Endogenous Productivity and Development Accounting[1]

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Abstract

We model an environment in which different vintages of capital with their different productivities coexist. A reduction in the cost of investment induces investment in new capital which raises both measured capital and measured productivity simultaneously. We calibrate this model to cross-country data on the price of investment goods and compare the resultant world distribution of per capita income with the actual distribution in the data. We find that the model does fairly well in quantitatively accounting for the observed dispersion in world income. In particular, the model generates 35-fold income gaps and 6-fold productivity differences between the richest and poorest countries in our sample.

1 Introduction

Cross-country data reveals that the per capita incomes of the richest countries in the world exceed those in the poorest countries by a factor of 35. In this paper we model an environment where new capital is more productive than older vintages of capital. In an environment where different vintages of capital coexist, a lower relative price of investment induces a higher steady state capital stock as well as a higher level of average productivity. We calibrate the model to cross-country data on the relative price of investment goods. The model can generate almost as much variation in cross-country relative income as is observed in the data. Moreover, the model generates 35-fold income gaps along with 6-fold productivity differences between the richest and poorest countries in our sample.

There is by now a large literature which examines the sources of differences in incomes across countries. There are two basic views that exist. One school of thought holds that most of the differences in incomes across nations are due to differences in productivity across nations. The most well known expressions of this view are Hall and Jones (1999) and Parente and Prescott (1994, 1999). A second view holds that differences in measured inputs can account for a significant component of the differences in incomes (e.g., see Chari, Kehoe and McGrattan (1997), Mankiw, Romer and Weil (1992), Kumar and Russell (2002), Young (1995)). In related work Klenow and Rodriguez (1998) attempt a systematic and careful decomposition of the data and conclude that productivity differences account for upwards of 60% of the income dispersion across nations with measured inputs accounting for the balance.

To put these conclusions in perspective,consider the standard one-sector neoclassical growth model with a production function $Y = AK^{\alpha}L^{1-\alpha}$ where K is capital, L is labor and A is a measure of total factor productivity (TFP). Letting $y = Y/L$ it is well known that this production function can be rewritten as

$$y = A^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}$$

According to Hall and Jones (1999), K/Y in the richest countries is about 3.6 times K/Y in the poorest countries. With a capital share of 1/3, this implies that A in the richest countries must be about 7 time the A in the poorest countries in order to explain income gaps of 35. Hence,explaining the source of potentially large prodctivity differences between the richest and poorest countries appears to be key to explaining the large income gaps observed in the data. This paper is an attempt to endogeneize productivity differences across countries.

The key starting point for our work is the well documented relationship between the relative price of investment and relative per capita income: poorer countries are also the countries where the price of capital goods (relative to the price of consumption goods) is higher (see, among others, Jones (1994), and Hall and Jones (1999)). This fact, combined with the documented importance of productivity differences across countries, suggests that the standard view of investment prices impacting income through their effect on capital accumulation (or more generally, measured inputs) can at best be a partial explanation for the observed income disparity across countries. The primary goal of our work is to formalize an environment wherein the price of capital affects the productivity of an economy over and above its standard effect on measured capital.

The main idea behind our work is that productivity and measured inputs are often determined jointly and they respond to the same set of economic decisions and incentives. In order to highlight this, we write down a growth model with embodied capital. We use a very simplified version of Hopenhayn (1992) in which in every period, potential producers of intermediate goods face a choice between two different types of capital (or machines) that they can invest in. The two types of new capital are distinct in their productivities with the more productive variety of capital being more expensive. Once a machine is in place its productivity remains constant over time. Hence, at any given time, the overall productivity of the economy reflects the mix of old and new capital as well as the mix of the two types of new capital. Changes in the relative price of new capital induce changes in not only the stock of new capital but also the average productivity of the economy due to the changing mix of new (high productivity) and old (low productivity) capital.

The model generates a steady-state distribution of relative incomes across countries as a function of, amongst other factors, the relative price of new capital. Using the observed relative price of capital from the PWT dataset, we generate a model-specific cross-country income distribution and compare its properties with the actual distribution in the data. We find that the model induces a cross-country distribution in which the per capita income of the richest 5% exceeds that of the poorest 5% of our sample of countries by a factor of 35 which is almost the same as that in the data. Moreover, the model can almost replicate the income dispersion in the data (as measured by the variance of relative per capita output). Lastly,the model generates a 6-fold productivity difference between the richest and poorest countries in our sample. Based on these results, we consider the model to be a qualified success.

Since we calibrate the model using the relative price of investment goods series from the PWT dataset, one key observation is in order before we proceed. In a recent paper, Hsieh and Klenow (2003) have argued that most of the observed variation in the relative price of investment goods in the PWT dataset is due to variations in the price of consumption across countries rather than variations in the price of investment goods. They interpret this result as suggesting that explanations of the world income dispersion that hinge on investment distortions in the form of import tariffs, taxes etc., are unlikely to be true. Instead, they argue the challenge is to explain the reasons for the low productivity of the investment goods sector in the poorer countries. Our model does not take a stand on whether the dispersion in the relative price of investment goods across countries is due to taxes or due to technology. All that is required for our results to go through is that there be observed variation in the cost of investment when expressed in terms of the domestic consumption good.

Two papers that closely relate to our work are Eaton and Kortum (2002) and Pessoa and Rob (2002). Eaton and Kortum develop a model with trade in capital goods. Their model predicts capital goods imports as a function of import prices of capital goods as well as other frictions to trade. They then use data on capital goods imports to derive a model implied series for the price of capital goods. Using this generated price series they show that the model can explain 25 percent of the cross-country variation in per capita income. The main difference of Eaton and Kortum's work from our's is that they do not focus on the cross-country differences in total factor productivity. While they allow productivity differences in the production technology for capital goods, these differences map into the price of capital goods, not the quality of the capital goods themselves. Thus, in their model a capital good which is cheaper to produce will be used more. However, the output produced by a given combination of that capital good and other factors will remain unaffected.

Pessoa and Rob (2002) have a motivation which is very similar to us. They write down a model of vintage capital with embodied technology and use it show that given variations in investment distortions across countries create larger income differences than in the standard model. However, their model has a much richer but more complicated structure than our's. They choose a production function from a class of CES functions by estimating the parameters of the function. Their model allows firms to destroy old technology, adopt new technology, and to choose the quantity of the new capital to buy. This richness of structure comes at a significant cost of tractability and simplicity. Our model, while missing these features, provides a much simpler environment to solve and quantify.[1,2]

The rest of the paper is organized as follows: In the next section we lay out the model while Section 3 characterizes the steady state of the model. Section 4 presents a special cases of the

[1] Moreover, the simplicity of our model implies that it is easy to deal with issues like transition dynamics and time series implications, most of which can be determined on pencil and pad.

[2] Our work is also related to Parente (1995) who develops a model of technology adoption. The key difference is that our framework formalizes environments with embodied technology while his work focuses on disembodied technology.

model in which there is only one type of new capital good. In Section 5 we calibrate the model and while the last section concludes.

2 Model

We consider a closed economy model. Time is discrete $t = 0, 1, ...$ The environment is characterized by perfect foresight: all agents know past, present, and future realizations of exogenous variables with probability one. At any time t, the economy is inhabited by L_t identical households who consume a final good and supply labor inelastically. We let the final good be the numeraire good so that all prices are in terms of the final good.

The final good is produced by a perfectly competitive representative firm by combining a list of differentiated intermediate goods. Each intermediate good is provided by a monopolistically competitive firm. Intermediate goods are produced by combining a machine with labor input. Entering intermediate good firms have then two options. They can either invest in the state of the art machine which embodies the frontier technology available; else they can invest in a machine whose productivity is the average productivity of the economy. The machine with the frontier technology comes at a higher cost than the machine with the average technology. Once a machine is bought/installed, its productivity remains fixed for the duration of the life of the machine. Lastly, productivity of the frontier technology is assumed to grow at an exogenous rate which is common to all economies of the world.

2.1 Final Goods Sector

The final good is produced by combining a set Ω_t of distinct intermediate goods according to

$$Y_t = \left[\int_{\Omega_t} \left[y_t \left(\omega \right) \right]^{\rho} d\omega \right]^{\frac{1}{\rho}}$$

where $0 < \rho < 1$.

A perfectly competitive final good firm chooses inputs $y_t(\omega)$ to maximize profits

$$\pi_t^f = Y_t - \int_{\Omega_t} p_t(\omega) y_t(\omega) d\omega$$

subject to the posted prices, $p_t(\omega)$, for each intermediate good $\omega \in \Omega_t$. The implied demand function for intermediate good ω is

$$y_t(\omega) = Y_t \left[p_t(\omega)\right]^{-\sigma}$$

where $\sigma = \frac{1}{1-\rho}$ denotes the elasticity of demand for good ω.

We find it useful to index intermediate goods by their technology as given by their labor productivity $\varphi \in \Re^+$. This turns out to be convenient as technology differences are the source of all the relevant firm heterogeneity in the model. In other words, all goods/firms ω which share the same technology φ are indeed identical.

Let $M_t(\varphi)$ be the measure of goods/firms with technology φ. We can then rewrite the final good production function as

$$Y_t = \left[\int \left[y_t(\varphi)\right]^{\rho} dM_t(\varphi) \right]^{\frac{1}{\rho}} \tag{1}$$

and the implied demand

$$y_t(\varphi) = Y_t \left[p_t(\varphi)\right]^{-\sigma}. \tag{2}$$

Since this sector is perfectly competitive, the representative final good firm must be making zero profits. Hence, at each date we have

$$1 = \left[\int \left[p_t(\varphi)\right]^{1-\sigma} dM_t(\varphi) \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

2.2 Intermediate goods firms

Intermediate goods firms in this economy produce output using a production technology that is linear in labor. Specifically, the production function is:

$$y_t(\varphi) = \varphi l_t(\varphi)$$

where φ is the productivity of the firm and $l_t(\varphi)$ its labor demand.[3] Hence, higher productivity is labor saving in that it lowers the labor required to produce the same unit of output.

Intermediate goods firms are monopolistically competitive and maximize profits at every date t by choosing the price of their good subject to the inverse demand function (equation (2)). Profits of firm φ at date t are given by

$$\pi_t(\varphi) = p_t(\varphi)y_t(\varphi) - (1-s)w_t l_t(\varphi)$$

where w_t is the wage and s is a per unit labor wage subsidy. The intermediate firm's problem implies an optimal pricing rule given by

$$p_t(\varphi) = \frac{(1-s)\,w_t}{\rho\varphi}.$$

We set the wage subsidy such that $1-s=\rho$. This eliminates the monopoly distortion. The optimal pricing rule then reduces to

$$p_t(\varphi) = \frac{w_t}{\varphi}. \tag{4}$$

Note that the pricing rule implies that higher productivity firms will charge a lower price.

Using the optimal pricing rule (4), it is straightforward to check that

$$\pi_t(\varphi) = \frac{1}{\sigma} p_t(\varphi) y_t(\varphi)$$

so profits are a share $\frac{1}{\sigma}$ of revenues. Note that relative profits are scaled by the level technology:

$$\frac{\pi_t(\varphi)}{\pi_t(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\sigma-1}.$$

2.3 Entry and Exit of Intermediate Good Firms

At every date there is a infinite pool of entrants. An entrant into the industry needs to purchase a machine in order to produce a new intermediate good. At date t, an entrant can either buy a

[3] We describe intermediate firms by their technology for expositional convenience. But it is important to keep in mind that every firm produces a distinct good even if they share the technology level.

machine with frontier productivity φ_t or it can buy a machine with average productivity $\tilde{\varphi}_t$. In the following we shall refer to the state-of-the-art machine as the frontier machine and the other one as the average machine. At date t the frontier machine costs f_t^d units of the final good while the average machine cost f_t^i. Once a machine is in place, the firm can hire labor and produce in that period itself. We assume that the productivity of the frontier machine evolves at the exogenous rate γ :

$$\frac{\varphi_{t+1}}{\varphi_t} = \gamma \tag{5}$$

We also assume that every period there is an exogenous exit rate δ of existing intermediate goods firms. Specifically, at the end of each period a fraction of δ of the existing stock of machines being used by intermediate goods firms in that period breaks down. Hence, the firms which own those machines go out of business.

It is assumed that every intermediate good firm is owned by the representative household. Let $v_t(\varphi)$ be the present value of a intermediate good firm with productivity φ operating at date t,

$$v_t(\varphi) = \sum_{j=0}^{\infty} (1-\delta)^j q_t^j \pi_{t+j}(\varphi).$$

A new frontier firm with productivity φ_t will start up if and only if $v_t(\varphi_t) \geq f_t^d$. Similarly, a new average firm with productivity $\tilde{\varphi}_t$ will start up at date t if and only if $v_t(\tilde{\varphi}_t) \geq f_t^i$. We assume that f_t^d and f_t^i are both proportional to the size of the economy as measured by the labor force. In particular, we assume that $f_t^d = f^d L_t$ and $f_t^i = f^i L_t$ where f^d and f^i are both constant over time.[4] Free entry into the industry implies that entry shall continue until

$$v_t(\varphi_t) \leq f^d L_t \tag{6}$$

$$v_t(\tilde{\varphi}_t) \leq f^i L_t \tag{7}$$

[4]This assumption formalizes the idea that a larger economy with more labor needs machines with bigger capacity (or equivalently, it needs a larger machine). Hence, the same productivity machine costs proportionately more in an economy with a larger labor force. This assumption ensures that the model does not generate any scale effects on development.

with strict equality if there is positive entry. Essentially, the two conditions state that entry will occur through the purchase of either type of machine as long as the present discounted value of profits are just sufficient to cover the fixed cost of buying the machine.

We assume that the machines are supplied by capital good producers. As described above, there are two types of capital goods: state-of-art machines and average machines. Both are produced by many competitive firms. Capital goods firms produce by converting the final good into machines. The production technology is linear with f_t^d units of the final good producing one unit of the state-of-art machine and f_t^i units of the final good producing one unit of the average machine. Perfect competition implies that prices of the frontier machine and the average machine (in terms of the final good) are f_t^d and f_t^i respectively.

2.4 Households

At every $t \geq 0$, the representative household maximizes the present discounted value of lifetime utility

$$\sum_{t=\infty}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + q_t b_t \leq w_t + d_t + b_{t-1} + \tau_t$$

for all $t \geq 0$, where c_t is consumption of the representative household and b_t are one-period bonds contracted at date t that pay one unit of the final good next period. Bonds are sold at discount at price q_t. Wages are given by w_t, and d_t and τ_t are dividends from firms and transfers from the government respectively. Note that d_t includes dividends from all the different types of firms in the economy. We assume that the economy has L_t households at time t.

The representative household inelastically supplies one unit of labor every period. The first order condition for the household leads to the standard Euler equation which prices the bonds

$$q_t = \beta \frac{u'(c_{t+1})}{u'(c_t)}. \tag{8}$$

2.5 Government

The government is this economy is assumed to follow a balanced budget period by period so that

$$\tau_t = s w_t L_t$$

Hence, the government finances its wage subsidy to intermediate firms through lump-sum taxes on households.

2.6 Market Clearing Conditions and Equilibrium Definition

In order to close the model, we need to introduce a few definitions and state the market clearing conditions. Let M_t be the aggregate measure of firms active at date t, i.e.,

$$M_t = \int dM_t(\varphi)$$

and $m_t(\varphi) = \frac{M_t(\varphi)}{M_t}$ the share of technology φ firms. Let N_t be the measure of new firms at date t

$$N_t = M_t - (1-\delta) M_{t-1}. \tag{9}$$

We denote as $\alpha_t \in [0,1]$ the fraction of new firms which opt for the frontier technology.

We define the average technology $\tilde{\varphi}_t$ of the sector at date t as

$$\tilde{\varphi}_t = \left[\int \varphi^{\sigma-1} dm_t(\varphi)\right]^{\frac{1}{\sigma-1}}.$$

Using the newly minted notation, we derive the law of motion for $\tilde{\varphi}_t$,

$$\tilde{\varphi}_t^{\sigma-1} = (1-\delta)\frac{M_{t-1}}{M_t}\tilde{\varphi}_{t-1}^{\sigma-1} + \left[\frac{N_t}{M_t}\right]\left[\alpha_t \varphi_t^{\sigma-1} + (1-\alpha_t)\tilde{\varphi}_t^{\sigma-1}\right]. \tag{10}$$

Equation (10) says that average productivity of this economy at any date is a function of two factors. First, it depends on the average productivity of the previous period weighted by the share of surviving firms from the last period (the first term on the right hand side (RHS)). Second, average productivity also depends on the share of new start-ups in the current period and the mix of frontier machines versus average machines amongst the new machines (the second term on the RHS).

Finally, a couple of market clearing conditions. First, the labor market clearing condition requires that we have

$$\int l_t(\varphi) dM_t(\varphi) = L_t \qquad \text{for all } t.$$

Second, equilibrium in the final good market gives the resource constraint for the economy:

$$c_t + [M_t - (1-\delta) M_{t-1}] \left[\alpha_t f^d + (1-\alpha_t) f^i \right] = Y_t / L_t. \tag{11}$$

We define equilibrium as:

Definition 1 *An equilibrium is a sequence of prices* $\{\{p_t(\varphi) : \varphi \in \Re^+\}, w_t, q_t\}_{t=0}^{\infty}$*, allocations*

$$\left[c_t, b_t, Y_t, M_t, N_t, \alpha_t, \left\{y_t(\varphi), M_t(\varphi), l_t(\varphi) : \varphi \in \Re^+\right\}\right]$$

and productivity distribution μ_t *such that for all* $t \geq 0$*, the allocations solve the household problem given the bond prices; all firms maximize profits; there are zero rents from entry; labor, bonds and good markets clear; and the evolution of intermediate firms* M_t *satisfies the corresponding law of motions.*

2.7 Solving for Equilibrium

We start by noting that zero profits for final goods firms implies that

$$Y = \int p_t(\varphi) y_t(\varphi) dM_t(\varphi).$$

Substituting the production technology for intermediate goods and the optimal pricing equation (4) gives

$$Y_t = w_t L_t.$$

Next, we can solve for equilibrium wages (and hence, income per person) by substituting the optimal intermediate goods pricing equation (4) into equation (3)

$$\begin{aligned} 1 &= \left[\int \left(\frac{w_t}{\varphi}\right)^{1-\sigma} dM_t(\varphi)\right], \\ w_t^{\sigma-1} &= \left[\int \varphi^{\sigma-1} dM_t(\varphi)\right], \end{aligned}$$

and factoring out M_t,

$$w_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}}. \tag{12}$$

The output of the final good in this economy is, trivially, given by

$$Y_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}} L_t. \tag{13}$$

Now, we can use equations (4), (2) and the labor market clearing condition to rewrite revenues of intermediate goods firms as

$$p_t(\varphi)\, y_t(\varphi) = \varphi^{\sigma-1} w_t^{2-\sigma} L_t$$

Substituting this expression for revenues into the expression for intermediate firms' profits gives

$$\pi(\varphi) = (1-\rho)\, \varphi^{\sigma-1} w_t^{2-\sigma} L_t \tag{14}$$

The conditions for entry by frontier firms and average firms imply that we must have

$$\begin{aligned} \varphi_t^{\sigma-1}\left[\sum_{j=0}^{\infty} q_t^j (1-\delta)^j w_{t+j}^{2-\sigma} \gamma_L^j\right] &= \sigma f^d, \\ \tilde{\varphi}_t^{\sigma-1}\left[\sum_{j=0}^{\infty} q_t^j (1-\delta)^j w_{t+j}^{2-\sigma} \gamma_L^j\right] &= \sigma f^i, \end{aligned}$$

where γ_L is the rate of growth of labor. Note that in deriving the above we have substituted equation (14) into equations (6) and (7). Hence, in equilibrium we must have

$$k_t^{1-\sigma} = \frac{f^d}{f^i} \tag{15}$$

where $k_t = \tilde{\varphi}_t/\varphi$ is the distance of the average technology of the economy from the frontier technology. Equation (15) makes clear that this distance depends on the cost of buying the latest machine relative to the average machine, which is constant over time.

3 Steady state

We now characterize the steady state of this economy. In particular, we look for paths along which $M_t, \varphi_t, \tilde{\varphi}_t, Y_t, c_t$ grow at a constant rate, and the share of frontier machines in new investment, α_t, is constant over time. In the following we shall use γ_j to denote the constant, steady state rate of growth of variable $j = M, Y, y, L$.

We start by noting that k_t is constant at all times (under our assumption of positive entry in both types of machines). Hence, $\gamma_{\tilde{\varphi}} = \gamma$. Second, along a balanced growth path, the price of the bonds will be constant, say $\tilde{\beta}$ and satisfy $q_t^j = \tilde{\beta}^j$ for all t and j.

From the free entry condition for frontier machine firms (equation (6)), we get

$$(\varphi_t)^{\sigma-1}\,\tilde{\varphi}_t^{2-\sigma} M_t^{\frac{2-\sigma}{\sigma-1}} \sum_{j=0}^{\infty} \left((1-\delta)\,\tilde{\beta}\gamma_L \gamma^{2-\sigma} \gamma_M^{\frac{2-\sigma}{\sigma-1}} \right)^j = \sigma f^d.$$

where we have used $w_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}}$ as well as the exogenous process for labor.[5] Since φ_t and $\tilde{\varphi}_t$ grow at the same rate, along a balanced growth rate path we must have

$$\gamma_M = \gamma^{\frac{\sigma-1}{\sigma-2}}. \tag{16}$$

As long as $\sigma > 2$, the growth rate of firms rises with the productivity growth. Throughout the rest of the paper we shall restrict attention to $\rho > 1/2$ so that $\sigma > 2$. This is a restriction on the elasticity of demand for intermediate goods. Later, we shall provide some discussion regarding estimates for this parameter.[6]

[5] We assume that $\beta\,(\gamma_L\gamma)^{(1-\nu)\frac{\sigma-1}{\sigma-2}}\,\gamma_F^{\frac{(\nu-1)}{\sigma-2}} < 1$. This condition ensures that the household problem is well defined. For the entry condition, note that

$$\tilde{\beta} = \beta\,(\gamma_L\gamma)^{-\nu\frac{\sigma-1}{\sigma-2}}\,\gamma_F^{\frac{\nu}{\sigma-2}}$$

Hence, a simple restriction on δ is sufficient to ensure that the entry condition is well defined as well.

[6] It is clear that for $\sigma \in (1,2)$, the number of intermediate goods firms is a decreasing function of productivity growth γ, i.e., it leads to immiserizing growth.

Next, in steady state the aggregate productivity index given by equation (10) can be rearranged and written as

$$\alpha = \left[\frac{(1-\delta)\left(1-\gamma^{1-\sigma}\right)}{\gamma^{\frac{\sigma-1}{\sigma-2}}-(1-\delta)}\right]\left[\frac{1}{\frac{f^d}{f^i}-1}\right] \tag{17}$$

where we have used equations (15) and (16). Hence, the share of frontier firms amongst new intermediate goods firms is constant in steady state.

Lastly, in steady state the economy's resource constraint can be written as

$$\frac{c_t}{w_t} = 1 - \frac{M_t\left[\alpha f^d + (1-\alpha) f^i\right]}{w_t}$$

where we have used the fact that α is constant in steady state. Using equation (12), it is easy to check that $\frac{M_t}{w_t}$ is constant along a steady state balanced growth path. Hence,

$$\gamma_c = \gamma_w = \gamma_M = \gamma^{\frac{\sigma-1}{\sigma-2}} \tag{18}$$

3.1 Cross-country comparisons

There are three main variables of interest for our cross-country comparisons: output per capita (Y_t/L_t), capital to output ratio (M_t/Y_t), and average productivity $\tilde{\varphi}_t$. In order to proceed we need some additional relationships. In the following, we shall compare two countries by following the notational convention of denoting the second country variables with primes.

In steady state, the free entry condition for average firms (equation (7)) in the two countries can be used to get

$$\frac{\tilde{\varphi}_t}{\tilde{\varphi}_t'}\left(\frac{M_t}{M_t'}\right)^{\frac{2-\sigma}{\sigma-1}} = \frac{f^i}{f^{i\prime}}$$

Since $\tilde{\varphi}_t/\tilde{\varphi}_t' = k/k'$, equation (15) implies that

$$\frac{\tilde{\varphi}_t}{\tilde{\varphi}_t'} = \left(\frac{f^{d\prime}/f^{i\prime}}{f^d/f^i}\right)^{\frac{1}{\sigma-1}} \tag{19}$$

which shows that the productivity gap between countries depends on the difference in the relative cost of frontier to average machines across countries. The higher the relative price of frontier machines the lower is the relative productivity level of the country.

Substituting equation (19) in the free entry condition gives

$$\frac{M_t}{M_t'} = \left(\frac{f^{d\prime}}{f^d}\right)^{\frac{1}{\sigma-2}} \left(\frac{f^{i\prime}}{f^i}\right) \tag{20}$$

This expression gives the ratio of machines at any given date along a steady state growth path. The ratio of machines depends in an obvious way on the cost of investing in both old and new machines – the higher the cost of a new machine (both f^d and f^i) the lower is M/M'.

Next, recall that per capita output is given by $Y/L = w = \tilde{\varphi} M^{\frac{1}{\sigma-1}}$. Hence,

$$\frac{w}{w'} = \frac{\tilde{\varphi}}{\tilde{\varphi}'} \left(\frac{M}{M'}\right)^{\frac{1}{\sigma-1}}$$

Using equations (15) and (20), this can be rewritten as

$$\frac{w}{w'} = \left(\frac{f^{d\prime}}{f^d}\right)^{\frac{1}{\sigma-2}} \tag{21}$$

Hence, the income gap across countries depends on the relative cost of frontier machines. In particular, the higher the relative cost of the frontier machine in a country the lower is its relative per capita income.[7]

3.2 Special Case: One capital good

We now study a special case of the model. In particular, we analyze the case where entrants into the intermediate goods sector can only purchase the frontier machine rather than choose between two types of machines. This case corresponds to setting $f^d = f^i \equiv f$ in the expressions that we derived above. The only caveat is that the free entry condition for firms which choose the average technology, equation 7, does not apply in this case since there is no such option.

[7] It is instructive to note that the ratio of per capita steady state incomes can also be written as $\frac{w}{w'} = \left(\frac{\varphi}{\varphi'}\right)^{\frac{\sigma-1}{\sigma-2}} \left(\frac{M/Y}{M'/Y'}\right)^{\frac{1}{\sigma-2}} \left(\frac{L}{L'}\right)^{\frac{1}{\sigma-2}}$. This expression looks very similar to the standard expression for the income ratio under the Solow model with a Cobb-Douglas production technology. The only difference is that in our case the last two terms on the right hand side (which are measured inputs) are raised to the power $(\sigma-2)^{-1}$ while in the Solow model they are raised to a power which is the ratio of the capital share to the labor share. Hence a $\sigma = 2.5$ would generate a fit for our model analogous to the fit of the neoclassical model with a capital share of 2/3.

The steady state expressions for relative productivity, relative numbers of machines and relative income are now given by, respectively,

$$\frac{\tilde{\varphi}_t}{\tilde{\varphi}'_t} = 1 \tag{22}$$

$$\frac{M_t}{M'_t} = \left(\frac{f'}{f}\right)^{\frac{\sigma-1}{\sigma-2}} \tag{23}$$

$$\frac{w}{w'} = \left(\frac{f'}{f}\right)^{\frac{1}{\sigma-2}} \tag{24}$$

Equations (22-24) reveal that the major difference between this version of the model and the more general model above is that this model cannot generate any productivity differences across countries in steady state. Recalling that per capita income is given by $w = \tilde{\varphi} M^{\frac{1}{\sigma-1}}$, it follows that all the predicted per capita income differences across countries in this model will come out of differences in the number of machines. Hence, this version of the model resembles the standard neoclassical growth model with disembodied technology.

The intuition behind this result is that in the one capital good case, entrants make only one decision: enter or not enter. Contingent on entering, all new intermediate firms have the frontier machine. Thus, countries with higher costs of machines have fewer entrants and thereby fewer new machines at each date. However, the quality of new machines is identical across countries. Since average productivity of a country is the average quality of new machines past and present, there is no difference in steady state productivity across countries.[8] This result also highlights the fact that in order to have permanent productivity differences across countries, one needs differences in the quality of new machines.

It is instructive to note that despite the fact in this version all cross-country relative income differences come from differences in machines (or capital) rather than the quality of capital, this model will nevertheless generate much larger income gaps for the same measured variations in

[8] Note though that this is a steady state result. Even in this one capital good case, there will be productivity differences across countries along the transition path.

capital. To see this recall that per capita output in this model is given by

$$w = \tilde{\varphi} M^{\frac{1}{\sigma-1}}$$

while in the standard neoclassical model it is given by

$$w = \tilde{\varphi} K^{\alpha}.$$

Now, the share of income going to capital owners in this model is $\sigma^{-1} = 1 - \rho$ while the capital share is α in the standard model. Hence, setting $\sigma = 1/\alpha$, per capita income in our model is given by

$$w = \tilde{\varphi} M^{\frac{\alpha}{1-\alpha}}$$

Hence, the same capital share and given cross-country variation in capital will generate larger a larger variation in per capita incomes in our model relative to the standard model.

4 A Quantitative Evaluation

We now calibrate the model to evaluate its quantitative ability to generate cross-country dispersion of relative income and productivity. Since the main driver of cross-country differences in the model is the variation in the price of capital goods across countries, this exercise is a test of the relevance of this margin for our theory of endogenous productivity and development. We focus on the stable income distribution associated with the balanced growth path. We assume that countries have the same human capital endowment, $L_i = L_j$ (since we do not have a theory for population dynamics, labor force participation or human capital accumulation). A period in our model is equal to one year.

We start by noting that our primary focus is on equations (19), (20), (21) and (**??**). These expressions give cross-country steady state comparisons of four key variables: productivity, capital, income and the capital-output ratio. Inspecting these equations shows that these variables are functions of f^i and f^d (which vary across countries) and parameters (which are invariant). To quantify the model we require cross-country data on f^i and f^d. The PWT dataset contains data on the price of investment goods. However, given our model, there are different ways of interpreting this data. The reported price of investment goods data in the PWT could either be the average price of new capital goods ($F = \alpha f^d + (1-\alpha)f^i$), or the price of frontier, capital goods (f^d), or the price of imitation capital goods (f^i). We adopt the first approach. In particular, we calibrate the model by equating the price of investment goods in the data to the average cost of new capital goods and then back out the individual prices of both types of machines.

Let the average price of investment goods be given by F where

$$F = \alpha f^d + (1-\alpha)f^i.$$

This expression can be rewritten as

$$F = f^i(1+a), \tag{25}$$

where $a \equiv \frac{(1-\delta)\left(1-\gamma^{1-\sigma}\right)}{\gamma^{\frac{\sigma-1}{\sigma-2}}-(1-\delta)}$. In deriving this expression we have used equation (17) which can be rewritten as $\alpha\left(\frac{f^d-f^i}{f^i}\right) = a$. Note that under our assumptions a is invariant across countries.

Our data implementation strategy is as follows: Take F from the cross-country data on the price of investment goods; then use equation (25) to compute an f^i for each country. Next, we assume that the dollar price of new, frontier capital goods is identical across countries. Hence, we assume that

$$f^d = \frac{\bar{f}}{p_c}$$

where p_c is the price of consumption goods and $\bar{f}$ is the common price of the frontier machine. This allows us to compute the price of frontier capital goods for each country. Hence, we now have a complete description of all prices for all countries which allows us to quantify the key cross-country expressions for relative productivity, income, capital and the capital-output ratio.[9]

[9] Our strategy for identifying f^d in the data is in the spirit of Hsieh and Klenow (2004) who find that there is not

A period in our model is equal to one year. The key parameter of the model is the elasticity of demand σ. For our baseline quantification of the model we set $\sigma = 2.6$ which is the value for the elasticity of demand for intermediate goods used by Acemoglu and Ventura (2003). The other parameters have no impact on income dispersion. We should note that since the capital income share in this model is σ^{-1}, setting $\sigma = 2.6$ implies a capital share of 0.38 which is close to the numbers reported by Gollin (2002).[10]

4.1 Results

We take data from year 2000. We measure income differences by using data on output per worker. Every country's income is expressed relative to the United States. The resulting estimates for income dispersion are reported in Table 1.

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much cross-country variation in the domestic price of investment goods measured in a common currency. Rather, the variation the price of investment relative to consumption is due to the variation in the price of consumption across countries. Note that in an environment where there was a global market for state-of-the-art capital goods, the price of the frontier machine would be tied down by the world price.

[10] Our model implies that the cross-country relative income ratio is given by $w/w' = (f^d/f^{d'})^{\frac{1}{\sigma-2}}$. Using this relationship, we also ran a simple linear regression

$$\log\left(\frac{y_{it}}{y_{jt}}\right) = b\log\left(\frac{f_i^d}{f_j^d}\right) + \varepsilon$$

and then use $b = \frac{1}{2-\sigma}$. The estimate is around $\sigma = 2.5$ which is very close to our baseline parameterization.

TABLE 1. **Predicted Values: GDP per worker**

Data: Penn World Tables, *Year*: 2000, $\sigma = 2.6$

	Std Dev		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.36	104	92	1.40	1.85
5 % censored	.23	.27	28	29	1.30	1.64
10 % censored	.20	.22	23	16	1.21	1.49
20 % censored	.12	.13	9	7	1.05	1.25

The first row of numbers in Table 1 shows the results for the full sample of 163 countries in our dataset. In the data the standard deviation of relative income per worker is 0.28 while the ratio of incomes of the richest (Luxembourg) to the poorest country (Zaire) in the sample is 104. The corresponding numbers generated by our model are 0.36 and 92. The second, third and fourth rows of the table show the results after dropping the richest and the poorest 5, 10 and 20 percent of countries from the sample, respectively. As the table makes clear, the results are surprisingly strong. The model reproduces almost exactly the income gap between the highest and the lowest income countries. On the income dispersion across countries as measured by the standard deviation, if anything, the model overshoots the data a little. We view these results as being supportive of the model.

As was pointed out above, the key parameter for our model is the elasticity of substitution between intermediate goods, σ. In Table 2 we report some robustness checks on our baseline results for GDP per worker for two different values: $\sigma = 2.5$, and 3. Table 2 shows two basic features. First, the ability of the model to reproduce the cross-country income dispersion is relatively robust to alternative values of σ. Even with $\sigma = 3$, the model generates a standard deviation of income which is almost the same as in the data. Contrarily, the fit of the model with respect to the income ratio of the richest to the poorest country in the sample declines as one increases the value of σ. Thus, for the 5 % percent censored sample, with $\sigma = 3$ the predicted max/min ratio of relative incomes from the model is 7 whereas in the data it is 28. This is easy to see equation (21) which says that $\frac{w}{w'} = \left(\frac{f^{d\prime}}{f^d}\right)^{\frac{1}{\sigma-2}}$. Hence, for $\sigma = 2.5$, the estimated relative price of frontier machines across countries is being raised to the power 2 whereas for $\sigma = 3$ the same relative price is only being raised to the power 1. Thus, the predicted income ratio under $\sigma = 3$ is only going to be the square root of the corresponding ratio under $\sigma = 2.5$.

Table 2. **Robustness: GDP per worker**

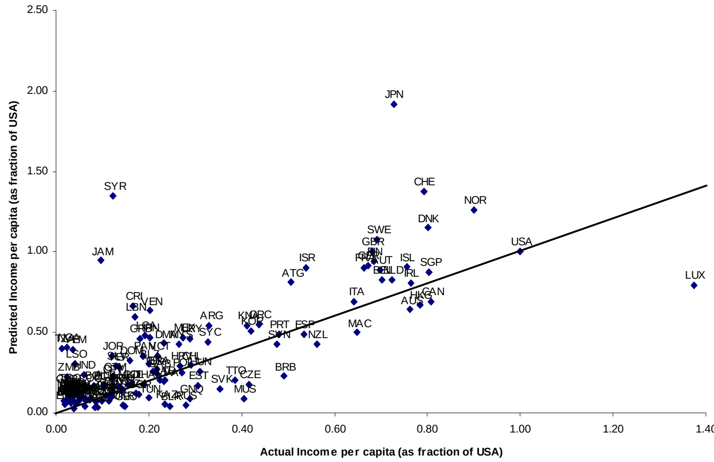
Data: Penn World Tables, *Year*: 2000

$\sigma = 2.5$	**Std Dev**		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.38	104	225	1.40	2.34
5 % censored	.23	.27	28	56	1.30	1.96
10 % censored	.20	.21	23	28	1.21	1.73
20 % censored	.12	.12	9	11	1.05	1.36

$\sigma = 3$	**Std Dev**		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.30	104	15	1.40	1.29
5 % censored	.23	.24	28	7	1.30	1.23
10 % censored	.20	.21	23	5	1.21	1.18
20 % censored	.12	.14	9	3	1.05	1.10

We study the fit of the induced world income distribution from the model in two additional ways. First, the last column of both Tables 1 and 2 report the ratio of the mean to the median of the relative income series in the data and from the model. The tables show that the fit of the model is good for almost all sub-samples for our baseline calibration as well as being robust to

Figure 1: Relative income per worker: Predicted vs data



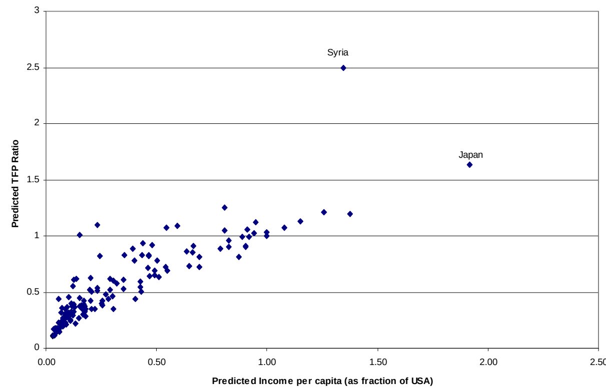
changes in the elasticity parameter σ. Essentially, the mean of the distribution is greater than the median both in the data and in the model with the magnitudes being pretty close.

The second method of evaluating the fit of the induced income distribution is to plot the relative income per person in the data against the predicted series from the model. Figure 1 shows the fit: the scatter points are pretty tightly concentrated around the 45-degree line. We conclude that the model fits the data quite well along this dimension as well.

4.2 TFP differences

An additional variable of interest to us is the predicted relationship between productivity and income. Figure 2 show the implied relationship. Predicted relative productivity is increasing with predicted relative income.

Figure 2: Predicted relative TFP vs predicted relative income per worker



Given that a key motivating factor for this paper was the 7-fold productivity difference between the richest and poorest countries implied by the standard one-sector groth model, what does our model imply about productivity differences between the richest and poorest countries of the world. As can be deduced from Figure 2, the implied productivity gap between the richest and poorest five countries in our sample is almost 6. We consider this quite promising given the higly simplified structure that we chose to work with.

5 Conclusion

In this paper we have formalized a model of embodied technology adoption which allows us to endogeneize total factor productivity (TFP). The main advantage of this approach is that it is able to generate larger cross-country income differences for the same given level of investment distortions. The primary mechansism is simple. A higher relative price of new capital goods reduces the purchases of new capital goods. This margin is the same as in the standard disembodied technology model. The larger effect on income differences comes from the fact that a smaller share of new capital goods also implies a lower quality of the average capital in the economy. This reduces average productivity and hence, per capita income. Intuitively, the mechanism of the model reduces per capita income both along the intensive margin (the number of capital goods) as well as the quality margin (the average productivity of installed capital).

Based on the measured prices of investment goods from the PWT, we find that the predicted relative income series from the model fits the data quite well. The model replicates both the cross-country variation in relative incomes as well as the income disparity between the richest and the poorest countries of our sample. We also find that the model generates productivity differences of the order of 6 between the richest and poorest five countries in our sample. We consider the quantitative results to be a qualified endorsement of the model.

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A A Vintage Capital Model

We produce a model where there is a richer menu of invesment choices and capital prices are endogenously determined. We show that the investment choices can be reduced to two, a frontier machine and an average machine, without loss of generality. In particular, the complete model features the same production function (13), output to machine ratios are proportional to the price of the average machine as in (20) and the distance to the frontier (15) is also given by the relative price of a frontier to an average machine.

The final good sector is identical. Output per capita is then given by

$$y_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}} \tag{26}$$

where $\tilde{\varphi}_t = \left[\int \varphi^{\sigma-1} dm_t^j(\varphi)\right]^{\frac{1}{\sigma-1}}$ and m_t^j is the share of firms of vintage j at date t, with correspondent technology φ_{t-j}. Hence the model trivially replicates the production function (13).

The price for the capital goods necessary to start-up a vintage j plant is given by

$$p_t^j = \tilde{A}_j \left(m_t^j\right)^{\psi} L_t \tag{27}$$

where $\psi > 0$, $\tilde{A}_j > 0$. Underlying expression (27) is a upward sloping supply curve, possibly arising from a factor in fixed supply.

The entry condition for vintage j plants at date t is

$$\varphi_{t-j}^{\sigma-1} \tilde{\varphi}_t^{2-\sigma} M_t^{\frac{2-\sigma}{\sigma-1}} = A_j \left(\frac{M_t^j}{M_t}\right)^{\psi} \tag{28}$$

where $A_j = \tilde{A}_j \sigma / \sum_{j=0}^{\infty} \left((1-\delta)\tilde{\beta}\gamma_L \gamma^{2-\sigma} \gamma_M^{\frac{2-\sigma}{\sigma-1}}\right)^j$. The strict sign reveals we are assuming positive entry of every vintage.

By the definition of $M_t = \sum_{j=0}^{\infty} M_t^j$, and therefore (28) implies

$$1 = \left(\tilde{\varphi}_t^{2-\sigma} M_t^{\frac{2-\sigma}{\sigma-1}}\right)^{\frac{1}{\psi}} \sum_{j=0}^{\infty} \left(\frac{\varphi_{t-j}^{\sigma-1}}{A_j}\right)^{\frac{1}{\psi}}$$

or

$$\begin{aligned} \frac{y_t}{M_t} &= \left[\sum_{j=0}^{\infty} \left(\frac{1}{A_j} \left(\frac{\varphi_{t-j}}{\tilde{\varphi}_t}\right)^{\sigma-1}\right)\right]^{\psi} \\ &= \left(\frac{\varphi_t}{\tilde{\varphi}_t}\right)^{(\sigma-1)\psi} \left[\sum_{j=0}^{\infty} \left(\frac{1}{A_j} \gamma^{(1-\sigma)j}\right)\right]^{\psi} \end{aligned}$$

or with the correspondent definition of the constant Γ_1.

$$\frac{y_t}{M_t} = \left(\frac{\varphi_t}{\tilde{\varphi}_t}\right)^{(\sigma-1)\psi} \Gamma_1. \tag{29}$$

It is easy to see that for two countries such that $A_j = \kappa A'_j$ for all $j \geq 0$, the ratio of machine per output would be given by κ. This is precisely what the model in the main text predicts.

Using (28) for a given vintage j, we solve for the share of vintage j plants at any date t,

$$\begin{aligned} \frac{M_t^j}{M_t} &= \left[\left(\frac{\varphi_{t-j}}{\tilde{\varphi}_t}\right)^{\sigma-1}\left(\frac{y_t}{M_t}\right)\frac{1}{A_j}\right]^{\frac{1}{\psi}} \\ &= \left(\frac{\varphi_t}{\tilde{\varphi}_t}\right)^{(\sigma-1)\frac{(1+\psi)}{\psi}} \Gamma_1^{1/\psi}\left[\gamma^{(1-\sigma)j}\frac{1}{A_j}\right]^{\frac{1}{\psi}}. \end{aligned} \tag{30}$$

We need only to solve now for the "distance to the frontier" term $\frac{\varphi_t}{\tilde{\varphi}_t}$. We take the definition of $\tilde{\varphi}_t$,

$$\begin{aligned} \left(\frac{\tilde{\varphi}_t}{\varphi_t}\right)^{\sigma-1} &= \sum_{j=0}^{\infty}\left(\frac{M_t^j}{M_t}\right)\gamma^{(1-\sigma)j} \\ &= \left(\frac{\varphi_t}{\tilde{\varphi}_t}\right)^{(\sigma-1)\frac{(1+\psi)}{\psi}} \Gamma_1^{1/\psi}\sum_{j=0}^{\infty}\gamma^{(1-\sigma)\left(\frac{1+\psi}{\psi}\right)j}A_j^{-\frac{1}{\psi}} \end{aligned}$$

or re-arranging terms,

$$\left(\frac{\tilde{\varphi}_t}{\varphi_t}\right)^{\sigma-1} = \Gamma_1^{\frac{1}{1+2\psi}}\left(\sum_{j=0}^{\infty}\gamma^{(1-\sigma)\left(\frac{1+\psi}{\psi}\right)j}A_j^{-\frac{1}{\psi}}\right)^{\frac{\psi}{1+2\psi}}. \tag{31}$$

From (31) we can solve for the y_t/M_t ratio and all shares M_t^j/M_t using (29) and (30).

To show how this relates to the model, take the ratio of the entry conditions for the latest technology and any given vintage j (28),

$$\left(\frac{\varphi_j}{\varphi_t}\right)^{\sigma-1} = \frac{p_t^j}{p_t^0}.$$

Constructing average

$$\tilde{\varphi}_t^{\sigma-1} = \sum_{j=0}^{\infty}\left(\frac{M_t^j}{M_t}\right)\left(\frac{p_t^j}{p_t^0}\right)\varphi_t^{\sigma-1}.$$

Because in the steady state the share of each vintage is constant,

$$\frac{M_t^j}{M_t} = \frac{M_{t-1}^j}{M_{t-1}}$$

all vintages grow at the same rate

$$\frac{M_t^j}{M_{t-1}^j} = \frac{M_t}{M_{t-1}} = \gamma^M.$$

Hence

$$\begin{aligned}\frac{N_t^j}{N_t} &= \frac{M_t^j - (1-\delta)\, M_{t-1}^j}{M_t - (1-\delta)\, M_{t-1}} \\ &= \frac{1-(1-\delta)\,\gamma_M^{-1}}{1-(1-\delta)\,\gamma_M^{-1}} \frac{M_t^j}{M_t}\end{aligned}$$

so

$$\left(\frac{\tilde{\varphi}_t}{\varphi_t}\right)^{\sigma-1} = \frac{\sum_{j=0}^{\infty}\left(\frac{N_t^j}{N_t}\right) p_t^j}{p_0}.$$

Hence, the endogenous level of productivity can be expressed as the function of the ratio of the price of an average machine and the frontier machine as in (15). Note that the price of the average machine is constructed using the investment shares which is consistent with our identification strategy.