A Habit-Based Explanation of the Exchange Rate Risk Premium

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ABSTRACT

This paper presents a fully rational general equilibrium model that produces a time-varying exchange rate risk premium and solves the uncovered interest rate parity (U.I.P) puzzle. In this two-country model, agents are characterized by slow-moving external habit preferences similar to Campbell & Cochrane (1999). Endowment shocks are *i.i.d* and real risk-free rates are time-varying. Agents can trade across countries, but when a unit is shipped, only a fraction of the good arrives to the foreign shore. The model gives a rationale for the U.I.P puzzle: the domestic investor receives a positive exchange rate risk premium when she is effectively more risk-averse than her foreign counterpart. Times of high risk-aversion correspond to low interest rates. Thus, the domestic investor receives a positive risk premium when interest rates are lower at home than abroad. The model is both simulated and estimated. The simulation recovers the usual negative coefficient between exchange rate variations and interest rate differentials. When the iceberg-like trade cost is

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taken into account, the exchange rate variance produced is in line with its empirical counterpart. A nonlinear estimation of the model using consumption data leads to reasonable parameters when pricing the foreign excess returns of an American investor.

Keywords: Exchange rate, Time-varying risk premium, Habits

JEL: F31, G12, G15.

According to the standard uncovered interest rate parity (U.I.P) condition, the expected change in exchange rate should be equal to the interest rate differential between foreign and domestic securities. Thus, borrowing funds where interest rates are low and lending where interest rates are higher should not create excess return. Assuming rational expectations, a simple regression of exchange rate variations on interest rate differentials should lead to a regression coefficient of 1. Instead, empirical work following Hansen & Hodrick (1980) and Fama (1984) consistently reveals a coefficient smaller than 1 and very often negative.[1] Froot & Thaler (1990) report that, in a survey of 75 published estimates, the slope coefficient of the regression of the nominal exchange rate appreciation on nominal interest rates is always below unity (positive in a very few cases, and -0.88 on average). The international economics literature refers to negative U.I.P slope coefficients as U.I.P puzzles or forward premium anomalies.

A U.I.P slope coefficient below 1 implies nonzero predictable excess returns for an investor borrowing funds at home, changing her currency for a foreign equivalent, lending on the corresponding foreign market for a fixed period and finally reconverting her earnings to the original currency.[2] There are two possible explanations for predictable excess returns: time-varying risk premia and/or expectational errors. Gourinchas & Tornell (2004) generate the characteristics of the U.I.P coefficient, but they depart from full rationality by assuming that agents systematically under-react to interest rate shocks. In this paper, I assume that expectations are rational. As a consequence, the currency excess return is a risk premium due to the unknown value of the exchange rate at the end of the foreign investment's period.

Most papers test the U.I.P condition on nominal variables. Yet, two recent studies relate the puzzle to real variables.[3] First, Hollifield & Yaron (2003) decompose the cur-

[1]The U.I.P condition appears to be a reasonable assumption only in three cases. Bansal & Dahlquist (2000) show that the U.I.P condition is not rejected at high inflation levels. Chaboud & Wright (2005) show that U.I.P is valid at very short horizons but is rejected for horizons above a few hours. Meredith & Chinn (2005) find that U.I.P cannot be rejected at horizons above 5 years. See Engel (1996) and Chinn (2006) for surveys.

[2]Predictability regressions are plagued with small sample bias and persistence in the right hand side variables. But Liu & Maynard (2005) and Maynard (2006) show that these biases can only explain part of the puzzle.

[3]Moreover, Verdelhan (2004) finds negative slope coefficients on regressions of real exchange rate changes on real interest rates differentials, with or without a constant term in the regressions.

rency risk premium into conditional inflation risk, real risk, and the interaction between inflation and real risk. They find evidence that real factors, not nominal ones, drive most of the predictable variation in currency risk premia.[4] Second, Lustig & Verdelhan (2005) show that the risk premia produced by asset pricing factors based on real consumption growth risk line up with predictable real excess returns in currency markets. But currency excess returns are the same - up to a first-order Taylor approximation - whether computed with real or nominal variables, because the inflation differential present in the change in the real exchange rate cancel out with the one present in the real interest rate differential.[5] Thus, theoretical explanations to the U.I.P puzzle may only rely on real variables. In this paper, I abstract from money and inflation and present a fully rational general equilibrium model that produces a time-varying exchange rate risk premium and solves the U.I.P puzzle.

Backus, Foresi, & Telmer (2001) describe the necessary features of a fully rational model that account for the forward premium anomaly: a negative correlation between the difference in interest rates and the difference in conditional variances of the two pricing kernels. The model presented in this paper replicates Backus et al. (2001)'s condition. In this two-country model, endowment shocks are *i.i.d* and agents are characterized by slow-moving external habit preferences similar to Campbell & Cochrane (1999).[6] I use Campbell & Cochrane (1999) as a device to capture time-variation in risk premia. These

[4]Hollifield & Yaron (2003) conclude that:

> Virtually none of the predictable variation in returns from currency speculation can be explained empirically by predictable variation in conditional inflation risk and in the interaction between conditional inflation and real risks. Models of a rational currency risk premium should focus on real risk.

[5]Let R^{real} be the real currency excess returns for a domestic investor. Let i and $i^\star$ be respectively the domestic and foreign nominal interest rates, and r and $r^\star$ their real counterpart. Let π and $\pi^\star$ be the domestic and foreign inflation rates. Finally, let e be the nominal interest rates, measured in units of domestic per foreign currency and q the real exchange rate measured in domestic good per foreign good. Using a first-order Taylor approximation of the real excess return leads to:

$$R^{real}_{t+1} = [(1+i^\star_t)\frac{e_{t+1}}{e_t} - (1+i_t)]\frac{1}{1+\pi_{t+1}} \simeq i^\star_t + \Delta e_{t+1} - \pi_{t+1} - (i_t - \pi_{t+1}) = r^\star_t + \Delta q_{t+1} - r_t.$$

[6]The habit literature has reproduced a wide variety of dynamic asset pricing phenomena. Major examples are Sundaresan (1989), Constantinides (1990), Campbell & Cochrane (1999) and Chen & Ludvigson (2004).

preferences entail a time-varying market price of risk. Moreover, real risk-free rates are low in bad times and high in good times because the precautionary savings motive is assumed to be greater than the inter-temporal consumption-smoothing motive. There is no friction on financial markets, which are characterized by the absence of arbitrage. In goods markets, agents can trade across countries, but when a unit is shipped, only a fraction of the good arrives to the foreign shore.

With this model, I obtain two theoretical results. First, the model gives a rationale for the existence of a currency risk premium and for its symmetry.[7] A domestic investor expects to receive a positive foreign currency excess return in times when she is more risk-averse than her foreign counterpart. Times of high risk-aversion correspond to low interest rates at home. Thus domestic investors enjoy positive foreign currency excess returns when domestic interest rates are low and foreign interest rates are high. This means that the U.I.P coefficient is below 1. Moreover, when the agent is risk-averse, a small consumption shock has a large impact on the change in marginal utility, and the stochastic discount factor has a considerable conditional variance. As a result, when interest rates are low, the conditional variance of the stochastic discount factor is high: Backus et al. (2001)'s condition is satisfied and the U.I.P puzzle rationalized.

Second, the introduction of international trade costs resolves the real exchange rate volatility quandary described by Brandt, Cochrane, & Santa-Clara (2006). In complete markets, the real exchange rate is theoretically equal to the ratio of foreign and domestic stochastic discount factors. We know since Mehra & Prescott (1985) and Hansen & Jagannathan (1991) that stochastic discount factors need to have a large variance to price stock excess returns. Taking into account the low correlation among consumption shocks across countries, and thus the low correlation of the stochastic discount factors, Brandt et al. (2006) show that the actual exchange rate is much smoother than the theoretical one. In this paper, the endowment shocks are uncorrelated across countries. But a finite trade cost allows countries to share risks. As a result, the variance of the theoretical exchange rate remains low.

To assess these theoretical results, two experiments are conducted. One, I calibrate and simulate a two-country model with habit preferences and finite or infinite trade costs.

[7] If a domestic investor gets a positive currency excess return by borrowing at home and lending abroad, her foreign counterpart's return is negative.

With infinite trade costs, I derive closed-form expressions for the U.I.P slope coefficient, the Sharpe ratio, the mean and variance of real interest rates. The simulation targets successfully all these moments and the mean and standard deviation of consumption growth. But the simulated exchange rate change varies three times more than in the data and it is too highly correlated to consumption growth. When the cost of trading is finite, the standard deviation of the real exchange rate decreases to its empirical counterpart. Thus this model gives a solution to the Brandt et al. (2006)'s quandary and produces reasonable volatilities of real interest rates and real exchange rates. Yet, the model cannot fully account for Backus & Smith (1993)'s puzzle; the correlation between differences in consumption growth and changes in real exchange rate is no longer equal to one as with CRRA preferences, but it remains higher than in the data.

Two, I estimate the model by considering the investment opportunities of an American investor in 14 other OECD countries. I then focus on all the cross-border investment opportunities of a German, Japanese and American investor. Following Hansen, Heaton, & Yaron (1996), a continuously-updating general method of moments (GMM) estimator is used. Estimates based exclusively on consumption data lead to reasonable parameters when pricing the excess returns of an American investor.

This paper is part of a large literature. Numerous studies have attempted to explain the U.I.P puzzle under rational expectations but few models can reproduce the negative U.I.P slope coefficient. Appendix (A) presents a literature review and Table (I) a synthetic view of the assumptions and results of these attempts. The three most successful studies are the following. First, Frachot (1996) shows that a financial two-country Cox, Ingersoll, & Ross (1985) framework can account for the U.I.P puzzle but he does not provide an economic interpretation of the currency risk premium. Second, Alvarez, Atkeson, & Kehoe (2005) use endogenously segmented markets. In their model, higher money growth leads to higher inflation, thus inducing more agents to enter the asset market because the cost of non-participation is higher, and leading to a decrease in risk premium. If segmentation is sufficiently large and sensitive to money growth, this time-varying risk qualitatively generates the forward premium anomaly. Yet, to reproduce quantitatively the U.I.P puzzle the model implies very large flows in and out of the asset markets. Third, Bacchetta & van Wincoop (2005) develop a model where investors face costs of collecting and processing information. Because of these costs, many investors optimally choose to only infrequently assess available information and revise their portfolios. Rational inattention produces a negative U.I.P coefficient along the lines suggested by Froot & Thaler (1990) and Lyons (2001): if investors are slow to respond to news of higher domestic interest rates, there will be a continued reallocation of portfolios towards domestic bonds and a appreciation of the currency subsequent to the shock. Bacchetta & van Wincoop (2005) obtain negative U.I.P slope coefficient for information and trading costs higher than 2 percents of total financial wealth.

The rest of this paper is organized as follows. Section II outlines the two-country two-good model. Section III derives a closed-form solution of the U.I.P slope coefficient in the special case of autarky. Section IV summarizes the simulation results with and without trade, and Section V presents the estimation exercises using consumption data. Section VI concludes.

I. Model

This paper builds on two strands of the literature. First, I assume iceberg-like shipping costs in international trade. Shipping costs were first proposed by Samuelson (1954), and then used by Dumas (1992), Sercu, Uppal, & Hulle (1995) and Sercu & Uppal (2003) to study real exchange rates. Obstfeld & Rogoff (2000) show that in a two-country two-good model a conservative trade cost of 25% (and a high price elasticity of import demand equal to 6) solve six major puzzles in international macroeconomics. Yet none of these papers tackle the forward premium puzzle, and Hollifield & Uppal (1997) show that proportional trade costs are not enough to reproduce the forward premium puzzle. They find that the implied U.I.P slope coefficient is never negative, not even for extreme levels of constant relative risk-aversion (CRRA) or trade costs. Here, I use trade costs as a simple device to model real exchange rates and I show that introducing time-variation in risk-aversion is a potential solution to the forward premium puzzle. Such a modeling simplification has a drawback: iceberg trade costs in a two country two-good model implies that there is no trade in some periods and only one-directional trade in the other periods, which is clearly counterfactual. Yet, accounting precisely for international trade is beyond the scope of this paper.

Second, I assume external habit preferences. Lustig & Verdelhan (2005) have shown that the U.I.P puzzle looks very much like a standard asset pricing puzzle, implying similar high degrees of risk-aversion. Such high risk-aversion coefficients would lead in the benchmark representative agent with CRRA preferences to counter-factually high risk-free rates. This is the classical Mehra & Prescott (1985)'s equity-premium puzzle.[8] Campbell & Cochrane (1999) preferences generate slow counter-cyclical variation in risk premia. In simulations, their model produces the pro-cyclical variation of stock prices, the long horizon predictability, the counter-cyclical variation of stock market volatility, the counter-cyclicality of the Sharpe ratio and the short- and long-run equity premium. Yet, Lettau & Ludvigson (2003) show that the Sharpe ratio implied by this model is smaller than its empirical counterpart. Tallarini & Zhang (2005) estimate the same model using US stock returns (assuming a constant risk-free rate) and find that the model performs reasonably well in matching the mean of returns but fails to capture the higher order moments. At the very least, Campbell & Cochrane (1999) show what is needed to solve the equity premium puzzle. In this paper, I show that the same feature, e.g an endogenous time-varying risk aversion, provides an interpretation to the U.I.P puzzle.[9] Moreover, these preferences are particularly appealing for currency excess returns as they imply a counter-cyclical Sharpe ratio, which is a feature of both stock and currency returns.[10]

This paper thus builds on the international economics and finance literature. I first focus on the trade aspect of the model. I then turn to the definition of the real exchange rate before describing in details a representative agent's preferences and their asset pricing implications.

[8]The problem worsens with the time-horizon considered. Cochrane & Hansen (1992) show that the performance of time-separable utility on bond pricing deteriorates as the horizon lengthens: on longer horizons, consumption is expected to grow, thus leading agents to try to borrow in the present. To counterbalance this behavior, risk-free rates must be counter-factually high in a general equilibrium model.

[9]Abandoning power utility and looking among successful asset pricing frameworks, several paths seem a-priori possible. These possibilities are based on one of the three following assumptions: the introduction of heterogeneity, state-nonseparability with Epstein-Zin preferences (Bansal & Yaron (2004)), or time-nonseparability in preferences. Sarkissian (2003) notes that heterogeneity alone can not produce a complete explanation of the U.I.P puzzle. Colacito & Croce (2005) study real exchange rates in the Epstein-Zin framework but do not test the U.I.P puzzle.

[10]Using data on OECD countries over the last twenty years, Verdelhan (2004) shows that Sharpe ratios of foreign currency excess returns are counter-cyclical.

A. International trade

There are two countries and two goods, identified by their locations. When a unit of the good is shipped across countries, only a fraction $1/(1+\tau)$ arrives. In each country, the representative agent is characterized by the same utility function as in Campbell & Cochrane (1999).[11] I abstract from the production side of each country and consider two endowment economies. I describe first the international trade mechanism and then preferences.

Let X_t denote the amount of the good exported from a domestic to a foreign country at time t. A superscript * refers to the same variable for the foreign country. The amount of exports $X_t \geq 0$ and $X_t^* \geq 0$ maximize the planing problem:

$$E\sum_{t=0}^{\infty}\beta^t \frac{(C_t - H_t)^{1-\gamma}-1}{1-\gamma} + E\sum_{t=0}^{\infty}\beta^t \frac{(C_t^\star - H_t^\star)^{1-\gamma}-1}{1-\gamma}. \tag{1}$$

subject to:

$$C_t = Y_t - X_t + \frac{X_t^*}{1+\tau} \text{ and } C_t^* = Y_t^* - X_t^* + \frac{X_t}{1+\tau} \tag{2}$$

where Y_t and Y_t^* denote the endowment, H_t and H_t^* the external habit level and C_t and C_t^* the amount of consumption in, respectively, the domestic and foreign country. The law of motion of the habit level in each country will be fully described in the next section. It does not depend on contemporaneous consumption and the planning problem reduces to a sequence of static problems. As a result, the external habit level can also be interpreted as a social externality or as a preference shock.

If one country exports, the other does not as there is only one kind of good in the model. Let us assume first that the domestic country exports ($X_t \geq 0$, $X_t^* = 0$). The first order condition is then:

$$\frac{C_t^* - H_t^*}{C_t - H_t} = (1+\tau)^{-\frac{1}{\gamma}} \tag{3}$$

[11] I focus on the habit's law of motion leading to a time-varying risk-free interest rate. The details of these preferences are presented in the next section.

And the optimal amount of exports is in this case:

$$X_t = \frac{Y_t - H_t - (1+\tau)^{\frac{1}{\gamma}}(Y_t^* - H_t^*)}{1 + (1+\tau)^{\frac{1}{\gamma}-1}}. \tag{4}$$

If $\frac{Y_t^*-H_t^*}{Y_t-H_t} < (1+\tau)^{-\frac{1}{\gamma}}$, then the domestic country exports the amount X_t above, otherwise there is no trade. Similarly, the foreign country exports when $\frac{Y_t^*-H_t^*}{Y_t-H_t}$ is above $(1+\tau)^{\frac{1}{\gamma}}$. In this case,

$$X_t^* = \frac{Y_t^* - H_t^* - (1+\tau)^{\frac{1}{\gamma}}(Y_t - H_t)}{1 + (1+\tau)^{\frac{1}{\gamma}-1}}. \tag{5}$$

Thus, there is a no-trade zone in which the marginal utility of shipping a good is more than offset by the trade cost. This happens when $(1+\tau)^{-\frac{1}{\gamma}} \leq \frac{Y_t^*-H_t^*}{Y_t-H_t} \leq (1+\tau)^{\frac{1}{\gamma}}$. I now turn to the assumptions on financial markets and their implications for the definition of the real exchange rate.

B. Real exchange rate

Complete financial markets I assume the absence of arbitrage and the completeness of the financial markets.[12] In each country, at each date, a representative investor has access to a domestic one-period risk-free asset, whose payoff is in terms of domestic consumption, and to a foreign one-period risk-free asset, whose payoff is in terms of foreign consumption. In complete markets, the change in the real exchange rate is defined as the ratio of the two stochastic discount factors at home and abroad:

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^{\star}}{M_{t+1}} \tag{6}$$

[12] Assuming the "law of one price on the asset markets" implies the existence of a stochastic discount factor M_{t+1}. Assuming the "absence of arbitrage" is stronger: it implies the existence of a *positive* M_{t+1}, see Cochrane (2001). I use the latter assumption because it also implies the uniqueness of M_{t+1} in complete markets. Note that the form of the utility function in this paper guarantees that $M_{t+1} > 0$.

where Q is expressed in domestic goods per foreign good.[13] Given Q_0 the exchange rate at date 0, equation (6) gives the entire path of Q. From this definition, one can compute the real exchange rate in case of trade or infinite trading costs, e.g autarky. In the latter case, the real exchange rate is the rate at which the two countries do not want to trade.[14]

Exchange rate When there is trade, the first-order condition (A) of the social planner's problem is satisfied, and the countries share risk. Thus, the real exchange rate does not depend on the endowments. When there is no trade, the real exchange rate is determined on the asset market and the ratio of the two marginal utilities of consumption move freely with the endowment shocks. To summarize, the real exchange rate Q_t can take the following values:

- If $\frac{V_t^\star - X_t^\star}{V_t - X_t} < (1+\tau)^{-\frac{1}{\gamma}}$, $Q_t = (1+\tau)$.
- If $\frac{V_t^\star - X_t^\star}{V_t - X_t} > (1+\tau)^{\frac{1}{\gamma}}$, $Q_t = \frac{1}{(1+\tau)}$.
- If $(1+\tau)^{-\frac{1}{\gamma}} \leq \frac{V_t^\star - X_t^\star}{V_t - X_t} \leq (1+\tau)^{\frac{1}{\gamma}}$, $Q_t = (\frac{V_t^\star - X_t^\star}{V_t - X_t})^{-\gamma}$.

Note that an increase in the trade cost τ or a decrease in the risk aversion coefficient γ enlarges the no-trade zone and thus increases the real exchange rate volatility as in Sercu & Uppal (2003).[15] The amount of trade and the real exchange rate depend on habit levels in each country.

[13] The Euler equation for a foreign investor buying a foreign bond is: $E_t(M_{t+1}^\star R_{t+1}^\star) = 1$. The Euler equation for a domestic investor buying a foreign bond is: $E_t(M_{t+1} R_{t+1}^\star \frac{Q(t+1)}{Q(t)}) = 1$. Because the stochastic discount factor is unique in complete markets, equation (6) follows.

[14] Yet, the exchange rate can be defined as in Alvarez et al. (2005) assuming that at date 0, each representative investor is endowed with claims on domestic and foreign consumptions. Let A_i and $A_i^\star$ be the initial claims of the domestic investor on respectively domestic and foreign consumption. Then, $Q(0) = (\overline{A} - A_i)/A_i^\star$, where $\overline{A}$ is the equilibrium asset holding. The numerator corresponds to the number of claims on domestic consumption that the domestic investor exchanged for claims on the foreign consumption (in the denominator).

[15] Sercu & Uppal (2003) study the impact of trade costs on exchange rate volatility and international trade using a power-utility framework for a two-country, two-good world. Assuming log-normal outputs, they show that a drop in shipping costs implies a decrease in the variance of the real exchange rate.

C. Habit-based preferences

In each country, the habit level is related to consumption through the following AR(1) process of the surplus consumption ratio $S_t \equiv (C_t - H_t)/C_t$:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - g).$$

Lowercase letters correspond to logs, $\lambda(s_t)$ is the sensitivity function, and g is the average growth rate of the log-normal endowment process:

$$\Delta v_{t+1} = v_{t+1} - v_t = g + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0, \sigma^2).$$

The same assumptions are made about the consumption process in both countries. Moreover, to keep the model simple and tractable, I assume that the two endowment shocks u_{t+1} and u^*_{t+1} are independent.

Chen & Ludvigson (2004) and Tallarini & Zhang (2005) provide some empirical support for using these preferences based on US domestic assets. Chen & Ludvigson (2004) estimate habit-based models without imposing the functional form of habit preferences. They conclude that in order to match moment conditions corresponding to Fama-French portfolios, habits should be nonlinear, internal, and equal to a huge fraction of current consumption (97% on average). Using a simulation-based method, Tallarini & Zhang (2005) estimate Campbell & Cochrane (1999)'s model on US domestic assets (assuming a constant real risk-free interest rate). They find that the persistence coefficient ϕ is significantly above 0.9 and the risk-aversion coefficient equal to 6.3.

External habits Habit is assumed to depend only on aggregate, not on individual, consumption.[16] Thus, the inter-temporal marginal rate of substitution is here:

$$M_{t,t+1} = \beta \frac{U_c(C_{t+1}, X_{t+1})}{U_c(C_t, X_t)} = \beta (\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t})^{-\gamma} = \beta G^{-\gamma} e^{-\gamma \left[(\phi - 1)(s_t - \bar{s}) + (1+\lambda(s_t)) v_{t+1} \right]},$$

where $\ln G = g$. Then the log risk free rate is:

$$r_t = -\ln(\beta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma^2}{2} \left[1 + \lambda(s_t)\right]^2.$$

Campbell & Cochrane (1999) suggest a sensitivity function of the following form:

(7) $$\lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1, \text{ when } s \leq s_{\max}, \text{ 0 elsewhere,}$$

with $\bar{S} = \sigma \sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$ and $s_{\max} = \overline{s} + (1 - \overline{S}^2)/2$. This sensitivity function leads to a linear risk-free rate:

(8) $$r_t = \overline{r} - B(s_t - \bar{s}),$$

where $\overline{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2 \sigma^2}{2 \bar{S}^2}$ and $B = \gamma * (1-\phi) - \frac{\gamma^2 \sigma^2}{S^2}$.[17]

[16] With internal habits, the marginal utility of consumption is:

$$MU_t = \frac{\partial U_t}{\partial C_t} = (C_t - X_t)^{-\gamma} - E_t(\sum_{t=0}^{\infty} \beta^t (C_{t+j} - X_{t+j})^{-\gamma} \frac{\partial X_{t+j}}{\partial C_t})$$

Consumption today raises future habits, lowering the overall marginal utility of consumption today. Thus, the marginal utility of consumption in case of internal habits has two terms. Figure (10) in Campbell & Cochrane (1999) shows that marginal utility with internal and external habits are proportional around the steady-state (but not when s tends to s_{max}). The real exchange rate depends on the ratio of two marginal utilities of consumption. The real exchange rate computed with internal habits thus should be very similar to the one computed with external habits.

[17] With these preferences, the variance of the log stochastic discount factor is equal to:

$$Var_t(\log M_{t,t+1}) = \frac{\gamma^2 \sigma^2}{\bar{S}^2} [1 - 2(s_t - \bar{s})].$$

Ljungqvist & Uhlig (2003) have shown that, in some cases, Campbell & Cochrane (1999)'s model produces a surprising result: habit levels may decrease following a sharp increase in consumption.[18] I recognize the theoretical possibility of Ljungqvist & Uhlig (2003)'s result, but I stick to Campbell & Cochrane (1999)'s habit model because Ljungqvist & Uhlig (2003)'s result does not appear in simulations based on actual data.

How does the risk-free rate react to domestic economic stance? Campbell & Cochrane (1999) emphasize the asset pricing results obtained with a constant, real, risk-free rate ($B = 0$). I prefer to assume a non-constant, risk-free rate for three reasons. First, real interest rate variances are significantly different from zero. Second, abandoning a constant interest rate may help reconcile theory and exchange rate estimations. If the interest rate is allowed to fluctuate in Campbell & Cochrane (1999)'s model, it closely resembles the framework proposed by Cox et al. (1985), which Frachot (1996) has shown reproduces the forward premium. Third, a growing literature studies real interest rate cyclicality. The assumption of a nonzero B has also been used by Buraschi (2004) and Wachter (2006) to model the US yield curve, and by Menzli, Santos, & Veronesi (2004) to study cross-sections of US assets.

What is the economic rationale behind the sign of B? Consumption smoothing and precautionary savings affect the real interest rate, and the parameter B here summarizes these two different effects.

- In good times, after a series of positive consumption shocks that result in a high surplus consumption ratio s, the agent wants to save more in order to smooth consumption. This leads to a decrease in the interest rate through an inter-temporal substitution effect.

- But, in good times, the representative agent is risk neutral (the local curvature of her utility function is $\frac{\gamma}{s_t}$). She is less interested in saving, leading to an increase in the real interest rate through a precautionary saving effect. Conversely, in bad times, when the surplus consumption ratio is low, the agent is very risk averse and saves more.

And the model produces, as mentioned earlier, a counter-cyclical Sharpe ratio equal to:

$$SR_t = \frac{\sigma_t(M_{t,t+1})}{E_t(M_{t,t+1})} = [e^{\frac{\gamma^2\sigma^2}{\bar{S}^2}(1-2(s_t-\bar{s}))} - 1]^{\frac{1}{2}} \simeq \frac{\gamma\sigma}{S}\sqrt{1-2(s_t-\bar{s})}.$$

[18]By construction, an infinitesimal rise in consumption always increases habit levels.

The case of $B < 0$ is thus the one in which the precautionary effect overcomes the substitution effect. A a result, interest rates are low in bad times and high in good times. This is in line with recent results found in the real interest rate literature. Challenging previous findings from Stock & Watson (1999), Dostey, Lantz, & Scholl (2003) conclude that the ex-ante real rate is contemporaneously positively correlated with GDP and with lagged cyclical output.

D. Exchange rate risk premium

The exchange rate risk premium is the excess return of a domestic investor who borrows funds at home, changes her currency to a foreign equivalent, lends on the foreign market for a defined period and finally reconverts her earnings to the original currency. Thus, using logs, the foreign currency excess return r^e_{t+1} is equal to:

$$r^e_{t+1} \simeq q_{t+1} - q_t + r^\star_t - r_t, \tag{9}$$

where r_t and $r^\star_t$ are respectively the domestic and foreign risk-free real interest rates. The domestic investor gains $r^\star_t$, but she has to pay r_t, and she loses if the dollar appreciates in real terms - q decreases - when her assets are abroad. Backus et al. (2001) show that foreign currency excess return is equal to:[19]

$$E_t(r^e_{t+1}) = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m^\star_{t+1}). \tag{10}$$

[19] I reproduce here Backus et al. (2001)'s proof in the case of complete markets. Assuming log-normal stochastic discount factors leads to: $r_t = -\log E_t M_{t,t+1} = -\ E_t \log M_{t,t+1} - \frac{1}{2}Var_t(\log M_{t,t+1})$, and $r^*_t = -\log E_t M^*_{t,t+1} = -\ E_t \log M^*_{t,t+1} - \frac{1}{2}Var_t(\log M^*_{t,t+1})$. The expected change in the exchange rate is then:

$$E_t(\log \frac{Q_{t+1}}{Q_t}) = E_t(\log M^*_{t,t+1}) - E_t(\log M_{t,t+1}) = -r^*_t + r_t - \frac{1}{2}Var_t(\log M^*_{t,t+1}) + \frac{1}{2}Var_t(\log M_{t,t+1}).$$

Equation (10) follows.

II. Theoretical results without trade

In this section, I abstract from trade and consider that each economy consumes its own endowment.[20] In this case, I can derive a closed-form expression for the currency excess return that highlights the rationale and mechanisms of the model.

A. An interpretation of the U.I.P puzzle

Using habit preferences and assuming that countries have the same characteristics and do not trade leads to the following expected currency excess return:[21]

$$E_t(r^e_{t+1}) = \frac{\gamma^2\sigma^2}{\bar{S}^2}(s^*_t - s_t).$$

This formulation of the exchange rate risk premium presents three interesting features.

First, it gives a rationale for the existence of the currency premium and for its symmetry. In this framework, the local curvature of the utility function is equal to $\frac{\gamma}{S_t}$, thus lower surplus consumption ratios entail more risk-averse agents. *The domestic investor gets a positive excess return at date t if she is more risk averse than her foreign counterpart.* The interpretation of teh risk premium is perfectly symmetric, thus taking into account that a positive excess return for the domestic investor means a negative one for he foreign counterpart.

In financial terms, an excess return is usually decomposed in terms of betas (i.e the covariances between returns and marginal utilities) and lambda (i.e how much return the investor gets per unit of risk).[22] The interpretation of the currency premium relies here on time-varying risk-aversion, or the market price of risk lambda, not on different exchange rate betas. In complete markets, when consumption growth shocks are uncorrelated across

[20] The endowment processes can be built to reflect actual post-trade consumptions. Introducing for example a low correlation between the two endowments processes does not overturn the results on U.I.P.).

[21] To simplify the closed-form solution of the currency excess return, I assume here that the domestic and foreign investors are characterized by the same underlying parameters: $\gamma = \gamma^*$, $S = S^*$, $\phi = \phi^*$, $\sigma = \sigma^*$ and $\tau = \infty$.

[22] For any excess return $R^{e,i}_{t+1}$, the Euler equation $E_t\left[M_{t+1}R^{e,i}_{t+1}\right] = 0$ can be restated as $E[R^{e,i}_{t+1}] = \left(-\frac{cov(m,R^{e,i})}{var(m)}\right)'\left(\frac{var(m)}{E(m)}\right) = \beta_i'\lambda$ where $\lambda = \frac{var(m)}{E(m)}$.

countries, the betas of both foreign and domestic investors are equal to one:

$$\beta_t = -\frac{cov_t(m_{t+1}, q_{t+1} - q_t)}{var_t(m_{t+1})} = -\frac{cov_t(m_{t+1}, m^{\star}_{t+1}) - var_t(m_{t+1})}{var_t(m_{t+1})} = 1.$$

Thus the interpretation of the risk premium here differs from the one emphasized in Lustig & Verdelhan (2005), which relies on a cross-section of betas. Assuming CRRA (i.e a constant price of consumption growth risk) and looking only at the domestic investor's pricing kernel, Lustig & Verdelhan (2005) show because high interest rate currencies depreciate on average when domestic consumption growth is low and low interest rate currencies appreciate under the same conditions, low interest rate currencies provide domestic investors with a hedge against domestic aggregate consumption growth risk.

Second, this proposed formulation also offers a possible explanation for the U.I.P puzzle. The expected change in exchange rate is equal to:

$$E_t(\frac{\Delta q}{q}) = [1 + \frac{1}{B}\frac{\gamma^2\sigma^2}{\bar{S}^2}]\left[r^f_t - r^{f,*}_t\right] = \gamma(\frac{1-\phi}{B})\left[r^f_t - r^{f,*}_t\right]. \tag{11}$$

In this framework, the U.I.P slope coefficient no longer needs to be equal to unity even if consumption shocks are simply *i.i.d. Since the risk premium depends on the interest rate gap, the coefficient α in a U.I.P regression can be below* 1 *and, if $B < 0$, even negative.*[23] The model reproduces the sufficient condition that Backus et al. (2001) outline for solving the U.I.P puzzle: a negative correlation between the difference in interest rates and the difference in conditional variance of the two pricing kernels. When the surplus consumption ratio s_t is low, the agent is very risk-averse. As the precautionary savings effect dominates the inter-temporal smoothing one (for a negative B), interest rates are low. A small consumption shock thus has a large impact on the change in marginal utility, and the stochastic discount factor has a considerable conditional variance $Var_t(\log M_{t,t+1})$. When interest rates are low, the conditional variance of the stochastic discount factor is high.

Third, *in the very long run, the risk premium disappears if the two countries have the*

[23]Since this model can reproduce a negative U.I.P coefficient, it can naturally satisfy the two Fama's conditions - presented in Appendix (A) - which were theoretically derived assuming $\alpha < 0$ for the first one and $\alpha < 1/2$ for the second one.

same intrinsic characteristics. If the two countries are similar (same average consumption growth rate g, risk-aversion γ, persistence ϕ and average surplus consumption ratio $\bar{S}$), then the average real risk free rate is the same in both countries. Taking the unconditional expectation of equation (11), the change in the real exchange rate is on average equal to zero. In the long run, two similar countries satisfy P.P.P convergence tests.[24]

B. Shortcomings

The model presented in this paper offers a fully rational general equilibrium explanation for the U.I.P puzzle. Yet it is overly simple and has three main shortcomings.

The condition outlined by Backus et al. (2001) for reproducing the U.I.P puzzle entails potentially negative interest rates. This is the case here. With a negative parameter B, real interest rates can be negative for very low values of the surplus consumption ratio.

In addition, Wachter (2006) argues that a positive parameter B is needed to obtain an upward sloping yield curve and to reproduce Campbell & Shiller (1991)'s results for the expectation hypothesis with the Campbell & Cochrane (1999) preferences. Thus it seems impossible to rationalize both the U.I.P and expectation hypothesis puzzles with only one set of parameters.[25] A small positive coefficient B produces a U.I.P coefficient which, while positive, still remains within one standard error of most empirical estimates. In this case, real interest rates are always positive and the yield curve is upward-sloping. Yet, as already mentioned in teh previous section, real interest rates are empirically found to be pro-cyclical, which implies a negative B.

Finally, this model produces a correlation between domestic and foreign consumption growth and the real exchange rate change that is too high. In complete markets and under autarky, the real exchange rate is equal to the ratio of domestic marginal utility of consumption to foreign marginal utility of consumption. This link is weakened only through the presence of trade and risk-sharing. Backus & Smith (1993) argue that empirically no correlation exists between exchange rate and consumption growth. The model presented here does not replicate this result. However, Lustig & Verdelhan (2005) show that the

[24]If the two countries have different structural parameters however, the change of the real exchange rate does not have to be zero in the long run: $E(\frac{\Delta q}{q}) = \overline{r} - \overline{r}^* + \frac{1}{2}\frac{\gamma^2\sigma^2}{\bar{S}^2} - \frac{1}{2}\frac{\gamma^{*2}\sigma^{*2}}{\bar{S}^{*2}}$.

[25]I would like to thank Charles Engel for pointing this out to me.

correlation between consumption growth and exchange rates is dependent on the interest rates differentials. Because the correlation switches sign when the interest rate differential fluctuates, a simple unconditional measure might not show a link between exchange rates and consumption growth.

III. Simulation

To better assess the performance of the model, I have performed two simulations. The first simulation assumes very high trade costs, thus prohibiting trade across countries. In this autarkic case, simple closed-form expressions can be obtained for a few interesting moments, thus making the calibration straightforward. The second simulation keeps the same set of parameters, but decreases the trade costs to levels advocated by the international trade literature. Below, I describe the calibration parameters and the simulation results. Finally, as a reality check for my parameters, I compute the time-series of the stochastic discount factor, the surplus consumption ratio and the local curvature using actual US consumption data.

A. Calibration

I assume that two countries, for example the United States and Germany, can be characterized by the same set of parameters (g, σ, $\overline{r}$, γ, ϕ and B) and that endowment shocks are not correlated across countries.

To determine the six independent parameters of the model, I target six simple statistics: the mean g and standard deviation σ of the consumption growth rate, the mean $\overline{r}$ and standard deviation of the interest rate σ_r, the U.I.P coefficient α, and the steady-state Sharpe ratio $\overline{SR}$. Under conditions of autarky, one can obtain closed-form expressions for the last three moments.[26] These six statistics are measured over the 1947:2-2004:3 period

[26] An exact closed-form expression for the standard deviation of the interest rate is difficult to obtain, but the choice of parameters can be based on a simple approximation: supposing that $\lambda(s_t)$ remains equal to its steady-state value ($\lambda(\overline{s}) = \frac{1-\bar{S}}{\bar{S}}$), the variance of the interest rate is close to $B^2(\frac{\sigma}{\bar{S}})^2\frac{1}{1-\phi^2}$, where $\bar{S}$ is defined in terms of σ, γ, ϕ and B. Adding the closed-form expression of the U.I.P coefficient ($\alpha = \frac{(1-\phi)\gamma}{B}$) and the Sharpe ratio at steady-state $\overline{SR} = \gamma\sigma/\bar{S}$ produces three conditions.

for the US economy. Per capita consumption data are from the BEA. US interest rates, inflation and stock market excess returns are from CRSP (WRDS). Expected inflation is computed using a one-lag two-dimension VAR (inflation and interest rate). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. The Sharpe ratio is obtained as the ratio of the unconditional mean of monthly stock excess returns on their unconditional standard deviation. The U.I.P coefficient is computed using the US-Germany exchange rate. German interest rates and inflation rates are from Global Financial Data. Table (II) compares the parameters used in this paper to the ones proposed by Campbell & Cochrane (1999) and Wachter (2006). The calibration choices outlined above lead to a reasonable risk-aversion coefficient of 2.2. Consumption is on average 8 percent above the habit level, with a maximum value of 12 percent.

I keep the same set of parameters when I open the model up to trade. Anderson & van Wincoop (2004) provide an extensive survey of the trade cost literature. They conclude that total international trade costs, which include transportation costs and border related trade barriers, represent an ad-valorem tax of about 74%.[27] Following Anderson & van Wincoop (2004), I assume a trade cost of 75% (which corresponds to $\tau = 3$). I also run the same simulation with a conservative trade cost of 25% as in Obstfeld & Rogoff (2000) (which corresponds to $\tau = 1/3$).

From the endowment shocks and the parameters above, I build surplus consumption ratios, stochastic discount factors, interest rates in both countries and their exchange rate. Appendix (D) details the procedure. I then regress the quarterly variation of the real exchange rate on the real interest rate differential to find the slope coefficient α from a U.I.P test.

B. Results under autarky

Results are summarized in Table (III). I first review the moments outlined in the calibration process and then turn to the properties of the implied real exchange rate.

[27] Border-related trade barriers represent a 44% cost and is a combination of direct observation and inferred costs. Transportation costs represent 21%.

U.I.P coefficient, variance of the interest rate and Sharpe ratio As expected, the U.I.P slope coefficient α is negative and in line with its empirical value. This is also the case for the interest rate standard deviation and the average Sharpe ratio targeted by the calibration. Thus Campbell & Cochrane (1999)'s preferences can, in a two-country model, reproduce the negative U.I.P slope coefficient without either endangering the stock market implications of the model or overshooting the mean and variance of real interest rates. The Sharpe ratio is sizable even with a reasonable risk-aversion coefficient. This does not mean that risk-aversion is always moderate. As in Campbell & Cochrane (1999), the local curvature coefficient $\eta_t = \frac{\gamma}{S_t}$ sometimes attains very high values, but this happens rarely, as shown in Figure (1).

Properties of the real exchange rate The distribution of the exchange rate - level and first-difference as presented in Figure (2) - reproduces the hump-shape found in the data. The model also delivers an autocorrelation coefficient of the exchange rate close to its empirical counterpart. The growth rate of the exchange rate however displays the main drawback of the autarkic model: simulated real exchange rate appreciation has a variance which is three times higher than the actual one. This result can be related to the very definition of the exchange rate in complete markets, which implies that its variance is equal to:

$$\sigma^2(\log \frac{q_{t+1}}{q_t}) = \sigma^2(m) + \sigma^2(m^*) - 2\rho(m, m^*)\sigma(m)\sigma(m^*).$$

But the variance of the stochastic discount factor deduced from asset pricing is high. Taking into account the low correlation among consumption shocks across countries, and thus the low correlation of stochastic discount factors, Brandt et al. (2006) show that the actual exchange rate is much smoother than the theoretical one implied by asset pricing models.[28] The same tension is present here, because the standard deviation of the change in exchange rate is proportional to the Sharpe ratio. Thus, one cannot obtain a high Sharpe ratio and a low exchange rate volatility at the same time.[29] Leaving autarky for

[28]Consumption shocks are not assumed correlated across countries in this paper. But the variance of the real exchange rate remains high even when the actual small correlation between domestic and foreign consumption processes is taken into account.

[29]The variance of real exchange rate appreciation is here at the steady-state:$\langle Var_t(q_{t+1} - q_t)\rangle_{Steady-state} = 2(\gamma\sigma\ \bar{S})^2 = 2\overline{SR}^2$.

a more realistic world in which trade is possible drastically changes this result.

C. Results with trade

Opening the model up to trade has an impact on both the real side of the economy and on asset prices.

U.I.P coefficient and trade When countries can trade, they share risk and the standard deviation of their consumption growth decreases.[30] This in turn decreases the standard deviation of real interest rates. The U.I.P coefficient remains negative. Its absolute value becomes smaller than under conditions of autarky, but remains in the 95% confidence interval of its empirical counterpart.

Finite trade costs imply non-zero simulated openness ratios, computed as the average of imports and exports divided by the endowment. For trade costs equal to 25% and 75%, these openness ratios are respectively equal to 3.5% and 10.1% on average with standard deviations of 1.7% and 15.2%. These statistics look reasonable when compared with real US data. The actual global openness ratio for the US is equal to 8.4% on average over the 1957:2-2004:4 period (with a standard deviation of 2.8%), but these figures take into account all international trade with the US and not only bilateral US-German trade.[31] One would expect the openness ratio to be smaller and more volatile for one particular bilateral trade than for the sum of all exports and imports.

Properties of the real exchange rate The most drastic change appears in relation to exchange rates. When compared with results obtained under autarky, the standard deviation of the change in simulated real exchange rates is divided by 8 at trade costs of 25% and by 3 at trade costs of 75%. As Sercu & Uppal (2003) noted, the lower the trade cost, the lower the exchange rate variance. The volatility of the simulated exchange rate thus appears below the post-war value for the US/German rate for a trade cost of 25% and broadly in line with its empirical counterpart for a trade cost of 75%. Despite this, the model with finite trade costs has two drawbacks. The distribution of the simulated

[30] The model still implies some high values for the curvature coefficient as shown in Figure (3).

[31] I used the IFS series 11190C.CZF... and 11198C.CZF... to measure imports and exports in US dollars and 11199B.CZF... for the gross domestic product in US dollars.

log exchange rate presented in Figure (4) has two peaks, as might be expected with this "sS"-type model. But the distribution of real exchange rates shown in figure (5) does not reproduce these clear features. In addition, while the trade model reduces the unconditional correlation between exchange rates and consumption growth by a factor of three, it remains too high.

D. Reality check

Figure (6) shows the time-series of the surplus consumption ratio, stochastic discount factor and local risk curvature for an American investor. The figure is based on the same set of parameters used in the simulation and presented in the first column of Table (II), but uses only actual US consumption data for the $1947:2-2004:3$ period. The stochastic discount factor and surplus consumption ratio look reasonable. The local curvature is much higher than the risk-aversion coefficient; Campbell & Cochrane (1999) designed their model for that purpose.

IV. Estimation

In this section, I present the results of the direct estimation of my model for foreign currency excess returns.

A. Method

The model can be estimated without linear approximation by computing the sample equivalent of the Euler equation. The estimation relies on the continuously-updating estimator studied by Hansen et al. (1996), which is here applied to conditional moments. I conduct two different experiments:

- first, I estimate the model using the moments implied by the pricing behavior of an American investing in other countries;[32]

[32] I consider the following 14 O.E.C.D countries: Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Mexico, Netherlands, Norway, Spain, Switzerland and the United Kingdom. The estimation is run over the $1981:3-2002:2$ period, for which consumption, interest rates and exchange rates are available for all countries considered.

- second, I suppose that American, Japanese and German investors can be characterized by the same set of structural parameters, and I take into account all the cross-investments opportunities between these three countries.

Theoretically, the model has only one kind of shock which drives both consumption and interest rate processes. The model can therefore be estimated in each case through either Treasury Bills or consumption data. The use of consumption data to explain foreign excess returns is naturally the most challenging case, and thus the one presented here.

The estimator is implemented over a grid of potential parameters. Possible ranges are deduced from the empirical literature on foreign exchange risk premia (the persistence coefficient ϕ should be above 0.8 and below unity), and on habit-based models (the steady-state surplus-consumption ratio $\overline{S} \in [0, 0.20]$). The risk-aversion coefficient γ varies between 0 and 10. For each variable, the range is divided into 100 steps, thus resulting in a 100x100x100 grid. For each value of the triplet, the sample equivalent ($\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$ where $f_t = m_t R_t$) of $E(MR)$ for the N excess returns (using the US lagged interest rate gap as an instrument) and the variance-covariance matrix[33] Ω lead to the estimator $J = T \times \overline{f} \times inv(\Omega) \times \overline{f}$.

B. Results

The American investor Let us first consider the different excess returns that an American investor enjoys when investing in the 14 other countries. This setup gives 14 conditional moments that allow for the estimation of the three parameters (γ, ϕ and $\overline{S}$). Figure (7) presents the graph of J_γ as a function of γ and Table (IV) reports minimization results. Standard errors are computed using GMM asymptotic theory (Hansen (1982)) for the three parameters of the model and by delta-method for the implied coefficients (see Appendix (E) for details).

The three parameters are estimated within their proposed ranges, and no corner solution is reached. The point estimates seem reasonable and they imply negative values

[33]Note here that Ω is computed for each set of parameters, and that Ω is the variance-covariance matrix not the spectral density matrix (This avoids the production of a non-positive definite matrix and takes into account the limited number of time periods in the estimation). Ω is sometimes singular to working precision. J is computed only for cases when the condition index ($RCOND$) is above $1e-7$ and the rank of Ω is N.

both for B and the U.I.P coefficient α. The p-value is equal to 0.42%.

American, German and Japanese investors It is tempting to push the model one step further by taking into account the opportunities facing every investor, not just the American. If each investor is characterized by a specific set of parameters γ_i, ϕ_i and $\overline{S}_i$, the number of parameters grows with the number of countries. As a result, I choose here to assume that all investors can be characterized by the same structural parameters, γ, ϕ and $\overline{S}$, and focus on only 3 countries (Germany, Japan and the United States).

Let M be a matrix of stochastic discount factors and R a matrix of real excess returns, where $R^{i,j}$ corresponds to the investment of i abroad in country j. In the case of the three countries, this leads to:

$$M = \begin{pmatrix} M^{GR} & M^{JP} & M^{US} \end{pmatrix} \text{ and } R = \begin{pmatrix} R^{GR,JP} & R^{GR,US} & R^{JP,GR} & R^{JP,US} & R^{US,GR} & R^{US,JP} \end{pmatrix}.$$

This setup gives 6 conditional moments that allow for the estimation of the three parameters, γ, ϕ and S. Figure (8) shows the minimized criterion as a function of the risk-aversion coefficient. The results are reported in table (IV), in the second column. Using consumption data only, the assumption that the three countries can be characterized by the same structural parameters is clearly rejected, the p-value rises up to 17%.[34] The difficulties encountered here are in line with Bams, Walkowiak, & Wolff (2004)'s conclusion that different foreign currencies' dollar risk premia respond to varying degrees to a common factor.[35] Thus assuming that all investors can be characterized by the same risk-aversion coefficient or the same habit process is far-fetched.

[34]Note that using interest rates instead of consumption data, the estimation leads to a much better fit with a p-value equal to 0.80%.

[35]Bams et al. (2004) note that a highly significant common component in the dollar risk premium exists. The dollar risk premia present a common pattern of positive serial correlation for the pound, the yen and the mark, and this common component explains most of the dynamics of the forward prediction error.

V. Conclusion

I have shown here that a two-country one-good model in which agents are characterized by slow-moving external habit preferences *a la* Campbell & Cochrane (1999) rationalizes the U.I.P puzzle. The model has two main features: a time-varying risk premium and an iceberg-like trade cost.

Because of the time-varying risk premium, the model reproduces the negative U.I.P slope coefficient. The failure of the U.I.P condition implies the existence of non-zero currency excess returns. But if a domestic investor receives a positive currency excess return, her foreign counterpart receives a negative one. The model rationalizes this stylized fact. In this model, the domestic investor gets positive excess returns in times when she is more risk-averse than her foreign counterpart. The same reasoning applies naturally to the foreign investor. Because of the trade cost, the variance of the real exchange rate is in line with its empirical counterpart. Thus, the introduction of trade cost is a potential solution to the exchange rate quandary described by Brandt et al. (2006): the theoretical real exchange rate is not too volatile even when the consumption processes among countries are uncorrelated.

Model simulations lead to the usual negative covariance between exchange rate variations and interest rate differentials, while simultaneously delivering a sizable Sharpe ratio. The model's estimation gives reasonable parameters, thus rationalizing the exchange rate risk premium.

All these results have been obtained for endowment economies. Could the same set of preferences be transposed into a production framework and thereby reconcile business cycle and asset pricing results? Recent attempts at using habit preferences in production economies have highlighted two difficulties: the variances of interest and consumption growth rates. Boldrin, Christiano, & Fisher (2001) highlight the first difficulty. They use habits in the representative agent's preferences and a "time-to-plan assumption"on investment and labor, but find that their model overestimates risk-free rate variability. By substituting Campbell & Cochrane (1999) for the Constantinides (1990)' form of habit preferences used by Boldrin et al. (2001), one can hope to overcome this difficulty. This habit form allows the parametrization of the interest rate's sensitivity to the economic stance, which impacts the variance of the risk-free rate. This comes at the price of decreasing the mean interest rate, which can be compensated for by a reasonable increase in risk-aversion. Lettau & Uhlig (2000) illustrate the second difficulty. They show that Campbell & Cochrane (1999) preferences deliver overly smooth consumption in a real business cycle framework. Agents are very risk-averse locally, meaning that the intertemporal elasticity of substitution is very low. This leads to a desire to use labor to radically smooth consumption. This difficulty might be overcome by introducing predetermined labor, a time-to-plan assumption and two separate sectors. Considering the many interesting results obtained in endowment economies, the transposition of this class of model onto a general equilibrium framework deserves some future work.

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A. Literature Review

The existence of the exchange rate risk premium stems from the empirical failure of uncovered interest rate parity (U.I.P).[36] Taking into account this empirical finding, expected exchange rate variations are assumed to be equal to their corresponding interest differentials up to a time-varying risk premium p:

$$\Delta e^e_{t,t+1} = i_t - i^*_t - p_{t,t+1}.$$

Fama (1984) highlights two characteristics of this risk premium.[37] Assuming rational expectations, he introduces a forecast error η_{t+1}, which is orthogonal to all information dated time t or earlier, defined as: $\Delta e_{t+1} = \Delta e^e_{t,t+1} + \eta_{t+1}$ where $\Delta e^e_{t,t+1}$ is the expected change in the spot rate. Then the U.I.P coefficient α is equal to:

$$\alpha = \frac{cov(\Delta e_{t+1}, i_t - i^*_t)}{var(i_t - i^*_t)} = \frac{var(\Delta e^e_{t,t+1}) + cov(p_{t,t+1}, \Delta e^e_{t,t+1})}{var(p_{t,t+1}) + var(\Delta e^e_{t,t+1}) + 2cov(p_{t,t+1}, \Delta e^e_{t,t+1})}.$$

Fama (1984) notes that this simple decomposition has two consequences: a negative U.I.P coefficient α entails a negative covariance between the risk premium and the expected variation in the exchange rate ($cov(p_{t,t+1}, \Delta e^e_{t,t+1}) < 0$); a U.I.P coefficient α less than $1/2$ entails a risk premium more volatile than the expected depreciation of the exchange rate, ($var(p_{t,t+1}) > var(\Delta e^e_{t,t+1})$.

Keynesian models *a la* Mundell-Fleming or Dornbusch, postulate U.I.P, as do target zone models *a la* Krugman. Flexible price monetary models usually start with the even stronger assumption of continuous purchasing power parity (P.P.P), leading to a constant real exchange rate. For all these models, the stylized fact on U.I.P constitutes a puzzle.

[36] The U.I.P puzzle has also been called "forward premium bias." The U.I.P condition leads to: $\Delta e^e_{t+1} = i_t - i^*_t$. Using the covered interest rate parity condition, $f_t - e_t = i_t - i^*_t$, one obtains a forward rate f_t that should be equal to market expectations of the future spot rate, $f_t = e^e_{t+1}$. Given rational expectations, the expected change in the exchange rate should differ from the realized one only by an expectational error, and the forward rate should hence be a good predictor of the future spot rate. Empirically, however, the forward rate is a very bad predictor of the spot rate; it cannot even correctly forecast the direction of the change in exchange rate.

[37] Note that the risk premium p defined by Fama (1984) is the opposite of the excess return used in this paper.

This paper builds on Backus et al. (2001), who describe the necessary features of a theory that accounts for the forward premium anomaly. When pricing kernels are log-normal, the risk premium equals the difference in their conditional variances. Thus, in order to satisfy Fama (1984)'s first condition and generate a negative U.I.P coefficient, one needs a negative correlation between the difference in conditional means and the difference in conditional variance of the two pricing kernels. To satisfy Fama (1984)'s second condition, one needs a great deal of fluctuation in conditional variances. These necessary features can be directly built in a financial model. For example, in Cox et al. (1985)'s model, the state variable is identified with the spot rate. It is assumed to follow a square-root process, in which the conditional expectation and variance of the short-term interest rates are assumed to be linear in the interest rate itself. Frachot (1996) shows that a two-country version of such a model produces, for certain parameter values, a negative U.I.P slope coefficient. This framework, however, offers no obvious economic explanation for the foreign currency risk premia.[38]

Recently, the development of dynamic stochastic equilibrium models has offered new opportunities for understanding the exchange rate behavior but Backus et al. (2001)'s conditions remain a challenge. In these newer models, the exchange rate risk premium is linked to the covariance of excess returns and stochastic discount factors. But one needs to depart from standard CRRA preferences and flexible prices to produce a sizeable time-varying risk premium. The proposed theoretical frameworks to date are the following (see table I for a summary):

- By assuming sticky prices and following Obstfeld & Rogoff (1995)'s pioneering work, Chari, Kehoe, & McGrattan (2002) produce volatile and persistent exchange rate fluctuations from the interaction of sticky prices and monetary shocks. They introduce "price-discriminating monopolists in order to get fluctuations in real exchange rate from fluctuations in the relative price of traded goods" and "staggered price-setting in order to get persistent real exchange rates." With prices fixed for one year, and a risk-aversion coefficient of 6, they obtain the real exchange volatility found in the data. Their model cannot, however, produce the right price volatility and the

[38]The U.I.P slope coefficient is equal to $(1-e^{-\lambda})/(1-\frac{\partial A^d(1)}{1+\frac{\alpha}{2}A^s A^d(1)})$ where λ, α and A^s are diffusion parameters, and A^d satisfies a unidimensional Riccati differential equation.

right persistence of the real exchange rate at the same time. And increasing the price-stickiness even to four years leads to an autocorrelation that is too low.[39]

- Alvarez et al. (2005) propose an interesting alternative to standard C.I.A models by introducing endogenously segmented markets: higher money growth leads to higher inflation, thus inducing more agents to enter the asset market because the cost of non-participation is higher. This leads to a decrease in risk premium. If segmentation is sufficiently large and sensitive to money growth, this time-varying risk generates the forward premium anomaly. But if inflation is high, for example above a cut-off value $\overline{\pi}$, all agents participate in the market and therefore consumption and risk premia remain constant. Thus, this model can qualitatively reproduce the U.I.P puzzle, while producing U.I.P for high inflation countries (as empirically noted by Bansal & Dahlquist (2000)). Yet, to reproduce quantitatively the U.I.P puzzle the model implies very large flows in and out of the asset markets.

- Moore & Roche (2002) introduce habit-persistence into the classical Lucas (1982) two-country monetary model. They are able to reproduce the relative volatilities of the exchange rate and the risk premium, but not the forward premium bias. Their sample estimates of the U.I.P coefficient α are all negative (the usual forward premium bias), but the results of their calibration experiments are all positive.

- Sarkissian (2003) addresses the issue within the framework of Constantinides & Duffie (1996), assuming heterogenous agents (here countries) that cannot perfectly insure themselves against consumption growth shocks. Two factors, world consumption growth and dispersion, produce a time-varying stochastic discount factor and lead to a negative covariance between the risk premium and depreciation rates. But none of these factors is significant in a beta-pricing framework, and the model can not reproduce the second Fama (1984) condition, $var(p_{t,t+1}) > var(\Delta e^e_{t,t+1})$. This means that the implied U.I.P coefficient is above 1/2.

[39] Chari et al. (2002) do not report results on the interest rate. Around the steady-state, a log-linearization leads to: $\widehat{q} = 5.94(\widehat{c}^* - \widehat{c}) + 0.06(\widehat{m}^* - \widehat{m})$ where a caret denotes the deviation from the steady-state of the log of each variable (resp. real exchange rate, consumption and real balances). On the one hand, if interest rates are pro-cyclical as in the data, the first term above leads to a positive U.I.P coefficient α. On the other hand, real balances decrease with interest rate (elasticity is equal to 0.39 in their model) and this effect pushes α down. The overall effect is therefore not clear.

- Lyons (2001) suggests that investors do not take advantage of arbitrage opportunities when the Sharpe ratio remains below unity. This behavior produces an "inaction zone"that can be matched in terms of the U.I.P coefficient. Hence this coefficient should vary between -1 and $+3$. To understand the negative value obtained on short horizons, one needs to introduce another friction that explains why investors do not fully adapt to changes in the interest rate gap. A limited adaptation hypothesis would predict that the U.I.P coefficient is first negative and then switches sign, tending towards unity as the horizon increases.

- Bacchetta & van Wincoop (2005) develop a model where investors face costs of collecting and processing information. Because of these costs, many investors optimally choose to only infrequently assess available information and revise their portfolios. Rational inattention produces a negative U.I.P coefficient along the lines suggested by Froot & Thaler (1990) and Lyons (2001): if investors are slow to respond to news of higher domestic interest rates, there will be a continued reallocation of portfolios towards domestic bonds and a appreciation of the currency subsequent to the shock. Bacchetta & van Wincoop (2005) obtain negative U.I.P slope coefficient for information and trading costs higher than 2 percents of total financial wealth.

- Departing from full rationality, Gourinchas & Tornell (2004) completely explains the forward premium by assuming that agents misperceive the persistence of interest rate shocks and learning effects. Using survey data, they argue that interest rate forecasts systematically under-react to interest rate innovations. They are able to reproduce both the sign and the magnitude of the U.I.P coefficient.

B. Tables

Table I: Summary of the literature

The table presents a survey of the results obtained on the UIP puzzle (empirically, slope coefficient α is often negative) and the volatility puzzle ($\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$ in the data, where $\sigma^2_{\Delta e}$ is the variance of the change in exchange rates, σ^2_p is the variance of the currency risk premium and $\sigma^2_{i-i^\star}$ is the variance of the interest rate differential).

Papers	*Features*	*UIP puzzle*	*Volatility puzzle*
Lucas (1982)	Two-country, cash-in-advance	$\alpha \simeq 1$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star} > \sigma^2_p$
Bekaert (1996)	Lucas (1982) + Habit persistence	$\alpha < 1/2$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star} > \sigma^2_p$
Moore and Roche (2002)	Lucas (1982) + Habit persistence + Limited participation	$0 < \alpha < 1$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Alvarez et al. (2005)	Lucas (1982) + Endogeneously segmented markets	$\alpha < 0$ for $\pi < \overline{\pi}$	$\sigma^2_{\Delta e} > \sigma^2_{i-i^\star}$
Sarkissian (2003)	Heterogeneity	$0 < \alpha < 1$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Lyons (2001)	Arbitrage zone + Limited adaptation	$-1 < \alpha < 3$ $-1 < \alpha < 0$	n.a
Bacchetta and van Wincoop (2005)	Information costs	$\alpha < 0$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Gourinchas and Tornell (2004)	Limited rationality	$\alpha < 0$	$\sigma^2_{\Delta e} > \sigma^2_p > \sigma^2_{i-i^\star}$
Obstfled and Rogoff (1995)	Monopolists + Sticky prices + UIP	$\alpha = 1$	$\sigma^2_{\Delta e} = \sigma^2_{i-i^\star}, \sigma^2_p = 0$
Chari et al. (2002)	Monopolists + Sticky prices	n.a	n.a

Table II
Calibration parameters

Data are quarterly. The reference period is here 1947:2-2004:3 (1947-1995 in Campbell & Cochrane (1999), 1952:2-2004:3 in Wachter (2006)). Per capita consumption data are from the BEA web site. Interest rates and inflation data are from CRSP(WRDS). Expected inflation is computed using a one-lag two-dimension VAR (inflation and interest rate). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation. The Sharpe ratio is obtained as the ratio of the unconditional mean of monthly stocks excess returns on their unconditional standard deviation. The U.I.P coefficient is computed using the US-German exchange rate. German interest rates, inflation rates and exchange rates are from Global Financial Data.

	My parameters	**Campbell & Cochrane (1999)**	**Wachter (2006)**
		Calibrated parameters	
$g(\%)$	0.53	0.47	0.55
$\sigma(\%)$	0.51	0.75	0.43
$\overline{r}(\%)$	0.34	0.23	0.66
γ	2.19	2.00	2.00
ϕ	0.99	0.97	0.97
B	-0.01	-	0.01
τ	0.33 / 3 / ∞	-	-
		Implied parameters	
β	1.00	0.97	0.98
$\overline{S}$	0.08	0.06	0.04
$S_{\max}$	0.12	0.09	0.06

Table III
Simulation Results

Quarterly values. The table presents the mean (g) and standard deviation (σ) of consumption growth, the mean ($\overline{r}$), standard deviation (σ_r) and autocorrelation ($\rho_{r_t,r_{t-1}}$) of the real interest rate and the standard deviation ($\sigma_{\frac{\triangle q}{q}}$) and autocorrelation ($\rho_{q_t,q_{t-1}}$) of the real exchange rate. α denotes the UIP slope coefficient and $s.e$ the associated standard error. $\overline{SR}$ denotes the mean Sharpe ratio. $\rho_{\triangle q_t,\triangle c_t}$ denotes the correlation between consumption growth and changes in exchange rate.

	Simulation Results			**Data**
	$\tau=\infty$	$\tau=1/3$	$\tau=3$	
$g(\%)$	0.53	0.53	0.53	0.53
$\sigma(\%)$	0.51	0.32	0.48	0.51
$\overline{r}$	0.34	0.31	0.30	0.34
σ_r (%)	0.54	0.40	0.44	0.57
$\rho_{r_t,r_{t-1}}$	0.99	0.99	0.99	0.62
$\sigma_{\frac{\triangle q}{q}}$ (%)	22.75	2.79	8.36	7.53
$\rho_{q_t,q_{t-1}}$	0.94	0.99	0.99	0.92
α	-1.42	-0.10	-0.61	-1.41
$(s.e.)$	(0.16)	(0.02)	(0.05)	(1.30)
$\overline{SR}$	0.13	0.15	0.13	0.15
$\rho_{\triangle q_t,\triangle c_t}$	0.60	0.17	0.34	0.09

Table IV
Estimation Results

Model estimated using consumption data for the stochastic discount factors. "G3 "corresponds to all the cross-border investment opportunities of American, German and Japanese investors. "1 US inv., 14 C"takes into account the possible excess returns of an American investor in 14 other O.E.C.D countries (Australia, Belgium, Canada, France, Germany, Italy, Japan, Korea, Mexico, Netherlands, Norway, Spain, Switzerland and the United Kingdom). Quarterly values, 1981:3-2002:2. Standard errors in parenthesis.

	G3	**1 US inv., 14 C**
Number of excess returns N	6	14
$J_{\min}$	5.30	11.23
$p - value = 1 - \chi^2(J_{\min}, N - p)$	0.17	0.42
$\widehat{\gamma}$	2.50 (1.80)	4.4 (1.0)
$\widehat{\phi}$	0.91 (0.13)	0.84 (0.16)
$\widehat{\overline{S}}$	0.018 (0.015)	0.022 (0.008)
$\widehat{B}_{implied}$	−0.18 (0.05)	−1.47 (0.26)
$\widehat{\alpha}_{implied}$	−1.99 (0.85)	−0.48 (0.05)

C. Figures

Figure 1. Histogram of the simulated surplus consumption ratio (in log) and the local curvature coefficient. Parameters presented in the first column of Table (II) with trade cost $\tau = \infty$ (no trade).

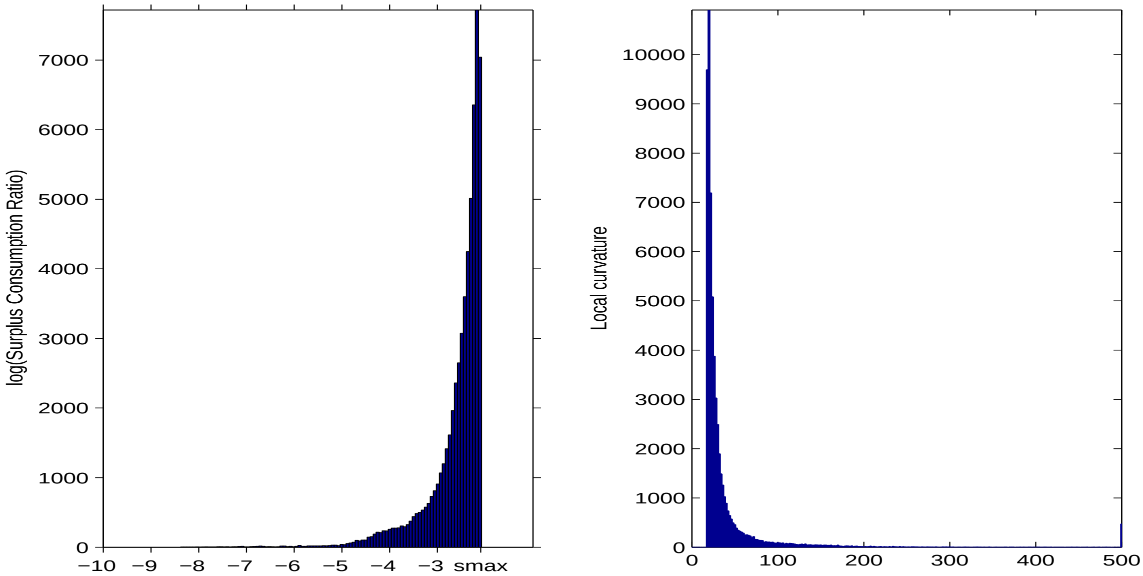


Figure 2. Histogram of the simulated exchange rate (level in log and quarterly change). Parameters presented in the first column of Table (II) with trade cost $\tau = \infty$ (no trade).

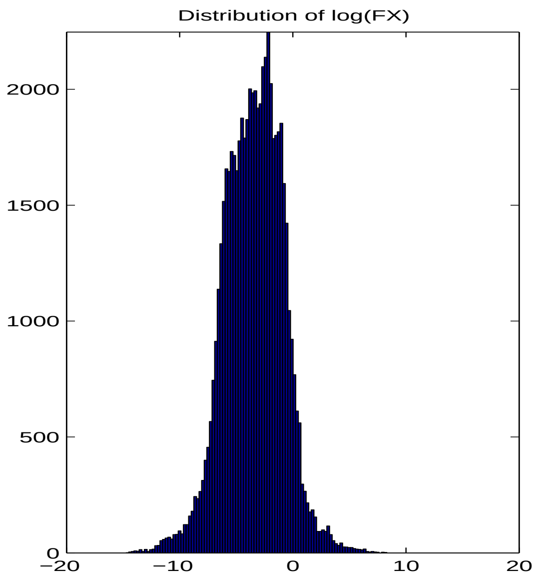


Figure 3. Histogram of the simulated surplus consumption ratio (in log) and the local curvature coefficient. Parameters presented in the first column of Table (II) with trade cost $\tau = 3$.

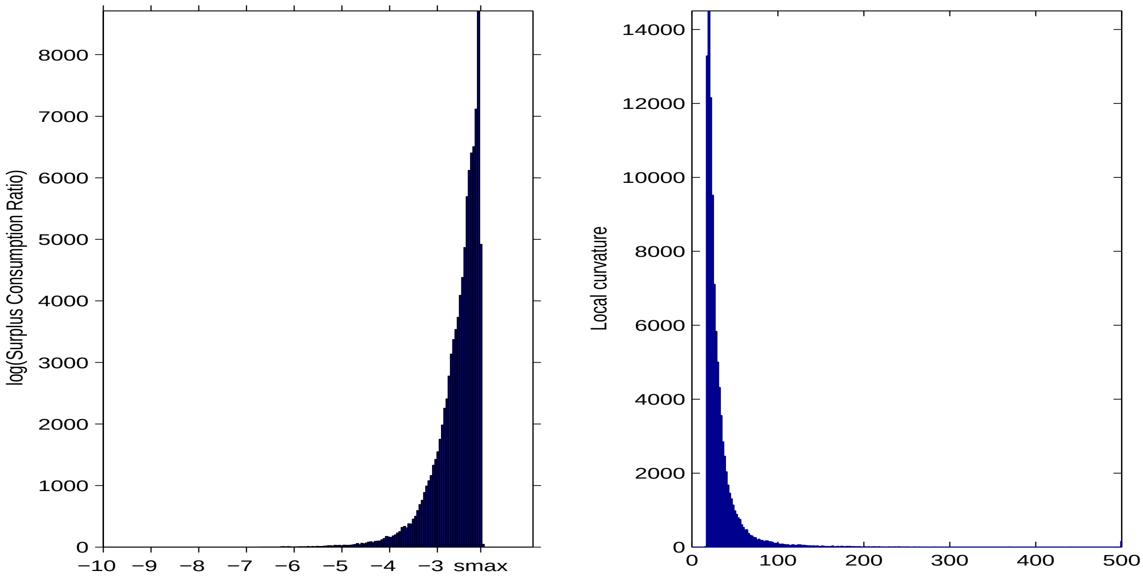


Figure 4. Histogram of the simulated exchange rate (level in log and quarterly change). Parameters presented in the first column of Table (II) with trade cost $\tau = 3$.

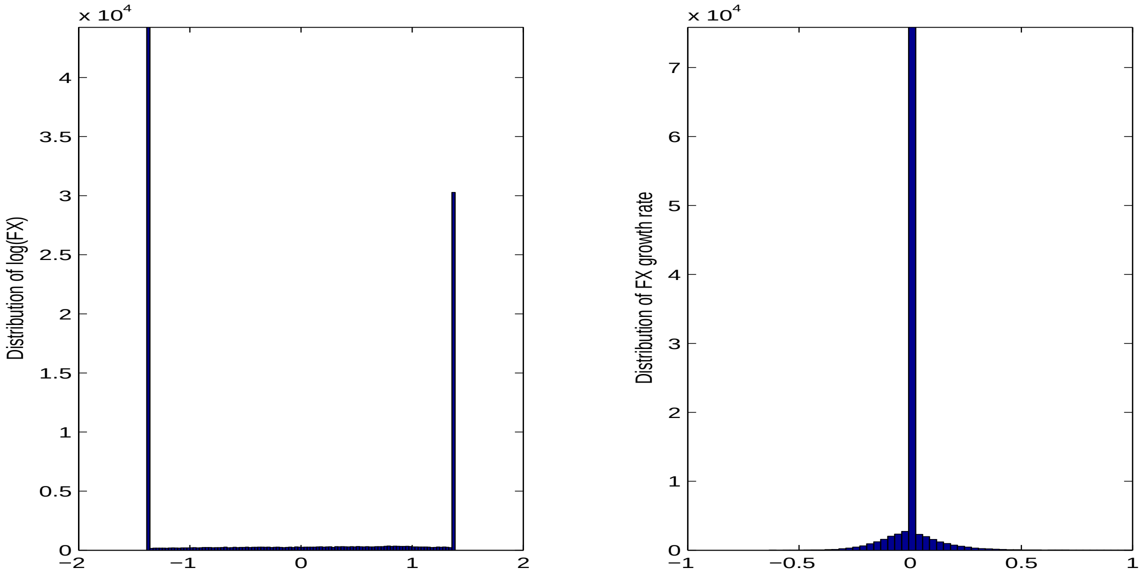


Figure 5. Histogram of the actual exchange rate (level in log and quarterly change) for US-Germany over the $1947:2-2004:3$ period.

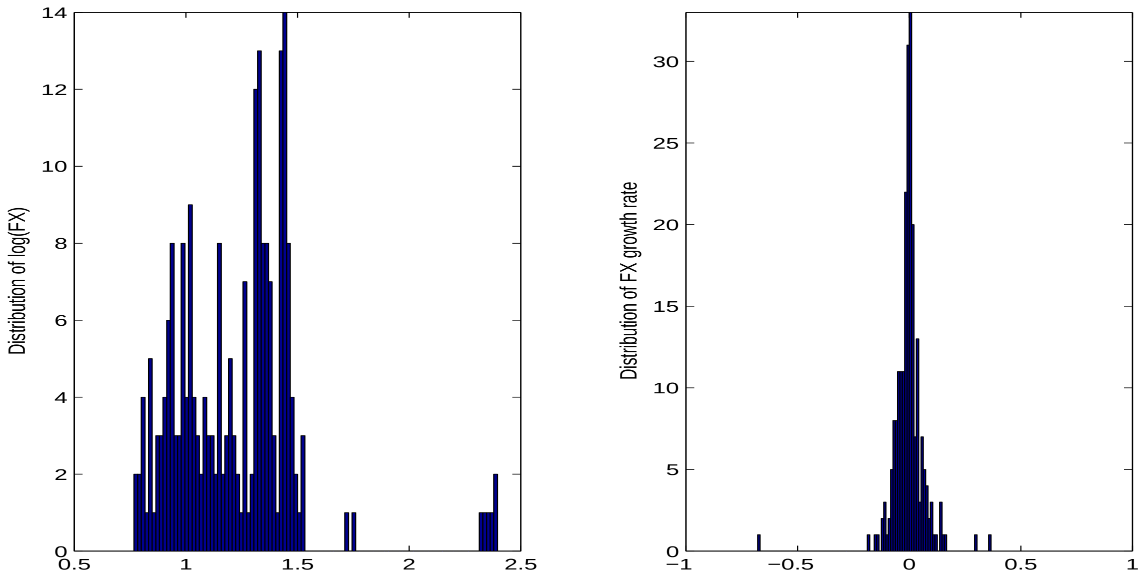


Figure 6. Reality check. Stochastic discount factor (SDF), surplus consumption ratio (SP) and local curvature for an American investor computed with actual US consumption data only over the $1947:2-2004:3$ period using the parameters presented in the first column of Table (II).

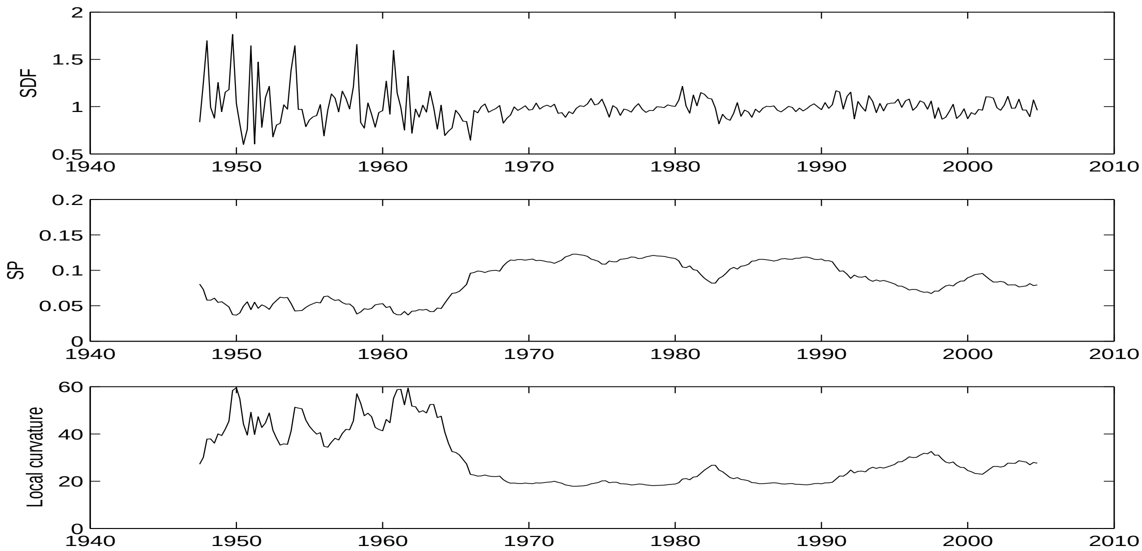


Figure 7. J criterion estimated on 14 moments from foreign investments of an American investor. Quarterly data $1981:3-2002:2$.

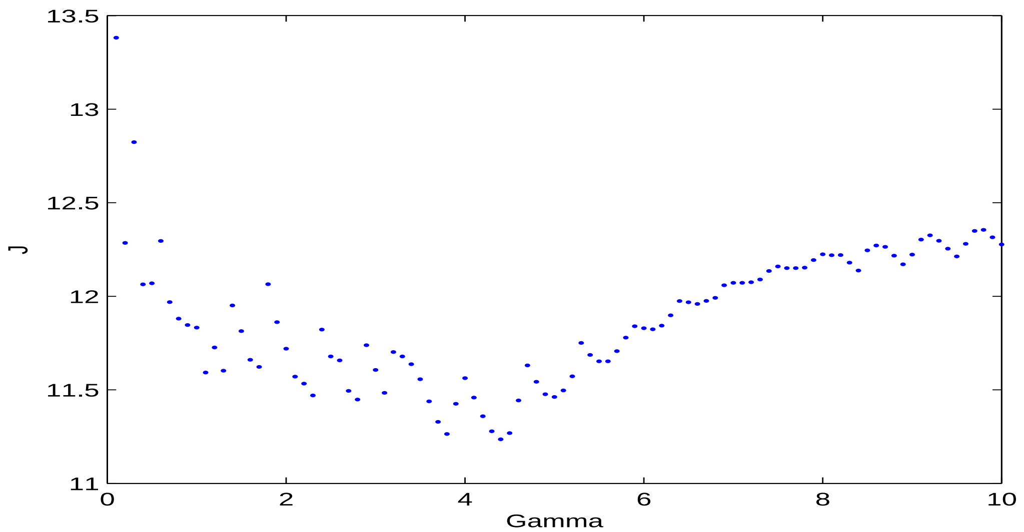
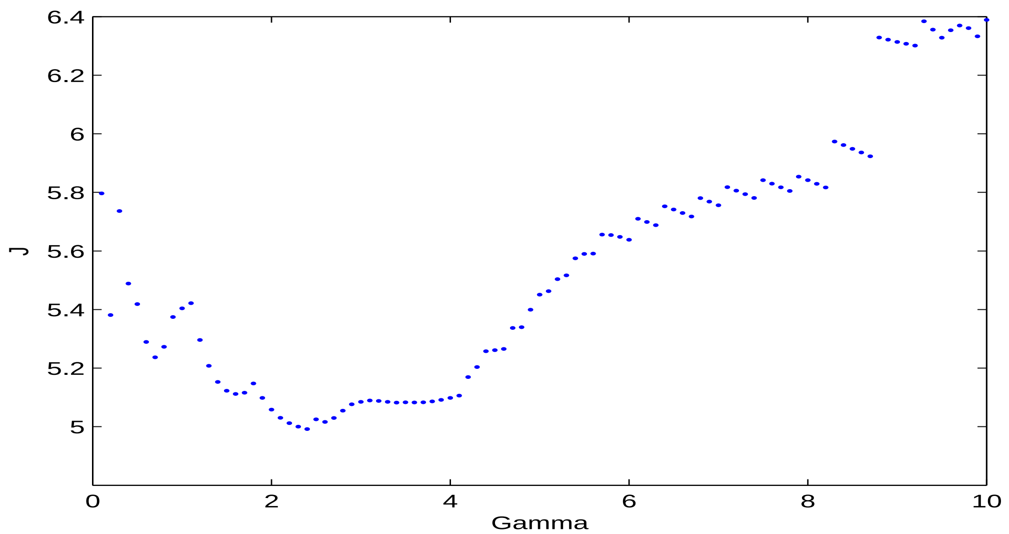


Figure 8. J criterion estimated on 6 moments from cross-investments of 3 countries (Germany, Japan, USA) using only consumption data. Quarterly data $1981:3-2002:2$.



D. Simulation Method

I first draw 100000 *i.i.d* endowment shocks. From the endowment shocks and the parameters of the model, I build the endowment process. Then, for each date, I compute the optimal amount of exports, imports and consumption. Solving the social planner program presents one difficulty. Trade at date $t+1$ in equations (4) and (5) depend on the habit level at date $t+1$. The habit level cannot be computed using the exact law of motion described in equation (C) because it seems to require the value of consumption at date $t+1$. But Campbell & Cochrane (1999) chose the sensitivity function (7) so that the habit level at date $t+1$ does not actually depend on consumption level on the same date. This can be shown using a first order Taylor approximation of the law of motion of the habit level x_{t+1} when s_t is close to its steady-state value $\bar{s}$ and the consumption growth $c_{t+1} - c_t$ is close to its average g. I use the same steps outlined in the NBER version of Campbell & Cochrane (1995) (footnote 1 page 6).

The log surplus consumption ratio is equal to:

$$s_t = \ln(\frac{e^{c_t} - e^{x_t}}{e^{c_t}}).$$

Let h be the steady-state value of $x_t - c_t$. Then a first-order Taylor approximation of s_t around $\bar{s}$ leads to:

$$s_t - \bar{s} \simeq (1 - \frac{1}{\bar{S}})(x_t - c_t - h).$$

Likewise,

$$\lambda(s_t)(c_{t+1} - c_t - g) \simeq \lambda(\bar{s})(c_{t+1} - c_t - g).$$

Equation (C) leads to:

$$(1 - \frac{1}{\bar{S}})(x_{t+1} - c_{t+1} - h) = \phi(1 - \frac{1}{\bar{S}})(x_t - c_t - h) + \lambda(\bar{s})(c_{t+1} - c_t - g).$$

Campbell & Cochrane (1999) chose the sensitivity function $\lambda(s_t)$ so that the habit level x_{t+1} does not depend on c_{t+1} ($\lambda(\bar{s}) = -(1 - \frac{1}{\bar{S}})$).

Thus,

$$x_{t+1} - h = \phi(x_t - c_t - h) + c_t + g$$

$$x_{t+1} = \phi x_t + [(1-\phi)h + g] + (1-\phi)c_t \tag{12}$$

Equation (D) gives a first guess for the habit level at date $t+1$, thus allowing the computation of trade and consumption at date $t+1$. This new estimate of consumption is used to compute the habit level using the exact law of motion and the process is iterated until convergence.

At each date, the real risk-free rates and the real exchange rate are computed from consumption data. I then regress the quarterly variation of the real exchange rate on the real interest rate differential to find the coefficient α from a U.I.P test.

E. Estimation Method

Let b be the vector of parameters to estimate, $b = [\gamma\ \phi\ \overline{S}]$ for example. The criterion is $J = T\ \times\ \overline{f_T}\ \times\ inv(\Omega)\ \times\ \overline{f_T}$ where $\overline{f_T}$ is the sample mean of f_t, Ω is the variance-covariance matrix of f_t ($\Omega = T f_t' f_t$) and f_t is the product of the stochastic discount factor and excess return at each date t:

$$f_t(b) = M_t R_t,$$

$$\overline{f}_T(b) = \frac{1}{T}\sum_{t=1}^{T} f_t.$$

For each couple (γ, ϕ), define the minimum value of $J_{\gamma,\phi} = \underset{\overline{S}}{Min}\ J(\gamma, \phi, \overline{S})$. For each value of γ, define $J_\gamma = \underset{\phi}{Min}\ J_{\gamma,\phi}$. Then consider $J_{\min} = Min\ J_\gamma$. The asymptotic distribution of the GMM estimate is (Hansen, 1982) :

$$\sqrt{T}(\widehat{b} - b) \to N[0, (ad)^{-1} a \Sigma a' (ad)^{-1\prime}],$$

where $a = \frac{\partial \overline{f}_T'(b)}{\partial b}\Omega^{-1}$ and $d = \frac{\partial \overline{f}_T(b)}{\partial b'}$. Σ is the spectral density matrix $\Sigma = \sum_{j=-\infty}^{\infty} E[f_t(b) f_{t-j}(b)]$ and the precision around b is given by $var(\widehat{b}) = \frac{1}{T}(ad)^{-1} a \Sigma a' (ad)^{-1\prime}$. Due to the sample's size, I use the variance-covariance matrix instead of the spectral density matrix in the estimation.

The stochastic discount factor can be expressed in terms of the parameters b to compute d:

$$M_{t+1} = \beta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \beta G^{-\gamma} e^{-\gamma\left[(\phi-1)(s_t-\bar{s})+(1+\lambda(s_t))v_{t+1}\right]}.$$

For each excess return R^i_{t+1}:

$$\frac{\partial f^i_t(b)}{\partial \gamma} = -[(s_{t+1} - s_t) + (c_{t+1} - c_t)] M_{t+1} R^i_{t+1},$$

$$\frac{\partial f^i_t(b)}{\partial \phi} = \gamma (s_t - \bar{s}) M_{t+1} R^i_{t+1},$$

$$\frac{\partial f_t^i(b)}{\partial S} = \left[\frac{\gamma(\phi-1)}{\overline{S}} - \gamma v_{t+1}\frac{\partial \lambda(s_t)}{\partial S}\right] M_{t+1} R_{t+1}^i,$$

where $\lambda(s_t) = \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})}-1$ and $\frac{\partial \lambda(s_t)}{\partial S} = (-\frac{1}{S^2})\sqrt{1-2(s_t-\bar{s})}+\frac{1}{S^2}[1-2(s_t-\bar{s})]^{-\frac{1}{2}} = [-\frac{1}{S}\ \lambda(s_t)+\frac{1}{S^3}\frac{1}{\lambda(s_t)}]$.