# Firms' Heterogeneous Sensitivities to the Business Cycle, and the Cross-Section of Expected Returns

François Gourio*

February 15, 2006

Abstract

**Note to the reviewer:** I am currently in the middle of a major revision of this paper. That is why in some places the paper is unfinished. Since this paper is due to be presented at a conference on firm dynamics in late April, I believe it will be greatly improved by the Summer.

**Abstract:** In this paper, I propose and test a simple technology-based theory of firms' sensitivities to aggregate shocks. I show that when the elasticity of substitution between capital and labor is below unity, low profitability firms are more sensitive to aggregate shocks, i.e. to the business cycle. Since the wage is smoother than productivity, revenues are more procyclical than costs, making profits, the residual procyclical. Firms with low profitability are more procyclical since the residual is smaller and the amplification greater. I study the asset pricing implications of this technology and find that it can explain the riskiness of small and "value" firms (Fama and French 1996). These firms are less profitable and are thus more procyclical. I find empirically that the cross-section of expected returns is well explained by differences in sensitivities of firms' earnings to GDP growth, or by differences in profitability. The model yields rich empirical implications by linking a firm's real behavior (the elasticity of output, employment and profits to an aggregate shock) to its financial characteristics (the firm's betas and its average return). I next embed my partial equilibrium model in a full DSGE model to conduct a GE analysis. Empirically I show that firms with low margins are indeed more sensitive to the business cycle in their employment, sales or profits.

Keywords: Cross-Section of Returns, Book-to-Market, Value Premium, Productivity Heterogeneity, Production-Based Asset Pricing.

JEL codes: E44, G12

*Boston University, Dept of Economics, 270 Bay State Road, Boston MA 02215. Email: fgourio@bu.edu. This paper is a major revision of one chapter of my University of Chicago dissertation, and is related to the cross-sectional results of my job market paper, "Operating Leverage, Stock Market Cyclicality, and the Cross Section of Returns". I am grateful to many seminar audiences and people for their comments, and especially to my dissertation committee Lars Hansen, Fernando Alvarez, John Cochrane, and Anil Kashyap.

# 1 Introduction

Is the business cycle a matter of all firms moving up and down in parallel, or do some firms react more to aggregate shocks? This paper studies the heterogeneity of firm sensitivities to the business cycle. There are two main motivations.

First, a large empirical literature in finance documents that some stocks earn high expected returns. This presumably reflects a compensation for risk. Since risk is driven by aggregate shocks, we need to explain why some firms are more sensitive to aggregate shocks, i.e. to the business cycle. But there is little work analyzing what real characteristics of the firms drive this sensitivity. Where does a stock's beta come from? The interest in this question is compounded by the finding by Fama and French (1992) that firms with high ratios of book value to market value have high average returns. The economic interpretation of their finding has remained elusive. Is book-to-market really an indicator of firm riskiness, if so why?

The second, more fundamental motivation, is that macroeconomists often use the fiction of a representative firm (i.e. an aggregate production function). But if some firms are more sensitive to the business cycle, as suggested by the empirical finance literature, this may be a poor approximation. In particular, some "marginal firms" would account for a large share of the variation in GDP or employment, and would play a determinant role in the business cycle. This could change the standard view of the business cycle.

In this paper, I propose and test a simple technology-based theory of firms' sensitivities to aggregate shocks. Firms differ in productivity, and low productivity firms are more sensitive to aggregate shocks, especially in terms of their earnings. The key empirical finding, summarized in Figure 1, is that the cross-sectional variation in expected returns is well explained by differences of earnings sensitivities to the business cycle. It is also well explained by differences in profitabilities. This is a solution to the long-standing puzzle, why are "value" stocks risky? The heterogeneity in sales or employment response is smaller than for earnings, but it is still significant.

The simple theory that I use is based on the idea of "operating leverage" or "labor leverage".[1] Consider a firm which has a fixed factor (capital) and which must decide on how much of the variable factor (labor) to hire. This firm is subjected to idiosyncratic and aggregate shocks. Profits are the difference between revenues and costs. In response to an aggregate shock, revenues and costs will not react similarly. Costs will tend to react less because the wage, which is the

[1] While the corporate finance literature also mentions an "operating leverage" effect, it is quite different from the mechanism of this paper, because it relies on fixed costs.

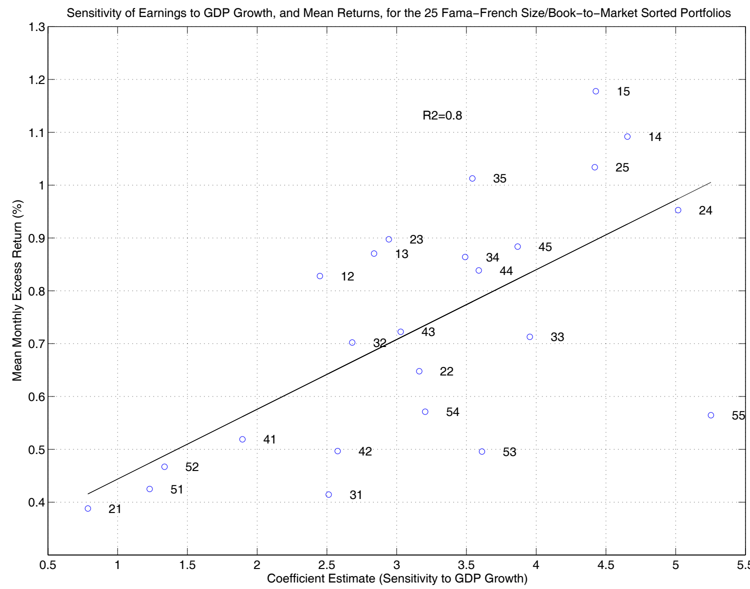



Figure 1: This figure plots, for each of the 25 portfolios of firms [sorted by size (market value) and book-to-market], the sensitivity of the earnings to GDP growth (estimated from a time-series regression) and the mean monthly excess return over the risk-free rate. See Section 4 for details.

cost of the most important input, is rather acyclical. Since costs move less than revenue, an increase in aggregate productivity will result in a more than one-for-one increase in profits. This amplification effect makes aggregate profits more cyclical than GDP; but the strength of the amplification differs across firms. Firms for which the average profit margin (i.e. the ratio of profit to output, or the capital share) is small will have a much higher amplification, since they "leverage" the fact that costs move less than revenues - costs are almost equal to revenues, so earnings increase greatly when (say) revenue rises 1% and costs 0.5%. This mechanism does not rely on costs being variable or fixed (the "old" operating leverage) but rather on productivity and input prices varying differently over the business cycle.

This explanation yields differences in sensitivities, to the extent that firms have different capital shares. I study the conditions under which heterogeneous capital shares arise. I find that this can occur either if the production function has an elasticity less than one, or if there are some fixed costs. The simple model thus links the differences in sensitivities to aggregate shocks with the literature on productivity heterogeneity. I develop the asset pricing implications with an exogenous pricing kernel: the model yields rich empirical implications by linking real behavior (the elasticity of output, employment and profits to an aggregate shock) with financial characteristics (the firm's betas and its average return). This model can explain why firms with small market capitalization or low book-to-market are more risky: they have lower profitability, and as a result they tend to have more procyclical earnings. Hence their cash flows are more volatile and/or more correlated with the business cycle. The logic of asset pricing based on macroeconomic risk implies that they earn higher expected returns.

Next I build a full DSGE model to embed my model of Section 2. Despite a cross-section of firms and aggregate risk, the model is tractable through a simple aggregation: only two endogenous state variables are necessary. With this model, I can analyze these heterogeneous sensitivities in general equilibrium.

I test the empirical implications of my model using Compustat. I find that they are overall supported: firms with high margins have less procyclical earnings, sales and employment. Regarding the value and size premium, I show that in the data, the book-to-market ratio is systematically related across firms to productivity and operating leverage. I also show that the pattern of cyclicality of earnings is close to the one predicted by the model: high book-to-market firms are more sensitive to GDP and to labor compensation than low book-to-market firms, and these estimates are of the order of magnitude predicted by the model. This holds also for the 25 portfolios of Fama and French, as illustrated in figure 1.

**Outline of the Paper**

The next section relates the paper to the existing research. Section 2 analyzes a partial equilibrium model and computes risk premia with exogenous prices. Section 3 introduces a special form of the model of Section 2 in a full dynamic general equilibrium model. Section 4 presents empirical evidence supporting the model, and Section 5 concludes.

## Relation to the Literature

This paper is related to three main strands of the literature and it is interesting to motivate the paper by reference to each.

**The Success of Factor models and the Missing Piece**

A large empirical literature in finance uses parsimonious factor models to fit the cross-section of expected returns. In this literature, the challenge is to find macroeconomic variables which proxy for the marginal utility of wealth, and which also covary strongly with the ex-post returns of some stocks (e.g., value stocks, with high book-to-market). This challenge has been met, and several variables, related to labor income, the consumption-wealth ratio, and durables or housing consumption have been proposed.[2] However this literature has not made any attempt

[2] A partial list includes Lettau and Ludvigson (2001) for the consumption-wealth ratio, Santos and Veronesi

at answering the natural question: why should stocks with high book-to-market covary strongly with durables consumption growth (or housing, etc.). The covariance is measured in the data but it does not have a reasonable interpretation: what is it about value firms that make them covary more with durables or housing? This is clearly an important question from a theoretical point of view. From a practical point of view, since these factor models are often rather atheoretical and there is a risk of "fishing" for the successful factor model or "overfitting" the Fama-French portfolios, understanding if the estimated loadings (betas) on the factors are reasonable is an important question.

**Assets in Place vs. Growth Options: Some Doubts**

Following the seminal work of Berk, Green and Naik (1999) and Gomes, Kogan and Zhang (2003), a small literature has emerged that addresses the question of the sources of firm riskiness (Carlson Fisher and Giammarino (2005,2006), Cooper (2006), Gala (2005) and Zhang (2005)). These papers all emphasize that firms differ in the mix of *growth options* and *assets in place*, and that these two components may have different riskiness.[3] Of course, the mechanisms that these papers propose are quite different. But a common theme is that firms have profit opportunities and that the net present value of investment projects is strictly positive. Economically, most of these models have (i) no entry, and (ii) firms operate decreasing return to scale production functions subject to idiosyncratic shocks. For instance, in the impressive paper by Zhang (2005), the production function has sharply decreasing returns to scale ($y = k^{\alpha}$ with $\alpha = 1/3$, where it should be 1 under constant returns). The rents, or economic profits are thus very large.

It seems doubtful that there are so large rents for new investment. Even if these rents are significant, is it so clear that these "growth options" are less risky than "assets in place"? The empirical work has been sometimes confusing, measuring growth options directly by Tobin $q$ and thus not testing in any way if the characteristics, and the behavior, of the firms with high book-to-market is different than the one of firms with low book-to-market.

This skepticism motivates my model: due to free entry, the value of growth options is nil: competition between firms drives the NPV of investment completely to zero. Variation over time in the value of existing production units arise because of the "operating leverage"/"labor leverage"

(2005) for labor income, Piazessi, Schneider and Tuzel (2006) and Lustig and Van Nieuwerburgh (2005) for housing, Pakos (2005) and Yogo (2005) for durables.

[3] The more precise definition of the value of assets in place is the value of the firm under the (generally non optimal) policy of not investing in the future. The value of growth options is then the residual = value of following the optimal policy minus the value of the assets in place. (This, by the way, does not imply in principle that the value of assets in place is capital or the book value.)

mechanism outlined in the introduction.

**Empirical Evidence on Heterogeneity in productivity and profitability**

One motivation for this paper is the empirical finding of a large cross–sectional heterogeneity in labor productivity (See Bartelman and Doms (2001) for a survey of the relevant empirical literature). Typically, the ratio of the labor productivity of the 25th centile producer to the 75th centile producer is about 2. The ratio of the labor productivity of the 90th centile producer to the 10th centile producer is about 4. If one uses TFP instead of labor productivity, the productivity differentials are now smaller, respectively about 1.4 and 2. (All these numbers are drawn from Syverson (2004) table 1.) Typically this literature has found that controlling for observables such as vintage or capital intensity does not explain the major share of productivity heterogeneity. My Section 2 asks under which conditions this heterogeneity in productivity can arise, given that firms can run their marginal cost up if they are more productive.

## 2 Partial Equilibrium Analysis of the Production Unit Problem and Value

In this section, I use a simple partial equilibrium model to examine how profits, sales, and employment vary for a production unit operating a constant return to scale technology with fixed capital. This section uses a simple model to show that the "labor leverage" mechanism does not rely on the lack of flexibility of the technology - the labor factor can be fully adjustable -, or on fixed costs, but rather on the fact that revenues and costs have different sensitivities to aggregate shocks.

The mechanism *does* require that there is heterogeneity in capital shares (aka profitabilities or operating margins), as in the data. However, the widely used Cobb-Douglas production function implies that capital shares are identical across firms, no matter what their productivity is. This is quite at odds with the data. I thus examine under which conditions firms with different productivity shocks have different capital shares. I find that this is true either if there are some fixed costs, or if the elasticity of substitution between capital and labor is less than unity. Under these assumptions, I show that low productivity firms have low capital shares and are more sensitive to aggregate shocks, especially in terms of earnings.

In Section 3, I insert a specific version of this firm model in a full dynamic stochastic general equilibrium model.

### A. Heterogeneous Sensitivities to Aggregate Shocks

**Setup**

Production takes place in units which are identical ex-ante but are hit upon creation and thereafter by productivity shocks. A unit operates a decreasing return to scales, labor-only production function subject to idiosyncratic and aggregate shocks: $y = zxF(k, n)$. Since capital is chosen before the shocks are realized, I assume it is the same in all units, which differ only by their $x$.

There are two possible interpretations of this technology:

- The first interpretation is that a project requires some irreversible capital investment, and that capital is fixed upon creation of the project. Think of Wal-Mart opening a new store in some location: the cost of building the store is probably largely both irreversible (sunk); moreover, whether the store is very successful or not, Wal-Mart will not probably increase its size. (Or in any case, this is not the interesting margin.) This interpretation, while attractive for some projects, can be problematic for others. For instance, a company experimenting with a new product would potentially adjust its scale and its capital stock very much in response to its success or failure.

- This leads to the second interpretation, which is that there are two types of capital: one which is fully adjustable and one which is not. Then one can handle the flexible capital just in the same way as I handle labor. Hence relabeling "flexible factors" for labor in the analysis below will yield

However I do need that some factor is fixed, or equivalently that there is some decreasing returns to scale, to avoid a degenerate distribution.[4]

Total operating income, or profits, or earnings, is:

$$\pi(z, x, k, w) = \max_{n \geq 0} \{zxF(k, n) - wn\}. \tag{2.1}$$

Since only the product $zx$ matters for the firm's decision, I will denote this $\pi(zx, k, w)$.

**Shocks**

For now, I do not make any specific assumption on the stochastic process governing the aggregate shocks $z$ or idiosyncratic shocks $x$. Later it will be useful to assume that $x$ is fixed and $\Delta \log z_t$ is a stationary process (so that I can use an infinite moving-average representation) to obtain some analytical results.

**Optimal employment decision**

Given current idiosyncratic and aggregate shocks $x$ and $z$, the unit chooses $n$ by equating the marginal product of labor and the market wage:

$$xzF'(k, n) = w.$$

The optimal labor demand takes the form $n = k \times g(xz/w)$. Let $y(xz, k, w) = zxkF(1, g(xz/w))$ be the total production (or sales), and let

$$s_K = \frac{\pi(xz, k, w)}{y(xz, k, w)} = \frac{\frac{zx}{w}F(1, g(xz/w)) - g(xz/w)}{\frac{zx}{w}F(1, g(xz/w))} \overset{def}{=} s_K(xz/w)$$

be the "profitability" i.e. the ratio of operating income (earnings) to sales. This is the share of output that is not paid to labor. This profit share is the capital share for the first interpretation (i.e. an ex-ante irreversible capital investment with constant returns). For the second interpretation, with decreasing returns to scale (or a fixed factor), it also contains a share of profits. I will refer to this number as the operating margin or profitability.

[4]If both factors are adjustable, there are differences in productivity, constant return to scales, and perfect competition, then only the unit with the highest $x$ would be operating. It would draw inputs from the whole economy. Factor adjustment costs would prevent this too.

The first result gives the response of each production unit to an aggregate productivity shock or to a change in the aggregate productivity $z$ or in the wage rate $x$.

**Result 1:** Let $\sigma$ be the (local) elasticity of substitution of $F$. Let $s_L$ be the share of output paid to labor. Then for all $x$,

$$\begin{aligned}
\frac{\partial \log n}{\partial \log z} &= \frac{xz/w}{g(zx/w)} g'(zx/w) = -\frac{F_n(k,n)}{nF_{nn}(k,n)} = \frac{\sigma}{s_K(xz/w)}, \\
\frac{\partial \log y}{\partial \log z} &= 1 + s_L \frac{\partial \log n}{\partial \log z} = 1 + s_L \frac{xz/w}{g(zx/w)} g'(zx/w) = 1 + \sigma \frac{s_L(xz/w)}{s_K(xz/w)}, \\
\frac{\partial \log n}{\partial \log w} &= -\frac{xz/w}{g(zx/w)} g'(zx/w) = -\frac{\sigma}{s_K(xz/w)}, \\
\frac{\partial \log y}{\partial \log w} &= -\frac{nF_n(k,n)}{F(k,n)} \frac{xz/w}{g(zx/w)} g'(zx/w) = -\sigma \frac{s_L(xz/w)}{s_K(xz/w)}.
\end{aligned}$$

The proof is simply obtained by differentiating labor demand and recalling that the elasticity of substitution $\sigma$ satisfies

$$\sigma = F_k(k,n)F_n(k,n)/F(k,n)F_{kn}(k,n).$$

This result shows that different production units (i.e. different $x$) will have different employment responses if and only if the labor demand $g$ has not a constant elasticity form. If $F$ has constant elasticity of substitution, differences in responses of employment or sales can occur only if the operating margin varies across firms. Of course we know that in the data, the operating margin varies significantly.

Of course in general equilibrium an increase in productivity will typically be associated with an increase in the wage. Putting these elasticities together, and for a given wage response to a productivity shock $\frac{\partial \log w}{\partial \log z}$, I infer the total responses of employment and output:

$$\begin{aligned}
\frac{d \log n}{d \log z} &= \frac{xz/w}{g(zx/w)} g'(zx/w) \left[1 - \frac{\partial \log w}{\partial \log z}\right] = \frac{\sigma}{s_K(xz/w)} \left[1 - \frac{\partial \log w}{\partial \log z}\right], \\
\frac{d \log y}{d \log z} &= 1 + \frac{nF_n(k,n)}{F(k,n)} \frac{xz/w}{g(zx/w)} g'(zx/w) = 1 + \frac{\sigma s_L(xz/w)}{s_K(xz/w)} \left[1 - \frac{\partial \log w}{\partial \log z}\right].
\end{aligned}$$

If the wage rises by the full amount of productivity, i.e. $\frac{\partial \log w}{\partial \log z} = 1$, employment does not respond and output increases only by the amount of the productivity increase. If the wage increases by less than productivity, there is a rise in employment in all units and a further rise in output. Moreover this rise may be different across units: if $g$ has a decreasing elasticity, this rise will be strongest in low productivity (low $x$) units. Alternatively, for a constant elasticity of substitution $\sigma$, the rise is strongest for low operating margins firms.

Turning to earnings (or operating profits), I first note that the envelope theorem applied to $\pi(xz,k,w) = \max_{n\geq 0}\{zxF(k,n) - wn\}$, yields the response of earnings to a change in aggregate productivity or in the wage:

$$\begin{aligned}\frac{\partial \log \pi(xz,k,w)}{\partial \log z} &= \frac{1}{s_K(xz,w)},\\ \frac{\partial \log \pi(x,z,w)}{\partial \log w} &= -\frac{s_L(xz/w)}{s_K(xz/w)}.\end{aligned}$$

Combining the two yields the total effect of a shock to aggregate productivity on current profits:

$$\begin{aligned}\frac{d \log \pi(x,z,w)}{d \log z} &= \frac{\partial \log \pi}{\partial \log z} + \frac{\partial \log \pi}{\partial \log w}\frac{\partial \log w}{\partial \log z},\\ \frac{d \log \pi(x,z,w)}{d \log z} &= \frac{1}{s_K(xz,w)}\left(1 - s_L(xz/w)\frac{\partial \log w}{\partial \log z}\right).\end{aligned}$$

This formula is the key element of this paper. It implies directly the following result:

**Result 2:**

- If $\frac{\partial \log w}{\partial \log z} = 1$, then $\frac{d \log \pi(x,z,w)}{d \log z} = 1$ is independent of $x$, i.e. all production units' earnings $\pi(xz,k,w)$ go up by one percent if aggregate productivity $z$ rises by one percent.

- If $\frac{\partial \log w}{\partial \log z} < 1$, then $\frac{d \log \pi(x,z,w)}{d \log z} > 1$ for all $x$ and this elasticity is inversely related to $s_K(xz/w)$ : if wages are "smoother" than productivity, earnings are more volatile than productivity, and this sensitivity is greater for the units which have a low operating margin.

In aggregate data, corporate profits are highly procyclical and volatile than GDP. A well-accepted reason for this is that labor compensation is relatively smooth and weakly correlated with GDP growth. Result 2 is the cross-sectional counterpart of this fact: firms which have high costs "leverage" the smoothness of wages. Their profits are more procyclical because they amplify the fact that costs do not respond much to macroeconomic conditions, whereas revenues do.

## B. Sources of differences in Operating Margins

For this mechanism to be interesting however, we need to choose a production function $F$ which generates heterogeneous operating margins i.e. a $F$ such that units with different productivities $x$ have different shares $s_K(xz/w)$. In particular, with a simple Cobb-Douglas production function, each units equates its marginal product of labor to the common wage, and since the marginal product of labor is proportional to the average product of labor, there is no cross-sectional heterogeneity either in labor productivity (output per employee) or in profitability!

In the data, as I explain below, firms which have low productivity (low output per capita $Y/N$) have also in general low TFP (low $x$) and low profitability. Hence it seems that an empirically successful production function should generate these two correlations. I first examine what conditions on $F$ are necessary to obtain these correlations.

Since $s_K(u) = \frac{uF(1,g(u))-g(u)}{uF(1,g(u))}$, a simple differentiation yields after some simplifications:

$$\frac{d\log s_K}{d\log u} = \frac{-g'(u)uF(1,g(u)) + g(u)F(1,g(u)) + g(u)ug'(u)F_2(1,g(u))}{\left(uF(1,g(u)) - g(u)\right)F(1,g(u))},$$

which is positive if and only if

$$-g'(u)uF(1,g(u)) + g(u)F(1,g(u)) + g(u)ug'(u)F_2(1,g(u)) > 0$$
$$1 - \frac{g'(u)u}{g(u)} > -\frac{ug'(u)}{g(u)}\frac{g(u)F_2(1,g(u))}{F(1,g(u))},$$

and given that $\frac{g'(u)u}{g(u)} = \frac{\sigma}{s_K}$ and $\frac{uF_2(1,g(u))}{F(1,g(u))} = s_L$ yields the condition

$$\begin{aligned} 1 - \frac{\sigma}{s_K} &> -\frac{\sigma}{s_K}s_L, \\ s_K &> \sigma(1-s_L), \\ 1 &> \sigma. \end{aligned}$$

Hence for an elasticity of substitution below unity, high productivity firms will have a higher operating margin. Under the same condition, output per capita is increasing in productivity since applying the elasticities found above yields

$$\begin{aligned} \frac{\partial \log y}{\partial \log x} - \frac{\partial \log n}{\partial \log x} &= 1 + \sigma\frac{s_L}{s_K} - \frac{\sigma}{s_K} \\ &= 1 + \sigma\left(\frac{s_L - 1}{s_K}\right) \\ &= 1 - \sigma, \end{aligned}$$

so that high TFP firms have high output per capita only if the elasticity of substitution is below one. Summarizing this section:

**Result 3:** Assume a constant return to scale production function with heterogeneous productivities $x$ and fixed capital $k$, i.e. $y = xF(k,n)$. Then high $x$ firms have higher output per worker and a higher capital share if and only if the elasticity of substitution is below one. Moreover, firms with low $x$ have a higher response to a 1% increase in productivity if and only the elasticity of substitution is below one.

It thus appears that this low elasticity of substitution case, which is empirically intuitive, is also the only case (within constant returns) that can generate the right correlation between TFP, labor productivity and operating margins (or capital shares). [One alternative might be to use decreasing returns, but they appear at face value inconsistent with the wide dispersion in size across firms.] I now consider a few simple examples to illustrate.

**Example 1. Cobb-Douglas** $F(k,n) = k^{1-\alpha} n^{\alpha}$.

The optimal labor demand is $n = k \times d \times (xz/w)^{\frac{1}{1-\alpha}}$ where $d$ is a constant. The supply is $y = e \times k \times (xz/w)^{\frac{1}{1-\alpha}}$ where $e$ is a constant. As a result the output per worker is $e/d$, independent of productivity. The operating margin (capital share) is $s = 1 - \alpha$, which is also independent of $x, z$, or $w$. Because the marginal product of labor is proportional to the average product of labor in this case, and the MPL is equated across firms, there is no heterogeneity in output per worker or in profitability. Hence, while this simple Cobb-Douglas production function is often used, it is at odds with the facts on the heterogeneity in measured productivity or capital shares.

**Example 2. Cobb-Douglas with Overhead labor** $F(n) = k^{1-\alpha} \left(n - \overline{n}\right)^{\alpha}$.

In this formulation, there is an overhead labor of $\overline{n}$, leading to a fixed cost (labelled in wages not units of output[5]). The optimal labor demand is $n = \overline{n} + d \times k \times (xz/w)^{\frac{1}{1-\alpha}}$ where $d$ is a constant. The output supply is $y = e \times k \times (xz/w)^{\frac{1}{1-\alpha}}$ where $e$ is a constant. Output per capita is thus $e/\left(d + \overline{n}\,(xz/w)^{-\frac{1}{1-\alpha}}\right)$ which is increasing and concave in $xz/w$. For very large $x$ the effect of $x$ on output per worker is small since fixed costs are a small share of output. The capital share $s$ is $1 - \frac{w}{e}\left(d + \overline{n}\,(xz/w)^{-\frac{1}{1-\alpha}}\right)$ which is increasing in $xz$ and decreasing in $w$. This formulation is in principle capable of capturing the two key correlations.

**Example 3. Putty-Clay** $F(k,n) = n1_{n \leq k} + k1_{n>k}$.

This production function has constant returns in labor up to a capacity constraint $\overline{n}$, after which the MPL is zero. This is an ex-post Leontief production function for individual units. One particular case of it is the Putty-Clay model of Gilchrist and Williams (2001), simplified in Gourio (2005,2006). With this formulation, two cases arise: either $zx \geq w$, and the optimal labor demand is $n = g(zx/w) = k$, or $zx < w$, and the optimal labor demand is $n = g(zx/w) = 0$. The output is similarly $y = zxk$ for $zx > w$, and 0 for $zx < w$. Finally, the profit share is $(zx - w)$ for $zx > w$, and 0 for $zx < w$. This setup can also generate the key correlations.

**Example 4. CES Production Function with Fixed Capital** $F(k,n)$.

As proved above, the each correlation is generated iff $\sigma < 1$.

**Conclusion from the four examples**

While the basic Cobb-Douglas production function does not generate the standard facts on

[5] The case where the fixed cost is in goods units yields similar results.

productivity, there are at least two reasonable setups that can: fixed costs or a "low" elasticity of substitution between capital and labor at the production unit level.

**Extensions: Intermediate Inputs, Variable Capital, Adjustment Costs, Monopolistic Competition**

It is possible to add other inputs, including variable capital, to this framework. Most simply, assume that the project has a production function $F(k,m,n)$. The production unit rents capital each period at the market price $R$, and buys materials at the cost $p_m$. The problem of the firm is thus now:

$$\pi(x,z,w,R,p_m) = \max_{n\geq 0, k\geq 0} \{zxF(k,m,n) - wn - Rk - p_m m\}.$$

If the production function has decreasing returns to scales, this problem is well defined. Applying the envelope theorem and totally differentiating with respect to $z$ yields:

$$\frac{d\log\pi}{d\log z} = \frac{1}{\pi/y}\left(1 - \frac{wn}{y}\frac{\partial \log w}{\partial \log z} - \frac{Rk}{y}\frac{\partial \log R}{\partial \log z} - \frac{p_m m}{y}\frac{\partial \log p_m}{\partial \log z}\right).$$

Hence we see that we can generalize this mechanism. Interestingly, the extent to which profits are procyclical does not depend on whether factors are adjustable or not. By the envelope theorem, factors are near their optimum use, so the effect on profits is second-order.

*[[to finish]]*

## C. Valuation and Risk Premia with Exogenous Prices

I now explore the asset pricing implications of the results of the previous section. In this Section, I take as exogenous the aggregate productivity process $\{z_t\}$, the wage process $\{z_t\}$, and the discount factor $\{m_t\}$, and I compute prices, betas and expected returns of the production units.

I start with the Campbell-Shiller (1988) log-linear approximation for the price-dividend ratio:

$$p_t - d_t = \frac{k}{1-\rho} + E_t \sum_{j\geq 0} \rho^j \left(\Delta d_{t+1+j} - r_{t+1+j}\right),$$

where $d_t =$ log dividend, $p_t =$ log price, $r_t =$ log return, and $k$ and $\rho$ are constants related to the average $p-d$ ratio.[6] Since $r_{t+1} = k + \rho p_{t+1} + (1-\rho)d_{t+1}$, I obtain as in Campbell (1991):

$$r_{t+1} - E_t r_{t+1} = (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j \Delta d_{t+j+1} - (E_{t+1} - E_t)\sum_{j\geq 1}\rho^j r_{t+j+1},$$

the usual decomposition of return innovations into innovations to current or future dividends and innovations to future returns. This is just a restatement of the "price equals present discounted

[6] $\rho = \frac{1}{1+\exp\left(\overline{d-p}\right)}$ and $k = -\log\rho + (1-\rho)\log(1/\rho - 1)$.

value of dividends" identity, without any behavioral implication. This equality holds for any log return. I first assume for simplicity that production units have a constant idiosyncratic shock $x$. Then if a production unit with productivity $x$ is traded, its return satisfies also

$$r_{t+1}(x) - E_t r_{t+1}(x) = (E_{t+1} - E_t) \sum_{j\geq 0} \rho^j \Delta d_{t+j+1}(x) - (E_{t+1} - E_t) \sum_{j\geq 1} \rho^j r_{t+j+1}(x),$$

Researchers have often used atheoretical VARs to measure these two components of unexpected returns. The novelty of my approach is to provide an explicit model for the first term, $\Delta d_{t+j+1}$, and link this term to observable characteristics of the firm. I believe this has the following advantages: (1) it delivers a more complete economic explanation; (2) we can circumvent the problem of measuring the betas, and (3) we obtain cross-equation restrictions, because the betas that determines the return is also directly determined by the earning beta of the firm.

In my model, the production unit yields a flow of earnings $\pi(xz, k, w)$. The formula above applies with $d_t(x) = \log \pi_t(x)$, so that $\Delta d_t(x) = \Delta \log \pi_t(x)$. Applying the analysis of the previous section, I have

$$\Delta \log \pi_t(x) \simeq \eta_z(x) \Delta \log z_t + \eta_w(x) \Delta \log w_t,$$

where $\eta_z(x)$ and $\eta_w(x)$ are the "elasticities" of profits to productivity and the wage:

$$\begin{aligned}
\eta_z(x) &= \frac{1}{s_K(x)}, \\
\eta_w(x) &= -\frac{s_L(x)}{s_K(x)}.
\end{aligned}$$

I assume constant expected returns.[7] Hence the second term $(E_{t+1} - E_t) \sum_{j\geq 1} \rho^j r_{t+j+1}$ is zero and:

$$r_{t+1}(x) - E_t r_{t+1}(x) = (E_{t+1} - E_t) \sum_{j\geq 0} \rho^j \Delta d_{t+j+1}(x).$$

**First Case: only productivity shocks and $\eta_w(x) = 0$**

I now take an exogenous log-normal discount factor. I assume that only the shock to $z$ is priced, and that the market price of risk is $\lambda_z > 0$. Hence,

$$\log m_{t,t+1} - E_t \log m_{t,t+1} = -\lambda_z \left( z_{t+1} - E_t z_{t+1} \right).$$

[7] Of course there is some consensus that expected returns are time-varying. My paper deals only with the cash flow part, and future work will address the questions of discount rate effects as emphasized by Campbell and Vuo (2004).

A positive innovation to aggregate productivity $z$ reduces the state-contingent price. To compute the risk premia, I need only to compute the covariance of the return with the innovation to $z_t$. The log of the risk premium is:[8]

$$\begin{aligned}
\log\left(\frac{E_t R_{t+1}(x)}{R^f_{t+1}}\right) &= E_t r_{t+1}(x) - E_t r^f_{t+1} + \frac{1}{2} V_t r_{t+1}(x) \\
&= -Cov_t\left(\log m_{t,t+1}, r_{t+1}(x)\right) \\
&= -Cov_t\left(\log m_{t,t+1} - E_t \log m_{t,t+1}, r_{t+1}(x) - E_{t+1} r_{t+1}(x)\right) \\
&= -Cov_t\left(\lambda_z\left(z_{t+1} - E_t z_{t+1}\right), \left(E_{t+1} - E_t\right)\sum_{j\geq 0}\rho^j \eta_z(x)\Delta \log z_{t+j+1}\right).
\end{aligned}$$

Note that only this last line uses that (i) the discount factor only prices shocks to $z$ and (ii) $\eta_w(x) = 0$ i.e. the profit does not react to wage shocks. These assumptions would be true for instance if the wage is constant.

To continue this computation, I assume that productivity growth follows some arbitrary stationary process, for which I write the Wold decomposition $\Delta \log z_t = A(L)\varepsilon^z_t$. I find that the log of the risk premium is

$$\begin{aligned}
\log\left(\frac{E_t R_{t+1}(x)}{R^f_{t+1}}\right) &= \eta_z(x)\lambda_z Cov_t\left(\left(E_{t+1} - E_t\right)\sum_{j\geq 0}\rho^j \Delta \log z_{t+j+1}, \left(z_{t+1} - E_t z_{t+1}\right)\right) \\
&= \eta_z(x)\lambda_z Cov_t\left(\left(E_{t+1} - E_t\right)\sum_{j\geq 0}\rho^j A(L)L^{-j}\varepsilon^z_{t+1}, \varepsilon^z_{t+1}\right)
\end{aligned}$$

Using the usual Hansen-Sargent (1980) formulas, I obtain an expression for the log risk premium $lrp(x)$ :

$$lrp(x) = \lambda_z \sigma_z^2 \eta_z(x) A(\rho),$$

i.e. the product of the market price of risk $\lambda_z \sigma_z^2$ times the sensitivity of the present discounted firm $x$ cash flows $A(\rho)\eta_z(x)$. For instance, if $A(L) = (1 - \kappa L)^{-1}$, then $lrp(x) = \lambda_z \sigma_z^2 \eta_z(x)(1 - \rho\kappa)^{-1}$.

**Second case: only productivity shocks are priced ($\lambda_w = 0$), but $\eta_w(x) \neq 0$.**

In Section 3, I introduce a full dynamic stochastic general equilibrium model where productivity shocks drive the economy. This motivates the interest in the case where the wage responds

[8] Assuming log-normality, I have the usual computations:

$$\begin{aligned}
E_t\left(m_{t,t+1} R_{t+1}(x)\right) &= 1 \\
E_t e^{\log m_{t,t+1} + r_{t+1}(x)} &= 1 \\
E_t \log m_{t,t+1} + E_t r_{t+1}(x) + \frac{1}{2} V_t \log m_{t,t+1} + \frac{1}{2} V_t r_{t+1} &= 1
\end{aligned}$$

to productivity shocks, but there are no intrinsic wage shocks, and so the wage does not enter the discount factor. To formalize that the wage responds to productivity, I first write the Wold decomposition for the $\Delta \log z$ process:

$$\Delta \log z_t = A_z(L)\varepsilon_t^z.$$

Next I project $\Delta \log w_t$ on the past history of $\varepsilon_t^z$, and call $\Delta \log \widetilde{w_t}$ the remainder, which is by construction uncorrelated with all the $\varepsilon_{t-k}^z$, for $k \geq 0$:

$$\Delta \log w_t = A_w(L)\varepsilon_t^z + \Delta \log \widetilde{w_t}.$$

Since I assume that only shocks to $\Delta \log z_{t+1}$ are priced, I have $\log m_{t,t+1} - E_t \log m_{t,t+1} = -\lambda_z (z_{t+1} - E_t z_{t+1}) = -\lambda_z \varepsilon_{t+1}^z$. In this case, I can again compute the key covariance:

$$\begin{aligned}
& Cov_t \left(\log m_{t,t+1}, r_{t+1}(x)\right) \\
= \ & Cov_t \left(\log m_{t,t+1} - E_t \log m_{t,t+1}, r_{t+1}(x) - E_{t+1} r_{t+1}(x)\right) \\
= \ & Cov_t \left(-\lambda_z \varepsilon_{t+1}^z, (E_{t+1} - E_t) \sum_{j \geq 0} \rho^j \left(\eta_z(x) \Delta \log z_{t+j+1} + \eta_w(x) \Delta \log w_{t+j+1}\right)\right) \\
= \ & Cov_t \left(-\lambda_z \varepsilon_{t+1}^z, (E_{t+1} - E_t) \sum_{j \geq 0} \rho^j \left(\eta_z(x) A_z(L) \varepsilon_{t+j+1}^z + \eta_w(x) \left(A_w(L) \varepsilon_{t+j+1}^z + \Delta \log \widetilde{w_{t+j+1}}\right)\right)\right),
\end{aligned}$$

and thus in this case $lrp(x) = \lambda_z \sigma_z^2 \left(\eta_z(x) A_z(\rho) + \eta_w(x) A_w(\rho)\right)$. This formula shows that the covariance with the asset pricing factor is now driven not only by the response of profits to productivity shocks, but also by the response of profits to the wages induced by the productivity shocks.

**Third case: both productivity and wage shocks are priced ($\lambda_w \neq 0$), and $\eta_w(x) \neq 0$.**

This more general case is interesting to see the impact of "wage shocks" on asset prices in this model. I thus assume that

$$\log m_{t,t+1} - E_t \log m_{t,t+1} = -\lambda_z \varepsilon_{t+1}^z - \lambda_w \varepsilon_{t+1}^w,$$

where $\left(\varepsilon_{t+1}^z, \varepsilon_{t+1}^w\right)$ are the innovations to the productivity and wage process. Again in anticipation of a general equilibrium model where productivity is exogenous and affects the wage, but not the reverse, I define first $\varepsilon_t^z$ as the Wold innovation of $\Delta \log z_t : \Delta \log z_t = A_z(L)\varepsilon_t^z$. Next I define $A_{wz}(L)$ as the projection of $\Delta \log w_t$ on $\left\{\varepsilon_{t-k}^z\right\}$. Let $\Delta \log \widetilde{w_t} = \Delta \log w_t - A_{wz}(L)\varepsilon_t^z$. Finally I apply a Wold decomposition to $\Delta \log \widetilde{w_t} = A_{ww}(L)\varepsilon_t^w$. By construction $\varepsilon_t^w$ is uncorrelated with $\varepsilon_t^z$ at all leads and lags. In the end I have:

$$\begin{aligned}
\Delta \log z_t &= A_z(L)\varepsilon_t^z, \\
\Delta \log w_t &= A_{wz}(L)\varepsilon_t^z + A_{ww}(L)\varepsilon_t^w,
\end{aligned}$$

and I can now again compute the covariance

$$
\begin{aligned}
& Cov_t\left(\log m_{t,t+1}, r_{t+1}(x)\right) = Cov_t\left(\log m_{t,t+1} - E_t \log m_{t,t+1}, r_{t+1}(x) - E_{t+1} r_{t+1}(x)\right) \\
= \; & Cov\left(-\lambda_z \varepsilon_{t+1}^z - \lambda_w \varepsilon_{t+1}^w, (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j\left(\eta_z(x)\Delta\log z_{t+j+1} + \eta_w(x)\Delta\log w_{t+j+1}\right)\right) \\
= \; & -Cov\left(\begin{array}{c} \lambda_z \varepsilon_{t+1}^z + \lambda_w \varepsilon_{t+1}^w, (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j\left(\eta_z(x)A_{zz}(L) + \eta_w(x)A_{wz}(L)\right)L^{-j}\varepsilon_{t+1}^z \\ + (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j \eta_w(x) A_{ww}(L) L^{-j}\varepsilon_{t+1}^w \end{array}\right) \\
= \; & -Cov\left(\lambda_z \varepsilon_{t+1}^z, (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j\left(\eta_z(x)A_{zz}(L) + \eta_w(x)A_{wz}(L)\right)L^{-j}\varepsilon_{t+1}^z\right) \\
& -Cov\left(\lambda_w \varepsilon_{t+1}^w, (E_{t+1} - E_t)\sum_{j\geq 0}\rho^j \eta_w(x) A_{ww}(L) L^{-j}\varepsilon_{t+1}^w\right).
\end{aligned}
$$

By the same logic as above, I obtain the log risk premium as

$$
lrp(x) = \lambda_z \sigma_z^2\left(\eta_z(x)A_{zz}(\rho) + \eta_w(x)A_{wz}(\rho)\right) + \lambda_w \sigma_w^2 \eta_w(x) A_{ww}(\rho).
$$

There are now two risk factors, and the loadings are endogenously determined by the model as a function of productivity $x$. Moreover, these loadings can be estimated by regressions of profits on aggregate productivity and the aggregate wage:

$$
\Delta \log \pi_t(x) = \eta_z(x)\Delta\log z_t + \eta_w(x)\Delta\log w_t.
$$

This is the sense in which this model provides *cross-equation restrictions.*

Future work will implement these formulas directly on the data. In light of the empirical success of Section 5, I believe these can explain a part of the size and value premium by a simple cash flow effect.

# 3 A General Equilibrium Model with Overhead Labor

The analysis of Section 2 shows that in order to replicate the heterogeneity of labor productivity and of profitability, and to obtain the heterogeneous sensitivities to aggregate shocks, one must depart from the standard Cobb-Douglas production function. In this section I use an extended version of the example 2 of Section 2: there are some fixed costs through overhead labor. There are three key elements to keep in mind as we extend the analysis from partial to general equilibrium:

(1) As should be clear from section 2, fixed costs are important in our analysis only in as much as they generate an heterogeneity of "profit shares", and not because of the inflexibility that they generate. Of course in general equilibrium, fixed costs may have additional implications.[9]

(2) My choice of using fixed costs in the general equilibrium model is driven by a desire for tractability. In this model with a non-trivial cross-section of firms and aggregate shocks, being able to aggregate the model and use only a few state variables is important for numerical analysis. Aggregation is not feasible for say, CES production functions subject to idiosyncratic shocks.

(3) The key condition that $\frac{\partial \log w_t}{\partial \log z_t} < 1$ which was noted in the partial equilibrium framework is here not an assumption. Rather, it is generated as an outcome of the labor market equilibrium. Here I assume a competitive labor market, which makes it hard to generate a smooth wage, since the wage here reflects the marginal product of labor (and does not incorporate any reason for "smoothness" such as risk-sharing). Future work will consider the implications of adding more wage rigidity exogenously in this model.

## A. Setup

**Preferences**

Preferences are of the usual expected utility type, defined over consumption and labor:

$$E \sum_{t \geq 0} \beta^t U(c_t, 1 - n_t). \tag{3.1}$$

I will parametrize the utility function directly as a function of the intertemporal elasticity of substitution $\sigma$ and the Frisch elasticity of labor supply $\varepsilon_{nw}$, as in Rotemberg and Woodford (1995).

**Technology**

[9] We know from the work of Jermann (1998) and Boldrin, Christiano and Fisher (2001) that an easy adjustment of labor makes it hard to generate a large equity premium.

The technology is the one of example 2, because it appears to be the most amenable to aggregation. Thus, production takes place in units which are identical at birth and have fixed capital. A unit operates a CRS production function subject to idiosyncratic and aggregate shocks: $y = zxF(k,n)$. I assume $F(k,n) = k^{1-\alpha}n^{\alpha}$. I assume for simplicity that $k$ does not change over time and cannot be chosen: it is an exogenous constant. Operating the unit requires paying fixed costs as well as the variable labor. I view the fixed cost as overhead labor: $c_f$ units of labor must be hired each period. Total operating income, or earnings, is thus

$$\pi(z,x,w) = \max_{n \geq 0} \left\{ zxk^{1-\alpha}n^{\alpha} - w\left(n + c_f\right) \right\}. \tag{3.2}$$

**Shocks**

There are three types of shocks in this economy.

*"Large idiosyncratic shock":* with probability $\delta$ each period, a unit receives a large negative shock that causes it to exit. This is the only reason for exit.

*"Small idiosyncratic shock":* the idiosyncratic productivity $x$ evolves according to a AR(1) process:

$$\log x_{i,t+1} = \rho \log x_{i,t} + \mu + \sigma_x \varepsilon_{i,t+1}, \tag{3.3}$$

with $\varepsilon_{i,t+1}$ is $N(0,1)$ independent across units and across time. I denote $F(x' \mid x)$ the transition function for this process.[10]

*"Aggregate shocks":* aggregate productivity follows a standard persistent AR(1) process:

$$\log z_{t+1} = \rho_z \log z_t + \sigma_z \varepsilon_{z,t+1}, \tag{3.4}$$

with $0 < \rho_z < 1$ and $\varepsilon_{z,t+1}$ is $N(0,1)$ iid over time and independent of all the $\varepsilon_{i,t}$. There is no growth in this economy.

**Production of new units**

New units are produced by a capital-good producing sector that can transform units of consumption good into capital goods at cost $\psi\left(I_t\right)$, where $I_t$ is the number of units created each period. $\psi$ is an "external adjustment cost" which satisfies $\psi' > 0$, $\psi'' > 0$. New units get an initial productivity draw $x$ from a distribution with p.d.f. $h$ and c.d.f. $H$.

**Resource constraints**

[10] Note that despite random walk shocks I have a stationary distribution (when I shut down the aggregate shocks) because of depreciation, i.e. the "death" shocks.

Let $G_t(x)$ be the measure of units with idiosyncratic productivity less than $x$, and let $g_t(x) = G_t'(x)$. Aggregating across units yields the goods resource constraint and the time resource constraint:

$$C_t + \psi\left(I_t\right) \leq Y_t = \int_0^\infty z_t x k^{1-\alpha} n_t(x)^\alpha dG_t(x), \tag{3.5}$$

$$\int_0^\infty n_t(x) dG_t(x) + c_f \int_0^\infty dG_t(x) = n_t. \tag{3.6}$$

The law of motion for the measure of productivity is, if $F(x' \mid x)$ is the transition c.d.f. of the idiosyncratic shock[11]:

$$G_{t+1}(x) = (1-\delta) \int_0^\infty F(x \mid s) dG_t(s) + I_t \times H(x), \tag{3.7}$$

which can equivalently be rewritten in terms of the underlying derivative:

$$g_{t+1}(x) = (1-\delta) \int_0^\infty \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds + I_t \times h(x). \tag{3.8}$$

**Social Planner Problem**

Since the welfare theorems hold in this economy, it is possible to use a social planning problem to find the competitive equilibrium.. The problem is

$$\max_{\{c_t, n_t, n_t(x), I_t, G_{t+1}(x)\}} E \sum_{t \geq 0} \beta^t U(c_t, 1-n_t)$$

$$\begin{aligned} c_t + \psi\left(I_t\right) &= \int_0^\infty z_t x k^{1-\alpha} n_t(x)^\alpha dG_t(x), \\ n_t &= \int_0^\infty n_t(x) dG_t(x) + c_f \int_0^\infty dG_t(x), \\ \forall x \geq 0 : G_{t+1}(x) &= (1-\delta) \int_0^\infty F(x \mid s) dG_t(s) + I_t \times H(x), \end{aligned}$$

given $G_0$, subject to the law of motion for the aggregate shocks.

It is convenient to divide this problem in two parts: the first one is the allocation of labor across firms (i.e., the computation of the aggregate production function), the second one is the choice of the total amount of labor, consumption and investment.

---

[11] Given our assumption that $x$ is in log a random walk with drift, we have

$$\begin{aligned} F(x \mid s) &= \Pr\left(\ln x_{t+1} \leq \ln x \mid \ln x_t = \ln s\right), \\ F(x \mid s) &= \Phi\left(\frac{\ln x - \ln s - \mu}{\sigma}\right), \end{aligned}$$

where $\Phi$ is the standard normal cdf. However I will not need to use this formula explicitely; I only need that the transition depends only on the ratio $x/s$.

**Allocation problem within the period**

Let $W(n_t)$ be the maximum feasible output given hours $n_t$, assuming perfect and costless reallocation of labor across units:

$$\begin{aligned} W(n_t) &= \max_{\{n_t(x)\}_{x\geq 0}} \int_0^\infty x k^{1-\alpha} n_t(x)^\alpha dG_t(x) \\ s.t. &: \quad n_t = \int_0^\infty n_t(x) dG_t(x) + c_f \int_0^\infty dG_t(x) \end{aligned}$$

The first-order condition with respect to $n_t(x)$ is:

$$\alpha x k^{1-\alpha} n_t(x)^{\alpha-1} = \mu_t, \tag{3.9}$$

where $\mu_t$ is the Lagrange multiplier on the constraint. Substituting back in the constraint yields

$$n_t = \left(\frac{\alpha}{\mu_t}\right)^{\frac{1}{1-\alpha}} k \int_0^\infty x^{\frac{1}{1-\alpha}} dG_t(x) + c_f \int_0^\infty dG_t(x).$$

Define $M_{1,t} = \int_0^\infty x^{\frac{1}{1-\alpha}} dG_t(x)$ and $M_{2,t} = \int_0^\infty dG_t(x)$. Then

$$n_t - c_f \times M_{2,t} = M_{1,t} k \left(\frac{\alpha}{\mu_t}\right)^{\frac{1}{1-\alpha}}$$

$$\mu_t = \frac{\alpha M_{1,t}^{1-\alpha} k^{1-\alpha}}{(n_t - c_f \times M_{2,t})^{1-\alpha}}.$$

Substituting back in (3.9) yields

$$\begin{aligned} n_t(x) &= k \left( \frac{\alpha x}{\frac{\alpha M_{1,t}^{1-\alpha} k^{1-\alpha}}{(n_t - c_f \times M_{2,t})^{1-\alpha}}} \right)^{\frac{1}{1-\alpha}}, \\ &= x^{\frac{1}{1-\alpha}} \left( \frac{n_t - c_f \times M_{2,t}}{M_{1,t}} \right). \end{aligned}$$

Thus the total production given aggregate labor input $n_t$ is

$$\begin{aligned} W(n_t) &= k^{1-\alpha} \int_0^\infty x^{\frac{1}{1-\alpha}} \left( \frac{n_t - c_f \times M_{2,t}}{M_{1,t}} \right)^\alpha dG_t(x) \\ W(n_t) &= k^{1-\alpha} \left( \frac{n_t - c_f \times M_{2,t}}{M_{1,t}} \right)^\alpha M_{1,t} \\ W(n_t) &= k^{1-\alpha} (n_t - c_f \times M_{2,t})^\alpha M_{1,t}^{1-\alpha}. \end{aligned} \tag{3.10}$$

**Sufficiency of the two moments of $G_t$**

This last expression shows that for the social planning problem, we need to know only two moments of $G_t$, the variables $M_{1,t}$ and $M_{2,t}$. In this section I show that these moments satisfy simple recursions where the distribution $G_t$ plays a role again only through these moments. This allows me to get rid of the cross-sectional distribution in solving the planner problem.

First let's write the law of motion for $M_{1,t}$ :

$$\begin{aligned} M_{1,t+1} &= \int_0^\infty x^{\frac{1}{1-\alpha}} dG_{t+1}(x) \\ &= \int_0^\infty x^{\frac{1}{1-\alpha}} g_{t+1}(x) dx \\ &= \int_0^\infty x^{\frac{1}{1-\alpha}} \left( \left( (1-\delta) \int_0^\infty \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds + I_t h(x) \right) \right) dx \\ &= (1-\delta) \int_0^\infty \int_0^\infty x^{\frac{1}{1-\alpha}} \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds dx + I_t \upsilon, \end{aligned}$$

where $\upsilon \stackrel{def}{=} \int_0^\infty x^{\frac{1}{1-\alpha}} h(x) dx$ is a constant.

*Case 1: No shock after the initial draw from h.*

In this case, $F(x \mid s) = 1_{x \geq s}$ i.e. the initial shock $h$ is permanent. Then $\frac{\partial F(x|s)}{\partial x} = \delta_{\{x=s\}}$ and

$$\begin{aligned} \int_0^\infty \int_0^\infty x^{\frac{1}{1-\alpha}} \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds dx &= \int_0^\infty x^{\frac{1}{1-\alpha}} \left( \int_0^\infty \delta_{\{x=s\}} g_t(s) ds \right) dx \\ &= \int_0^\infty x^{\frac{1}{1-\alpha}} g_t(x) dx \\ &= M_{1,t}, \end{aligned}$$

hence the law of motion for $M_{1,t}$ is simply:

$$M_{1,t+1} = (1-\delta) M_{1,t} + I_t \upsilon.$$

*Case 2: Unit root shock.*

If the idiosyncratic shock evolves according to: $\ln x_{t+1} = \ln x_t + \mu + \sigma \varepsilon_{t+1}$, then $F(x \mid s) = \Phi\left(\frac{\ln x - \ln s - \mu}{\sigma}\right)$ and

$$\frac{\partial F(x \mid s)}{\partial x} = \frac{1}{\sigma x} \phi \left( \frac{\ln x - \ln s - \mu}{\sigma} \right).$$

Thus

$$\begin{aligned} \int_0^\infty \int_0^\infty x^{\frac{1}{1-\alpha}} \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds dx &= \int_0^\infty \int_0^\infty x^{\frac{1}{1-\alpha}} \frac{1}{\sigma x} \phi \left( \frac{\ln x - \ln s - \mu}{\sigma} \right) g_t(s) ds dx \\ &= \int_0^\infty \int_0^\infty \left( \frac{x}{s} \right)^{\frac{1}{1-\alpha} - 1} s^{\frac{1}{1-\alpha} - 1} \frac{1}{\sigma} \phi \left( \frac{\ln x - \ln s - \mu}{\sigma} \right) g_t(s) ds dx \\ &= \int_0^\infty s^{\frac{\alpha}{1-\alpha}} s g_t(s) \left( \int_0^\infty v^{\frac{\alpha}{1-\alpha}} \frac{1}{\sigma} \phi \left( \frac{\ln v - \mu}{\sigma} \right) dv \right) ds \\ &= \int_0^\infty s^{\frac{1}{1-\alpha}} g_t(s) \left( \int_0^\infty v^{\frac{1}{1-\alpha}} \frac{1}{\sigma v} \phi \left( \frac{\ln v - \mu}{\sigma} \right) dv \right) ds \end{aligned}$$

where $v = x/s$.

But $\int_0^\infty v^{\frac{1}{1-\alpha}} \frac{1}{\sigma v}\phi\left(\frac{\ln v-\mu}{\sigma}\right) dv = E\left(V^{\frac{1}{1-\alpha}}\right)$ where $\ln V$ is $N(\mu, \sigma^2)$ thus this integral can be evaluated:

$$\zeta = \int_0^\infty v^{\frac{1}{1-\alpha}} \frac{1}{\sigma v}\phi\left(\frac{\ln v-\mu}{\sigma}\right) dv = e^{\frac{\mu}{1-\alpha}+\frac{\sigma^2}{2(1-\alpha)^2}},$$

and finally

$$\int_0^\infty \int_0^\infty x^{\frac{1}{1-\alpha}} \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds dx = \zeta M_{1,t},$$

so that the law of motion for $M_{1,t}$ is:

$$M_{1,t+1} = (1-\delta)\zeta M_{1,t} + I_t \upsilon.$$

A similar (simpler) calculation shows that $M_{2,t}$ satisfies

$$\begin{aligned}
M_{2,t+1} &= \int_0^\infty g_{t+1}(x) dx \\
&= \int_0^\infty \left(\left((1-\delta)\int_0^\infty \frac{\partial F(x \mid s)}{\partial x} g_t(s) ds + I_t h(x)\right)\right) dx \\
&= (1-\delta)\int_0^\infty \left(\int_0^\infty \frac{\partial F(x \mid s)}{\partial x} dx\right) g_t(s) ds + I_t,
\end{aligned}$$

$$M_{2,t+1} = (1-\delta) M_{2,t} + I_t.$$

**Simplified social planner problem**

I can thus rewrite the social planner problem as:

$$\max_{\{c_t, n_t, I_t, M_{1,t+1}, M_{2,t+1}\}_{t=0}^\infty} E \sum_{t \geq 0} \beta^t U(c_t, 1-n_t)$$

$$c_t + \psi(I_t) = z_t W(n_t) = z_t k^{1-\alpha}\left(n_t - c_f \times M_{2,t}\right)^\alpha M_{1,t}^{1-\alpha}.$$

$$M_{1,t+1} = (1-\delta)\zeta M_{1,t} + I_t \upsilon,$$

$$M_{2,t+1} = (1-\delta) M_{2,t} + I_t,$$

given $M_{1,0}$ and $M_{2,0}$. In the appendix I derive the first-order conditions, the steady-state and the log-linearized version which I use to conduct business cycle analysis.

**Asset Prices: Pricing a Production Unit**

I know compute the value of a production unit of productivity $x$. First assume for simplicity that $x$ is constant. The cum-dividend asset price of a production unit with productivity $x$ in this economy is:

$$
\begin{aligned}
P_t(x) &= E_t \sum_{j\geq 0} \frac{\beta^j \lambda_{t+j}}{\lambda_t}(1-\delta)^j \max_{n\geq 0}\left\{z_{t+j}xn^{\alpha} - w_{t+j}\left(n+c_f\right)\right\} \\
&= E_t \sum_{j\geq 0} \frac{\beta^j \lambda_{t+j}}{\lambda_t}(1-\delta)^j \left(\alpha^{\frac{\alpha}{1-\alpha}}\left(1-\alpha\right)\left(\frac{z_{t+j}}{w_{t+j}^{\alpha}}\right)^{\frac{1}{1-\alpha}} x^{\frac{1}{1-\alpha}} - w_{t+j}c_f\right) \\
&= q_{1,t}x^{\frac{1}{1-\alpha}} - q_{2,t}. \qquad (3.11)
\end{aligned}
$$

This is just breaking down the present value of cash flows as the present value of revenues and variable labor costs, minus the present value of fixed costs, where I have defined

$$
\begin{aligned}
q_{1,t} &= dE_t \sum_{j\geq 0} \frac{\beta^j \lambda_{t+j}}{\lambda_t}(1-\delta)^j \left(\frac{z_{t+j}}{w_{t+j}^{\alpha}}\right)^{\frac{1}{1-\alpha}}, \\
q_{2,t} &= c_f E_t \sum_{j\geq 0} \frac{\beta^j \lambda_{t+j}}{\lambda_t}(1-\delta)^j w_{t+j},
\end{aligned}
$$

or recursively:

$$
\begin{aligned}
q_{1,t} &= E_t\left[d\left(\frac{z_t}{w_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} + \frac{\beta\lambda_{t+1}}{\lambda_t}\left(1-\delta\right)q_{1,t+1}\right], \\
q_{2,t} &= E_t\left[c_f w_t + \frac{\beta\lambda_{t+1}}{\lambda_t}\left(1-\delta\right)q_{2,t+1}\right].
\end{aligned}
$$

The free-entry condition (derived from the FOCs of the DGSE model) equates the marginal cost of creating one additional unit and the marginal value:

$$
\psi'(I_t) = q_{1,t}\upsilon - q_{2,t}.
$$

To generate our key cross-sectional prediction, we need that when a positive TFP shock $z$ hits the economy, the present value of revenues minus variable costs rises by more than the present value of fixed costs (which depends on the wage), i.e.

$$
\frac{\partial \log q_{1,t}}{\partial \log z_t} > \frac{\partial \log q_{2,t}}{\partial \log z_t}. \qquad (3.12)
$$

While it is hard to analyze exactly under which conditions this inequality will hold, I will provide parameters that satisfy this condition in the numerical simulations of the next subsection.

Note that since all units have the same quantity of capital $k$, they will all have the same book value $k$, and there will be heterogeneity in book-to-market according to heterogeneity in $x$. Hence high $x$ firms (which have had lucky draws of productivity) have low book-to-market. If the condition 3.12 is satisfied, they have lower sensitivity (in terms of prices) to aggregate shocks. This is because the sensitivity is

$$\begin{aligned}\frac{d\log P_t(x)}{d\log z_t} &= \frac{x^{\frac{1}{1-\alpha}}q_{1,t}}{x^{\frac{1}{1-\alpha}}q_{1,t}-q_{2,t}}\frac{\partial\log q_{1,t}}{\partial\log z_t}+\frac{-q_{2,t}}{x^{\frac{1}{1-\alpha}}q_{1,t}-q_{2,t}}\frac{\partial\log q_{2,t}}{\partial\log z_t},\\ &= \alpha(x)\frac{\partial\log q_{1,t}}{\partial\log z_t}+(1-\alpha(x))\frac{\partial\log q_{2,t}}{\partial\log z_t},\end{aligned}$$

with $d\alpha/dx<0$, and

$$\frac{\partial}{\partial x}\frac{d\log P_t(x)}{d\log z_t}=\alpha'(x)\left(\frac{\partial\log q_{1,t}}{\partial\log z_t}-\frac{\partial\log q_{2,t}}{\partial\log z_t}\right)<0,$$

ensuring that firms with high $x$ (which according to our previous analysis have high TFP, high output per worker and high profitability) are less responsive to aggregate shocks than firms with low $x$. Since $x$ is positively correlated with market size according to (3.11), the model reproduces the value and size premium. However in this model all return differentials are perfectly justified by the correct beta (with respect to marginal utility of consumption $U_c(c_t,1-n_t)$.

**Introducing Firms**

In my model, productivity on existing units does not carry over to new units. Hence a production unit with a good shock can scale up instantaneously through labor but only up to a point. The allocation of new projects to firms can then be random, to generate firms.

## B. Numerical Results from the DSGE model

**Calibration and Numerical Methods**

I use the following parameters to simulate my model. Many of these parameters are standard in the business cycle literature. (To be explained and finished.)

| Calibration of the DGSE model ("Annual Data") | | |
|---|---|---|
| Parameter | Symbol | Value |
| 1/Intertemporal Substitution of Consumption | $\gamma$ | 20 |
| Intertemporal Substitution of Leisure | $\varepsilon_{hw}$ | 100 |
| Share of variable labor in output | $\alpha$ | 0.50 |
| Death rate (=depreciation) | $\delta$ | 0.15 |
| Trend of idiosyncratic productivity process | $\mu$ | 0.0 |
| Standard Deviation of idiosyncratic productivity process | $\sigma$ | 0.2 |
| Standard Deviation of aggregate productivity process | $\sigma_z$ | .02 |
| Persistence of aggregate shock | $\rho_z$ | .9 |
| Fixed cost in labor term | $c_f$ | 1 |
| Discount factor | $\beta$ | 0.95 |
| Mean initial productivity | $\upsilon$ | 1 |
| Curvature of adjustment cost function | $\psi''$ | 2 |

Table 1. List of parameters used in the Calibration.

| % | Variable Labor | Fixed Labor | Capital |
|---|---|---|---|
| Share of each factor | 50.0 | 29.3 | 20.7 |

Table 2. Decomposition of Value Added in Steady-State.

**Impulse Response Functions**

Figure 2 display the response to a persistent TFP shock $z$ of aggregate consumption, output, investment, and employment. Note that the model generates a hump-shape pattern for employment. The model matches roughly the relative volatility of investment, output and consumption roughly but underestimates the volatility of employment.

Figure 3 plots the IRF of $q_{1,t}$ and $q_{2,t}$ to a persistent TFP shock. For this calibration, the present value of revenues minus variable costs $q_{1,t}$ increases by more than the present value of fixed costs $q_{2,t}$, which generates the value and size premium.

## C. Simulated Data

TBA. As in my job market paper I will present results from a simulated panel mimicking the empirical results of Fama and French (1992, 1996)

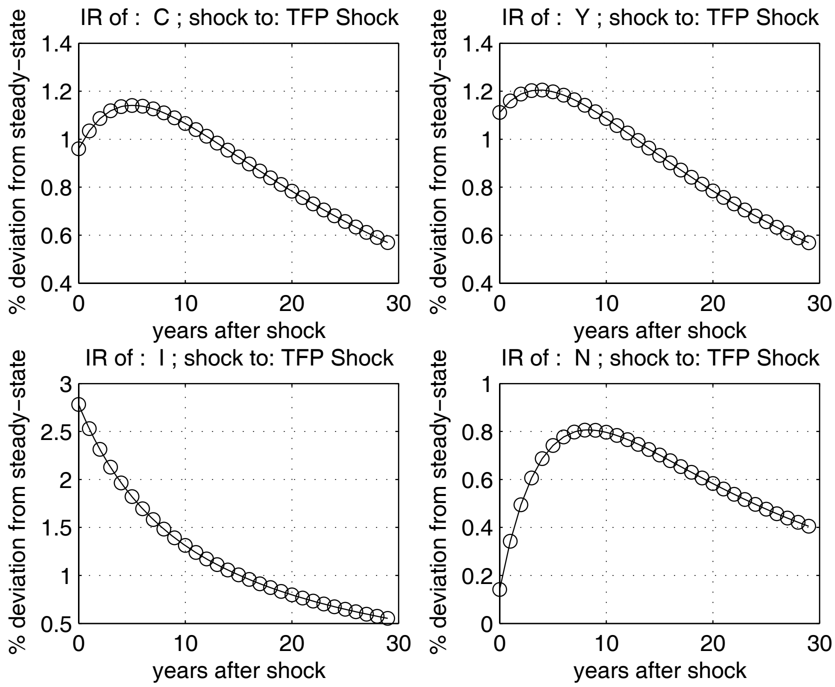



Figure 2: IRF of Consumption, Output, Investment and Employment to a persistent TFP shock.

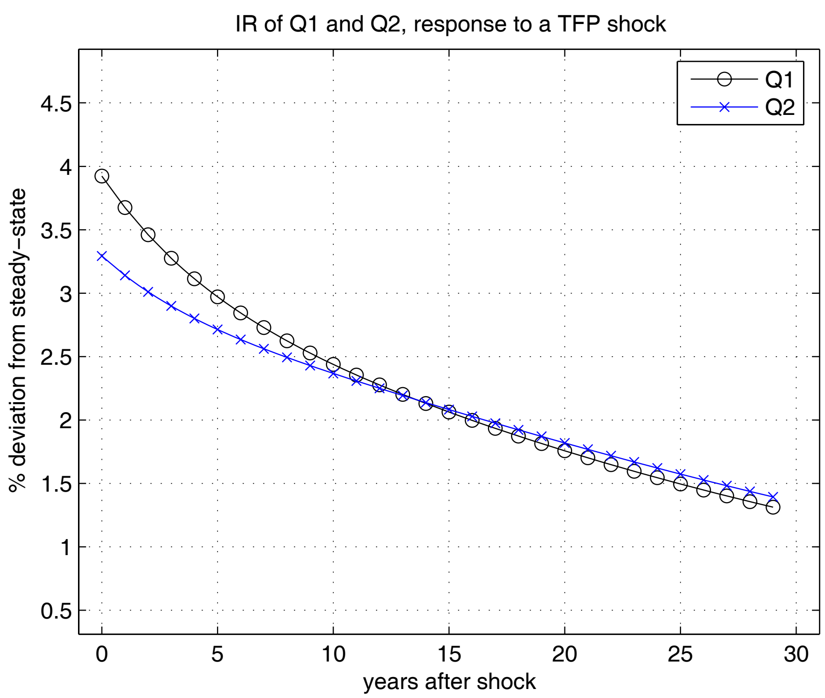



Figure 3: IRF of $q_1$ and $q_2$ to a persistent TFP shock.

## 4 Empirical Evidence

This section documents two main empirical facts. The first one is that firms with high margins or high productivity have in general more procyclical earnings, sales and employment as predicted by the model. The second fact is that small firms and value firms, including the portfolios studied in the asset pricing literature, have lower margins and productivity, and are more procyclical than large or growth firms. This is especially true for the earnings (or profits) of these firms. Hence, small and value stocks are clearly more risky in term of cash flows. As I show, this can explain why these stocks have higher expected returns. Indeed, this direct evidence is consistent with recent indirect evidence such as Campbell and Vuolteenaho (2004) who find that value stocks have higher cash flow betas.

**Methodology**

I measure the effect of a firm characteristic $x_{j,t}$ (e.g. productivity) on the sensitivity of firm variable $Z_{j,t}$ (e.g. sales) to the business cycle. To do so, I run the following regression:

$$\Delta \log Z_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}. \tag{4.1}$$

Note first that I use GDP growth to measure the business cycle. I present two types of results: (1) results from running this regression on the unbalanced Compustat panel of firms (a pooled time-series and cross-section regression), (2) results from running this for portfolios formed on book-to-market or size and book-to-market as in Fama and French (1996).

**Data and Variables**

I use Compustat data from 1963 to 2004. I will often present robustness checks that include running this on the sample 1978-2004, and/or only for the manufacturing sector (SIC between 2000 and 3990), for which the physical production interpretation is probably more reasonable.

As left-hand side variables $Z_{j,t}$, I use (i) real earnings, where earnings are measured as Compustat's item 13 (income before taxes, interests and depreciation), (ii) real net income, where net income is item 172 (income after taxes, interest and depreciation), (iii) real sales (item 12), (iv) employment (item 29). In appendix I give some results for investment growth (item XX). In all cases the deflator is the CPI, except for investment for which I use the GDP deflator for non-residential fixed investment.

As variable $x_{j,t}$, I use three measures. First, I use the "operating margin" (or profitability): the ratio of operating income (item 13 in Compustat) to sales. Second, I use the book-to-market ratio as constructed by Fama and French (1992, 1996; it is actually well approximated by the ratio of item 60 to the product of item 25 and item 199). Third, I use a productivity measure, the sales-to-employee ratio (item 12 and item 29). For each case, I take the variable normalized in each year relative to the average of Compustat, and in logs. For instance, for the first variable, the exact definition is $x_{j,t} = \log\left(\frac{OI_{j,t}/S_{j,t}}{\sum_k OI_{k,t}/\sum_k S_{k,t}}\right)$. These variables have all a mean approximately zero and a standard deviation near one. Hence, the coefficient $\delta$ in the regression can be interpreted as the effect of an increase of one standard deviation of margin, book-to-market or productivity on the sensitivity to GDP growth.

Future work will involve creating a more precise firm-level productivity variable. I will also examine the effect of aggregate productivity shocks (e.g. measured as innovations to TFP growth) firm-level variables.

**Descriptive Statistics**

Table 3 presents some summary statistics on the right-hand side sorting variable $x_{j,t}$ before normalization. There is a wide dispersion in profitability,

book-to-market and productivity. Table 4 shows that high profitability is associated with high output per capita and low book-to-market (i.e. high Tobin $q$). The correlation between book-to-market and labor productivity outside manufacturing is zero however. Some of this reflects differences in industry composition, with some industries having on average higher book-to-market: adding industry controls make this correlation negative.

| $x_{j,t} =$ | Coverage | Median | 10th centile | 90th centile |
|---|---|---|---|---|
| Margin $OI/S$ | Manuf. | 0.117 | -0.053 | 0.235 |
| | All | 0.133 | -0.016 | 0.493 |
| Book-to-market | Manuf. | 0.571 | 0.178 | 1.531 |
| | All | 0.590 | 0.182 | 1.498 |
| Productivity $S/E$ | Manuf. | 0.92 | 0.46 | 1.96 |
| | All | 1 | 0.42 | 3.05 |

Table 3: Summary Statistics for the variables used as $x_{j,t}$
$OI/S$ and $B/M$ are not normalized, and productivity $S/E$
is normalized by the cross-sectional median in each year.
Pooled cross-section and time-series statistics.

| $x_{j,t} =$ | $OI/S$ | $B/M$ | $S/E$ | $OI/S$ | $B/M$ | $S/E$ |
|---|---|---|---|---|---|---|
| Margin $OI/S$ | 1 | | | 1 | | |
| Book-to-market $B/M$ | -0.38 | 1 | | -0.24 | 1 | |
| Productivity $S/E$ | 0.10 | -0.11 | 1 | 0.18 | 0.01 | 1 |

Table 4: Correlation matrixes for the variables used as $x_{j,t}$.
After normalization by Compustat mean, and in logs, e.g. $x_{j,t} = \log\left(\frac{OI_{j,t}/S_{j,t}}{\sum_k OI_{k,t}/\sum_k S_{k,t}}\right)$
Manufacturing sample (left panel), and Whole sample (right panel).

## A. The Sensitivity of Earnings and Profits to GDP growth as a function of Margin, Productivity or Book-to-Market

The left panel of Table 5 gives the results of running the equation (4.1) on Compustat for $Z =$ earnings, and the right panel gives the results of running the same regression for $Z =$net income (which I call "profit"). In both cases, results are presented for the three sorting variables: operating margins (or profitability, i.e. operating income over sales), book-to-market, and productivity (sales per employee). In each case, I give results for the manufacturing sample and for the entire sample of firms, and I give results for a specification which includes industry dummies (both in levels and interacted with GDP growth, i.e. the industry dummies are included in $x_{j,t}$). These dummies control for industry-level differences in cyclicality. For conciseness the tables report only $\widehat{\delta}$, and the associated standard error and $R^2$.

| $x_{j,t} =$ | Coverage | Industry | $\widehat{\delta}$ | t-stat | $R^2$ | $\widehat{\delta}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| $OI/S$ | Manuf. | yes | **-1.57** | 5.2 | 0.13 | **-2.13** | 3.8 | 0.04 |
| | | no | **-1.73** | 5.9 | 0.11 | **-2.25** | 4.2 | 0.03 |
| | All | yes | **-0.62** | 3.2 | 0.09 | **-1.10** | 3.2 | 0.03 |
| | | no | **-1.30** | 7.9 | 0.06 | **-2.08** | 7.5 | 0.02 |
| $B/M$ | Manuf. | yes | **1.15** | 4.9 | 0.03 | **1.32** | 3.3 | 0.02 |
| | | no | **0.76** | 3.0 | 0.03 | **1.79** | 4.9 | 0.02 |
| | All | yes | **0.78** | 4.3 | 0.03 | **1.40** | 4.8 | 0.02 |
| | | no | **0.93** | 5.4 | 0.02 | **1.51** | 5.5 | 0.02 |
| $S/E$ | Manuf. | yes | 0.65 | 1.6 | 0.02 | -0.18 | 0.3 | 0.03 |
| | | no | -0.24 | 0.8 | 0.02 | -0.81 | 1.7 | 0.02 |
| | All | yes | 0.29 | 1.37 | 0.03 | 0.38 | 1.2 | 0.02 |
| | | no | -0.64 | 3.7 | 0.01 | -0.91 | 3.5 | 0.01 |

Table 5: Results from the Pooled Panel OLS Regressions:

LEFT PANEL: $\Delta \log OI_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$

RIGHT PANEL: $\Delta \log NI_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$

Compustat 1963-2004. OLS Standard Errors.

LHS is real % change of operating income (item 13; left panel) or real % change of net income (item 172; right panel). The RHS $x = OI/S, B/M$ or $S/E$ is in logs, standardized by the cross-sectional mean in year $t$. (see text)

In interpreting the results of Table 5, one must keep in mind that the average sensitivity of a firm's earning to GDP growth is about 3.4, so that for a typical firm and in a typical year, a 1% increase in GDP increases earnings by 3.4%. Table 1 shows first that firms with higher margins tend to have a lower sensitivity of their earnings to GDP growth. For instance, for manufacturing and with industry controls, a one standard deviation increase in the margin of the firm would make it less sensitive by 1.57 point, so that its sensitivity would be only about 3.4-1.57=1.83. This is a sizeable effect. It is also robust across samples and specifications and always significant, though its size varies a bit: the effect is stronger in manufacturing, and smaller when one accounts for different industry cyclicalities.

Table 5 next reveals that firms with high book-to-market have more procyclical earnings as well, and significantly so: a one-standard deviation of book-to-market increases the sensitivity of profits by slightly less than one.

Finally, the effect of productivity as measured by sales over employees is negative indeed before the industry controls are introduced, but turns positive and barely significant with the industry controls. Less productive industries are more procyclical. (By this measure of productivity, manufacturing is less productive, since sales per employee is smaller in manufacturing in table 3.) This may be because the productivity proxy is poor, or it may reflect a more direct failure of my model.

In Appendix, I show that these results are stable to a change in sample to 1978-2004 (thus eliminating possible compositional effects in the smaller Compustat sample 1963 to circa 1975). These results are also robust to the inclusion of fixed effects. Note finally that these regressions are run only for firms which have positive earnings in two consecutive years, so that these regressions exclude many interesting firms in difficulties. In a given year, there are about 11% of observations with negative earnings. These firms will be included however in our analysis using portfolios.

The right panel of Table 5 presents the results for the same regression with net income - "profit"- instead. This variable is after tax, after interest and after depreciation so it is even more volatile. For this variable the typical sensitivity is 4.5, and again firms with lower margins or higher book-to market are significantly more procyclical: a one standard deviation increase in book-to-market increases the sensitivity by about 1.5, and a one standard deviation decrease in margin increases the sensitivity by about 2 (except for one estimate, for which it is close to 1). The effect of productivity remains negative before industry controls but insignificant after the industry effects are accounted for.

| $x_{j,t} =$ | Coverage | Ind. | $\widehat{\delta}$ | t-stat | $\widehat{\xi}$ | t-stat | $R^2$ | $\widehat{\delta}$ | t-stat | $\widehat{\xi}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $OI/S$ | Manuf. | yes | **-0.92** | 3.0 | -3.52 | 8.1 | 0.14 | **-2.18** | 3.7 | **-0.29** | 0.3 | 0.05 |
| | | no | **-1.17** | 3.9 | -3.20 | 7.5 | 0.11 | **-2.39** | 5.0 | **0.11** | 0.1 | 0.04 |
| | All | yes | **-0.38** | 1.9 | -1.49 | 5.7 | 0.10 | **-1.28** | 3.6 | **0.76** | 1.6 | 0.03 |
| | | no | **-1.09** | 6.4 | -1.23 | 5.4 | 0.06 | **-2.28** | 7.9 | **0.85** | 2.1 | 0.02 |
| $B/M$ | Manuf. | yes | **1.06** | 3.9 | -1.02 | 2.6 | 0.04 | **1.92** | 4.4 | **-1.44** | 2.2 | 0.03 |
| | | no | **1.50** | 5.9 | -1.19 | 3.2 | 0.03 | **2.47** | 6.1 | **-1.90** | 3.1 | 0.03 |
| | All | yes | **0.92** | 4.8 | -0.58 | 2.2 | 0.03 | **1.68** | 5.4 | **-0.84** | 1.9 | 0.02 |
| | | no | **1.07** | 5.9 | -0.54 | 2.1 | 0.02 | **1.69** | 5.8 | **-0.55** | 1.3 | 0.01 |
| $S/E$ | Manuf. | yes | 0.93 | 2.3 | -1.25 | 2.5 | 0.04 | -0.17 | 0.3 | 0.70 | 0.8 | 0.03 |
| | | no | 0.09 | 0.3 | -1.63 | 3.5 | 0.02 | -0.72 | 1.5 | -0.05 | 0.1 | 0.02 |
| | All | yes | 0.27 | 1.2 | 0.12 | 0.4 | 0.03 | 0.48 | 1.4 | -0.39 | 0.9 | 0.02 |
| | | no | -0.55 | 3.1 | -0.43 | 1.9 | 0.02 | -0.86 | 3.2 | -0.26 | 0.7 | 0.01 |

Table 6: Results from the Pooled Panel OLS Regression:

Left panel: $\Delta \log OI_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \zeta \Delta \log w_t + \xi x_{j,t} \Delta \log w_t + \varepsilon_{j,t}$

Right panel: $\Delta \log NI_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \zeta \Delta \log w_t + \xi x_{j,t} \Delta \log w_t + \varepsilon_{j,t}$

Compustat 1963-2004. OLS standard errors.

Ind. = yes if industry controls (levels and interacted with GDP) are computed.

LHS is real % change of operating income (item 13; left panel)

or real % change of net income (item 172; right panel).

The RHS $x = OI/S, B/M$ or $S/E$ is in logs, standardized by the cross-sectional mean in year $t$.

**The "Old" Operating Leverage Does Not Explain the Value Premium**

Corporate finance textbooks mention a different "operating leverage" amplification mechanism which relies on fixed costs. The intuition is that firms differ in their amount of fixed costs. The higher the share of fixed costs, the more volatile the profits for a given change in sales, since a high share of costs do not vary with sales. To evaluate if this can account for the value premium, I run the following regressions:

$$\Delta \log OI_{j,t} = \alpha + \beta \Delta \log S_{j,t} + \gamma x_{j,t} + \delta x_{j,t} \Delta \log S_{j.t} + \varepsilon_{j,t}. \tag{4.2}$$

Table 4 shows that the coefficient $\delta$ is generally economically small and sometimes insignificant. This I interpret as evidence against the "old" operating leverage. My mechanism is different in two respects: (1) I examine how costs and revenue respond to aggregate shocks, and (2) whether costs are fixed or variable is irrelevant.

| $x_{j,t} =$ | Coverage | Industry Controls | $\widehat{\delta}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|
| Book-to-Market | Manuf. | yes | 0.06 | 3.6 | 0.31 |
| | | no | 0.01 | 0.4 | 0.30 |
| | All | yes | 0.02 | 1.8 | 0.30 |
| | | no | -0.01 | 1.2 | 0.29 |

Table 7: Tests of the "Old" Operating Leverage.

**Balanced Panel Regressions**

In ongoing work, I use a balanced panel of Compustat firms. The results are similar for these regression. This balanced panel will allow me in the future to study the dynamic response of a firm's output and profits to a macroeconomic shock, i.e. to go beyond one-period responses.

### B. The Sensitivity of Real Variables to GDP growth as a function of Margins, Productivity and Book-to-Market

Macroeconomists care more about production and employment decisions than distribution of income between workers and capitalists. The model of Section 2 predicts that firms with low productivity have higher responses of sales and employment to an increase in productivity. To test this, I run the same regression (4.1) with $Z$ = sales or employment. (A table in appendix describes the results for $Z$ = investment). Table 8 gives the results.

The typical sensitivity of sales (resp. employment) to GDP for Compustat firms is 1.9 (resp. 1.6), but there is significant heterogeneity in responses. For instance, a firm with a margin higher than the mean by one standard deviation has a sensitivity of sales which is smaller by about 0.3, and a manufacturing firm with a book-to-market higher than the mean by one standard deviation has a sensitivity of sales greater by about 0.3. For employment, the numbers are a bit smaller for book-to-market. Results with sales per employee as right-hand-side sorting variable remain fragile.

| $x_{j,t} =$ | Coverage | Industry | $\widehat{\delta}$ | t-stat | $R^2$ | $\widehat{\delta}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| Margin $OI/S$ | Manuf. | yes | **-0.19** | 1.5 | 0.05 | **-0.14** | 1.1 | 0.04 |
| | | no | **-0.32** | 2.6 | 0.03 | **-0.26** | 2.2 | 0.03 |
| | All | yes | **-0.15** | 1.5 | 0.03 | **-0.19** | 11.5 | 0.03 |
| | | no | **-0.58** | 7.1 | 0.02 | **-0.50** | 5.8 | 0.01 |
| Book-to-market | Manuf. | yes | **0.31** | 2.0 | 0.05 | **0.10** | 0.9 | 0.05 |
| | | no | **0.40** | 2.8 | 0.04 | **0.13** | 1.3 | 0.04 |
| | All | yes | **0.06** | 0.5 | 0.05 | **0.05** | 0.5 | 0.04 |
| | | no | **0.08** | 0.7 | 0.04 | **-0.03** | 0.3 | 0.03 |
| Productivity $S/E$ | Manuf. | yes | 0.07 | 0.4 | 0.10 | 0.26 | 1.8 | 0.03 |
| | | no | -0.32 | 1.90 | 0.08 | -0.07 | 0.6 | 0.01 |
| | All | yes | 0.20 | 1.60 | 0.07 | -0.08 | 0.7 | 0.03 |
| | | no | -0.39 | 3.70 | 0.05 | -0.51 | 5.6 | 0.01 |

Table 8: Results from the Pooled Panel Regression:

LEFT PANEL: $\Delta \log S_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$,

RIGHT PANEL: $\Delta \log N_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$.

Compustat 1963-2004. OLS Standard errors.

$\Delta \log S_{j,t} =$ real sales growth (item 12) and $\Delta \log N_{j,t} =$ employment growth (item 29)

## C. Explaining Portfolio Returns

I now examine if the model can explain the differences of expected returns across stocks by differences in their cash flow betas. Specifically, for each portfolio $i$ of stocks, I run the time-series regression:

$$\Delta \log OI_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \varepsilon_{i,t}, \tag{4.3}$$

and I test whether the $\beta_i$ line up against the average returns as Section 2 leads us to expect.[12]

I apply this analysis first to book-to-market sorted portfolios, and next to size- and book-to-market sorted portfolios. This is motivated by a large empirical literature in financial economics, centered around the contributions of Fama and French (1992, 1996) who show that these sorts generate puzzlingly large variations in expected returns.

[12] Of course, according to Section 2, this cash flow betas differences should also lead to return betas differences, but this requires us to make stronger assumption on the pricing kernel.(...)

To be able to do this regression, I re-construct the portfolios using Compustat as Fama and French did.[13] I compute for each portfolio the sum of the operating income (for instance) of all the firms in portfolio $i$ at time $t$, and I construct the sum of the operating income at year $t+1$ of all the firms that were in portfolio $i$ at time $t$.[14] This allows me to define the growth rate of real variables of the value-weighted portfolios.

**10 Book-to-Market Sorted Portfolios**

Table 9 shows the regression coefficient estimates of (4.3). I also provide estimates for a similar regression with net income[15], employment, investment and sales. The estimates of sensitivities are increasing in book-to-market, especially for earnings and net income. For these two variables, the spread in sensitivities is economically and statistically important: an increase of 1% of GDP increases earnings of the low book-to-market firms by about 1.5%, while it increases the earnings of high book-to-market firms by 5.5%. This spread of earnings explains well the differences in average returns as shown in Figure 4: the $R^2$ of a cross-sectional regression of average returns on the sensitivity is 0.93. The variable behind the sensitivity, according to Section 2, is the profitability. Figure 5 shows that differences in profitabilities also explain well the differences in average returns: in this case, the $R^2$ is 0.97. These plots suggest strongly that "value" stocks are risky simply because they have high cash flow risk.

The sensitivities of the "real" variables are also generally increasing, especially for the extreme portfolios, but less clearly than for earnings or net income. This is quite consistent with the model which says that the responses of sales and employment are multiplied by $\sigma$, which we assumed to be less than one in this paper following Result 3.

Table 10 adds the wage in this sensitivity regression, to test the independent effect of productivity and wages: I run

$$\Delta \log Z_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \gamma_i \Delta \log w_t + \varepsilon_{i,t}.$$

[13] In particular, I follow their definition of book-to-market and their definitions of deciles based on the NYSE stocks.

[14] Since firms change portfolios over time, I need to keep track of where the firms were in year $t$ to compute the growth between $t$ and $t+1$ of the operating income of firms in portfolio $i$ at $t$. I drop firms which disappear between $t$ and $t+1$. This could bias my results if these firms are more cyclical and there is more attrition in low book-to-market portfolios (and small) portfolios. This seems rather unlikely, but will be examined in future work.

[15] Running this regression requires that $Z_{i,t} > 0$ for all time periods. This is not true for net income in this case, which is negative in some periods for various portfolios. I restrict the sample of the net income regression to 1963-1991 and do not run it for portfolio 10. For these portfolios and this sample, I have positive net income at all times.

Overall, the sensitivities on GDP are not much affected. The high book-to-market portfolios are on average more sensitive (negatively) to the wage, as predicted by Section 2. This pattern is however not monotonic, with a couple of portfolios behaving differently.

| Portfolio $i =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_i$ for $Z_{i,t}$ = Earnings | **1.42** | **1.42** | **1.73** | **1.79** | **3.40** | **3.06** | **2.93** | **3.92** | **3.29** | **5.52** |
| t-stat | 3.42 | 2.72 | 1.85 | 2.87 | 5.10 | 6.64 | 4.21 | 4.73 | 5.39 | 7.20 |
| R2 | 0.24 | 0.16 | 0.11 | 0.16 | 0.37 | 0.29 | 0.30 | 0.42 | 0.28 | 0.41 |
| $\beta_i$ for $Z_{i,t}$ = Net Income | **1.79** | **2.75** | **3.06** | **2.67** | **6.28** | **7.49** | **6.88** | **9.26** | **17.97** | **na** |
| t-stat | 2.51 | 3.53 | 2.40 | 3.58 | 4.38 | 3.41 | 3.79 | 2.88 | 1.73 | na |
| R2 | 0.20 | 0.32 | 0.20 | 0.20 | 0.36 | 0.41 | 0.42 | 0.31 | 0.19 | na |
| $\beta_i$ for $Z_{i,t}$ = Employment | **0.86** | **1.23** | **0.84** | **1.40** | **1.34** | **1.20** | **1.30** | **1.37** | **1.17** | **1.69** |
| t-stat | 2.96 | 4.39 | 3.86 | 7.18 | 7.26 | 5.77 | 5.23 | 5.21 | 5.96 | 6.47 |
| R2 | 0.10 | 0.30 | 0.20 | 0.49 | 0.49 | 0.28 | 0.42 | 0.39 | 0.32 | 0.49 |
| $\beta_i$ for $Z_{i,t}$ = Investment | **2.12** | **2.06** | **1.70** | **1.63** | **1.86** | **2.31** | **2.49** | **3.51** | **3.20** | **5.34** |
| t-stat | 2.16 | 2.04 | 1.83 | 2.22 | 3.07 | 3.29 | 3.80 | 4.01 | 3.25 | 4.34 |
| R2 | 0.08 | 0.13 | 0.08 | 0.07 | 0.12 | 0.10 | 0.12 | 0.22 | 0.20 | 0.27 |
| $\beta_i$ for $Z_{i,t}$ = Sales | **0.81** | **0.93** | **0.78** | **0.60** | **1.25** | **1.14** | **1.46** | **1.32** | **1.28** | **2.39** |
| t-stat | 2.49 | 3.13 | 1.65 | 1.24 | 3.59 | 4.07 | 3.14 | 3.28 | 4.31 | 8.08 |
| R2 | 0.08 | 0.18 | 0.07 | 0.05 | 0.20 | 0.18 | 0.19 | 0.20 | 0.20 | 0.44 |

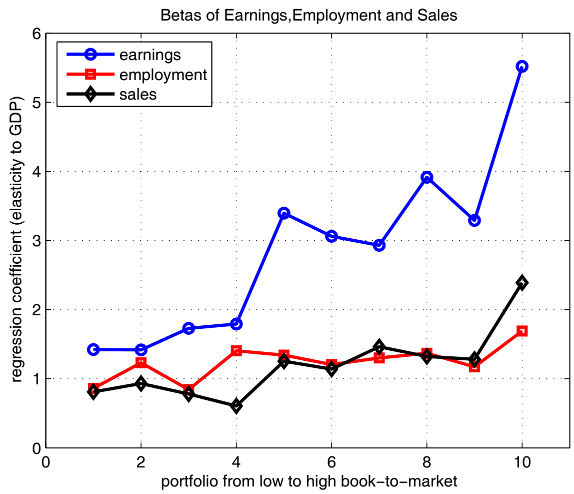
Table 9: Point estimates $\widehat{\beta}_i$, t-stats and $R^2$ of the time series regressions:

$$\Delta \log Z_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \varepsilon_{i,t}.$$

This OLS regression is run for each portfolio $i$

and for each of the variable $Z$ on the columns.

t-stat computed with Newey-West standard errors and 3 lags




$\beta_i$ of earnings, sales and employment on GDP, for each of the 10 book-to-market sorted portfolios. The $\beta_i$ are from Table 9.

| Portfolio $i =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_i$ for $Z_{i,t}$ = Earnings | **1.42** | **1.54** | **1.81** | **2.09** | **3.61** | **3.30** | **3.21** | **3.63** | **3.81** | **5.97** |
| t-stat | 3.43 | 3.29 | 2.10 | 3.76 | 4.90 | 7.91 | 5.40 | 4.71 | 5.57 | 7.07 |
| $\gamma_i$ for $Z_{i,t}$ = Earnings | **0.02** | **-0.61** | **-0.42** | **-1.46** | **-1.07** | **-1.16** | **-1.38** | **1.39** | **-2.57** | **-2.23** |
| t-stat | 0.04 | 0.78 | 0.41 | 1.17 | 1.08 | 0.87 | 1.25 | 0.98 | 3.00 | 1.89 |
| R2 | 0.24 | 0.17 | 0.11 | 0.19 | 0.39 | 0.30 | 0.32 | 0.44 | 0.34 | 0.44 |
| $\beta_i$ for $Z_{i,t}$ = Net Income | **2.04** | **2.78** | **3.82** | **2.83** | **6.26** | **8.03** | **7.73** | **10.59** | **21.15** | **na** |
| t-stat | 3.00 | 3.60 | 2.33 | 2.93 | 4.25 | 3.38 | 4.65 | 2.98 | 1.85 | na |
| $\gamma_i$ for $Z_{i,t}$ = Net Income | **-1.32** | **-0.14** | **-4.03** | **-0.83** | **0.11** | **-2.86** | **-4.46** | **-7.03** | **-16.71** | **na** |
| t-stat | 1.23 | 0.08 | 1.65 | 0.41 | 0.05 | 1.23 | 1.78 | 2.22 | 1.81 | na |
| R2 | 0.23 | 0.32 | 0.29 | 0.20 | 0.36 | 0.42 | 0.46 | 0.35 | 0.23 | na |

Table 10: Point estimates $\widehat{\beta}_i$ and $\widehat{\gamma}_i$, t-stats and $R^2$ of the time series regressions:

$$\Delta \log Z_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \gamma_i \Delta \log w_t + \varepsilon_{i,t}.$$

This OLS regression is run for each portfolio $i$, for $Z$ = operating income (earnings) and $Z$ = net income. Sample 1963-2004 for earnings, and 1963-1991 for net income. T-stat computed with Newey-West standard errors and 3 lags

### The 25 Fama-French Portfolios

I apply the same analysis to the 25 size- and book-to-market sorted portfolios constructed by Fama and French (1996; see Ken French's website). These portfolios have attracted tremendous

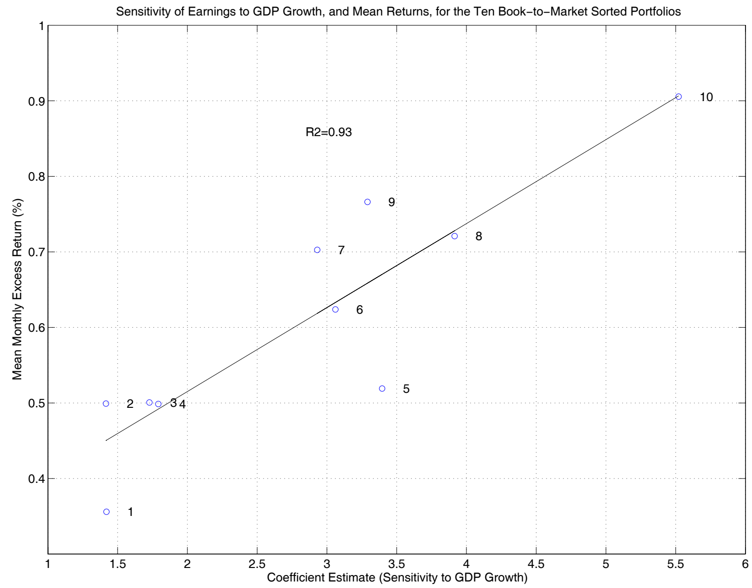



Figure 4: This figure plots the estimates of earnings sensitivity to GDP growth (Table 9), for each of the 10 book-to-market sorted portfolios, and the mean excess return per month. (Data from Ken French's website and from Compustat.)

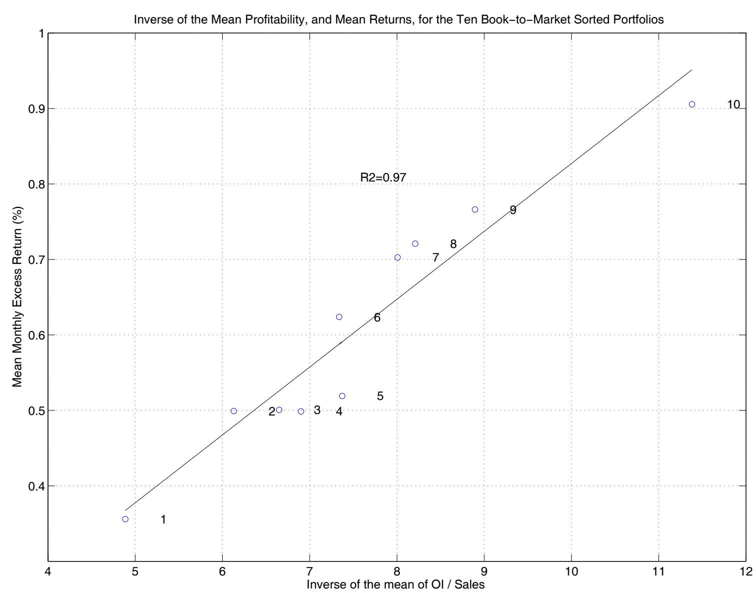



Figure 5: This figure plots for each of the 10 book-to-market sorted portfolios, the inverse of the mean over time of the ratio operating income over sales, against the mean monthly excess return. (Data from Ken French's website and from Compustat.)

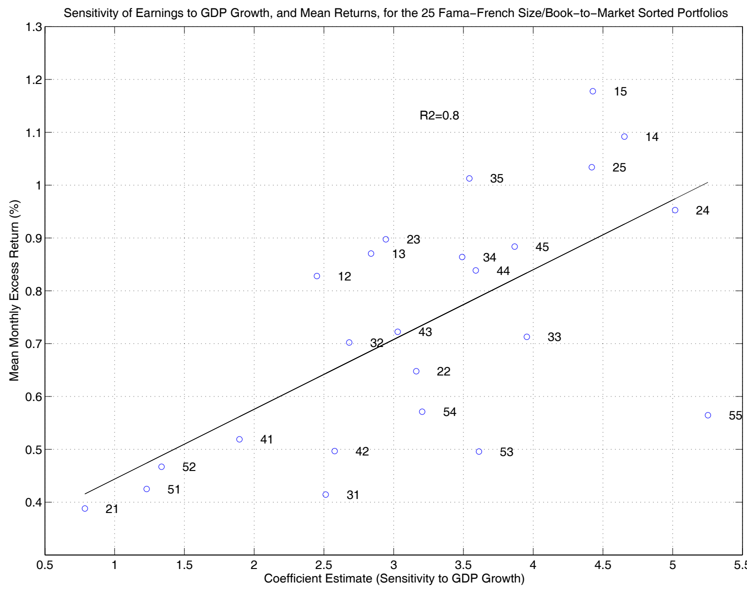



Figure 6: This figure plots the estimates of earnings sensitivity to GDP growth (Table A3), for each of the 25 Fama-French portfolio, and the mean excess return per month. (Data from Ken French's website and from Compustat.)

interest in the empirical finance literature. Tables A3 and A4 in appendix give the estimated $\beta_i$.[16]

Just as I did with the ten book-to-market sorted portfolios, Figures 6 and 7 summarize the association between earning sensitivity to GDP growth (resp. profitability) and mean returns. Again, the model explains very well

This explanation leaves us with a puzzle however. Why don't these differences in cash flow sensitivities show up in differences in betas? One possibility is that, as in Campbell and Vuolteenaho (2004), there is a second risk factor with a smaller market price of risk, for which these stocks have loadings in the opposite direction. It would be interesting to explain this second risk factor in term of the model.

[16] In this case, many stocks have negative net income, and so I am not able to run the regression with net income growth. As for earnings, only portfolio 1 (the small growth portfolio) has negative income, so it is (unfortunately) excluded from this picture.

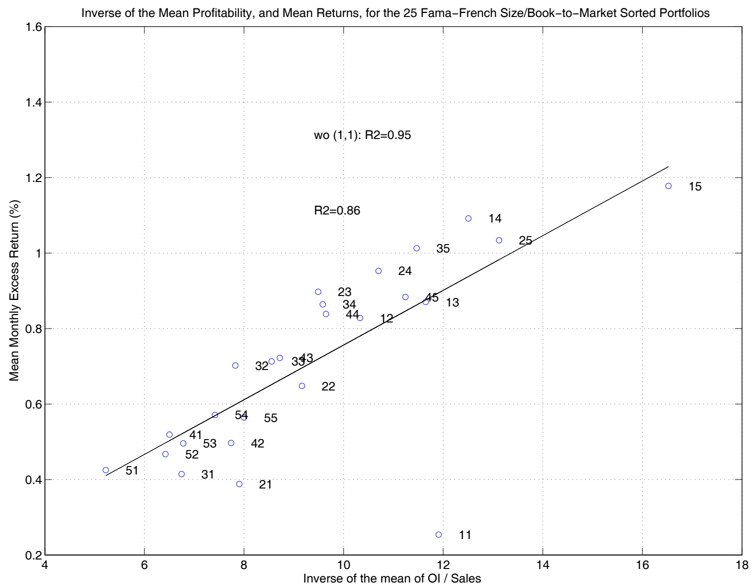



Figure 7: This figure plots for each of the 25 size and book-to-market sorted portfolios, the inverse of the mean over time of the ratio operating income over sales, against the mean monthly excess return. (Data from Ken French's website and from Compustat.)

## 5 Conclusions and Work in Progress

**Key conclusions:**

- This paper gives a full story of why some firms have higher reactions to macroeconomic shocks $\rightarrow$ why they have higher loadings on the factors $\rightarrow$ why the stocks have higher returns.

- Mechanism emphasized: difference in business cycle response of revenue and costs.

- "Old" operating leverage of corporate finance: costs are fixed b/c inputs are fixed. The analysis here makes clear that what matters is not whether costs are fixed or variable as a function of output but *how these different costs respond to aggregate shocks.*

- No *need* for growth options.

- Lots of testable empirical predictions, some right, some not so right - but this is in contrast with factor models which typically do not provide additional testable implications.

- Macro implications to explore.

**Work in progress:**

- More empirical work to bring directly the asset pricing relations of Section 2B to the data. More robustness analysis for the regressions presented here.
- Try factor pricing with aggregate productivity.
- Calibration and numerical simulations to improve and finish (Fama-French portfolios, etc.).
- Work with different assumptions than competitive wage-setting in the DGSE model.
- Numerical solution using non-linear methods to verify and add some results for the DSGE model.

**Future Work:**

- Modeling the different sensitivities to discount rate changes, an empirically important component of returns (Campbell and Vuolteenaho 2004).
- Using portfolios sorted on margins.
- Across industries patterns.

## References


1. Baily, Martin, Eric Bartelsman and John Haltiwanger. 2001. "Labor Productivity: Structural Change and Cyclical Dynamics". Review of Economics and Statistics, August.

2. Bansal, Ravi, Bob Dittmar and Chris Lundblad. 2004. "Consumption, Dividends, and the Cross-Section of Equity Returns". Journal of Finance, forthcoming.

3. Bartelsman, Eric, and Dunne. 2001. "Understanding Productivity: Lessons from Longitudinal Microdata" Journal of Economic Literature.

4. Bresnahan, Timothy, and Daniel Raff. 1991. "Intra-Industry Heterogeneity and the Great Depression: The American Motor Vehicles Industry, 1929-1935". The Journal of Economic History, 51:2, 317-331.

5. Campbell, John: "A Variance Decomposition for Stock Returns". Economic Journal, 101 (405): March 1991, pp. 157-179.

6. Campbell, John and Robert Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors". Review of Financial Studies, 1(3) Autumn, 1988, pp. 195-228.

7. Campbell, John and Tuomo Vuolteenaho. 2004. "Bad Beta, Good Beta". American Economic Review, forthcoming.

8. Carlson, Murray, Adlai Fisher and Ron Giammarino. 2004. "Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns ". Journal of Finance, 59, 2577-2603.

9. Carlson, Murray, Adlai Fisher and Ron Giammarino. 2005. Corporate Investment and Asset Price Dynamics: Implications for SEO Event Studies and Long-Run Performance, with M. Carlson and R. Giammarino. Forthcoming, Journal of Finance.

10. Cooper, Ian. 2006. "Asset Pricing Implications of Non-Convex Adjustment Costs of Investment". Forthcoming, Journal of Finance.

11. Dwyer, Douglas. 2001. "Plant-level productivity and the Market Value of a Firm". Census Bureau CES working paper.

12. Fama, Eugene and Kenneth French. 1992. "The Cross-Section of Expected Stocks Returns". Journal of Finance, 47:2, 427-465.

13. Fama, Eugene and Kenneth French. 1996. "Multifactor Explanations of Asset Pricing Anomalies". Journal of Finance, 51:1, 55-84.

14. Fama, Eugene and Kenneth French. 1997. "Industry costs of equity". Journal of Financial Economics, 43:2, 153-193.

15. Gala, Vito. 2005. "Investment and Return". Manuscript.

16. Gilchrist, Simon, and John Williams. 2000. "Putty-Clay and Investment: A Business Cycle Analysis", Journal of Political Economy, 108:5, 928-960.

17. Gomes Joao, Leonid Kogan, and Lu Zhang. 2003. "Equilibrium Cross Section of Returns". Journal of Political Economy, 114:4, 693-732.

18. Gomme, Paul and Jeremy Greenwood. "On the cyclical allocation of risk". JEDC, 19:1-2, 91-124.

19. Gourio, François. "Putty-Clay Technology and Stcock Market Volatility". Manuscript. 2005.

20. Gourio, François. "A Tractable Putty-Clay Model, with Applications to Capital Taxation and Labor Demand". Manuscript. 2006.

21. Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. 1997. "Long-Run Implications of Investment-Specific Technological Change". American Economic Review, 87:3, 342-362.

22. Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. 2001. "The role of investment-specific technological change in the business cycle". European Economic Review, 40:1, 91-115.

23. Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation". Econometrica, 50:1, 213-224.

24. Hansen, Gary and Edward Prescott. 2000. "Capacity Constraints, Asymmetries, and the Business Cycle". Working paper.

25. Hansen, Lars and Thomas Sargent. 1980. "Formulating and Estimating Dynamic Linear Rational Expectations Models", Journal of Economic Dynamics and Control, 2: 7-46.

26. Lettau Martin and Sydney Ludvigson. 2001. "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying". Journal of Political Economy. 109:6, 1238:1287.

27. Pakos, Michal. 2004. "Asset Pricing with Durable Goods and Non-Homothetic Preferences". Manuscript.

28. Piazzesi, Monika, Martin Schneider and Selale Tuzel. 2004. "Housing, consumption, and asset pricing". Manuscript.

29. Rotemberg, Julio and Michael Woodford. " ". in "Frontiers of Business Cycles"

30. Santos, Tanos and Pietro Veronesi. 2005. "Labor Income and Predictable Stock Returns". Review of Financial Studies.

31. Syverson, Chad. 2004. "Product Substitutability and Productivity Dispersion". Review of Economics and Statistics, May.

32. Yogo, Motohiro. 2004. "A Consumption-Based Explanation of Expected Stock Returns". Manuscript.

33. Zhang, Lu. 2005. "The Value Premium". Journal of Finance.

## 6 Appendix

### A. Algebraic reminders for Section 2

**Campbell and Shiller (1988)**

$$\begin{aligned} r_{t+1} &= \log(1+R_{t+1}) \\ &= \log\left(P_{t+1}+D_{t+1}\right)-\log(P_t) \\ &= p_{t+1}+\log\left(1+\frac{D_{t+1}}{P_{t+1}}\right)-p_t \\ &= p_{t+1}-p_t+\log\left(1+\exp\left(d_{t+1}-p_{t+1}\right)\right) \end{aligned}$$

$$\log\left(1+\exp\left(d_{t+1}-p_{t+1}\right)\right) \simeq k+(1-\rho)\left(d_{t+1}-p_{t+1}\right)$$

$$\begin{aligned} \rho &= \frac{1}{1+\exp\left(\overline{d-p}\right)} \\ k &= -\log\rho+(1-\rho)\log(1/\rho-1). \end{aligned}$$

$$r_{t+1}=\rho p_{t+1}-p_t+k+(1-\rho)d_{t+1}$$

$$p_t=\rho p_{t+1}+k+(1-\rho)d_{t+1}-r_{t+1}$$

$$\begin{aligned} p_t &= \frac{k}{1-\rho}+\sum_{j\geq 1}\rho^{j-1}\left((1-\rho)\,d_{t+j}-r_{t+j}\right) \\ &= \frac{k}{1-\rho}+\sum_{j\geq 0}\rho^{j}\left((1-\rho)\,d_{t+1+j}-r_{t+1+j}\right) \end{aligned}$$

**Hansen and Sargent (1980) Prediction Formulas**

First compute the expectation of a discounted sum. The $[.]_+$ operator deletes terms with negative powers.

$$\begin{aligned} E_t\sum_{j\geq 0}\beta^j A(L)\varepsilon_{t+j} &= E_t\sum_{j\geq 0}\beta^j A(L)L^{-j}\varepsilon_t \\ &= \left[\sum_{j\geq 0}\beta^j A(L)L^{-j}\right]_+\varepsilon_t \\ &= \left[\frac{LA(L)}{L-\beta}\right]_+\varepsilon_t \\ &= \frac{LA(L)-\beta A(\beta)}{L-\beta}\varepsilon_t. \end{aligned}$$

Apply this formula to compute the expectation as of $t-1$ :

$$E_{t-1}\sum_{j\geq 0}\beta^j A(L)\varepsilon_{t+j} = E_{t-1}E_t\sum_{j\geq 0}\beta^j A(L)\varepsilon_{t+j}$$

$$\begin{aligned}
&= E_{t-1}\frac{LA(L)-\beta A(\beta)}{L-\beta}\varepsilon_t \\
&= \frac{\frac{LA(L)-\beta A(\beta)}{L-\beta} - \frac{-\beta A(\beta)}{-\beta}}{L}\varepsilon_{t-1} \\
&= \frac{LA(L)-\beta A(\beta)-A(\beta)(L-\beta)}{L\,(L-\beta)}\varepsilon_{t-1} \\
&= \frac{A(L)-A(\beta)}{L-\beta}\varepsilon_{t-1}.
\end{aligned}$$

Thus

$$\begin{aligned}
(E_t - E_{t-1})\sum_{j\geq 0}\beta^j A(L)\varepsilon_{t+j} &= \left(\frac{LA(L)-\beta A(\beta)}{L-\beta} - \left(\frac{A(L)-A(\beta)}{L-\beta}\right)L\right)\varepsilon_t \\
&= A(\beta)\varepsilon_t.
\end{aligned}$$

## B. Computations for the DGSE model of Section 3

Bellman Equation

$$\begin{aligned}
V(M_1, M_2, z) &= \max_{c,n,I}\left\{U(c, 1-n) + \beta E_{z'/z}\left[V(M_1', M_2', z')\right]\right\} \\
s.t. &: \; c + \psi(I) = z(n - c_f \times M_2)^{\alpha} M_1^{1-\alpha} \\
M_1' &= (1-\delta)\zeta M_1 + I\upsilon \\
M_2' &= (1-\delta)M_2 + I
\end{aligned}$$

**Social Planner Problem and First-Order Conditions**

Social Planner Problem:

$$\max_{\{c_t, n_t, I_t, M_{1,t+1}, M_{2,t+1}\}_{t=0}^{\infty}} E\sum_{t\geq 0}\beta^t U(c_t, 1-n_t)$$

$$c_t + \psi\left(I_t\right) = z_t W(n_t) = z_t\left(n_t - c_f \times M_{2,t}\right)^{\alpha} M_{1,t}^{1-\alpha}.$$

$$M_{1,t+1} = (1-\delta)\zeta M_{1,t} + I_t\upsilon,$$

$$M_{2,t+1} = (1-\delta)M_{2,t} + I_t,$$

given $M_{1,0}$ and $M_{2,0}$.

Lagrangrean:

$$L = E\sum_{t\geq 0}\beta^t\begin{pmatrix} U(c_t, 1-n_t) + \lambda_t\left(z_t\left(n_t - c_f \times M_{2,t}\right)^{\alpha} M_{1,t}^{1-\alpha} - c_t - \psi\left(I_t\right)\right) \\ +\mu_{1,t}\left((1-\delta)\zeta M_{1,t} + I_t\upsilon - M_{1,t+1}\right) \\ +\mu_{2,t}\left(M_{2,t+1} - (1-\delta)M_{2,t} - I_t\right)\end{pmatrix}$$

First-order conditions:

$$\begin{aligned}
U_c(t) &= \lambda_t, \\
U_l(t) &= \lambda_t \alpha z_t \left(n_t - c_f \times M_{2,t}\right)^{\alpha-1} M_{1,t}^{1-\alpha}, \\
\psi'(I_t)\lambda_t &= \mu_{1,t}\upsilon - \mu_{2,t}, \\
\mu_{1,t} &= \beta(1-\delta)\zeta E_t\left(\mu_{1,t+1}\right) + \beta(1-\alpha)E_t\left(\lambda_{t+1}z_{t+1}\left(n_{t+1} - c_f \times M_{2,t+1}\right)^{\alpha} M_{1,t+1}^{-\alpha}\right), \\
\mu_{2,t} &= \beta(1-\delta)E_t\left(\mu_{2,t+1}\right) + \beta\alpha c_f E_t\left(\lambda_{t+1}z_{t+1}\left(n_{t+1} - c_f \times M_{2,t+1}\right)^{\alpha-1} M_{1,t+1}^{1-\alpha}\right).
\end{aligned}$$

**Steady-state**

The nonstochastic steady-state (i.e. setting $z = 1$) steady-state variables $(c, n, \lambda, \mu_1, \mu_2, I, M_1, M_2)$ satisfy:

$$\begin{aligned}
U_c(c, 1-n) &= \lambda, \\
U_l(c, 1-n) &= \lambda\alpha\left(n - c_f \times M_2\right)^{\alpha-1} M_1^{1-\alpha}, \\
\psi'(I)\lambda &= \mu_1\upsilon - \mu_2, \\
\mu_1\left(1 - \beta(1-\delta)\zeta\right) &= \beta(1-\alpha)\lambda\left(n - c_f \times M_2\right)^{\alpha} M_1^{-\alpha}, \\
\mu_2\left(1 - \beta(1-\delta)\right) &= \beta\alpha c_f\lambda\left(n - c_f \times M_2\right)^{\alpha-1} M_1^{1-\alpha}, \\
c + \psi(I) &= \left(n - c_f \times M_2\right)^{\alpha} M_1^{1-\alpha}, \\
M_1\left(1 - (1-\delta)\zeta\right) &= I\upsilon, \\
M_2\delta &= I.
\end{aligned}$$

Rewriting the system:

$$\begin{aligned}
U_c(c, 1-n) &= \lambda, \\
\frac{U_l(c, 1-n)}{U_c(c, 1-n)} &= \alpha\left(\frac{n}{M_1} - c_f \times \frac{M_2}{M_1}\right)^{\alpha-1}, \\
\psi'(I) &= \frac{\mu_1}{\lambda}\upsilon - \frac{\mu_2}{\lambda}, \\
\frac{\mu_1}{\lambda}\left(1 - \beta(1-\delta)\zeta\right) &= \beta(1-\alpha)\left(\frac{n}{M_1} - c_f \times \frac{M_2}{M_1}\right)^{\alpha}, \\
\frac{\mu_2}{\lambda}\left(1 - \beta(1-\delta)\right) &= \beta\alpha c_f\left(\frac{n}{M_1} - c_f \times \frac{M_2}{M_1}\right)^{\alpha-1}, \\
\frac{c + \psi(I)}{M_1} &= \left(\frac{n}{M_1} - c_f \times \frac{M_2}{M_1}\right)^{\alpha}, \\
M_1\left(1 - (1-\delta)\zeta\right) &= I\upsilon, \\
M_2\delta &= I.
\end{aligned}$$

One possible algorithm to solve this steady-state: first, pick a guess $n/M_1$. Then compute $M_2/M_1 = \frac{1-(1-\delta)\zeta}{\delta\upsilon}$ and $\frac{n}{M_1} - c_f \times \frac{M_2}{M_1}$. Thus obtain $\mu_1/\lambda$, $\mu_2/\lambda$ and $I$. Thus get $M_1, M_2, n$, and $\lambda$. Use the labor supply equation to check that the guess $n/M_1$ is correct. If not, adjust the guess.

**Log-linearization**

Let $w_t = \alpha z_t \left(n_t - c_f \times M_{2,t}\right)^{\alpha-1} M_{1,t}^{1-\alpha}$. Log-linearizing the system around this nonstochastic steady-state yields:[17]

$$
\begin{aligned}
\widehat{c}_t &= \varepsilon_{c\lambda}\widehat{\lambda}_t + \varepsilon_{cw}\widehat{w}_t \\
\widehat{n}_t &= \varepsilon_{n\lambda}\widehat{\lambda}_t + \varepsilon_{nw}\widehat{w}_t \\
\widehat{u}_t &= \left(\frac{n^*}{n^* - c_f M_2^*}\widehat{n}_t + \frac{-c_f M_2^*}{n^* - c_f M_2^*}\widehat{M}_{2,t}\right) \\
\widehat{w}_t &= \widehat{z}_t + (1-\alpha)\widehat{M}_{1,t} + (\alpha - 1)\widehat{u}_t \\
\frac{I^*\psi''(I^*)}{\psi'(I^*)}\widehat{I}_t + \widehat{\lambda}_t &= \frac{\mu_1^*\upsilon}{\mu_1^*\upsilon - \mu_2^*}\widehat{\mu}_{1,t} + \frac{-\mu_2^*}{\mu_1^*\upsilon - \mu_2^*}\widehat{\mu}_{2,t}, \\
\widehat{\mu}_{1,t} &= \beta(1-\delta)\zeta E_t\left(\widehat{\mu}_{1,t+1}\right) + (1 - \beta(1-\delta)\zeta)\, E_t\left(\widehat{\lambda}_{t+1} + \widehat{z}_{t+1} + \alpha\widehat{u}_{t+1} - \alpha\widehat{M}_{1,t+1}\right), \\
\widehat{\mu}_{2,t} &= \beta(1-\delta)E_t\left(\widehat{\mu}_{2,t+1}\right) + (1 - \beta(1-\delta))\, E_t\left(\widehat{\lambda}_{t+1} + \widehat{z}_{t+1} + (\alpha-1)\widehat{u}_{t+1} - (\alpha-1)\widehat{M}_{1,t+1}\right).
\end{aligned}
$$

$$
\frac{c}{c+\psi(I)}\widehat{c}_t + \frac{\psi(I)}{c+\psi(I)}\frac{I\psi'(I)}{\psi(I)}\widehat{I}_t = \widehat{z}_t + \alpha\widehat{u}_t + (1-\alpha)\widehat{M_{1,t}}
$$

$$
\widehat{M}_{1,t+1} = (1-\delta)\zeta\widehat{M}_{1,t} + (1-(1-\delta)\zeta)\,\widehat{I}_t,
$$

$$
\widehat{M}_{2,t+1} = (1-\delta)\widehat{M}_{2,t} + \delta\widehat{I}_t.
$$

## C. Data Sources for Section 5

The macroeconomic data comes from the BEA (www.bea.gov) and the BLS (ww.bls.gov). The stock returns and betas are from Ken French's website. I used the Compustat database and formed portfolios on book-to-market and on size and book-to-market as in Fama and French (1996).

**Sample Construction in Compustat**

I keep only firms with a december fiscal-year, to make the timing comparable to macroeconomics aggregates. I also keep only firms listed on the NYSE, the Nasdaq or the American Exchange. I do keep foreign-owned firms. I show the results both for the manufacturing sample and the full sample. I define book-to-market as Fama and French (1992, 1996) and drop outliers (book-to-market $> 20$ or $< 0.005$).

[17] I use the Rotemberg-Woodford (1995) notation for the system $U_c = \lambda$, $U_l = w\lambda$.

**Industry Controls**

The industry controls are industry dummies and industry dummies interacted with GDP growth. The industries are the 30 industry classifications used by Fama and French (see prof. French's website), which closely follow the 2 or 4-digit SIC codes.

D. Additional Empirical Results for Section 5

Table A1 gives the results of the earning sensitivity regression (4.1) for the sample 1978-2004. Table A2 gives the result of the regression (4.1) for $Z =$ investment.

| $x_{j,t} =$ | Coverage | Industry | $\widehat{\delta}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|
| $OI/S$ | Manuf. | yes | -2.56 | 6.3 | 0.14 |
| | | no | -2.86 | 7.1 | 0.12 |
| | All | yes | -0.98 | 3.9 | 0.10 |
| | | no | -1.71 | 7.8 | 0.07 |
| $B/M$ | Manuf. | yes | 1.36 | 3.6 | 0.04 |
| | | no | 1.93 | 9.6 | 0.03 |
| | All | yes | 1.10 | 4.5 | 0.03 |
| | | no | 1.19 | 5.1 | 0.02 |
| $S/E$ | Manuf. | yes | 1.22 | 2.3 | 0.03 |
| | | no | 0.38 | 0.9 | 0.02 |
| | All | yes | 0.34 | 1.3 | 0.02 |
| | | no | -0.40 | 1.8 | 0.01 |

Table A1: Results from the Pooled Panel Regression:

$$\Delta \log OI_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$$

**Compustat 1978-2004.**

| $x_{j,t} =$ | Coverage | Industry | $\widehat{\delta}$ | t-stat | $R^2$ |
|---|---|---|---|---|---|
| $OI/S$ | Manuf. | yes | -0.89 | 2.6 | 0.02 |
| | | no | -1.28 | 3.8 | 0.02 |
| | All | yes | -0.86 | 3.5 | 0.02 |
| | | no | -1.58 | 7.7 | 0.01 |
| $B/M$ | Manuf. | yes | 0.44 | 1.4 | 0.03 |
| | | no | 0.63 | 2.2 | 0.03 |
| | All | yes | 0.52 | 2.3 | 0.03 |
| | | no | 0.60 | 2.8 | 0.03 |
| $S/E$ | Manuf. | yes | -2.01 | 4.9 | 0.02 |
| | | no | -2.22 | 6.5 | 0.01 |
| | All | yes | -1.11 | 4.1 | 0.02 |
| | | no | -1.92 | 8.7 | 0.01 |

Table A2: Results from the Pooled Panel Regression:

$$\Delta \log I_{j,t} = \alpha + \beta \Delta \log GDP_t + \gamma x_{j,t} + \delta x_{j,t} \Delta \log GDP_t + \varepsilon_{j,t}$$

Compustat 1963-2004.

| # | $\beta_i$ Earnings | t-stat | R2 | $\beta_i$ Empl. | t-stat | R2 | $\beta_i$ Invt | t-stat | R2 | $\beta_i$ Sales | t-stat | R2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **na** | na | na | 1.32 | 2.02 | 0.04 | 3.36 | 1.81 | 0.07 | 0.88 | 1.23 | 0.02 |
| 2 | **2.45** | 0.61 | 0.02 | 1.89 | 1.33 | 0.11 | 2.83 | 0.74 | 0.03 | 1.69 | 0.46 | 0.01 |
| 3 | **2.84** | 3.50 | 0.24 | 2.17 | 6.45 | 0.37 | 5.31 | 6.13 | 0.36 | 1.94 | 4.47 | 0.30 |
| 4 | **4.65** | 3.69 | 0.27 | 1.75 | 5.85 | 0.40 | 5.93 | 3.57 | 0.41 | 2.59 | 6.64 | 0.46 |
| 5 | **4.43** | 7.32 | 0.47 | 1.71 | 5.55 | 0.46 | 5.86 | 3.59 | 0.38 | 2.39 | 4.88 | 0.44 |
| 6 | **0.79** | 0.71 | 0.02 | 1.56 | 2.24 | 0.09 | 2.58 | 1.68 | 0.05 | 2.02 | 2.08 | 0.12 |
| 7 | **3.16** | 5.13 | 0.45 | 2.26 | 5.03 | 0.42 | 5.17 | 5.15 | 0.35 | 1.56 | 3.84 | 0.33 |
| 8 | **2.94** | 3.42 | 0.32 | 2.39 | 8.33 | 0.45 | 5.37 | 3.80 | 0.26 | 2.33 | 8.14 | 0.46 |
| 9 | **5.02** | 4.71 | 0.35 | 2.33 | 4.75 | 0.32 | 7.81 | 6.85 | 0.47 | 2.99 | 3.74 | 0.19 |
| 10 | **4.42** | 7.52 | 0.37 | 1.47 | 5.68 | 0.40 | 3.50 | 2.83 | 0.12 | 1.72 | 6.07 | 0.28 |
| 11 | **2.51** | 5.10 | 0.24 | 2.47 | 3.86 | 0.10 | 1.29 | 1.01 | 0.02 | 1.66 | 4.31 | 0.23 |
| 12 | **2.68** | 5.77 | 0.35 | 1.24 | 5.16 | 0.21 | 3.12 | 3.49 | 0.16 | 1.64 | 6.47 | 0.32 |
| 13 | **3.95** | 4.51 | 0.37 | 1.27 | 5.89 | 0.33 | 4.46 | 4.20 | 0.27 | 1.65 | 4.19 | 0.28 |
| 14 | **3.49** | 5.60 | 0.43 | 1.75 | 3.87 | 0.34 | 5.18 | 5.08 | 0.31 | 1.89 | 5.30 | 0.39 |
| 15 | **3.54** | 4.96 | 0.25 | 1.20 | 2.87 | 0.21 | 2.83 | 2.54 | 0.10 | 1.89 | 3.38 | 0.25 |
| 16 | **1.89** | 2.93 | 0.31 | 1.92 | 4.97 | 0.38 | 3.25 | 3.53 | 0.19 | 1.36 | 4.42 | 0.20 |
| 17 | **2.58** | 5.65 | 0.37 | 1.60 | 5.85 | 0.37 | 3.24 | 2.94 | 0.19 | 1.45 | 4.55 | 0.22 |
| 18 | **3.03** | 8.12 | 0.44 | 1.20 | 6.20 | 0.38 | 3.32 | 3.90 | 0.21 | 1.25 | 5.56 | 0.29 |
| 19 | **3.59** | 4.42 | 0.32 | 0.86 | 2.91 | 0.16 | 3.50 | 5.99 | 0.16 | 1.45 | 4.51 | 0.29 |
| 20 | **3.87** | 4.86 | 0.25 | 1.01 | 3.53 | 0.22 | 2.62 | 1.95 | 0.07 | 1.10 | 3.46 | 0.12 |
| 21 | **1.23** | 3.67 | 0.22 | 0.63 | 4.06 | 0.10 | 1.92 | 2.02 | 0.09 | 0.69 | 3.53 | 0.12 |
| 22 | **1.34** | 1.76 | 0.08 | 0.82 | 3.73 | 0.24 | 1.16 | 1.80 | 0.04 | 0.32 | 0.81 | 0.02 |
| 23 | **3.61** | 6.18 | 0.41 | 1.18 | 4.64 | 0.36 | 1.48 | 2.23 | 0.07 | 1.16 | 3.04 | 0.15 |
| 24 | **3.20** | 6.28 | 0.32 | 1.17 | 6.67 | 0.38 | 2.52 | 3.84 | 0.14 | 1.13 | 4.10 | 0.16 |
| 25 | **5.25** | 4.52 | 0.32 | 1.56 | 7.81 | 0.40 | 4.65 | 5.08 | 0.24 | 1.90 | 5.64 | 0.28 |

Table A3: Point estimates $\widehat{\beta}_i$, t-stats and $R^2$ of the time series regressions:

$$\Delta \log Z_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \varepsilon_{i,t}.$$

This OLS regression is run for each portfolio $i$

and for each of the variable $Z$ on the columns.

t-stat computed with Newey-West standard errors and 3 lags

| # | $\widehat{\beta}_i$ | t-stat | $\widehat{\gamma}_i$ | t-stat | R2 |
|---|---|---|---|---|---|
| 1 | na | na | na | na | na |
| 2 | 2.34 | 0.70 | 0.54 | 0.13 | 0.02 |
| 3 | 2.98 | 3.85 | -0.72 | 0.70 | 0.25 |
| 4 | 4.72 | 3.48 | -0.34 | 0.35 | 0.27 |
| 5 | 4.54 | 7.58 | -0.55 | 0.57 | 0.47 |
| 6 | 0.87 | 0.97 | -0.42 | 0.24 | 0.02 |
| 7 | 3.17 | 5.63 | -0.04 | 0.04 | 0.45 |
| 8 | 3.03 | 3.17 | -0.43 | 0.39 | 0.32 |
| 9 | 4.89 | 4.05 | 0.64 | 0.41 | 0.35 |
| 10 | 5.28 | 8.41 | -4.20 | 2.66 | 0.49 |
| 11 | 2.70 | 4.99 | -0.90 | 0.91 | 0.26 |
| 12 | 2.79 | 6.35 | -0.53 | 0.78 | 0.36 |
| 13 | 4.11 | 4.05 | -0.78 | 0.68 | 0.38 |
| 14 | 3.45 | 4.83 | 0.18 | 0.13 | 0.43 |
| 15 | 4.11 | 5.52 | -2.77 | 2.30 | 0.31 |
| 16 | 1.92 | 3.16 | -0.13 | 0.21 | 0.31 |
| 17 | 2.72 | 6.86 | -0.68 | 0.69 | 0.38 |
| 18 | 3.28 | 6.38 | -1.25 | 1.40 | 0.47 |
| 19 | 3.58 | 5.03 | 0.06 | 0.05 | 0.32 |
| 20 | 4.69 | 6.68 | -4.04 | 3.19 | 0.35 |
| 21 | 1.27 | 4.12 | -0.21 | 0.37 | 0.22 |
| 22 | 1.54 | 2.27 | -1.00 | 0.93 | 0.09 |
| 23 | 3.88 | 5.83 | -1.29 | 1.17 | 0.43 |
| 24 | 3.18 | 6.03 | 0.12 | 0.09 | 0.32 |
| 25 | 5.83 | 4.35 | -2.81 | 2.14 | 0.36 |

Table A4: Estimates of $\Delta \log Z_{i,t} = \alpha_i + \beta_i \Delta \log GDP_t + \gamma_i \Delta \log w_t + \varepsilon_{i,t}$, for the 25 Fama-French portfolios. This OLS regression is run for each portfolio $i$, for $Z =$ operating income (earnings). Sample 1963-2004, T-stat computed with Newey-West standard errors and 3 lags