality (A1) in the Appendix ensures that. Second, to obtain local stability of the recursive equilibrium around the deterministic steady state it is necessary to impose an additional restriction on the model parameters. This restriction is given by inequality (A2) in the Appendix. Under these two restrictions the following proposition holds.

[9] Here, we are assuming that the average $k_{i,t}$ for the fraction $(1-\gamma)$ of old entrepreneurs who do not die in period t, is equal to K_t. That is, we are assuming that an appropriate law of large numbers apply to our continuum of entrepreneurs.

Proposition 4 *Suppose the parameters* $\left\{\alpha, \xi, \theta, \gamma, \beta, \beta_E, \hat{A}, l_E\right\}$ *satisfy conditions (A1) and (A2) in the Appendix. Then the economy with constant productivity* $A(s) = \hat{A}$ *has a deterministic steady state with* $\beta_E R > 1$*. Furthermore, there is a* $\Delta > 0$ *such that if the process* $A(s)$ *satisfies* $\overline{A} - \underline{A} < \Delta$*, then there exists a recursive competitive equilibrium where the financial constraint is always binding.*

4.1 Average q and marginal q

We are now in a position to define the *financial value* of a representative firm. The value of the firm is simply the sum of all the claims on the firm's future profits, held by insiders and outsiders. That is, it is equal to the net present value of the payments $c^E_{i,t+s}$ and $d_{i,t+s}$.[10] This leads us to the following expression for the value of the firm:

$$s_{i,t} = W\left(v_{i,t}, b_{i,t}; \chi_{i,t}, X_t\right) + b_{i,t} - d_{i,t} - c^E_{i,t}.$$

Where W corresponds to the net present value of the payments to the insider (including current payments) and $b_{i,t}$ corresponds to the net present value of the payments to outsiders (including current payments). We subtract the current payments, $d_{i,t} + c^E_{i,t}$, to obtain the end-of-period (ex-dividend) value of the firm.

Normalizing the financial value of the firm by the total capital invested we obtain our definition of *average q*

$$q_{i,t} \equiv \frac{s_{i,t}}{k_{i,t+1}}.$$

Note that, for an entrepreneur in the last period of activity, both $s_{i,t}$ and $k_{i,t+1}$ are zero, so $q_{i,t}$ is not well defined. On the other hand, for continuing entrepreneurs, it is possible to show $q_{i,t}$ is the same for all agents, and we denote it simply by q_t.

Proposition 5 *Average q is greater or equal than marginal q,* $q_t \geq q^m_t$ *with a strict inequality if the financial constraint is binding.*

Proof. Given that $\phi_t \geq 1$ we have

$$s_{i,t} = \phi_t\left(v_{i,t} - b_{i,t}\right) + b_{i,t} - d_{i,t} - c^E_{i,t} \geq v_{i,t} - b_{i,t} - c^E_{i,t} = q^m_t k_{i,t+1}.$$

■

Notice that, absent financial constraints we have $\phi_t = 1$ and $q_t = q^m_t$. In this case the model boils down to the Hayashi (1982) model, and q_t is a sufficient statistic for investment, given that q^m_t is, as we observed in 3.1.

On the other hand, in presence of financial frictions there is a wedge between the value of the entrepreneur's claims in case of liquidation $(v_{i,t} - b_{i,t})$ and the value of the claims he holds to future profits.

If we used the factor ϕ_t to discount the value of future profits that go to the entrepreneurs, we would get

$$\frac{\phi_t\left(v_t - b_t\right)}{\phi_t} + b_t - d_t - c^E_t = q^m_t k_{t+1}$$

[10] Note that for an entrepreneur it is optimal to invest all his wealth in his own firm, and he receives no labor income after his first period of life. Therefore all his consumption, $c^E_{i,t+s}$, is financed by pay-outs from the firm.

and average and marginal q will be equal. However, all the firm's claims are priced at market prices, i.e. using the discount factor of *outside* investors. Then, the presence of $\phi_t > 1$ introduces a form of mis-measurement in a fraction of the firm's current value and creates a wedge between q_t and q_t^m.

The presence of $\phi_t > 1$ is closely related to the presence of a positive external finance premium as defined in 3.3. The recursive relation 10 shows that ϕ_t is a forward looking variable that cumulates the expected values of the future returns on leveraged net worth $\tilde{R}_{t+s}$ which, in turns, are closely related to the external finance premium. Therefore, the value of ϕ_t will be larger when entrepreneurs expect a positive external financial premium in future periods. This generates a source of variability in q_t, which is unrelated to movements in q_t^m, and hence in investment. This variability is analyzed in the following section.

As a side remark, notice that in this model there is a one-to-one relation between q_t^m and q_t^o, therefore if the price of used capital was observed it would be a sufficient statistic for total investment. The price q_t^o is the price at which liquidating entrepreneurs sell used capital, so its empirical counterpart are the prices paid for acquisitions and for sales of used capital equipment. This points to a potential alternative way of measuring q_t that does not rely on financial market data. The presence of this relation, however, relies heavily on the absence of adjustment costs for the transfer of used capital and on the way in which we model firms' exit. In this model firms are all identical and exit is an exogenous event, therefore q_t^o for exiting firms corresponds to q_t^o for all firms. In a more realistic model, the value of q_t^o for exiting firms would not be representative of the shadow value for other firms. Therefore, this alternative empirical strategy is also subject to serious measurement problems.

5 Investment Dynamics

5.1 Calibration

In this section we examine the quantitative implications of the model, looking at the behavior of investment, average q and cash-flow. We focus on economies where the financial constraints is always binding, i.e. where Proposition (4) applies. The production function is Cobb-Douglas and adjustment costs are quadratic, as specified in (15) and (16). The baseline parameters are:

$$
\begin{aligned}
\alpha &= 0.33; \qquad \delta = 0.05; \qquad \xi = 5; \\
\beta &= 0.97; \qquad \beta_E = 0.969; \\
\theta &= 0.6; \qquad \gamma = 0.12; \qquad l_E = 0.3.
\end{aligned}
$$

The values for α and δ are standard. The time period represents a year, and we set β to match a risk-free interest rate of 3%. The adjustment cost ξ is set to 5. This value is much smaller than the values usually derived from q theory equations. Absent financial frictions, the coefficient for q in an investment regression is equal to $1/\xi$. Therefore, to match the low value of the coefficient empirically estimated —typically smaller than 0.1— one needs to assume a large value for ξ, which implies unrealistic levels of the average adjustment costs. Setting $\xi = 5$ means that in absence of financial frictions the coefficient on q would be 0.2. The value for θ is set to 0.6. The parameter θ is approximately equal to the fraction of investment financed with outside funds. Fazzari et al. (1988) report that 30% of manufacturing investment is financed externally. Therefore, by choosing $\theta = 0.6$ we choose a very conservative value for this parameter, biasing our results in the direction of the case of no financial constraints. The parameters γ and l_E are chosen to give an outside finance premium of 3%, close to the one in Bernanke et al. (2000). We experimented with different values of γ and l_E and found out that, as long as the finance premium remains constant, the specific choice of these parameters has little effect on our results. The choice of β_E is conventional. By experimenting with different values of β_E we found that the value of β_E only affects the average value of q, but has no effect on the correlations we consider. On the other hand, the choice of β_E is relevant to check that the conditions of Proposition 4 are verified. In particular, a value of β_E closer to β means that the range of A_t has to be smaller.

The productivity parameter A_t is given by

$$A_t = e^{a_t}$$

where a_t follows the process

$$\begin{aligned} a_t &= x_t + \eta_t \\ x_t &= \rho x_{t-1} + \epsilon_t \end{aligned}$$

The shocks η_t and ϵ_t are Gaussian, i.i.d. shocks, and $\rho = 0.95$. Allowing for both temporary and persistent shocks turns out to be relevant, especially when we look at the univariate correlation between investment and q. To apply proposition 4 we need bounded values for A_t. This is achieved simply by truncating the values for A_t, obtained from the process above. Clearly, for small levels of σ_η^2 and σ_ϵ^2, the truncation is immaterial.

5.2 Impulse-response functions

Figure 1 shows the impulse-response functions for investment, q and cash-flow, following a persistent shock, ϵ_t. The investment rate is given by:

$$i_t \equiv \frac{I_t - \delta K_t}{K_t},$$

q_t is reported in log deviations from the steady state, and cash flow (normalized by the capital stock) is defined as:

$$cf_t \equiv \frac{A_t F\left(K_t, L_t\right) - w_t L_t}{K_t} = \frac{\alpha A_t K_t^{\alpha}}{K_t} = \alpha A_t K_t^{\alpha - 1}.$$

In the right column we also report, for reference, the impulse-response functions for productivity, A_t, for the "wedge" between average and marginal q, defined as $q_t - q_t^m$, and for the capital stock (in log deviations from the steady state).

Following a persistent technology shock, investment, average q, and cash flow increase on impact, as in the standard model without financial frictions. However, now q increases for two reasons. First, marginal q is increasing with investment. On top of that, ϕ_t is increasing, since firms anticipate that the financial constraint will be tighter in the periods immediately following the shock. This increases the wedge between average q and marginal q and magnifies the response of q to the shock.

Why is the financial constraint is tighter in the periods following the shock? Due to the persistent nature of the shock, future profitability increases and this increases the expected returns, R_{t+s}. At the same time, entrepreneurs' inside funds have increased because of the increase in current cash flow. However, initially, the first effect dominates and the rate of return on entrepreneurial net worth increases.

In later periods, investment and cash flow remain above their steady state levels. However, now q decreases below its steady state level. In this phase, the wedge is moving in the opposite direction. This happens because future profits are now smaller, as productivity is going back towards its steady state value. At the same time, the entrepreneurs' net worth has increased thanks to the profits accumulated in the early phase. Therefore, in later periods the financial constraint is less tight and the wedge falls. This accounts for the fall in average q in the later periods.

This dynamic responses illustrate the fact that the movements in the wedge depend on the tension between the desired level of investment (driven by expected productivity) and the availability of funds (driven by past productivity). As the wedge varies over time, the one-to-one relation between q and investment is broken.

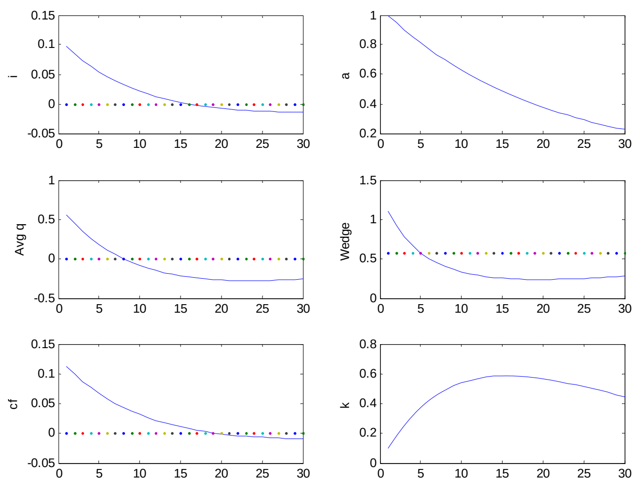

Figure 1. Impulse response functions to a persistent shock.

A temporary productivity shock provides an even starker example of the effects of the time-varying wedge. The impulse-responses for the shock η_t are plotted in Figure 2. In this case investment and cash flow jump temporarily above their steady state levels. However, average q jumps below its steady state level and then gradually adjusts back.

A temporary shock can be viewed as a pure wealth shock to entrepreneurs. This shock has no effect on future profitability, but changes the availability of funds to invest. The direct result of the shock is that entrepreneurs are less financially constrained and increase investment. As the capital stock moves towards its first best level the return on entrepreneurial capital falls, and so does the wedge between average and marginal q. Marginal q is still increasing, due to the increase in investment. However, the net effect is to decrease average q.

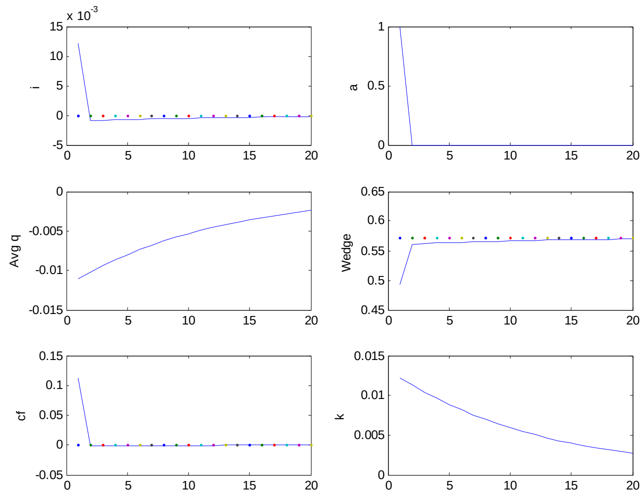

Figure 2. Impulse response functions to a temporary shock.

5.2.1 Raw correlations

Before turning to regressions and to conditional correlations, it is useful to address a basic question. Can our model can replicate the very weak observed correlation between q and investment? That is, can we replicate the simple univariate relation between q and investment?

In Figure 3 we plot the relationships between investment and q, and investment and cash flow, for an economy with only persistent shocks. There is almost a perfect linear relationship between investment and cash flow, and there is a noisy relationship between investment and q. The reason for the noisy relationship between investment and q is the time variation in the wedge described above.

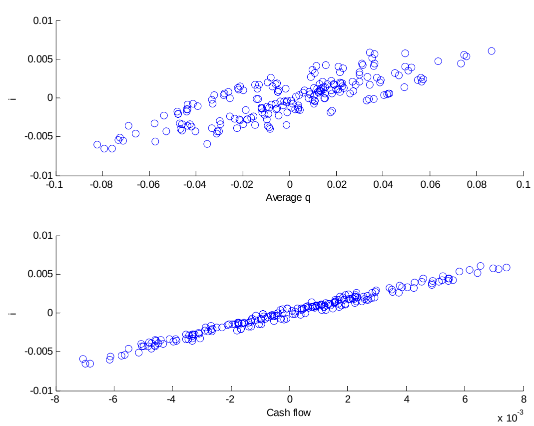

Figure 3.

Simulated values for (i, q) and (i, cf), with only persistent shocks.

However, the correlation between investment and q is still quite strong. If we regress investment on q we obtain a coefficient of 0.07 and an R^2 close to 0.60.

This correlation disappears once we consider the polar case of an economy with only temporary shocks, which is plotted in Figure 4. In this case, average q actually displays a weak negative relationship with investment. At the same time, the relation between investment and cash flow still displays an almost perfect fit.

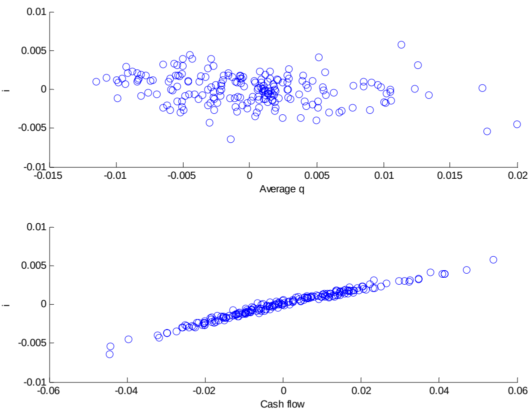

Figure 4.

Simulated values for (i, q) and (i, cf), with only temporary shocks.

Now, if we regress investment on q we obtain a coefficient of -0.07, and, more importantly, an R^2 close to zero.

By experimenting with the case where both temporary and persistent shocks are present, we found out that there is a simple monotone relation between the relative size of temporary shocks and the correlation (and R^2) between investment and q. That is, the model is able to replicate the weak correlation between q and investment observed in the data, if we introduce sufficiently large temporary shocks in the model.

5.2.2 Multivariate regression

Now we turn to standard investment regressions, and we ask whether our model can replicate the coefficients on q and cash flow observed in the data. To do so, we simulate our model and we generate 500 sample series of 200 periods of artificial data. For each simulated series, we run the standard investment regression:

$$i_t = a_0 + a_1 q_t + a_2 cf_t + e_t.$$

Existing empirical estimates of this regression are mostly based on panel data, while our model immediate implications are in terms of time series. However, it is not difficult to extend our model to a version with heterogeneous productivity shocks, and local input and capital markets. If we assume that firms in "sector" j are hit by the productivity shock A_{jt}, and that labor and old capital are immobile across sectors, the panel implications of our model are identical to its time series implications. One can rewrite the model introducing a sector-specific wage rate w_{jt} and a sector-specific price of old capital q^o_{jt}, and the results obtained above carry over at the sector level. An alternative approach would be to introduce heterogeneity *and* decreasing returns at the individual level, instead of the "external" decreasing returns that we have assumed so far. The reason we do not go that way, for the moment, is that other papers (Gomes (2001), Cooper and Ejarque (2001, 2003)) have shown that decreasing returns can weaken the relation between q and investment. Therefore, by keeping our assumption of "external" decreasing return, we study the effect of financial frictions in isolation.

As a reference point, we consider the coefficients obtained in Gilchrist and Himmelberg (1995), which are reported in Table 1.

q (s.e.)	cf (s.e.)
0.033	0.242
(0.016)	(0.038)

Table 1. GH (1995) Empirical multivariate investment regressions.

Multivariate regression results for the simulated model are presented in Table 2. We consider the case where both temporary and persistent shocks are present and we report the results for different values of the ratio $\sigma^2_{pers}/\sigma^2_{tot}$. For a high fraction of persistence shocks (80%) we can approximately match the empirical coefficients of Gilchrist and Himmelberg (1995). Also note that the coefficient on cash flow is at least twice as large as the coefficient on investment, regardless of the mix of shocks.

$\sigma^2_{pers}/\sigma^2_{tot}$	q (s.d.)	cf (s.d.)
1	0.016	0.73
	(.00)	(.01)
0.8	**0.048**	**0.26**
	(.01)	(0.05)
0.6	0.056	0.15
	(.01)	(.01)
0.4	0.057	0.12
	(.01)	(.00)
0.2	0.055	0.12
	(.01)	(.00)
0	0.050	0.12
	(.00)	(.00)

Table 2. Multivariate regression results. Average coefficients and standard deviations.

6 Extensions

6.1 Occasionally binding constraints

So far we have assumed that shocks are small enough that Proposition 4 applies and the financial constraint is always binding. This assumption is a useful simplification for two reasons. First, it reduces the state space for a recursive equilibrium to K_t, while in general the state space is given by both B_t and K_t. Second, the optimal financial contracts is simply given by

$$B_{t+1} = \theta R_{t+1} K_{t+1}$$

while in the general case, the optimal financial contract is described by a function of the type

$$D\left(s_{t+1}, B_t, K_t\right)$$

which can specify different payments for each realization of the exogenous state s_{t+1} next period.

As a first step in the analysis of the general case, we consider here the case of an economy that is hit by a single temporary shock at time τ. After time τ, the productivity level is deterministic and equal to A. If the temporary shock is sufficiently large, the firms enter a path in which the financial constraint is not binding for the first $T - \tau$ periods, and is binding again afterwards. In particular, condition (11) holds as an equality for t, $\tau \leq t \leq T$, and we have

$$\phi_t = (1 - \gamma) \phi_{t+1} + \gamma$$

Condition (10) tells us that in this case investment is determined by the condition

$$q_t = \beta R_{t+1}$$

and the evolution of entrepreneurial wealth is given by

$$N_{t+1} = \frac{1 - \gamma}{\beta} N_t + \gamma w_t L_E$$

Since the rate of return on entrepreneurial wealth is lower than in steady state, entrepreneurial wealth declines over time up to the point where the financial constraint is binding again.

The dynamics of investment in this case are illustrated in Figure 5 for different values of the temporary shock. In the first panel of Figure 5 we report the dynamics of entrepreneurial wealth N, in percentage deviations from the steady state. In the second panel, we report the dynamics of the investment rate. The figure shows that the relation between cash-flow shocks and investment is non-linear and that the propagation depends on the size of the shocks. For small shocks the firm uses all the extra cash flow to invest in physical capital, so the effect at impact (normalized by the size of the shock) is large, but it dies out quickly. For large shocks the firm instead invests only a fraction of the extra cash flow in physical capital, and invests the rest at the risk free rate. The firm effectively is accumulating cash reserves to be used for investment in the following periods. The impact effect is smaller (when normalized by the size of the shock) but the response of investment is more persistent. These dynamics are driven by the interaction of the financial constraint and the adjustment cost. With no financial constraint the temporary shock would have no effect on investment, and with no adjustment cost the temporary shock would only have temporary effects on investment.

The interest in this result lays in the fact that with occasionally binding constraints, we can reduce the effect of cash flow on investment and we can increase the serial correlation of the investment rate after a temporary shock. If one goes back to the simulations reported in Table 2 and compares them with the Gilchrist and Himmelberg (1995) coefficients reported in Table 1 one can see that both effects help match the empirical evidence for low levels of $\sigma^2_{pers}/\sigma^2_{tot}$.

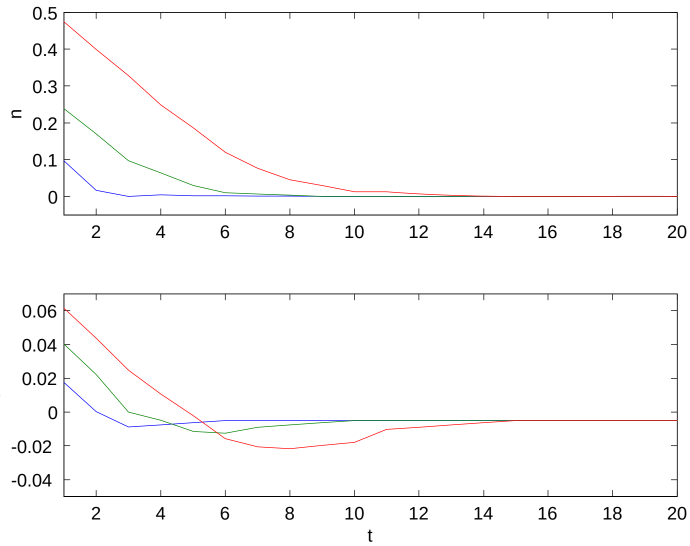

Figure 5. Impulse response functions of entrepreneurs' wealth N (top panel) and investment rate I (bottom panel) to temporary shocks of various sizes.

7 Conclusions

In this paper we have developed a tractable framework for thinking about the effect of financial frictions on asset prices and Tobin's q. The main conclusion is that, in the presence of financial frictions q may reflect some of the future rents that will go to the insider. Since the insider's shadow discount factor is different from the market discount factor, these future rents are "mispriced," and the financial value of the firm appears larger than the value of installed capital. As a consequence Tobin's q is larger than one.

Using a calibrated version of our model, we have explored its quantitative implications for the correlations between investment, q and cash flow measures. The model can replicate the low correlation between q and investment observed both in aggregate and in micro data. Moreover, the model can replicate the coefficients on q and cash flow obtained in panel data regressions.

The model is stylized in many respects. We have decided to stay as close as possible to the original Hayashi (1982) environment, to focus on the "pure" effect of the financial friction. On the other hand, to retain the constant returns to scale features of the Hayashi (1982) model, we have built a model where heterogeneity plays a very limited role. For example, in our model, reallocating funds across entrepreneurs would have no effects on aggregate investment dynamics. It would be interesting to extend the analysis to the case of decreasing returns to scale, where heterogeneity plays a much richer role.

Also, to simplify the analysis, we have considered the case of risk neutral consumers with a constant discount factor. This means that we have ruled out shocks that affect the supply of funds on financial markets. If we interpret our model as a model of a single "sector" (as we did in section 5.2.2), then the assumption of constant interest rates and risk premia may be reasonable. On the other hand, if we interpret the model as an aggregate model, then the interaction between the consumers' stochastic discount factor and q might have interesting consequences for aggregate behavior.

8 Appendix

Proof of Proposition 3

Using the fact that $W(v,b,0,X) = v-b$, and $W(v,b,1,X) = \phi(X)(v-b)$, the first order conditions of problem (P) can be written as:

$$
\begin{aligned}
&1-\lambda+\xi=0\\
&-\beta_E(1-\gamma)H\left(X'|X\right)\phi\left(X'\right)+\lambda\beta(1-\gamma)H\left(X'|X\right)-\mu\left(1,X'\right)=0\\
&-\beta_E\gamma H\left(X'|X\right)+\lambda\beta\gamma H\left(X'|X\right)-\mu\left(0,X'\right)=0\\
&\beta_E\sum_{X'\in-(X)}\left[\left(\gamma+(1-\gamma)\phi\left(X'\right)\right)R\left(X'\right)H\left(X'|X\right)+\mu\left(1,X'\right)\theta R\left(X'\right)+\right.\\
&\qquad\qquad+\mu\left(0,X'\right)\theta R\left(X'\right)-\lambda q^m(X)+\psi=0
\end{aligned}
$$

where λ is the Lagrange multiplier on the resource constraint **??**, $\mu(\chi',X')$ is the Lagrange multipliers on the collateral constraint in state (χ',X'), and ξ and ψ are the Lagrange multiplier on the non-negativity constraint for, respectively, c^E and k' (implicit in problem (P)). Let $\Gamma(X)$ be the set of realizations of X' that have positive probability according to $H(X'|X)$, this set is assumed to be finite, which is consistent with the recursive equilibria studied. The envelope condition implies that

$$\phi(X)=\lambda.$$

Therefore, the conditions $\mu\geq 0$ imply that, if $H(X'|X)>0$, then:

$$
\begin{aligned}
\beta_E\phi\left(X'\right) &\leq \beta\phi(X), &(17)\\
\beta_E &\leq \beta\phi(X). &(18)
\end{aligned}
$$

Substituting the μ and rearranging the last optimality condition we obtain:

$$
\begin{aligned}
&\beta_E\sum_{X'\in-(X)}\left[\left(\gamma+(1-\gamma)\phi\left(X'\right)\right)(1-\theta)R\left(X'\right)\right]H\left(X'|X\right)+\\
&-\lambda\left(q^m(X)-\beta\theta\sum_{X'\in-(X)}R\left(X'\right)H\left(X'|X\right)\right)+v=0
\end{aligned}
$$

which, together with the envelope condition gives:

$$\phi(X)=\frac{\beta_E(1-\theta)\mathbb{E}\left[\left(\gamma+(1-\gamma)\phi(X')\right)R(X')\,|X\right]+\psi}{q^m(X)-\beta\theta\mathbb{E}\left[R(X')\,|X\right]}.$$

Definition of Recursive Competitive Equilibrium

A recursive competitive equilibrium, with linear policies for the entrepreneurs and risk neutral asset pricing, is given by:

(i) a transition probability $H(X'|X)$, where $X=\{K,B,s\}$;
(ii) pricing functions $R(X), q^m(X), w(X)$; and
(iii) policy functions $c^E(v,b,\chi,X), k'(v,b,\chi,X)$, $d(v,b,\chi,X)$ and $b'(\chi',X';v,b,\chi,X)$, that are linear in $v-b$;[11]

[11] The first two arguments of the b' function reflect the state contingent nature of the optimal contract chosen in state (v,b,χ,X).

The restriction to policy functions that are linear in $v-b$ is justified, given Proposition (2).

which satisfy the following conditions:

(a) the policies in (iii) are optimal for problem (P) in section 3.2, given the transition H;

(b) the functions $R(X), q^m(X)$ and $w(X)$ satisfy the following equations (these conditions embed market clearing in the used capital market and in the labor market):

$$\begin{aligned} R(X) &= A(s) F_1(K,1) - G_2\left(k'(V,B,1,X),K\right), \\ q^m(X) &= G_1\left(k'(V,B,1,X),K\right), \\ V &= R(X)K, \\ w(X) &= A(s) F_2(K,1); \end{aligned}$$

(c) the following inequality is satisfied (this condition ensures market clearing in the consumption goods' market, with $c_t > 0$)

$$\begin{aligned} & A(s) F_1(K,1) - G\left(k'(R(X)K,B,1,X),K\right) + \\ - \quad & \gamma c^E(R(X)K,B,0,X) - (1-\gamma) c^E(R(X)K,B,1,X) + \\ - \quad & \gamma d(R(X)K,B,0,X) - (1-\gamma) d(R(X)K,B,0,X) > 0 \end{aligned}$$

(d) the transition for s' is consistent with $\pi(s'|s)$; the transition probabilities for K' and B' are consistent with the following:

$$\begin{aligned} K' &= k'(R(X)K,B,1,X) \text{ with probability 1}, \\ B' &= (1-\gamma) b'\left(1,\left\{K',B',s'\right\};V,B,1,X\right) - \gamma w(X) l_E \text{ with probability } \pi\left(s'|s\right). \end{aligned}$$

Proof of Proposition 4

Part I. Deterministic steady state

Consider the case of a deterministic steady state. Let productivity be constant $A_t = \hat{A}$. In this case, we have $q_t^m = 1$ and $q_t^o = 1-\delta$. The steady state capital stock $\hat{K}$ and gross return $\hat{R}$ can be found as the solution of:

$$\begin{aligned} \left(1-\beta\theta\hat{R}\right)\hat{K} &= (1-\gamma)(1-\theta)\hat{R}\hat{K} + \gamma\hat{w}L_E \\ \hat{R} &= \hat{A}F_K\left(\hat{K},1\right) + 1 - \delta \end{aligned}$$

It is straightforward to show that $\hat{K}$ is an increasing function of θ, that as $\theta \to 0$ also $\hat{K} \to 0$ and that there exists a $\theta^* < 1$ such that $\beta_E\hat{R} = 1$.[12] The marginal utility of entrepreneurial wealth, $\hat{\phi}$, satisfies

$$\frac{\hat{\phi}}{\gamma + (1-\gamma)\hat{\phi}} = \frac{(1-\theta)\beta_E\hat{R}}{1-\theta\beta\hat{R}}.$$

If the following condition is satisfied

$$\beta_E\hat{R} > 1,$$

[12] In the case of a Cobb-Douglas production function the steady state capital stock can be obtained analytically and is equal to:

$$\hat{K} = \left(\frac{\alpha\beta\theta + \alpha(1-\gamma)(1-\theta) + \gamma(1-\alpha)L_E}{1-(\beta\theta+(1-\gamma)(1-\theta))(1-\delta)}\right)^{\frac{1}{1-\alpha}}.$$

then $\phi > 1$ and both (17) and (18) are satisfied. Given the discussion above this condition is satisfied as long as

$$\theta < \theta^{*}. \tag{A1}$$

Part II. Stability

To analyze the stability properties of the steady state we can linearize the transition equation (14) and use the definition of R_t. We get the following second order equation for $k_{t+1} = \ln K_t - \ln \hat{K}$,

$$\alpha_2 k_{t+2} + \alpha_1 k_{t+1} + \alpha_0 k_t = 0$$

where

$$\begin{aligned}
\alpha_2 &= \beta\theta\xi \\
\alpha_1 &= -\left[\xi + 1 - \beta\theta R + \beta\theta\left(\xi + \alpha\left(1-\alpha\right)\hat{K}^{\alpha-1}\right) - \left(1-\gamma\right)\left(1-\theta\right)\xi\right] \\
\alpha_0 &= \left[\xi + \alpha\left(1-\alpha\right)\left(\gamma L_E - \left(1-\gamma\right)\left(1-\theta\right)\right)\hat{K}^{\alpha-1} + \left(R-\xi\right)\left(1-\gamma\right)\left(1-\theta\right)\right]
\end{aligned}$$

Provided that

$$\alpha_1^2 - \alpha_0\alpha_2 > 0 \tag{A2}$$

it is possible to show that the steady state $\hat{K}$ is saddle-path stable. Then with bounded shocks we can construct a stable stochastic steady state $\mathcal{K}$. One can then establish the continuity of the function ϕ with respect to the parameters $A\left(s\right)$ and show that ϕ_t is bounded in $[\underline{\phi}, \overline{\phi}]$. Finally, it is possible to find a small enough value of $\overline{A} - \underline{A}$ such that the bounds for ϕ_t satisfy

$$\underline{\phi}\beta > \beta_E\overline{\phi}.$$

This guarantees that the financial constraint is always binding.

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