Population Policy through Tradable Procreation Rights*

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Abstract

Tradable permits are now widely used to control pollution. We investigate the implications of setting up such a system in another field – population control –, either domestically or at the global level. We first generalize the framework with both tradable procreation allowances and tradable procreation exemptions, in order to tackle both over- and under-population problems. The implications of procreation rights for income inequality and education are contrasted. With procreation exemptions or procreation allowances that would be expensive enough, resources are redistributed from the rich to the poor. In contrast, cheap procreation allowances redistribute resource towards the rich. As far as human capital is concerned, natalist policy would be bad for education, while population control would be good. If procreation rights are granted in proportion to existing fertility levels (grandfathering) instead of being allocated uniformly, population control can be made more redistributive. On the whole, procreation rights offer an interesting alternative to both coercive population control in developing countries and pronatalist policies in the developed world.

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Introduction

In many countries, a gap obtains between the size of the existing population and what is perceived as the optimal population size. A significant number of developed countries have been adopting pronatalist policies since the 1930's e.g. through family benefits. Conversely, a country like China has been implementing coercive measures since the 1970's in order to reduce its population. The reasons why the existing population may be deemed sub-optimal vary both historically and geographically. In developed countries, the post-WW II population was judged too small because [***] And today, the size imbalance between the active population and those who depend on it is due to an increasing life expectancy and a decreasing fertility rate. Such a size difference allegedly threatens the financial viability of our pension schemes and health care systems. Turning to the developing world, the reason why Chinese authorities aim at stabilizing their population is [***] And it is certain that at a global level, one of the central reasons why one may want to consider the Earth as overpopulated has to do with a concern for natural resources depletion, environmental degradation and an alleged inability of the planet to feed and provide sufficient space to such a population.

Not only can the idea of an optimal population be invoked with various economic and non-economic reasons in mind. Philosophers have also shown that such an idea is not unproblematic, which becomes clear once we ask ourselves: "optimal for whom?". In the present paper, we shall leave such questions aside, assuming that in a given country at a given point in time it can be meaningful to aim at a population size that would be judged optimal. Rather, we want to focus on another dimension of the problem, i.e. the means that should be used to reach such a desired population level. More specifically, we want to apply the system of tradable rights to procreation, focusing on a few of its possible effects. Given that population policies are likely to affect an essential dimension of people's life, it is especially important to design policies that are both effective and as little coercive as possible. [1]

Tradable quotas schemes have been promoted as a policy tool for several decades. they have typically been proposed and widely implemented to combat air pollution,[2] over-production (e.g. tradable milk quotas in the EU), overexploitation of natural resources (e.g. individual transferable fish quotas). It has also been proposed in other areas - while never being implemented - such as inflation control (Lerner and Colander 1980), and, more recently, asylum policy (Schuck (97) and Hathaway and Neve (1997)) or deficit control (Casella 1999). The idea is always to agree on a cap to reduce the extent of a given problem (over-production, over-inflation, pollution, excessive unemployment,), to allocate the corresponding rights to the various actors involved (states, firms and/or individuals), and to allow for tradability of such rights between the actors, in order to

[1] Actual policies are often criticized because either they are coercive (Sen 1996), or they are not very effective (Demeny 1987).

[2] The 1990 Clean Air Act Amendments initiated the first large-scale use of the tradable permit approach to pollution control. The empirical analysis of Joskow, Schmalensee, and Bailey (1998) shows that the emission rights market created in 1990 had become reasonably efficient within four years.

take into account differences in marginal reduction costs. One of the oldest of such proposals is ?)'s idea of tradable procreation licenses to combat overpopulation.

After reviewing the literature in Section 1, we describe our benchmark model with endogenous fertility and education choice in Section 2. We introduce tradable procreation allowances granted either to households or countries in Section 3. We also generalize the system to accommodate the reverse situation, when they are tradable procreation exemptions. We look at how these rights modify the arbitrage conditions. Next we study the existence of equilibrium prices for these rights. We then address in Section 4 one of the possible objections against tradable rights: the fact that tradability will impoverish the poor even further. Consequences in terms of education are analyzed in Section 5. Finally, in Section 6, we study the redistributive impact of an alternative allocation rule of procreation rights, granting rights in proportion to existing fertility levels (grandfathering). Section 7 concludes.

1 Literature Review

Boulding's Proposal

Since the middle of the 20th century there is a growing anxiety that earth may not be able to sustain an ever increasing population. In an attempt to address this issue, Kenneth Boulding proposed (1964: 135-136):

> I have only one positive suggestion to make, a proposal which now seems so far-fetched that I find it creates only amusement when I propose it. I think in all seriousness, however, that a system of marketable licenses to have children is the only one which will combine the minimum of social control necessary to the solution to this problem with a maximum of individual liberty and ethical choice. Each girl on approaching maturity would be presented with a certificate which will entitle its owner to have, say, 2.2 children, or whatever number would ensure a reproductive rate of one. The unit of these certificates might be the "deci-child," and accumulation of ten of these units by purchase, inheritance, or gift would permit a woman in maturity to have one legal child. We would then set up a market in these units in which the rich and the philoprogenitive would purchase them from the poor, the nuns, the maiden aunts, and so on. The men perhaps could be left out of these arrangements, as it is only the fertility of women which is strictly relevant to population control. However, it may be found socially desirable to have them in the plan, in which case all children both male and female would receive, say, eleven or twelve deci-child certificates at birth or at maturity, and a woman could then accumulate these through marriage.
>
> This plan would have the traditional advantage of developing a long-run tendency toward equality in income, for the rich would have many children

> and become poor and the poor would have few children and become rich. The price of the certificate would of course reflect the general desire in a society to have children. Where the desire is very high the price would be bid up; where it was low the price would also be low. Perhaps the ideal situation would be found when the price was naturally zero, in which case those who wanted children would have them without extra cost. If the price were very high the system would probably have to be supplemented by some sort of grants to enable the deserving but impecunious to have children, while cutting off the desires of the less deserving through taxation. The sheer unfamiliarity of a scheme of this kind makes it seem absurd at the moment. The fact that it seems absurd, however, is merely a reflection of the total unwillingness of mankind to face up to what is perhaps its most serious long-run problem.

Design issues

Heer (1975) and Daly(1991,1993) discuss Boulding's proposal. They both propose amendments or complements to the scheme's design. Such proposals essentially revolve around four issues: the need for continuous adjustments of the birth rate target, the issue of shifting up the reproduction age through the system, the problem of early mortality and the definition of the license beneficiaries.

As to the first issue, Heer suggests that the government (and not only the individual permit users) could be allowed to buy such permits. A similar issue arises in the field of pollution permits, where e.g. environmental NGOs, despite not being permit users, are allowed to influence the global cap through buying permits. In the present case, Heer identifies two reasons why it may be worth allowing the government to be a permit buyer as well. On one hand, in order to deal with the problem of partial non-compliance, the government could buy on the market a number of permits corresponding with the number of unlicensed babies (Heer, 1975: 4). On the other hand, in contrast with the case of pollution permits, procreation rights are allocated for life rather than for a given period. And other factors than birth rate affect the size of a given population, most notably mortality rate and geographical mobility (immigration/emigration). As Heer writes, "(...) the original Boulding proposal guarantees only that fertility will vary narrowly around replacement level and cannot guarantee that the rate of natural increase (the crude birth rate minus the crude death rate) will be nil nor, a fortiori, can it provide for zero population growth (which would be obtainable with a zero rate of natural increase only provided there was also no net immigration from abroad)" (1975: 4). A government may thus want to adjust the amount of birth licenses to the evolution of these other factors. One could argue that allowing the government to buy permits all along would make it less necessary to adjust the amount of permits allocated to each birth cohort.

Heer focuses on a second set of amendments aimed at influencing the birth rate through raising the women's age of reproduction. Two avenues are proposed, a incentive-based one and a more standard one. As to the former, one could "allow for the possibility that individuals, until they reached age 35, could loan their license units to the government and receive interest during such time as their license units were on loan to the government" (Heer, 1975: 6). There could thus be a financial incentive to delay reproduction, which would certainly have an impact on the birth rate. The other way in which Heer proposes to influence the age of reproduction (hence indirectly the lifelong reproduction rate) consists in "stipulating that licenses to bear children be granted only at age 18 and that individuals under this age neither be allowed to purchase licenses nor to be given licenses by other persons" (Heer, 1975: 8). The two avenues (loan and minimum age) are of course not exclusive and could therefore be combined.

Child mortality is another concern for both Heer and Daly. They want to prevent the system from disadvantaging (often poorer) parents experiencing child loss. This leads to a third set of proposals. Heer proposes that "a woman losing a child before that child reached its eighteen's birthday would be given a sufficient number of license units to bear an additional child; but if a child died after its eighteenth birthday, its mother would receive no additional units" (1975: 8). As to Daly, his approach to the issue of child mortality implies both that licenses be granted at birth and that they be bequeathable. This is such that "if a female dies before having a child, then her certificate becomes part of her estate and is willed to someone else, for example, her parents, who either use it to have another child or sell it to someone else" (1993: 336). If on top, permits were allocated both to girls and boys (rather than to girls only), Daly's allocation at birth would offer a solution to the problem of early mortality of both girls and boys, allowing the parents to give birth to more than two children in such a case without having to buy extra permits. What is clear from this is that the answer provided to the "beneficiary definition" question has a clear impact on the ability of the scheme to address the "child mortality" challenge.

Finally, who should receive the licenses: women only, men only, both men and women? Boulding's proposal consists in a "ladies-only" allocation. For Heer, the possible merits of a men-only scheme include "a considerable reduction in the incidence of illegitimate births" (1975: 8) for reasons that are not entirely clear though. But other reasons are suggested as possible grounds for granting them to women only, such as the need for compensating the discriminations that women suffer. In contrast, issues such as whether single-sex allocation would not be discriminatory from a gender-orientation perspective, or as to how to deal with the licenses in case of divorce once they are granted to both parents, are not examined by these authors.

Other considerations

Besides these design issues, other problems are considered by the three authors. The three most significant ones are: the question of the scheme's distributive impact, the problem of enforcement, and the examination of alternative means to reach the same goal.

Regarding distributive impact, Daly, as Boulding, discusses the issue. He traces possible injustices arising from the scheme back to background distributive injustices that are present anyway. Not only does he claim that existing injustices will not be worsened by the introduction of the scheme. He even argues that inequalities will be reduced, for two reasons. First, the "new marketable asset is distributed equally" (336), which does not tell us why this would reduce inequalities. Second, "as the rich have more children, their family per capita incomes are lowered; as the poor have fewer children their family per capita incomes increase. From the point of view of the children, there is something to be said for increasing the probability that they will be born richer rather than poorer" (1993: 336). It is not clear of course why the richer would tend to have more children as a result of the scheme. It is easier to understand why the poorer would tend to have less.

[To be continued. Comparison with alternative policies. Institutional Arrangement and Enforcement Rule. Infertility.]

2 Benchmark Model

To analyze whether procreation rights allow to reach a target fertility rate and what are their effect on inequality in the society, We consider a model inspired from de la Croix and Doepke (2003). They propose a simple framework where households are heterogeneous in terms of human capital, and low-skilled households choose to have more children than skilled ones. This theoretical set-up reflects the well-documented fact that fertility is inversely related to the education level of the mother (Kremer and Chen 2002). Differential fertility will turn to be a key element in the analysis of procreation rights. We first present the benchmark model without procreation rights.

The model economy is populated by overlapping generations of people who live for two periods, childhood, and adulthood. Time is discrete and runs from 0 to ∞. All decisions are made in the adult period of life. There are two types of agents, indexed by i, unskilled (group $i = A$) and skilled (group $i = B$), who differ only in their wage w_t^i. The size of each group is denoted N_t^i. Agents can be interpreted either as households within a country, or as countries within the global economy. Adults care about consumption c_t^i, their number of children n_t^i, and the probability $\pi(e_t^i)$ that their children become skilled. This probability depends on the education e_t^i they receive. The utility function is given by:

$$\ln[c_t^i] + \gamma \ln[n_t^i \pi(e_t^i)]. \tag{1}$$

The parameter $\gamma > 0$ is the weight attached to children in the households' objective. Notice that parents care both about child quantity n_t^i and quality $\pi(e_t^i)$. As we will see below, the tradeoff between quantity and quality is affected by the human capital endowment of the parent. Notice also that parents do not care about their children utility, as it would be the case with dynastic altruism, but they care about their future human capital.

To attain human capital, children have to be educated. Parents freely choose the education spending per child e_t^i. Apart from the education expenditure, raising one child also takes fraction $\phi \in (0,1)$ of an adult's time.

Parents provide education to their children because it raises the probability that there children will be skilled. Specifically, given education e, the probability $\pi^i(e)$ of becoming skilled is given by:

$$\pi^i(e) = \tau^i \ (\theta + e)^\eta, \quad \eta \in (0,1).$$

The parameter θ measures the education a child obtains in the absence of education spending by the parents. The parameter η measures the elasticity of success with respect to education $\theta + e$. The parameter τ^i depends on the type i, and we assume the children of skilled parents have, ceteris paribus, a greater chance of becoming skilled themselves, i.e. $\tau^B > \tau^A$. Note that, in what follows, e is always going to be bounded from above; hence we can always define the constant term τ^i as a function of the other parameters of the model such that the function $\pi^i()$ returns values in the interval $[0,1]$.

The budget constraint for an adult with wage w_t^i is given by:

$$c_t^i = \left[w_t^i (1 - \phi n_t^i) - n_t^i e_t^i \right]. \tag{2}$$

The aggregate production function for the consumption good is linear in the total input of both types of workers. We have:

$$Y_t = \omega^A L_t^A + \omega^B L_t^B.$$

Here the marginal products of each type are constant and equal to ω^A and $\omega^B > \omega^A$ respectively. The total input of the groups are given by L_t^A and L_t^B. The equilibrium condition on the labor markets $N_t^i(1 - \phi n_t^i) = L_t^i$ will imply that wages are equal to marginal productivity:

$$w_t^i = \omega^i.$$

Denoting the equilibrium outcome in the benchmark case with hatted variables, we obtain the following definition:

Definition 1 (Equilibrium)
Given initial population sizes N_0^A and N_0^B, an equilibrium is a sequence of individual quantities $(\hat{c}_t^i, \hat{e}_t^i, \hat{n}_t^i)_{i=A,B.t \geq 0}$ and group sizes $(\hat{N}_t^i)_{i=A,B.t \geq 0}$ such that

- *Consumption, education and fertility maximize households' utility (1) subject to the budget constraint (2);*
- *Group sizes evolve according to:*

$$\begin{bmatrix} \hat{N}_{t+1}^A \\ \hat{N}_{t+1}^B \end{bmatrix} = \begin{bmatrix} \hat{n}_t^A (1 - \pi^A(\hat{e}_t^A)) & \hat{n}_t^B (1 - \pi^B(\hat{e}_t^B)) \\ \hat{n}_t^A \pi^A(\hat{e}_t^A) & \hat{n}_t^B \pi^B(\hat{e}_t^B) \end{bmatrix} \begin{bmatrix} \hat{N}_t^A \\ \hat{N}_t^B \end{bmatrix} \tag{3}$$

- *Labor market clears, i.e.*

$$\hat{N}_t^i(1-\phi\hat{n}_t^i) = L_t^i \qquad \forall i. \tag{4}$$

Let us now analyze the solution to the individual maximization problem. Parents (/countries) face a tradeoff between the number of children they have, and the amount of resources they spend on the education of each child. Since for educated women, the opportunity cost of child-rearing time is high, they will prefer to invest in the education or "quality" of a small number of children. For less educated women, in contrast, the opportunity cost of raising children is low, while providing education is expensive relative to their income. Mothers with little education and low income would therefore prefer to have many children, but invest less in the education of each child. This notion of a quantity-quality tradeoff in the decisions on children was first introduced by Becker (1960) and is supported by empirical evidence on the cross-sectional distribution of fertility and education. Maximizing utility (1) subject to the constraint (2) leads to the following conditions. If $w > \theta/(\eta\phi)$ [interior regime],

$$\hat{e} = \frac{\eta\phi w - \theta}{1-\eta}, \qquad \text{and:} \tag{5}$$

$$\hat{n} = \frac{(1-\eta)\gamma w}{(\phi w - \theta)(1+\gamma)}. \tag{6}$$

otherwise,

$$\hat{e} = 0, \qquad \text{and:} \tag{7}$$

$$\hat{n} = \frac{\gamma}{\phi(1+\gamma)} \tag{8}$$

This simple model displays the two important properties of quantity-quality tradeoff models: $\partial\hat{e}/\partial w \geq 0$, i.e. parental education spending increases in income, and $\partial\hat{n}/\partial w \leq 0$, i.e. fertility decreases in income. Since income in this model reflects human capital, fertility is a decreasing function of the human capital of the parents.

Observed income inequality Δ^B (B for benchmark) can be measured by the difference between high skilled and low skilled income:

$$\Delta^B = \omega^B(1-\phi\hat{n}^B) - \omega^A(1-\phi\hat{n}^A). \tag{9}$$

This measure will be used later to assess the effect of procreation rights on income differences.

The long-run properties of the model can be analyzed by defining the following population ratio:

$$z_t = \frac{N_t^A}{N_t^B}.$$

The dynamic system (3) is reduced to a first-order recurrence equation $z_{t+1} = f(z_t)$. In Appendix A, we show that the function $f(.)$ satisfies: $f(0) > 0$, $f'(z) > 0$, $f''(z) < 0$. The last two results are guaranteed by the fact that $\tau^B > \tau^A$. The dynamics of z_t admit a single positive steady state which is globally stable.

3 Implementing Tradable Procreation Rights

The government has a fertility objective of ν children per person. We do not question this objective, but only impose the reasonable condition that it should be biologically feasible, i.e.

$$0 < \nu < \frac{1}{\phi}. \tag{C1}$$

At her majority, each women receives for free ν procreation allowances from the State. We assume that each procreation allowance corresponds with the right to procreate one child. Each time she gives birth to a child, a mother has to cede one procreation allowance back to the State and as soon as n_t becomes larger than ν, she will receive free of charge from the State one exemption right per additional child. At the standard menopausal age, each women having less biological children than ν can sell on the market the non used procreation allowances at a price $g_t \geq 0$, and has to give the State $\nu - n$ exemptions, which she will have purchased on the procreation exemptions market at a price $q_t \geq 0$. If she has more biological children than ν, she has to buy additional procreation allowances, and can sell the un-used exemptions. Procreation and exemption rights can be sold and purchased at any moment in time. We assume that fines are such that nobody is violating such rules at equilibrium.

In our model there is no mortality or infertility risk. In a more general set-up, we can address these issues by assuming a perfect insurance market which would cover those risks.

The budget constraint for an adult becomes:

$$c_t^i = \left[w_t^i(1 - \phi n_t^i) - n_t^i e_t^i\right] + g_t(\nu - n_t^i) + q_t(n_t^i - \nu). \tag{10}$$

The variable g_t is the price of one procreation allowance, while q_t is the price of one procreation exemption. Obviously only the difference $g_t - q_t$ matters. We call this difference "procreation price" and accordingly define

$$p_t = g_t - q_t.$$

Equilibria with procreation price p_t positive would corresponds to situations where fertility should be discouraged, while p_t negative would mean that fertility should be promoted. The following definition stresses that there is one additional market compared to Definition 1.

Definition 2 (Equilibrium with Procreation Rights)
Given initial population sizes N_0^A and N_0^B, an equilibrium is a sequence of individual quantities $(c_t^i, e_t^i, n_t^i)_{i=A,B.t\geq 0}$, group sizes $(N_t^i)_{i=A,B.t\geq 0}$, and prices $(p_t)_{t\geq 0}$ such that

- *Consumption, education and fertility maximize households' utility (1) subject to the budget constraint (2);*

- *Group sizes evolve according to (3).*
- *Labor market clears, i.e. Equation (4) holds.*
- *Asset market clears, i.e.*

$$\sum_i (n_t^i - \nu) N_t^i = 0 \tag{11}$$

Fertility and Education Choices

In this section we drop the time index to save notation. A condition for the problem to be well defined requires the endowment of the household to be positive: $w + p\nu > 0$. This condition will hold for all households if the equilibrium price satisfy:

$$\omega^A + p\nu > 0. \tag{C2}$$

This condition is always satisfied when fertility is discouraged ($g > 0$ and $q = 0$). When fertility is promoted, condition (C2) imposes a lower bound on the price of exemptions rights. The condition says that a poor household wanting to have zero children should be able to pay it.

The solution to the household decision problem can either be interior, or at a corner. There is an additional difficulty compared to the problem without procreation rights. If fertility is strongly encouraged by a negative procreation price p, the biological constraint $n < 1/\phi$ might be binding, i.e. some households may want to have more children that what is biologically feasible. In Appendix B we characterize the optimal solution. It appears that they are four different regimes depending on the procreation price and the income, which are:

- R4 It corresponds to the interior solution. It is obtained when the price of the procreation rights is not too low. If it was, either households would like to offer no education at all to their children (the constraint $e \geq 0$ binds and $p > \hat{p}(w)$ is violated), or their economically optimal fertility would be above the biological maximum (the constraint $n \leq 1/\phi$ binds and $p > \tilde{p}(w)$ is violated).
- R3 It corresponds to the corner regime with no education that we already find in the model without procreation rights.
- R2 p is very negative, the poor will entirely specialize into the production of children, and give no education.
- R1 p is very negative, the poor will entirely specialized into the production of children. This activity is so well paid that they can afford to give some education.

From the results in Appendix B, we can establish the properties of fertility and education as a function of income and procreation prices.

Corollary 1 (Fertility, Education and Income)

- *For low income and low procreation price (regimes R1 and R2), neither education nor fertility are affected by income.*
- *For low income and high procreation price (R3), education is independent from income and fertility decreases in income.*
- *For high level of income (R4), spending on education increases with income, fertility decreases in income if and only if the price of procreation is low enough:*

$$\frac{\partial n}{\partial w} < 0 \Leftrightarrow p < \frac{\theta}{1 - \phi\nu}.$$

Proof: see Appendix B. ■

This stresses that if $p = 0$ (no procreation rights), the usual result that high income parents have fewer children applies. If procreation is sufficiently taxed through a positive price of procreation, then the usual pattern is turned upside down, and high income parents have more children because they can afford it.

Corollary 2 (Fertility and Procreation Price)
The individual fertility rate n is a decreasing function of procreation prices p $\in (-1/\nu, +\infty[$.

Proof: see Appendix B. ■

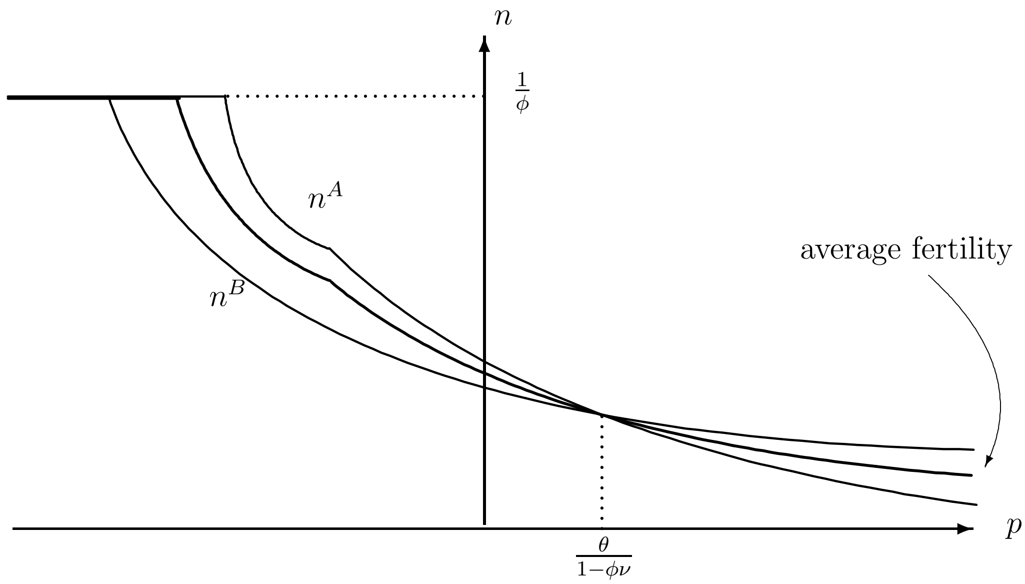
Corollary 2 simply reflects that children are a non-Giffen good. Figure 1 represents an example of fertility for the two groups A and B as a function of the procreation price. As p increases, group A goes through regimes R1→ R2 → R3 → R4. In R1 and R2 fertility is constant at $1/\phi$. At the point where the regime shifts from R3 and R4 we see a non differentiability in the fertility function. Group B goes from regime R1 to R4 as p rises. We also see that, as long as p is below the threshold defined in Corollary 1, fertility of group A is equal or greater than fertility of group B. For large p however, the differential fertility is reversed.

Equilibrium

The fact that groups' fertility are decreasing function of the procreation price implies that average fertility is also a decreasing function of p. Under the condition that for the lowest possible p (see condition (C2)) all groups are in regime R1, average fertility goes from $1/\phi$ for very low p to 0 for large p. This is enough to establish the existence and uniqueness of an equilibrium procreation price, which implies that the target fertility rate ν is reached.

Proposition 1 (Existence and Uniqueness of Equilibrium)
If $\tilde{p}(\omega^B) > -1/\nu$ *the equilibrium procreation price exists and is unique.*

Figure 1: Fertility as a function of income and procreation price



Proof: See Appendix C. ■

Income inequality Δ^T (T for tradable) is given by:

$$\Delta^T = \omega^B(1 - \phi n^B) + p(\nu - n^B) - \left[\omega^A(1 - \phi n^A) + p(\nu - n^A)\right]. \tag{12}$$

4 Effects on Income Inequality

In order to assess the distributive impact resulting from the introduction of a tradable procreation scheme, it is insightful to first look at a simpler case, which is the impact of fixed quotas.

The Model with Fixed Quotas

Examining the impact of tradable quotas schemes would not make much sense without providing a comparison of such effects with either those of a business-as-usual situation, or those of alternative measures aimed at reaching the same demographic target. There are two families of such alternative policy options. Either, we go for a measure that, while being quantity-oriented as well, would be of a more rigid type, i.e. fixed rather than tradable quotas. Or, we go for a variety of price-oriented measures, such as family allowances, free education,... (in case of underpopulation) or for taxation (in case of overpopulation). In the case of price-oriented methods, potential parents are totally free to chose the number of children they wish to have, under the pressure of incentives or disincentives set up by regulatory authorities. Here, we only compare tradable quotas with fixed ones, both because this is the closest realistic alternative method, as the Chinese example suggests, and because a comparison with price-oriented methods requires a much richer analytic aparatus than the one we wish to rely upon here. Let us stress as well the fact that when comparing fixed and tradable quotas, we should assume the same type of initial allocation (here: equality per head). We will examine later an alternative type of initial allocation.

Formally, fixed quotas imposes an additional constraint $\nu \geq n$ to the maximization problem studied in Section 2. Provided that the constraint $\nu \geq n$ binds for both groups, which is true if

$$\nu \leq \frac{(1-\eta)\gamma\omega^A}{(\phi\omega^A-\theta)(1+\gamma)},$$

the solution to the maximization problem is: If $w > \theta/(\gamma\eta(1/\nu-\phi))$ [interior regime],

$$e = \frac{\gamma}{\eta}w(1/\nu-\phi) - \theta 1 + \gamma\eta$$

Otherwise

$$e = 0.$$

It is trivial to show that education spending are fostered by the quantitative constraint on the number of children. Families with fewer children can afford to spend more on each child.

Income inequality Δ^F (F for fixed) as measured by the difference between high skilled and low skilled income is given by:

$$\Delta^F = \omega^B(1-\phi\nu) - \omega^A(1-\phi\nu). \tag{13}$$

Accordingly, we compute the change in income difference between the two types of equilibria, which leads to:

$$\frac{\Delta^F-\Delta^B}{\phi} = \underbrace{(\hat{n}-\nu)(\omega^B-\omega^A)}_{\text{productivity effect}>0} + \underbrace{\omega^B(\hat{n}^B-\hat{n})-\omega^A(\hat{n}^A-\hat{n})}_{\text{differential fertility effect}<0}$$

where $\tilde{n}$ is average fertility in the benchmark case. The first effect, labeled "productivity effect" can be understood as follows. Let us envisage a business-as-usual situation in which high-income and low-income people have the same fertility level $\tilde{n}$. This level is higher than the one required by our demographic target ν. With the introduction of non-tradable quotas, granting each person a procreation right lower than the actual procreation level, the extent to which the rich will procreate less than the poor is identical. Both the high-income and the low-income will increase their income as a result of the time made available by a lower fertility. However, since the hourly wage (and underlying it, the productivity) of the high- income is higher than the one of the low-income person, the income of the rich will increase relatively more than the one of the poor. In short, the introduction of fixed quotas to fight overpopulation in a world in which the fertility rate does not vary with the level of income, will make the poor-income relatively poorer than the high-income. Let us refer to this effect as the productivity (or hourly income) effect on income inequality.

The second effect, labeled "differential fertility effect" relaxes the assumption regarding the absence of fertility differential. In the benchmark situation low-income people tend to have more children than high-income people. Here, a second type of effect can be isolated, of a redistributive type rather than of an anti-distributive type this time. It can be explained as follows. If the fixed quotas scheme requires the same fertility level from the poor and the rich, the poor will have to reduce her fertility level much more than the rich. As a result, she will also increase her working time more than the rich. This effect referred to here as the "initial differential fertility" effect, will clearly reduce inequalities between the rich and the poor, when compared with the income differential in the business-as-usual situation.

At this stage, it is crucial to consider the two effects together. For if the productivity differential is such that the hourly income differential is high, in the presence of a fertility differential that would not be especially high, then the productivity effect might end up dominating the differential fertility effect, leading to income differentials that are higher at the end than in the business-as-usual situation. While the reverse situation might occur, isn't it plausible for a variety of reasons to suppose that the income differential will generally tend to be stronger than the fertility differential, leading to a dominance of the HP effect over the IDF effect? One variable that may affect the relative weight of the two effects is the elasticity of educational outcomes over investment in education, represented b y η. Indeed, computing the extreme hypothetical fertility rates of households with zero income and infinite income leads to:

$$n_{\max} = \lim_{x \to 0} \hat{n} = \frac{\gamma}{\phi(1+\gamma)}$$

$$n_{\min} = \lim_{x \to \infty} \hat{n} = \frac{\gamma(1-\eta)}{\phi(1+\gamma)}$$

The maximum differential fertility is therefore:

$$\frac{n_{\max}}{n_{\min}}.$$

If this elasticity η is large, the fertility differential will tend to be large as well, to such an extent that the IDF effect may actually dominate the HP effect. The reason underlying this connection between outcome-investment elasticity and fertility differential is the following. The poor is equally concerned about education as the rich. However, for the poor, the cost of investing in education as well as the opportunity cost of having children is lower than for the rich. A higher outcome investment elasticity will not affect much the poor, but will push the rich the substitute even more quantity of children for quality.

Effect of tradability

Our third step consists in the replacement of fixed quotas with tradable quotas. Accordingly we compute the change in income difference between the benchmark and the model with tradeable rights, which leads to:

$$\frac{\Delta^T - \Delta^B}{\phi} = \underbrace{(\hat{n} - \nu)(\omega^B - \omega^A)}_{\text{productivity effect } >0} + \underbrace{\left[\omega^B(\hat{n}^B - \hat{n}) - \omega^A(\hat{n}^A - \hat{n})\right] - \left[\omega^B(n^B - \nu) - \omega^A(n^A - \nu)\right]}_{\text{differential fertility effect}} + \underbrace{\frac{p}{\phi}(n^A - n^B)}_{\text{tradability effect}} \tag{14}$$

Here, we have a third type of effect, referred to as the procreation price effect, that is not one-directional. In other words, depending on the price of the procreation permits, tradability will lead either to a redistributive or to an anti-distributive effect.

This third term $p(n^A - n^B)/\phi$ is the transfer going from the poor to the rich through the tradable rights. To assess the effect of tradability we must sign this term. Tradability will be said to be redistributive when the $p(n^A - n^B)/\phi$ is negative, implying that the income gap is reduced.

Proposition 2 (Redistributive Nature of Tradability)
(i) If $\theta > 0$, tradability is redistributive if and only if

$$p < 0 \text{ or } p > \min\left\{\frac{\theta}{1 - \phi\nu}, \hat{p}(\omega^A), \tilde{p}(\omega^A)\right\}.$$

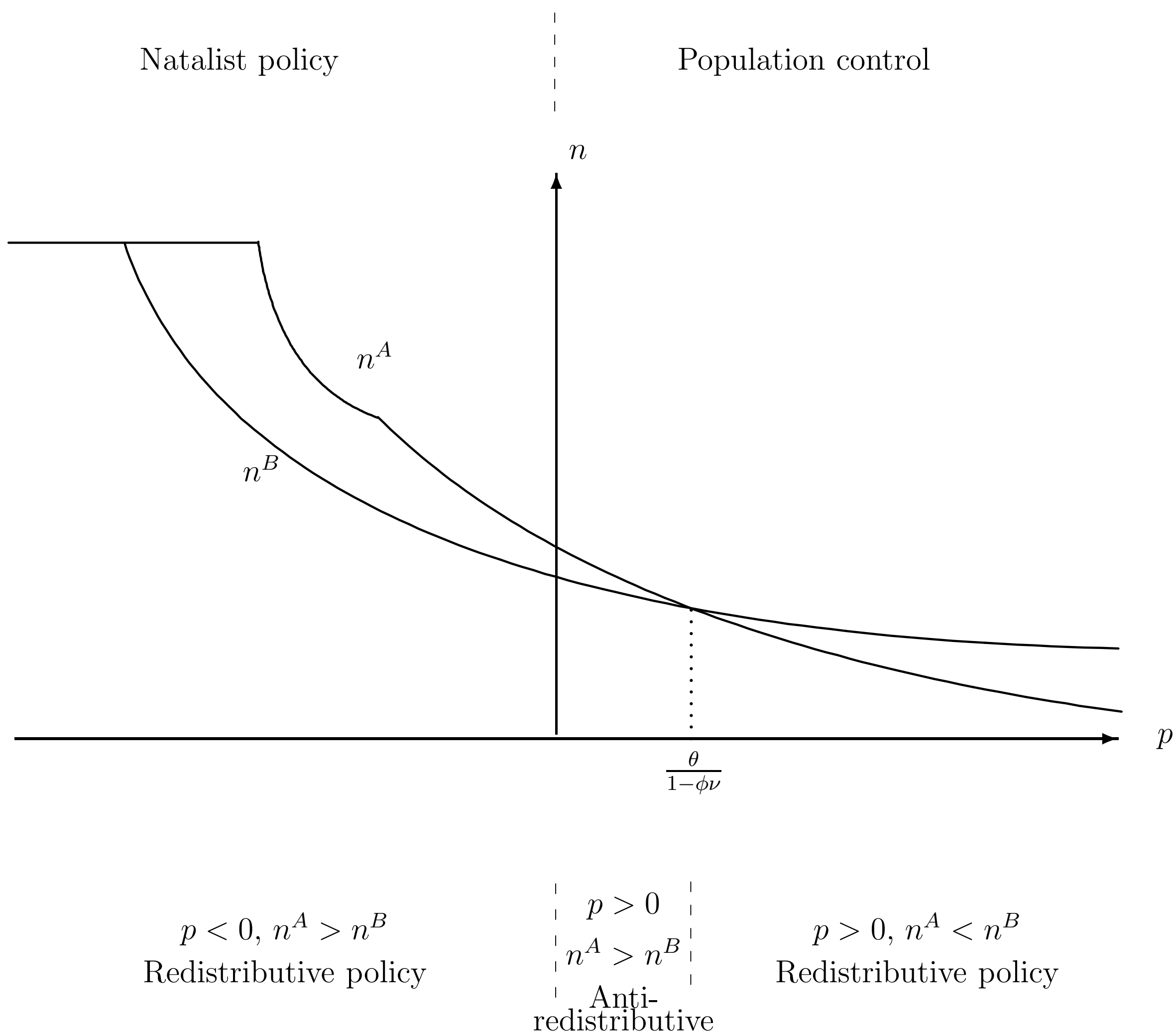
(ii) If $\theta = 0$ tradability is always redistributive.

Proof: See Appendix D. ■

Figure 2 summarizes the results in the case of our example. The intuition behind the proposition goes as follows. When $p < 0$, we are in a world where fertility is encouraged. Poor households have more children and benefit from the existence of procreation rights. When p is positive and large, having children is so expensive that rich people have more children than poor ones. Tradability is also redistributive in that situation (this is the case imagined by Boulding). It is only when the procreation price is modestly positive that poor households have more children than rich and have to pay for it. Tradability is then anti-redistributive. Notice that if θ is close to zero, this eventuality disappears.

One important issue is whether practical indications can be provided as whether an actual scheme is likely to fall within the anti-redistributive zone identified in Figure 2. There is at least one such consideration that is relevant here. If we focus on the distributive impact of tradability alone, if ν is lower but close to the existing procreation level, the price of the procreation rights will not be large enough and we are likely to

Figure 2: Redistributive Nature of Tradability



fall within the anti-redistributive zone. In other words, from the point of view of the tradability effect alone, the best guarantee for the scheme to have a distributive impact consists in adopting a radical reform, i.e. one involving a ν that is diverges enough from the existing procreation rate. This is significant as gradual reforms in this field are more likely to be politically feasible than radical ones. Two conclusions can be drawn from this. First, if we take the tradability effect alone as illustrated in Figure 2, schemes implementing the proposed model are likely in practice to fall within the anti-redistributive zone, since radical changes are less politically feasible. Second, whether this would tend to make the whole scheme anti-redistributive all-things-considered does not necessarily follow. And this makes it all the more important to consider later appropriate initial allocation rules such as grandfathering that are likely to increase the distributive nature of the scheme (see Section 6).

5 Effects on Education

Let us now turn our attention to the effect of procreation rights on education. This is an important question because both social mobility and long-run income are positively affected by education spending. The following proposition shows that education spending depend in a non trivial way on procreation price.

Proposition 3 (Education and Procreation Price)
There is a threshold $\dot{p} = \min\{\breve{p}, \tilde{p}(w)\} < 0$ such that:

- *if $\dot{p} \leq -1/\nu$, investment in education e is increasing in p;*
- *if $\dot{p} > -1/\nu$, individual investment in education e is decreasing in p for $p < \dot{p}$ and increasing in p for $p > \dot{p}$.*

Proof: From the expression for e given in Proposition 4 we find that

$$\text{R1: } \frac{\partial e}{\partial p} < 0, \qquad \text{R2 \& R3: } \frac{\partial e}{\partial p} = 0, \qquad \text{R4: } \frac{\partial e}{\partial p} > 0.$$

■

A special case arises when p is extremely low and $\dot{p} > -1/\nu$. Then, poor households are in regime R1; they are entirely specialized into the production of children ($n = 1/\phi$). A small rise in p has no effect on fertility, but has a negative income effect, and so education spending are reduced.

The most intuitive case is when $\dot{p} \leq -1/\nu$. An increase in the procreation price reduces fertility (Corollary 2) and increases education, which is the usual quantity-quality tradeoff facing a rise in the cost of children. In this case, natalist policy would be bad for education, social mobility and long-run income, while population control would be good.

One of the interesting effects of a pro-natalist policy is intergenerational and illustrates as well the interaction between the income and the educational impact of demographic policies. Let us envisage two generations: P (parents) and C (children). And let us envisage a population target such that the procreation price is negative. Such a pro-natalist policy tends to reduce income inequalities within generation P due to the tradability effect (and possibly as well to the grandfathering effect if it arises - see below). However, this raises a serious difficulty. For the very same pro-natalist policy, while reducing income inequalities within generation P, will also reduce the average level of education (hence, the income level as well) of generation C. This is due to the fact that the pro-natalist subsidy is insufficient to compensate the income loss resulting from the fact that people with more children tend to work less, which entails that they will earn less and have less money to invest in education both in total and, a fortiori, per capita. In other words, a pro-natalist policy will end up leading to a situation such that, while being redistributive for generation P, it will tend to increase income inequalities between generation P and C. This is a problem e.g. because we will then end up with a world in which the worst off people are worse off than in the business-as-usual scenario (i.e. in the absence of pro-natalist policy). Does this constitute a sufficient reason to reject our proposal altogether? We do not think so. It rather stresses on the need to couple demographic policy with educational policy. We could then both get the redistributive impact at the generation P level without generating the negative impact on education (and income) for generation C. In practice, this could take the form of education subsidies or publicly provided education.

6 Procreation Rights with Grandfathering

In the previous section, we asked ourselves whether a tradable procreation quotas scheme would necessarily have anti-redistributive impacts. The intuition underlying such a worry is that because of tradability, the wealthiest people (or countries) may be able to further increase the gap separating them from poorer people (or countries). Our model demonstrates however that there is only one area in table x in which such anti-redistributive effects arise. Outside this zone, the scheme would rather have a redistributive impact. The key practical question will then be to identify where the practical scheme envisaged in specific real-life circumstances would be located on this table. In this section, we would like to have a closer look at still another theoretical option that could have different distributive properties. Rather than implementing an equal per capita initial allocation of procreation quotas, we would do so on a grandfathering basis.

Grandfathering is a concept that originates from the late 19th century in the southern US states. It consists in an attempt at further delaying the electoral enfranchisement of black people. As the suffrage became formally broadened, extending to both white and black men, the introduction of poll tax or literacy requirements was supposed to slow down the access of most black people to the suffrage. It was also excluding some white men however, which arouse concern in the white community. This led to the introduction of the so-called "grandfather clause" in the electoral regime of some of these southern US states, stating that those whose grandfather or father was already enfranchised would be exempted from poll tax and/or literacy requirements. In practice, this meant that all white males would have the right to vote while still preserving the exclusion of most black males through poll tax and/or literacy requirements, from which they could not be exempted since none had had a grandfather entitled to vote.

By extension, the idea of grandfathering usually refers nowadays to the exemption from new regulation granted on a temporary basis to actors already involved in a given activity, for products that already exited the product on chain (in case of product standards). More interestingly for us, it is used in the tradable emission permits context (such as the Kyoto context) to refer to one mode of initial permits allocation, i.e. one that grants relatively larger shares of emission rights to those who already emit relatively more. This means that when facing a given global emission reduction target, the larger polluters will have to reduce their emissions in the same proportion as the lesser polluters, which means that the larger polluters are partly exempted from the application of the new rule. A grandfathering-base initial allocation thus amounts in such a tradable quotas cases to a relative exemption.

In the pollution reduction case, grandfathering can prima facie be expected to lead to largely anti-redistributive consequences, larger per capita polluters being also generally richer actors (individuals or countries). In the procreation case, the relationship between fertility rate and wealth is not so straightforward and might actually be the reverse. This suggests that grandfathering in the case of tradable procreation quotas could well have a distributive impact that differs both from the one exhibited in the case of grandfathering for tradable emission quotas and from the one unveiled in the previous section for tradable procreation quotas allocated on a per capita basis. We now provide the analysis needed to operate the latter comparison.

We assume that countries receive an initial endowment of rights proportional to their fertility rate in the absence of procreation rights:

$$\nu^i = \mu n^i \,|_{p=0} \,.$$

If the parameter μ is larger than one, policy is pronatalist. If it is lower than one, population policy is restrictive. The average fertility target is still ν which relates to the ν^i through:

$$N^A \nu^A + N^B \nu^B = (N^A + N^B)\nu.$$

Since endowments are now different across agents, the resource gap between agents becomes:

$$\Delta^G = \omega^B - \omega^A + p(\nu^B - \tilde{n}^B) - p(\nu^A - \tilde{n}^A).$$

where fertility levels with a tilde denote fertility in the grandfathering case. Computing the change in income difference between the benchmark case and grandfathering, one gets:

$$\frac{\Delta^G - \Delta^B}{\phi} = \underbrace{(\hat{n} - \nu)(\omega^B - \omega^A)}_{\text{productivity effect} >0} + \underbrace{\left[\omega^B(\hat{n}^B - \hat{n}) - \omega^A(\hat{n}^A - \hat{n})\right] - \left[\omega^B(\tilde{n}^B - \nu) - \omega^A(\tilde{n}^A - \nu)\right]}_{\text{differential fertility effect}} + \underbrace{\frac{p}{\phi}(\tilde{n}^A - \tilde{n}^B)}_{\text{tradability effect}} \underbrace{-\frac{p\mu}{\phi}(\hat{n}^A - \hat{n}^B)}_{\text{grandfathering effect}} \quad (15)$$

where the hatted variables represent fertility with grandfathering. We can compare this expression with the one in equation (14).

The productivity effect is unchanged. The differential fertility effect and the tradability effect have the same form as before, and would play the same role provided that fertility behaviors are unaltered by grandfathering. The last effect, labeled grandfathering effect, is a transfer from the rich to the poor in case of a positive procreation price, and reflects the fact that poor households obtained more procreation rights than the rich ones.

To evaluate whether grandfathering modifies the fertility behavior in a quantitative significant way, we have proceeded with numerical simulations. It appears that the difference in fertility levels between the model with equal allocation of rights and the one with grandfathering is very small, typically less than one percent for any level of the procreation price. Education too is almost unchanged. The reason is that grandfathering acts as a lump-sum transfer, does not modify the price of procreation directly (only through general equilibrium effects), and has finally little impact on fertility.

We conclude from this that the essential difference between (15) and (14) is the last effect, which is a pure transfer. Grandfathering has a redistributive effect in the case of population control, simply by implementing a redistributive initial allocation of rights.

7 Conclusion

[*** to be written]

In this paper, we study and defend a modified version of Boulding's idea of tradable procreation permits. We envisage a scheme with both tradable procreation allowances and tradable procreation exemptions, able to deal with both problems of over- and under-population, either domestically or at the global level. This reflects the fact that population control is a two-side problem: both too few children or too many is bad (while less pollution is always better).

We show that an equilibrium with such assets exists and that it can implement any desired growth rate of population.

We then address the worries as to the possible anti-redistributive nature of such a scheme. The implications of procreation rights for income inequality and education are contrasted. With procreation exemptions or procreation allowances that would be expensive enough, resources are redistributed from the rich to the poor. In contrast, cheap procreation allowances redistribute resource towards the rich.

As far as human capital is concerned, natalist policy would be bad for education, while population control would be good.

We next show that an alternative allocation rule of procreation rights, granting rights in proportion to existing fertility levels (grandfathering), can make population control redistributive.

On the whole, we claim that procreation rights offer an interesting alternative to both coercive population control in developing countries and pronatalist policies in the developed world. In case of pronatalist policies, sustaining education through additional measures is necessary.

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A Dynamics in the benchmark case

Dynamics are described by:

$$z_{t+1} = \frac{n^A(1-\pi^A)z_t + n^B(1-\pi^B)}{n^A\pi^A z_t + n^B\pi^B} \equiv f(z_t).$$

The function f has the following properties:

$$\begin{aligned} f(0) &= \frac{1-\pi^B}{\pi^B} > 0 \\ f'(z) &= \frac{n^A n^B(\tau^B - \tau^A)}{(n^A\pi^A z + n^B\pi^B)^2} > 0 \\ f''(z) &= \frac{2(n^A)^2\pi^A n^B(\tau^A - \tau^B)}{(n^A\pi^A z + n^B\pi^B)^3} < 0 \end{aligned}$$

The dynamics of z_t admit a single positive steady state:

$$z = \frac{n^A(1-\pi^A) - n^B\pi^B\sqrt{(n^B\pi^B - n^A(1-\pi^A))^2 + 4n^A\pi^A n^B(1-\pi^B)}}{2n^A\pi^A},$$

which is globally stable thanks to the properties of $f(.)$.

Long-run income per capita is given by

$$\frac{Y}{N^A+N^B} = \frac{\omega^A N^A(1-\phi n^A) + \omega^B N^B(1-\phi n^B)}{N^A+N^B} = \frac{\omega^A z(1-\phi n^A) + \omega^B(1-\phi n^B)}{z+1},$$

and is a negative function of long-run z.

B Solution to the Household Maximization Problem

We study the following Kuhn-Tucker problem:

$$\max_{c,e,n} \{\ln[c] + \gamma\ln[n] + \gamma\eta\ln[\theta+e]; w(1-\phi n) - ne + p(\nu - n) \geq c \geq 0, 1/\phi \geq n, e \geq 0\}$$

It is obvious from the properties of the utility function (non satiety and $u'(0) = +\infty$) that optimal consumption is positive and that the budget constraint holds with equality;we can thus substituting c from the budget constraint into the utility function. The problem becomes:

$$\max_{e,n} \{\ln[w(1-\phi n) - ne + p(\nu - n)] + \gamma\ln[n] + \gamma\eta\ln[\theta+e]; 1/\phi \geq n, e \geq 0\}$$

The Kuhn-Tucker conditions are:

$$
\begin{aligned}
a+\frac{\gamma\,\eta}{e+\theta}-\frac{n}{-(e\,n)+w\,(1-n\,\phi)+p\,(-n+\nu)}&=0\\
-b+\frac{\gamma}{n}+\frac{-e-p-w\,\phi}{-(e\,n)+w\,(1-n\,\phi)+p\,(-n+\nu)}&=0\\
a\,e&=0\\
b\left(-n+\frac{1}{\phi}\right)&=0\\
a&\geq 0\\
b&\geq 0\\
n&\leq\frac{1}{\phi}\\
e&\geq 0
\end{aligned}
$$

The first-four equations define a system that we can solve for (a,b,n,e). There are four solutions:

$$
\begin{aligned}
b=&\frac{\phi\left((1+\gamma)\,\theta-w\,(1+\gamma\eta)\,\phi-p\,(1+\gamma-\gamma\,(1-\eta)\,\nu\phi)\right)}{\theta-p\,(1-\nu\phi)},\ a=0,\\
e=&\frac{-\theta-p\gamma\eta\,(1-\nu\phi)}{1+\gamma\eta},\ n=\frac{1}{\phi} && (16)\\
b=&\frac{\phi\,(w\phi+p\,(1+\gamma(1-\nu\phi)))}{p(1-\nu\phi)},\ a=-\frac{\gamma\eta}{\theta}-\frac{1}{p(1-\nu\phi)},\ e=0,\ n=\frac{1}{\phi} && (17)\\
b=&0,\ a=\gamma\left(\frac{1}{p+w\phi}-\frac{\eta}{\theta}\right),\ e=0,\ n=\frac{\gamma\,(w+p\nu)}{(1+\gamma)\,(p+w\phi)} && (18)\\
b=&0,\ a=0,\ e=\frac{\eta\,(p+w\phi)-\theta}{1-\eta},\ n=\frac{\gamma\,(1-\eta)\,(w+p\nu)}{(1+\gamma)\,(p-\theta+w\phi)} && (19)
\end{aligned}
$$

From these equations we can fully characterize the solution to the individual problem.

Proposition 4 (Solution to the Individual Problem) *Define the following threshold values for the procreation price:*

$$\hat{p}(w)\equiv\frac{\theta-\eta\phi w}{\eta} \tag{20}$$

$$\tilde{p}(w)\equiv\frac{\theta(1+\gamma)-(1+\eta\gamma)\phi w}{\gamma(1-(1-\eta)\phi\nu)+1} \tag{21}$$

$$\bar{p}(w)\equiv\frac{-\phi w}{\gamma(1-\phi\nu)+1}<0 \tag{22}$$

$$\breve{p}\quad\equiv\frac{-\theta}{\gamma\eta(1-\phi\nu)}<0 \tag{23}$$

Assume that (C1) and (C2) hold.

R1 If $p < \breve{p}$ and $p < \tilde{p}(w)$, $n = 1/\phi$, and

$$e = \frac{-\gamma\eta(1+\phi\nu)p - \theta}{1+\gamma\eta} > 0. \tag{24}$$

R2 If $p > \breve{p}$ and $p < \bar{p}(w)$, $e = 0$ and $n = 1/\phi$.

R3 If $p < \hat{p}(w)$ and $p > \bar{p}(w)$, $e = 0$ and

$$n = \frac{\gamma(w+\nu p)}{(\phi w + p)(1+\gamma)} < \frac{1}{\phi}. \tag{25}$$

R4 If $p > \hat{p}(w)$ and $p > \tilde{p}(w)$,

$$e = \frac{\eta\phi w - \theta + \eta p}{1-\eta} > 0, \qquad \textit{and:} \tag{26}$$

$$n = \frac{(1-\eta)\gamma(w+\nu p)}{(\phi w - \theta + p)(1+\gamma)} < \frac{1}{\phi}. \tag{27}$$

The function $\hat{p}(w)$ is obtained by solving for p the condition $e = 0$ in Equation (19) or the condition $a = 0$ in Equation (18). The function $\tilde{p}(w)$ is obtained by solving for p the condition $n = 1/\phi$ in Equation (19) or the condition $b = 0$ in Equation (16). The function $\bar{p}(w)$ is obtained by solving for p the condition $b = 0$ in Equation (17) or the condition $n = 1/\phi$ in Equation (18). The threshold $\breve{p}$ is obtained by solving for p the condition $a = 0$ in Equation (17) or the condition $e = 0$ in Equation (16).

The threshold procreation prices defined in Proposition 4 display two interesting properties. First, we can unambiguously rank them when the wage is equal to zero:

$$\hat{p}(0) > \tilde{p}(0) > \bar{p}(0) = 0 > \breve{p}.$$

Second, they all intersect at the same point:

$$\text{at } w^\star = \frac{\theta(1+\gamma(1-\phi\nu))}{\gamma\eta\phi(1-\phi\nu)}, \qquad \hat{p}(w^\star) = \tilde{p}(w^\star) = \bar{p}(w^\star) = \breve{p}.$$

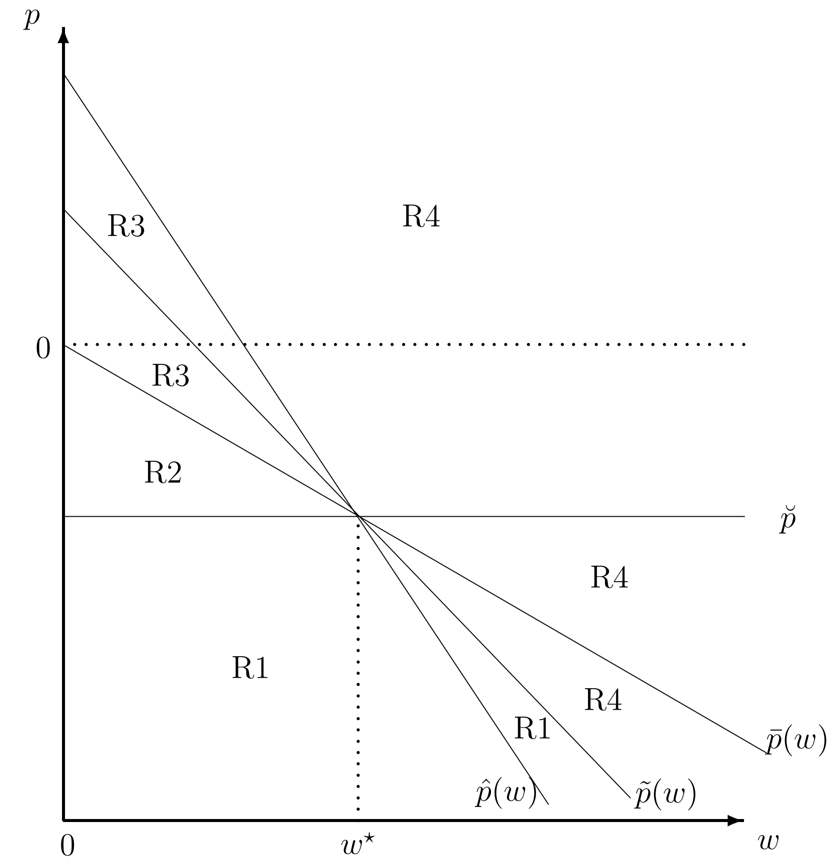
We represent these four lines and the corresponding regimes R1 to R4 in Figure 3. Let us now consider a household with wage w. Starting from a very high price p, the solution will be in regime R4. Letting the price drop, and abstracting from the constraint (C2), two situations can arise. If $w < w^\star$, the succession of regimes will follow R4 → R3 → R2 → R1 as prices drop; if $w \geq w^\star$, we will pass from R4 to R1 directly. When we take into account the constraint (C2), regime R1 will never be a possible outcome for households with a very high wage.

All claims in Corollary1 are a direct consequence of the results of Proposition 4. From Equation (27), $\partial n/\partial w = -\gamma(1-\eta)(\theta - p(1-\phi\nu))/[(1+\gamma)(p-\theta+\phi w)^2]$.

Corollary2: From the expression for n given in Proposition 4 we find that

$$\text{R1 \& R2: } \frac{\partial n}{\partial p} = 0, \qquad \text{R3 \& R4: } \frac{\partial n}{\partial p} < 0.$$

Figure 3: Solution to the individuals problem: regimes R1 to R4



C Proof of Proposition 1

If $\tilde{p}(\omega^B) > -1/\nu$, both fertility levels tend to $1/\phi$ when p approaches $-1/\nu$. Hence, for p going from $-1/\nu$ to $+\infty$, total fertility $(N^A n^A + N^B n^B)/(N^A + N^B)$ goes monotonically from $1/\phi$ to zero. A each date, there exist therefore a price p_t for which

$$(N_t^A n_t^A + N_t^B n_t^B)/(N_t^A + N_t^B) = \nu \in]0, 1/\phi[,$$

which is the equilibrium price satisfying equation (11). This price is unique because fertility is a monotonous function of p.

D Proof of Proposition 2

(i) To analyze the sign of Δ, we should consider the different possible regimes. From Figure 3, if the poor is in R1, the rich can be either in R1 or in R4. If the poor is in

R2, the rich can be in R2, R3 or in R4. If the poor is in R3, the rich can be in R3 or R4. If the poor is in the interior regime R4, the rich is in R4 too. We thus have eight situations to consider.

Cases were both types of households are either in R1 or in R2 are excluded by assumption (C1). Indeed, in that case, aggregate fertility per person would be $1/\phi$ and the equilibrium condition (11) would be violated. This leaves us with six different cases.

When the poor is in R2, the procreation price is necessarily negative, and $n^A = 1/\phi > n^B$. When both individuals are in R3, the procreation price can be either positive or negative, but we always have $n^A > n^B$ because fertility decreases in income in regime R3. Hence, $p < 0 \rightarrow \Delta < 0$ in this regime. When the poor is in R3 and the rich in R4, we also always have $n^A > n^B$ (compare (25) to (27)), which implies in this regime $p < 0 \leftrightarrow \Delta < 0$. Finally, if both individuals are in the interior regime R4, we can use Proposition 1 to infer that if $p < 0$ ($n^A > n^B$) or if $p > \frac{\theta}{1-\phi\nu}$ ($n^B > n^A$), then $\Delta < 0$. The min term in the condition of the proposition gathers the requirement of being in regime R4 with the condition of Proposition 1.
(ii) If $\theta = 0$ the expression

$$min \left\{ \frac{\theta}{1-\phi\nu}, \hat{p}(\omega^A), \tilde{p}(\omega^A) \right\}$$

is negative and the condition in (i) is always true.