# Fiscal Policy and Macroeconomic Uncertainty in Emerging Markets: The Tale of the Tormented Insurer

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**Abstract**

Governments in emerging markets often behave like a "tormented insurer", trying to use non-state-contingent debt instruments to avoid sharp adjustments in payments to private agents despite large fluctuations in public revenues. In the data, their ability to sustain debt is inversely related to the variability of their revenues, and their primary balances and current expenditures follow a cyclical pattern that contrasts sharply with the countercyclical behavior observed in industrial countries. This paper proposes a dynamic, stochastic equilibrium model of a small open economy with incomplete markets that can rationalize this behavior. In the model, a fiscal authority that chooses optimal expenditure and debt plans given stochastic revenues interacts with private agents that also make optimal consumption and asset accumulation plans. The competitive equilibrium of this economy is solved numerically as a Markov perfect equilibrium using parameter values calibrated to Mexican data. If perfect domestic risk pooling were possible, the ratio of public-to-private expenditures would be constant. With incomplete markets, however, this ratio fluctuates widely and results in welfare losses that dwarf conventional estimates of the benefits of risk sharing and consumption smoothing. The model generates a negative relationship between average public debt and revenue variability similar to the one observed in the data, and matches the GDP correlations of government purchases and the primary balance found in Mexican data.

**Keywords**: debt sustainability, public debt, fiscal solvency, procyclical fiscal policy, incomplete markets.
**JEL classification codes**: E62, F34, H63

# 1 Introduction

The empirical regularities of fiscal policy in developing countries, and particularly in emerging economies, differ sharply from those observed in the industrial world. Three key stylized facts summarize the differences:

(1) The ratios of public revenue to GDP are significantly smaller on average and substantially more volatile (as indicated by coefficients of variation) in developing economies, as Figures 1 and 2 show.

(2) The ability of countries to support higher ratios of public debt to GDP on average is inversely related to the volatility of their public revenue ratios (see Figure 3): The higher coefficients of variation of public revenue in developing countries are associated with lower average debt ratios.

(3) Fiscal policy is clearly countercyclical in industrial countries but acyclical or procyclical in developing countries. Talvi and Végh (2005) and Gavin and Perotti (1997) first documented this fact, and recent studies by Catao and Sutton (2002), Kaminsky, Reinhart and Végh (2004) and Alessina and Tabellini (2005) provide detailed cross-country evidence confirming it. These studies show that GDP and the primary fiscal balance (government expenditures) are positively (negatively) correlated over the business cycle in industrial countries, while in developing countries the GDP correlation of the primary balance (government expenditures) is close to zero or slighlty negative (positive).

This paper argues that the striking differences in the stylized facts of fiscal policy in emerging economies may result from the combination of frictions in the financial markets they have access to with imperfections in their own structure of government revenues and outlays. In particular, this paper models fiscal authorities in developing countries as playing the role of a "tormented insurer," who tries to maintain a relatively smooth stream of current expenditures and transfer payments for the private sector (i.e., provide a form of social insurance) in the face of substantial, non-insurable fiscal revenue risk and having access only to a non-state-contingent debt instrument.

Mendoza and Oviedo (2004) show how a simple, partial-equilibrium model of a tormented insurer that tries to keep fiscal outlays constant at an ad-hoc exogenous level, given a

Markov process of fiscal revenues, delivers results that combine the notion of a "natural debt limit" from the precautionary savings literature with Barro's (1979) prediction regarding the Random Walk behavior of public debt in his tax smoothing framework. The government keeps its outlays constant as long as the history of revenue realizations and the dynamics of debt result in a debt level above the annuity value of the "worst," or "catastrophic," level of fiscal revenues (i.e., the government's natural debt limit). When, and if, this limit is reached, government outlays adjust to a "fiscal crisis" (or "lowest tolerable") level.[1] In the long run, the public debt-GDP ratio has dynamics that are determined by initial conditions and lacks a well-defined limiting distribution. Depending on the initial debt ratio, public debt ends up hitting either the natural debt limit or being retired entirely (with zero debt imposed as a lower bound at which the government effectively rebates its fiscal revenue to the private sector).

This basic setup cannot say much about procyclical fiscal policy, since both revenues and outlays are exogenous, and cannot say much about the implications of the tormented insurer's behavior for the private sector's plans, equilibrium allocations, and the overall welfare of the economy. Yet, it does illustrate the potential for the tormented insurance framework to account for the observed negative relationship between the volatility of government revenues and average debt ratios: the governments of countries with more volatile revenues have tighter natural debt limits and require more precautionary savings, so they tolerate less debt. This simple model is also helpful for showing that traditional approaches to evaluate fiscal solvency ignoring aggregate uncertainty can be very misleading.

This paper reformulates the tormented insurer's problem in a dynamic, stochastic general equilibrium framework. The paper focuses in particular on the competitive equilibrium of an incomplete-markets economy in which the government chooses optimal plans for public debt and government expenditures facing two exogenous sources of revenue volatility. The first source are the cyclical variations in the economy's output. The second source are fluctuations

[1]This level of outlays can be set to zero without loss of generality, and in this case the government's natural debt limit allows the largest debt and satisfies the same definition as in Aiyagari (1994). A positive (and far more realistic) crisis level of government outlays results in a tighter natural debt limit, and the limit is tighter the smaller the cut in outlays, so that countries that are perceived as capable of stronger fiscal adjustment can support higher debt ratios.

in an "implied" tax process that captures both the volatility of tax policy and, perhaps of just as much relevance for developing countries, the volatility of key exogenous determinants of fiscal revenues. Among the latter fluctuations in real world commodity prices figure prominently because commodity exports are an important source of government revenues for many developing nations. The government issues one-period, non-state contingent bonds with a return perfectly arbitraged with the return on foreign bonds of the same type. Domestic agents make optimal consumption and savings plans facing the volatility of their after-tax income and using as vehicles of saving domestic public debt and international bonds. In contrast with the basic setup in Mendoza and Oviedo (2004), the model features well-defined limiting distributions of public and foreign debt, and can be used to examine the cyclical co-movement of the primary balance and government purchases, as well as the positive and normative implications of the tormented insurer setup for equilibrium allocations and welfare.

This paper's tormented insurer setup has two forms of asset market incompleteness. The first one is the standard "external" asset market incompleteness typical of small open economy models: the economy as a whole experiences idiosyncratic income fluctuations and has only access to a world market of non-state-contingent bonds as an insurance mechanism. The second one is "domestic" asset market incompleteness. If the government could issue state contingent debt, or enact state-contingent, non-distorting taxes, it could attain a domestic social planner's optimum in which the incomes of the government and the private sector are pooled (so that the relevant constraint is the resource constraint of the economy as a whole) and the marginal utilities of public and private spending are equalized across time and states of nature. Hence, cyclical fluctuations in fiscal revenues and after-tax private income do not alter the distribution of wealth between the government and the private sector.[2] In the tormented insurer's world, however, the implied tax process splits the economy's output across the private and public sectors, and the government can only issue non-state-contingent debt. Hence, the fiscal authority cannot replicate the domestic

[2]Because the first form of market incompleteness is not eliminated by allowing the government to issue domestic debt contingent on fiscal revenues, this social optimum does not correspond to the Arrow-Debreu complete markets equilibrium.

social optimum and must operate in a second-best environment.

The competitive equilibrium of this second-best outcome is represented in recursive form as a Markov perfect equilibrium (MPE): The government (private sector) formulates its optimal spending and financing plans taking as given a conjecture of the private sector's (government's) optimal plans, but behaving competitively so that both agents move simultaneously and take all relevant prices as given. The MPE is attained when the optimal plans chosen by the government (private sector) are consistent with the conjecture of the government's (private sector's) optimal plans under which the private sector (government) formulates its plans. In this MPE, fluctuations in fiscal revenue and after-tax private income result in changes in the distribution of wealth across the private and public sectors.

The quantitative predictions of the model are examined by conducting a set of numerical experiments in a version of the model calibrated to Mexican data. The results show that the model, when calibrated to capture the low average and high volatility of Mexico's public revenues, makes progress in explaining the other stylized facts that distinguish fiscal policy in developing countries from that of industrial nations. In particular, the model produces an inverse relationship between average debt ratios and fiscal revenue volatility similar to the one found in international data, and generates GDP correlations for government purchases and the primary balance very similar to the ones found in Mexican data. Moreover, a comparison of the domestic social optimum with the MPE shows that domestic asset market incompleteness has important implications for equilibrium allocations and welfare. The volatility of expenditures is significantly higher in the MPE than in the social optimum, even though both result in very similar long-run averages of private and public expenditures, and this translates into welfare costs of market incompleteness that are several orders of magnitude larger than standard results in the literature. The costs range from 1.6 to 3.5 percent in terms of the long-run average of a compensating variation in a time-and-state-invariant level of private consumption that equates national expected lifetime utility across the social optimum and the MPE. In the literature, the welfare costs of eliminating asset markets for purposes of consumption smoothing and/or risk sharing with standard preferences are generally below 1/10th of a percent (see, for example, Lucas (1987) or Mendoza (1991)) .

This paper forms part of a growing literature examining fiscal policy in environments with incomplete asset markets, including, among others, the studies by Aiyagari, Marcet, Sargent, and Seppälä (2002), Aguiar, Amador and Gopinath (2005), Celasun, Durdun and Ostry (2005), Schmitt-Grohé and Uribe (2002), and Yakadina and Kumhof (2005). The contribution of this paper to the literature is the focus on the domestic side of the asset market incompleteness, in a setup that shifts attention from the study of the Ramsey optimal taxation problem to the study of the optimal debt and expenditure policies for a given stochastic process of fiscal revenues. The latter is taken as an institutional feature of developing countries, intended to represent their heavy reliance on commodity exports as a large source of revenue and these countries' limited ability to fine-tune conventional direct and indirect tax rates with the aim to improve risk sharing.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the competitive equilibrium. Section 3 describes the calibration of the model to the Mexican economy and derives the model's quantitative predictions. Section 4 concludes.

# 2 The Model

Consider a small open economy with stochastic endowment income and limited opportunities for risk sharing and consumption smoothing because financial markets are incomplete. This economy is inhabited by two infinitely-lived agents: a representative household and a government. The government receives stochastic public revenues that are affected by two sources of uncertainty: fluctuations in the economy's endowment income and fluctuations in an implied tax rate that represents fluctuations in tax policy and in other key exogenous determinants of fiscal revenues. The government's total non-interest outlays include current expenditures (i.e., purchases of goods and services), which will be chosen optimally, and transfer payments to the private sector, which are kept at a deterministic, constant level for simplicity. The government can sell one-period, non-state contingent bonds to the private sector to finance primary fiscal deficits. On the side of the private sector, households collect stochastic after-tax income, which is affected by the same sources of uncertainty as fiscal revenues. Households make optimal intertemporal consumption plans and they have access to the domestic market of public bonds and to a world market of one-period, non-state contingent bonds. These two bonds are perfect substitutes from the standpoint of households, and the gross real rate of return on both equals $R$. The combination of market incompleteness and income uncertainty induces both agents to undertake precautionary saving in order to self-insure against endowment and tax shocks.

The variables that describe the state of the economy at any point in time are the stock of public debt, $b^g_{t-1}$, the net foreign asset position, $b^I_{t-1}$ and a pair of realizations of the shocks, $\epsilon_t = (\epsilon^y_t, \epsilon^\tau_t)$, where $\epsilon^y_t$ is the shock to national income and $\epsilon^\tau_t$ is the implied tax shock. Thus, the state of the economy at date $t$ is given by the triple $s_t = (b^g_{t-1}, b^I_{t-1}, \epsilon_t)$.

The economy's endowment income, $y_t \exp(\epsilon^y_t)$, is the product of a deterministic trend component, $y_t$, and a cyclical component, $\exp(\epsilon^y_t)$. The trend component grows at the constant, exogenous, gross rate $\gamma$ and all equilibrium allocations follow this common trend, as in the standard exogenous balanced-growth setup. Following Carroll (2004), the analysis focuses, without loss of generality, on a detrended representation of the model in which: (a) all allocations are expressed as ratios of $y_t$ and (b) the subjective discount factor and the gross return on assets are adjusted so that the solutions of the detrended model can be mapped into the equivalent solutions of the growing economy.[3] These adjustments imply an effective discount factor given by $\beta\gamma^{1-\sigma}$, where $\beta$ is the subjective discount factor and $\sigma$ is the coefficient of relative risk aversion, and an effective gross return on assets $\mathcal{R} \equiv R/\gamma$.

The implied tax shocks will be calibrated so that, taking GDP as the overall tax base, the stochastic process of total public revenues in the model matches one observed in actual data. The stochastic tax *rate* is $\tau^y \exp(\epsilon^\tau_t)$ and includes a (detrended) average tax rate $\tau^y$ and a cyclical tax shock $\exp(\epsilon^\tau_t)$. Note that, to the extent that this implied tax shock reflects fluctuations in world commodity prices, it is analogous to a terms-of-trade shock.

The shocks to GDP and implied taxes are represented by a joint Markov process defined by an $NS \times 2$ matrix $\mathcal{E}$ containing $NS$ realizations of the pair $\epsilon = (\epsilon^y, \epsilon^\tau)$ and an $NS \times NS$ transition probability matrix $P$ with typical element $P_{ij} = \text{Prob}(\epsilon_t = \epsilon_j | \epsilon_{t-1} = \epsilon_i)$, where

[3]Carroll also shows how to generalize this setup to include i.i.d. shocks to the trend component, which would add to the model permanent shocks as those studied by Aguiar and Gopinath (2004).

$i, j = 1, ..., NS$ index the $NS$ realizations of the pair $\epsilon$, and $\sum_{j=1}^{NS} P_{ij} = 1$, $\forall i = 1, ..., NS$.

## 2.1 The Government's Problem

The government chooses sequences of (detrended) expenditures and debt issues $\{g_t, b_t^g\}_{t=0}^{\infty}$ so as to maximize a standard CRRA expected utility function:

$$E_0 \left[ \sum_{t=0}^{\infty} \left(\beta \gamma^{1-\sigma}\right)^t \frac{g_t^{1-\sigma}}{1-\sigma} \right]; \qquad \sigma \neq 1; \quad \sigma > 0 \tag{1a}$$

subject to the following government budget constraint:

$$g_t + z + \mathcal{R} b_{t-1}^g \leq b_t^g + \exp(\epsilon_t^y) \tau^y \exp(\epsilon_t^\tau) \tag{1b}$$

where $g_t$ represents government expenditures at time $t$, $z$ are time-and-state-invariant transfer payments to the private sector, $\mathcal{R} \equiv R/\gamma$ is the growth-adjusted interest rate, and $b_t^g$ is the stock of public debt chosen at time $t$. The budget constraint (1b) states that total government outlays, consisting of expenditures, transfers, and gross debt repayments, must not exceed the total government resources, consisting of issues of new debt and fiscal revenues.

Given that the marginal utility of public expenditures goes to infinity as $g_t$ approaches zero from above, the government never chooses a plan that leaves it exposed to the risk of facing less than strictly positive expenditures in all dates and states of nature. As a result, the government imposes on itself a "natural debt limit," which is given by the annuity value of the lowest Markov realization of fiscal revenues net of transfers $\frac{\min[\exp(\epsilon^y)\tau \exp(\epsilon^\tau)]-z}{\mathcal{R}-1}$. This upper bound on debt is exactly analogous to the concept of the natural debt limit introduced by Aiyagari (1994) in the heterogenous agents-precautionary savings literature. Following Aiyagari, the debt constraint faced by the government can be expressed more generally as an upper bound $\phi^g$ that satisfies:

$$b_t^g \leq \phi^g \leq \frac{\min\left[\exp(\epsilon^y)\tau \exp(\epsilon^\tau)\right] - z}{\mathcal{R} - 1} \tag{1c}$$

Hence, the government's debt constraint can be set at the natural debt limit, or at an arbitrarily tighter limit, which Aiyagari (1994) labeled an ad-hoc debt limit. This ad- hoc debt limit can be justified as a form of natural debt limit implied by a constraint requiring government purchases not to fall below an exogenous minimum level at any date and state of nature. The quantitative solutions of the MPE will focus on cases in which $\phi^g$ is set equal to the natural debt limit.

In the context of the tormented insurer's framework, Mendoza and Oviedo (2004) show that the government's natural debt limit also implies a credible commitment to remain "able" to repay (i.e., to have enough resources to repay) the debt at all times. This is because the natural debt limit represents the stock of debt the government can honor even if it draws the worst realization of fiscal revenue "almost surely." Mendoza and Oviedo obtain this debt limit as an exogenous requirement that an effective insurer, defined as one that wants to remain able to keep government outlays smooth for the longest possible time, would want to meet. In this case, situations where levels of debt higher than this debt limit were to be allowed correspond to situations in which there exist sequences of fiscal revenues $\{(\exp \epsilon_t^y)\tau \exp(\epsilon_t^\tau)\}_{t=0}^T$ with non-zero probability of occurrence under which the government cannot repay its debt even by setting $g_T = 0$ at some date $T$. In the model of this paper, the CRRA form of the government's payoff function rules out this possibility.

The optimality conditions of the government's problem are the budget constraint (1b), the debt constraint (1c), and the following Euler equation:

$$g_t^{-\sigma} = \beta \mathcal{R} \mathrm{E}\left[g_{t+1}^{-\sigma}\right] + \mu_t^g \tag{1d}$$

where $\mu_t^g$ is the non-negative Lagrange multiplier on the debt constraint. The above Euler equation has the standard interpretation of equating the marginal cost and benefit of sacrificing a unit of public expenditures at date $t$, with the caveat that if the debt constraint binds the multiplier in the right-hand-side is positive. Note, however, that when $\phi^g$ in (1c) is set equal to the natural debt limit, the CRRA utility function implies that $\mu_t^g = 0$ at equilibrium for all $t$.

## 2.2 The Household's Problem

The household 's problem is analogous to the government's problem, except that the household is the holder of the public debt and can also trade in the world's bond market. The representative household's choice variables are stochastic sequences of consumption and bond holdings $\{c_t, b_t^g, b_t^I\}_{t=0}^{\infty}$ that maximize expected lifetime utility:

$$E_0\left[\sum_{t=0}^{\infty}\left(\beta\gamma^{1-\sigma}\right)^t \frac{c_t^{1-\sigma}}{1-\sigma}\right] \tag{2a}$$

subject to the following budget constraint:

$$c_t + x + (b_t^g + b_t^I) \leq \exp(\epsilon_t^y)\left[1 - \tau\exp(\epsilon_t^\tau)\right] + \mathcal{R}(b_{t-1}^g + b_{t-1}^I) + z \tag{2b}$$

This budget constraint restricts the sum of consumption, an exogenous, invariant level of private absorption $x$ (which is used in the calibration to represent investment expenditures), and purchases of public and foreign bonds not to exceed the household's resources. The latter are given by the sum of after-tax private income, financial income, and government transfers.

Since the household's payoff function has the same CRRA form as the government's, the household's problem features a similar natural debt limit on its total bond position $(b_t^g + b_t^I)$ given by the annuity value of the lowest Markov realization of after tax income, adjusted to add transfers and subtract $x$. Alternatively, for a given Markov plan of domestic public debt given by the policy function $b_t^g = \hat{b}^g(b^g, b^I, \epsilon)$, the household faces a natural limit on external debt that results in the following constraint on foreign bond holdings:

$$b_t^I \geq \phi^I \geq -\frac{\min\left[\exp(\epsilon^y)\left[1 - \tau\exp(\epsilon^\tau)\right] + \mathcal{R}b^g - b^{g\prime}(b^g, b^I, \epsilon)\right] + z - x}{\mathcal{R} - 1} \tag{2c}$$

The right-most expression in this constraint is the natural debt limit on foreign debt. Hence, condition (2c) allows for the external debt constraint to be set at its natural debt limit or at a tighter ad-hoc debt limit. As before, ad-hoc debt limits can be thought of as implied by an exogenous constraint requiring consumption never to fall below a pre-determined minimum level. As explained in section ??, the quantitative experiments use an ad-hoc debt limit for the household's problem to calibrate the baseline simulation to match the average consumption-output ratio observed in actual data.

The set of optimality conditions of the private sector are the flow budget constraint (2b), the external debt constraint (2c), and the following Euler equation:

$$c_t^{-\sigma} = \beta \mathcal{R} \mathrm{E}\left[c_{t+1}^{-\sigma}\right] + \mu_t^c \tag{2d}$$

where $\mu_t^c$ is the non-negative Lagrange multiplier on the external debt constraint that, as in the case of the government, is equal to zero for all $t$ if $\phi^I$ is set at the natural debt limit. This Euler equation is also standard, with the caveat of the multiplier on the borrowing constraint, and it states that the private sector equates the marginal benefit and cost of an additional unit of consumption expenditures.

Given the perfect substitutability between $b_t^g$ and $b_t^I$, the household is indifferent between the two assets and their returns are perfectly arbitraged. The government has, however, a well-defined supply of government debt that it desires to place each period according to its optimal plans. Hence, the model assumes that the household, given its infinitely-elastic demand for domestic public debt at the world real interest rate, is willing to hold the amount of this debt that the government issues. This does not introduce any wealth or price distortions on the optimality conditions of the household's problem and is consistent with the competitive equilibrium conditions. Foreign bonds and domestic public debt markets could be segmented by introducing additional frictions in the financial setup of the model, but the aim of the analysis is to highlight the implications of the incompleteness of asset markets that emerge in the tormented insurer's framework even when domestic public debt and foreign bonds are perfect substitutes.

## 2.3 Competitive Equilibrium

**Definition (CE)** The competitive equilibrium of the economy is characterized by stochastic sequences representing the allocations of private and public expenditures, government debt, and private net foreign asset holdings, $\{c_t, g_t, b_t^g, b_t^I\}_{t=0}^{\infty}$, such that:

i) $\{g_t, b_t^g\}_{t=0}^{\infty}$ solve the government's problem.

ii) $\{c_t, b_t^g, b_t^I\}_{t=0}^{\infty}$ solve the household's problem.

iii) The following economy-wide resource constraint holds:

$$c_t + x + g_t \leq \exp(\epsilon_t^y) + \mathcal{R}b_{t-1}^I - b_t^I \tag{3}$$

## 2.4 Markov Perfect Equilibrium

The above competitive equilibrium cannot be represented as the solution to a social planner's problem because the incompletness of asset markets prevents the private and public sectors from pooling risk. In particular, the non-insurable, idiosyncratic income shocks faced by the private and public sectors lead to fluctuations in the "domestic distribution of wealth" (i.e., the wealth distribution of the government vis-a-vis the private sector). The absence of risk pooling across the two sectors is reflected in fluctuations in the ratio of marginal utilities of expenditures, or, given the CRRA structure of preferences, in the ratio $c_t/g_t$.

Under these conditions, solving for the competitive equilibrium requires adopting a solution strategy that can capture the state-contingent wealth dynamics accurately. The adopted strategy solves for the competitive equilibrium as a Markov perfect equilibrium where the government and the private sector behave competitively by taking all relevant prices as given and by making simultaneous moves. Each agent formulates optimal plans taking as given a conjecture of the other agent's optimal plans, and none of the agents internalizes the effect of its actions on each other's choices. The equilibrium is a Nash equilibrium of a two-player dynamic game with uncertainty; the solution strategy involves representing the game in recursive form and solving for its equilibrium by backward induction.

The two agent's optimization problems are expressed in recursive form as follows. At the beginning of each period, agents observe the state of the economy $s = (b^g, b^I, \epsilon)$. The state space of the Markov shocks includes the $NS$ possible realizations of the pair $\epsilon =$

$(\epsilon^y, \epsilon^\tau)$ defined earlier. The state space of domestic and external debt is defined by discrete grids with NBG and NBI nodes respectively. Hence, the state space of public debt is $b^g \in B^g = \{b_1^g < b_2^g < ... < b_{NBG}^g = \phi^g\}$ and the one for net foreign assets is $b^I \in B^I = \{b_1^I = \phi^I < b_2^I < ... < b_{NBI}^g\}$. Each agent takes as given a conjectured decision rule for the other agent's optimal plans. The government conjectures that the household's decision rule is $b^{I\prime} = \tilde{b}^I(b^g, b^I, \epsilon)$ and the household conjectures that the government's decision rule is $b^{g\prime} = \tilde{b}^g(b^g, b^I, \epsilon)$. Given these, each agent finds an optimal decision rule that solves the Bellman equation representing their individual optimization problems.

The Bellman equation for the government's optimization problem, given the conjecture $\tilde{b}^I(b^g, b^I, \epsilon)$, is:

$$V(b^g, b^I, \epsilon) = \max_{b^{g\prime} \in B^g, g} \left\{ \frac{g^{1-\sigma}}{(1-\sigma)} + \beta\gamma^{1-\sigma}\mathrm{E}\left[V(b^{g\prime}, \tilde{b}^I(b^g, b^I, \epsilon), \epsilon')\right] \right\} \tag{4}$$

$$\text{s.t.:} \quad g + z + \mathcal{R}b^g \leq b^{g\prime} + \exp(\epsilon^y)\tau^y \exp(\epsilon^\tau)$$

$$b^{g\prime} \leq \phi^g$$

where the debt limit $\phi^g$can be set at the government's natural debt limit or at a tighter ad-hoc debt limit. The solution to this dynamic programming problem yields a decision rule for the government's debt $\hat{b}^g(b^g, b^I, \epsilon)$.

The Bellman equation for the private sector's optimization problem, given the conjecture $\tilde{b}^g(b^g, b^I, \epsilon)$, is:

$$W(b^g, b^I, \epsilon) = \max_{b^{I\prime} \in B^I, c} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta\gamma^{1-\sigma}\mathrm{E}\left[W(\tilde{b}^g(b^g, b^I, \epsilon), b^{I\prime}, \epsilon')\right] \right\} \tag{5}$$

$$\text{s.t.:} \quad c + x + \tilde{b}^g(b^g, b^I, \epsilon) + b^{I\prime} \leq \exp(\epsilon^y)\left[1 - \tau\exp(\epsilon^\tau)\right] + \mathcal{R}(b^g + b^I) + z$$

$$b^{I\prime} \geq \phi^I$$

where the debt limit $\phi^I$can be set at the household's natural debt limit or at a tighter ad-hoc debt limit. The solution to this dynamic programming problem yields a decision rule for the private sector of the form $\hat{b}^I(b^g, b^I, \epsilon)$.

**Definition (MPE)** A Markov perfect equilibrium for the small open economy is a pair of value functions $V$ and $W$ and a pair of decision rules $\hat{b}^g$ and $\hat{b}^I$ such that:

i) Given the conjecture $\tilde{b}^I$, $V$ solves the Bellman equation (4), and $\hat{b}^g$ is the associated optimal policy rule.

ii) Given the conjecture $\tilde{b}^g$, $W$ solves the Bellman equation (5), and $\hat{b}^I$ is the associated optimal policy rule.

iii) The conjectured and optimal decision rules satisfy $\hat{b}^g(\cdot) = \tilde{b}^g(\cdot)$ and $\hat{b}^I(\cdot) = \tilde{b}^I(\cdot)$

Conditions i) to iii) imply that, at equilibrium, each agent's conjecture of the other agent's optimal decisions matches the decisions that the other agent actually finds optimal to choose. Note that the computation of this MPE is simplified by the fact that, while the government's actions affect the household's dynamic programming problem, the household's choices do not affect the government's optimal plans. Hence, the MPE can be computed by solving first the government's Bellman equation, and then imposing the resulting public debt decision rule on the household's problem. This feature of the model follows from the lack of feedback from actions of the private sector on the government's payoff and constraints, which in turn results from the simplifying assumptions making fiscal revenues independent of the actions of the private sector. The rationale for these assumptions is to show that even in this case, in which the strategic interaction between the two agents is simplified substantially, the incompleteness of domestic financial markets has important consequences. Extending the analysis to the case in which there is two-way feedback between the government's and the private sector's plans is straightforward, albeit computationally intensive.

It is straightforward to show that a Markov perfect equilibrium, if it exists, is a competitive equilibrium for the small open economy. Consider first the Bellman equations (4) and (5). Using the standard Benveniste-Sheinkman equation (or the Envelope theorem), it follows that the first order conditions of the Bellman equations imply that the Euler equations of the competitive equilibrium, eqs. (1d) and (2d), hold. Moreover, the budget constraints of the Bellman equations yield the economy-wide resource constraint (3) of the competitive equilibrium.

The numerical solutions to the MPE are obtained by iterating to convergence on the Bellman equations (4) and (5) on the discrete state space containing NBG possible public debt positions, NBI possible net foreign asset positions, and NS pairs of income and tax shocks. In the implementation of the solution algorithm, NBG=200, NBI=200 and NS=9, so the discrete state space is of dimensions $200 \times 200 \times 9$.[4] As long as there is no feedback from the private sector choices to the government's Bellman equation, the algorithm can be simplified by solving first the government's problem as a stand-alone value-function-iteration problem, and then using the resulting optimal decision rule for public debt as the conjectured decision rule for the same variable in the Bellman equation of the private sector.

## 2.5 Domestic Social Optimum

As explained in the Introduction, the main distortion preventing the tormented insurer from implementing fiscal policies that support perfect risk pooling across the private and public sectors is the incompleteness of domestic asset markets. The quantitative analysis of the next section explores the effects of this distortion on allocations and welfare by comparing the outcome of the MPE with that of a social optimum in which domestic asset markets are assumed to be rich enough to support perfect domestic risk pooling. The domestic economy as a whole still faces incomplete markets vis-a-vis the rest of the world because GDP fluctuations still represent non-insurable, idiosyncratic income shocks for the small open economy.

The social planner's problem that yields the domestic social optimum is similar to the standard Negishi-Mantel social planner's representation of the complete markets equilibrium as a weighted sum of individual utilities subject to the resource constraint. The only caveat is that, because the international asset markets remain incomplete, the outcome of the social planner's problem defined here does not correspond to the complete-markets equilibrium of the small open economy. The domestic social optimum is defined as the sequences of allocations $\{c_t, g_t\}_{t=0}^{\infty}$ that maximize the weighted sum of the government and

[4] We explore the robustness of the results to enlarging the state space or conforming it with finer grids by solving the model with a state space of dimensions $400 \times 400 \times 9$. The results are largely robust, with the ergodic means, standard deviations, and autocorrelations differing by at most 5 percent.

private payoffs,

$$E_0 \left\{ \eta \sum_{t=0}^{\infty} \left(\beta\gamma^{1-\sigma}\right)^t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-\eta) \sum_{t=0}^{\infty} \left(\beta\gamma^{1-\sigma}\right)^t \frac{g_t^{1-\sigma}}{1-\sigma} \right\}, \tag{6a}$$

where $\eta > 0$ and $(1-\eta)$ are the weights assigned to the private and public sector payoffs respectively, subject to the small open economy's resource constraint:

$$c_t + g_t + x + b_t^I \leq \exp(\epsilon^y) + \mathcal{R} b_{t-1}^I \tag{6b}$$

and the following borrowing constraint on net foreign assets:

$$b^{I\prime} \geq \phi^{SO} \geq -\frac{\min\left[\exp(\epsilon^y)\right] - x}{\mathcal{R} - 1} \tag{6c}$$

where the quotient in the right-hand-side of this borrowing constraint is the social planner's natural debt limit.

In this equilibrium with perfect domestic risk pooling, the marginal utilities of public and private expenditures are equalized across states and over time. Given CRRA utility functions, this translates into an equilibrium condition that determines a time- and state-invariant expenditure ratio:

$$\frac{c_t}{g_t} = \left[\eta/(1-\eta)\right]^{1/\sigma} \tag{6d}$$

Note that $c_t$ and $g_t$ still fluctuate because the external market incompleteness still exists and it does not allow the small open economy to fully insure away its macroeconomic risk.

The recursive form of the social planner's problem is given by the following dynamic programming problem:

$$V^{so}(b^I, \epsilon^y) = \max_{b^{I\prime} \in B^I, c, g} \left[ \eta \left( \frac{c^{1-\sigma}}{1-\sigma} \right) + (1-\eta) \left( \frac{g^{1-\sigma}}{1-\sigma} \right) + \beta\, \gamma^{1-\sigma} \mathrm{E}\left[ V^{so}(b^{I\prime}, \epsilon^{y\prime}) \right] \right] \tag{6e}$$

subject to the resource constraint (6b) and the external debt constraint (6c).

If the government had access to state contingent tax or debt instruments, it could implement the above domestic social optimum as a competitive equilibrium. State contingent taxes could work as follows: Assume that the public and private debt limits are set at their natural debt limits, public debt is set to zero at all times, and the government introduces a set of state-contingent income taxes $\tau(\epsilon_t^y, \epsilon_t^\tau) = (g_t^* + z) / (\exp(\epsilon_t^y)\tau^y \exp(\epsilon_t^\tau))$, where starred variables represent optimal allocations of the social planner's problem. This policy would support the social optimum because: (a) $\{g_t^*\}_{t=0}^\infty$ would satisfy the government budget constraint and the government's Euler equation; (b) $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$ would satisfy the private sector's budget constraint and its Euler equation; and (c) the resource constraint holds. That the Euler equations are satisfied is obvious from the fact that the same Euler equations hold for the social planner. The government budget constraint obviously holds given the definition of the tax rule. The budget constraint of the private sector holds for $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$ because the resource constraint holds for the social optimum, the government's budget constraint holds, and there is no public debt.

Similarly, the social optimum could be obtained as a competitive equilibrium with a policy setting income taxes to zero and issuing state-contingent public debt (i.e., domestic one-period Arrow securities) to effectively implement state contingent lump-sum taxes $LST_t = g_t^* + z$. This policy would tax away from households, in a lump-sum fashion, exactly the resources needed to pay for the time-invariant transfers and the social optimum sequence $\{g_t^*\}_{t=0}^\infty$. The private sector would then find it optimal to choose the allocations $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$ because $\{c_t^*\}_{t=0}^\infty$ satisfies its Euler equation and the lump-sum tax leaves just enough disposable income for the private sector to satisfy its budget constraint by choosing $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$.

# 3 Quantitative Findings

## 3.1 Baseline Calibration and Discrete State Space

This section's quantitative analysis uses a baseline calibration of the model to Mexican data at an annual frequency. The calibration has two components. First, a set of parameter values set so that variables in the model match their counterparts in Mexican data or taken as standard from the Real Business Cycle (RBC) literature. Second, a discrete Markov representation of the stochastic process of output and implied taxes observed in the data. Table 1 summarizes the calibration parameters.

The growth rate is set to $\gamma = 1.0088$, which is the average growth rate of Mexico's real GDP per capita between 1980 and 2000 computed using data from the World Bank's *World Development Indicators*. In the same source and sample, the average investment-GDP ratio yields $x = 0.226$. The mean income tax rate is set to $\tau = 0.256$, which is the 1980-2002 average ratio of total public revenue to GDP using data at current prices from Mexico's *INEGI Bank of Economic Information* (at *http://dgcnesyp.inegi.gob.mx/cgi-win/bdieintsi.exe*). The time-invariant government transfers are set at $z = 0.111$, which is the difference between the average ratio of total non-interest government outlays to GDP (from the same *INEGI* source) and the average government expenditures-GDP ratio, $g = 0.0978$, from *World Development Indicators*.

The real interest rate is set to $R = 1.0986$, which is the average EMBI+ real return on Mexican sovereign debt for the 1994-2002 period computed using the data from Neumeyer and Perri (2005). This rate includes both the risk-free rate as well as the default risk premium. The model does not consider default explicitly, but using an interest rate that is more representative of the actual rate at which the Mexican government borrows is more reasonable than simply applying the real rate on U.S. T-bills.

Regarding preference parameters, the value of the coefficient of relative risk aversion is $\sigma = 2$, which is the usual value in RBC models. Unlike in the RBC literature, however, the subjective discount factor $\beta$ cannot be set simply to match the inverse of the growth-adjusted gross interest rate because precautionary savings makes asset holdings diverge to infinity in this case, as agents try to attain a non-stochastic consumption stream in the face of non-diversifiable income shocks. In models without long-run growth (see Aiyagari (1994), Hugett (1993) and Ljungqvuist and Sargent (2000, ch. 14)), the condition $\beta R < 1$ yields a well-defined unique invariant distribution of asset holdings. In models with growth, Carroll (2004) showed that the condition required to prevent asset holdings from diverging to infinity is: $\beta R \times \max[(1/R)^{\sigma}, (1/\gamma)^{\sigma}] < 1$. Precautionary savings also implies that the averages of the model's stochastic stationary state vary with the choice of parameters (particularly those that appear in Carroll's condition, the Markov process of shocks, and the choice of $\phi^g$).

Figure 4 shows how the mean level of public debt obtained from the MPE falls as $\beta$ increases so that $\beta R \times \max[(1/R)^\sigma, (1/\gamma)^\sigma]$ converges to 1 from below (using the values of $R$, $\gamma$ and $\sigma$ set above, and keeping $\phi^g$ at the level of the government's natural debt limit obtained using (1c) and the Markov process of shocks described below). The Figure is plotted with the horizontal axis inverted to show that it yields the same concave relationship as the Aiyagari-Hugget class of models (with the elasticity of average public "assets" going to infinity as $\beta$ increases).[5] Each average level of debt has associated with it an average level of public expenditures obtained from the budget constraint, and this level is lower the higher the debt because of the extra income allocated to debt service. Hence, the baseline calibration of the model uses a value of $\beta$ such that, given the calibrated values of $R$, $\gamma$ and $\sigma$, and with $\phi^g$ set at the government's natural debt limit, Carroll's condition holds and the long-run average of government purchases equals Mexico's average of 9.78 percent. The resulting value is $\beta = 0.925$ (for which Carroll's condition yields $\beta R \gamma^{-\sigma} = 0.9984 < 1$).

The joint Markov process driving the shocks to endowment income and implied taxes is constructed using annual data for Mexico's real GDP and total fiscal revenues for the period 1980-2004. The implied tax-rate series, $TX$, is constructed by computing the ratio of total public revenue to $GDP$. The data for $GDP$ and $TX$ are then expressed in per capita terms, logged, and detrended using the Hodrick-Prescott filter to obtain their cyclical components, $\widehat{GDP}$ and $\widehat{TX}$. Panel (a) of Table 2 reports the unconditional moments of these two time series. The joint process driving the endowment and implied tax shocks is obtained by estimating a VAR(1), $\widehat{X}_t = \Phi \widehat{X}_{t-1} + \zeta_t$, where $\widehat{X}_t = (\widehat{GDP}_t, \widehat{TX}_t)'$, $\Phi$ is the 2×2 matrix of autocorrelation coefficients, and $\zeta_t$ is an i.i.d. random vector with covariance matrix $\Sigma_\zeta$. The estimation results of this VAR are as follows:[6]

[5] Formally, public debt should go to -$\infty$ as Carroll's condition approaches 1 from below. Since the upper bound of public debt is set to zero, however, the entire mass of the ergodic distribuion of debt concentrates at this upper bound as $\beta$ raises to levels for which Carroll's condition is "close enough" to 1.

[6] t-statistics are shown in parenthesis.

$$\hat{\Phi} = \begin{pmatrix} 0.2789 & 0.4389 \\ (1.4495) & (1.2614) \\ & \\ \text{-}0.1489 & 0.6004 \\ (\text{-}1.5643) & (3.4867) \end{pmatrix}; \qquad \hat{\Sigma}_\zeta = \begin{pmatrix} 0.000727 & \text{-}0.000268 \\ \text{-}0.000268 & 0.002378 \end{pmatrix}$$

Since the MPE is solved using a discrete state space, the VAR representation of the shocks needs to be converted into a discrete Markov process. This is done using Tauchen's (1991) quadrature method, setting the Markov chain to carry nine *pairs* of shocks to GDP and taxes (i.e., nine states in total). Given that the off-diagonal elements of $\hat{\Phi}$ are not statistically different from zero, we use the diagonal version of $\hat{\Phi}$ and $\hat{\Sigma}_\zeta$ as inputs for Tauchen's algorithm. The resulting set of Markov realizations and their associated long-run probabilities are reported in the Appendix. Panel (b) of Table 2 shows the unconditional moments of GDP and taxes produced by the Markov chain. These do not match exactly the moments from the data in Panel (a) because of the approximation error of the Markov chain with 9 states, but the standard deviation, correlation and autocorrelation coefficients are close to their empirical counterparts.[7]

The grids of public debt and foreign assets are specified as follows. The upper bound of the public debt grid is its natural debt limit. Given the values of $R$, $\gamma$, $z$ and the lowest realization of government revenue supported by the states of income and tax shocks in the Markov process, equation (1c) yields $b^g_{200} = \phi^g = 1.318$ (or about 132 percent of GDP). The lower bound of public debt is set to zero ($b^g_1 = 0$) which implies that the government cannot become a net creditor (i.e., hold negative debt positions). This constraint binds with 0.51 percent probability in the long run.

Given the optimal decision rule for public debt, the values of $R$, $\gamma$, $z$ and $x$, and the lowest realization of private disposable income (defined as $\exp(\epsilon^y)\left[1 - \tau\exp(\epsilon^\tau)\right] + \mathcal{R}b^g - b^{g\prime}(b^g, b^I, \epsilon)$) supported by the states in the Markov process, equation (2c) yields a natural debt limit on net foreign assets of -7.105 (more than 7 times larger than GDP). However, the

[7]The approximation improves as $NS$, but at a high cost in computing time. The drawback of working with a limited number of states is that, by construction, natural debt limits are produced with the lowest realizations of income supported by the Markov chain, and the extent to which these realizations reflect adverse outcomes that are truly relevant (i.e., that have nontrivial probability) depends on the number of states in the chain.

MPE obtained with such a large *maximum* external debt ratio yields an *average* foreign debt ratio that is also too large (at 200 percent of GDP), and consequently the corresponding long-run average consumption ratio is too low relative to the average in Mexican data (53 percent in the MPE v. 66.9 percent in the data). Hence, instead of using the natural debt limit for the lower bound of external assets, we set an ad-hoc debt limit such that the baseline MPE yields an average private consumption ratio consistent with the data. The ad-hoc debt limit that satisfies this criterion is $b_1^I = \phi^I = -0.5$. The upper bound for $b^I$ is chosen so that its long-run probability is approximately zero (without any binding constraint) and has no effect on the moments of the ergodic distribution. The resulting upper bound is $b_{200}^I = 0.1$.

## 3.2 Cyclical Co-movements in the Baseline Calibration

Table 3 shows the statistical moments that characterize cyclical co-movements in the competitive equilibrium (i.e., moments computed with the ergodic distribution of the MPE). All moments in the table correspond to the model's detrended variables, which are ratios relative to GDP. The mean value of GNP is lower than that for GDP (which by construction is equal to 1) because the economy is paying interest to the rest of the world on a stock of net foreign assets of about 36 percent of GDP (in line with the estimates for Mexico obtained by Lane and Milesi-Ferretti (1999)). This debt position and the value of $R$, imply that the economy runs a trade surplus of 3.2 percent of GDP on average in the long run. The average public debt ratio is equal to 53 percent of GDP, and hence the mean *total* asset position of the household (i.e., $\mathrm{E}[b_t^g + b_t^I]$) is equal to 17 percent of GDP.

Figures 5.a and 5.b show the limiting distributions of public debt and international assets. These figures show that the support of the equilibrium allocations of public debt and net external assets differ sharply from the corresponding debt limits. The long-run mean of public debt (external assets), is 79 (14) percentage points lower (higher) than its natural debt limit. The substantial difference between the mean of public debt and the government's natural debt limit shows that it is optimal for a government acting as a tormented insurer, aiming to do its best to keep its total outlays smooth given the volatility of its revenues and the incompleteness of asset markets, *not* to use a large portion of its borrowing capacity on average. There are non-zero-probability histories of exogenous adverse revenue outcomes that can lead the government to hold large amounts of debt in the stochastic steady state, of up to 130 percent of GDP, but the endogenous probability of observing these outcomes is very low (see Figure 5a). Outcomes with higher debt ratios are not consistent with optimal behavior in the economy's long-run competitive equilibrium.

The variability and co-movement indicators in Table 3 illustrate the challenges that the private and public sectors face in trying to smooth consumption in the competitive equilibrium. Private consumption is more volatile than after-tax income and government expenditures are more volatile than fiscal revenues. Private and public expenditures are positively correlated with the relevant income measures (after-tax income and fiscal revenues respectively), but the correlations are low. Precautionary savings helps agents lower the correlation between their incomes and their expenditures in the long run, but still, because asset markets are incomplete, they cannot fully hedge idiosyncratic income risk. The end results are significantly higher levels of variability for expenditures than for incomes, even when the correlations between expenditures and incomes are not very high.

The effects of imperfect risk pooling can also be observed in the relative variability of private and public expenditures. When the government has access to state-contingent fiscal instruments, the ratio of private to public expenditures is constant across time and states of nature, and hence government expenditures and private consumption have identical coefficients of variation. In contrast, in the model's competitive equilibrium the coefficient of variation of government expenditures is nearly 6 times larger than that for private consumption. The coefficient of variation of the ratio $c_t/g_t$ itself is 42.4 percent.

The low, positive correlation between aggregate income and government expenditures (at 0.02) matches the estimate of this correlation observed in the data (see Appendix Table 5 in Kaminsky et al. (2004)). This pattern of non-negative correlations between government purchases and GDP is one of the defining features of the puzzling phenomenon of procyclical fiscal policies that characterize developing economies. For the United States, for example, Kaminsky et al. report an estimate of the same correlation of -0.37, and the average of their estimates for G7 countries is about -0.2. Thus, the model is successful at explaining the absence of a clear countercyclical pattern in the behavior of government purchases observed in emerging economies, and it mimics very accurately Mexico's correlation between government expenditures and output.

The model is also able to replicate a second key feature of procyclical fiscal policies in developing countries: the primary fiscal balance is nearly uncorrelated with output, instead of displaying the marked positive correlations observed in industrial countries. Alesina and Tabellini (2005) estimate regressions of the (differenced) primary balance-GDP ratio on the cyclical components of GDP and terms of trade, and the lagged primary balance-GDP ratio, and find that the "beta" coefficients on GDP are significantly higher for industrial economies than for developing countries. The average of their beta coefficients for industrial OECD countries is 0.26, while their average for Latin American and the Caribbean is -0.13 (see Table 1 of Alessina and Tabellini's paper). Their beta coefficient for Mexico is -0.094. Using a long stochastic simulation of the model's decision rules to generate data for 20,000 periods, and estimating the analog of their regression, the model produces a beta coefficient of 0.097 (with a negligible standard error of 0.00326). It is difficult to assess the accuracy of this match between the betas produced by the model and the data because Alessina and Tabellini do not report country-specific standard errors. They do note that their beta coefficients have generally large standard errors, and that many of the developing country betas are not statistically different from zero. Hence, this suggests that it is possible that again the model matches not just the general feature that developing countries tend to have betas much closer to zero than industrial countries, but that the model could match closely Mexico's true beta.

The autocorrelations of asset holdings (both public debt and net foreign assets) are very close to 1, and in addition, public and private expenditures are highly serially correlated. These results are consistent with the high autocorrelation of assets typical of models of incomplete markets and precautionary saving with non-state-contingent assets, and they are also in line with the findings of Aiyagari et al. (2002), who found that the solution to the Ramsey optimal taxation problem with incomplete markets and exogenous government expenditures yields near-random-walk behavior in optimal taxes and debt. Here, tax revenue is exogenous and hence optimal public expenditures, instead of optimal taxes, and debt display near-unit-root behavior.[8] As Aiyagari et al. also note, the high autocorrelation of public debt is a feature that is in line with the predictions of Barro's (1979) classic work on optimal debt under tax smoothing.

It is interesting to compare the outcome of the model's competitive equilibrium with the predictions of the simpler partial equilibrium model of Mendoza and Oviedo (2004). In the partial equilibrium setup, the government keeps outlays constant at an exogenous ad-hoc level, except when a sufficiently long sequence of adverse revenue realizations puts the economy in a state of "fiscal crisis", at which the natural debt limit binds and the fiscal authority therefore cuts its outlays to an ad-hoc crisis level. Given a Markov process of revenue realizations, simulated time paths of public debt always diverge eventually to either zero (i.e., the no-assets constraint) or to the natural debt limit, depending on the initial debt ratio–truly matching Barro's (1979) result stating that the long-run behavior of debt is fully determined by initial conditions. Since there is no well-defined stochastic stationary equilibrium, however, the partial equilibrium model is of limited use for assessing the long-run dynamics of public debt. In contrast, the competitive equilibrium of the model of this paper has a unique, invariant limited distribution to which the economy converges regardless of initial conditions. When perturbed by an initial shock, all endogenous (detrended) variables eventually revert to their means as the effect of the shock vanishes. As shown below, this unique distribution yields precise predictions about the expected value and the time series dynamics of public debt, and the rest of the model's endogenous variables, in the long-run and the short-run for any given set of initial conditions. It is also true, however, that the near-random-walk patterns of public debt, net foreign assets and expenditures implies that the mean-reverting behavior of these variables will take a very long time to be felt after an exogenous shock to GDP or implied taxes hits the economy.

Figure 6 plots the Markov forecast functions of public debt and net foreign assets starting

[8] Aiyagari et al. also need limits on government assets (i.e., negative debt) to recover Barro's predictions, because otherwise the optimal taxes are set to zero an all expenditures are financed with a large enough "war chest" of precautionary savings. In contrast, the model of this paper needs only to satisfy Carroll's stationarity condition to have a well-defined ergodic distribution of assets. Limits on government assets are required only if one is interested in particular long-run equilibria that match particular features of the data (as was the case in the baseline calibration that uses the natural debt limit as a lower bound for debt and 0 as an upper bound to match the observed average GDP share of government expenditures).

from the mean value of the latter and an initial public debt ratio of 0.634 (roughly 10 percentage points above the mean in the ergodic distribution). The plots show nine forecast functions (i.e., non-linear impulse responses conditional on the initial asset ratios and a date-0 pair of shocks) corresponding to each of the nine pairs of shocks in the states of the Markov chain of GDP and implied tax shocks. Thus, each plot has nine forecast functions, one for each of the nine possible initial states: $s = (b_0^g = 0.634,\ b_0^I = \mathrm{E}[b^I], \epsilon_j)$ for $j = 1, .., 9$. Although the nine forecast functions start at $b^g = 0.634$ (which is visually imperceptible in the graph given that the time horizon goes through $t = 2000$) the nine realizations of $\epsilon$ give rise to nine different values of $b^g$ in the second period of the figure. One way to interpret these forecast functions is to view them as plotting the expected responses of public debt and foreign assets given a date-0 unanticipated shock (e.g. the rescue of failing banks) that puts the stock of public debt 10 percentage points above its long-run average.

Figures 6a and 6b illustrate the effects of the high persistence of assets discussed above, in the form of the very low speed of convergence of the public debt and net foreign asset ratios to their corresponding long-run averages. This is near-Random-Walk behavior, but is clearly *not* a true Rando Walk, as in Barro (1979), because the competitive equilibrium of the tormented insurer model features a unique invariant distribution of assets, even though the effects of purely transitory shocks on asset holdings can take very long to fade away.

Figure 7 shows a sample of 10 simulated time series of the public debt-GDP ratio for the same starting state $s_0$ described above. For each simulation, a 50-period time series of realizations of $\epsilon_t$ is drawn from the Markov matrix of realizations $\mathcal{E}$ (which is detailed in the Appendix). As the length of the simulations grows sufficiently large, the expected value of $b_t^g$ computed with each of the 10 time series converges to the mean public debt ratio in the limiting distribution. As each time series fluctuates over time, however, the effect of the high persistence of asset holdings is reflected in the wide range of debt ratios that are consistent with the competitive equilibrium starting from the same initial conditions.

Figures 8-10 show impulse-response functions to tax and GDP shocks constructed by estimating a standard, unrestricted VAR(1) model using data generated from the model in a stochastic time-series simulation that runs for 20,100 periods, discarding the first 100 periods. This simulation uses the optimal asset accumulation rules of the MPE and the

Markov chain of the exogenous shocks. The impulse response functions show responses to Cholesky one-standard-deviation, positive innovations to the tax and GDP processes (starting from initial conditions equal, by construction, to the averages of public debt and foreign assets). Figures 8 and 9 also include asymptotic two-standard-error bands for each variable's impulse response.

Figure 8 shows the impulse responses of the $c/g$ ratio for tax and output shocks. The plots are truncated at 20 periods, even though if plotted for a sufficiently long sample, the two of them return to the zero line because of the mean-reverting nature of the model's stochastic steady state. The positive output shock has a small impact effect that lowers the $c/g$ ratio below its long run average. In contrast, the positive tax shock has a negative impact effect on the $c/g$ ratio that is more than 4 times larger than that for the output shock, and this effect grows to be nearly 6 times larger by the 6th period, before beginning its slow-moving reversion to the zero line. These results reflect the fact that the positive output shock increases the incomes of both the private and public sector, but the tax shock moves the two agent's incomes in opposite directions. Hence, the tax shock implies larger foregone opportunities for efficient risk sharing because of the incompleteness of asset markets. Government revenues rise while private disposable income falls, and this opposing moves result in persistent differences in expenditure patterns that imply relatively higher (lower) government (private) expenditures for a long period of time. In contrast, the domestic social optimum would imply a constant $c/g$ ratio, which in the plots of Figure 8 corresponds to the zero line.

Figure 9 provides further details on the impulse response functions of the rest of the model's endogenous variables to a positive tax shock. Public debt and net foreign assets exhibit highly persistent declines. Private consumption declines on impact and then recovers very gradually, while public expenditures rise on impact and continue to increase until they level off about 6 periods after the tax shock hits. The current account falls as private agents borrow to moderate the consumption effect of the adverse shock to disposable income. Conversely, the primary fiscal balance increases on impact and then falls gradually, as the government reacts to its positive revenue shock by saving to transfer part of its extra income for future expenditures. The persistence of the effects shown in the impulse response functions of assets and expenditures illustrate once more the wealth effects induced by the incompleteness of asset markets. The tax shock plays no role in the domestic social optimum, since the social planner pools the income of the public and private sectors, thus effectively providing full insurance against implied tax shocks, so again the zero line is the relevant point for comparing the responses of the competitive equilibrium with those of the social optimum.

Figure 10 shows the impulse-response functions that follow a positive, one-standard-deviation innovation to the output process in the competitive equilibrium and the domestic social optimum. The output shock is non-insurable risk for both economies, because both have access to the same set of incomplete international asset markets. Hence, the output shock does not result in large differences in the responses of macroeconomic variables. Still, the plots show that the wealth effects of the asset market incompleteness are not exactly identical in the two economies. In particular, private consumption (public consumption) is always lower (higher) in the competitive equilibrium than in the social optimum. Note also that public expenditures increase on impact, illustrating again the procyclical pattern of government purchases predicted by the model.

## 3.3 Welfare Costs of the Tormented Insurer's Problem

This section compares the competitive equilibrium allocation and welfare resulting from the MPE with those produced by the domestic social optimum. Welfare comparisons are conducted following the approach introduced by Lucas (1987), which is based on computing compensating variations in time- and state-invariant (i.e., stationary) consumption levels that represent particular levels of expected lifetime utility under alternative environments (in this case, between the MPE and the domestic social optimum). The aim is to convert the ordinal units of the payoff functions into cardinal measures that can be used for quantitative welfare comparisons.

To construct the welfare measure for the domestic social optimum, define $\check{c}^{SO}(b^g, b^I, \epsilon)$ as a stationary consumption function that represents the *collective* welfare of the public and private sectors. This welfare measure satisfies:[9]

$$\frac{\check{c}^{SO}(b^g, b^I, \epsilon)^{1-\sigma}}{(1-\sigma)\,(1-\beta)} = V^{so}(b^g, b^I, \epsilon)$$

A comparable measure of *collective* welfare of the two sectors in the MPE is constructed by defining the stationary consumption function $\bar{c}^{MPE}(b^g, b^I, \epsilon)$ that satisfies the following condition:

$$\frac{\bar{c}^{MPE}(b^g, b^I, \epsilon)^{1-\sigma}}{(1-\sigma)(1-\beta)} = (1-\eta)V(b^g, b^I, \epsilon) + \eta W(b^g, b^I, \epsilon)$$

where $V$ and $W$ are the value functions defined in (4) and (5). There are also stationary consumption functions that measure the *individual* welfare of each sector, and these are given by the function $\bar{g}(b^g, b^I, \epsilon)$ that yields the same payoff as $V(b^g, b^I, \epsilon)$ and the function $\bar{c}(b^g, b^I, \epsilon)$ that yields the same payoff as $W(b^g, b^I, \epsilon)$.

The key variable to evaluate the welfare costs of the domestic asset market incompleteness is the percentage difference bewteen $\check{c}^{SO}(b^g, b^I, \epsilon)$ and $\bar{c}^{MPE}(b^g, b^I, \epsilon)$. The minimum and maximum differences across the 360,000 triples $(b^g, b^I, \epsilon)$ in the state space are 0.07 and 41.45 percent respectively. These figures do not mean much, however, because they do not take into account the long-run probability of the particular coordinate in the state space that produced them. To address this issue, it makes more sense to evaluate welfare effects by comparing expected welfare costs computed using the limiting distributions of the MPE or the domestic social optimum. Using the limiting distribution of the MPE, the expected welfare cost is:

$$\sum_{(b^g, b^I, \epsilon)\in B^g\times B^I\times s} \Pi_{\infty}^{MPE}(b^g, b^I, \epsilon)\left(\frac{\check{c}^{SO}(b^g, b^I, \epsilon)}{\bar{c}^{MPE}(b^g, b^I, \epsilon)} - 1\right)$$

where $\Pi_{\infty}^{MPE}(b^g, b^I, \epsilon)$ is the endogenous ergodic distribution of public debt, foreign assets and the shocks in the MPE. The second alternative measure of expected welfare costs uses the limiting distribution of the social optimum, $\Pi_{\infty}^{SO}(b^g, b^I, \epsilon)$, instead of $\Pi_{\infty}^{MPE}(b^g, b^I, \epsilon)$. A problem with this second measure is that these probabilities are not defined for the grid $B^g$,

[9] These welfare-equivalent stationary consumption levels are computed for each pair $(b^I, \epsilon^y)$ in the social optimum, and for each triple $(b^g, b^I, \epsilon)$ in the MPE. To express the stationary consumption levels of the social optimum in terms of the state triples of the MPE, note that neither $b^g$ nor $\epsilon^\tau$ affect the social planner's problem so that $\check{c}^{SO}(b^g, b^I, \epsilon)$ could equivalently be defined as $\check{c}^{SO}(b^I, \epsilon^y)$.

since public debt does not enter into the social planner's problem. To make this probability measure cover the state space for all triples $(b^g, b^I, \epsilon)$, the probability of each pair $(b^I, \epsilon)$ is distributed evenly across each of the $NBG$ elements of the $B^g$ grid.

The average welfare costs yield striking results. Using the ergodic distribution of the domestic social optimum, the benefits of eliminating the tormented insurer's problem are equivalent to an increase of 3.52 percent in the trend level of consumption per capita. If we use the ergodic distribution of the MPE instead, the increment in trend consumption per capita reaches 1.57 percent. These welfare gains from improving risk sharing dwarf the negligible measures of the benefits of consumption smoothing and risk sharing obtained in standard RBC models, and are comparable with those obtained in the quantitative analysis of the efficiency gains of replacing capital income taxes with consumption taxes (see, for example, Lucas (1996), Mendoza (1991), Mendoza and Tesar (1998)). The main reason for the marked difference with previous measures of the welfare gains of risk sharing is that the tormented insurer's problem deviates sharply from the representative agent environment of the standard RBC models. In the competitive equilibrium examined here, the income process of the government is significantly different from that faced by the private sector, and the two agents can only used non-state-contingent debt to smooth consumption and self-insure. As a result, the ratio of $c_t/g_t$ (i.e., the proxy for shifts in the distribution of wealth between the private and public sectors) fluctuates widely over the business cycle, as documented earlier, and the fluctuations in government expenditures are particularly costly (since the average of $g_t$ is nearly 7 times smaller than the average of $c_t$, and hence the curvature of the CRRA payoff function, with the same $\sigma$ parameter, is more pronounced and yields larger utility changes for the fluctuations in $g_t$).

Table 4 compares the long-run moments of macroeconomic aggregates in the MPE and the social optimum to offer more insight on the determinants of the large welfare costs of imperfect risk sharing in the model. The mean values of the variables are almost identical in the two economies, so the welfare costs do not arise because the social planner has access to resources not available to the agents in the competitive equilibrium. The welfare costs arise instead because, when the government can access state-contingent debt or taxes, the ability to implement perfect risk pooling results in public and private expenditure allocations that fluctuate much less than when the economy lacks state-contingent fiscal instruments. As Table 4 shows, the standard deviation of public expenditures (private consumption) is 13.5 (2.3) times higher in the competitive equilibrium than in domestic social optimum.

## 3.4 The Revenue Process and Public Debt Dynamics

The properties of the stochastic process of public revenue are a key determinant of the equilibrium dynamics of public debt that the model produces. The significantly larger effects of the tax shock compared to those of the GDP shock found in the impulse response analysis already illustrate this fact. We explore further the relationship between public debt and the revenue process by analyzing the model's ability to account for the inverse relationship between average public debt ratios and coefficients of variability of public revenues found in the data (recall Figure 3). The model has the potential to account for this pattern of the data because higher revenue variability reduces the mean debt ratio, as it strengthens the incentives for the government to build precautionary savings given the incompleteness of asset markets. This is a result well-known in the precautionary savings literature. Chapter 14 of Ljungqvist and Sargent (2000) shows, for example, that the concave curve mapping the relationship between mean asset holdings and the risk free interest rate in the Aiyagari-Hugett class of models shifts to the right as the variance of endowment shocks increases because of stronger incentives for precautionary savings by private agents.

Figure 3 shows a set of 45 pairs of exogenous coefficients of variation of public revenues and endogenous mean public debt ratios in the model's competitive equilibrium produced as numerical solutions to variations from the baseline calibration of the Markov process of shocks.[10] These model pairs are identified by asterisks in the scater diagram of Figure 3, and the continuous curve represents the corresponding logartihmic regression line. As the figure shows, the model is consistent with the data in predicting that (a) the long-run average debt ratio is a negative function of the variability of fiscal revenues, and (b) this relationship is non-linear, with the ability to sustain average debt ratios declining at a faster

[10] The 45 coefficients of variation of public revenues were generated by applying Tauchen's algorithm to the 45 combinations of 5 values of the autocorrelation coefficient of taxes in the VAR ($\hat{\Phi}_{(2,2)} = ...$) and 9 values of the variance element of tax innovations ($\hat{\Sigma}_{\epsilon(2,2)} = ...$).

rate as the coefficient of variation of public revenues increases. A comparison of the model's regression line with that produced with actual data shows, however, that the model's mean debt-revenue variability curve is steeper than that observed in the data.

The negative relationship between the variability of government revenues and average debt holds even when the government's debt limit remains unaltered. Consider, for example, an experiment in which the autocorrelation coefficient of implied taxes of the VAR (i.e., the element $\hat{\Phi}_{2,2}$ of the matrix $\hat{\Phi}$) increases to 0.9 from 0.6 in the baseline calibration.[11] With $\hat{\Phi}_{2,2}$ =0.6, the Markov representation of the VAR yields a coefficient of variation of fiscal revenue of 6.13, while $\hat{\Phi}_{2,2}$ =0.9 yields a coefficient of variation of revenue of 6.68. The natural debt limit remains constant at $\phi^g = 1.318$, but the long-run average of the public debt ratio falls by 12 percentage points from 53 to 41 percent.

# 4 Concluding Remarks

Fiscal policy in developing countries, particularly in emerging economies, differs sharply from that of industrial countries in three key respects: (1) public revenue-GDP ratios are much smaller and significantly more volatile, (2) countries with more variable revenue ratios support lower average debt ratios, and (3) fiscal policy follows acyclical or procyclical patterns, with GDP correlations of the primary balance (government expenditures) close to zero or slighlty negative (positive).

This paper proposes a model of fiscal policy in small open economies with incomplete asset markets that can rationalize these facts. The model shifts the focus from the study of Ramsey optimal taxation problems toward the study of optimal debt and expenditure policies in an environment where the government tends to act as a "tormented insurer," seeking to keep payments to the private sector smooth despite low and volatile revenues and debt markets limited to non-state-contingent debt. The competitive equilibrium that characterizes the interaction between this tormented insurer and the private sector of the

[11] Note that, using Tauchen's method to generate the Markov representation of the VAR, changes in the autocorrelation matrix $\hat{\Phi}$ change the transition probabilities of the Markov but not the vector of realizations of the shocks. Since the natural debt limit on public debt depends on the worst revenue realization, changes in $\hat{\Phi}$ do not change the government's debt limit even though they change the coefficient of variation of fiscal revenues.

economy is modeled in a dynamic, stochastic, general equilibrium framework and solved numerically as a Markov perfect equilibrium.

Domestic asset market incompleteness has important implications for welfare and for optimal public debt and government expenditure choices. The welfare costs arising from this form of market incompleteness are derived by comparing the MPE with the domestic social optimum attained by a planner who can pool the fiscal risk, and thus equate the marginal utilities of public and private expenditures across states and over time. The average welfare costs of imperfect domestic risk sharing are large, with an average gain of 3.5 percent in a utility-equivalent compensating variation in stationary consumption calculated with the limiting distribution of the MPE, or 1.6 percent if the stationary distribution of the social optimum is used instead. Costs of these magnitude dwarf the negligible costs of imperfect risk sharing and cyclical variability of consumption obtained with conventional RBC models.

The model is able to explain why countries suffering from higher fiscal risk support lower average public debt-GDP ratios. In particular, the model is consistent with international data in producing a negative, non-linear relationship between the variability of fiscal revenues and the average public debt ratios. The model also matches the GDP-correlations of government expenditures and the primary fiscal balance found in data for Mexico.

The implications of a modification of the MPE in which the government shifts from the implied tax on GDP to a non-state-contingent consumption tax are studied in an ongoing extension of this paper. This modification adds complexity to the MPE by introducing two-way feedback between the plans of the private and public sector via an endogenous tax base determined by the private sector's consumption plans. The presumption is that the consumption tax may be more desirable from a social welfare perspective because private consumption smoothing can act as an endogenous stabilizer that yields more stable fiscal revenues than the implied tax (i.e., than commodity export revenues). In turn, reduced revenue variability implies more access to debt for the government and improved ability to self-insure and smooth expenditures.

# Appendix: Markov Chain for GDP and Implied Tax Rate

The Markov chain representing the dynamics of the exogenous state variables, aggregate income (or GDP) and implied tax rates, is characterized by the $(9 \times 9)$ state-transition matrix $P$, and the $(9 \times 2)$ matrix $\mathcal{E}$ containing all possible realizations of the income and tax shocks, $\epsilon = (\epsilon^y, \epsilon^\tau)$. These two matrices along with the vector $\pi_\infty^\mathcal{E}$ that represents the unconditional stationary probability distribution of the $\epsilon$ pairs are shown next:

$$P = \begin{pmatrix} 0.1853 & 0.3210 & 0.0348 & 0.1496 & 0.2592 & 0.0281 & 0.0075 & 0.0131 & 0.0014 \\ 0.0988 & 0.3950 & 0.0988 & 0.0652 & 0.2609 & 0.0652 & 0.0027 & 0.0108 & 0.0027 \\ 0.0412 & 0.3807 & 0.2197 & 0.0223 & 0.2057 & 0.1187 & 0.0008 & 0.0069 & 0.0040 \\ 0.0464 & 0.0804 & 0.0087 & 0.2268 & 0.3929 & 0.0425 & 0.0693 & 0.1201 & 0.0130 \\ 0.0278 & 0.1111 & 0.0278 & 0.1111 & 0.4444 & 0.1111 & 0.0278 & 0.1111 & 0.0278 \\ 0.0130 & 0.1201 & 0.0693 & 0.0425 & 0.3929 & 0.2268 & 0.0087 & 0.0804 & 0.0464 \\ 0.0040 & 0.0069 & 0.0008 & 0.1187 & 0.2057 & 0.0223 & 0.2197 & 0.3807 & 0.0412 \\ 0.0027 & 0.0108 & 0.0027 & 0.0652 & 0.2609 & 0.0652 & 0.0988 & 0.3950 & 0.0988 \\ 0.0014 & 0.0131 & 0.0075 & 0.0281 & 0.2592 & 0.1496 & 0.0348 & 0.3210 & 0.1853 \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} -0.0467 & -0.0655 \\ 0.0000 & -0.0827 \\ 0.0467 & -0.0999 \\ -0.0467 & 0.0172 \\ 0.0000 & 0.0000 \\ 0.0467 & -0.0172 \\ -0.0467 & 0.0999 \\ 0.0000 & 0.0827 \\ 0.0467 & 0.0655 \end{pmatrix}; \quad \pi_\infty^\mathcal{E} = \begin{pmatrix} 0.0397 \\ 0.1477 \\ 0.0434 \\ 0.0968 \\ 0.3448 \\ 0.0968 \\ 0.0434 \\ 0.1477 \\ 0.0397 \end{pmatrix}$$

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Table 1: Calibration of the Model to the Mexican Economy

| Notation | Parameter / Variable | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.925 |
| $\gamma$ | Gross growth rate | 1.036 |
| $\phi^g$ | Natural debt limit on public debt | 1.318 |
| $\phi^I$ | Ad-hoc debt limit on international debt | -0.500 |
| $\sigma$ | Coefficient of relative risk aversion | 2.000 |
| $\tau$ | Mean income-tax rate | 0.256 |
| $R$ | Gross world interest rate | 1.0986 |
| $x$ | Private investment expenditures | 0.226 |
| $z$ | Government transfers | 0.111 |
| | | |
| | Minimum value government debt | 0.000 |
| | Maximum value of international assets | 0.100 |

Table 2: Unconditional Moments of GDP and the Implied Tax Rate. Mexican Annual Data, 1980-2004 and Unconditional Moments of the Markov Chain

| | Mexican data (a) | | Markov chain (b) | |
|---|---|---|---|---|
| Statistic | GDP | Implied tax rate | GDP | Implied tax rate |
| Standard deviation | 0.02948 | 0.06027 | 0.02781 | 0.05689 |
| Minimum | -0.07073 | -0.12294 | -0.04670 | -0.09991 |
| Maximum | 0.05018 | 0.01080 | 0.04670 | 0.09991 |
| Cross correlation | -0.24172 | -0.24172 | -0.19786 | -0.19786 |
| Autocorrelation | 0.351 | 0.535 | 0.278 | 0.576 |

Table 3: Moments of Macroeconomic Aggregates in the Ergodic Distribution of the Markov Perfect Equilibrium (Baseline Calibration)

| Variable ($x$) | E[$x$] | $\sigma(x)$ | cv($x$) | $\rho(x)$ | $\rho(x, y_i)$, where $y_i =$ GDP | after tax inc. | fiscal revenue |
|---|---|---|---|---|---|---|---|
| GDP | 1.00 | 2.80 | 2.80 | 0.28 | 1.00 | 0.86 | 0.28 |
| GNP | 0.97 | 3.03 | 3.13 | 0.42 | 0.95 | 0.82 | 0.26 |
| After-tax income | 0.74 | 2.78 | 3.73 | 0.38 | 0.86 | 1.00 | -0.25 |
| Consumption | 0.64 | 3.20 | 4.97 | 0.97 | 0.20 | 0.24 | -0.07 |
| Gov. expenditures | 0.10 | 2.83 | 29.07 | 1.00 | 0.02 | -0.04 | 0.12 |
| Tax rate | 0.26 | 1.47 | 5.73 | 0.58 | -0.20 | -0.68 | 0.88 |
| Fiscal revenue | 0.26 | 1.50 | 5.86 | 0.53 | 0.28 | -0.25 | 1.00 |
| Primary fiscal balance | 0.05 | 3.05 | 64.47 | 0.90 | 0.12 | -0.09 | 0.38 |
| Current account | 0.00 | 2.27 | - | 0.24 | 0.96 | 0.80 | 0.31 |
| Trade balance | 0.03 | 2.55 | 79.43 | 0.34 | 0.82 | 0.68 | 0.28 |
| Public debt | 0.53 | 30.51 | 57.37 | 1.00 | 0.00 | 0.02 | -0.05 |
| International assets | -0.36 | 10.72 | -29.72 | 0.98 | 0.08 | 0.08 | 0.00 |

Note: For each variable $x$, E[$x$] is the mean, $\sigma(x)$ is the percentage standard deviation, cv($x$) is the percentage coefficient of variation, $\rho(x)$ the autocorrelation, and $\rho(x, y_i)$ is the cross-correlation with the variable $y_i$ indicated in the heading of the columns.

Table 4: Comparison of Limiting Moments of Macroeconomic Aggregates: Markov Perfect Equilibrium versus Social Optimum

| | Markov Perfect Equilibrium | | | | | Social Optimum | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable ($x$) | E[$x$] | $\sigma(x)$ | cv($x$) | $\rho(x)$ | $\rho(x, y)$ | E[$x$] | $\sigma(x)$ | cv($x$) | $\rho(x)$ | $\rho(x, y)$ |
| GDP | 1.00 | 2.80 | 2.80 | 0.28 | 1.00 | 1.00 | 2.80 | 2.80 | 0.28 | 1.00 |
| GNP | 0.97 | 3.03 | 3.13 | 0.42 | 0.95 | 0.97 | 3.01 | 3.11 | 0.41 | 0.96 |
| Consumption | 0.64 | 3.20 | 4.97 | 0.97 | 0.20 | 0.64 | 1.39 | 2.16 | 0.89 | 0.46 |
| Gov. expenditures | 0.10 | 2.83 | 29.07 | 1.00 | 0.02 | 0.10 | 0.21 | 2.16 | 0.89 | 0.46 |
| Current account | 0.00 | 2.27 | - | 0.24 | 0.96 | 0.00 | 2.25 | - | 0.23 | 0.95 |
| Trade balance | 0.03 | 2.55 | 79.43 | 0.34 | 0.82 | 0.03 | 2.51 | 75.35 | 0.33 | 0.82 |
| International assets | -0.36 | 10.72 | -29.72 | 0.98 | 0.08 | -0.37 | 10.01 | -26.76 | 0.97 | 0.08 |

Note: For each variable $x$, E[$x$] is the mean, $\sigma(x)$ is the percentage standard deviation, cv($x$) is the percentage coefficient of variation, $\rho(x)$ the autocorrelation, and $\rho(x, y)$ is the cross-correlation with GDP.

Figure 1: Average Public Revenue-GDP Ratios in Emerging and Industrial Countries: 1990-2002

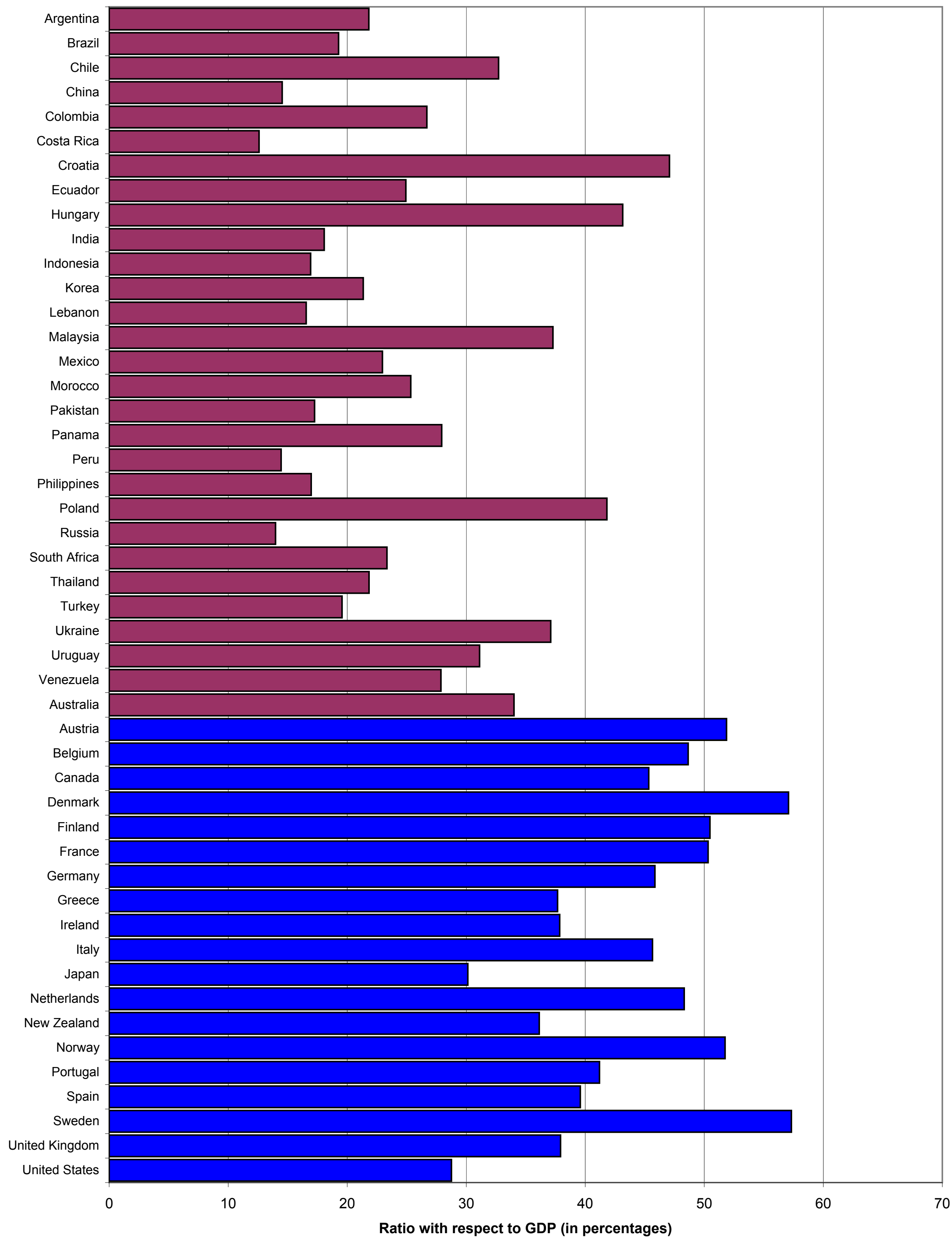


Figure 2: Coefficients of Variation of Public Revenue-GDP Ratios in Emerging and Industrial Countries: 1990-2002

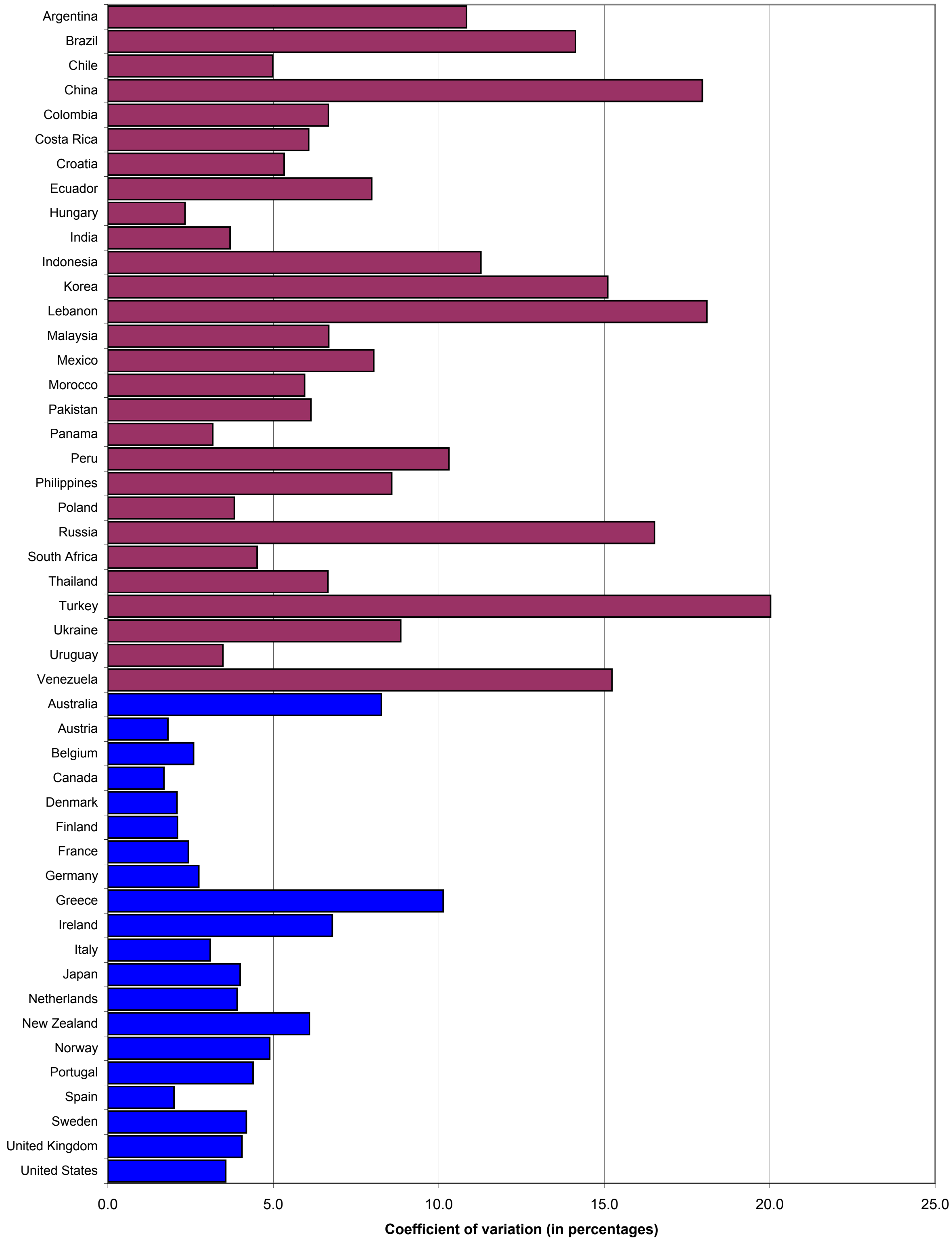


Figure 3: Volatility of Fiscal Revenues and Average Public Debt Ratios

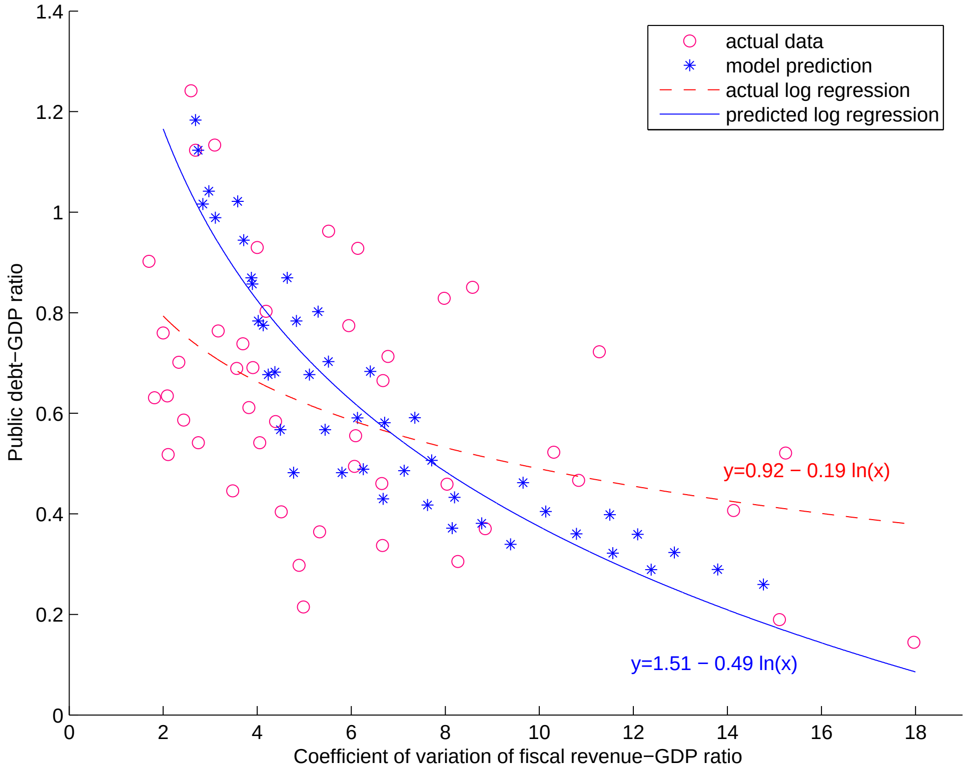


Figure 4: Carroll's Condition, Discount Factor and Mean Value of Public Debt-GDP Ratio

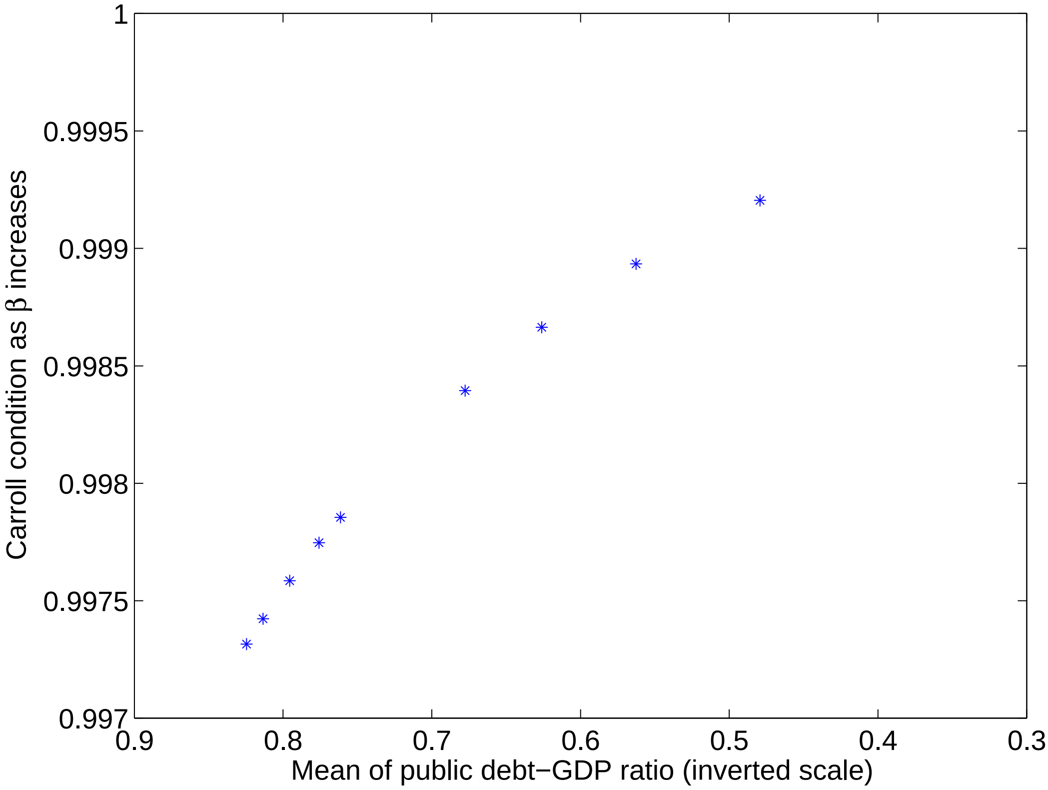
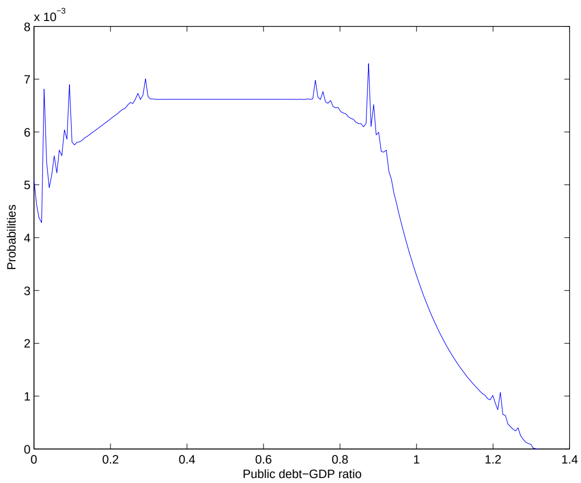
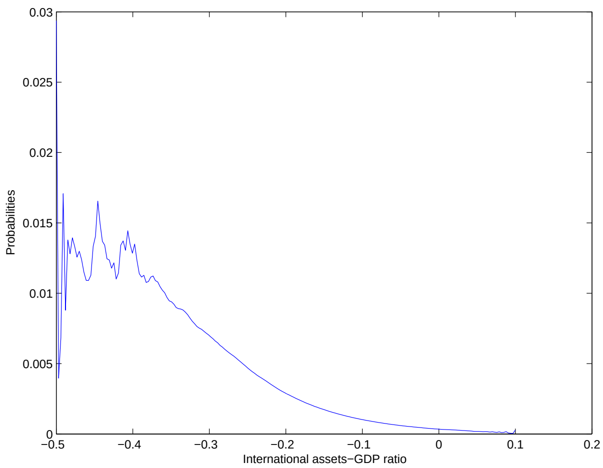



Figure 5: Marginal Distribution of Public Debt and International Assets in the Limiting Distribution of the MPE under Baseline Calibration

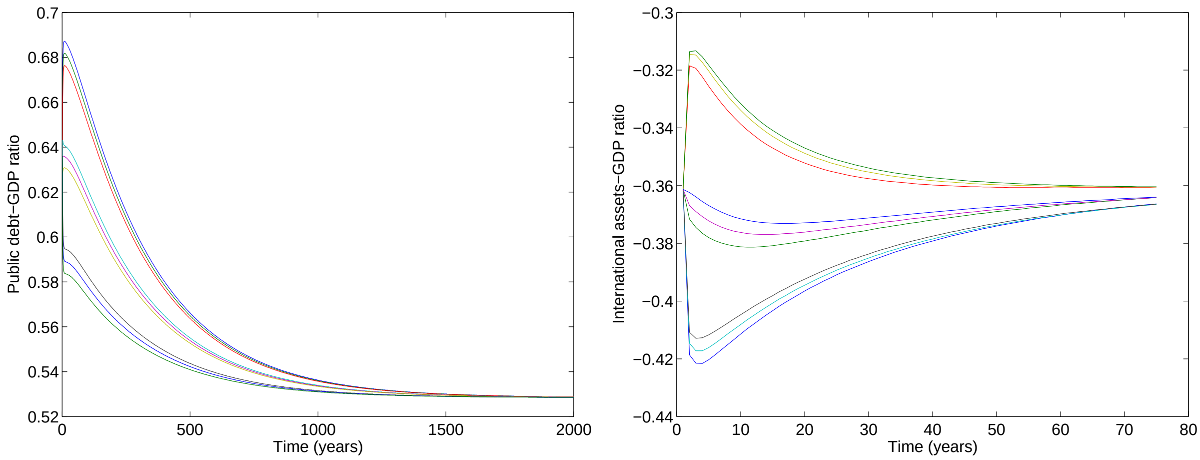



(a) Public Debt




(b) International Assets

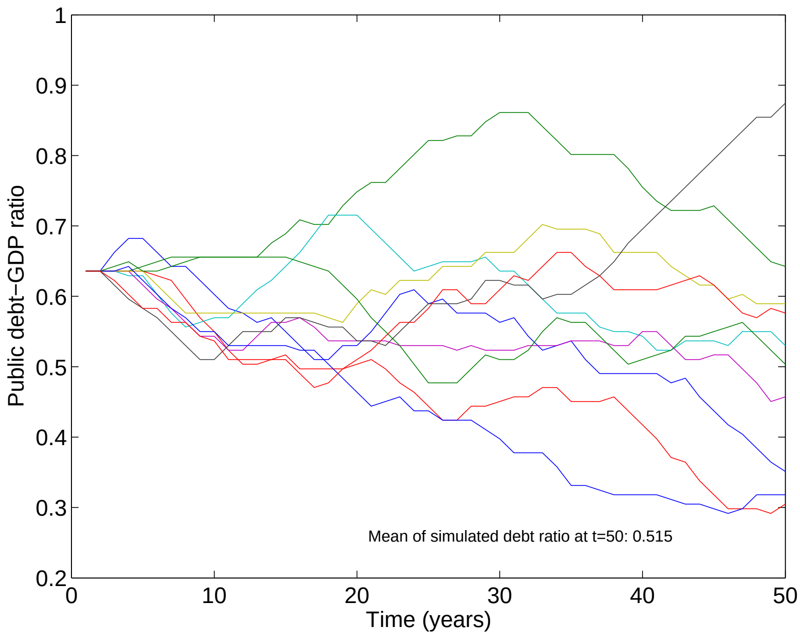
Figure 6: Forecasting Functions of Assets




(a) Public Debt (b) International Assets

Note: Starting value of public debt is equal to 0.634 (120% of its mean value in the limiting distribution); starting value of international assets is equal to its mean value in the limiting distribution.

Figure 7: Simulations of Public Debt Using Simulations of the Markov Chain and Optimal Policy Functions




Note: Starting value of public debt is equal to 0.634 (120% of its mean value in the limiting distribution).

Figure 8 Impulse Response Functions of the Consumption-Government Expenditures Ratio
(Cholesky One S.D. Innovations ± 2 S.E.)

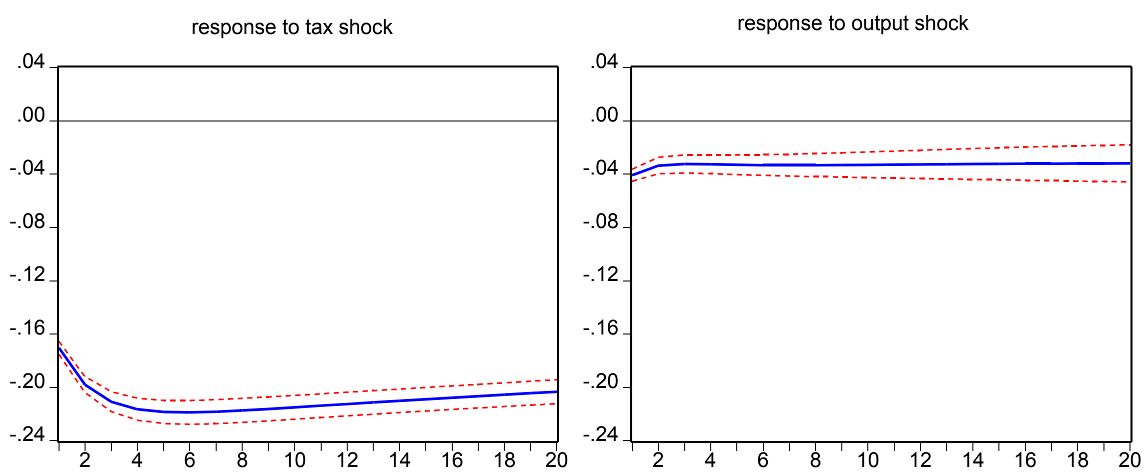


Figure 9 Impulse Response Functions to Tax Shock in the Competitive Equilibrium
(Cholesky One S.D. Innovations ± 2 S.E.)

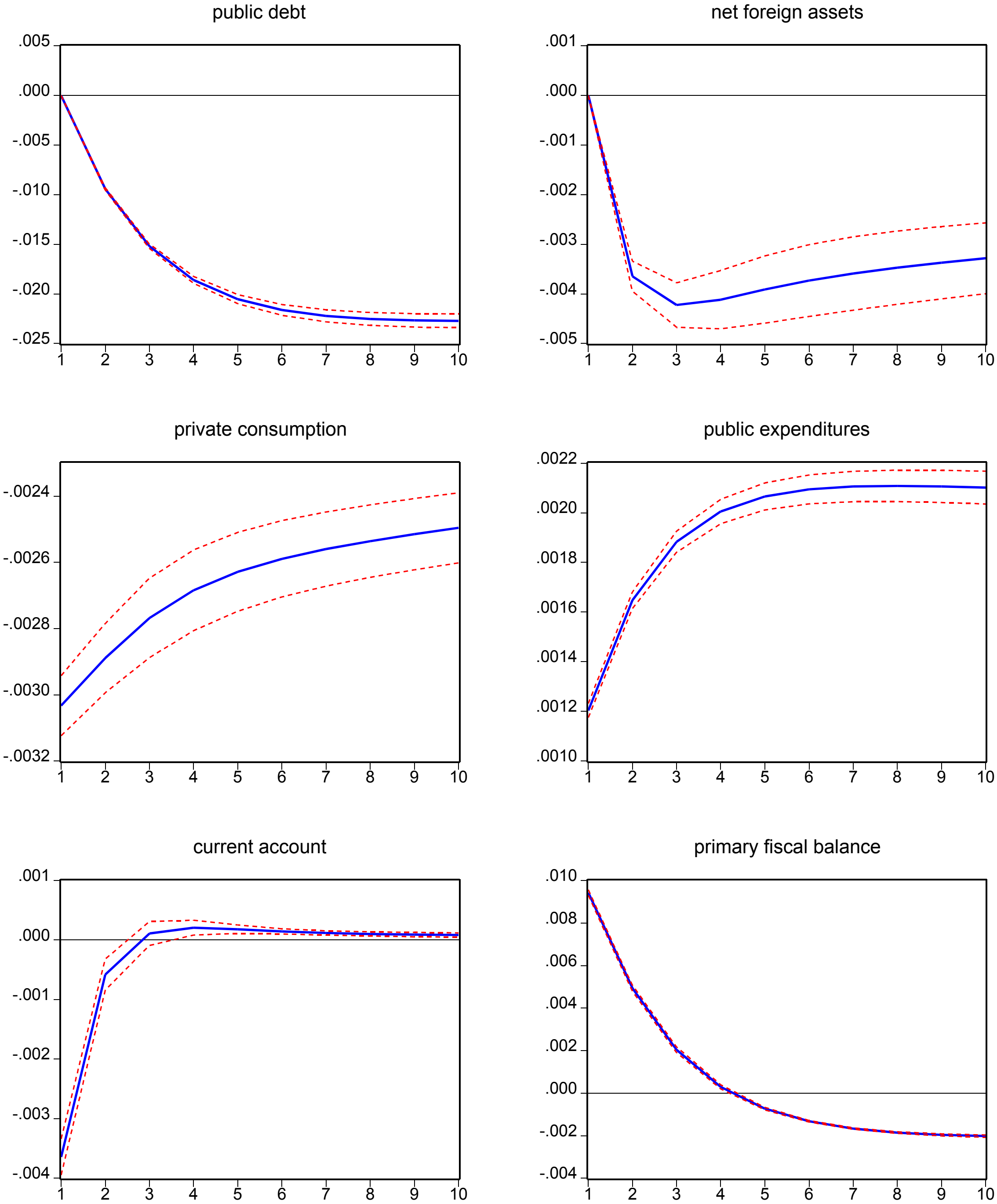


Figure 10 Impulse Response Functions to an Output Shock in the Competitive Equilibrium and the Domestic Social Optimum (responses to Cholesky One S.D. Innovations)

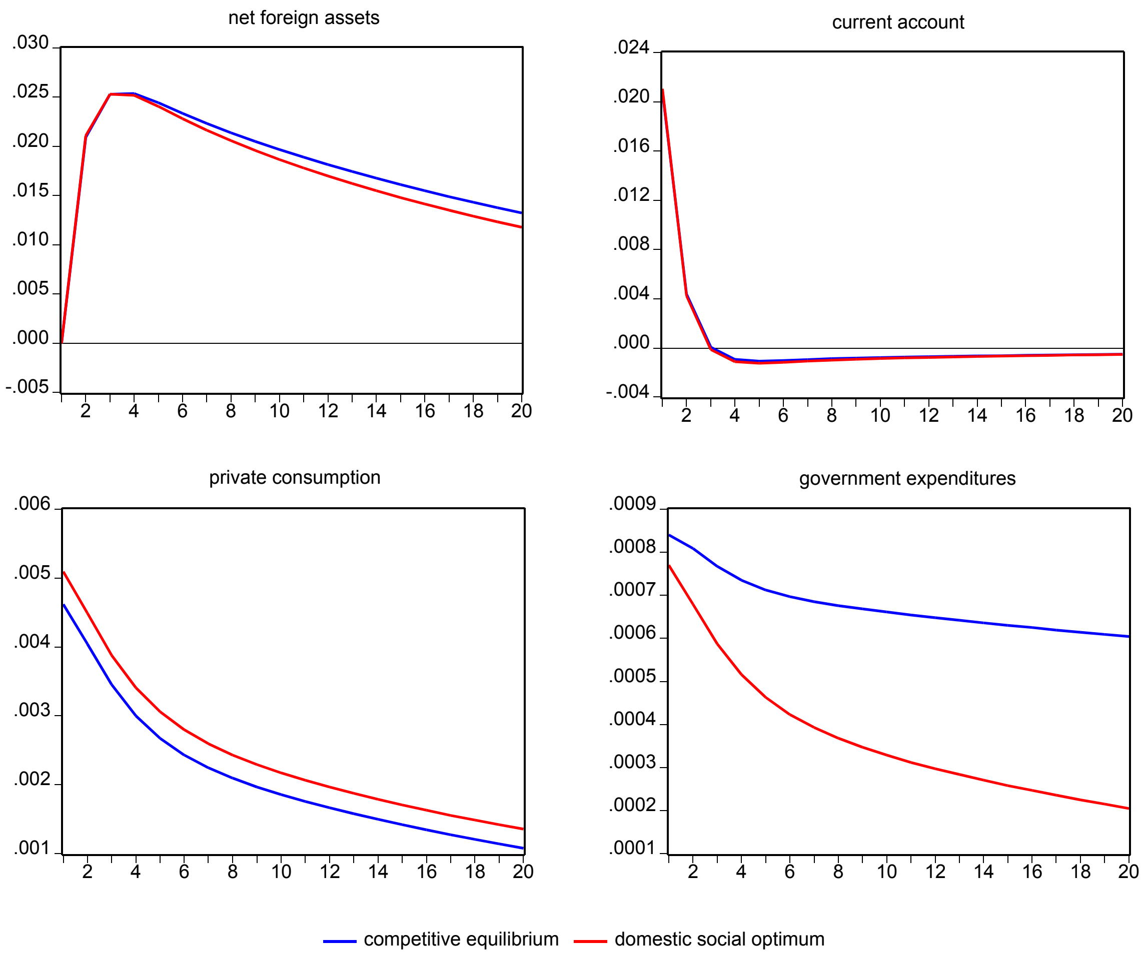


Figure 11: Mean of Implied Income-Tax Rate and Cross Correlation between Government Expenditures and Fiscal Revenue