

Abstract

This paper examines the business cycle properties of business cycle models with search frictions and wage bargaining which rely not only on labor, but also on capital in the production function. In the presence of capital, the choice of bargaining framework matters, even under perfect competition and constant returns to scale. In particular, under individual bargaining, the welfare theorems do not hold, due to a hold-up effect in capital and a hiring externality, so that solving a planner's problem is not sufficient. I examine the business cycle properties of the decentralized model with individual bargaining under alternative calibration strategies.

Individual Wage Bargaining and Business Cycles

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Recently, there has been renewed interest in the business cycle properties of models with search frictions and wage bargaining. Shimer (2005) shows that the carefully calibrated baseline model fails to match the data along several dimensions, while Hall (2005) emphasizes the role that rigid wages can play in reconciling the model with business cycle facts. These models are characterized by perfect competition, constant returns to scale and the absence of capital. In such a setting, the well-known equivalence between one-worker and large firms holds (Pissarides (2000)) and the distinction between individual and collective bargaining is not important.

This paper examines the business cycle properties of models which rely not only on labor, but also on capital in the production function. In the presence of capital, the choice of bargaining framework does matter, even under perfect competition and constant returns to scale.

The first generation of business cycle models with capital and wage bargaining were solved by reliance upon the welfare theorems. Merz (1995) and Andolfatto (1996) solve the planner's problem, and then show that wages and prices exist which support the planner's solution as a decentralized recursive competitive equilibrium. They do not, however, specify the wage-bargaining mechanism which leads to these wages.

The first contribution of this paper is to show that individual bargaining leads to a competitive equilibrium solution which does not correspond to the planner's problem. The reason is that under individual bargaining, capital is predetermined at the time of wage bargaining (having been chosen optimally by the firm in the previous period, as is standard in RBC models) and is subject to a hold-up problem. The hold-up problem leads to an inefficiently low equilibrium capital-labor ratio.¹ In addition, the firm's optimal capital-labor ratio is distorted by a hiring externality which leads firms to hire workers beyond the point at which their employment costs are recouped by their marginal products.

The presence of inefficiencies in the decentralized model invalidates the use of the second welfare theorem when bargaining is individual. This result has important consequences for the business cycle properties which can be accounted for by models with wage bargaining. Since the dominant labor market institution in the US is individual bargaining², it seems sensible to focus on individual bargaining when trying to match business cycle facts of the US economy.

The second contribution of this paper is to examine the business cycle properties of the decentralized individual bargaining model with capital. The quantitative section of this paper explores alternative calibration strategies and examines their success at matching key business cycle facts. The crucial parameters are workers' bargaining power μ and the output elasticities of labor $1 - \alpha$ and capital α . Workers' bargaining power controls the deviation of the capital-labor

¹Cahuc, Marque and Wasmer (2004) also make the point that hold-up effects are present in a model with a predetermined capital stock, constant returns to scale and perfect competition. However, these authors restrict attention to the steady-state of a partial equilibrium model, while I study a general equilibrium model with technology shocks.

²Less than 10% of American private sector workers are currently covered by a collective bargaining agreement, according to CPS data compiled by Hirsch and Macpherson (2003).

ratio from its efficient value, while both μ and α are involved in determining the factor shares³. In particular, there is a continuum of pairs (μ, α) which lead to the observed factor shares in the US economy. Two alternative strategies for pinning down the (μ, a) pair are explored. I find values of worker's bargaining power between 0.15 and 0.28, lower than those traditionally used in the literature, but more than two to four times higher than the value of 0.06 favored by Hagedorn and Manovskii (2005). For robustness, I also consider values of μ equal to 0.72 and 0.50. Finally, since the flow value of unemployment is the subject of such controversy, I vary it widely, using both 0.40 as favored by Shimer (2005) and most of the literature and the higher values used by Hagedorn and Manovskii (2005).

The main results are as follows: lower values of worker bargaining power do succeed in bringing down the volatility of the wage, but at the cost of also bringing down the volatility of consumption and investment to levels which are too low compared to the data. Following Hagedorn and Manovskii's strategy of increasing the flow value of unemployment to extremely high levels does help bring the labor market variables into line with the data, but the performance of the model in accounting for the relative volatilities of the real variables consumption and investment remains poorer than that of Shimer (2005).

The paper is organized as follows: Section 2 presents the model, and equilibrium and the steady-state are found in section 3. Alternative calibration strategies are discussed in section 4, while quantitative results are presented in section 5 and section 6 concludes.

1 Model

This section presents the basic model. It is a standard neo-classical growth model, augmented by labor market frictions and wage bargaining. The bargaining setup involves firms bargaining individually with each worker, and renegotiation of wages is possible at each date.

1.1 Household's Problem

Each household consists of a number of individuals which is large enough to guarantee perfect insurance over consumption. Each household member supplies one unit of labor inelastically, and the household maximizes its discounted expected utility from consumption as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

³In contrast, in models with marginal product wages, it is well-known that CRS Cobb-Douglas production under perfect competition leads to factor shares which are governed exclusively by α .

subject to the large-family budget and time constraints

$$h_t w_t + \pi_t + r_t k_{t-1} \geq c_t + i_t \quad (2)$$

$$h_t + u_t = 1 \quad (3)$$

where h_t family members earn the wage w_t and u_t are unemployed, π_t is a share of firms' profits, r_t is the return on capital and i_t is investment. Capital accumulation follows the law of motion:

$$k_t = (1 - \delta) k_{t-1} + i_t \quad (4)$$

where δ is the rate of depreciation. The solution to the family's problem takes the form of an Euler equation:

$$u_c(c_t) = \beta E_t \{u_c(c_{t+1}) [1 - \delta + r_{t+1}]\} \quad (5)$$

subject to the budget constraint

$$h_t w_t + \pi_t + (r_t + 1 - \delta) k_{t-1} \geq c_t + k_t \quad (6)$$

1.2 Search and Matching in the Labor Market

The labor market is characterized by a standard search and matching framework. Unemployed workers U_t and vacancies V_t are converted into matches by a constant returns to scale matching function $m(U_t, V_t) = s \cdot U_t^\eta V_t^{1-\eta}$. Defining labor market tightness as $\theta_t \equiv \frac{V_t}{U_t}$, the firm meets unemployed workers at rate $q_t = s\theta_t^{-\eta}$, while the unemployed workers meet vacancies at rate $f_t = s\theta_t^{1-\eta}$. Aggregate unemployment evolves as

$$U_{t+1} = U_t + [1 - f_t - \chi] U_t \quad (7)$$

where χ is the exogenous match destruction rate.

Workers are identical and bargaining is individual. Define $\tilde{\beta}_{t+1} \equiv \beta \frac{u_c(c_{t+1})}{u_c(c_t)}$ to be the households' stochastic discount factor. A worker's value of employment is:

$$V_t^E = w_t + E_t \left\{ \tilde{\beta}_{t+1} [(1 - \chi) V_{t+1}^E + \chi V_{t+1}^U] \right\}$$

The value of unemployment is standard.

$$V_t^U = b + E_t \left\{ \tilde{\beta}_{t+1} [f_t V_{t+1}^E + (1 - f_t) V_{t+1}^U] \right\} \quad (8)$$

where b denotes some non-tradeable flow value to being unemployed, expressed in units of output. Defining $V_t^W \equiv V_t^E - V_t^U$ yields an expression for the worker's surplus to employment:

$$V_t^W = w_t - b + (1 - \chi - f_t) E_t \left\{ \tilde{\beta}_{t+1} V_{t+1}^W \right\} \quad (9)$$

1.3 Firm's Problem

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and produce using a constant returns to scale technology. We abandon the one-worker-per-firm assumption in favor of a more general framework with multiple-worker firms. In the absence of capital, the one-worker firm assumption would be harmless, since both individual and collective bargaining would yield the same results. In the presence of installed capital, however, individual bargaining leads to a hold-up problem, while efficient bargaining (i.e. collective bargaining over both wages and employment) does not. Hence, it is not possible to appeal to the welfare theorems to solve a planner's problem for the individual bargaining model, and its business cycle properties will differ from those of the planner's problem.

Firms maximize the discounted value of future profits. Consistent with stylized facts we assume that firms adjust employment by varying the number of workers [extensive margin] rather than the number of hours per worker. Firm i 's state variable is the number of workers currently employed, h . The firm's key decision is the number of vacancies v . Firms open as many vacancies as necessary to hire in expectation the desired number of workers next period, while taking into account that the real cost to opening a vacancy is Φ_V . The firm's problem becomes:

$$V^J(z_t, h_t) = \max_{v_t, k_t} \left[y_t - w(z_t, h_t, k_{t-1}) h_t - r_t k_{t-1} - \Phi_V v_t + E_t \left\{ \tilde{\beta}_{t+1} V^J(z_{t+1}, h_{t+1}) \right\} \right]$$

subject to

$$\begin{aligned} \text{production function:} & \quad y_t = A e^{z_t} k_{t-1}^\alpha h_t^{1-\alpha} \\ \text{transition function:} & \quad h_{t+1} = (1 - \chi) h_t + q_t v_t \\ \text{wage curve} & \quad : \quad w(z_t, h_t, k_{t-1}) \\ \text{technology shock:} & \quad z_t = \rho z_{t-1} + \varepsilon_t \end{aligned}$$

where the wage curve is the result of individual bargaining as described in the following sub-section. When optimizing, firms take into account the impact of their capital and labor input choices on the bargained wage. In the following sub-section, the bargained wage will indeed turn out to depend upon these input choices.

The first-order condition for capital equates the marginal product of capital to the cost of capital, where the latter includes both the rental cost per unit of capital r_t and the marginal impact of the choice of capital on the wage bill.

$$\frac{\partial y_t}{\partial k_{t-1}} = h_t \frac{\partial w_t}{\partial k_{t-1}} + r_t \quad (10)$$

The first order condition for vacancies states that the marginal value of an additional worker must equal the cost of searching for him/her:

$$\frac{\Phi_V}{q(\theta_t)} = E_t \left\{ \tilde{\beta}_{t+1} \frac{\partial V^J(z_{t+1}, h_{t+1})}{\partial h_{t+1}} \right\}. \quad (11)$$

Combining (11) with the envelope condition for employment h_t leads to an optimality condition for the firm's choice of labor input:

$$\frac{\Phi_V}{q_t} = E_t \left\{ \tilde{\beta}_{t+1} \left[\frac{\partial y_{t+1}}{\partial h_{t+1}} - h_{t+1} \frac{\partial w_{t+1}}{\partial h_{t+1}} + (1 - \chi) \frac{\Phi_V}{q_{t+1}} - w(z_{t+1}, h_{t+1}, k_t) \right] \right\} \quad (12)$$

Equation (12) equates the cost of hiring a worker (left hand side) to the discounted expected marginal benefits of hiring that worker. These marginal benefits are the worker's marginal product net of wages, taking into account the impact of hiring an additional worker on the bargained wage, and that a worker will remain with the firm with probability $(1 - \chi)$.

Finally, the envelope condition for employment h_t and (11) also lead to an expression for the marginal value of a worker:

$$\frac{\partial V^J(z_t, h_t)}{\partial h_t} = \frac{\partial y_t}{\partial h_t} - w(z_t, h_t, k_{t-1}) - h_t \frac{\partial w}{\partial h_t} + (1 - \chi) \frac{\Phi_V}{q_t} \quad (13)$$

Equation (13) will be the firm's surplus when bargaining with each worker.

1.4 Individual Wage Bargaining

In this section I describe the wage bargaining. The key assumption of the individual bargaining framework is that firms cannot commit to long-term employment contracts, and may renegotiate wages with each worker at any time, making each worker effectively the marginal worker.⁴ Hence, the firm's outside option is not remaining idle, but rather producing with one worker less, so that firm's surplus is the marginal value of a worker. Also, individual bargaining involves bargaining over wages only, since an individual worker can only deprive the firm of her own marginal product, which does not give the worker sufficient leverage to negotiate hiring.

Individual bargaining is the appropriate bargaining setup when studying the business cycle properties of the US economy for two further reasons. First, "employment at will" is dominant in US labor markets, which are hence better characterized by individual than by collective bargaining. Currently, less than 10% of private sector workers are covered by a collective bargaining agreement, according to CPS data reported in Hirsch and Macpherson (2003). Second, on theoretical grounds, individual bargaining is the natural extension of the Mortensen-Pissarides framework to multi-worker firms, because it ensures that Nash-bargaining over wages is fully microfounded. In particular, Stole and Zwiebel (1996) show that individual bargaining may be understood as a Binmore-Rubinstein-Wolinsky (1986) alternating offer game. Hence the wage curve (18) can be obtained either by fully modeling the pairwise bargaining structure, or by solving a standard generalized Nash bargaining problem.

⁴The individual bargaining framework was introduced by Stole and Zwiebel (1996, 1996a). It has previously been applied to settings with decreasing returns to scale by Smith (1999), to multiple worker types by Cahuc et. al. (2004) and to settings with monopolistic competition in goods markets by Ebell and Haefke (2004, 2005).

The individual Nash bargaining problem maximizes the weighted sum of log surpluses

$$\max_{w_t} \mu \ln V_t^W + (1 - \mu) \ln \frac{\partial V^J(z_t, h_t)}{\partial h_t}$$

subject to firm surplus (13) and worker's surplus (9).⁵ Worker's bargaining power is given by μ . The solution takes the form of a first-order linear differential equation in the wage:

$$\begin{aligned} w_t = & \mu \left[(1 - \alpha) A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha - h_t \frac{\partial w}{\partial h_t} + (1 - \chi) \frac{\Phi_V}{q_t} \right] \\ & + (1 - \mu) \left[b - (1 - \chi - f_t) E_t \left\{ \tilde{\beta}_{t+1} V_{t+1}^W \right\} \right] \end{aligned} \quad (14)$$

The differential equation (14) is solved in the appendix. Its solution is:

$$\begin{aligned} w(h_t, k_{t-1}, z_t) = & \mu \left[\frac{1 - \alpha}{1 - \alpha\beta} A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha + (1 - \chi) \frac{\Phi_V}{q_t} \right] \\ & + (1 - \mu) \left[b - (1 - \chi - f_t) E_t \left\{ \tilde{\beta}_{t+1} V_{t+1}^W \right\} \right] \end{aligned} \quad (15)$$

1.5 Firm's Problem Redux: The hold-up problem

As posited earlier, the bargained wage does turn out to depend upon the firm's input choices. Differentiating (15) and substituting into (10) and (12) yields optimality conditions for the firm's choices of labor and capital.

$$\underbrace{r_t}_{\text{rental cost of capital}} = \underbrace{\left[\frac{1 - \mu}{1 - \alpha\mu} \right]}_{<1} \underbrace{\alpha A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^{\alpha-1}}_{\text{marginal product of capital}} \quad (16)$$

$$\underbrace{w_t + \frac{1}{\beta_t} \frac{\Phi_V}{q_{t-1}} - (1 - \chi) \frac{\Phi_V}{q_t}}_{\text{employment costs}} = \underbrace{\frac{1}{1 - \alpha\mu} (1 - \alpha) A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha}_{>1 \text{ marginal product of labor}} \quad (17)$$

The firm's optimality conditions show that both the firm's choice of capital and of labor inputs are distorted under individual bargaining. First, focus on the firm's choice of labor inputs. Firms find it optimal to hire workers beyond the point at which their employment costs are covered by their marginal product. The reason is that $\alpha > 0$ and constant returns to scale imply that the marginal product of labor is decreasing. Since worker's wages include a fraction μ of their marginal product, this implies that increasing labor input h_t depresses the wage. Because each worker is treated as the marginal worker, increasing

⁵Due to the large family assumption, there is no feedback from the bargained wage to current and future marginal utilities of consumption. Also, we can take V_{t+1}^W as constant, as it will turn out to depend only on aggregate labor market tightness.

h_t decreases the wage for all workers, which Stole and Zwiebel (1996) term a 'hiring externality'.

In contrast, the firm's optimal choice of capital is *lower* than its efficient level, due to the presence of a hold-up effect.⁶ To see this note that firms optimally choose a capital stock which is lower than that that would equate the marginal product of capital to its rental cost. The reason is that capital is predetermined at the time of bargaining, having been installed one period previously as is standard in RBC models. This implies that workers are able to obtain a fraction of the rents to the capital through the wage bargaining, decreasing the returns to capital obtained by the firm. This causes firms to employ less capital than would be efficient.⁷

The combination of underaccumulation of capital and overhiring of workers leads to an inefficiently low capital-labor ratio, given by:

$$\frac{k_{t-1}}{h_t} = (1 - \mu) \underbrace{\frac{\alpha}{1 - \alpha} \frac{w_t + \frac{1}{\beta_t} \frac{\Phi_V}{q_{t-1}} - (1 - \chi) \frac{\Phi_V}{q_t}}{r_t}}_{\text{efficient capital-labor ratio}}$$

Hence, the higher is worker's bargaining power, the greater is the deviation of the capital-labor ratio from its efficient level.

Finally, use (9) in conjunction with (15) and (17) to obtain the wage curve:

$$w(h_t, k_{t-1}, z_t) = \mu \left[\frac{1 - \alpha}{1 - \alpha\mu} A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha + \Phi_V \theta_t \right] + (1 - \mu) b \quad (18)$$

1.6 Free Entry

Under free entry, the net value of starting a firm is zero. A firm founded at date t has no workers and hence does not produce in its initial period. It does post enough vacancies $v_{0,t}$ to hire the desired workforce at $t + 1$. The firm's problem in its initial period is:

$$V^J(0, z_t) = \max_{v_{0,t}} \left[-\Phi_V v_{0,t} + E_t \left\{ \tilde{\beta}_{t+1} V^J(z_{t+1}, h_{t+1}) \right\} \right]$$

subject to

$$\text{initial hiring: } h_{t+1} = q_t v_{0,t}$$

This leads to the following first order condition for initial vacancies:

$$\frac{\Phi_V}{q_t} = E_t \left\{ \tilde{\beta}_{t+1} \frac{\partial V^J(z_{t+1}, h_{t+1})}{\partial h_{t+1}} \right\}$$

which is identical to the first order condition for vacancies at later dates given by equation (11). Since the net profits of all firms are zero, aggregate profits are also zero.

⁶This hold-up effect is also described in Cahuc et. al. (2004). These authors, however, concentrate exclusively on the steady state and do not examine the cyclical implications of the inefficient capital and hiring decisions.

⁷Note that the firm chooses efficient labor and capital inputs when $\mu = 0$ so that worker's bargaining power is zero or when $\alpha = 0$, so that no capital is used in production.

2 Equilibrium

An equilibrium is defined as sequences of prices and labor market tightnesses which solve the firm's, the household's and the bargaining problem and which let markets clear. The equilibrium is a tuple (w_t, r_t, θ_t) which satisfies the household's Euler equation (5) and the budget constraint (6), the firm's optimality conditions (16) and (17), the wage curve (18), free entry $\Pi_t = \pi_t = 0$, the transition equation for aggregate unemployment (7), and in which all markets clear.

This definition of equilibrium yields a system of eight equations in the eight unknowns $(y_t, \theta_t, h_t, k_{t-1}, w_t, R_t, c_t, z_t)$. All equilibrium equations are listed in the appendix.

3 Calibration

The period length is one month. There are nine parameters to pin down: vacancy costs Φ_V , worker's bargaining power μ , the output elasticity of capital α , the flow value of unemployment b , the depreciation rate δ , the match destruction rate χ , the technology parameter A , the matching elasticity η and the matching scale parameter s . I follow Shimer (2005) in setting the exogenous separation rate χ to 0.034 and the job-finding rate f_t to 0.45, both based on their values in the data. I set the match elasticity of unemployment to be 0.5, as is standard in the literature and in line with the estimates in the range (0.5,0.7) reported in Petrongolo and Pissarides (2003).⁸ The job finding rate q_t is set so that vacancy duration is 1.5 months, as reported in Ridder and van Ours (1991). The choices of vacancy duration and job-finding rate lead to a value for steady-state labor market tightness $\theta = \frac{f_t}{q_t} = 0.68$ and hence pin down the scale parameter of the matching function as $s = \frac{f_t}{\theta^{1-\eta}} = 0.55$.

The steady-state capital share $\bar{\alpha}$ is determined by the steady-state versions of (16) and (17) as:

$$\text{capital share} = \frac{(1 - \mu) \alpha}{(1 - \mu) \alpha + 1 - \alpha}$$

There is a continuum of pairs (α, μ) which lead to the target capital share of 0.36, which satisfy $0 \leq \mu \leq 1$ and $0.36 \leq \alpha \leq 1$. I examine two alternative calibration strategies for pinning down the (α, μ) pair. First, note that the ratio of TFP to labor productivity growth along a balanced growth path must equal $1 - \alpha$. Wolff (1991) finds this ratio to be 0.59 in post-war data (and 0.60 in a longer data set covering 1880-1979), pinning down α at 0.41. This pins down the worker's bargaining power at $\mu = 0.19$, the value necessary to obtain a capital share of 0.36.

Alternatively, one can use the capital-labor ratio to pin down α and μ . Again, note that the ratio of TFP to capital-labor ratio growth along a balanced growth

⁸I also examine calibrations with alternative values of matching elasticity of $\eta = 0.72$ and $\eta = 0.19$.

path is $1 - \alpha$. Wolff (1991) reports average post-war TFP growth of 1.36% and average capital-labor growth rate of 2.44% over the same period, corresponding to a value of $\alpha = 0.44$ and hence μ must be 0.28 to achieve the targeted capital income share. Using the average growth rates of TFP and capital-labor ratio over the period 1880-1979 yields an α of 0.40 and a worker's bargaining power of $\mu = 0.16$. Hence, both calibration strategies yield relatively low values of worker's bargaining power, which are intermediate between those commonly used in the literature (between 0.40 and 0.72), and the very low values used by Hagedorn and Manovskii (2005), who set $\mu = 0.06$.⁹

The remaining parameter values are standard. The monthly depreciation rate of $\delta = 0.007$ corresponds to a quarterly rate of 0.022. The monthly discount factor β is chosen as 0.997 to yield a steady-state return to capital of 4.0 % annually. The technology parameter is normalized so that steady-state output is unitary.

The final free parameter is b , the flow value of unemployment. Since this parameter has been the source of some amount of controversy, I examine the business cycle properties of the individual bargaining model for two values of $b \in \{0.40, 0.64\}$, corresponding to replacement rates of 0.65 and 0.94 respectively.

4 Results

I run five experiments, summarized in Table 1. Experiment I replicates as closely as possible Shimer (2005)'s calibration, choosing $\eta = \mu = 0.72$ and $b = 0.40$. Experiment II sets worker's bargaining power μ to 0.19, as described above, while allowing matching elasticity η to take its standard value of 0.50, while keeping b at its standard value of 0.40. Experiment III repeats Experiment II but with the high value of b . Experiments IV and V set μ to 0.19, and impose the Hosios condition to obtain matching elasticity $\eta = 0.19$.

Table 2 gives the volatilities of key variables as fractions of the volatility of the technology shock. The results of experiment I are (not surprisingly) similar to those of Shimer (2005): unemployment, vacancies and tightness are not volatile enough, while wages are too volatile. In addition, we see that the calibrated model of experiment I generates consumption and investment paths that are considerably less volatile than the data. Experiment II reduces the bargaining power of workers to $\mu = 0.19$ and match elasticity to $\eta = 0.50$. This actually worsens the performance of the model along all dimensions except wage volatility, which decreases to be more closely aligned with the data.

Experiment III replicates the calibration strategy of Hagedorn and Manovskii (2005). In a model with capital, the ability of a high replacement rate to match the volatilities of vacancies, unemployment and tightness is dampened quite substantially. More worryingly, although low values of bargaining power μ in conjunction with high values of b do improve the performance of the model with

⁹ Also note that in our setting, μ is the percent deviation of the steady-state capital-labor ratio from its efficient level. Very high values for μ would imply very large deviations from efficient capital-labor ratios.

respect to the volatilities of the labor market variables w , u , v and θ , they actually worsen the performance of the model in matching the volatilities of the real variables consumption and investment.

Experiment IV and V repeat experiments II and III when the value of the matching elasticity η is chosen at 0.19 to satisfy the Hosios condition. Overall, experiment V performs best in accounting for the behavior of both the labor market and the real variables. However, experiment V relies on matching elasticities that are considerably lower than those measured in the data.

In progress: elastic labor supply, non-linear vacancy-posting costs.

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5 Appendix

5.1 Solving the Differential Equation for the Bargained Wage

Begin by noting that one can disregard the constant terms (those terms which do not depend on h_t) when solving (14), and simply add them back in later. Hence, one is looking for a solution to:

$$w_t = \mu \left[(1 - \alpha) A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha - h_t \frac{\partial w}{\partial h_t} \right] \quad (19)$$

Rearranging slightly gives:

$$\frac{w_t}{\mu h_t} + \frac{\partial w_t}{\partial h_t} - (1 - \alpha) e^{z_t} A k_{t-1}^\alpha h_t^{-\alpha-1} = 0 \quad (20)$$

Next, write down the homogeneous version:

$$\frac{w(h_t)}{\mu h_t} + \frac{\partial w_t}{\partial h_t} = 0 \quad (21)$$

which has well-known solution

$$w_t = K h_t^{-\frac{1}{\mu}} \quad (22)$$

Take the derivative, using the fact that K might depend upon h_t :

$$\frac{\partial w_t}{\partial h_t} = -K \frac{1}{\mu} h_t^{-\frac{1}{\mu}-1} + h_t^{-\frac{1}{\mu}} \frac{\partial K}{\partial h_t} \quad (23)$$

Substitute (22) and (23) back into (20):

$$\frac{\partial K}{\partial h} = (1 - \alpha) e^{z_t} A k_{t-1}^\alpha h_t^{-\alpha-1+\frac{1}{\mu}} \quad (24)$$

Taking integral over both sides yields

$$K = \frac{(1 - \alpha) e^{z_t} A k_{t-1}^\alpha h_t^{-\alpha+\frac{1}{\mu}}}{-\alpha + \frac{1}{\mu}} + J \quad (25)$$

where J is a constant of integration. Now substitute (25) into (22) to obtain:

$$\begin{aligned} w_t &= \left[\frac{(1 - \alpha) e^{z_t} A k_{t-1}^\alpha h_t^{-\alpha+\frac{1}{\mu}}}{-\alpha + \frac{1}{\mu}} + J \right] h_t^{-\frac{1}{\mu}} \\ w_t &= \frac{\mu(1 - \alpha)}{1 - \alpha\mu} A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^\alpha + J h_t^{-\frac{1}{\mu}} \end{aligned}$$

Finally, need to pin down J using a terminal condition. Following Cahuc, et. al. (2004), choose the condition that $\lim_{h_t \rightarrow 0} h_t w_t = 0$, that is, the firm-level wage should not explode as firm-level employment h_t approaches zero. This implies that $J = 0$. Adding back in the constant term yields (15).

5.2 Equilibrium System of Difference Equations

$$c_t^{-\gamma} = \beta E_t \{c_{t+1}^{-\gamma} R_{t+1}\} \quad (26)$$

$$c_t = h_t w_t + (r_t + 1 - \delta) k_{t-1} - k_t \quad (27)$$

$$y_t = A e^{z_t} k_{t-1}^\alpha h_t^{1-\alpha} \quad (28)$$

$$r_t = \frac{1 - \mu}{1 - \alpha\mu} \alpha A e^{z_t} \left(\frac{k_{t-1}}{h_t} \right)^{\alpha-1} \quad (29)$$

$$\frac{\Phi_V}{s\theta_t^{-\eta}} = E_t \left\{ \beta \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \left[\frac{1 - \alpha}{1 - \alpha\mu} A e^{z_{t+1}} k_t^\alpha h_{t+1}^{-\alpha} + (1 - \chi) \frac{\Phi_V}{s\theta_{t+1}^{-\eta}} - w_{t+1} \right] \right\} \quad (30)$$

$$w_t = \mu \frac{1 - \alpha}{1 - \alpha\mu} \frac{y_t}{h_t} + \mu \Phi_V \theta_t + (1 - \mu) b \quad (31)$$

$$h_t = (1 - h_{t-1}) s \theta_{t-1}^{1-\eta} + h_{t-1} (1 - \chi) \quad (32)$$

$$z_t = \rho z_{t-1} + \varepsilon_{t+1} \quad \text{where } E(\varepsilon_{t+1}) = 0 \quad (33)$$

5.3 Steady-state

- Rental rate of capital

$$\bar{r} = \frac{1}{\beta} - (1 - \delta)$$

- Labor market tightness

$$(1 - \alpha) \left(\frac{A}{1 - \alpha\mu} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{r}} (1 - \mu) \right)^{\frac{\alpha}{1-\alpha}} = b + \frac{\mu}{1 - \mu} \Phi_V \bar{\theta} + \frac{\tilde{r} + \chi}{1 - \mu} \frac{\Phi_V}{\bar{q}} \quad (34)$$

- Job finding and filling rates

$$\begin{aligned} \bar{q} &= s \bar{\theta}^{-\eta} \\ \bar{f} &= s \bar{\theta}^{1-\eta} \end{aligned}$$

- Unemployment rate

$$\bar{u} = \frac{\chi}{\chi + \bar{f}}$$

- Employment rate

$$\bar{h} = 1 - \bar{u}$$

- Capital-labor ratio

$$\frac{\bar{k}}{\bar{h}} = \left(\frac{A\alpha}{\bar{r}} \frac{1 - \mu}{1 - \alpha\mu} \right)^{\frac{1}{1-\alpha}}$$

- Capital stock

$$\bar{k} = \frac{\bar{k}}{\bar{h}}$$

- Investment

$$\bar{i} = \delta \bar{k}$$

- Wage

$$\bar{w} = b + \frac{\mu}{1-\mu} \Phi_V \bar{\theta} + \frac{\mu}{1-\mu} \frac{\Phi_V}{\bar{q}} (\bar{r} + \chi)$$

- Consumption

$$\bar{c} = \bar{w} \bar{h} + (r - \delta) \bar{k}$$

5.4 Log-linearized System of Difference Equations

Define $R_t = r_t + 1 - \delta$:

$$E_t \left\{ \gamma \hat{c}_t - \gamma \hat{c}_{t+1} + \hat{R}_{t+1} \right\} = 0 \quad (35)$$

$$-\bar{c} \hat{c}_t + \bar{h} \bar{w} (\hat{h}_t + \hat{w}_t) + \bar{R} \bar{k} (\hat{R}_t + \hat{k}_{t-1}) - \bar{k} \hat{k}_t = 0 \quad (36)$$

$$\hat{y}_t - z_t - \alpha \hat{k}_{t-1} - (1 - \alpha) \hat{h}_t = 0 \quad (37)$$

$$-\bar{R} \hat{R}_t + \frac{1-\mu}{1-\alpha\mu} \alpha \frac{\bar{y}}{\bar{k}} (\hat{y}_t - \hat{k}_{t-1}) = 0 \quad (38)$$

$$E_t \left\{ \begin{aligned} & -\bar{w} \hat{w}_{t+1} + \frac{1-\alpha}{1-\alpha\mu} \frac{\bar{y}}{\bar{h}} (z_{t+1} + \alpha \hat{k}_t - \alpha \hat{h}_{t+1}) \\ & + (1-\chi) \frac{\Phi_V}{\bar{q}} \eta \hat{\theta}_{t+1} - \frac{\Phi_V}{\beta \bar{q}} (\gamma \hat{c}_{t+1} - \gamma \hat{c}_t + \eta \hat{\theta}_t) \end{aligned} \right\} = 0 \quad (39)$$

$$-\bar{w} \hat{w}_t + \mu \frac{1-\alpha}{1-\alpha\mu} \frac{\bar{y}}{\bar{h}} (\hat{y}_t - \hat{h}_t) + \mu \Phi_V \bar{\theta} \hat{\theta}_t = 0 \quad (40)$$

$$-\bar{h} \hat{h}_t + (1 - \bar{h}) (1 - \eta) s \bar{\theta}^{1-\eta} \hat{\theta}_{t-1} + (1 - \chi - s \bar{\theta}^{1-\eta}) \bar{h} \hat{h}_{t-1} \quad (41)$$

$$z_t = \rho z_{t-1} + \varepsilon_{t+1} \quad (42)$$

6 Tables

Table I: Parameter Values for Experiments

	I	II	III	IV	V
μ	0.72	0.19	0.19	0.19	0.19
b	0.40	0.40	0.64	0.40	0.64
η	0.72	0.50	0.50	0.19	0.19

Table II: Relative Volatilities

	I	II	III	IV	V	data
θ	2.0	1.8	8.9	2.4	10.6	19.1
v	1.7	1.3	6.4	1.5	6.8	9.5
u	0.5	0.8	3.7	1.6	7.1	10.1
w	0.9	0.7	0.6	0.8	0.8	0.4
c	0.2	0.1	0.1	0.1	0.2	0.7
i	3.6	3.0	3.4	3.4	4.2	4.8

Values in the table are the ratio of the volatility of variable x to the volatility of the technology shock. Data values for θ , v and u are taken from Shimer (2005), while data values for w , c and i come from Cooley and Prescott (1995). Note that the real data gives volatilities relative to output volatility.