REPUTATION AND CAREER CONCERNS*

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Abstract

This paper studies Holmstrom's [1999] seminal model of career concerns, but considers that a small change in the beliefs about the agent's future productivity may imply a large change in his compensation—because, for example, the agent may be fired or promoted. This allows us to study how the agent's effort decision depends on his current reputation—with reputation we refer to the beliefs about the agent's future productivity. We shall show that the market's and the agent's problems can be written recursively. We find that the relationship between the agent's decisions and his current reputation is typically nonmonotonic: equilibrium effort is hump-shaped over reputation. Furthermore, equilibrium effort may be higher if there is less dispersion in the distribution of abilities; it may be higher later in the agent's career; and it may be higher than the efficient effort level.

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1 Introduction

Fama (1980) suggests that agents are disciplined by the opportunities provided by the *markets* for their services, both within and outside the firm: agents are disciplined by their *career concerns.* These incentives are present in situations where the labor market does not know the agent's future productivity, but it learns about it by observing performance. In general, the employer has to pay more to the agent when the agent is believed to be more productive because otherwise another firm would offer more. Moreover, if it is believed that the agent has higher productive ability, this may also allow him to work in a "better" position. Thus, the agent's compensation depends on the *labor market's belief about his future productivity,* and his decisions are influenced by his aspiration of affecting his *future reputation*—with reputation we refer to the beliefs about the agent's future productivity.

Career-concern incentives matter in many lines of work. For example, an assistant professor exerts effort writing papers in part because the decision on his tenure and his future salaries depend on the beliefs about his future productivity—determined by his past production. Studying these incentives is necessary for understanding the dynamics of the agent's decisions over his career, and for designing optimal compensation contracts that complement these incentives (see Gibbons and Murphy [1992]).

This paper presents a framework that allows us to study how the agent's reputation affects his decisions.[1] Would the agent exert more effort when his *current reputation* is bad than when his *current reputation* is good? If the agent makes decisions to influence his *future reputation*, it seems natural to expect that his decision would depend on his *current reputation*. Since the agent's reputation most likely changes over his career, one cannot characterize the dynamics of the agent's decisions without understanding how his reputation affects his decisions. Understanding how the agent's reputation affects his decisions is also a first step toward understanding how contracts complementing career-concern incentives should depend on the agent's reputation (or his past performance).

As a natural first step toward understanding how the agent's current reputation affects the strength of

[1] Carefully chosen assumptions allow previous studies to sidestep this. See, for example, Holmstrom [1999], Gibbons and Murphy [1992], Besley and Case [1995], Prendergast and Stole [1996], Dewatripont, Jewitt, and Tirole [1999a, 1999b], Persson and Tabellini [2000], Shi and Svensson [2002], Alesina and Tabellini [2003], Le Borgne and Lockwood [2004], Ahmad and Martinez [2005], and Eggertsson and Le Borgne [2005].

his incentives, this paper considers only one departure from Holmstrom [1999] that makes effort depend on reputation. We shall study a different compensation scheme in a T-period version of Holmstrom's [1999] seminal model. That is, with the exception of the assumption on the compensation scheme, the framework studied here is exactly the framework in Holmstrom [1999]. It will be shown that departing from Holmstrom's [1999] framework only in the compensation scheme considered allows us to study how equilibrium effort depends on reputation. The paper shows that we can easily represent recursively dynamic models of career concerns and characterize the effort decisions. The dynamic effects explained in the paper are robust to changes in our (or Holmstrom's) assumptions.

First, we shall consider a general career-concern compensation scheme in which the agent's compensation depends on the market's belief about the agent's future productivity in an unspecified way. It is shown that the players' (the market's and the agent's) problems can be set up recursively. In other words, for all histories of the game that imply the same beliefs about the agent's ability, compensation and equilibrium effort are the same. This insight greatly facilitates the study of career concerns when we consider that equilibrium effort depends on the agent's current reputation. The existence and uniqueness of equilibrium effort is also discussed.

Second, as a benchmark, we shall present a framework in which the agent's compensation is given by his expected productivity as in previous studies of career concerns. With this assumption, the expected compensation is linear in effort and the marginal gain from exerting effort does not depend on the agent's reputation. Thus, the agent's equilibrium decisions do not depend on his reputation.

Finally, we shall investigate a discontinuous compensation scheme. That is, we shall consider the case where a small change in the agent's reputation implies a large change in his compensation. There are many situations where the agent's compensation is not a continuous function of his reputation. For example, the agent may be assigned to different levels in a hierarchy or to different sectors in the economy according to his reputation, and these reassignments often imply a discontinuous change in compensation.[2] As an illustration, consider that there is an important difference between the lowest salary in the NBA (National Basketball Association) and the highest salary in the NBA Development League. With the discontinuous

[2] This idea is formalized by, for example, MacDonald [1982], Bernhardt [1995], Gibbons and Waldman [1999], and Persson and Tabellini [2000].

compensation scheme, we can study how the agent's reputation influences his equilibrium decisions.[3]

Would an agent with a better reputation face stronger career-concern incentives, or would such an agent be more difficult to control? Consequently, should contract incentives be stronger if the agent's reputation is better? We shall show that the relationship between the strength of career-concern incentives and reputation is typically nonmonotonic: equilibrium effort is hump-shaped over reputation.

We shall also show that some of the conclusions presented in previous studies of career concerns (and replicated here) are not robust to changes in the career-concern compensation scheme considered. First, the agent faces an intertemporal effort-smoothing decision, in contrast to the intratemporal effort decision more commonly studied. Second, equilibrium effort may be higher if there is less dispersion in the distribution of abilities. Third, the agent may decide to exert more effort later in his career. Fourth, the equilibrium effort level may be higher than the efficient effort level.

The rest of this paper is organized as follows. Section 2 presents the framework. Section 3 shows that the players' problems can be set up recursively. Section 4 characterizes equilibrium effort when the agent's compensation is given by his expected productivity. Section 5 characterizes equilibrium effort for a discontinuous compensation scheme. Section 6 concludes and suggests possible extensions.

2 Framework

This paper studies different compensation schemes in a T-period version of the main model in Holmstrom's [1999] seminal paper. For simplicity, we shall focus on the stationary case of that model. It will be shown that departing from Holmstrom's [1999] framework only in the compensation scheme considered, allows us to study how the agent's decisions depend on his reputation. With the exception of the assumption on the compensation scheme, the framework studied here is exactly the framework in Holmstrom [1999].

For now, we assume only that the compensation is a function of the market's belief about the agent's future productivity (as in every model of career concerns).[4] With this assumption, Section 3 shows that

[3] Reputation would also affect effort with non-linear compensation schemes. The framework presented here would also be useful to study these situations. The dynamic effects would be the same (but reputation would also affect effort through other channels).

[4] As in previous studies of career concerns, in this paper, the only role of the employer (or principal) is to learn about the

the players' problems can be set up recursively. Examples of such compensation scheme are presented in sections 4 and 5.

2.1 The environment

Consider a dynamic game played by the agent and the market for his services. Time is discrete and indexed by $t \in \{0, 1, ..., T\}$. At the beginning of period t, the market decides on the agent's compensation w_t, and the agent consumes w_t. The agent then decides on his effort level, $a_t \geq 0$. Output y_t is a stochastic function of the agent's ability, $\bar{\eta}_t$, and of a_t. In particular,

$$y_t = a_t + \bar{\eta}_t + \varepsilon_t, \qquad (1)$$

where ε_t is a normally distributed random variable with expected value 0 and precision h_ε (the variance is $\frac{1}{h_\varepsilon}$). After the agent chooses his effort level, ε_t and $\bar{\eta}_t$ are realized.

The agent's ability evolves as a random walk. In particular, $\bar{\eta}_{t+1} = \bar{\eta}_t + \beta_t$, where β_t is assumed to be normally distributed with mean 0 and precision h_β.[5]

Neither the market nor the agent knows the agent's ability.[6] At the beginning of the game, the market's

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agent's future productivity and to determine the compensation according to his belief. The exact relationship between the principal's belief about the agent's future productivity and the compensation depends on the labor market structure considered (see, for example, MacDonald [1982], Bernhardt [1995], Gibbons and Waldman [1999], and Persson and Tabellini [2000]). The analysis of this relationship is beyond the scope of this paper. Here, we want to focus on the incentives generated when the agent's compensation depends on his future productivity.

[5] As is explained later, assuming that ability evolves over time allows us to focus on the case in which the precision of the beliefs about the agent's ability does not depend on the number of periods the agent worked. Moreover, there are many situations in which a worker's ability changes over time (this is the case, for example, in professional sports). This assumption may also represent situations in which the agent's tasks are changing over time and his ability depend on the tasks he is focusing on (for example, a manager who is concerned primarily about decreasing production costs may be forced to focus on marketing because of a new competitor). Martinez [2004] presents a firing model of career concerns in which an agent's ability does not change over time and the main results presented here are not affected.

[6] An agent may be ignorant of his ability when met with new tasks. This assumption also deepens the understanding of situations where an agent's success does not only depend on his individual ability but also on the ability of others working with him.

and the agent's beliefs about the agent's ability are normally distributed with mean b_0 and precision $h_{\bar{\eta}}$.

Define $\eta_t \equiv \bar{\eta}_t + \varepsilon_t = y_t - a_t$. That is, η_t is the stochastic component of y_t. It is the signal about the agent's ability extracted from observing output in period t when it is believed that the agent exerted effort a_t.

There is a cost to exerting effort, $c(a_t)$, with $c'(a_t) \geq 0$, $c''(a_t) > 0$, and $c'(0) = 0$. Each period t, the agent's utility equals $w_t - c(a_t)$. Let $\delta \in (0,1)$ denote the players' discount factor.

Each period, players observe y_t while $\bar{\eta}_t$ and ε_t are unobservable. The market does not observe the agent's effort level (which is of course known by the agent).[7]

It is assumed that the agent plays a pure strategy. In period t, when the agent decides on his effort level, his information includes the history of output realizations, $y^{t-1} = (y_0, y_1, ..., y_{t-1})$, and the history of effort levels, $a^{t-1} = (a_0, a_1, ..., a_{t-1})$. For all $t > 0$, let $a_t\left(y^{t-1}, a^{t-1}\right)$ denote the agent's period-t optimal effort level with information $\left(y^{t-1}, a^{t-1}\right)$.[8] Let a_0^* denote the first-period equilibrium effort. When the market determines the agent's compensation, his information is given by y^{t-1}. Let $w_t\left(y^{t-1}\right)$ denote period-t compensation when the market's information is given by y^{t-1}.[9]

2.2 Equilibrium concept

Models of career concerns can be formalized as dynamic games of incomplete information or Bayesian games. We shall use Perfect Bayesian Equilibrium as the equilibrium concept.

2.3 The learning process

From this point forward, *belief* refers to *belief about the agent's ability* unless stated otherwise (as when referring to the market's belief about the effort level the agent exerted).

[7] Alternatively, it can be assumed that the agent's action is observable but the principal is uninformed (see, for example, Shi's and Svensson's [2002] political-budget-cycle model of career concerns).

[8] Previous studies present assumptions such that $a_t\left(y^{t-1}, a^{t-1}\right)$ does not depend on y^{t-1} and a^{t-1}. We shall study situations in which $a_t\left(y^{t-1}, a^{t-1}\right)$ depends on y^{t-1} and a^{t-1}.

[9] Given that the compensation depends on the history of outputs, by exerting effort at t (and changing y_t according to equation 1) the agent may affect his compensation in every future period.

Players learn about the agent's ability using Bayesian learning. For simplicity, the precision of the noise in the random walk ability process β_t is chosen to make the mean of the distribution sufficient for characterizing beliefs. Thus, we assume that

$$h_\beta = \frac{h_{\bar{\eta}}^2 + h_{\bar{\eta}} h_\varepsilon}{h_\varepsilon}.$$

With this assumption, the precision of the period-$t+1$ beliefs about the signal η_{t+1} is always equal to the precision of the period-t beliefs about the signal η_t and does not depend on the number of observations of the agent's output. This precision is given by

$$H \equiv \frac{h_{\bar{\eta}} h_\varepsilon}{h_\varepsilon + h_{\bar{\eta}}}. \tag{2}$$

Consequently, Holmstrom's [1999] tenure effect in the determination of players' decisions is not present. Here, the focus is on the effects of reputation which previous studies left unexplored.

Let b_{mt} and b_{at} denote the mean of the market's and the agent's beliefs *at the beginning of period* t (from here on, *at period* t). We shall refer to *belief with mean* b as *belief* b. When the players' beliefs are coincidental at t, let $b_t = b_{mt} = b_{at}$ denote their beliefs.

2.4 Equilibrium learning

In a Perfect Bayesian Equilibrium, the market always believes it is on the equilibrium path, i.e., it assigns probability one to the agent exerting the equilibrium effort level in every period.

The market is rational, and understands the game. In particular, the market can infer the agent's strategy, $a_t\left(y^{t-1}, a^{t-1}\right)$. The only information available to the market is the history of outputs y^{t-1}. Using y^{t-1} and its previous inferences regarding the effort levels exerted by the agent, the market infers that the effort level exerted by the agent in period t is given by

$$a_t\left(y^{t-1}\right) \equiv a_t\left(y^{t-1}, a^{t-1}\left(y^{t-2}\right)\right)$$

where

$$a^{t-1}\left(y^{t-2}\right) \equiv \left(a_0^*, a_1\left(y_0, a_0^*\right), ..., a_{t-1}\left(y^{t-2}, a^{t-2}\left(y^{t-3}\right)\right)\right)$$

denotes the history of past effort levels inferred by the market when y^{t-2} was observed. Recall that at the beginning of the game, the market and the agent have the same information, and, therefore, the market can infer a_0^* correctly.

Observing y_t allows the players to infer η_t (and to update their beliefs) by using their knowledge about the effort exerted by the agent (either a_t or $a_t\left(y^{t-1}\right)$) and the production function. The agent knows the effort he exerted, and he is always able to infer the signal $\eta_t = y_t - a_t$. The signal inferred by the market is given by

$$\eta_{mt}\left(y^t\right) \equiv y_t - a_t\left(y^{t-1}\right) = \eta_t + a_t - a_t\left(y^{t-1}\right).$$

On the equilibrium path, the effort level expected by the market is equal to the effort level exerted by the agent, and the signal the market infers is equal to the signal the agent infers. The market's inference is wrong, however, when the agent deviates from equilibrium behavior.

According to Bayes' rule, the mean of the beliefs at $t+1$ is a weighted sum of the mean at t and the inferred period-t signal where the weight of the mean at t is given by

(3)
$$\mu = \frac{h_{\bar{\eta}}}{h_{\bar{\eta}} + h_{\varepsilon}}.$$

Thus, the agent's belief at $t+1$ is characterized by

$$b_{at+1} = B\left(b_{at}, \eta_t\right) \equiv \mu b_{at} + (1-\mu)\,\eta_t,$$

and the market's belief at $t+1$ is characterized by

(4)
$$b_{mt+1} = B\left(b_{mt}, \eta_{mt}\left(y^t\right)\right) = \mu b_{mt} + (1-\mu)\left(\eta_t + a_t - a_t\left(y^{t-1}\right)\right).$$

The market's belief at $t+1$ is a function of its belief at t, the true signal, the effort level exerted by the agent, and the effort level expected by the market. Let $B_m\left(b_{mt}, \eta_t, a_t, a_t\left(y^{t-1}\right)\right)$ denote this function. On the equilibrium path, given that the signal inferred by the market is equal to the signal inferred by the agent, the market's and the agent's beliefs are coincidental.

Of course, to find the agent's equilibrium strategy, off equilibrium play has to be considered. However, the structure of the game (one cannot infer from observables that the agent has deviated) and Perfect Bayesian Equilibrium imply that the market puts full probability on equilibrium play by the agent. Consequently, the market believes that the agent's belief coincides with his belief.

The agent knows that the market believes that the agent always exerted effort according to $a^{t-1}\left(y^{t-2}\right)$. Several histories of output realizations and effort levels result in the same beliefs. Let $Y^t\left(b, a^t\right)$ denote the set of histories y^t that imply belief b when the updating was done considering that effort was exerted according to a^t. For all $y^{t-1} \in Y^{t-1}\left(b, a^{t-1}\left(y^{t-2}\right)\right)$, the agent believes that the market's belief is characterized by $b_{mt} = b$.

3 A recursive formulation

In this section, we shall show that the beliefs (and t) are the relevant state variables. As explained in Section 2, at time t, the agent's information is given by a^{t-1} and y^{t-1} while the market's information is given by y^{t-1}. This section shows that for all histories y^{t-1} that imply b_{mt}, the compensation is the same. Similarly, for all histories y^{t-1} and a^{t-1} that imply b_{at} and b_{mt}, the optimal effort level is the same. This allows us to present a recursive formulation of the model that facilitates the study of career concerns when we consider that the agent's decisions depend on his reputation. We shall also discus the existence and uniqueness of the agent's equilibrium effort strategy.

This section considers a general career-concern compensation scheme in which the compensation depends on the market's belief about the agent's future productivity in an unspecified way. We shall show that for this general compensation scheme the players' problems can be set up recursively. Sections 4 and 5 present examples of such a compensation scheme. Therefore, in these sections, we can characterize the optimal strategies as functions of the beliefs.

For characterizing the agent's strategies, it is assumed that the compensation scheme is such that the agent's maximization problems are strictly concave. In sections 4 and 5, when a specific compensation scheme is introduced, assumptions that assure the concavity of the agent's problems are discussed. It is also assumed that the compensation is weakly increasing with respect to the agent's expected productivity. In general, the employer has to pay more to the agent when the agent is believed to be more productive because otherwise another firm in the market would offer more to the agent. Moreover, if it is believed that the agent has higher productive ability, this may also allow him to work in a "better" position.

We use backward induction. We show that in period T, the agent exerts no effort, and, therefore, the compensation is determined by the market's belief. Then it is shown that in period $T-1$, the equilibrium effort level is determined by beliefs. Given that the agent's future effort levels depend only on beliefs, in period $T-1$, the compensation depends only on the market's belief. Then it is shown that in period

$T-2$, the equilibrium effort level is determined by beliefs and so on.

3.1 Period-$T-1$ equilibrium effort strategy

In period $T-1$, the agent exerts effort to influence his period-T compensation. Thus, for characterizing the period-$T-1$ equilibrium effort strategy, it is necessary to characterize the period-T compensation. In period T, the agent cannot influence any future compensation, and, therefore, he always chooses $a_T = 0$. Consequently, the expected productivity of the agent in period T is determined by his expected ability. Hence, in deciding w_T, the market only considers the ability he expects the agent to have in period T. The market's information is the history of outputs y^{T-1}. However, because of the recursiveness of Bayesian learning, the agent's excepted ability is given by b_{mT}. Let $\omega_T(b_{mT})$ denote the period-T compensation. Given that the compensation is weakly increasing with respect to the agent's expected productivity, $\omega_T(b_{mT})$ is weakly increasing with respect to b_{mT}. These conclusions are summarized in the following lemma:

Lemma 1 *For all histories y^{T-1} and a^{T-1}, $a_T\left(y^{T-1}, a^{T-1}\right) = 0$. Therefore, for any b_{mT}, the period-T compensation is the same for all $y^{T-1} \in Y^{T-1}\left(b_{mT}, a^{T-1}\left(y^{T-2}\right)\right)$. Moreover, the period-$T$ compensation is weakly increasing with respect to b_{mT}.*

Consequently, at period $T-1$, for all histories y^{T-2} and a^{T-2} that imply b_{aT-1} and b_{mT-1}, the agent's maximization problem is given by

$$(5) \qquad \max_{a_{T-1}} \left\{ \delta E_{\eta_{T-1}} \left[\omega_T \left(B_m \left(b_{mT-1}, \eta_{T-1}, a_{T-1}, a_{T-1}\left(y^{T-2}\right)\right)\right) \middle| \, b_{aT-1} \right] - c(a_{T-1}) \right\}$$

where E denotes the expectation operator (the agent is uncertain about η_{T-1}, and he believes that η_{T-1} is distributed according to b_{aT-1}).

This maximization problem shows that the optimal effort level may depend on the agent's belief, b_{aT-1}, on the market's belief, b_{mT-1}, and on the effort level expected by the market, $a_{T-1}\left(y^{T-2}\right)$. In particular, for any b_{aT-1} and b_{mT-1}, if the effort level expected by the market depends on y^{T-2}, then the optimal effort level could indeed depend on y^{T-2}.

Assuming that the compensation $\omega_T(b_{mT})$ is such that problem 5 is strictly concave assures that for a given effort level expected by the market, $a_{T-1}\left(y^{T-2}\right)$, there exists a unique optimal effort level given by the first-order condition of problem 5. This does not guarantee that the *equilibrium* effort strategy exists and is unique. Recall that in order to find the equilibrium effort strategy, we have to solve a fixed point problem. The effort level expected by the market has to be equal to the effort level the agent exerts in equilibrium. It could be that there is no equilibrium effort level such that when the market expects $a_{T-1}\left(y^{T-2}\right)$, the agent chooses $a_{T-1}\left(y^{T-2}\right)$. It could be more than one $a_{T-1}\left(y^{T-2}\right)$ such that when the market expects $a_{T-1}\left(y^{T-2}\right)$, the agent's optimal effort level is given by $a_{T-1}\left(y^{T-2}\right)$. However, the next lemma shows that a unique equilibrium effort strategy exists. Moreover, the equilibrium effort level is the same for all histories y^{T-2} that imply the same equilibrium beliefs (the proof is provided in Appendix A).[10]

Lemma 2 *In period $T-1$, on the equilibrium path, for any b_{T-1}, an equilibrium effort level exists and is unique. Consequently, the equilibrium effort level is the same for all $y^{T-2} \in Y^{T-2}\left(b_{T-1}, a^{T-2}\left(y^{T-3}\right)\right)$.*

The intuition behind this result is clear. Asides from determining beliefs, y^{T-2} only affects problem 5 through the effort level expected by the market, $a_{T-1}\left(y^{T-2}\right)$. This effort level affects the marginal benefit of exerting effort through the signal inferred by the market, $\eta_{mT-1}\left(y^{T-2}\right)$. However, in equilibrium, the effort level exerted by the agent is the effort level expected by the market, and, therefore, $\eta_{mT-1}\left(y^{T-2}\right) = \eta_{T-1}$, which does not depend on y^{T-2}. Thus, the equilibrium effort strategy only depends on y^{T-2} through the beliefs.

Lemma 2 shows that we can characterize the agent's equilibrium effort at $T-1$ as a function of the players' beliefs (that in equilibrium are coincidental). Let $\alpha_{T-1}\left(b_{T-1}\right)$ denote this function where $\alpha_{T-1}\left(b_{T-1}\right)$ is defined by

$$(6) \qquad c'\left(\alpha_{T-1}\left(b_{T-1}\right)\right) = \delta \frac{\partial E_{\eta_{T-1}}\left[\omega_T\left(\mu b_{T-1} + (1-\mu)\,\eta_{T-1}\right)\middle|\, b_{T-1}\right]}{\partial a_{T-1}}.$$

The agent exerts effort because the marginal cost of exerting effort is compensated by an expected increase in next-period compensation (a higher a_{T-1} implies a higher b_{mT}, and, therefore, implies that the agent's

[10] Martinez [2004] discusses a firing model of career concerns in which the convexity of the agent's problem implies that the agent's equilibrium strategy does not exist even though an optimal effort level exists for each effort expected by the principal. It is also shown that in a more general framework, if the agent's problem is strictly concave, the agent's equilibrium action exists and is unique.

expected compensation is higher). In equation 6, effort does not appear in the right-hand side because, in equilibrium, the agent chooses the effort expected by the market. Therefore, the signal inferred by the market is the true signal ($\eta_{mT-1}\left(y^{T-2}\right) = \eta_{T-1}$), and $b_{mT} = B\left(b_{T-1}, \eta_{T-1}\right) = \mu b_{T-1} + (1-\mu)\,\eta_{T-1}$.

3.2 Period-$T-2$ equilibrium effort strategy

For period $T-2$, the same logic applies. Asides from determining beliefs, the history y^{T-3} only affects the agent's maximization problem through the effort level expected by the market, $a_{T-2}\left(y^{T-3}\right)$ (see problem 7 below). This effort level affects the marginal benefit of exerting effort through the signal inferred by the market, $\eta_{mT-2}\left(y^{T-3}\right)$. However, in equilibrium, the effort level exerted by the agent is equal to the effort level expected by the market, and, therefore, the signal inferred by the market is the true signal, and is independent of y^{T-3}.

In order to find the period-$T-2$ equilibrium effort strategy, the $T-1$ compensation needs to be characterized. In period $T-1$, the market knows that future effort levels only depend on the history of the game through the agent's reputation. Therefore, the agent's expected productivity only depends on b_{mT-1}, and, consequently, the compensation only depends b_{mT-1}. This is summarized in the following corollary of lemma 2.

Corollary 1 *In period $T-1$, for any market's belief b_{mT-1}, the compensation is the same for all output histories $y^{T-2} \in Y^{T-2}\left(b_{mT-1}, a^{T-2}\left(y^{T-3}\right)\right)$.*

In period $T-2$, when deciding his effort level, the agent considers how his effort would affect his next-period expected lifetime utility. Therefore, before writing the agent's problem, it is useful to consider his expected lifetime utility at the beginning of $T-1$ and how it depends on a_{T-2}. We shall show that the agent's expected lifetime utility depends on the history of the game only through the beliefs.

Lemma 2 shows that in period $T-1$, on the equilibrium path, the agent's equilibrium effort strategy, $\alpha_{T-1}\left(b_{T-1}\right)$, only depends on the history of the game through the players' beliefs, b_{T-1}. Therefore, the market's belief is sufficient for determining the effort it expects. Consequently, in period $T-1$, off the equilibrium path, the agent's optimal effort only depends on y^{T-2} through the beliefs (recall that aside from determining beliefs, y^{T-2} only affects problem 5 through the effort expected by the market). This is summarized in the following corollary of Lemma 2.

Corollary 2 *In period $T-1$, for any b_{mT-1} and b_{aT-1}, the optimal effort level is the same for all y^{T-2} and a^{T-2} such that $y^{T-2} \in Y^{T-2}\left(b_{mT-1}, a^{T-2}\left(y^{T-3}\right)\right)$ and $y^{T-2} \in Y^{T-2}\left(b_{aT-1}, a^{T-2}\right)$.*

Given that the optimal effort at $T-1$ only depends on the history of the game through the beliefs, the agent's expected lifetime utility at the beginning of $T-1$ only depends on the beliefs. Let

$$Z_{T-1}(b_a, b_m) = \omega_{T-1}(b_m) - c(\alpha_{T-1}(b_a, b_m)) + \delta E_{\eta_{T-1}}\left[\omega_T\left(B_m\left(b_m, \eta, \alpha_{T-1}(b_a, b_m), \alpha_{T-1}(b_m)\right)\right) \middle| b_a\right]$$

denote this utility where b_a and b_m characterize the agent's and the market's beliefs, respectively, $\alpha_{T-1}(b_a, b_m)$ denotes the optimal effort for these beliefs, and $\omega_{T-1}(b_m)$ denotes the period-$T-1$ compensation for b_m. Thus, in period $T-2$, for all histories that imply b_{aT-2} and b_{mT-2}, the agent's maximization problem is given by

$$(7) \quad \max_{a_{T-2}} \left\{\delta E_{\eta_{T-2}}\left[Z_{T-1}\left(B\left(b_{aT-2}, \eta_{T-2}\right), B_m\left(b_{mT-2}, \eta_{T-2}, a_{T-2}, a^{T-2}\left(y^{T-3}\right)\right)\right) \middle| b_{aT-2}\right] - c(a_{T-2})\right\}.$$

We shall focus on situations in which the agent would exert effort because of career concerns. Therefore, the following discussion assumes that the agent expects Z_{T-1} to increase with respect to b_{mT-1} (specific assumptions are discussed when the compensation scheme is specified in sections 4 and 5).[11]

Following the same logic used for period $T-1$, one can prove that at $T-2$, a unique equilibrium effort exists. Moreover, the equilibrium effort is the same for all histories y^{T-3} that imply the same equilibrium belief b_{T-2}. This is summarized in the following lemma:

Lemma 3 *In period $T-2$, on the equilibrium path, for any b_{T-2}, an equilibrium effort level exists and is unique. Consequently, the equilibrium effort level is the same for all $y^{T-3} \in Y^{T-3}\left(b_{T-2}, a^{T-3}\left(y^{T-4}\right)\right)$.*

The equilibrium effort strategy, $\alpha_{T-2}(b_{T-2})$, satisfies the first-order condition of problem 7 when the effort expected by the market $a_{T-2}\left(y^{T-3}\right)$ equals $\alpha_{T-2}(b_{T-2})$, i.e., it satisfies

$$(8) \qquad c'(\alpha_{T-2}(b_{T-2})) = \delta \frac{\partial E_{\eta_{T-2}}\left[Z_{T-1}\left(B\left(b_{T-2}, \eta_{T-2}\right), B\left(b_{T-2}, \eta_{T-2}\right)\right) \middle| b_{T-2}\right]}{\partial a_{T-2}}.$$

[11] The agent's effort increases b_{mT-1} (see equation 4). Consequently, the agent exerts effort if and only if he expects Z_{T-1} to be increasing with respect to b_{mT-1}. Section 5 explains why Z_{T-1} may be decreasing with respect to b_{mT-1}.

In equation 8, as in equation 6, effort does not appear in the right-hand side because, in equilibrium, the agent chooses the effort level expected by the market. Moreover, the right-hand side of equation 8 is positive (as explained above, career concerns are only relevant if it is positive). Thus, equation 8 shows that a unique equilibrium effort level exists for all $y^{T-3} \in Y^{T-3}\left(b_{T-2}, a^{T-3}\left(y^{T-4}\right)\right)$.

3.3 Period-t equilibrium effort strategy

The analysis of any period before $T-2$ parallels that above. In period $T-2$, the market knows that future effort levels only depend on the agent's reputation, and, therefore, the period-$T-2$ compensation only depends on b_{mT-2}. The agent's period-$T-2$ expected lifetime utility only depends on the history of the game through beliefs, and, therefore, the equilibrium effort level in period $T-3$ only depends on the history of the game through beliefs. The same is true for any period before $T-3$. This is summarized in the following proposition:

Proposition 1 *In period t, for any b_{at} and b_{mt}, and for all y^{t-1} and a^{t-1} such that $y^{t-1} \in Y^{t-1}\left(b_{mt}, a^{t-1}\left(y^{t-2}\right)\right)$ and $y^{t-1} \in Y^{t-1}\left(b_{at}, a^{t-1}\right)$, the optimal effort level, $\alpha_t\left(b_{at}, b_{mt}\right)$, is the same . The compensation, $\omega_t\left(b_m\right)$, is the same for all $y^{t-1} \in Y^{t-1}\left(b_{mt}, a^{t-1}\left(y^{t-2}\right)\right)$. Moreover, the optimal effort level exists, is unique, and is given by*

$$c'(\alpha_t\left(b_{at}, b_{mt}\right)) = \delta \frac{\partial E_\eta\left[Z_{t+1}\left(B\left(b_{at}, \eta\right), B_m\left(b_{mt}, \eta, \alpha_t\left(b_{at}, b_{mt}\right), \alpha_t\left(b_{mt}\right)\right)\right) \middle| b_{at}\right]}{\partial a_t} \tag{9}$$

where

$$Z_t\left(b_a, b_m\right) = \omega_t\left(b_m\right) - c(\alpha_t\left(b_a, b_m\right)) + \delta E_{\eta_t}\left[Z_{t+1}\left(B\left(b_a, \eta\right), B_m\left(b_m, \eta, \alpha_t\left(b_a, b_m\right), \alpha_t\left(b_m\right)\right)\right) \middle| b_a\right].$$

Given equation 9, if the agent's period-$t+1$ strategy is known, his period-t strategy can easily be obtained. Thus, it is easy to solve the model by backward induction.

4 A benchmark

This section provides a benchmark following previous studies of career concerns in assuming that the compensation equals the agent's expected productivity.[12] Following Section 3, we can write the equilibrium

[12] Holmstrom [1999] explains that this compensation scheme would result if the principal is a risk-neutral firm in a competitive market and the history of outputs produced by the agent is known by every firm in the market. However, this compensation

effort as a function of the beliefs. Thus, the compensation scheme is given by

$$\omega_t(b_{mt}) = b_{mt} + \alpha_t(b_{mt}). \tag{10}$$

Following section 3, we know that in period T, the agent always choose $a_T = 0$. Therefore, $\omega_T(b_{mT}) = b_{mT}$, and the agent's $T-1$ maximization problem can be written as

$$\max_{a_{T-1}} \left\{ \delta E_{\eta_{T-1}} \left[\mu b_{mT-1} + (1-\mu)\left(\eta_{T-1} + a_{T-1} - \alpha_{T-1}(b_{mT-1})\right) \middle| b_{aT-1} \right] - c(a_{T-1}) \right\}. \tag{11}$$

In this problem, the expected compensation is linear in effort and the marginal gain from exerting effort does not depend on the beliefs. The agent's incentives are independent of how talented he believes he is, b_{aT-1}, how talented the market believes the agent is, b_{mT-1}, and the effort expected by the market, $\alpha_{T-1}(b_{mt-1})$. This is the case in previous studies of career concerns: the agent's decisions do not depend on his reputation.

Problem 11 is concave, and the optimal effort level $\hat{a}_{T-1}$ is characterized by

$$c'(\hat{a}_{T-1}) = \delta(1-\mu). \tag{12}$$

With his effort level, the agent affects his future compensation through the signal inferred by the market. Therefore, equilibrium effort is higher if the agent is more concerned about the future (δ is higher), or the weight of the signal in b_{mT} is higher ($1-\mu$ is higher).

The agent's $T-1$ compensation is given by $\omega_{T-1} = b_{mT-1} + \hat{a}_{T-1}$. The agent's expected utility at the beginning of $T-1$ is given by

$$Z_{T-1}(b_{aT-1}, b_{mT-1}) = \omega_{T-1}(b_{mT-1}) - c(\hat{a}_{T-1}) + \delta E_{\eta_{T-1}} \left[\omega_T\left(\mu b_{mT-1} + (1-\mu)\eta_{T-1}\right) \middle| b_{aT-1} \right].$$

Following Section 3, the agent's maximization problem in period $T-2$ can be written as

$$\max_{a_{T-2}} \left\{ \delta E_{\eta_{T-2}} \left[Z_{T-1}\left(B\left(b_{aT-2}, \eta_{T-2}\right), B_m\left(b_{mT-2}, \eta_{T-2}, a_{T-2}, \alpha_{T-2}(b_{mT-2})\right)\right) \middle| b_{aT-2} \right] - c(a_{T-2}) \right\}. \tag{13}$$

scheme is not consistent with compensation schemes observed in reality (see, for example, Bernhardt [1995], and Gibbons and Waldman [1999]). Section 5 incorporates a realistic feature to the compensation scheme considered that allows us to study how the agent's decisions depend on his reputation.

Problem 13 is concave, and the optimal effort level $\hat{a}_{T-2}$ is characterized by

$$c'(\hat{a}_{T-2}) = \delta(1-\mu) + \delta^2\mu(1-\mu). \tag{14}$$

The equilibrium effort is independent of the agent's reputation.

As illustrated by problem 13, in this framework, the agent's problems are static. There is no link between the agent's decision in the current period and his future decisions. In Section 5, this is not the case, and the agent faces an effort-smoothing decision.

The same logic allows us to find the equilibrium strategies for periods before $T-2$. These findings are summarized in the following proposition (that gives the finite-horizon version of the result in Holmstrom [1999]).

Proposition 2 *In period t, the compensation scheme is given by $\omega_t(b_{mt}) = b_{mt} + \hat{a}_t$ where the equilibrium effort level, $\hat{a}_t$, is given by*

$$c'(\hat{a}_t) = (1-\mu)\,\delta\frac{1-(\delta\mu)^{T-t}}{1-\mu\delta}. \tag{15}$$

The following corollaries of Proposition 2 (discussed in Section 5.3) characterize the equilibrium effort decision in equation 15.

Corollary 3 *The equilibrium effort level decreases with respect to t.*

Corollary 4 *The equilibrium effort level is decreasing with respect to $h_{\bar{\eta}}$, and it is increasing with respect to h_{ε}.*

Corollary 5 *The equilibrium effort level is never greater than the efficient effort level, $\bar{a}$, defined by $c'(\bar{a}) = 1$. It is only equal to $\bar{a}$ in the infinite-horizon version of the model with $\delta = 1$.*

The next section shows that some of the results in previous studies of career concerns are not robust to changes in the career-concern compensation scheme considered. In particular, the agent faces an intertemporal effort smoothing decision; the equilibrium effort level depends on the agent's reputation; and none of the corollaries presented in this section hold.

5 A discontinuous compensation scheme

If the agent exerts effort to influence his future reputation, it seems natural to expect that his decisions would depend on his current reputation. Section 3 shows that in general, this is indeed the case in models of career concerns. Therefore, considering this should be an integral part of studying the evolution of the agent's decisions over his career as well as the design of contracts that complement career-concern incentives. Section 4 shows how in previous models of career concerns, under carefully chosen assumptions, the agent's decisions do not depend on his reputation. In particular, as illustrated in problem 11, previous studies assume that the expected compensation is linear in effort and the marginal gain from exerting effort is independent from the agent's reputation. In order to study how the agent's decisions depend on his reputation, this section moves away from this assumption considering a realistic feature of many compensation schemes: the compensation scheme may present discontinuities.[13] That is, a small change in the agent's reputation may imply a large change in his compensation.[14] Discontinuous compensation schemes are widely observed.

First, as documented by the empirical literature, the agent may be assigned to different levels in a hierarchy according to his reputation, and these reassignments often imply a discontinuous change in the agent's compensation. For example, Murphy [1985] finds that *"presidents promoted to chief executive officer received one-time salary increases of 14.3%, and vice presidents promoted to president or chief ex-*

[13] Problem 11 shows that there are a number of assumptions in the benchmark that could be modified to obtain a relationship between the agent's decision and his current reputation. As a natural first step toward understanding how the agent's reputation affects his incentives, this paper considers only one departure from this framework that makes effort depend on reputation (a departure that we consider particularly interesting). The framework presented in the paper would be useful to study the effect of changing other assumptions (see Martinez [2004]). The dynamic effects explained in the paper are robust to changes in other assumptions.

[14] Previous models of career concerns considering discontinuous compensation schemes present either two-period frameworks or repeated two-period relationships (for example, studying term limits). Therefore, in these studies, the agent's equilibrium action does not depend on his reputation. This is the case in the large literature on political agency started by Barro [1973] and discussed by Besley [2005] (Besley and Case [1995] and Hess and Orphanides [1995, 2001] present empirical evidence supporting this theory). An exception to this is the work by Ashworth [2001] who presents a three-period-political-agency model. However, his paper does not consider that the principal's belief and the agent's belief may be different off equilibrium.

ecutive officer received average increases of 20.9% and 42.9%, respectively." Kwon [2005] also finds that incentives are largely provided by promotions, and the probability of a promotion depends on the agent's performance. There is a theoretical literature explaining why a firm would choose this compensation structure. For example, Bernhardt [1995] accounts for these observations in a framework in which the agent's promotion signals his ability to other firms, and implies a discontinuous increase in his compensation. In this section, these situations are represented by an exogenous discontinuous compensation scheme.[15]

Furthermore, capacity constraints (for example, there is a finite number of CEOs, governors, and NBA players) imply that the employer replaces the incumbent agent if the employer expects to be better off with a replacement (see, for example, Martinez [2005]).[16] In general, the agent is not indifferent about losing his position. For example, Diermeier, Keane, and Merlo [2005] find that the means of the monetized values of a House seat and a Senate seat in 1995 dollars are equal to $616,228 and $1,673,763, respectively. Murphy [1985] finds that *"An executive who loses his CEO status but remains as chairman of the board receives, on average, a 16.1% cut in pay."* Firing incentives are especially important for the positions that are crucial for the performance of an economy because these positions usually have a unique character that gives value to keeping the job.[17] Thus, the insights provided in this section allow several applications. One is the study of the more typical employees' effort-choice decisions (as in Holmstrom [1999]). An alternative is the study of the conflict of interest between politicians and voters, as applied to pure rent seeking (as in the models in Persson and Tabellini [2000]), to the cyclical manipulation of fiscal policy (as in Shi and Svensson [2002]), or to monetary policy (as in Eggertsson and Le Borgne [2005]).

[15] The compensation scheme could be made endogenous with a model of the labor market similar to the one presented by Bernhardt [1995]. For expositional simplicity, this paper does not to do so, and focuses on the career-concern incentives implied by this scheme that, for example, Bernhardt [1995] left unexplored.

[16] In a model without learning about ability where the principal uses long-term contracts for providing incentives to the incumbent agent on the job, Spear and Wang [2005] show that the principal may want to replace the incumbent because it may be more costly to induce the incumbent to exert effort than to induce a new agent to exert effort. In a model of career concerns that allows for firing, if incentive contracts were considered, the firing motives in Spear and Wang [2005] could appear.

[17] Empirical studies document the way in which the turnover of politicians (for a review, see Lewis-Beck and Stegmaier [2000]) and managers (see, for example, Khorana [1996], and Mian [2001]) is related with past performance.

Similarly, a discontinuous change in the employee's compensation may be observed if he is assigned to different sectors in the economy according to his reputation. MacDonald (1982) presents a model in which workers are assigned to different tasks depending on their reputation, and their productivity depends on the task they are assigned to.

In other to study the agent's career-concern incentives in these situations (in which he faces a discontinuous compensation scheme), we assume that

$$\omega_t(b_{mt}) = \begin{cases} w_G \text{ if } b_{mt} \geq b_G \\ w_B \text{ otherwise} \end{cases} \tag{16}$$

where $w_G > w_B$.[18] This compensation scheme may be interpreted as the agent being assigned to the good occupation if his reputation is good enough, and to the bad occupation otherwise (and receiving a higher compensation while in the good occupation).[19]

5.1 Equilibrium effort strategy

We focus on situations in which the agent would exert effort because of career concerns, and, therefore, we assume that the agent expects his next-period lifetime utility to increase if he exerts a higher effort in the current-period (specific assumptions are discussed below).

[18] The results presented here do not change much if w_G and w_B depend on the agent's reputation. Even if this is not the case, if by improving his reputation the agent increases the probability of receiving w_G in every future period, he always benefits from a better reputation (as when w_G and w_B depend on the agent's reputation). Assuming that w_G and w_B do not depend on reputation simplifies the analysis and allows us to focus on the incentives generated by a discontinuity in the compensation scheme.

[19] The model studied in this section assumes that the learning process about the agent's ability is not affected by the occupation in which the agent is working (this is also the case in previous studies; see, for example, Gibbons and Waldman [1999]). It could be assumed, for example, that the agent has a different ability for each occupation and the bad-occupation output is a worse signal of the good-occupation ability than the good-occupation output. For expositional simplicity, this is not considered. Alternatively, previous studies on career concerns considering firing present a compensation rule similar to the one studied here but assume that, after the agent is fired, there is no more learning about his ability and, therefore, the agent can never go back to the job (or the good occupation). These intertemporal effects of firing incentives do not much affect the results presented here (see Martinez [2005]). As in previous studies of career concerns, the agent participation decision is not studied here. It is assumed that the agent always works and he works in the occupation he is assigned to.

This section proceeds by characterizing the agent's equilibrium strategy through the first-order condition of the agent's problem.[20] Let f_x denote the density function for a normally distributed random variable with mean x and precision H. Let

$$\eta_G(b) \equiv \frac{b_G - \mu b}{1-\mu}$$

denotes the signal η required for $B(b,\eta) = b_G$. The following proposition shows how we can represent the agent's incentives in the Euler equation corresponding to the agent's problem (see Appendix B for the proof).

Proposition 3 *In period $t < T$, for any b_t, the equilibrium effort level, $\alpha_t(b_t)$, is given by*

$$c'(\alpha_t(b_t)) = \delta(w_G - w_B) f_{b_t}(\eta_G(b_t)) + \delta \int_{-\infty}^{\infty} r_t(B(b_t,\eta_t)) c'(\alpha_{t+1}(B(b_t,\eta_t))) f_{b_t}(\eta_t) d\eta_t \quad (17)$$

where

$$r_t(b_m) \equiv \frac{dB\left(b_m, \eta_{mt+1}(b_m)\right)}{db_m} = \mu - (1-\mu)\alpha'_{t+1}(b_m), \quad (18)$$

$$\eta_{mt}(b_m) \equiv y_t - \alpha_t(b_m), \quad (19)$$

and $\alpha'_t(b)$ denotes the derivative of $\alpha_t(b)$ with respect to b.

A unique period-t equilibrium strategy, α_t, can easily be obtained from equation 17 once a unique period-$t+1$ equilibrium strategy, α_{t+1}, is known. Therefore, the unique equilibrium strategies can easily be obtained by solving the model backwards.

[20] Assumptions are needed to guarantee the concavity of the agent's problem. For example, the probability of receiving w_G next period is not a globally concave function of effort. In order to assure the global concavity of the agent's problem, it is sufficient to assume enough convexity in the cost function. For example, one could find an upper bound for the slope of the marginal benefit curve and assume that the slope of the marginal cost curve is always higher (this is particularly easy for $T-1$). Another alternative is to assume the standard exponential cost function, $c(a) = a^n$, and to assume that n is high enough. Consequently, the marginal cost is very low for a low a and, for a high enough a, it starts increasing very rapidly, assuring that the marginal cost curve crosses the marginal benefit curve only once (from below) and the problem is globally concave (see Martinez [2005]).

The first term in the right-hand side of equation 17 represents the *next-period gain* from exerting effort. It represents the gain from increasing the probability of receiving w_G next period. The next-period gain is given by the change in the occupational compensation, $w_G - w_B$, multiplied by the change in the probability of receiving w_G next period, $f_{b_t}(\eta_G(b_t))$, and discounted by δ.

The second term in the right-hand side of equation 17 represents the *job-value gain* from exerting effort. The job-value gain describes the typical intertemporal tradeoff in dynamic models: having less utility today allows an agent to have more utility next period. For example, to influence his compensation at $t+2$ (and at every period after $t+2$), the agent can exert effort in t and in $t+1$. Thus, the agent compares the costs and the *effectiveness* of exerting effort in t and in $t+1$. Previous studies on career concerns consider only situations in which the agent faces an *intratemporal* decision each period. This paper presents a tractable framework that allows us to analyze the agent's *intertemporal* decisions as well.

In equation 17, r_t represents the *relative effectiveness* in changing the market's future beliefs (and, therefore, the probability of receiving w_G in the future) of exerting effort in t (compared with exerting effort in $t+1$). For example, to influence his compensation at $t+2$, the agent needs to affect b_{mt+2}. The agent's effort in t affects b_{mt+1} directly, and affects b_{mt+2} through b_{mt+1} (as indicated in equation 4). His effort in $t+1$ affects b_{mt+2} directly. Thus, the relative effectiveness of a_t (compared with a_{t+1}), r_t, is the derivative of the market's period-$t+2$ belief, $B\left(b_{mt+1}, \eta_{mt+1}(b_{mt+1})\right)$, with respect to b_{mt+1}, where $\eta_{mt}(b_{mt}) \equiv y_t - \alpha_t(b_{mt})$ denotes the signal inferred by the market. If r_t is lower than one, it implies that a_t was (relatively) less effective than a_{t+1} in changing b_{mt+2}.

In previous studies, because the effort expected by the market does not depend on the agent's reputation, a higher b_{mt} always implies that for a given effort level, a higher b_{mt+1} is more likely. In this paper, this does not need to be the case. For example, suppose that the effort expected by the market is increasing with respect to the reputation b_{mt} (see Section 5.2). Then, at t, if b_{mt} is higher, and the market believes the agent exerted a higher effort level, $\alpha_t(b_{mt})$, for any output y_t, the market infers a lower signal $\eta_{mt}(b_{mt}) \equiv y_t - \alpha_t(b_{mt})$. The market thinks that y_t is the result of a high effort and a low signal. Consequently, the agent's reputation at $t+1$ may be worse if his reputation at date t is better, i.e., $b_{mt+1} = B(b_{mt}, \eta_{mt}(b_{mt}))$ may be decreasing with respect to b_{mt} (and the relative effectiveness of a_{t-1} may be negative).[21]

[21] A negative expected relative effectiveness may imply a negative job-value gain. If the job-value gain is negative enough,

Equation 17 shows that equilibrium effort depends on the agent's reputation (in contrast to the benchmark case presented in Section 4). Next, we shall discuss how the agent's reputation affects his decisions.

5.2 Incentives and reputation

Would an agent with a better reputation face stronger career-concern incentives, or would such an agent be more difficult to control? This section helps answer this question by studying the relationship between the strength of career-concern incentives and the agent's reputation.

First, let us focus on the next-period gain. Reputation only affects the next-period gain through the density function for signals evaluated at the minimum signal required for the agent to have an expected ability equal to the occupational threshold b_G.

Even though on the equilibrium path the players' beliefs are always coincidental, the role of each of these beliefs on the agent's decision can be studied separately. The market's belief determines the minimum signal required for obtaining w_G next period. For example, if at the beginning of the period the market believes the agent is very talented, the agent may receive w_G next period even if the current-period signal is low (because the market's next-period belief may still be good enough). The agent's belief determines the signal density function he uses for evaluating his problem. Loosely speaking, it determines how likely he thinks it is that a certain signal is realized. In the next-period-gain term, the density reads $f_{b_{at}}\left(\eta_G(b_{mt})\right)$ (but on the equilibrium path $b_{at} = b_{mt}$). The next lemma describes the relationship between the agent's belief and the strength of the next-period gain incentives (see Appendix C for the proof).

Lemma 4 *On the equilibrium path, $f_{b_{at}}\left(\eta_G(b_{mt})\right)$ is increasing with respect to b_{at} if and only if $b_{mt} < b_G$. Moreover, $f_{b_{at}}\left(\eta_G(b_{mt})\right)$ is decreasing with respect to b_{at} if and only if $b_{mt} > b_G$, and $\frac{\partial f_{b_{at}}(\eta_G(b_{mt}))}{\partial b_{at}} = 0$ if and only if $b_{mt} = b_G$.*

effort decreases the agent's expected lifetime utility, and, therefore, the agent may choose not to exert effort. Furthermore, if $\alpha_t(b_t)$ is decreasing with respect to b_t, the expected y_t can be decreasing with respect to b_t. With an endogenous compensation scheme, this would imply that $\omega_t(b_{mt})$ is decreasing with respect to b_{mt}, and, therefore, the agent could choose not to exert effort (see Martinez [2005]). An interior solution to the agent's problem can be guaranteed with assumptions that limit the responsiveness of effort to changes in reputation. The relationship between equilibrium effort and reputation is discussed in section 5.2. It is easy to see that for example, a low enough value of $w_G - w_B$ and/or a low enough value of the precision in the signal distributions, H, can guarantee an interior solution.

The intuition for this result is as follows. The next-period-gain benefits of exerting effort are given by the change in the probability of receiving w_G next period multiplied by $w_G - w_B$ and the discount factor. This probability change is represented by $f_{b_{at}}(\eta_G(b_{mt}))$ that loosely speaking, represents the probability of having the minimum signal required for receiving w_G next period, $\eta_G(b_{mt})$. When the market's belief is high ($b_{mt} > b_G$), the current-period minimum signal the agent needs for receiving w_G next period is low (i.e., given a high b_{mt}, $\eta_G(b_{mt})$ is low). When the agent believes he is better (i.e., when b_{at} is higher), he believes that a low signal is less likely (i.e., $f_{b_{at}}(\eta_G(b_{mt}))$ is lower). Consequently, when the agent believes he is better, he has weaker incentives to exert effort. A parallel argument applies when $b_{mt} < b_G$.[22]

The next lemma characterizes the relationship between the market's belief and the next-period gain (the proof is provided in Appendix D).

Lemma 5 *On the equilibrium path,* $f_{b_{at}}(\eta_G(b_{mt}))$ *is increasing with respect to* b_{mt} *if and only if* $b_{at} < b_G$*. Moreover,* $f_{b_{at}}(\eta_G(b_{mt}))$ *is decreasing with respect to* b_{mt} *if and only if* $b_{at} > b_G$ *and* $\frac{\partial f_{b_{at}}(\eta_G(b_{mt}))}{\partial b_{mt}} = 0$ *if and only if* $b_{at} = b_G$*.*

This lemma says that when the agent expects his ability to be high (low), he has weaker (stronger) incentives to exert effort when the market believes the agent is better. The intuition behind this result is straightforward. Under the normality assumption for the signal density functions, more extreme signal values are less likely (the result holds for all density functions with this property). In other words, if the value of signal is more extreme (i.e., if the signal is further from the mean), the density function evaluated at the signal is lower. Suppose the agent expects his ability to be high ($b_{at} > b_G$). Recall that on the equilibrium path the market's belief coincides with the agent's belief, and, therefore, the market's belief is high ($b_{mt} = b_{at} > b_G$). Then, the minimum signal realization that would allow the agent to receive w_G next period is low ($\eta_G(b_{mt}) < b_G$). In particular, the minimum signal is lower than the expected signal, b_{at}. If the market believes the agent is better, the minimum signal is lower and further from the expected signal, and, therefore, is less likely. That is, when the agent expects his ability to be high ($b_{at} > b_G$), $f_{b_{at}}(\eta_G(b_{mt}))$ is decreasing with respect to b_{mt}, and the agent has weaker incentives to exert effort if the market believes he is better. A parallel argument applies when $b_{at} < b_G$.

[22] As this intuition suggests, this result holds under more general assumptions about $f_{b_a}(\eta)$ (it is only required that low signals are less likely for better agents).

In period $T-1$, given that the only incentives are those captured by the next-period gain, lemmas 4 and 5 imply that equilibrium effort is hump-shaped over reputation, as stated in the following proposition:

Proposition 4 *In period $T-1$, an agent who is believed to be more (less) talented exerts more effort if and only if his reputation is worse (better) than b_G.*

In general, the job-value gain preserves the nonmonotonicity of the next-period gain. We will explain the intuition for period $T-2$ but the same logic applies for all periods. We are interested in describing how effort in period $T-2$ depends on reputation. Recall that the job-value gain is represented (in equation 17) by the expected marginal cost of exerting effort next period (weighted by the relative effectiveness). Suppose the agent's reputation is represented by b at $T-2$. Then, the agent knows that his reputation at $T-1$ is likely to be close to b. If for reputations close to b equilibrium effort is high (low) at $T-1$, then the expected marginal cost of exerting effort at $T-1$ is high (low), the job-value gain at $T-2$ is high (low) for b, and the agent has incentives to exert high (low) effort at $T-2$ for b. Thus, typically, for any reputation b such that effort is high (low) at $T-1$, it follows that effort is also high (low) at $T-2$. In other words, the relationship between reputation and the job-value gain at $T-2$ follows the same pattern of the relationship between reputation and equilibrium effort at $T-1$. In particular, at $T-2$, the job-value gain is hump-shaped over reputation, and equilibrium effort is hump-shaped. The same reasoning applies for any period $t < T-1$. Hence, in general, equilibrium effort is hump-shaped for all t. While equation 17 shows that this general intuition is complicated by the relationship between the agent's reputation and the expected relative effectiveness, the main force in play is the one explained above (for numerical examples see Martinez [2005]).[23]

5.3 The importance of the compensation scheme

In this section, we show that some of the conclusions presented in previous studies of career concerns, and replicated in Section 4, are not robust to changes in the career-concern compensation scheme considered.

[23] For example, if b is low enough, $\alpha_{T-1}(b)$ is a convex function of b. Therefore, $r_{T-2}(b) = \mu - (1-\mu)\,\alpha'_{T-1}(b)$ is decreasing with respect to b. Thus, an agent with a better reputation expects the relative effectiveness of his effort to be lower, and could choose to exert lower effort. In this case, this force contradicts the general intuition described above (recall that if b is low enough, the expected $T-1$ effort is increasing with respect to b).

More specifically, with the compensation scheme considered in this section, none of the corollaries presented in Section 4 hold.

5.3.1 The effect of tenure

Would an agent exert less effort later in his career? Consequently, is the need for contract incentives more important later in the agent's career?

Holmstrom [1999] studies an infinite-horizon version of the model presented in Section 4 in which ability does not change over time, and, therefore, the weight of the period-t belief in the period-$t+1$ belief, μ, is increasing with respect to the number of observations regarding the agent's performance. He shows that the equilibrium effort level is lower for an agent who has been working longer. This is the case because with more observations of the agent's performance, the signal has a lower weight in the market's next-period belief (μ is higher), and the agent affects the signal with his effort.

This channel is not present in the framework studied in Section 4 (ability is assumed to change in a way that makes μ constant). Nevertheless, in the finite-horizon framework studied in Section 4, it remains that equilibrium effort is decreasing with respect to the number of periods the agent worked. This is the case because closer to T (that may represent, for example, retirement) improving reputation is less beneficial because it affects compensations in fewer periods.

On the other hand, in this section, considering a discontinuous compensation scheme allows us to identify two reasons why an agent may decide to exert more effort later in his career. First, relative effectiveness may be negative. For example, consider the effort decision in two consecutive periods. Suppose the equilibrium effort level in the second period is increasing in the agent's reputation (as is the case for low-reputation agents). If the agent exerts more effort in the first period, he improves his second-period reputation and, therefore, he makes the market believe that he would exert a higher effort in the second period. Thus, in the second period, for any output the agent produces, the market believes that this output was produced with a higher effort and, therefore, the market infers a lower signal of ability. That is, the first-period effort hurts the agent in the second period. In fact, it is easy to construct examples in which this effect is important enough to make the agent exert no effort in the first period.

Second, it is necessary to consider that an agent's reputation may change over his career. For example, suppose that the agent's expected ability is high, and, therefore, he exerts more effort when his reputation is worse (see Section 5.2). Then, if the agent's reputation deteriorates over his career (but it is still high), the agent may decide to exert more effort later in his career.[24]

5.3.2 The effectiveness of career-concern incentives

Based on corollary 4, Holmstrom [1999] concludes that career-concern incentives "*will work more effectively if the ability process is more stochastic* (the precision in the ability distribution, $h_{\bar{\eta}}$, is lower) *or if the observations on outputs are more accurate* (the precision in the production noise, h_{ε}, is higher)."

The next proposition shows that this conclusion about the ability process is not robust to changes in the career-concern compensation scheme under consideration (see Appendix E for the proof).

Proposition 5 *For any b_{T-1} sufficiently close to b_G, $\alpha_{T-1}(b_{T-1})$ is increasing with respect to the precision in the ability distribution, $h_{\bar{\eta}}$.*

In models of career concerns where the agent's compensation is given by his expected productivity, an increase in uncertainty about ability (a decrease in $h_{\bar{\eta}}$) only brings about an increase in the quality of the signal as an indication of ability (and, therefore, an increase in the weight of the current-period signal in the next-period belief, $1-\mu$). Given that the agent affects the signal inferred by the market with his effort level, if the signal weight in the next-period belief is higher, the agent exerts more effort. This explains the result replicated in Section 4.

When discontinuities in the compensation scheme are considered, the density function for signals determines the marginal benefit of exerting effort, and $h_{\bar{\eta}}$ affects incentives through this density function. Proposition 5 shows that the effect on the density function may imply a positive relationship between effort and $h_{\bar{\eta}}$, and, furthermore, may dominate the mechanism described in previous studies.

The same two forces are present when changes in the degree of uncertainty in the production process are considered. However, the next proposition shows that the effect on the signal weight is dominant (see Appendix F for the proof).

[24] Martinez [2005] shows that the agent's effort may be lower further from T in a model in which the compensation is not decided in every period. In a firing model of career concerns in which a fired agent never comes back to the job, Martinez [2004] discusses a third reason for the agent's effort to be lower in periods further from T: the value of staying on the job could be lower further from T.

Proposition 6 *For any b_{T-1}, $\alpha_{T-1}(b_{T-1})$ is increasing with respect to the precision in the production noise distribution, h_ε.*

5.3.3 Efficiency

If the compensation scheme presents a discontinuity, the result in corollary 5 does not hold either. The agent's effort may be higher than the efficient effort. For example, at $T-1$, the marginal benefit of exerting effort is given by the change in the occupational compensation, $w_G - w_B$, multiplied by the change in the probability of receiving w_G next period, and discounted by δ. This marginal benefit may be higher than one (the marginal product of effort), and, therefore, equilibrium effort may be higher than the efficient effort—in Section 4, the marginal benefit of exerting effort is given by the marginal product of effort discounted by μ and δ.[25]

6 Conclusions and Extensions

We studied Holmstrom's [1999] seminal model of career concerns, but considered that a small change in the agent's reputation may imply a large change in his compensation—because, for example, the agent may be fired and/or promoted. This allowed us to study how the agent's decisions depend on his reputation. We showed that if the compensation scheme presents a discontinuity, the agent faces an intertemporal effort-smoothing decision, and equilibrium effort is hump-shaped over reputation. The players' problems can be set up recursively. That is, for all histories that imply the same beliefs, the players' optimal actions are identical. This facilitates the study of career concerns when we consider that the agent's decisions depend on his reputation. It was also shown that some of the conclusions presented in previous studies are not robust to changes in the career-concern compensation scheme considered: career-concern incentives may work more effectively if there is more dispersion in the distribution of abilities; the agent may decide to exert more effort later in his career; and the equilibrium effort level may be greater than the efficient effort level.

[25] Holmstrom [1999] discusses other variations of the model in section 4 that may imply that equilibrium effort is higher than the efficient effort level.

Other modifications of the model presented in this paper would provide interesting insights on agency relationships. For example, one could study situations in which the agent's compensation is not decided after every output observation (see Martinez [2005]). Moreover, because previous work on career concerns do not study how reputation affects incentives, one wonders what could be learned from considering this relationship. Therefore, there are many other possible applications for the framework presented here.[26] Furthermore, it seems natural to test the empirical implications of the model. Previous empirical studies on career concerns follow the theoretical literature and focus on the relationship between incentives and the number of observations of the agent's output without considering reputation (see, for example, Chevalier and Ellison [1999], Stiroh [2003], and Wilczynski [2004]). For empirical work, past performance could be used as an indication of reputation, and, for empirical studies about the career-concern incentives of politicians, approval ratings can be used as a measure of reputation (for example, job approval ratings for U.S. governors, senators, and presidents are available at http://www.unc.edu/~beyle/jars.html). As a natural first step toward understanding how the agent's reputation affects his incentives, this paper considers only one departure from Holmstrom [1999] that makes effort depend on reputation. Analyzing other channels (for example, introducing wealth effects in the agent's utility function) may deepen the understanding of the role of reputation.[27]

[26] For example, in Prendergast and Stole [1996], the agent makes investment decisions in order to affect the principal's perception about the agent's ability to learn. They show how the agent's incentives depend on the number of periods he has been working but they do not consider how reputation would affect incentives. Variations of the model presented here could also be used for studying the role of reputation in monopoly regulation and the ratchet effect (see, for example, Meyer and Vickers [1997]).

[27] For example, Martinez [2004] shows that, if there is a complementarity in production between effort and ability, this complementarity could imply an inverse relationship between the strength of next-period-gain incentives and reputation. However, this effect is dominated by the nonmonotonicity described in this paper (Holmstrom [1999] indicates that with a multiplicative production function, the marginal productivity of effort is higher when ability is higher and, therefore, we could expect equilibrium effort to be increasing in reputation). The framework presented here is useful to study these other channels. The dynamic effects are the same (although the relationship between effort and reputation could be affected).

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Appendix A: Proof of Lemma 2

On the equilibrium path, for any history y^{T-2}, expected effort $a_{T-1}\left(y^{T-2}\right)$, and beliefs b_{T-1}; the optimal effort is characterized by the first-order condition

$$c'(a_{T-1}) = \delta \frac{\partial E_{\eta_{T-1}}\left[\omega_T\left(\mu b_{T-1} + (1-\mu)\left(\eta_{T-1} + a_{T-1} - a_{T-1}\left(y^{T-2}\right)\right)\right)\middle| b_{T-1}\right]}{\partial a_{T-1}}. \tag{20}$$

Let $\hat{a}_{T-1}$ denote the equilibrium effort level ($\hat{a}_{T-1}$ satisfies the first-order condition in equation 20 when the market believes that the agent exerts $\hat{a}_{T-1}$). The equilibrium effort is given by

$$c'(\hat{a}_{T-1}) = \delta \frac{\partial E_{\eta_{T-1}}\left[\omega_T\left(\mu b_{T-1} + (1-\mu)\eta_{T-1}\right)\middle| b_{T-1}\right]}{\partial a_{T-1}}. \tag{21}$$

The right-hand side in equation 21 does not depend on $\hat{a}_{T-1}$ and is nonnegative (because ω_T is a weakly increasing function). Therefore, there exists a unique $\hat{a}_{T-1}$ that satisfies equation 21. Consequently, for each b_{T-1}, the equilibrium effort is the same for all $y^{T-2} \in Y^{T-2}\left(b_{T-1}, a^{T-2}\left(y^{T-3}\right)\right)$.

Appendix B: Proof of Proposition 3

Let $W_t(b_a, b_m)$ denote the agent's expected lifetime utility at the moment he decides on his period-t effort (i.e., after he received the period-t compensation). In period $t < T$, the agent's maximization problem is given by

$$W_t(b_t, b_t) = \max_{a_t}\left\{\begin{array}{l} \delta w_B + \delta\left(w_G - w_B\right)\left[1 - F_{b_t}\left(\eta_G\left(b_t\right) - a_t + \alpha_t\left(b_t\right)\right)\right] + ... \\ +\delta\int_{-\infty}^{\infty} W_{t+1}(B\left(b_t, \eta_t\right), B_m\left(b_t, \eta_t, a_t, \alpha_t(b_t)\right)) f_{b_t}\left(\eta_t\right) d\eta_t - c(a) \end{array}\right\}.$$

For all $t < T$, the first-order condition for this problem, evaluated in equilibrium (imposing $a_t = \alpha_t\left(b_t\right)$), reads

$$c'\left(\alpha_t\left(b_t\right)\right) = \delta\left(w_G - w_B\right) f_{b_t}\left(\eta_G\left(b_t\right)\right) + \delta\int_{-\infty}^{\infty} \left.\frac{\partial W_{t+1}}{\partial b_m}\right|_{B(b_t,\eta_t),B(b_t,\eta_t)} (1-\mu) f_{b_t}\left(\eta_t\right) d\eta_t \tag{22}$$

where, $\frac{\partial W_T}{\partial b_m} = 0$, and, for all $t < T-1$

$$\begin{aligned} W_{t+1}(b_{at}, b_{mt}) \quad = \quad & \delta w_B + \delta\left(w_G - w_B\right)\left[1 - F_{b_{at}}\left(\eta_G\left(b_{mt}\right) - \alpha_{t+1}\left(b_{at}, b_{mt}\right) + \alpha_{t+1}\left(b_{mt}\right)\right)\right] + ... \\ & +\delta\int_{-\infty}^{\infty} W_{t+2}(B\left(b_{at}, \eta_t\right), B_m\left(b_{mt}, \eta_t, \alpha_{t+1}\left(b_{at}, b_{mt}\right), \alpha_{t+1}(b_{mt})\right)) f_{b_{at}}\left(\eta_t\right) d\eta_t - ... \\ & -c\left(\alpha_{t+1}\left(b_{at}, b_{mt}\right)\right). \end{aligned}$$

By the envelope theorem, for all $t < T-1$,

$$\left.\frac{\partial W_{t+1}}{\partial b_m}\right|_{b_a,b_m} = \delta\left(w_G - w_B\right) f_{b_a}\left(\eta_G\left(b_m\right) - \alpha_{t+1}\left(b_a, b_m\right) + \alpha_{t+1}\left(b_m\right)\right)\left(\frac{\mu}{1-\mu} - \alpha'_{t+1}\left(b_m\right)\right) + ... \\ +\delta \int_{-\infty}^{\infty} \left.\frac{\partial W_{t+2}}{\partial b_m}\right|_{B(b_a,\eta),B_m(b_m,\eta,\alpha_{t+1}(b_a,b_m),\alpha_{t+1}(b_m))} \left(\mu - (1-\mu)\,\alpha'_{t+1}(b_m)\right) f_{b_a}\left(\eta\right) d\eta.$$

Evaluated at $b_a = b_m = B\left(b, \eta\right)$,

$$\left.\frac{\partial W_{t+1}}{\partial b_m}\right|_{B(b,\eta),B(b,\eta)} = \left(\frac{\mu}{1-\mu} - \alpha'_{t+1}\left(B\left(b,\eta\right)\right)\right)\left[\begin{array}{l} \delta\left(w_G - w_B\right) f_{b_a}\left(\eta_G\left(B\left(b,\eta\right)\right)\right) + ... \\ \delta \int_{-\infty}^{\infty} \left.\frac{\partial W_{t+2}}{\partial b_m}\right|_{B(B(b,\eta),\eta'),B(B(b,\eta),\eta')} \left(1-\mu\right) f_{B(b,\eta)}\left(\eta'\right) d\eta' \end{array}\right].$$

By equation 22,

$$\left.\frac{\partial W_{t+1}}{\partial b_m}\right|_{B(b,\eta),B(b,\eta)} = \left(\frac{\mu}{1-\mu} - \alpha'_{t+1}\left(B\left(b,\eta\right)\right)\right) c'\left(\alpha_{t+1}\left(B\left(b,\eta\right)\right)\right).$$

Substituting into equation 22 yields

$$c'\left(\alpha_t\left(b_t\right)\right) = \delta\left(w_G - w_B\right) f_{b_t}\left(\eta_G\left(b_t\right)\right) + \delta \int_{-\infty}^{\infty} \left[\mu - \left(1-\mu\right)\alpha'_{t+1}\left(B\left(b_t,\eta_t\right)\right)\right] c'\left(\alpha_{t+1}\left(B\left(b_t,\eta_t\right)\right)\right) f_{b_t}\left(\eta_t\right) d\eta_t.$$

Appendix C: Proof of Lemma 4

Recall that $f_{b_{at}}$ denotes the density for a normally distributed random variable with mean b_{at}, and that $\eta_G(b_{mt})$ is decreasing with respect to b_{mt}. Moreover, $\eta_G(b_G) = b_G$. On the equilibrium path, $b_{at} = b_{mt}$. Consequently, if $b_{at} = b_{mt} > b_G$, then $\eta_G(b_{mt}) < \eta_G(b_G) = b_G < b_{at}$. Therefore, $f_{b_{at}}\left(\eta_G\left(b_{mt}\right)\right)$ is decreasing with respect to b_{at}. If $b_{at} = b_{mt} < b_G$, then $\eta_G(b_{mt}) > \eta_G(b_G) = b_G > b_{at}$. Therefore, $f_{b_{at}}\left(\eta_G\left(b_{mt}\right)\right)$ is increasing with respect to b_{at}. If $b_{at} = b_{mt} = b_G$ and, consequently, $\eta_G\left(b_{mt}\right) = \eta_G\left(b_G\right) = b_{at}$, then $\frac{\partial f_{b_{at}}(b_{at})}{\partial b_{at}} = 0$.

Appendix D: Proof of Lemma 5

Recall that $f_b\left(\eta\right)$ increases with respect to η if and only if $\eta < b$. On the equilibrium path, $b_{at} = b_{mt}$. If $b_{at} = b_{mt} > b_G$, then $\eta_G(b_{mt}) < b_G < b_{at}$, and $f_{b_{at}}\left(\eta_G(b_{mt})\right)$ is decreasing with respect to b_{mt}. If $b_{at} = b_{mt} < b_G$, then $\eta_G(b_{mt}) > b_G > b_{at}$, and $f_{b_{at}}\left(\eta_G(b_{mt})\right)$ is increasing with respect to b_{mt}. If $b_{at} = b_{mt} = b_G$, $\eta_G(b_{mt}) = b_G = b_{at}$, and $\frac{\partial f_{b_{at}}(b_{mt})}{\partial b_{mt}} = 0$.

Appendix E: Proof of Proposition 5.

The next-period gain can be written using the density function for a standard normal random variable, f, i.e.,

$$f_{b_{T-1}}\left(\frac{b_G - \mu b_{T-1}}{1-\mu}\right) = \sqrt{H} f\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)$$

where $\sqrt{H}$ is increasing with respect to $h_{\bar{\eta}}$, and

$$\frac{\partial f\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)}{\partial \frac{\sqrt{H}}{1-\mu}} = f'\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)(b_G - b_{T-1}).$$

For b_{T-1} close to b_G, this derivative is small enough to be dominated by $\sqrt{H}$ being increasing with respect to $h_{\bar{\eta}}$, making the next-period gain increasing with respect to $h_{\bar{\eta}}$. Given that the marginal cost of exerting effort is increasing, the equilibrium effort level is increasing with respect to $h_{\bar{\eta}}$.

Appendix F: Proof of Proposition 6

Recall that $\sqrt{H}$ is increasing with respect to h_{ε}. Moreover,

$$\frac{\partial f\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)}{\partial \frac{\sqrt{H}}{1-\mu}} = f'\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)(b_G - b_{T-1}).$$

If $b_{T-1} \leq b_G$,

$$f'\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right) \leq 0$$

and the derivative is not positive. If $b_{T-1} > b_G$,

$$f'\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right) > 0$$

and the derivative is negative. Furthermore,

$$\frac{\sqrt{H}}{1-\mu} = \sqrt{\frac{h_{\bar{\eta}}}{h_{\varepsilon}} + h_{\bar{\eta}}}$$

is decreasing with respect to h_{ε}, and, therefore,

$$f\left(\sqrt{H}\frac{b_G - b_{T-1}}{1-\mu}\right)$$

is non-decreasing with respect to h_{ε}. Given that the marginal cost of exerting effort is increasing, equilibrium effort increases with h_{ε}.