Divisible Money with Partially Directed Search

Dror Goldberg

Department of Economics

Texas A&M University

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Abstract

Monetary search models are difficult to analyze unless the distribution of money holdings is made degenerate. Popular techniques include using an infinitely large household (Shi 1997) and adding a centralized market with quasi-linear utility (Lagos and Wright 2005). Wallace (2002) suggests as an alternative to have two-member households who can somehow direct their search, thus creating a degenerate distribution in a different way. This idea is modelled here for the first time by modifying the partially directed search model of Goldberg (forthcoming). The Friedman rule is optimal, but the costs of deviating from it are different from the above mentioned models.

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1 Introduction

In recent years attempts have been made to make monetary search models more suitable for policy analysis and for an investigation of broad issues in macroeconomics. Although these models provide microfoundations for the use of money, they naturally imply a non-degenerate distribution of money holdings across agents. This makes them difficult to analyze. The challenge is to transform the disorder created by random matching into order, into a representative agent framework. Several ways to solve this problem have been suggested. Shi (1997) uses an infinitely large household (and the law of large numbers) to create a degenerate distribution. Lagos and Wright (2005, henceforth LW) add a centralized market at the end of every period, in which a quasi-linear utility function realigns the distribution of money among agents when they exit that market. Laing, Li, and Wang (2003) assume that a buyer meets a continuum of sellers every period, with the law of large numbers again creating a degenerate distribution.

While these models have already been applied to address many issues in monetary economics, Wallace (2002) offers a dissenting view:

> The goal in these endeavors seems to be to make degenerate distributions consistent with trade in pairs and the essentiality of money. If that is the goal, then the following is a simpler and less contrived scheme that achieves the goal. Consider [a random matching model] but with each agent being a single shopper-producer pair. At each date, let each type s shopper meet a randomly drawn type $s-1$ producer. Then all meetings are single-coincidence meetings and there are symmetric outcomes in which all households hold the same amount of money. Another version, of course, would have all the type s shoppers meet together, in a market, with all the type $s-1$ producers. Call these models *directed-search* versions (pp. 5-6, original italics).

To the best of my knowledge, neither Wallace nor anyone else has yet turned this idea into a formal model. Three reasons come to mind. First, Wallace's main point in that unpublished paper is actually that allowing a *non-degenerate* distribution is better for analyzing some important issues in monetary economics. Second, the nature of the directed search suggested by Wallace may seem too organized, too close to trading post models. Finally, it may not seem worthwhile to construct a model simply for "esthetical" reasons.

Kocherlakota (2005), however, suggests that the assumptions that create degenerate distributions in the models mentioned above may generate spurious results. Such concerns regarding these increasingly popular models suggest that it may be useful to explore the alternative suggested by Wallace. This is the goal of the current paper.

I use the off the shelf, partially directed search model of Goldberg (forthcoming). That model modifies Kiyotaki and Wright (1989) in a fairly minimal and realistic way. Agents live and produce in fixed structures, with all production structures of a particular good concentrated in the same geographic location. This can be thought of literally, as in the case of a fishermen's village, or abstractly, as in the case of producers listed together in the Yellow Pages. Either way, these locations are common knowledge and accessible to all other agents who can choose to go to any particular location. It is called partially directed search because a shopper's choice of a particular producer is random. The model remains as close as possible to standard bilateral matching by assuming that within each location the short side of the market is fully matched. As opposed to trading post models any trade is allowed at any location. In this, not too-organized economy, indeed many equilibria are possible, including autarky, commodity money equilibria, Walrasian equilibria, and a pure fiat money equilibrium.

Modifying that model with two-member households, as suggested by Wallace (also see Lucas 1980), is the only thing needed to create a degenerate distribution. Every period, all type s shoppers (who consume only good s) visit location $s-1$, where all type $s-1$ producers wait for them, ready to produce good s. The matching rule indeed guarantees that everyone gets matched and trades. In a symmetric equilibrium, the prices and quantities in the two transactions performed by each household are identical, and so money holdings remain fixed for each household. If, in addition, the initial distribution of money is degenerate across households, it will remain so in the following periods.

The model is applied to optimal, deterministic long-run monetary policy. Berentsen and Rocheteau (2003) show that in regard to this question the Lagos-Wright model reaches the same results as Shi's model, if one only makes the same assumptions about bargaining: Can a producer observe the shopper's money holdings (thus giving rise to a holdup problem)? And what is the division of bargaining power? (See also Rauch 2000). I therefore refer to these basic models as SLW when comparing their results to those of the current model. As in SLW the Friedman rule is optimal; with the holdup problem it yields the first-best allocation only if shoppers have all the bargaining power. The current model differs from SLW in three aspects. First, it can be solved just as easily even without assuming exogenous breakdowns. Second, the welfare costs of deviating from the Friedman rule are different because producers know for sure that their households will spend all the money next period. Third, with the holdup problem, these welfare costs are different also because the value function is generally non-linear, whereas in the other models it is linear (LW) or seems so to any atomistic agent (Shi). The two latter differences have opposite effects and may even cancel out.

The paper is organized as follows. Section 2 describes the model. Section 3 defines the equilibrium and solves the model without the holdup problem. The holdup problem is discussed in Section 4. Section 5 concludes.

2 The Model

2.1 Standard Features

This is a discrete time, infinite horizon economy. As in Trejos and Wright (1995) and Shi (1995), there are $s = 1, ..., S$ types of divisible goods. There is a continuum of infinitely lived households with unit mass. There are equal proportions of households $s = 1, ..., S$. A type s household derives utility $u(q)$

from consuming q units of good s and disutility $c(q)$ from producing q units of good $s+1$ (mod S). As usual, $u'(q) > 0$ and $c'(q) > 0$ for all $q > 0$; $u'(0) > c'(0) = 0$; $u''(q) \leq 0$, $c''(q) \geq 0$, with at least one strict inequality; $u(0) = c(0) = 0$; and there exists a finite q^* such that $u'(q^*) = c'(q^*)$. The discount factor is $\beta \in (0,1)$. All goods perish at the end of their production period. Each household is endowed with m divisible units of intrinsically useless money (henceforth, \$). A household that enters a period with $\$m$ has an expected lifetime utility $v(m)$. Variables and functions of other households are denoted with capital letters. A government periodically changes the money supply with a constant lump-sum transfer τ. Agents are anonymous so credit is impossible.

2.2 The Geographical Environment

As in Goldberg (forthcoming), good s can be produced only in a specific location, called City $s-1$, and this fact is common knowledge. In this city there is a continuum of non-tradable houses, in which all type $s-1$ households live and produce good s. Each household consists of two agents – a seller and a buyer, who share the above utility and disutility[1]. The seller always stays home. At the beginning of a period, each buyer chooses which city to visit. There are no travel costs. In each city, visiting buyers are randomly matched with sellers: No buyer is matched with more than one seller, and no seller is matched with more than one buyer of a given type. The short side of the market is fully matched[2]. Matched agents bargain over terms of trade. At the end of the period, all buyers go back home, regardless of whether they traded or not[3].

[1] Wallace's (2002) terminology is changed here to "seller" and "buyer" to facilitate comparison of the notation to other models, as explained below.

[2] This constraint does not bind in equilibrium as the measures of sellers and buyers are the same in every city.

[3] As for similar models, Lucas' (1980) story is only a justification of the cash in advance constraint. The model itself is completely Walrasian, with no issues of matching and bargaining. Later models based on it drop the two-member household from the description of the environment without affecting the equations. Trejos and Wright (1995) actually do place sellers

2.3 Bargaining

The use of a household, rather than a single agent, suggests staying as close as possible to the exceptionally clear exposition of Berentsen and Rocheteau (2003, henceforth BR). They nest Shi (1997) and Rauch (2000) and derive the results in a way that is directly comparable to LW. They show that all these models actually achieve the same results regarding the Friedman rule if they make the same bargaining assumptions. As in BR, the formulation of the bargaining problem here is flexible enough to nest the holdup problem as an optional feature. Unlike BR, it is also flexible enough to nest the case of no exogenous breakdowns.

Bargaining is conducted with alternating offers. A buyer proposes to buy a quantity q^b for a payment x^b to a seller whose value function is $V(M)$. The benefit to the seller from accepting the offer is the discounted value function with the new money holdings $M + \tau + x^b - P^b$, minus the threat point T^s. P^b is the amount of money spent that period by the seller's spouse in another city, and is taken as given. The seller has a reservation value $R^s(M, y)$ which could depend on its household's money holdings[4] and the amount of money brought by the buyer (y). An optimal offer makes the seller indifferent between accepting and rejecting:

$$-c(q^b) + \beta[V(M + \tau + x^b - P^b) - T^s] = R^s(M, y). \tag{1}$$

As in Trejos and Wright (1995), if the bargaining process has exogenous breakdowns then the threat point is $T^s = V(M + \tau - P^b)$; otherwise, $T^s = 0$ (also see Shi 1995 for the latter case).

Similarly, a seller optimally proposes to produce a quantity q^s for a payment x^s such that the buyer

in fixed locations throughout most of their paper, but this is done only to prevent direct barter between sellers. Their buyers do not know which good is produced in any location, and so randomness is still significant.

[4] In BR the reservation value depends on the seller's money holdings, because with random matching any money-holder agent can be either a buyer or a seller. Here it is known before matches whether any agent will be a buyer or a seller. Thus, the seller's money holdings is irrelevant (and equal to zero under most policies, as shown below).

is indifferent between accepting and rejecting:

$$u(q^s) + \beta[V(M + \tau - x^s + P^s) - T^b] = R^b(M), \tag{2}$$

where P^s is the amount of money earned by the buyer's spouse that period (and taken as given), and $R^b(M)$ is the buyer's reservation value. With breakdowns $T^b = V(M + \tau + P^s)$ and otherwise $T^b = 0$. A household's next period's money holdings is denoted m_{+1}. The discounted marginal value of money is $\omega \equiv \beta v'(m_{+1})$.

Note that with breakdowns, (1) has the term $\beta[V(M + \tau + x^b - P^b) - V(M + \tau - P^b)]$. In SLW it is linear in the proposed payment and equal to $x^b\omega$. In Shi (1997) it happens because the buyers and sellers are atomistic. The contribution of each sale to the household's money holdings is negiligible, and so is any agent's share in any increase in the household's welfare. The change in the value function is therefore the payment multiplied by the derivative of the value function at the equilibrium's money holdings, or $x^b\omega$ (see derivation in BR, p. 4, n. 7). In LW it happens because agents who exit this meeting continue to the night market with its linear value function. The vertical distance between any two points on that value fucntion is simply the horizontal distance multiplied by the fixed slope, or $x^b\omega$.

Here, in contrast, there are only one buyer and one seller in each household. The contribution of each one of them to the household's money holdings is anything but negligible, and so is their share in the household's increase in welfare. Also, there is no night market here with a linear value function. In general, the value fucntion of the current model is not linear in money holdings. Moreover, the current model does not even have to assume exogenous breakdowns.

Breakdowns happen after a rejection of a buyer's offer with probability $\theta\Delta$ and after a rejection of a seller's offer with probability $(1 - \theta)\Delta$, where $\theta \in (0, 1]$ can be thought of as the buyer's bargaining power, and Δ is the length of subperiods during the bargaining. If a seller rejects an offer it offers to produce Q^s for a payment X^s with probability $1-\theta\Delta$. Therefore the reservation value satisfies

$$R^s = (1-\theta\Delta)\{-c(Q^s) + \beta[V(M+\tau+X^s-P^b) - T^s]\}. \tag{3}$$

Similarly, the buyer may be able to offer to pay $\$X^b$ for Q^b goods, so

$$R^b = [1-(1-\theta)\Delta]\{u(Q^b) + \beta[V(M+\tau-X^b+P^s) - T^b]\}. \tag{4}$$

There are some feasibility restrictions. The buyer cannot offer to pay more than the $\$y$ it brought to the match ($y \geq x^b$), the seller cannot ask for more than the $\$Y$ the buyer brought ($Y \geq x^s$), and the buyer cannot take for shopping more money than the household has ($m \geq y$).

2.4 Dynamic Programming Problem

The main difference between this problem and BR's problem is that here every meeting is a single-coincidence meeting, because search is directed and there are no matching frictions within a city. In BR's notation, the probability of trade in a given meeting is $z = 1$. A household chooses offers of quantities and prices, how much money to take when going shopping, and how much money to hold next period:[5]

$$\begin{aligned} v(m) \quad = \quad & \max_{q^b,q^s,x^b,x^s,y,m_{+1}} u(q^b) - c(q^s) + \lambda(y-x^b) + \pi(Y-x^s) + \phi(m-y) \\ & +\alpha\{-c(q^b) + \beta[V(M+\tau+x^b-P^b) - T^s] - R^s(M,y)]\} \\ & +\delta\{u(q^s) + \beta[V(M+\tau-x^s+P^s) - T^b] - R^b(M)\} \\ & +\beta v(m_{+1}), \end{aligned} \tag{5}$$

subject to the law of motion for the household's money holdings:

$$m_{+1} = m + \tau + x^s - x^b. \tag{6}$$

[5]As in BR, (5) assumes that the particular household analyzed gets to make offers both as a buyer and a seller. Since the time between offers $\Delta \to 0$, the offers will be equal in equilibrium and it does not matter who makes the first offer.

3 Equilibrium

Definition 1: A symmetric stationary monetary equilibrium is the value function v and the match-specific terms of trade q^b, q^s, x^b, x^s such that:

1. Given quantities, prices and the distribution of money, $v = V$ solves the dynamic programming problem.

2. Quantities of goods produced and payments are positive and equal in all matches: $q = q^b = q^s = Q^b = Q^s > 0$ and $x = x^b = x^s = X^b = X^s = P^b = P^s > 0$.

3. The distribution of money is consistent with initial endowments and the evolution of money holdings implied by trades.

4. The real value of money ωm is constant over time.

As explained in the Introduction, if all buyers visit the appropriate sellers then there is a degenerate distribution of money, and thus the model can be solved as a representative household model[6].

The first order conditions with respect to q^b, q^s, x^b, x^s, and y are:

$$\alpha = \frac{u'(q^b)}{c'(q^b)} \tag{7}$$

$$\delta = \frac{c'(q^s)}{u'(q^s)} \tag{8}$$

$$\lambda + \omega = \alpha\beta \frac{\partial[V(M + \tau + x^b - P^b) - T^s]}{\partial x^b} \tag{9}$$

[6] In principle, a model of this type might also have an equilibrium in which sellers travel and visit buyers at their homes. In Goldberg (2006) it is defined as a door-to-door equilibrium and it does not exist. Here, the necessity of production at home, and the determination of quantity after bargaining, make this equilibrium even less likely.

$$\pi - \omega = \delta\beta \frac{\partial[V(M + \tau - x^s + P^s) - T^b]}{\partial x^s} \tag{10}$$

$$\phi = \lambda - \alpha R^s_y(M, y), \tag{11}$$

where $R^s_y(M, y) \equiv \frac{\partial R^s}{\partial y}$.

The envelope condition is

$$\frac{\omega_{-1}}{\beta} = \phi + \omega. \tag{12}$$

Because $T^s = V(M + \tau - P^b)$ with breakdowns and 0 otherwise, $\frac{\partial T^s}{\partial x^b} = 0$ either way. Then (9) can be rewritten as $\lambda + \omega = \alpha\beta V'(M + \tau + x^b - P^b)$. In equilibrium the right hand side equals $\alpha\omega$, so

$$\omega = \alpha\omega - \lambda. \tag{13}$$

Plug this and (11) in (12) to get $\frac{\omega_{-1}}{\beta} = -\alpha R^s_y(M, y) + \alpha\omega$. Plug (7) here to get

$$\frac{\omega_{-1}}{\beta} = \frac{u'(q)}{c'(q)} \left[\omega - R^s_y(M, y)\right]. \tag{14}$$

Multiply both sides by $\frac{\beta}{\omega\gamma}$, where $\gamma \equiv \frac{m}{m_{-1}}$ is the money growth rate. Then

$$\frac{(m\omega)_{-1}}{m\omega} = \frac{\beta}{\gamma} \frac{u'(q)}{c'(q)} \left[1 - \frac{R^s_y(M, y)}{\omega}\right] \tag{15}$$

In a stationary equilibrium the left hand side equals 1, so (15) can be rewritten as

$$\frac{\gamma}{\beta} = \frac{u'(q)}{c'(q)} \left[1 - \frac{R^s_y(M, y)}{\omega}\right] \tag{16}$$

Equation (16) is identical to BR's equation (19) when the probability of a single coincidence of wants in that model is $z = 1$. This probability is 1 in the current model because search is partially directed. I return to the implications of that below.

As a first new result, note that this solution holds regardless of whether or not there are exogenous breakdowns. SLW do not show solutions for that case.

The first-best allocation is q^* which satisfies $u'(q^*) = c'(q^*)$. As seen in (16), the Friedman rule, $\gamma = \beta$, yields that if and only if the holdup term $R^s_y(M, y) = 0$. The following results follow BR once again.

Proposition 1: a. If there is no holdup problem, for every $\gamma > \beta$ there is a unique monetary equilibrium and $q \to q^*$ as $\gamma \to \beta$; **b.** in such an equilibrium the buyer brings all its household's money to the match and spends it all; **c.** there is no equilibrium iff $\gamma < \beta$; **d.** if $\gamma = \beta$, the buyer may leave money at home and not spend all it takes.

All proofs are in the Appendix.

Without the holdup problem, BR's equation (19) can be rewritten as

$$\frac{\gamma - \beta}{z\beta} = \frac{u'(q)}{c'(q)} - 1, \tag{17}$$

where $z \in (0, 1/3]$ measures the probability of a single-coincidence meeting. Note that z is irrelevant iff the government follows the Friedman rule. Deviations from that rule make z significant in determining the welfare cost of such deviations. The smaller is z, the smaller is q: Sellers have a smaller probability of spending in the near future the money they can earn today, so they are discouraged from producing today. The current model forces $z = 1$ for analytical tractablity. A rudimentary simulation of (17) for different values of z shows the significance of this parameter's value. Figures 1 and 2 show the welfare *per match* (i.e., $u(q) - c(q)$) for different values of γ and z.[7] The line $z = 1$ refers to the value forced by the current model and by Walrasian models (and also one of the values used by LW); $z = 1/3$ is from Kiyotaki and Wright (1989) and is also BR's upper bound; $z = .066$ is LW's lowest estimate. Future

[7] The preferences are $u(q) = q^{2/3}/(2/3)$ and $c(q) = q$, following LW's calibration. The results are essentially the same for the algebraically simplest functional forms, $u(q) = 2q$ and $c(q) = q^2$. The discount factor is monthly, $\beta = .9966$, following LW's shortest length of a period. Welfare under the Friedman rule is normalized to 1 in the figures.

work should examine empirically what is the probability that a consumer finds his or her preferred good in the first attempt and how much time it takes.

4 The Holdup Problem

LW emphasize how the holdup problem affects the welfare costs of inflation in order to illustrate why it is important to analyze monetary policy with models that have solid microfoundations. Walrasian models cannot generate results like this because the problem obviously arises only in bargaining. Rauch (2000) achieves a similar result. Here the holdup problem also serves to highlight the role of SLW's technical assumptions in generating their substantive results. For comparison with SLW's results, I assume exogenous breakdowns from now on, and continue with BR's solution method.

Analysis of the holdup term requires finding the derivative of the seller's reservation value with respect to the buyer's money holdings. From (3) the derivative is

$$R_y^s = (1-\theta\Delta)\left[\begin{array}{c} -c'(Q^s)\frac{\partial Q^s}{\partial y} + \beta\frac{\partial V(M+\tau+X^s-P^b)}{\partial(M+\tau+X^s-P^b)}\frac{\partial(M+\tau+X^s-P^b)}{\partial y} \\ -\beta\frac{\partial V(M+\tau-P^b)}{\partial(M+\tau-P^b)}\frac{\partial(M+\tau-P^b)}{\partial y} \end{array}\right]. \tag{18}$$

In the bargaining M, τ and P^b are taken as given. Only X^s depends on y so (18) becomes $R_y^s = (1-\theta\Delta)\left[-c'(Q^s)\frac{\partial Q^s}{\partial y} + \beta V'(M+\tau+X^s-P^b)\frac{\partial X^s}{\partial y}\right]$. Conjecture that as in Proposition 1, in all equilibria buyers spend all the money they bring (i.e., $X^s = y$). Only then $Q^s = q(y)$. With $\Delta \to 0$,

$$R_y^s = -c'(q)q'(y) + \omega. \tag{19}$$

Equation (19) is identical to BR's (23). Divide by ω to get

$$\frac{c'(q)q'(y)}{\omega} = 1 - \frac{R_y^s}{\omega}. \tag{20}$$

Plug (20) in (16) to get

$$\frac{\gamma}{\beta} = \frac{u'(q)q'(y)}{\omega}. \tag{21}$$

To find $q'(y)$, start by equating (1) to (3) and (2) to (4):

$$\begin{aligned}&-c(q^b) + \beta[v(m+\tau+x^b-P^b) - v(m+\tau-P^b)] = \\ &(1-\theta\Delta)\{-c(q^s) + \beta[v(m+\tau+x^s-P^b) - v(m+\tau-P^b)]\}\end{aligned} \tag{22}$$

$$\begin{aligned}&u(q^s) + \beta[v(m+\tau-x^s+P^s) - v(m+\tau+P^s)] \\ &= [1-(1-\theta)\Delta]\{u(q^b) + \beta[v(m+\tau-x^b+P^s) - v(m+\tau+P^s)]\}\end{aligned} \tag{23}$$

If $\Delta \to 0$, $x^b \to x^s$ and $q^b \to q^s$ solve both equations. With $x^b \to x^s$, rewrite them as

$$c(q^s) - c(q^b) = -\theta\Delta\{-c(q^s) + \beta[v(m+\tau+x^s-P^b) - v(m+\tau-P^b)]\} \tag{24}$$

$$u(q^s) - u(q^b) = -(1-\theta)\Delta\{u(q^b) + \beta[v(m+\tau-x^b+P^s) - v(m+\tau+P^s)]\}. \tag{25}$$

Divide to get

$$\frac{c(q^s) - c(q^b)}{u(q^s) - u(q^b)} = \frac{\theta\{-c(q^s) + \beta[v(m+\tau+x^s-P^b) - v(m+\tau-P^b)]\}}{(1-\theta)\{u(q^b) + \beta[v(m+\tau-x^b+P^s) - v(m+\tau+P^s)]\}}. \tag{26}$$

In the limit both q^b and q^s converge to q, so

$$\frac{c'(q)}{u'(q)} = \frac{\theta\{-c(q) + \beta[v(m+\tau+x^s-P^b) - v(m+\tau-P^b)]\}}{(1-\theta)\{u(q) + \beta[v(m+\tau-x^b+P^s) - v(m+\tau+P^s)]\}}. \tag{27}$$

In equilibrium $x^b = x^s = y$:

$$\frac{c'(q)}{u'(q)} = \frac{\theta\{-c(q) + \beta[v(m+\tau+y-P^b) - v(m+\tau-P^b)]\}}{(1-\theta)\{u(q) + \beta[v(m+\tau-y+P^s) - v(m+\tau+P^s)]\}}. \tag{28}$$

In LW the discounted increase in the value function for a seller, which is similar to $\beta[v(m+\tau+y-P^b) - v(m+\tau-P^b)]$, is the constant slope of the value function times the payment, or $\beta v'(m_{+1})y = \omega y$. In Shi (1997) it is the derivative of the value function at the equilibrium's money holdings times the payment, which is again ωy. Here, in contrast, the value function is not necessarily linear and every agent is a significant member of its household. The discounted increase in the value function for a seller is then $\omega y + s$, with the term $s \geq 0$ capturing the fact that the value function is weakly concave (see Figure 3). Similarly, the discounted decrease for a buyer is $\beta[v(m+\tau+P^s) - v(m+\tau-y+P^s)] = \omega y - b$, where $b \geq 0$. Therefore,

$$\frac{c'(q)}{u'(q)} = \frac{\theta[-c(q) + \omega y + s]}{(1-\theta)[u(q) - (\omega y - b)]}. \tag{29}$$

This can be rearranged as

$$\omega y = \frac{(1-\theta)c'(q)[u(q) + b] + \theta u'(q)[c(q) - s]}{\theta u'(q) + (1-\theta)c'(q)}. \tag{30}$$

Following BR's notation, let $g(q;\theta) \equiv \frac{(1-\theta)c'(q)u(q)+\theta u'(q)c(q)}{\theta u'(q)+(1-\theta)c'(q)}$. Then (30) becomes

$$\omega y = g(q;\theta) + D(q;\theta), \tag{31}$$

where $D(q;\theta) \equiv \frac{b(1-\theta)c'(q)-s\theta u'(q)}{\theta u'(q)+(1-\theta)c'(q)}$.

Totally differentiating (31) yields

$$\frac{q'(y)}{\omega} = \frac{1}{g'(q;\theta) + D'(q;\theta)}. \tag{32}$$

Plug (32) in (21) to get

$$\frac{\gamma}{\beta} = \frac{u'(q)}{g'(q;\theta) + D'(q;\theta)}. \tag{33}$$

Note that $D'(q;\theta) = \frac{\theta(1-\theta)(b+s)[c''(q)u'(q)-c'(q)u''(q)]}{[\theta u'(q)+(1-\theta)c'(q)]^2} \geq 0$. This is the difference between this model and SLW.

Proposition 2: The current model replicates the SLW results iff at least one of the following holds: the value function is linear; the buyers have all the bargaining power. Then for all $\gamma > \beta$ there is a unique equilibrium and $\lim_{\gamma \to \beta} q = q^*$.

A linear value function means $s = b = 0$. Given the standard assumptions made above regarding the properties of the utility and cost functions, there is no reason to expect a linear value function.

The techincal assumptions used by SLW to create a degenerate distribution are equivalent to assuming complete linearity. I assume from now on that the value function is strictly concave ($b > 0$, $s > 0$).

As for the case where the holdup problem bites, there is only one difference from BR's Proposition 1, due to $D'(q;\theta)$. Let $\Phi(q;\theta) \equiv \frac{u'(q)}{g'(q;\theta)+D'(q;\theta)}$ and let $\bar{q}$ solve $\Phi(\bar{q};\theta) = 1$.

Proposition 3: If $\theta < 1$ and $\Phi(0;\theta) = \infty$, then: **a.** for every $\gamma > \beta$ there is a monetary equilibrium with $q \to \bar{q} < q^*$ as $\gamma \to \beta$; **b.** in such an equilibrium the buyer brings all its money to the match and spends it all; **c.** there is no equilibrium iff $\gamma < \beta$; **d.** if $\gamma = \beta$, the buyer may leave money at home, but spends all it takes.

Proposition 3 also verifies the conjecture used to derive (19). An important qualitative result is the same as in SLW.

Theorem 1: The Friedman rule is optimal.

There is a quantitative difference from SLW.

Proposition 4: If $D'(q;\theta) \neq 0$ the money growth rate required for the first-best allocation is lower than in SLW.

This tends to increase the welfare costs of inflation compared to those estimated by LW (see their Figure 1). The other difference from SLW (discussed in Section 3), which tends to make the costs in the current model lower than in SLW, still exists here. It is not obvious which effect dominates.

Note that the difference term $D'(q;\theta)$ increases in both s and b, which reflect the value function's concavity. Both represent gains from trade that are absent in SLW. For a seller, the gain from trade is not just ωy, as it is in SLW; there is also the gain s. The marginal value of money is higher for a seller than at the equilibrium point because it knows how critical its sale is for the household's welfare. For a buyer, the loss from trade is not ωy, as it is in SLW. The loss is lower than that by the magnitude b. The marginal value of money is lower for a buyer than at the equilibrium point because it knows that its spouse is earning money at the same time back home. Another way to demonstrate it is to insert $g'(q;\theta)$

and $D'(q;\theta)$ explicitly in (33), set $u'(q) = c'(q)$, and invert the equation. The result is

$$\frac{\beta}{\gamma} = 1 + \frac{(1-\theta)\theta[c''(q) - u''(q)][u(q) + b - c(q) + s]}{u'(q)^2}. \tag{34}$$

The large term on the right hand side, which prevents the Friedman rule from achieving the first-best allocation, includes the net gains from trade. In SLW these gains are $u(q) - c(q)$, while here they are $u(q) + b - c(q) + s$. Either one of these neglected gains reduces the money growth rate required for achieving the first-best allocation.

5 Conclusion

It was often argued that early monetary search models were not suitable for policy analysis due to the restrictions on asset holdings. These restrictions keep the distributions of assets analytically tractable. Shi (1997) pioneeres the idea of adding another ingredient to the random matching model so that the decision-making unit will be able to both buy and sell every period. Such an ability can create a degenerate distribution even without restrictions on asset holdings, thus making the model as solvable as any representative agent model. Shi's extra ingredient is the law of large numbers. It imposes discipline on the chaotic results caused by random matching. Lagos and Wright (2005) use a special Walrasian night market to impose discipline on the distribution that is unavoidably distorted in the random matching day market. Wallace (2002), although far from endorsing the goal of degenerate distributions, fixes the "mess" caused by random matching by going to the root of the problem, namely the randomness itself, instead of allowing it and fixing its consequences later. The directedness of search allows a substitution of Shi's law of large numbers with a two-member household.

The current paper formalizes Wallace's two-member household suggestion in a model that already has partially directed search. Its essential assumption, that buyers know how to access non-specific sellers of the goods they want, is not only more realistic than random matching but also produces results that are more consistent with monetary history (see Goldberg forthcoming).

Esthetics aside, there are differences in the welfare costs of inflation between the current model and the models of Shi and Lagos and Wright. First, since a seller knows for sure that its household will succeed tomorrow in spending all the money it earns today, the cost of deviating from the Friedman rule is lower than in random matching models. Second, when the holdup problem exists the money growth rate needed for the first-best allocation is even lower and further away from the Friedman rule than the one found in random matching models. This probably implies higher welfare costs of inflation and has to do with the non-linearity of the value function. The current model also provides a solution without exogenous breakdowns in bargaining.

These three differences require further investigation of the three issues involved. What is the probability of a single-coincidence meeting? How large is the quantitative difference attributed to linearity? Is it always reasonable to assume exogenous breakdowns in bargaining?

It remains to be seen if the current model differs from the others in other applications. Clearly, each model has its advantages. Shi's model is flexible, e.g., in letting the household choose how to split the money among its potential buyers. The Lagos-Wright model has the great advantage of having centralized markets inherent in it. For those not pleased with any household construct, a variant of the current model has roommates instead of a household. Each period they must alternate in who produces and who goes shopping and they do not share the same value function. That variant is just as tractable as the one above (details available by request). The current model is useful even when one analyzes *changes* in the distribution. By creating a degenerate distribution in steady state, the propagation of shocks is then much easier to analyze. See Goldberg (2005) for an application in the context of real sectoral shocks.

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Appendix

Proof of Proposition 1: a. With $R^s_y(M,y) = 0$, the right hand side of (16) becomes $\Pi(q) \equiv \frac{u'(q)}{c'(q)}$. Because $c'(0) = 0$, $\Pi(0) = \infty$. By definition, $\Pi(q^*) = 1$. Together with continuity of $\Pi(q)$, this establishes existence of an equilibrium for every $\gamma > \beta$, with $q < q^*$. It is easy to show that $\Pi(q)$ is strictly decreasing in q.. This establishes uniqueness. Since $\Pi(q)$ is strictly increasing in γ, this also means that q is strictly decreasing in γ. In the limit, as $\gamma \to \beta$, $q \to q^*$. **b-d.** Using (7) and (13), (16) can be rewritten as $\frac{\gamma}{\beta} = \frac{\lambda}{\omega} + 1$. Then $sign(\gamma - \beta) = sign(\lambda)$, and (11) implies $\lambda = \phi$. The results follow from λ and ϕ being multipliers. QED

Proof of Proposition 2: $(1-\theta)(b+s) = 0$ means $D'(q;\theta) = 0$, and then (33) is identical to BR's equation (30) (for $z = 1$). Then the first part of BR's Proposition 1 applies. The proof is similar to that of my Proposition 1 above and Proposition 3 below, and is therefore omitted. QED

Proof of Proposition 3: While $g'(0;\theta) = 0$, it is not obvious that $D'(0;\theta) = 0$ (it is with $c(q) = q^a$, with $a > 1$). So it needs to be assumed that $\Phi(0;\theta) = \infty$. **a.** With $\Phi(0;\theta) = \infty$, $\Phi(\bar{q};\theta) = 1$, and the continuity of $\Phi(q;\theta)$, there is an equilibrium for every $\gamma > \beta$, with $q \to \bar{q}$ as $\gamma \to \beta$.. It is easy to show that $\Phi(q^*;\theta) < 1$. Therefore, $\bar{q} < q^*$. **b-d.** Using (13) and (19), (11) can be rewritten as $\phi = \alpha c'(q)q'(y) - \omega$. Using (7) and (32), this becomes $\phi = \omega\left[\Phi(q;\theta) - 1\right]$. From (33) this means $\phi = \omega\left[\frac{\gamma}{\beta} - 1\right]$, so $sign(\phi) = sign(\gamma - \beta)$. The latter must be positive in equilibrium since ϕ is a multiplier. Using (7), (13) becomes $\lambda = \omega\left[\frac{u'(q)}{c'(q)} - 1\right]$. Then $sign(\lambda) = sign[u'(q) - c'(q)]$. Part a implies that $u'(q) > c'(q)$ if $\gamma = \beta$. QED

Proof of Proposition 4: With $D'(q;\theta) > 0$ the denominator of (33)'s right hand side is larger than in SLW. This means that γ/β is even smaller than in SLW. QED

Figure 1: Welfare Costs of Low Inflation

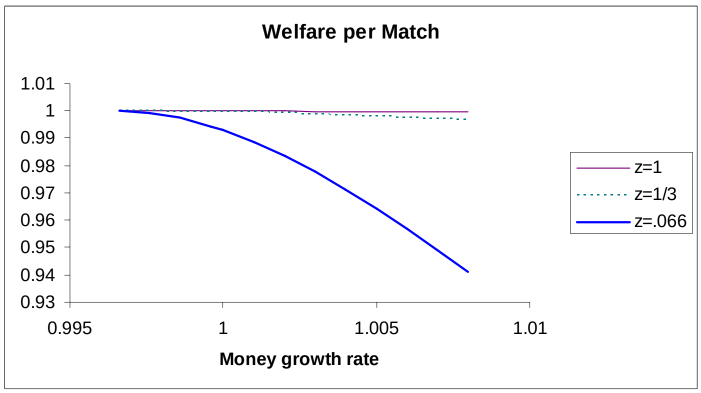

Figure 2: Welfare Costs of High Inflation

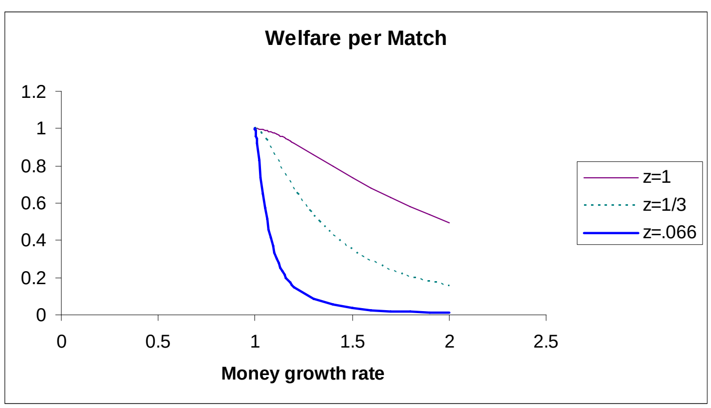

Figure 3: The Value Function