Asymmetric Information and the Lack of International Portfolio Diversification*

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Abstract

There is pervasive evidence that individuals invest primarily in domestic assets and thus hold poorly diversified portfolios. Empirical studies suggest that informational asymmetries may play a role in explaining the bias towards domestic assets. In contrast, theoretical studies based on asymmetric information fail to produce significant quantitative effects. The present paper develops a theoretical model in which the presence of informational asymmetries explains a significant fraction of the home equity bias observed in the data. The main departure from previous theoretical work is the assumption that local investors outperform foreign investors in identifying the correct ranking of local investment opportunities instead of possessing superior information about the aggregate performance of the domestic stock market. The other key assumption is based on the evidence that short-selling is a costly activity. This paper studies the case of a two-country world. There are two assets in each country. Only local investors receive informative signals about local assets. Thus, domestic agents have an incentive to concentrate their investments in the local asset favored by the signal realization, and reduce the position held in the other local asset. When the signal is sufficiently informative and short-sales are costly, local investors decide not to finance purchases of the perceived "good" local asset by selling short the perceived "bad" local asset. Instead they invest a lower fraction of their portfolio in foreign securities. This liberates resources that can be allocated in the local asset perceived to pay higher expected returns.

Keywords: *International portfolio diversification, home bias, asymmetric information.*

JEL Classification: *D82, F30, G11, G15*

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1 Introduction

The last decades have witnessed a remarkable increase in international capital flows. Gross cross-border transactions in bond and equity of US residents represented 4 percent of the GDP in 1975, but increased to 320 percent by 2003. Yet the fraction of the US equity portfolio invested overseas has remained quite low. Figure 1 provides a measure of the bias towards domestic stocks. The dashed line describes the fraction of foreign stock holdings in the equity portfolio of US residents. In order to assess whether those values are high or low, we need a theory. The solid line describes an index that interprets the data from the perspective of the International version of the Capital Asset Pricing Model (ICAPM). The latter predicts that in a frictionless world, the composition of every investor's portfolio coincides with the world market portfolio composition.[1] By construction, the index takes a value of one when the fraction of foreign stocks is zero, and takes a value of zero when the fraction of foreign equity holdings coincides with the prediction of the ICAPM. The graph shows that even though the bias has been receding over time, it is still significant. This discrepancy between the theory and the data is known in the literature as the *home equity bias puzzle*. It was initially documented by French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995).

This finding has motivated a vast body of literature. There may be various reasons why domestic investors are reluctant to invest abroad. For example, there are domestic regulations that limit the foreign exposure of institutional investors. Some foreign countries impose limits on the fraction of a firm that can be owned by non-nationals. Transaction costs may be higher for cross-border transactions. Exchange rate fluctuations increase the risk of investing in foreign assets if domestic investors care only about returns nominated in domestic currency. However, none of these factors have offered a satisfactory explanation. Lewis (1999) and Karolyi and Stulz (2003) offer a detailed survey of this literature.

An alternative explanation rests on the conjecture that local investors can collect more precise information about domestic assets.[2, 3] The typical setup in this literature features a two-country world with a single asset per country. Domestic investors receive informative signals about domestic and foreign assets, but the signal about the local asset is more precise. The reason why this generates home equity bias is that the informational disadvantage about the foreign asset turns it into a more

[1] Baxter and Jermann (1997) argue that the amount invested overseas should be even larger than that, given that the returns on human capital are correlated with the returns on domestic financial assets.

[2] See Gehrig (1993), Zhou (1998) and Coval (2000).

[3] Several papers find empirical evidence in favor of the presence of informational asymmetries in financial markets. A survey is provided in the next section.

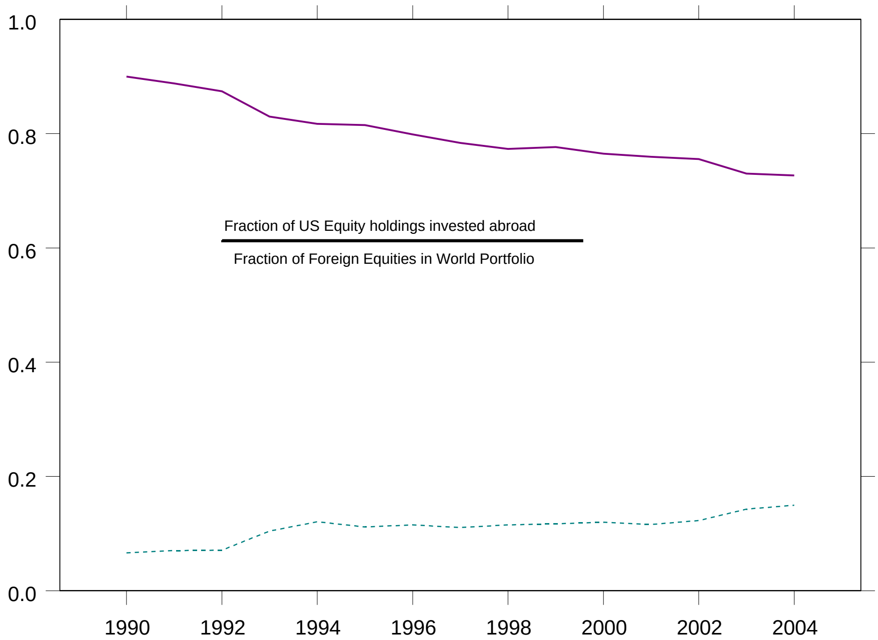

Figure 1: **Measures of foreign investment positions. Sources: Department of Commerce and International Federation of Stock Exchanges**

risky investment option from the perspective of local investors. However, this explanation faces two limitations. First, it relies on effects that are of second order importance. Once the parameters in the model are replaced with realistic values, the fraction of the bias that can be explained with the model is quite low. Second, this approach produces high volatility in the fraction invested overseas, which is not observed in the data.

The distinctive feature of this paper is that the information asymmetry between domestic and foreign investors relates to the performance of individual stocks rather than to the market portfolio. This paper assumes that domestic investors do not outperform foreign investors in predicting the performance of aggregate variables, but they do enjoy an advantage in identifying the best individual investment opportunities. The latter is motivated on the grounds that there is more scope for the existence of disparities in information about individual firms than about aggregates like the stock market. The second assumption that separates this work from previous theoretical papers is based on the extensive evidence that shows that short-selling is a costly activity.[4]

This paper maintains the two-country world setup but with two technologies in each country. All technologies are publicly listed. Each technology receives either a high productivity shock (high returns) or a low productivity shock (low returns). The probability distribution over the next period's productivity shock is not publicly observed. Instead, domestic investors receive an informative signal about local assets. A majority of domestic investors receives a signal favorable to the local asset that it is more likely to pay high returns. The remaining local investors receive an incorrect signal, encouraging investment in the local asset that pays high returns with the lowest probability. Each investor does not know whether the signal received is correct or not.[5]

Agents diversify their portfolios in two dimensions: across countries and across assets within each country. The information structure described above has two implications regarding the portfolio composition. First, agents want to concentrate the local component of their portfolios in the local technology favored by the signal realization (the "good" asset). Second, since the information received is uncorrelated with any index of aggregate performance, local investors do not have an incentive to invest more heavily in the domestic or foreign country. This implies that when the degree of information is such that local agents still demand a positive amount of both local assets, the home equity bias is nil. The result changes when the degree of information conveyed by the signal is such that agents are willing to finance the purchases of the perceived "good" local stock by selling short the perceived "bad" local stock. When short-selling is costly, local agents find it optimal to finance the purchases of the "good" local asset by investing less in foreign stocks, instead of short-selling the perceived "bad" local stock. This mechanism is what generates home bias in the model. The result is driven by an effect of first order importance, i.e., differences in expected returns. This allows us to explain a high fraction of the bias observed in the data for realistic parameters values. In addition, and in line with what we observe in the data, the fraction of foreign investments generated by the model is stable over time.

The paper is also consistent with other dimensions of the portfolio behavior of US investors. The paper predicts that the foreign component of the equity portfolio is more diversified than the local component. In addition, if the precision of the signal was allowed to vary across agents, the model would state that the fraction invested in foreign stocks is inversely proportional to the degree of concentration

[4]See section 2.1 on page 10.

[5]For simplicity, it is assumed that local agents do not receive information about foreign assets. The results will not be affected as long as the foreign signal is sufficiently less informative than the local signal.

of the portfolio of local stocks. This coincides with the findings of Albuquerque et al. (2005a). The present paper predicts that investors not only invest more heavily in local stocks but they also enjoy higher returns on their trades of these stocks. This is consistent with Coval and Moskowitz (1999), Coval and Moskowitz (2001), and Ivkovich and Weisbenner (2005). In other studies, Ivkovick et al. (2004), and Kacperczyk et al. (2004) find that agents with more concentrated portfolios earn abnormal returns (after controlling for risk). The present paper offers a theoretical explanation for this.

The remainder of the paper is organized as follows. The next two subsections review related empirical and theoretical literature. Section 2 introduces the model and discusses the assumptions. Section 3 describes the testable implications of our model. Section 4 extends the model to a case with positive cross-asset return correlation. Section 5 allows for endogenously determined stock prices in a context with partially revealing prices. Section 6 computes the shadow price of the short-sales constraint. Section 7 provides some analytical characterization of our main results using the CARA-Gaussian setup. Finally, Section 8 concludes.

1.1 Related empirical literature

This paper relies on the assumption that domestic investors have better information about domestic stocks than foreign investors. This can be justified on many grounds. Equity investment in foreign companies requires understanding different accounting practices and legal environments. Domestic investors are exposed to a wide array of sources of local news that can convey useful information about the performance of domestic companies. In addition, the geographic proximity allows for face-to-face contacts with local corporate executives, employees and other individuals that may have valuable private information.

Coval and Moskowitz (1999, 2001) study the portfolio composition of more than 2,000 mutual funds in the US. They find evidence of home equity bias within US boundaries. Fund managers invest more in companies with headquarters located near the fund's offices. Moreover, they earn substantial abnormal returns in nearby investments, while at the same time stocks held predominantly by local investors tend to show higher expected returns. The evidence strongly suggests that fund managers are exploiting an informational advantage in their selection of nearby stocks. Ivkovich and Weisbenner (2005) study a sample of 78,000 households and also find evidence of a strong preference for local stocks. In addition, the excess return on local investments is larger among companies not listed in the Standard and Poor

500 index. The latter are presumably firms with wider informational asymmetries between local and non-local investors.[6]

In a similar vein, Ivkovick et al. (2004) show that the stocks purchased by individuals with concentrated portfolios display higher returns than the stocks purchased by individuals with more diversified portfolios. A similar finding is reported by Kacperczyk et al. (2004) for a sample of US equity mutual funds. Both papers argue that this is consistent with the presence of informational asymmetries across market participants.

It should be stressed that these studies analyze the behavior of agents who are trading in the most developed and transparent financial market in the world. If informational differences can persist in this environment, the case for asymmetric information across country boundaries is even stronger. The barriers that can account for the information asymmetries in the US must be lower than the barriers that prevent information from flowing across countries. Surprisingly, the empirical evidence in this area is not conclusive.

On the one hand, several papers find evidence supporting the hypothesis that local investors are better informed than foreign investors. Kang and Stulz (1997) study foreign stock ownership in Japanese firms. They find that foreign investors concentrate their portfolios in large firms, firms with good accounting performance, and firms with high exports. These are the types of companies where the information asymmetries are presumed to be the lowest. Dahlquist and Robertsson (2001) find qualitatively similar results for Swedish firms. Choe et al. (2004) analyze foreign trades in Korea and find that foreign investors buy at higher prices than domestic investors, and sell at lower prices. A similar result is found by Dvořák (2001) for Indonesia. Shukla and van Inwegen (1995) provide evidence that UK mutual funds obtain lower returns from their investments in the US compared to US funds. Ahearne et al. (2004) study the home bias of US investors against specific countries. They find that the home bias decreases with the fraction of the foreign country market value that is cross-listed in the US stock market. Frankel and Schmukler (1996) show that domestic investors were “front-runners” in the Mexican crisis of 1994: they tried to sell their local investments before foreign investors did. Portes and Rey (1999) analyze the determinants of cross-border transaction flows. They find that distance

[6] Hubermann (2001) also documents that investors tilt their portfolio composition toward local companies. But he argues that this behavior is due to the fact that investors prefer to invest in firms that are familiar to them, independently of their prospects. If that were the case, we should not expect to observe abnormal returns on local investments. However, the evidence provided by Coval and Moskowitz (1999, 2001) and Ivkovich and Weisbenner (2005) suggests that a significant fraction of investors indeed behave rationally.

has a significant negative impact, which they argue is a proxy for informational asymmetries. Hau (2001) finds that traders located in Frankfurt and in German speaking cities in Europe show higher proprietary trading profits on German stocks. There is also evidence of higher profits for traders located near corporate headquarters.

On the other hand, other papers find evidence that foreign investors outperform local investors, which implies that either foreign agents do not have less information than local residents, or that the informational disadvantage does not play a significant role. Karolyi (2002) shows that foreign investors obtain higher returns in Japan. Grinblatt and Keloharju (2000) reach the same conclusion for Finland. Seasholes (2004) provides evidence that foreign investors in Taiwan buy before price increases and sell before price decreases. Froot and Ramadorai (2003) use evidence of changes in close-end country fund prices, and the net value of the underlying assets. They find that cross-border flows positively forecast changes in both prices. This is interpreted as evidence favoring the assumption that foreigners are better informed about fundamentals than domestic investors are.

One reason behind the mixed results is that it is not easy to isolate the role played by differences in information. Even when the comparison is made across similar classes of agents, there may be other factors affecting the different behavior of domestic and foreign agents. In this sense, one advantage of the samples used by Coval and Moskowitz (1999, 2001), Ivkovick et al. (2004), and Kacperczyk et al. (2004) is that they consist of a homogenous set of actors who are subject to the same legal environment.

1.2 Related theoretical literature

The above evidence suggests that the assumption of a local information advantage has an intuitive appeal, and is consistent with most empirical studies. From a theoretical point of view, it has already been said that the present paper is not the first attempt to explain the home bias assuming informational asymmetries between domestic and foreign investors. Gehrig (1993) and Brennan and Cao (1997) use the workhorse model of rational expectations equilibrium developed by Grossman (1976), Grossman and Stiglitz (1980) and Admati (1985). They assume that every agent receives informative signals about the future performance of domestic and foreign assets. The domestic signal conveys more information than the foreign one. This leads to more imprecise assessments about future performance of the foreign asset. Domestic agents thus perceive the foreign stock as more risky than the local stock, and reduce their holdings of the foreign asset. However, Glassman and Riddick (2001) and Jeske (2001) argue that the implied risk aversion needed to generate quantitatively significant results is unreasonably high.

Zhou (1998) considers a two asset model with a more sophisticated learning process. Agents face the so-called "infinite regress" problem: forecasting the forecasts of forecasts ... of others. But that feature does not help him to obtain any sizeable effect. Coval (2000) extends the framework in Zhou (1998) by introducing direct investment decisions and simplifies the learning process. He also obtains a small impact on the home bias.

In addition to the poor quantitative performance, Jeske (2001) argues that the previous modelling strategies do not seem suitable to address the home bias puzzle. Since domestic agents hold better information about domestic assets, sufficiently low expected local dividends induce residents to liquidate their local positions in favor of foreign assets. On the other hand, foreign investors unaware of the poor expected performance of local assets may find it convenient to purchase local stocks at a discount. As a consequence, these models predict unrealistic fluctuations of the fraction of foreign investments (which can turn into foreign bias for certain shock realizations). These limitations lead him to conclude that asymmetric information does not stand up as a compelling theoretical explanation for the home equity bias.

The reason behind the lack of success of previous attempts is that the burden of the explanation relies on effects that are of second order importance. In the setups considered, agents can be neither systematically pessimistic nor optimistic with respect to any asset. The explanation for why agents show a preference for domestic assets is that they are perceived to be less risky or that they provide better hedge against consumption risk (see Coval (2000)). But this plays a secondary role in the standard expected utility framework with the HARA utility functions commonly assumed for macroeconomic analysis.

Van Nieuwerburgh and Veldkamp (2005) consider a CARA-Gaussian structure but allow for a more complex learning process. Agents decide how much to reduce the variance of the signal they learn from. They face a limited "learning capacity". Thus, the decision of how much to learn from each asset is non-trivial. They show that when local investors enjoy an information advantage in local assets, the optimal strategy is to concentrate the learning capacity in those assets. Thus, allowing agents to learn from local and foreign assets not only does not reduce the information asymmetry between domestic and foreign investors, but also magnifies it. In contrast to the previous literature, they obtain quantitatively significant results, but their model is ill-suited to deal with the second critique –the excess volatility in the fraction invested abroad.

Epstein and Miao (2003) and Alonso (2005) assume preferences that allow for ambiguity aversion and are able to explain a significant fraction of the bias. The reason is that they introduce an effect of first order importance: domestic agents are systematically pessimistic about foreign stocks. The present paper relies also on effects of first order importance, yet shows that similar quantitative results can be obtained without a major departure from the mainstream model.

2 The model

We consider a two-country world. Each country is inhabited by a large number of infinitely-lived, identical agents. Agents have preferences defined over a stream of tradable consumption goods:

$$E\left[\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t}\right)\mid\mathcal{I}_{0}\right],$$

with

$$u\left(c\right)=\frac{c^{1-\sigma}-1}{1-\sigma}.$$

The perceived probability distribution of future consumption flows depends on each agent's initial information set, denoted by $\mathcal{I}_0$.

Each country hosts two risky technologies. Each technology produces the same consumption and investment good. The output provided by each technology depends on the capital allocated in the previous period and the current productivity shock. The productivity shocks vary across technologies. For simplicity, it is assumed that productivity shocks may take either a high value (A_h) or a low value (A_l). Technologies do not require labor as an input and display constant returns to scale, i.e., they are of the AK type. The probability that technology i is hit with a high productivity shock in period t is denoted by ν_{it}, where $i=1,\ 2,\ 1^{*},\ 2^{*}$. The superscript "*" is used to denote foreign variables. The probability values are drawn from a joint distribution with density $f\left(\nu_{1t},\nu_{2t},\nu_{1^{*}t},\nu_{2^{*}t}\right)$. The density function f is time invariant.

Investors are not able to observe the probabilities that govern the distribution of productivity shocks, i.e., they do not observe the realizations of ν_{it+1}. Instead, they receive informative signals about the relative expected performance of local technologies. A fraction ϕ of domestic investors receive a signal favorable to the local technology that it is more likely to receive a high productivity shock, while a fraction $1-\phi$ receive an incorrect signal, suggesting to invest in the technology that is less likely to receive a high productivity shock. Each investor does not know whether the signal received is correct or not. The value of ϕ is assumed to be larger than 0.5 (the signals are informative). Finally, it is assumed that agents are not allowed to hold short positions.

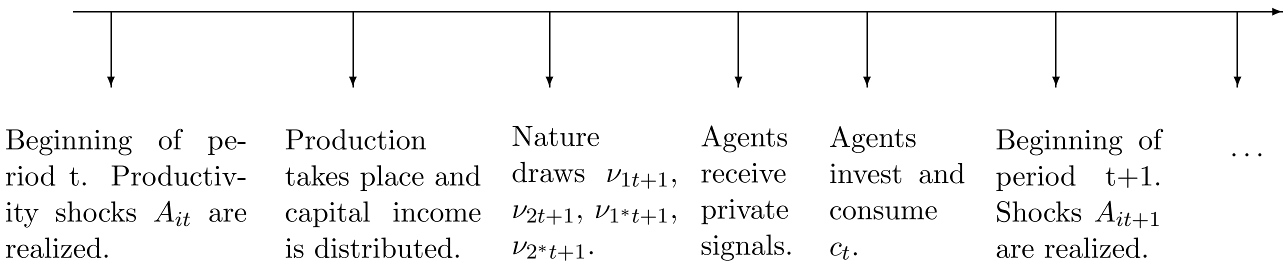

Figure 2: **Order of events in period t.**

The timing of the model is described in Figure 2. Productivity shocks are realized at the beginning of each period. Then, the production process takes place and agents receive income $y = \sum_i k_{it}A_{it}$. Recall that capital is the only factor input. After that, Nature draws the vector $(\nu_{1t+1}, \nu_{2t+1}, \nu_{1^*t+1}, \nu_{2^*t+1})$, that determines the probability distribution over next period's productivity shocks. This vector also conditions the distribution of signals received by domestic and foreign agents. Each agent uses the signal realization to update the probability distribution over next period's productivity shocks. At the end of the period, agents decide how much to invest in each of the four technologies, and how much to consume.

The investor's problem has a recursive structure. This is due to the fact that the density function f is time invariant, and does not depend on previous shock realizations.[7] From the perspective of an individual investor, the relevant state variables are his wealth level (ω)and the signal received: (s). The last piece of information is helpful to forecast the probability distribution for next period's shocks. The agent's optimization problem can then be summarized by the following Bellman equation.

$$V(\omega, s) = \underset{k'_1, k'_2, k'_{1^*}, k'_{2^*}}{Max} \left\{ u(c) + \beta E\left[V\left(\omega', s'\right)\right] \right\} \quad (1)$$

[7]If the density depended on past realizations of ν_i and ν_{i^*}, agents would typically need to keep track of the entire history of shocks in order to update their beliefs.

subject to

$$c + k_1' + k_2' + k_{1^*}' + k_{2^*}' = \omega,$$
$$\omega' = \sum_{i=1,2,1^*,2^*} k_i' A_i',$$
$$\text{and } k_i' \geq 0 \quad \text{for } i = 1, 2, 1^*, 2^*.$$

The last inequality rules out short-selling. An alternative modelling strategy would be to impose a “fee” when agents decide to short-sell one of the assets. The actual volume of short-sales in the US market is very low. This means that the short-selling constraint is not a very restrictive assumption.

2.1 Discussion of assumptions

The signal structure captures the hypothesis that local investors do not possess significantly better information about the future performance of the local stock market compared to what foreign investors know, but they do outperform foreign investors in spotting the best domestic investment opportunities. For instance, it is not clear why American investors should do systematically worse at predicting the performance of the German stock market when compared to German investors. In this case, the relevant information set consists mostly of public news and past performance of aggregate variables, which are readily available to every investor at the same time. It may be possible that some local investors get privileged access to information about policy decisions (like a proximate declaration of default), but this relates to rare events and should not play a significant role in developed countries, with good institutions. On the contrary, domestic investors are exposed to a wide array of sources of local news that can convey useful information about the performance of domestic companies. In addition, the geographic proximity allows for face-to-face contacts with local corporate executives, employees and other individuals that may have valuable private information.

One shortcoming of the present paper is that the allocation of information is exogenously given. This contrasts with the fact that individuals can actually acquire information from various sources: Investing in a portfolio managed by a Mutual Fund is probably the easiest one. But individuals can also pay for expert investment advice.[8] In fact, the last two decades have witnessed a sensible decrease in the home bias, and a simultaneous increase in the fraction of equity investments managed by institutional investors. One possible interpretation is that a higher participation of institutional investors has helped

[8] The expansion of the world wide web has also made possible access to an enormous amount of data. On the other hand, we could reasonably conjecture that this type of information cannot be used to design profitable investment strategies.

to reduce the information asymmetry between investors in different countries, inducing a more diversified portfolio.[9] In spite of this, several studies surveyed in the previous section find evidence consistent with the fact that information is not evenly distributed.

Several reasons prevent information from flowing freely across agents. It is in the best interest of any investor to maintain an information advantage if he can profit from it. But even financial firms may find it convenient to hide some of the information they possess. For example, a Mutual Fund that has invested heavily in certain stocks may not be interested in spreading bad news about those stocks. In addition, in most cases it is not easy to verify the quality of the information received. This favors agents who do not want to truthfully disclose all the information they have.

This paper does not explore the causes that explain the presence of informational asymmetries.[10] Instead, we take as given the fact that some investors know more about certain assets than other investors.

The second key assumption in the model is that agents face a short-sale constraint. There is plenty of evidence that short-selling is more costly than buying a stock. The common method of shorting an equity is to borrow the security and sell it. Later, the short-seller needs to buy it back to return it to the lender. The explicit cost of the transaction is the fee that the short-seller pays for the loan. But there are other costs. The standard practice is that the equity lender can ask the loan to be repaid ("recall" the shares) at any time, which exposes the short-seller to risk. There are also various regulations that increase the cost of short-selling: the proceeds from short-selling are taxed at the short-term capital gain tax rate independently of how long the short position is open; sell orders that are short sales can be executed only after the stock price has increased (on an "uptick"), etc.[11] Some institutional investors are even prohibited from taking short positions. Almazan et al. (2004) find that by 2000, 69 percent of US equity funds were not permitted to engage in short-selling practices. Among the ones that were not constrained, only 10 percent held short positions.[12] All of the above may help to explain why the

[9] Another interpretation is that the more active participation of Mutual Funds, and other institutional investors, has decreased the cost of foreign equity investments. Nonetheless, previous empirical studies have not found convincing evidence in favor of the transaction cost explanation of the home bias.

[10] Van Nieuwerburgh and Veldkamp (2005) provide an explanation for why local agents may find it optimal to learn mostly about local stocks.

[11] See D'Avolio (2002), Dechow et al. (2001) and Duffie et al. (2002) for a description of the institutional details of equity lending markets and regulations applied to short-selling. An additional implicit cost of short-selling is described in Lamont (2004). He argues that firms have incentives to impede short sales of their stock. He analyzes a sample of 266 firms who threatened, took action against, or accused short sellers of illegal activities. His findings suggest that those firms succeeded in raising the costs of short selling.

[12] Koski and Pontiff (1999) find that 79 percent of equity mutual funds make no use of derivatives suggesting that funds are also not finding synthetic ways to take short positions.

market for equity loans is so thin. D'Avolio (2002) reports that in June 2001 the total amount of stocks shorted represented 1.7 percent of the market capitalization. Dechow et al. (2001) document that short positions represented only 0.2 percent of the market value in 1976 but increased to 1.4 percent in 1993. The previous evidence shows that short-selling activities are quite limited in the US, the most developed financial market. This suggests that although the assumption of no-short-sales constraints made is this paper may appear extreme, it is in line with the trading limitations observed in actual markets.

Other features of the model require further explanation. Notice that agents can only invest in four assets while there are sixteen possible state realizations, depending on the realization of the productivity shocks. This means that markets are incomplete. Not only is this a realistic assumption, but it is also necessary in order to have a well defined measure of the home bias. Otherwise, it would not be clear how to classify assets that pay contingent on joint domestic and foreign state realizations. Besides, under complete markets there would be fifteen endogenous prices. That is more than enough to reveal all the information agents need to know.

The model laid down above does not feature endogenous factor prices. The return on capital is exogenously given by productivity shocks, and the share prices of the four technologies are equal to one.[13] The sectors' sizes fully adjust in every period in response to the aggregate demand of each stock. Assuming a more standard production process with decreasing marginal productivity on capital and labor would allow for endogenous factor prices. But the presence of incomplete markets implies that the model cannot be solved as if each economy was inhabited by a representative agent. In order to forecast future factor prices, individuals will typically need to keep track of the wealth distribution in each country. Even though this is an interesting extension, it involves a high level of complexity and is not necessary to illustrate the main point of the paper.

The constant-share-price result can be relaxed by imposing a sluggish adjustment in the supply of stocks. The standard model with asymmetric information used in the finance literature assumes a constant asset supply over time, except for shocks due to liquidity trading. We choose the opposite extreme in order to prevent investors from extracting valuable information from prices. If relative stock prices are allowed to differ, it would be necessary to allow for additional sources of uncertainty in order to mask the expected relative performance of each asset. This is not a trivial extension. To the best of our knowledge, the only model structure with multiple assets and partially revealing prices corresponds

[13]This is because there is only one good in the economy, and there are no adjustments costs.

to the one developed by Admati (1985). Her results relies on the Gaussian-CARA framework. However, it is not possible to accommodate that modelling strategy in the present work. The assumption of no-short-sales breaks down the Bayesian updating scheme over normally distributed variables, making the problem analytically intractable. Section 5 develops a simple environment that allows for endogenously determined stock prices. It shows that the main conclusion of the paper is not affected as long as prices do not reveal too much information.

2.2 Implications for the home bias

This section makes an extra simplification to the model introduced above. The probability distribution from which the values of ν_i are drawn is assumed to be independent across technologies and across time, and follow a uniform distribution with support $[0,1]$. The previous simplification implies that there is no persistence in returns.[14] The second implication induced by this simplified setup is that signals about domestic technologies do not convey information about foreign assets. It is straightforward to generalize the model and allow domestic agents to receive some information about foreign assets. The main results will not be affected as long as foreign signals are sufficiently less informative than local ones.

The conditional expectation of the value function can then be expressed as

$$\begin{aligned} E\left[V\left(\omega', s'\right) \mid s,\right] &= E\left[Pr\left(h,h,h,h,1\right) \mid s\right] V\left(\omega'_{h,h,h,h},1\right) + \cdots + E\left[Pr\left(l,l,l,l,1\right) \mid s\right] V\left(\omega'_{l,l,l,l},1\right) + \\ & \quad E\left[Pr\left(h,h,h,h,2\right) \mid s\right] V\left(\omega'_{h,h,h,h},2\right) + \cdots + E\left[Pr\left(l,l,l,l,2\right) \mid s\right] V\left(\omega'_{l,l,l,l},2\right) \end{aligned} \quad (2)$$

where

$$\omega_{i',j',i^{*\prime},j^{*\prime}} = A_{i'}k_1 + A_{j'}k_2 + A_{i^{*\prime}}k_{1^*} + A_{j^{*\prime}}k_{2^*}.$$

The term $E\left[Pr\left(i,j,i^*,j^*,s'\right) \mid s\right]$ denotes the conditional joint probability of receiving a signal s' in the following period and observing a future vector of productivity shocks equal to (i,j,i^*,j^*). The first two components in this vector refer to productivity shocks to domestic technologies 1 and 2 respectively. The last two components denote shock realizations of foreign technologies 1 and 2 respectively. The conditional expectation is computed using Bayes' rule.

[14] There is mixed evidence in this respect: some authors find that past prices do not convey useful information about future returns while other papers find some effect. But even the latter do not find a large effect. This suggests that eliminating serial correlation in returns is not a very restrictive assumption. See Malkiel (2003) for a discussion on the topic.

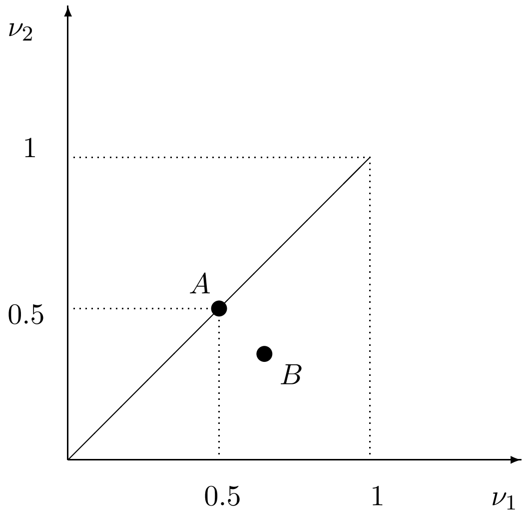

Figure 3: **Change in perceived distribution of returns upon the arrival of a signal favoring local asset 1.**

Figure 3 illustrates how a domestic investor updates his beliefs. Before receiving any signal, the only information the agent possesses is that a set of values of ν_i' has been drawn from a uniform distribution. Thus, the expected probability of receiving a high return when investing in domestic technology i coincides with the unconditional mean of ν_i. In this case, the latter is equal to 0.5. Thus, the beliefs of both domestic technologies are represented by point A. The perceived return process of foreign stocks is not affected by the signal realization, so point A also illustrates the beliefs of a local agent with respect to foreign stocks. The perceived distribution of returns of local stocks changes upon the receipt of a signal. For example, if an investor receives a signal favoring technology 1, he believes that it is more likely that the actual realization of (ν_1', ν_2') is below the diagonal, than above the diagonal. This explains why the conditional expectation of (ν_1', ν_2') lies on a point like B. The agent becomes optimistic about asset 1 and pessimistic about asset 2. This explains the incentive to concentrate the local component of his portfolio in one local stock.

Formally,

$$
\begin{aligned}
E\left[\Pr\left(i',j',i^{*\prime},j^{*\prime},s'\right)\mid s\right] &= \frac{\Pr(s')\,E\left[\Pr\left(i',j',i^{*\prime},j^{*\prime},s\right)\right]}{\Pr(s)} \\
&= \frac{\Pr(s')\,E\left[\Pr\left(A'_{1*}=A_{i^{*\prime}}\right)\right]E\left[\Pr\left(A'_{2*}=A_{j^{*\prime}}\right)\right]E\left[\Pr\left(A'_1=A_{i'},A'_2=A_{j'}\right)\mid s\right]}{\Pr(s)},
\end{aligned}
$$

where the second equation makes use of the assumptions stated at the beginning of this section. More explicitly, the previous expectation is computed as follows:

$$
E\left[\Pr\left(i',j',i^{*\prime},j^{*\prime},s'\right)\mid s\right] = \left[\begin{array}{c}\Pr\left(\nu'_1>\nu'_2\right)\Pr\left(s'\mid\nu'_1>\nu'_2\right)+\\ \Pr\left(\nu'_1<\nu'_2\right)\Pr\left(s'\mid\nu'_1<\nu'_2\right)\end{array}\right] E\left[\Pr\left(A'_{1*}=A_{i^{*\prime}}\right)\right]\times
$$

$$
\frac{E\left[\Pr\left(A'_{2*}=A_{j^{*\prime}}\right)\right]\left\{\begin{array}{c}\Pr\left(s\mid\nu_1>\nu_2\right)\int_0^1\int_0^{\nu_1}\Pr^{i'}(\nu_1)\Pr^{j'}(\nu_2)\,f(\nu_1)\,f(\nu_2)\,d\nu_2 d\nu_1+\\ \Pr\left(s\mid\nu_1<\nu_2\right)\int_0^1\int_{\nu_1}^1\Pr^{i'}(\nu_1)\Pr^{j'}(\nu_2)\,f(\nu_1)\,f(\nu_2)\,d\nu_2 d\nu_1\end{array}\right\}}{\Pr\left(\nu_1>\nu_2\right)\Pr\left(s\mid\nu_1>\nu_2\right)+\Pr\left(\nu_1<\nu_2\right)\Pr\left(s\mid\nu_1<\nu_2\right)},
$$

where ν'_i denotes the probability with which technology i will pay high returns on the following period. $\Pr^{i'}(\nu)$ denotes the probability that technology i receives productivity shock $A_{i'}$ if the actual probability of being hit with a high productivity shock is ν. Formally,

$$
\Pr^{i'}(\nu) = \left\{\begin{array}{ccc}\nu & if & i'=h,\\ 1-\nu & if & i'=l.\end{array}\right.
$$

The dynamic optimization problem is solved by finding policy functions that satisfy the Euler Equations. It is easy to check that the assumptions of a constant relative risk aversion utility function and linear technology lead to individual policy functions that are linear in wealth. However, it is not possible to find an explicit expression for the slope coefficients, so the problem is solved using numerical techniques.[15] Appendix 7 considers a model with CARA utility function and a Gaussian process for the productivity shocks. That framework allows us to fully characterize the optimal investment policy. We find qualitatively similar results to the ones described in this section.

The parameter values are chosen in such a way that the domestic return process resembles some key statistics of the US stock market. The assumption that foreign assets share the same characteristics

[15]Equation 2 shows that each individual needs to allocate future consumption across 32 states. The assumption that technologies are identical means that the signal received does not affect the perceived discounted utility of future consumption streams. This reduces the future state space by half. In addition, investors do not receive any information that can help them differentiate between the two foreign assets so the latter are perceived to be equivalent. In practice, then, it is only necessary to solve for three policy functions: the investments in the two domestic technologies and the foreign one. Also, the state space can be reduced to 12 possible realizations. With logarithmic utility function, the solution for the policy functions consists of the root of a system of three polynomials of 12th order!

with domestic assets is made for simplicity. A period in the model corresponds to one year. The value for the high productivity shock (A_h) is set equal to 1.27. The value for the low productivity shock (A_l) is set equal to 0.85. This yields an average return on each stock market of 6 percent, with a standard deviation of 14.8 percent.[16] Finally, we choose standard preference parameters: a logarithmic utility function and a subjective discount factor (β) of 0.96.

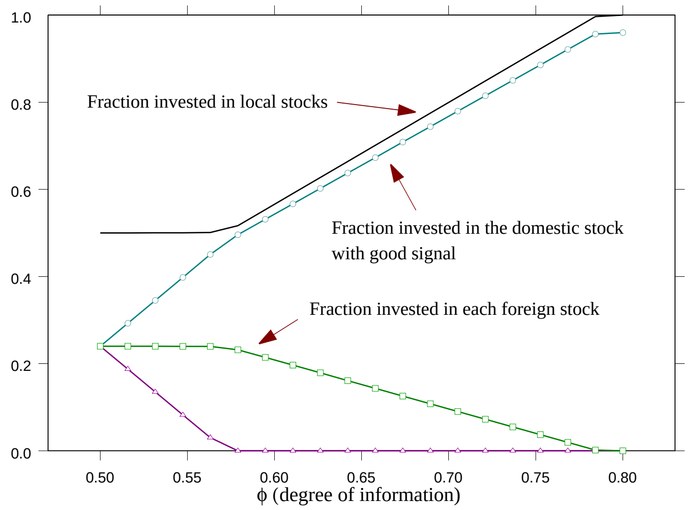

Figure 4: **Fractions invested in each local and foreign asset, and home bias as functions of the precision of the signal**

Figure 4 describes the main result of the paper. The graph shows how investors' policy functions depend on the precision of the signal. A completely uninformative signal corresponds to the case where $\phi = 0.5$. Half of domestic agents receive a signal favorable to technology 1 and the other half receive a signal favorable to technology 2, regardless of the actual probability distribution of domestic returns. In this case the signal is pure noise and agents hold a perfectly diversified portfolio: they invest a quarter of their savings in each asset. As the signal becomes more informative, the signal realization has an effect on the investment decisions. Every local investor increases his position in the local technology favored by the signal realization and decreases his position in the other local technology. The overall proportion of local assets in his portfolio is barely affected: He still invests roughly half of his savings in foreign assets.

[16]The statistics correspond to the case where both technologies receive the same weight.

When the signal is sufficiently informative, local investors want to finance purchases of the perceived high-returns asset by selling short the perceived low-returns asset. When short-selling is a costly activity, the optimal strategy is to lower the fraction invested in foreign stocks. This liberates resources that can be invested in the local asset with higher expected returns. It is in this range of values of ϕ where the model can generate a significant level of home bias. For instance, when the proportion of local investors that receive the "correct" signal is around 65 percent, domestic investors always hold 70 percent of their portfolios in local assets. More precisely, 65 percent of domestic investors invest 70 percent of their savings in the local asset that is actually paying higher returns with higher probability. The remaining 30 percent of their portfolio is split between the two foreign stocks. The remaining local investors are investing 70 percent of their savings in the local asset with worse prospects (though they do not know this), and split the remaining 30 percent of their savings between the two foreign stocks.

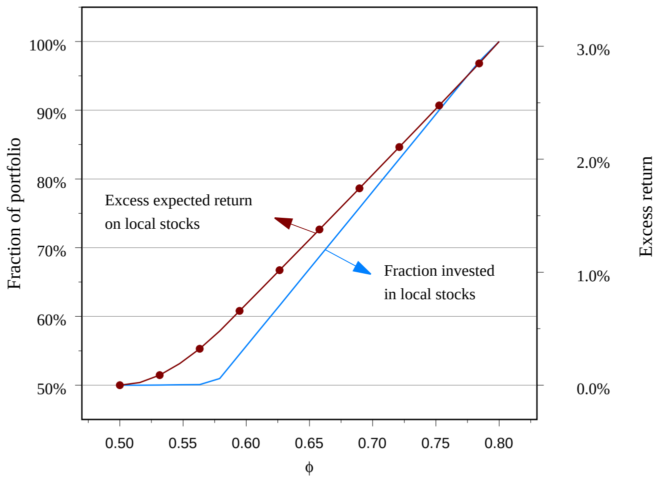

Figure 5: **Home bias and risk adjusted excess return on local portfolio.**

The reason why local investors tilt their portfolio toward domestic assets is because they know that, on average, they earn abnormal returns on the local portfolio. Figure 5 illustrates the risk adjusted difference between the expected return of the local portfolio, versus the expected return of the foreign portfolio.[17] The figure shows that the range of values of excess returns required to explain a significant

[17]We do not model explicitly the market for risk-free bonds. However, all agents in the model are identical except for the signal realization. This implies that no agent, either local or foreign, is willing to borrow or lend at the equilibrium risk-free

fraction of the bias is not unreasonable. For example, Coval and Moskowitz (2001) find that mutual funds earn an excess return of 1.18 percent on local stocks compared to what they earn on non-local stocks. Ivkovich and Weisbenner (2005) find that households earn an additional 3.2 percent on their local stocks.

The model does a good job in explaining a significant fraction of the bias we observe in the data. It has also a richer structure compare to previous papers in the literature. Thus, if we wanted to run additional tests, we should check whether the portfolio implications are aligned with what we observe in the data. This is discussed in the next section.

Some comments on the robustness of the results are in order. This section builds on several restrictive assumptions. Some of them are abandoned in later sections. Section 4 considers the case where there is positive cross-asset return correlation between local stocks.[18] Not only this is a more realistic assumption, but it also reduces the role played by differences in information about the relative performance of local assets. For instance, when the returns of local assets are perfectly correlated, the model collapses to the case of one asset per country. Section 5 considers a different setup in which local and foreign stocks are in fixed supply –except for unobserved liquidity shocks. This introduces a nontrivial signal extraction problem: agents learn from the signals and market prices. The result described in this section is robust to the previous generalizations.

3 Testable implications

This section lays down the main empirical implications of the model described before. The fact of considering a framework with multiple assets distinguishes this paper from previous work and allows for a richer set of implications. We find that not only is the present model able to explain a significant fraction of the home equity bias observed in the data, but it is also consistent with several patterns

rate –denoted by R^f. This simplifies the computation of the equilibrium risk-free rate. Denote by R_L^e the expected return on the portfolio of local stocks, σ_L^e the standard deviation of the return on the portfolio of local stocks, R_F^e the expected return on the portfolio of foreign stocks, σ_F^e the standard deviation of the returns on the portfolio of foreign stocks. The risk adjusted excess return on the local portfolio (ER_L)is computed as follows:

$$ER_L = \frac{R_L^e - R^f}{\sigma_L^e}\sigma_F^e - \left(R_F^e - Rf\right).$$

The previous equation computes the abnormal returns earned on a portfolio that has the same risk as the foreign portfolio and the same Sharpe ratio as the local portfolio.

[18]Another extension would be to consider the scenario where there is positive cross-country return correlation. This will reduce the incentives to hold a diversified portfolio and therefore, it will help the model to explain a higher fraction of the bias.

of foreign investment behavior. Albuquerque et al. (2005b) analyze the equity portfolios of a set of individuals who traded through a large investment broker between 1991 and 1996. They find evidence suggesting that the portfolio of foreign stocks is more diversified than the portfolio of local stocks. Local agents hold a larger fraction of their foreign equity investments through mutual funds, compared to the fraction of US stocks held through institutional investors. They also find that the share of foreign investments is negatively correlated with the degree of concentration of the portfolio of local stocks. US investors who invest heavily in a few domestic firms tend to allocate a lower fraction of their equity investments in foreign stocks.

For simplicity, the present paper restricts attention to the case where domestic agents do not receive information about foreign assets. In this scenario, local investors specialize in one domestic stock but hold a perfectly diversified foreign portfolio: they invest the same amount in each foreign stock.[19] This is in line with the first finding described above. With respect to the second finding, our model predicts that agents who receive precise information about the relative performance of local stocks display a bias towards local assets. In contrast, local investors who receive less informative signals hold a more diversified local portfolio and display no bias: they invest half of their wealth in foreign assets.

A general prediction of the current setup is that more concentrated portfolios show better performance. When there is heterogeneity in the quality of the information received, the model predicts that individuals with more precise information show a higher specialization and on average, enjoy higher returns. This is consistent with the evidence found by Ivkovick et al. (2004) for a sample of US households. They show that the stocks purchased by individuals with concentrated portfolios display higher returns than the stocks purchased by individuals with diversified portfolios. A similar finding is reported by Kacperczyk et al. (2004) for a sample of US equity mutual funds.

Tesar and Werner (1995) report that the turnover rate on foreign equity portfolios is significantly higher than the turnover rate on domestic equity portfolios. This finding has been used as evidence against theoretical explanations that rely on informational asymmetries. If domestic agents receive more precise information about domestic firms than foreign firms, they should trade more intensively on local stocks.[20] However, Warnock (2002) argues that these findings are based on data published

[19]Allowing domestic agents to receive signals about foreign stocks induces a less diversified foreign portfolio. However, it does not affect the proportion invested in foreign equity as long as the foreign signal is not so informative that local investors fully specialize in one foreign asset.

[20]The exception is Brennan and Cao (1997). They develop a model with asymmetric information that generates higher turnover rates on foreign equity portfolios.

before reliable cross-border holdings data were available. He uses more accurate data and finds foreign turnover rates significantly lower than the ones reported in Tesar and Werner (1995), and roughly comparable to domestic turnover rates. Our model features lower foreign turnover rates but given the previous evidence, we do not interpret this fact as a severe limitation of the model.

4 The case with positive return correlation between domestic assets

Given that the signals observed by local agents relate only to future relative performance of domestic assets, the role played by differences in information decreases with the correlation of local returns. For example, if the returns of both assets were perfectly correlated, there would be no scope for differences in performance. That case would resemble the framework analyzed in previous studies (one asset per country), which we already know does not help to explain the lack of international portfolio diversification.

In order to allow for cross-asset return correlation, it is assumed that the values of ν_1 and ν_2 are drawn in two steps. The steps are summarized in Figure 6. First, Nature draws a value ν, which can be interpreted as an aggregate shock. The variable ν satisfies $\nu = \frac{\nu_1+\nu_2}{2}$. A realization of ν corresponds to a point on the main diagonal in Figure 6. For instance, suppose that a point like A has been drawn. In a second stage, Nature draws a value η that determines the relative performance of domestic assets. In this case, η corresponds to a point on the line that goes through A, has a slope equal to -1, and is contained in the unit square.

This approach introduces two mechanisms that help to generate positive cross-asset return correlation. One is to allow for a higher probability mass on the extreme values of ν. The other is to allow for a distribution of η such that it increases the probability mass of realizations of (ν_1, ν_2) close to the diagonal. When this happens, local assets tend to share a similar probability distribution and therefore display unconditional return correlation.[21] We maintain the assumption that local and foreign stocks are ex ante identical. A formal description proceeds in the following paragraphs.

For a given realization of ν and η, ν_1 is given by the following equations:

$$\nu_1 = \begin{cases} 2\nu\eta & if \;\; \nu < 0.5, \\ (2\nu - 1)(1-\eta) + \eta & if \;\; \nu > 0.5. \end{cases}$$

The value taken by ν_2 is just $2\nu - \nu_1$. This means that when the realization of η equals 0.5, the

[21] It should be noticed that the return of both assets are uncorrelated once we condition on the realizations of ν_1 and ν_2.

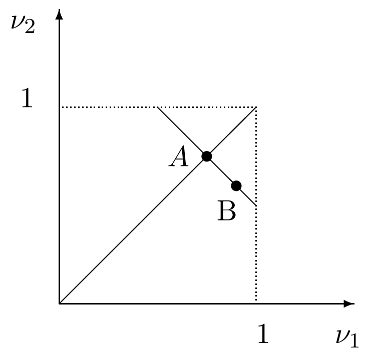

Figure 6: **How positive cross-asset return correlation is generated.**

returns of both domestic technologies share the same probability distribution. For values of η larger than 0.5, technology 1 yields higher expected returns. For values of η below 0.5, technology 2 yields higher expected returns. The random variable η follows a Beta distribution. This section maintains the assumption that local assets have the same ex ante distribution of returns, which implies that η has a mean of 0.5. This pins down one of the parameters of the Beta distribution. The remaining parameter is used to control for the volatility of η.

The random variable ν has support [0, 1] and density $f(\nu;\alpha)$, with

$$f(\nu;\alpha)=\begin{cases}\frac{2\alpha\nu+1-\alpha}{1-\frac{\alpha}{2}} & if \;\; \nu<0.5,\\ \frac{2\alpha(1-\nu)+1-\alpha}{1-\frac{\alpha}{2}} & if \;\; \nu>0.5.\end{cases}$$

.

The above distribution collapses to the Uniform distribution when $\alpha=0$, and displays a probability distribution shifted towards the corners when $\alpha<0$.

The random variables that determine aggregate and relative performance are assumed to be independent. Thus, the density function of ν and η can be written as

$$h(\nu,\eta)=\begin{cases}g^{\beta}(\eta;\sigma_\eta)\,\frac{2\alpha\nu+1-\alpha}{1-\frac{\alpha}{2}} & if \;\; \nu<0.5,\\ g^{\beta}(\eta;\sigma_\eta)\,\frac{2\alpha(1-\nu)+1-\alpha}{1-\frac{\alpha}{2}} & if \;\; \nu>0.5.\end{cases}.$$

The function $g^{\beta}(\eta;\sigma_\eta)$ denotes the density function of a random variable with Beta distribution and parameters $\frac{1}{8}\left[\frac{1}{\sigma_\eta^2}-4\right]$. The second term allows us to determine how much probability mass is allocated near the extremes, i.e., when both assets pay high or low returns with certainty. The density function over ν_1 and ν_2 is obtained after a change of variables.

$$f(\nu_1,\nu_2) = \begin{cases} g^{\beta}\left(\frac{\nu_1}{\nu_1+\nu_2};\sigma_\eta\right)\frac{2\alpha(\nu_1+\nu_2)+1-\alpha}{1-\frac{\alpha}{2}}\frac{1}{2(\nu_1+\nu_2)} & if \;\; \nu_1+\nu_2<1, \\ g^{\beta}\left(\frac{1-\nu_2}{2-\nu_1-\nu_2};\sigma_\eta\right)\frac{\alpha(2-\nu_1-\nu_2)+1-\alpha}{1-\frac{\alpha}{2}}\frac{1}{2(2-\nu_1-\nu_2)} & if \;\; \nu_1+\nu_2>1. \end{cases}$$

This section maintains the assumption that both countries are identical. This means that foreign assets are subject to the same return process as local assets. It is also assumed that returns from domestic and foreign assets are uncorrelated. Before choosing the values of σ_η and α, it is necessary to determine what degree of correlation is consistent with the data. The capital asset pricing model (CAPM) provides a simple framework in order to retrieve a sensible value. The CAPM states that:

$$R_i = R_f + \beta_i (R_m - R_f) + \epsilon_i, \tag{3}$$

where R_i denotes the return of asset i, R_f denotes the risk free interest rate, R_m denotes market return and ϵ_i denotes the idiosyncratic shock to asset i. This setup captures a simple mechanism that generates cross-asset return correlation: The return of every asset in the economy depends on a single aggregate variable, i.e., the excess return of the market portfolio. The next step is to provide an interpretation for the assets in our model. If the model were followed literally, each asset would correspond to a portfolio of local firms. This approach implies a high level of aggregation, so we would expect to obtain a strong cross-asset correlation. However, the same reason we utilized to abstract from informational asymmetries about aggregate variables could be applied to those portfolios. That is why we would like to interpret local and foreign assets as firms or specific industries. The fact that the paper considers only two stocks per country is just a simplification necessary for tractability purposes.

If each asset stands for a firm, the results in Fama and French (1992) imply a zero cross-asset correlation.[22] They study a sample of 2,267 stocks and conclude that the average estimation of β is not significantly different than zero. Fama and French (1993) sort individual stocks into 25 portfolios according to firm sizes and book-to-market ratio. Their estimation of the single factor model produces a mean return correlation across portfolios of 78 percent.[23] Fama and French (1997) sort stocks according to the industry they belong to. They construct 48 industry-portfolios and obtain a mean cross-asset correlation of 63 percent.

[22]The implicit assumption is that idiosyncratic shocks (denoted by ϵ_i) are independent across assets.

[23]The mean R^2 is taken as the estimated cross-asset correlation.

An alternative procedure to estimate the cross-asset return correlation is to use Equation (3) and assume that both assets have a β of one, i.e., they are two representative assets. Then,

$$Corr\left(R_i, R_j\right) = \frac{Cov\left(R_i, R_j\right)}{\sigma\left(R_i\right)\sigma\left(R_j\right)} = \frac{Var\left(R_m - R_f\right)}{Var\left(R_m - R_f\right) + Var\left(\epsilon\right)} = \frac{1}{1 + \frac{Var(\epsilon)}{Var\left(R_m - R_f\right)}},$$

which shows that cross-asset correlation depends on the ratio of idiosyncratic risk to aggregate risk. Campbell et al. (2001) use the CAPM structure to estimate return volatility at the market, industry, and firm levels. Using their estimates, a correlation of 0.6 is obtained at the industry level, similar to the value obtained in Fama and French (1997). As expected, the correlation at the firm level is significantly lower, ranging from 0.19 to 0.25 depending on whether returns are computed on a daily or weekly basis.

The previous evidence illustrates that the correlation can take almost any positive value between 0 and 0.8, depending on how assets are defined. We follow Campbell et al. (2001) and choose a correlation of 0.25 as the benchmark value. The baseline value of α is set to 0 and the baseline value of σ_η is set to 0.25. We also consider the case $\alpha = -6$ and $\sigma_\eta = 0.1$, which generates a correlation of 0.45. Figure 7 reports the results. As expected, the fraction invested overseas decreases as the correlation increases. But the bias can still be significant for reasonable levels of cross-asset return correlation. However, when the cross-asset correlation is relatively high, there is less room for disagreement about asset returns and agents tend to hold a more diversified portfolio.

5 Endogenous asset prices

The objective of this section is to illustrate that the main result of the paper does not depend on the assumption that the supply of stocks is infinitely elastic, which implies constant stock prices. When the last assumption is abandoned, equilibrium prices typically reveal valuable information. In order to prevent prices from being fully revealing, it is necessary to allow for additional sources of uncertainty. This is not an easy task once we depart from the standard environment with a CARA utility function and Gaussian returns. In accordance with most of the previous finance literature, the model below assumes that prices are only partially revealing because of the existence of supply shocks (noise traders). The difference with respect to the previous literature is that we restrict attention to an ad hoc structure of shocks that has the advantage of reducing the dimensionality of the price realizations that can be observed in equilibrium. Instead of extracting information from a multidimensional continuum space,

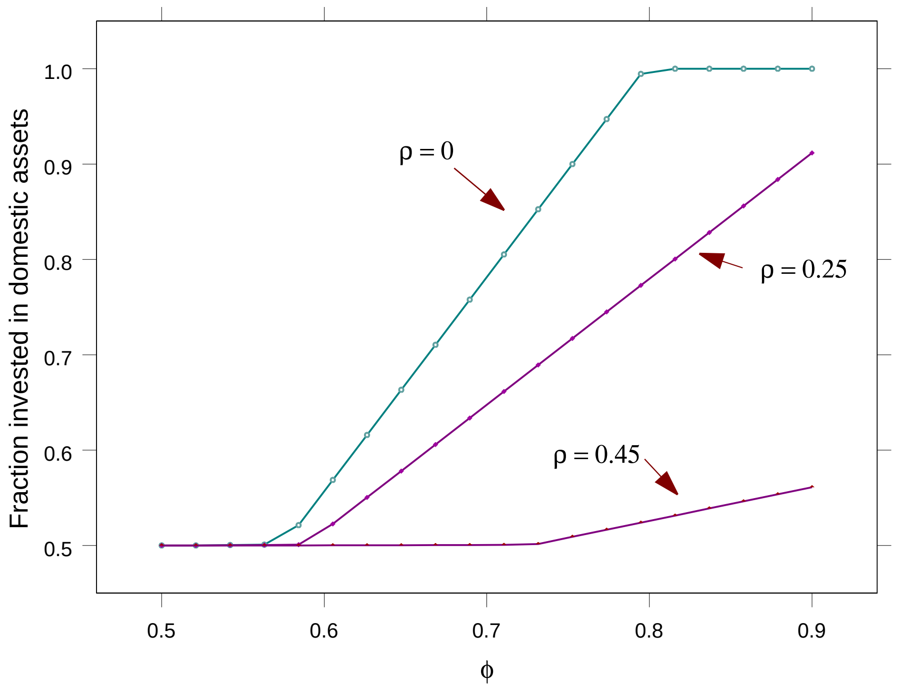

Figure 7: **Sensitivity of home equity bias to cross-asset return correlation**

agents learn from a finite set of prices.[24] We conclude the section by showing that a significant home bias can still be observed as long as prices do not convey too much information.

The main features of the model are the following ones: As before, the world is composed of two countries. There are two trees in each country. The trees display the same unconditional distribution of dividends, i.e., they are ex ante identical. For simplicity, it is assumed that agents live for two periods.[25] Agents are initially endowed with exogenous income and shares of trees. It is assumed that every agent is entitled to an equal amount of shares of local and foreign trees. There is a measure 1 of agents in each economy. The last two assumptions imply that every agent is endowed with 0.5 shares of each tree. Consumption goods are perishable. Agents can only allocate consumption across time and states by trading shares of trees. As before, short-sales are not allowed.

[24]The approach taken in this section is similar to Wallace (1992).

[25]It easy to verify that the modeling strategy followed in the previous section implies that the fraction of savings invested in local stocks is invariant to the time horizon. This allows us to compare the results of the present section with the findings in the case of constant stock prices.

Trees pay dividends in the second period and then die. Tree i pays high dividends d_h with probability ν_i, and low dividends d_l with probability $1-\nu_i$. Dividend payoffs are independent across assets. Each ν_i is drawn from a uniform distribution with support [0,1]. Agents do not observe the actual realizations of ν's but receive informative signals. The signal structure is the same as the one defined in Section 2.

The consumer's optimization problem can be stated as follows:

$$\underset{a_1,a_2,a_{1^*},a_{2^*}}{Max} \left\{ u\left(c_0\right) + \beta \sum_{i=l,h} \sum_{j=l,h} \sum_{i^*=l,h} \sum_{j^*=l,h} \Pr\left(i,j,i^*,j^* \mid \mathcal{I}\right) u\left(c_{i,j,i^*,j^*}\right) \right\} \tag{4}$$

subject to

$$\begin{aligned} c_0 &= y + p_1(0.5-a_1) + p_2(0.5-a_2) + p_{1^*}(0.5-a_{1^*}) + p_{2^*}(0.5-a_{2^*}), \\ c_{i,j,i^*,j^*} &= d_i a_1 + d_j a_2 + d_{i^*} a_{1^*} + d_{j^*} a_{2^*} \qquad for \;\; i,j,i^*,j^* \in \{h,l\}, \\ a_m &\geq 0 \quad for \; m \in \{1,2,1^*,2^*\}, \end{aligned}$$

where a_m denotes holdings of asset m, p_m denotes the market price of asset m, y denotes the exogenous income received in the first period, and $\mathcal{I}$ denotes the agent's information set.

It is necessary to differentiate between two types of state realizations. There is a current state and a future state. The current state is determined by the realization of ν_1, ν_2, ν_{1^*}, and ν_{2^*}. Given the signal structure, if agents could pool all the information available, they would only be able to differentiate between four possible state configurations, depending on which is the best asset in each country.[26] This means that there are four possible observable current states. They are described in Table 1. On the other hand, the future state realization is determined by the actual dividend shocks experienced by each of the four trees. As before, there are 16 possible future states.

Current state	Description
I	$\nu_1 > \nu_2;\ \nu_{1^*} > \nu_{2^*}$
II	$\nu_1 > \nu_2;\ \nu_{1^*} < \nu_{2^*}$
III	$\nu_1 < \nu_2;\ \nu_{1^*} > \nu_{2^*}$
IV	$\nu_1 < \nu_2;\ \nu_{1^*} < \nu_{2^*}$

Table 1: **Partitions of pooled information**

[26]This is due to the fact that the fraction ϕ does not depend on the absolute values of (ν_1, ν_2) or (ν_{1^*}, ν_{2^*}). In other words, the intensity of the signal is independent from the actual gap in expected relative performances.

5.1 A heuristic description of the model with partially revealing prices

Without additional sources of uncertainty, the current setup features fully revealing prices. For instance, suppose that current state I has taken place. Thus, a majority of domestic agents has received a signal favoring asset 1. The resulting higher demand of that asset translates into a higher relative price of local stock 1. Thus, domestic and foreign agents would be able to infer which one is the best domestic asset just by looking at market prices. A similar result would apply to foreign assets. This implies that there would be no heterogeneity across agents. On top of the egalitarian distribution of endowments assumed in this section, agents would share the same information set. Prices would adjust in such a way that agents decide not to trade and keep half of their wealth in foreign assets. Figure 8 illustrates the mapping from states to price vectors in this economy. Depending on the signal realization, each asset can have either a high, or a low price, The former is denoted by $\bar{p}^{FR}$, and the latter is denoted by $\underline{p}^{FR}$.

$$I = (\nu_1 > \nu_2,\ \nu_{1^*} > \nu_{2^*}) \longrightarrow \vec{p}_I^{FR} = \left(\bar{p}^{FR}, \underline{p}^{FR}, \bar{p}^{FR}, \underline{p}^{FR}\right)$$

$$II = (\nu_1 > \nu_2,\ \nu_{1^*} < \nu_{2^*}) \longrightarrow \vec{p}_{II}^{FR} = \left(\bar{p}^{FR}, \underline{p}^{FR}, \underline{p}^{FR}, \bar{p}^{FR}\right)$$

$$III = (\nu_1 < \nu_2,\ \nu_{1^*} > \nu_{2^*}) \longrightarrow \vec{p}_{III}^{FR} = \left(\underline{p}^{FR}, \bar{p}^{FR}, \bar{p}^{FR}, \underline{p}^{FR}\right)$$

$$IV = (\nu_1 < \nu_2,\ \nu_{1^*} < \nu_{2^*}) \longrightarrow \vec{p}_{IV}^{FR} = \left(\underline{p}^{FR}, \bar{p}^{FR}, \underline{p}^{FR}, \bar{p}^{FR}\right)$$

Figure 8: **Fully revealing prices.**

$$\begin{tikzcd}
I \arrow[r] \arrow[dr, dotted] \arrow[ddr, dotted] \arrow[dddr, dotted] & \vec{p}_I^{PR} \\
II \arrow[r] & \vec{p}_{II}^{PR} \\
III \arrow[r] & \vec{p}_{III}^{PR} \\
IV \arrow[r] & \vec{p}_{IV}^{PR}
\end{tikzcd}$$

Figure 9: **Partially revealing prices.**

In order to make the problem more interesting, the current section features a non-trivial information structure. It assumes that the supply of trees is subject to shocks. These shocks can be thought of as asset demand that arises from unmodelled agents, or due to non-informational reasons. The typical interpretation in the literature is that they reflect trades of investors faced with liquidity shocks. In this scenario, a high price of one of the assets does not necessarily signal high expected dividends. It is also possible that the asset has become valuable because its demand was hit with a large inelastic component that left few shares available to the remaining agents.

Figure 9 illustrates how the model works in the case where the current state is characterized by higher expected returns of local and foreign tree 1. This section studies a case in which there are four possible combinations of supply shocks. One is that there is no noise, i.e., no shocks have taken place. The remaining three alternatives are such that the equilibrium prices observed in those cases mimic the equilibrium prices observed in the remaining states with no shocks. A solid line is used in Figure 9 to represent the mapping from states to equilibrium prices when the shocks have taken zero value. A dashed line represents the mapping from states to prices when the shocks are non-zero. Agents face a non-trivial signal extraction problem. They observe a vector of equilibrium prices but cannot realize whether the prices reveal the actual ranking of local and foreign stocks, or they are just the result of noise.

5.2 A formal description of the model and the equilibrium concept

Formally, each supply shock consists of a four dimensional vector. Component i represents the shock to stock i. There are four possible supply shocks for each current state realization. This means that, unconditionally, there are sixteen possible supply shocks. One of the four possible shock vectors is the null vector, This is independent of the initial state that has been realized. When the null vector is realized, the asset supplies are unaffected. However, the remaining shocks are such that the resulting equilibrium prices can mimic prices observed in other states with zero shocks. Formally, denote by $\{\vec{\mu}_{ij}\}_{j=1}^{4}$ the set of possible supply shocks in current state i. Notice that $\vec{\mu}_{ij} \in \mathbb{R}^4$ $\forall i, j$, and $\vec{\mu}_{ii} = \vec{0}$ $\forall i$. Let us denote by $\vec{p}_i \in \mathbb{R}^4$ the equilibrium price vector in state i with zero shocks. When the vector of supply shocks takes a value $\vec{\mu}_{ij}$, the equilibrium price vector equals $\vec{p}_j$, independently of the state i that has been realized.

There is a probability $q + \frac{1-q}{4}$ that the supply shock takes null values. All other shock realizations occur with probability $\frac{1-q}{4}$. The degree of informativeness of market prices is summarized in the value taken by q. If $q = 0$ prices are fully uninformative. If $q = 1$, prices are fully informative. Prices are partially revealing in all other cases.

Denote by $\vec{\lambda}_i \in \mathbb{R}^4$ the vector of measures of agents in current state i. The first two components of vector $\vec{\lambda}_i$ correspond to the fractions of domestic agents receiving signals 1 and 2, respectively. The last two components correspond to the fractions of foreign agents receiving signals 1^* and 2^* respectively. These measures are summarized in Table 2.

Let $\vec{a}\,(\vec{p}, s)$ denote the vector of asset demands that solves optimization problem (4). Namely,

Current state	Local agents		Foreign agents	
	Signal 1	Signal 2	Signal 1*	Signal 2*
I	ϕ	$1-\phi$	ϕ	$1-\phi$
II	ϕ	$1-\phi$	$1-\phi$	ϕ
III	$1-\phi$	ϕ	ϕ	$1-\phi$
IV	$1-\phi$	ϕ	$1-\phi$	ϕ

Table 2: **Measure of agents depending on the current state realization**

$\vec{a}(\vec{p}, s) = [a_1(\vec{p}, s), a_2(\vec{p}, s), a_{1^*}(\vec{p}, s), a_{2^*}(\vec{p}, s)]$. Let $Z_{ij}^k(\vec{p})$ denote the excess demand of asset k in state i with supply shock ij and price vector $\vec{p}$. Formally,

$$Z_{ij}^k(\vec{p}) = \sum_{l=1}^{4} \vec{\lambda}_i(l)\, \vec{a}_k(\vec{p}, \vec{s}(l)) - (1 + \vec{\mu}_{ij}(l)) \text{ for } i, j \in \{I, II, III, IV\}, \quad k \in \{1, 2, 1^*, 2^*\},$$

where $\vec{x}(l)$ denotes the component l of vector $\vec{x}$. The term $\vec{s}$ denotes the vector of possible signal realizations. The current state, indexed by i, determines the measure of agents receiving a signal favoring local tree 1, as well as the possible values that the supply shocks may take. The shocks, indexed by j, determine the available net supply of assets once the inelastic component has been incorporated.

Definition 1 *A rational expectations equilibrium (REE) consists of a set of price vectors $\vec{p}_I$, $\vec{p}_{II}$, $\vec{p}_{III}$, $\vec{p}_{IV}$, and individual demands $\vec{a}(\vec{p}, s)$ such that:*

(i) $\vec{a}(\vec{p}, s)$ solves each consumer's optimization problem for market prices $\vec{p}$ and individual signal s.

(ii) Markets Clear: $Z_{ij}^k(\vec{p}) = 0\ \forall\, i, j \in \{I, II, III, IV\}$ where $k \in \{1, 2, 1^*, 2^*\}$.

(iii) Agents update their beliefs using their private signal and market prices according to Bayes' rule.[27]

Given the particular uncertainty structure assumed in this section, the equilibrium satisfies an extra condition:

- Prices are partially revealing: $\vec{p} = \vec{p}_j$ whenever $\vec{\mu} = \vec{\mu}_{ij} \quad \forall i, j \in \{I, II, III, IV\}$.

[27] Appendix contains a formal description of the beliefs' updating scheme.

Unfortunately, the above problem does not allow for a closed-form solution. This implies that the equilibrium must be found using numerical techniques. Given that the trees are ex ante identical and that the optimization problems of domestic and foreign investors are entirely symmetric, it is sufficient to solve for equilibrium prices in two cases: first, when the price of domestic asset 1 is higher than the price of domestic asset 2 and $\nu_1 > \nu_2$; second, when the same ranking of local prices is combined with $\nu_1 < \nu_2$. Local agents do not receive signals about foreign assets, so the results are invariant to the ranking of prices in foreign markets. We show the results for the first case. The second one features higher levels of home bias in the range of values we are interested in, i.e., when prices do not reveal too much information.[28]

The model is solved assuming that agents share a logarithmic utility function. The low dividend value is set to 0.8 and the high dividend value is set to 1.2. Finally, q and ϕ are left as free parameters. The first one controls for the degree of informativeness of market prices. The second one determines how informative individual signals are. The results are shown in Figure 10.

If prices are very informative or individual signals are not sufficiently informative, local agents invest roughly half of their portfolios in domestic assets. This corresponds to values of q close to 0, or values of ϕ close to 0.5. As prices become less informative or individual signals become more informative, investors start to bias their portfolio toward domestic securities. The graph shows that the home equity bias can be quite significant when prices are not very informative.

The graph also shows that the bias may decrease with the precision of the signal if the latter is already sufficiently informative. The reason is the following. Agents that display the strongest preference toward domestic assets are the ones receiving an incorrect signal. Their beliefs mirror the beliefs of agents receiving the correct signal, but the price of the asset for which they expect higher dividends is lower. As the signal becomes more precise (ϕ increases), the fraction of individuals with this strong preference for local assets decreases, driving down the overall fraction invested in local assets.

Table 3 illustrates the magnitude of supply shocks for a case where 65 percent of domestic portfolios are composed of local assets. It shows that it is not necessary to consider extreme shocks in order to observe a significant level of home bias. Even though the model presented in this section relies on ad-hoc assumptions, we conjecture that extending the model to less arbitrary distributions of shocks or allowing for other sources of uncertainty that mask the current state realization, will not lead to

[28]The reason is that a majority of domestic agents receive a signal favoring the cheapest local asset, which reinforces the desire to invest locally

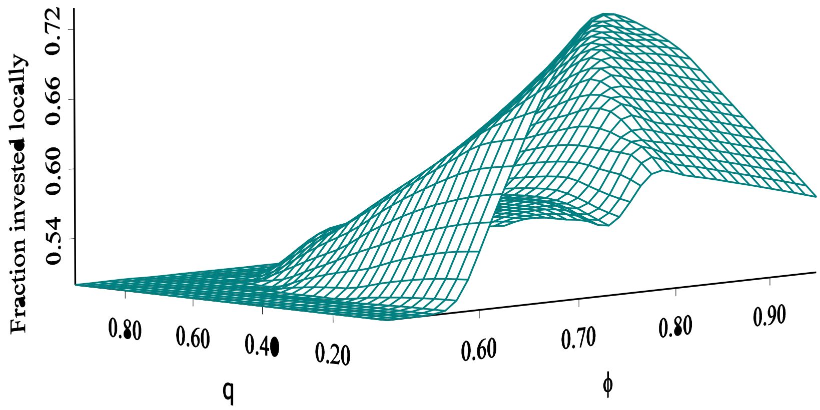

Figure 10: **Fraction invested in local assets as a function of the information conveyed by prices and individual signals**

different qualitative conclusions. The difference is that in a more general case, agents need to learn over a fourth dimensional space. This is due to the fact that there are four prices that convey valuable information. The mechanism leading to partially revealing prices would not be qualitatively different from the one assumed in this section, but the level of complexity would be significantly larger.

6 The shadow price of the short-sales constraint

The restriction on short-sales captures in a simple way the fact that short-selling is a costly activity. A more general formulation can be developed assuming, for example, that agents are required to pay a fee in order to hold short positions. The fee should include not only the direct cost derived from the equity loan, but also the implicit cost due to legal restrictions and the extra risk incurred by short-selling (like an early recall). This section finds the implicit fee that prevents agents from selling short.

It is assumed that agents pay a fee τ whenever they short-sell. The fee is proportional to the amount

Stated mimicked	Asset 1	Asset 2	Asset 1^*	Asset 2^*
I	0.000000	0.000000	0.000000	0.000000
II	-0.080300	0.010677	-0.358955	0.437354
III	-0.358955	0.437354	-0.080300	0.010677
IV	-0.439255	0.448031	-0.439255	0.448031

Table 3: **Supply shocks to domestic and foreign assets when $q = 0.1$ and $\phi = 0.65$ Expressed as a fraction of the average supply of each asset.**

sold short. The investor's optimization problem is set out below. The only difference with respect to the optimization problem in the baseline model is the individual's budget constraint. Without loss of generality, we consider the problem of an agent with a signal realization that favors asset 1.

$$V(\omega) = \underset{k'_1, k'_2, k'_{1*}, k'_{2*}}{Max} \left\{ u(c) + \beta E \left[V\left(\omega' \mid s = 1\right) \right] \right\} \tag{5}$$

subject to

$$c + k'_1 \left(1 + I\left(k'_1\right)\tau\right) + k'_2 \left(1 + I\left(k'_2\right)\tau\right) + k'_{1*} \left(1 + I\left(k'_{1*}\right)\tau\right) + k'_{2*} \left(1 + I\left(k'_{2*}\right)\tau\right) = \omega,$$

where

$$I(x) = \begin{cases} -1 & if \;\; x < 0, \\ 0 & if \;\; x > 0. \end{cases}$$

The value of τ consistent with the observation that agents do not hold short positions can be retrieved from the first order conditions in the problem with short-sales constraints.[29] The results shown in Figure 11 are based on the benchmark parameterization used in Section 4, where a cross-asset return correlation of 0.25 is assumed. The graphs illustrate that the model is capable of generating significant levels of home bias without imposing high costs on short sales. For instance, when the fraction invested in local stocks is around 75 percent, the implicit fee is 2.5 percent.

[29] As it was said before, policy functions are linear in wealth. Thus, the policy function of asset i can be written as $k'_i(\omega) = \alpha_i \omega$ with $i \in \{1, 2, 1^*, 2^*\}$. There are 16 future state realizations depending on the productivity shock faced by each local and foreign technology. Let A_i^h denote the productivity shock received by technology i in state h. Since we consider the case where the local signal favors technology 1, the short-sales constraint binds when the investor wants to short sell stocks of technology 2. The implicit value of τ consistent with no-short-sales is therefore obtained as follows:

$$\tau = 1 - \beta \sum_{h=1}^{16} \Pr(j \mid s = 1) \frac{A_2^h}{\sum\limits_{i=1,2,1^*,2^*} \alpha_i A_i^h}.$$

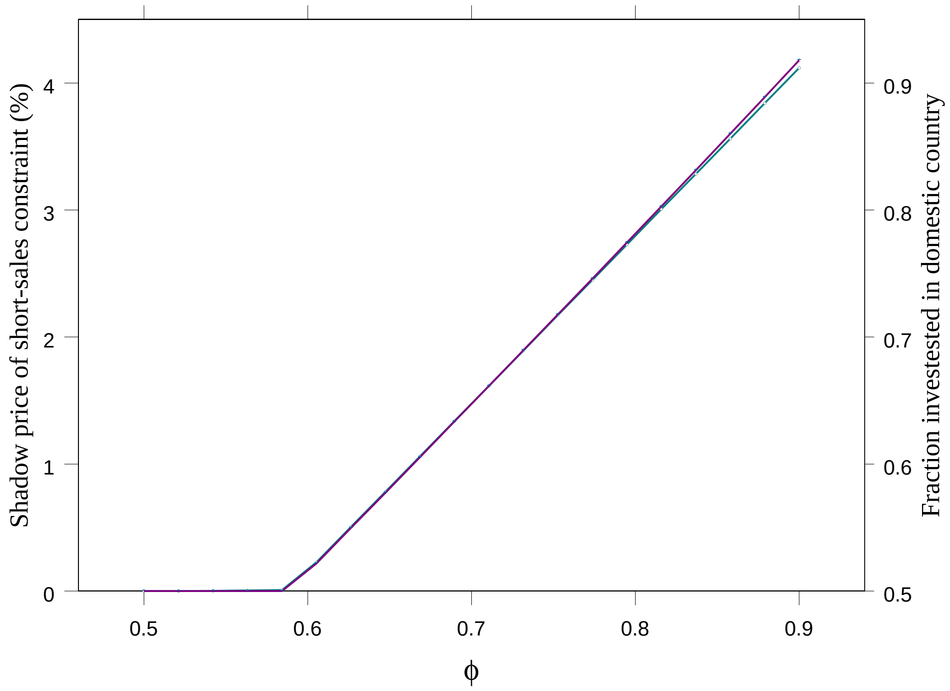

Figure 11: **Home bias and shadow price of short-sales constraint in the baseline model with cross-asset return correlation of 0.25.**

7 Analytical characterization in the CARA-Gaussian framework

This section considers a model that shares the blueprints of the framework laid down in Section 2 and has the advantage of being analytically tractable. However, it uses a more complex and nonstandard structure for macroeconomic analysis. In addition, it generates a volatile fraction of foreign investments. For these reasons it is not taken as our benchmark model.

The setup analyzed in this section assumes that agents live for two periods. They consume at the end of the second period. Only investment activities take place in the first period. As before, there are two technologies available in each country. Production technology is of the "AK" type, but the productivity shock follows a different process. The shock to technology i (A_i) consists of two parts,

$$A_i = \mu_i + \epsilon_i,$$

where both μ_i and ϵ_i are normally distributed.[30] More precisely,

$$\begin{aligned}\mu_i &\sim N\left(\theta, \sigma_\mu^2\right), \\ \epsilon_i &\sim N\left(0, \sigma_\epsilon^2\right) \quad \forall i = 1, 2, 1^*, 2^*.\end{aligned}$$

.

This section maintains the assumption that the signal received by local agents reveals information about relative performance of local technologies, but not about the aggregate performance of the home country. Formally, each local agent observes a private signal s that satisfies the following:

$$s = \mu_1 - \mu_2 + \xi,$$

$$\xi \sim N\left(0, \sigma_\xi^2\right).$$

The realization of ξ is idiosyncratic. It is assumed that agents cannot pool the signals. They only observe their own signal. Each foreign investor receives a private signal s^*, with

$$s^* = \mu_1^* - \mu_2^* + \xi.$$

For simplicity, it is assumed that all normal variables introduced before are uncorrelated. The final modification with respect to our benchmark framework is that investors display preferences with constant coefficient of absolute risk aversion. The utility function of local and foreign agents has the following form:

$$u(c) = -e^{-\lambda c}.$$

Consider now the problem of a local investor endowed with ω units of the good and signal s. His objective is to maximize his expected utility of consumption, i.e.

$$E\left[-e^{-\lambda[k_1 A_1 + k_2 A_2 + k_{1^*} A_{1^*} + (\omega - k_1 - k_2 - k_{1^*}) A_{2^*}]} \mid s\right],$$

subject to

$$k_i \geq 0 \quad \forall i = 1, 2, 1^*.$$

The demand of the fourth asset (foreign technology 2) is obtained as a residual. The preferences, dividend distribution, and signal structure assumed in this section allow us to reduce the optimization

[30] Since the only information agents receive is about μ_i, the component ϵ_i determines how useful that information is.

problem to maximizing the certainty equivalent of consumption. After rearranging terms, the objective function simplifies to the following expression:

$$\begin{bmatrix} k_1 & k_2 & k_{1^*} & \omega \end{bmatrix} \begin{bmatrix} E\left[A_1 - A_{2^*} \mid s\right] \\ E\left[A_2 - A_{2^*} \mid s\right] \\ E\left[A_{1^*} - A_{2^*} \mid s\right] \\ E\left[A_{2^*} \mid s\right] \end{bmatrix} - \frac{\lambda}{2} \begin{bmatrix} k_1 & k_2 & k_{1^*} & \omega \end{bmatrix} \Sigma \begin{bmatrix} k_1 \\ k_2 \\ k_{1^*} \\ \omega \end{bmatrix}.$$

The matrix Σ denotes the covariance matrix of $[A_1 - A_{2^*}, A_2 - A_{2^*}, A_{1^*} - A_{2^*}, A_{2^*}]$ conditional on the information conveyed by the signal. The conditional expectations and covariance matrix can be found using the projection theorem.[31]

$$\Sigma = \begin{bmatrix} 2\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) - \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\xi^2} & \sigma_\mu^2 + \sigma_\epsilon^2 + \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\xi^2} & \sigma_\mu^2 + \sigma_\epsilon^2 & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \\ \sigma_\mu^2 + \sigma_\epsilon^2 + \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\xi^2} & 2\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) - \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\xi^2} & \sigma_\mu^2 + \sigma_\epsilon^2 & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \\ \sigma_\mu^2 + \sigma_\epsilon^2 & \sigma_\mu^2 + \sigma_\epsilon^2 & 2\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \\ -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & \sigma_\mu^2 + \sigma_\epsilon^2 \end{bmatrix}$$

and

$$\begin{bmatrix} E\left[A_1 - A_{2^*} \mid s\right] \\ E\left[A_2 - A_{2^*} \mid s\right] \\ E\left[A_{1^*} - A_{2^*} \mid s\right] \\ E\left[A_{2^*} \mid s\right] \end{bmatrix} = \begin{bmatrix} \frac{\sigma_\mu^2 s}{2\sigma_\mu^2 + \sigma_\epsilon^2} \\ \frac{-\sigma_\mu^2 s}{2\sigma_\mu^2 + \sigma_\epsilon^2} \\ 0 \\ \theta \end{bmatrix}.$$

The expressions for the conditional expectations show that the expected return of asset 1 is corrected upward upon the arrival of a positive signal, while the expected return of asset 2 is corrected downward. Also, the main diagonal of Σ shows that the availability of some information about domestic assets drives their perceived variance down compared to that of foreign assets. The domestic signal does not carry any information about foreign assets, so its perceived probability distribution coincides with the unconditional distribution.

[31] Consider two normally distributed random vectors, say X and S.

$$\begin{bmatrix} X \\ S \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_X \\ \mu_S \end{bmatrix}, \begin{bmatrix} \Sigma_{X,X} & \Sigma_{X,S} \\ \Sigma_{S,X} & \Sigma_{S,S} \end{bmatrix}\right)$$

The distribution of X given $S = s$ is also normal.

$$(X \mid S = s) \sim N\left(\mu X + \Sigma_{X,S}\Sigma_{S,S}^{-1}\left(s - \mu_S\right), \Sigma_{X,X} - \Sigma_{X,S}\Sigma_{S,S}^{-1}\Sigma_{S,X}\right)$$

Equation (6) reports the optimal investment behavior in the unconstrained problem, i.e., when agents do not face short-sales constraints.

$$\begin{bmatrix} k_1 \\ k_2 \\ k_{1^*} \end{bmatrix} = \frac{\omega}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{s}{\lambda} \begin{bmatrix} \frac{\sigma_\mu^2}{(\sigma_\mu^2+\sigma_\epsilon^2)(2\sigma_\mu^2+\sigma_\xi^2)-2\sigma_\mu^4} \\ \frac{-\sigma_\mu^2}{(\sigma_\mu^2+\sigma_\epsilon^2)(2\sigma_\mu^2+\sigma_\xi^2)-2\sigma_\mu^4} \\ 0 \end{bmatrix}. \tag{6}$$

If the signal is not very informative (σ_ξ^2 is high), asset holdings resemble the perfectly diversified portfolio, where an equal amount is invested in each asset. A similar result holds if investors are highly risk averse (high λ) or returns are volatile (high $\sigma_\mu^2 + \sigma_\epsilon^2$). However, it is easy to verify that agents always allocate half of their portfolios in domestic assets, regardless of the signal realization.

In the constrained problem, the solution coincides with (6) whenever the signal does not take extreme values. Equation (7) describes the solution in the case where the short-sales constraint is binding for one of the local assets.

$$\begin{pmatrix} k_i \\ k_{1^*} \end{pmatrix} = \frac{1}{3\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right)-2\sigma_\mu^4} \left[\frac{\sigma_\mu^2}{\lambda}\begin{pmatrix} 2 \\ -1 \end{pmatrix}|s| + \omega\left(2\sigma_\mu^2+\sigma_\xi^2\right)\begin{pmatrix} \sigma_\mu^2+\sigma_\epsilon^2 \\ \sigma_\mu^2+\sigma_\epsilon^2-\frac{\sigma_\mu^4}{2\sigma_\mu^2+\sigma_\xi^2} \end{pmatrix}\right], \tag{7}$$

where

$$\begin{aligned} i &= 1 \text{ if } s \in \left(\frac{\omega}{4}\frac{\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right)-2\sigma_\mu^4\right]}{\sigma_\mu^2}, \frac{\omega\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right)-\sigma_\mu^4\right]}{\sigma_\mu^2}\right), \\ i &= 2 \text{ if } s \in \left(-\frac{\omega}{4}\frac{\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right)-2\sigma_\mu^4\right]}{\sigma_\mu^2}, -\frac{\omega\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right)-\sigma_\mu^4\right]}{\sigma_\mu^2}\right). \end{aligned}$$

It is easy to check from the previous equation that the fraction invested in local assets grows as the signal increases in absolute value. Agents fully specialize in one of the domestic assets when the signal received is sufficiently large in absolute value.

The previous solution shows a result also observed in the benchmark model: The bias decreases with the degree of risk aversion (λ). Similarly, we may also expect to observe more diversified portfolios as asset returns become more volatile. The following proposition shows that this is not always the case.

Proposition 2 *The home equity bias increases with* σ_μ^2 *if* $2\left[\left({\sigma_\mu}^2+{\sigma_\epsilon}^2\right)\left(2{\sigma_\mu}^2+{\sigma_\xi}^2\right)-{\sigma_\mu}^4\right] \times \left(3{\sigma_\epsilon}^2{\sigma_\xi}^2-4{\sigma_\mu}^4\right)+2{\sigma_\mu}^2\left[{\sigma_\mu}^2{\sigma_\xi}^2+2\left({\sigma_\mu}^2+{\sigma_\xi}^2\right){\sigma_\epsilon}^2\right]>0.$

Lemma 3 *When σ_μ^2 is sufficiently small, the home equity bias increases with σ_μ^2 when $\sigma_\mu^2 < \sqrt{\frac{3}{4}}\sigma_\epsilon\sigma_\xi$ and decreases with σ_μ^2 when $\sigma_\mu^2 > \sqrt{\frac{3}{4}}\sigma_\epsilon\sigma_\xi$.*

The explanation is that changes in σ_μ^2 induce a horse race between two effects: On the one hand, as the component of the return from which agents learn become more volatile, there is more room for a wider dispersion of beliefs at the individual level. This increases the bias. On the other hand, more volatile returns induce a stronger desire to hold diversified portfolios. This discourages full specialization in one of the local stocks and, henceforth, it reduces the bias.

8 Conclusions

There is pervasive evidence that individuals and institutional investors favor stocks of their own country. In addition, empirical studies show that there exists a home equity bias within US boundaries. Households and mutual funds prefer stocks of proximate companies. These studies also show that the returns agents enjoy on local stocks exceed the returns on non-local stocks. This suggests that the lack of portfolio diversification is based on rational behavior. It also points towards the presence of informational asymmetries in financial markets.

This paper develops a theoretical model that can explain a significant fraction of the bias observed in the data. The model differs from the standard theory in three aspects. First, it considers the case of multiple stocks per country. Second, it assumes that local investors are able to collect more precise information about the ranking of local stocks than that of foreign stocks. Third, it assumes that short-sales are costly. In this environment, each domestic investor displays a strong preference for certain local stocks. When the information collected is sufficiently precise, local investors find it convenient to finance purchases of the perceived good local stocks by selling short the perceived bad local stocks. However, if the cost of short-selling is high, domestic investors decide to sacrifice the diversification services provided by their foreign investments in order to concentrate their equity portfolio in the (domestic) stocks that are thought to offer higher expected returns. Unlike previous papers in the literature, the underlying mechanism that explains the bias for local stocks is based on first order effects, i.e., differences in expected returns. This explains the ability of the model to generate significant quantitative results. In addition, the model has several testable implications regarding portfolio behavior that are in line with previous empirical studies.

We show that the strong home equity bias implied by the model is robust to several changes in the baseline specification. However, there was one extension that was not pursued in the paper: the case of persistence in stock dividends. The introduction of permanent shocks has a double effect. On the one hand, it increases the power of private information. The latter can now be used to forecast the future stream of returns. Without persistence, it only helps to predict next period returns. This effect strengthens the mechanism that generates home bias. On the other hand, there is more public information available. If dividends have a persistent component, agents can learn from past realizations of dividends. This reduces the role of private information and undermines the incentives to invest heavily in local stocks.[32] However, this extension poses two challenges from a technical point of view. First, it can only be solved under a recursive structure. The problem becomes intractable if agents need to keep track of all past dividend realizations and signals received in order to compute their beliefs. Second, it requires dealing with multiple state variables with continuous domain.

[32] This conclusion depends on the fact that stock prices adjust to the information contained in past dividend realizations. If prices are constant over time, as in the "AK" model, the adjustment is made through quantities. In this case, the fraction invested overseas is low on average, but can display a high volatility.

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A Beliefs updating scheme when agents learn from signals and prices

Agents need to infer the probability distribution over future states before solving their optimization problem. They receive two pieces of information: prices and individual signals. Both of them reveal information regarding the relative values of ν_1, ν_2, ν_{1^*} and ν_{2^*}. Agents' beliefs consist of the expected probability distribution over future states conditional on the information received.[33] In order to simplify the exposition, it is assumed that the market price vector corresponds to the equilibrium prices agents would observe in current state I without supply shocks, i.e., $\vec{p} = \vec{p}_I$. It is straightforward to generalize the formulas to other cases. The formal expression for the expected probability of future state i, j, i^*, j^* given prices $\vec{p}$ and private signal s is illustrated below.

$$E\left[Pr\left(i,j,i^*,j^*\right) \mid \vec{p}, s\right] = \frac{E\left[\Pr\left(i,j,i^*,j^*,\vec{p},s\right)\right]}{\Pr\left(\vec{p},s\right)} =$$

$$\frac{\begin{array}{c}\Pr\left(\nu_1 > \nu_2; \nu_{1^*} > \nu_{2^*}\right)\left(\frac{1-q}{4}+q\right)\Pr\left(s \mid \nu_1 > \nu_2\right) E\left[\Pr\left(i,j,i^*,j^*\right) \mid \nu_1 > \nu_2; \nu_{1^*} > \nu_{2^*}\right]+\\ \Pr\left(\nu_1 > \nu_2; \nu_{1^*} < \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 > \nu_2\right) E\left[\Pr\left(i,j,i^*,j^*\right) \mid \nu_1 > \nu_2; \nu_{1^*} < \nu_{2^*}\right]+\\ \Pr\left(\nu_1 < \nu_2; \nu_{1^*} > \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 < \nu_2\right) E\left[\Pr\left(i,j,i^*,j^*\right) \mid \nu_1 < \nu_2; \nu_{1^*} > \nu_{2^*}\right]+\\ \Pr\left(\nu_1 < \nu_2; \nu_{1^*} < \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 < \nu_2\right) E\left[\Pr\left(i,j,i^*,j^*\right) \mid \nu_1 < \nu_2; \nu_{1^*} < \nu_{2^*}\right]\end{array}}{\begin{array}{c}\Pr\left(\nu_1 > \nu_2; \nu_{1^*} > \nu_{2^*}\right)\left(\frac{1-q}{4}+q\right)\Pr\left(s \mid \nu_1 > \nu_2\right)+\\ \Pr\left(\nu_1 > \nu_2; \nu_{1^*} < \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 > \nu_2\right)+\\ \Pr\left(\nu_1 < \nu_2; \nu_{1^*} > \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 < \nu_2\right)+\\ \Pr\left(\nu_1 < \nu_2; \nu_{1^*} < \nu_{2^*}\right)\frac{1-q}{4}\Pr\left(s \mid \nu_1 < \nu_2\right)\end{array}}$$

The second equation above uses the law of conditional probabilities and the third one uses Bayes' rule. Every current state realization could lead to the observed market price $\vec{p}$. Thus, when agents compute their beliefs, they span over the four possible current states. The first element in each term on the numerator denotes the a priori probability of being in each current state. The second and third components capture the probability of observing prices $\vec{p}$ and signal s for each current state. Finally, the fourth component computes the expected probability that the future dividend shocks take values i, j, i^*, j^* for each current state realization.

[33] It is sufficient to compute the expectation of these probabilities because the latter enter linearly in the individual's first order conditions.

B Proof of proposition 2

Denote by Φ the fraction invested in local assets. There is a home bias when $\Phi > 0.5$. The aggregate fraction invested in local assets depends on the actual realizations of μ_1 and μ_2. The latter conditions the distribution of information across agents. For instance, if the difference between these two variables is large, a high fraction of local investors will receive extreme signals. In order to allow for a general statement, we consider the ex ante expectation of Φ. That is, the unconditional expected fraction invested in local assets. The latter is computed as follows:

$$E(\Phi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{{\sigma_\mu}^2 2\pi} exp\left[-\frac{(\mu_1-\theta)^2}{2{\sigma_\mu}^2} - \frac{(\mu_2-\theta)^2}{2{\sigma_\mu}^2}\right] \times$$

$$\left\{ \begin{array}{l} \int_{-\infty}^{-\bar{s}} f(s \mid \mu_1,\mu_2)\, ds d\mu_1 d\mu_2 + \int_{-\bar{s}}^{-\underline{s}} g(s)\, f(s \mid \mu_1,\mu_2)\, ds d\mu_1 d\mu_2 + \int_{-\underline{s}}^{\bar{s}} \frac{f(s|\mu_1,\mu_2)}{2} ds d\mu_1 d\mu_2 + \\ \int_{\underline{s}}^{\bar{s}} g(s)\, f(s \mid \mu_1,\mu_2)\, ds d\mu_1 d\mu_2 + \int_{\bar{s}}^{\infty} f(s \mid \mu_1,\mu_2)\, ds d\mu_1 d\mu_2 \end{array} \right\},$$

where

$$g(s) = \frac{1}{3\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right) - 2\sigma_\mu^4}\left[\frac{\sigma_\mu^2}{\omega\lambda} 2|s| + \left(2\sigma_\mu^2+\sigma_\xi^2\right)\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\right]$$

$$\underline{s} = \frac{\omega}{4}\frac{\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right) - 2\sigma_\mu^4\right]}{\sigma_\mu^2}$$

$$\bar{s} = \frac{\omega\lambda\left[\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right) - \sigma_\mu^4\right]}{\sigma_\mu^2}.$$

We are interested in the derivative

$$\frac{\partial E(\Phi)}{\partial {\sigma_\mu}^2}. \tag{B.1}$$

It is easy to verify that the derivatives of the threshold values ($\underline{s}$ and $\bar{s}$) with respect to σ_μ^2 cancel out and therefore, do not play a role. This implies that the sign of Equation (B.1) depends on the sign of the derivatives of the values taken by $g(s)$. Without loss of generality, we consider now the case where $s > 0$.

$$\frac{\partial g(s)}{\partial \sigma_\mu^2} = \frac{\frac{2s}{\lambda\omega}\left(3\sigma_\epsilon^2\sigma_\xi^2 - 4\sigma_\mu^4\right) + 2\sigma_\mu^2\left[\sigma_\mu^2\sigma_\xi^2 + 2\left(\sigma_\mu^2+\sigma_\xi^2\right)\sigma_\epsilon^2\right]}{\left[3\left(\sigma_\mu^2+\sigma_\epsilon^2\right)\left(2\sigma_\mu^2+\sigma_\xi^2\right) - 2\sigma_\mu^4\right]^2}$$

The sign of the expression above is ambiguous. However, it is possible to find a sufficient condition that can help us to identify cases where the derivative takes positive values. If the sign is positive for

$s = \bar{s}$, then it must be positive for all possible signals belonging to the range $[\underline{s}, \bar{s}]$. This substitution yields the sufficient condition stated in the text. From the previous equation, it is easy to see that when σ_μ^2 is sufficiently small, the first term in the numerator dominates the entire expression and, therefore it determines the sign of the derivative. ■