Growth-Indexed Bonds in Emerging Markets: a Quantitative Approach

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Abstract

In the aftermath of sovereign defaults and financial crises in the 1990s, there have been calls for the widespread use by sovereigns of equity-like financial instruments, in particular, of GDP-indexed bonds. This paper calibrates a general equilibrium model with endogenous default to a typical emerging market economy and evaluates whether the existence of a (partially) GDP-indexed bond, as proposed in the literature, is quantitatively important in what concerns spreads, debt to GDP ratios, and the likelihood of default.

Keywords: Sovereign debt, GDP-indexed bonds; Spreads; Emerging markets; Default.

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1 Introduction

The sovereign defaults and financial crises of the 1990s renewed the calls for the establishment of institutions that would minimize the likelihood of occurrence of these events.

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According to Rogoff (1999), "[t]he main problem with the present system is that it contains strong biases towards debt finance [...] and does not adequately support equity finance and direct investment," which would enhance enhance risk sharing, leading to more efficient allocations and higher growth (Obstfeld, 1994). Though debt contracts tend not to be explictly state contingent, the efficient contract may implicitly provide partial insurance to the borrower, allowing for repayments contingent upon the realization of certain states of the nature, thus making default excusable when the borrower is hit by certain shocks (Grossman and Van Huyck, 1988; Bulow and Rogoff, 1989a). However, evidence shows that default is often associated with output drops and may indeed be at the origin of such losses (Chuhan and Sturzenegger, 2003, revise some of the empirical evidence). An explicit risk-sharing mechanism that would minimize the likelihood of default would also be likely to eliminate the amplification effects on output associated with default episodes.

The sovereign is still the most important external debtor in most of the emerging market countries [check, build table with GDF data]. This poses an additional difficulty to the creation of risk-sharing-enhancing financial instruments because of the well-known conflict between incentives and insurance motives. A natural candidate would be revenue-contingent claims; however, moral hazard and the high cost of monitoring to avoid the sovereign shirking on its job of revenue collection or mis-reporting that collection make the existence of revenue-contingent financial claims very hard to sustain. As a way of circumventing the inability of sovereigns to issue equity, Borensztein and Mauro (2004) ressuscitated the proposal to create of GDP-indexed bonds.

The idea to create GDP-indexed bonds comes on the tradition of several proposals to create explicit contingent debt contracts. The main common idea of these proposals is that debt contracts should be indexed to a variable that is outside of the control of the sovereign (like commodity prices), otherwise the moral hazard problem would subside. Among all the possible benefits related with the creation of GDP-indexed bonds, probably the most important is the insurance it provides to borrower countries, as GDP growth works as an aggregator of several types of shocks (commodity prices, terms of trade, catastrophes, etc) that can hit the economy. The problems of adoption of low-growth policies and mis-reporting however still persist. To this objection, Borensztein and Mauro (2004) counter that, if anything, it is high growth, not low growth, that leads politicians to re-election; that is, the interests of politicians may indeed be aligned with those of the international investors. However, as recognized by these authors, financial instruments indexed to a statistical indicator would be easier to create by countries that comply with reliable statistics and that are credibly commited to sound policies. Financial instruments of this tiype are not unheard off: inflation-indexed bonds are an example of the feasibility to index bonds to statistical indicators that are, most of the times, produced by government agencies.[1]

This paper examines whether the quantitative implications of (partially) GDP-indexed bonds for spreads, debt to GDP ratios, and probabilities of default are substantially different from those implied by standard debt instruments. With this purpose, this paper develops a general equilibrium model with endogenous default thatbuilds on the seminal work by Eaton and Gersovitz (1981) and more recent work by Arellano (2005), and Aguiar and Gopinath (2006). A quantitative exercise as the one presented in this paper is especially relevant in a setup in which the underlying stochastic process for output appears as a crucial component to evaluate the benefits of risk sharing (Aguiar and Gopinath, 2005; Jeanne and Gourinchas, 2005), the decisions to default and the implied interest rates (Arellano, 2005; Aguiar and Gopinath, 2006). Moreover, this paper can be seen as a complement to previous work by Chamon and Mauro (2005); however, whereas their paper aimed to develop a pricing model that would be easy and ready to use by market practitioners, this paper aims to evaluate whether the existence of (partially) GDP-indexed bonds makes a difference from a quantitative point of view. In particular, this paper recognizes the endogenous nature of default decisions, whereas Chamon and Mauro (2005) use ad-hoc trigger rules of default.[2]

[1] Borensztein and Mauro (2004) do an extensive review of the history of the ideas behind their proposal as well as the history of financial innovation at the sovereign level, providing some examples of existing equity-like instruments for sovereigns. They also review in a comprehensive way the benefits, obstacles, and possible solutions associated with the creation of GDP-indexed bonds. They also briefly cover the alternative of indexing bonds to exports.

[2] General equilibrium models have been widely used for asset pricing since the seminal work by Lucas (1978). For ane xtensive survey of the literature, see Cochrane (2001).

The remainder of the paper is organized as follows. Section II describes the basic model, with the standard one-period uncontingent bond when ouput processes allow for a stochastic trend. Section III extends the model to incorporate (partially) GDP-indexed bonds. Section IV calibrates and computes the model for several emerging market economies. Section V concludes.

2 The model

2.1 The basic model

Consider a small open economy whose sovereign is the only agent with access to international capital markets. The sovereign is a benevolent planner who maximizes the expected lifetime utility of a risk averse representative agent

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

whose instantaneous utility function $u(\bullet)$ satisfies the standard assumptions in the literature: it is twice continuously differentiable, stricly increasing, and stricly concave.

The sovereign can issue one period bonds at face value against the promise to pay an interest rate defined at the date of issue. However, contracts are not enforceable. At the beginning of each period, the sovereign observes the endowment, y_t, and then decides whether to default or to honor its obligations:

$$V(b_t, y_t) = \max \left\{ V^C(b_t, y_t), V^D(y_t) \right\} \tag{2}$$

If the sovereign decides not to default, it can borrow (or save) by selling (buying) bonds $b_{t+1} > 0$ (< 0) in the international markets at a gross interest rate $R(b_{t+1}, y_t)$; the promised interest rate depends on the amount borrowed (b_{t+1}), and on the endowment shock at time t (y_t). For all $b_{t+1} \leq 0$, $R(b_{t+1}, y_t) = R_{t+1}$, regardless of the endowment shock and the amount borrowed, where R_{t+1} is the world risk-free (gross) interest rate

(U.S. treasury bonds). The value of not defaulting for the sovereign is then given by

$$V^C(b_t, y_t) = \max_{c_t, b_{t+1}} \left\{u(c_t) + \beta E_t \max \left\{V^C(b_{t+1}, y_{t+1}), V^D(y_{t+1})\right\}\right\}$$
$$s.t.$$
$$c_t + b_{t+1} = y_t + R(b_t, y_{t-1}) b_t.$$

If a country decides to default, then it is excluded from the international markets. However, there is a constant exogenous probability θ that the country main regain access to the international capital markets.[3] Once in autarky, the country can only consume a fraction α of its endowment, αy_t, $0 \leq \alpha \leq 1$.[4] Therefore, the value of defaulting is

$$V^D(y_t) = u(\alpha y_t) + \beta(1-\theta) E_t V^D(y_{t+1}) + \beta\theta E_t V^C(0, y_{t+1}).$$

International investors are risk-neutral agents who compete in an environment without barriers to entry. Therefore, the return they get from lending to this sovereign must equal, in expected terms, the return they would get from investing in risk-free assets:

$$R_t = (1 - \pi(b_{t+1}, y_t)) R(b_{t+1}, y_t),$$

where

$$\pi(b_{t+1}, y_t) = \Pr\left\{V^D(y_{t+1}) \geq V^C(b_{t+1}, y_{t+1}) / y_t\right\}$$

is the probability the sovereign defaults at date $t+1$ given she borrowed b_{t+1} and endowment at date t was y_t.

[3] Here, I follow the current strand of literature on sovereign debt. The literature on sovereign default models postulates the exclusion of a defaulting country from international capital markets from that moment on, with an exogenous probability of re-entry (see, for example, Aguiar and Gopinath, 2006, and Arellano, 2005; Eaton and Gersovitz, 1981, build an extreme of this case, excluding defaulters forever).

[4] Reductions in output as a consequence of defaults have been identified by Chuan and Sturzenegger (2002), and incorporated in sovereign debt models by Cole and T. Kehoe (1998), Alfaro and Kanczuk (2005), Arellano (2005), and Aguiar and Gopinath (2006). Dooley (2000) builds a model that provides micro-foundations for this fact.

2.2 The nature of output shocks

Emerging market economies are characterized by a higher volatility of output than developed economies. Moreover, as shown by Aguiar and Gopinath (2005), emerging markets experience extremely volatile shocks to a stochastic trend, that is, business cycle fluctuations in these countries are explained mostly by permanent shocks, instead of by transitory shocks.[5] In another paper, Aguiar and Gopinath (2006) show that a model with shocks to the trend (permanent shock) is able to match the pattern of default of emerging market economies, a feature that a model with only transitory shocks is not able to replicate. Transitory shocks do not substantially affect either the value of defaulting, the value of financial integation, or the relative value of both decisions, and so the decision to default comes mostly from the level of debt outstanding; knowing this fact, international investors demand an interest rate schedule steep enough such that sovereigns seldom find it optimal to hold levels of debt that would induce them to default (Arellano, 2005; Aguiar and Gopinath, 2006). However, as explained by Aguiar and Gopinath (2006), shocks to the trend substantially affect both the value of defaulting and the value of financial integration, and, more importantly, the relative value of the two decisions. The decision to default is then more sensitive to the realization of shocks, and less so to the amount of debt held, thus generating an interest rate schedule less sensitive to debt holdings, which leads to sovereigns issuing debt amounts that make default more likely.

Given the evidence on the importance of shocks to the trend for emerging markets, this section revamps the model presented above to accomodate output processes with both transitory and permanent shocks. The notation and the modelling options follow closely Aguiar and Gopinath (2006). The endowment has two components, a transitory element z_t and a trend Γ_t,

$$y_t = e^{z_t}\Gamma_t.$$

[5] Most of these shocks have their origins in changes in government policy. This is an important argument against the applicability of GDP-indexed bonds to these countries. However, as Borensztein and Mauro (2004) emphasized, it may be a good way to borrow for emerging market countries credibly commited to good policies. It is out of the scope of this paper to discuss the the nature and existence of commitment mechanisms that would tie governments with a good policy.

The transitory shock follows an autoregressive process $AR(1)$ around a mean μ_z

$$z_t = \mu_z (1 - \rho_z) + \rho_z z_{t-1} + \varepsilon_t^z,$$

with $|\rho_z| < 1$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$. The growth rate of the trend, g_t, has a mean μ_g

$$\Gamma_t = g_t \Gamma_{t-1},$$

and its logarithm follows an autoregressive process $AR(1)$

$$\ln(g_t) = \left(1 - \rho_g\right)\left(\ln\left(\mu_g\right) - \upsilon\right) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g,$$

with $\left|\rho_g\right| < 1$, $\varepsilon_t^g \sim N\left(0, \sigma_g^2\right)$, and $\upsilon = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.

To accomodate these stochastic processes for the transitory component and for the trend, the state variable y_t of the basic model is replaced by the pair (z_t, g_t), and so the sovereign's value function at date t takes the form:

$$V(b_t, z_t, g_t) = \max \left\{V^C(b_t, z_t, g_t), V^D(z_t, g_t)\right\},$$

where

$$\begin{aligned} V^C(b_t, z_t, g_t) &= \max_{c_t, b_{t+1}} \{u(c_t) + \beta E_t V(b_{t+1}, z_{t+1}, g_{t+1})\} \\ s.t.& \\ c_t + b_{t+1} &= y_t + R(b_t, z_{t-1}, g_{t-1})\, b_t, \end{aligned}$$

and

$$V^D(z_t, g_t) = u(\alpha y_t) + \beta(1-\theta) E_t V^D(z_{t+1}, g_{t+1}) + \beta\theta E_t V^C(0, z_{t+1}, g_{t+1}).$$

As before, the non-arbitrage condition for the interest rates holds

$$R_t = (1 - \pi (b_{t+1}, z_t, g_t)) R (b_{t+1}, z_t, g_t),$$

with

$$\pi (b_{t+1}, z_t, g_t) = \Pr \left\{ V^D (z_{t+1}, g_{t+1}) \geq V^C (b_{t+1}, z_{t+1}, g_{t+1}) / z_t, g_t \right\}$$

being the probability the sovereign defaults at date $t+1$ given she borrowed b_{t+1} and endowment shocks at date t were z_t and g_t.

As Cochrane (2001) emphasizes, any pricing equation takes the form of

$$E_t \left[m_{t+1} x_{t+1}^i \right] = q_t^i,$$

where m_{t+1} is the stochastic discount factor, x_{t+1}^i is the payoff of security i, and q_t^i is its price; in terms of returns, this same expression takes the form of

$$E_t \left[m_{t+1} R_{t+1}^i \right] = 1,$$

where R_{t+1}^i is the *marginal* return of security i. For standard asset pricing models, agents are *small* in the sense that they take the returns as given, so total returns are linear in quantities, thus making indifferent the distinction between marginal and average return. However, in the present model, the sovereign is not small as she is aware that the interest rate charged depends on the amount borrowed, so marginal and average returns are *different*: the average expected return is $(1 - \pi (b_{t+1}, z_t, g_t)) R (b_{t+1}, z_t, g_t)$ whereas the marginal return, assuming differentiability of $\pi (\bullet, z_t, g_t)$ for the sake of expositional simplicity, is $\frac{\partial (1-\pi(b_{t+1}, z_t, g_t)) R(b_{t+1}, z_t, g_t) b_{t+1}}{\partial b_{t+1}}$. Therefore, the pricing equation writes as

$$E_t \left[m_{t+1} \frac{\partial (1 - \pi (b_{t+1}, z_t, g_t)) R (b_{t+1}, z_t, g_t) b_{t+1}}{\partial b_{t+1}} \right] = E_t [m_{t+1} R_{t+1}] = 1,$$

where $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$.

3 The model with (partially) GDP-indexed bonds

Consider now an economy in all identical to the economy described in the above, with a single exception: whereas in the previous economy the sovereign only has access to one period uncontingent bonds, in this economy the sovereign can issue bonds whose payoffs are a function of the state of the nature. In particular, bonds issued at date t pay an interest $\tilde{R}(b_{t+1}, z_t, g_t)$ plus a differential $\gamma_{t+1} - \bar{\gamma}$, where γ_{t+1} is the growth rate of output from date t to date $t+1$, and $\bar{\gamma}$ is a reference value (for example, a historical average) above (below) which the borrower pays a premium (discount) on the interest rate contracted at date t.[6] As suggested by Borensztein and Mauro (2004), a cap is imposed on the discount such that the (net) interest rate paid at date $t+1$ is never negative:

$$\hat{R}\left(b_{t+1}, z_t, g_t, \gamma_{t+1}\right) = \max\left\{1, \gamma_{t+1} - \kappa\left(b_{t+1}, z_t, g_t\right)\right\}, \text{ for } b_{t+1} > 0, \text{ and} \tag{3}$$

$$\hat{R}\left(b_{t+1}, z_t, g_t, \gamma_{t+1}\right) = \tilde{R}\left(b_{t+1}, z_t, g_t\right) = R_t \text{ for } b \leq 0, \tag{4}$$

where

$$\gamma_{t+1} = \frac{y_{t+1}}{y_t},$$

and

$$\kappa\left(b_{t+1}, z_t, g_t\right) = \bar{\gamma} - \tilde{R}\left(b_{t+1}, z_t, g_t\right).$$

Whereas in the previous economy interest rates to be paid in date $t+1$ are contracted and known at date t, in this economy, for $b_{t+1} > 0$, the interest rate to be effectively paid at date $t+1$ is unknown at date t. In this case, the sovereign sells a call option with a unit price and a strike price that is a function of the amount borrowed and the state of the economy $\kappa(b_{t+1}, z_t, g_t) = \bar{\gamma} - \tilde{R}(b_{t+1}, z_t, g_t)$.[7] Whenever the economy grows above the strike price, $\gamma_{t+1} > \kappa(b_{t+1}, z_t, g_t)$, the option pays a positive payoff. If the economy grows exactly at the reference level $\bar{g}$, then the option pays the interest rate contracted at date

[6] The interest rate $\tilde{r}(b_{t+1}, y_t)$ does not have to coincide with $r(b_{t+1}, y_t)$, so a different notation applies.

[7] In this paper, the strike price is endogenous. This is another difference with relation to the work by Chamon and Mauro (2005): in their paper the interest rate contracted at date t is assumed, and imposed to be the same as that of an uncontingent one period bond, i.e., $r(b_{t+1}, y_t) = \hat{r}(b_{t+1}, y_t)$.

t, $\tilde{R}(b_{t+1}, z_t, g_t)$. If the economy grows above the reference level, then the option pays the interest rate contracted at date t, plus a premium equal to the growth rate differential, $\gamma_{t+1} - \bar{\gamma}$.

As before, the sovereign decides whether to default or to honor its obligations after observing the realization of shocks to the growth rate

In this economy, the value of not defaulting is then given by

$$V^C(b_t, y_t, g_t) = \max_{c_t, b_{t+1}} \left\{ u(c_t) + \beta E_t \max \left\{ V^C(b_{t+1}, y_{t+1}, g_{t+1}), V^D(y_{t+1}) \right\} \right\}$$

$$s.t.$$

$$c_t + b_{t+1} = y_t + (1 + \hat{r}(b_t, y_{t-1}, g_t))\, b_t.$$

As before, the value of defaulting is

$$V^D(y_t) = u(\alpha y_t) + \beta(1-\pi) E_t V^D(y_{t+1}) + \beta\pi E_t V^C(0, y_{t+1}, g_{t+1}),$$

and so after observing the endowment y_t and the growth rate g_t, the sovereign decides whether to default or to nor its obligations

$$V(b_t, y_t, g_t) = \max \left\{ V^C(b_t, y_t, g_t), V^D(y_t) \right\} \tag{5}$$

4 Data and calibration

Calibration and data follow previous work by Arellano (2005) and Aguiar and Gopinath (2006).

5 Conclusions

References

[1] Aguiar, Mark, and Gita Gopinath, 2005, "Emerging Market Business Cycles: The Cycle is the Trend," NBER Working Paper No. 10307 (Cambridge, Massachusetts: National Bureau of Economic Research).

[2] Aguiar, Mark, and Gita Gopinath, 2006, "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, forthcoming.

[3] Alfaro, Laura, and Fabio Kanczuk, 2005, "Sovereign Debt as a Contingent Claim: A Quantitative Approach," *Journal of International Economics 65*: 297-314.

[4] Arellano, Cristina, 2005, "Default Risk, the Real Exchange Rate, and Income Fluctuations in Emerging Economies,"unpublished, University of Minnesota.

[5] Borensztein, Eduardo, and Paolo Mauro, 2004, "The Case for GDP-Indexed Bonds," *Economic Policy, Vol. 38, April*: 167-216.

[6] Borensztein, Eduardo, Marcos Chamon, Olivier Jeanne, Paolo Mauro, and Jeromin Zettelmeyer, 2004, *Sovereign Debt Structure for Crisis Prevention*, IMF Occasional Paper 237 (Washington: International Monetary Fund).

[7] Bullow, Jeremy, and Kenneth Rogoff,1989a, "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review, 79 (1)*: 43-50.

[8] Bullow, Jeremy, and Kenneth Rogoff, 1989b, "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy 97*: 155-78.

[9] Chamon, Marcos, and Paolo Mauro, 2005, "Pricing Growth-Indexed Bonds," IMF Working Paper No. 05/216 (Washington, District of Columbia: International Monetary Fund).

[10] Chuhan, Punam, and Federico Sturzenegger, 2003, "Default Episodes in the 1980s and 1990s: What Have We Learned?" unpublished, World Bank and Universidad Torcuato di Tella.

[11] Cochrane, John H., 2001, *Asset Pricing* (Princeton, New Jersey: Princeton University Press).

[12] Dooley, Michael P, 2000, "Can Output Losses Following International Financial Crises be Avoided?" NBER Working Paper No. 7531 (Cambridge, Massachusetts: National Bureau of Economic Research).

[13] Eaton, Jonathan, and Mark Gersovitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies 48 (2)*: 289-309.

[14] Grossman, Herschel I., and John B. Van Huyck, 1988, "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review 78 (5)*: 1088-97.

[15] Jeanne, Olivier, and Pierre-Olivier Gourinchas, 2005, "The Elusive Gains of Financial Integration," *Review of Economic Studies*, forthcoming.

[16] Lucas, Robert E., Jr., 1978, "Asset Pricing in an Exchange Economy," *Econometrica, Vol. 46 (6)*: 1429-45.

[17] Neumeyer, Pablo A., and Fabrizio Perri, 2005, "Business Cycles in Emerging Economies: The Role of Interest Rates," *Journal of Monetary Economics 52 (2)*: 345-80.

[18] Obstfeld, Maurice, 1994, "Risk-Taking, Global Diversification, and Growth," *American Economic Review, Vol. 85 (5)*: 1310-29.

[19] Reinhart, Carmen M., Kenneth Rogoff, and Miguel A. Savastano, 2003, "Debt Intolerance," in William Brainard and George Perry (eds.), *Brookings Papers on Economic Activity 1*: 1-74.

[20] Reinhart, Carmen M., and Kenneth Rogoff, 2004, "Serial Default and the 'Paradox' of Rich to Poor Capital Flows," *American Economic Review 94 (2)*: 52-8.

[21] Rogoff, Kenneth, 1999, "Institutions for Reducing Global Financial Instability," *Journal of Economic Perspectives Vol. 13*: 21-42.

[22] Uribe, Martin, and Vivian Z. Yue, 2005, "Country Spreads and Emerging Countries: Who Drives Whom?" *Journal of International Economics*, forthcoming.

[23] Yue, Vivian Z, 2005, "Sovereign Default and Debt Renegotiation," unpublished, New York University.