Optimal Capital Taxation under Limited Commitment

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Abstract

We study optimal capital taxation under limited commitment. We prove that the optimal tax rate on capital income should be positive in steady state and should be increasing over time provided that full risk-sharing is not feasible.

In a limited commitment environment, a one unit increase of capital investment by an agent increases all individuals' autarky values in the economy and generates externality costs in the economy. This externality cost provides a rationale for positive capital taxation even in the absence of government expenditure. Moreover, we show that both this externality cost of capital investment and the optimal tax rate are potentially much bigger than one might expect.

1 Introduction

In the Ramsey literature of capital taxation, Chamley (1986) and Judd (1985) argue for zero capital taxation in the long run. Chari and Kehoe (1999) show that the capital tax rate should be high initially and decrease to zero. Moreover, Atkeson, Chari and Kehoe (1999)

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show that the zero capital taxation result is robust to a wide range of the assumptions. Finally, Lucas (1990) argues that for the U.S. economy there is a significant welfare gain to be realized in switching to this policy. In sum, the zero capital taxation argument suggests that the current capital stock in the U.S. economy is too low since the capital tax rate is too high, and that decreasing the tax rate can lead to large welfare gains.

There is a competing literature that argues that certain frictions can rationalize capital taxation. Aiyagari (1996) shows that with incomplete markets, agents have a precautionary savings motive which leads them to overinvest in capital. He proves that the optimal capital income tax should be positive in the long run so that this over-investment is reduced.

Golosov, Kocherlakota and Tsyvinski (2003) obtain a nonzero optimal capital taxation result by introducing incentive constraints, which arise from the private information of an individual's idiosyncratic shock. To motivate high-skilled agents to reveal their type, they argue that the tax burden of high-skilled agents should be lighter than that of low-skilled agents

We study another type of economy that is closely related to the recent literature[1] of endogenous incomplete markets where there is a continuum of households with idiosyncratic shocks and there exists a complete set of contingent claims, but financial contracts are not perfectly enforceable. As in Kehoe and Levine (1993) and Alvarez and Jermann (2000), we have endogenous debt limits in the form of enforcement constraints so that households are not able to accumulate more debt than they are willing to pay back. If a household defaults on a financial contract, he can be excluded from future contingent claims markets trading and can have his assets seized. The private sector that faces a possibility of being debt-constrained in the future has a higher discount factor for one unit of future consumption than does the planner, who does not face this constraint.

Chien and Lee (2005) argue that a positive capital income tax is required to achieve optimal capital levels as a result of deviations of the market return from the time preference rate. They set up a model inhabited by a continuum of heterogeneous agents; there is a complete set of contingent claims, but financial contracts are not perfectly enforceable. As in Aiyagari (1996), they also have a government as an unconstrained investor for physical capital. A capital income tax is required to equate the pre-tax return on capital with the

[1] Alvarez and Jermann (2000, 2001), Kehoe and Levine (1993), Kehoe and Perri (2002, 2004), Kocherlakota (1996), Krueger (1999), Lustig (2005) among others.

time preference rate. The steady state capital income tax rate is chosen as follows:

$$\tau_K = 1 - \frac{1/q - 1}{1/\beta - 1},$$

where β is the time discount factor and q is the price of risk-free debt.

This paper considers a different role for the government. We now assume that the government plays only a minimal role in the economy: it collects tax revenues and simply transfers it back to the households in a lump-sum fashion. Otherwise, this paper considers the same economic environment as Chien and Lee (2005). There is no aggregate shock in the economy; however agents are exposed to an idiosyncratic labor shock. We assume that once an agent defaults on a financial contract, the punishment is that he is permanently excluded from the financial market and his only source of income is labor income. Both of these assumptions are common in the literature.

In such an environment, we re-evaluate the zero capital taxation result in the planner's problem and find that there is in fact a role for capital income taxes. We prove that the optimal capital tax rate should be strictly positive and should increase over time up to a certain point. To do this, we first solve for the constrained efficient allocations. Then we introduce capital taxation in order to decentralize the economy: the tax on capital income is needed to make private agents internalize an additional cost of capital investment.

This additional cost obtaining from the limited commitment environment as follows. A higher level of capital stock (capital investment) increases labor income (marginal product of labor in equilibrium), which in turn increases the value of autarky. Higher autarky values increase costs to the planner: the planner must increase compensation to agents with binding enforcement constraints since these agents might otherwise to be tempted to leave the risk sharing pool.

This can be seen by examining the effect of an additional unit of capital at time T: increasing the capital stock at period T by one unit improves the marginal productivity of labor, which directly improves autarky values in periods $t = 1, \ldots, T$; thus all earlier enforcement constraints (from $t = 1, ..., T$) become tighter.

We call this additional compensation an "externality cost". This positive externality cost implies non-zero capital taxation in the decentralized economy. Moreover, the externality costs accumulate over time, and this cumulative externality cost implies that the capital tax rate should be increasing over time.

We decentralize the constrained efficient allocations with solvency constraints and a capital tax. Solvency constraints are constructed in the same way as in Alvarez and Jermann

(2000). Our construction of the capital tax is different from that of Kehoe and Perri (2004), since we introduce financial intermediaries which enable us to have a linear capital tax, whereas Kehoe and Perri's capital tax rate was agent-specific[2] .

We do not address how to design a labor income tax that supports the constrained efficient allocation. This is for the following reasons. First, traditional Ramsey literature argues that the capital tax rate should be zero even in the absence of a labor tax. Second, introducing a labor tax does not change our result of positive capital taxation as long as full risk-sharing is not feasible. Labor tax, however, may lower the steady state tax rate on capital income in the quantitative analysis since a positive labor tax will relax enforcement constraints[3].

This paper is organized as follows. Section 2 describes the model economy and characterizes the constrained efficient allocations of this economy. Section 3 characterizes the steady state constrained efficient allocations. Section 4 decentralizes the allocations with capital taxation and solvency constraints. Section 5 carries out the quantitative analysis for our calibrated model. Section 6 concludes.

2 Model

2.1 Environment

There is a continuum of agents of measure one. They receive an initial promised utility (ν_0) and initial idiosyncratic shock (s_0) over an initial joint distribution Φ_0. There is a single, non-storable, consumption good. The agents rank consumption streams $\{c_t\}$ according to the following preference:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\middle|\, s_0\right)\frac{c_t\left(\nu_0,s^t\right)^{1-\gamma}}{1-\gamma}$$

where we assume the power utility with risk-aversion coefficient γ.

There is no aggregate uncertainty. The only event that each household faces is a stochastic idiosyncratic labor supply shocks. Each event s_i takes on values on a discrete grid $S\equiv\{s_1,...,s_i,...,s_I\}$. The idiosyncratic shock s follows a Markov process with a transition probability $\pi\left(s'|s\right)$. We assume the law of large numbers holds so that the transition probabilities can be interpreted as the fractions of agents making the transition from one state to

[2]In theirs, there are two countries (agents) and tax rates are different for each country (agent).

[3]See Chien and Lee (2005)

another. In addition, we assume that there is a unique invariant distribution $\bar{\Pi}(s)$ in each state s. Again, by the law of large numbers $\bar{\Pi}(s)$ is the fraction of agents drawing s in every period. We denote s_t as the history of shock realizations:

$$s^t = (s_0, s_1, ..., s_{t-1}, s_t)$$

We assume that an aggregate labor supply is perfectly inelastic for all periods and denote it as $\bar{L}$. An agent's labor supply (hours worked) is denoted by

$$\bar{L} \cdot s$$

We normalize the average idiosyncratic labor supply shock to be one.

$$1 = \int s_t d\Phi_t$$

The output in the economy is produced using a single technology that exhibits constant returns to scale:

$$\begin{aligned} Y_t &= F(K_t, L_t) \\ &= K_t^{\alpha} L_t^{1-\alpha} \end{aligned}$$

where $F(\cdot, \cdot)$ is a production function, and K_t and L_t denote the aggregate capital input and the aggregate labor input respectively. We use a Cobb-Douglas production function with capital income share $\alpha \in [0, 1]$.

The feasibility constraint for the economy is that the output can either be consumed or invested in the capital stock of the next period:

$$\sum_{s^t} \int c_t\left(W_0, s^t\right) \pi\left(s^t \middle| s^0\right) d\Phi_0 + K_{t+1} = K_t^{\alpha} \bar{L}^{1-\alpha} + (1-\delta) K_t, \text{ for } \forall t$$

where $\delta \in [0, 1]$ denotes the depreciation rate of capital.

2.2 Enforcement Technology

Following Kehoe and Levine (1993) and Kocherlakota (1996), this literature commonly assumes that in the decentralized economy, households are excluded from financial markets

forever when they default. We assume a more severe punishment upon default: households are not only excluded from the contingent claims markets forever but also (1) its current wealth is seized by the creditor and (2) it cannot receive any lump-sum transfer from the government. That is, the household loses all of its assets and income flows but its labor income cannot be garnished by the creditor. Hence, its only source of income beginning from the default period will be its labor income. The household who defaults at period t will have the following simple budget constraints for $\forall\ \tau \geq t$:

$$c_\tau = w_\tau s_\tau \overline{L}$$

The autarky value V_{aut} at period t can therefore be written as:

$$V_{aut}\left(s_t, \{w_\tau\}_{\tau=t}^{\infty}\right) = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi\left(s^\tau | \, s^t\right) u\left(w_\tau \cdot \bar{L} s_\tau\right)$$

where w_t denotes the wage rate.

The households face an enforcement constraint. That is, the allocations are constrained so that planner makes them better off than autarky in every possible node in history:

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi\left(s^\tau | \, s^t\right) u\left(c_\tau\right) \geq \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi\left(s^\tau | \, s^t\right) u\left(F_L\left(K_\tau, \bar{L}\right) \cdot \bar{L} s_\tau\right), \ \forall t \geq 0, \ \forall s^t \quad (1)$$

For the autarky value in the planner's problem, we substitute the marginal product of labor $F_L\left(K_\tau, \bar{L}\right)$ for the wage rate w_τ from the equilibrium condition.

2.3 Planner's problem

As in Kochelakota (1996) and Alvarez and Jermann (2000, 2001) , we set up the planner's problem to discuss the constrained efficient allocations.

Planner is assumed to be benevolent so that he maximizes the social welfare:

$$\max_{\{c_t, K_{t+1}\}} \int \sum_{t}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t \middle| \, s^0\right) a_0\left(\nu_0, s_0, K_0\right) u\left(c_t\left(\nu_0, s^t, K_t\right)\right) d\Phi_0$$

subject to

$$\sum_{s^t} \int c_t\left(\nu_0, s^t, K_t\right) \pi\left(s^t \middle| s^0\right) d\Phi_0 \leq K_t^{\alpha} \bar{L}^{1-\alpha} - K_{t+1} + (1-\delta) K_t \ , \forall t \tag{2a}$$

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi\left(s^\tau \middle| s^t\right) u\left(c_\tau\left(\nu_0, s^\tau, K_\tau\right)\right) \geq \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi\left(s^\tau \middle| s^t\right) u\left(F_L\left(K_\tau, \bar{L}\right) \cdot \bar{L} s_\tau\right), \tag{2b}$$

$$\forall t, \forall\left(\nu_0, s^t\right)$$

where a_0 is a Pareto weight assigned to the each agent by the planner and ν_0 is initial promised utility.

Planner maximizes the social welfare subject to constraints $(2a)$ and $(2b)$. Constraint $(2a)$ is a feasibility constraint which must hold for all t and constraint $(2b)$ is an enforcement constraint which must hold for all t and all (ν_0, s^t) and implies that each agent's continuation value in the risk-sharing pool (i.e. each agent's continuation value of staying in the economy) should be at least as large as the value of autarky for all t and nodes. Let the Lagrangian multipliers on $(2a)$ and $(2b)$ be $\theta_t(K_t)$ and $\beta^t \pi\left(s^t \middle| s^0\right) \mu_t\left(\nu_0, s^t, K_t\right)$ respectively.

In order to make the problem recursive, we can define cumulative multipliers: $\xi_t\left(\nu_0, \eta^t, K_t\right)$[4]

$$\xi_t\left(\nu_0, s^t, K_t\right) = a_0\left(\nu_0, s_0, K_0\right) + \sum_{s^r \preceq s^t} \mu_r\left(\nu_0, s^r, K_r\right),$$

where s^r is a subsequent history of s^t. We can rewrite cumulative multiplier recursively

$$\begin{aligned} \xi_t\left(\nu_0, s^t, K_t\right) &= \xi_{t-1}\left(\nu_0, s^{t-1}, K_{t-1}\right) + \mu_t\left(\nu_0, s^t, K_t\right), \\ \xi_0\left(\nu_0, s_0, K_0\right) &= a_0\left(\nu_0, s_0, K_0\right) \end{aligned}$$

where $\{\xi_t\left(\nu_0, s^t, K_t\right)\}$ is a non-decreasing stochastic process.

The Lagrangian can now be written as

$$\begin{aligned} L = & \int \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[\begin{array}{l} \xi_t\left(\nu_0, s^t, K_t\right) u\left(c_t\left(\nu_0, s^t, K_t\right)\right) \\ -\left[\xi_t\left(\nu_0, s^t, K_t\right) - a_0\left(\nu_0, s_0, K_0\right)\right] u\left(F_{\bar{L},\tau}\left(K_\tau, \bar{L}\right) \cdot \bar{L} s_\tau\right) \end{array}\right] d\Phi_0 \\ & + \sum_{t=0}^{\infty} \theta_t\left(K_t\right) \left[K_t^{\alpha} \bar{L}^{1-\alpha} + (1-\delta) K_t - K_{t+1} - \sum_{s^t} \int c_t\left(\nu_0, s^t, K_t\right) \pi\left(s^t \middle| s^0\right) d\Phi_0\right] \end{aligned}$$

[4]Marcet and Marimon (1999)

The next step is to derive the first-order necessary conditions. The first-order conditions are the following:

$$\theta_t = \beta^t \xi_t u'(c_t) \tag{3}$$

$$\theta_t = \theta_{t+1}\left[F_{K_{t+1},t+1} + (1-\delta)\right] - \underbrace{\left[\sum_{s^{t+1}} \int \beta^{t+1} \pi\left(s^{t+1}\right) \cdot \left[\xi_{t+1} - a_0\right] \cdot \frac{\partial u\left(F_{\bar{L},t+1}\left(K_{t+1},\bar{L}\right)\cdot \bar{L} s_{t+1}\right)}{\partial K_{t+1}} d\Phi_0\right]}_{\text{Externality Cost}} \tag{4}$$

Equation (3) is the first-order condition with respect to an individual agent's consumption. θ_t is an aggregate variable since it is the shadow price of the feasibility condition. ξ_t is a summary statistic of an agent's history. It measures how severely and how many times the agent has been constrained in his history. Therefore, equation (3) implies that the agent's consumption is history-dependent and that the agent's consumption should be higher if he has a higher ξ_t. We will characterize the agent's consumption allocation in the next section.

Equation (4) is the first-order condition with respect to aggregate capital investment K_{t+1}. The first line of the equation is a standard Euler equation. This Euler equation, however, contains the second term which we call the externality cost. This is the additional cost that the planner must pay in order to keep the agent from defaulting. This term contains (1) all the multipliers on enforcement constraints as summarized by the shadow prices (costs) of the enforcement constraints $\xi_t - a_0 \ \left(= \mu_1 + \cdots + \mu_{t+1}\right)$, and (2) an increment in per-period autarky value when the capital stock is increased by one unit. To see how one unit of capital affects the externality cost term, consider the effect of such a change on capital stock at period T. This change will increase marginal product of labor, thereby increasing autarky values (that are solely dependent upon labor income) and making autarky more tempting. This change will affect all autarky values – and hence all enforcement constraints – prior to period T, and as a result increase the cumulative multiplier. We will also characterize the capital allocation in the next section.

2.4 Characterization of Constrained Efficient Allocations

In this section, we characterize the constrained efficient allocations. We will first characterize an agent's consumption allocation and the shadow price of one unit of consumption next period. Second, we will discuss the capital allocation and externality cost that the planner encounters when he makes a capital investment decision.

2.4.1 Consumption Allocations[5]

Enforcement constraints introduce a stochastic element into the consumption share of each household. The household's initial promised utility, v_0 determines its initial Pareto weight a_0 and this weight governs the household's consumption share in all future states of the world. When there are no enforcement constraints, the household's consumption share is constant over time:

$$\begin{aligned} c_t\left(\nu_0, s^t, K_t\right) &= \frac{a_0^{1/\gamma}}{E\left[a_0^{1/\gamma}\right]} C\left(K_t\right), \\ \text{where } C\left(K_t\right) &= \sum_{s^t} \int c_t\left(\nu_0, s^t, K_t\right) \pi\left(s^t \middle| s^0\right) d\Phi_0, \\ E\left[a_0^{1/\gamma}\right] &= \int a_0^{1/\gamma} d\Phi_0 \end{aligned}$$

However, when enforcement constraints exist, the Pareto weights become stochastic and so does the household's consumption share. The household's consumption is characterized by the same linear risk sharing rule:

$$\begin{aligned} c_t\left(\nu_0, s^t, K_t\right) &= \frac{\xi_t^{1/\gamma}\left(w_0, s^t, K_t\right)}{E\left[\xi_t^{1/\gamma}\left(\nu_0, s^t, K_t\right)\right]} C\left(K_t\right), \\ \text{where } C\left(K_t\right) &= \sum_{s^t} \int c_t\left(\nu_0, s^t, K_t\right) \pi\left(s^t \middle| s^0\right) d\Phi_0 \end{aligned} \tag{5}$$

The household's consumption share is stochastic. Recall that $\xi_t\left(w_0, s^t, K_t\right)$ is the sum of all enforcement constraints in history s^t plus the initial Pareto weight, a_0, and that these cumulative multipliers stay constant until the household switches to a state with a binding enforcement constraint. When this occurs, the multipliers increase so that the enforcement constraint is satisfied with equality.

Let $h_t\left(K_t\right)$ denote the $1/\gamma$th cross sectional moment of the cumulative multiplier:

$$h_t\left(K_t\right) = E\left[\xi_t^{1/\gamma}\left(\nu_0, s^t, K_t\right)\right]$$

This process $h_t\left(K_t\right)$ is also a non-decreasing process and measures how many agents become constrained and how severely they are constrained.

[5] See Lustig (2005)

This risk-sharing rule implies that when the household does not switch to a state with an enforcement constraint, its consumption share drifts downwards at the rate of the growth rate of $h_t(K_t)$. The derivation of the risk sharing rule is found in Lustig (2005).

Next, we discuss the period t shadow price of one unit of consumption at $t+1$.

$$\begin{aligned} q_t(K_t) &= \frac{\theta_{t+1}(K_{t+1})}{\theta_t(K_t)} \\ &= \beta\left[\frac{C(K_{t+1})}{C(K_t)}\right]^{-\gamma}\left[\frac{h_{t+1}(K_{t+1})}{h_t(K_t)}\right]^{\gamma} \\ &= \frac{1}{R_t(K_t)} \end{aligned} \tag{6}$$

This shadow price is the same as the one in Lustig (2005) and this contains the multiplicative adjustment cost of $\left[\frac{h_{t+1}(K_{t+1})}{h_t(K_t)}\right]^{\gamma}$. It measures the shadow cost of the enforcement constraints. As the growth rate of $h(K_t)$ in the economy increases, the cost of the enforcement constraints increases and the more the planner needs to compensate at $t+1$. Hence the price of one unit of consumption increases.

2.4.2 Capital Allocations

In this part, let us characterize the capital allocation. We can rewrite equation (4) together with equations (3), (5) and (6) as follows:

$$1 = q_t(K_t)\left[F_{K,t+1} + (1-\delta)\right] - \chi_{t+1}$$

where

$$\chi_{t+1} = \frac{1}{\beta^t C_t^{-\gamma} h_t^{\gamma}} \sum_{s^{t+1}} \int \beta^{t+1}\left[\xi_{t+1} - a_0\right] \frac{du\left(F_{\bar{L},t+1}\bar{L}\eta_{t+1}\right)}{dK_{t+1}} \pi\left(s^{t+1}\right) d\Phi_0$$

The above Euler equation would be a standard one if it did not contain the positive term χ_{t+1} on the right hand side. As a result of this extra term, the standard marginal return of investing one unit of capital exceeds the marginal cost of giving up one unit of consumption in constrained efficient allocations. Hence, there is an additional cost to the planner such that in equilibrium, the marginal benefit is equal to the marginal cost. We call the term χ_{t+1} the "externality cost of capital investment". We argue that this externality cost of capital investment induces the need for a tax on capital income in order to make private agents internalize the externality in the decentralized economy.

Externality Cost of Capital Investment χ_{t+1}*:* we now focus on the externality cost which is the last term on the right hand side of the Euler equation:

$$\begin{aligned}\chi_{t+1} &\equiv \frac{1}{\beta^t C_t^{-\gamma} h_t^{\gamma}} \sum_{s^{t+1}} \int \beta^{t+1} \left[\xi_{t+1} - a_0\right] \frac{du\left(F_{\bar{L},t+1}\bar{L}\eta_{t+1}\right)}{dK_{t+1}} \pi\left(s^{t+1}\right) d\Phi_0 \\ &= \frac{1}{\beta^t C_t^{-\gamma} h_t^{\gamma}} \sum_{s^{t+1}} \int \beta^{t+1} \left[\mu_1 + \cdots + \mu_t + \mu_{t+1}\right] \frac{du\left(F_{\bar{L},t+1}\bar{L}\eta_{t+1}\right)}{dK_{t+1}} \pi\left(s^{t+1}\right) d\Phi_0 \quad (7)\end{aligned}$$

This externality cost of capital investment consists of three parts. First, there is the incremental per-period autarky value $\frac{du\left(F_{\bar{L},t+1}\bar{L}\eta_{t+1}\right)}{dK_{t+1}}$ of a one unit increase of the capital stock. Second, the first part is multiplied by the all the earlier multipliers as a shadow price (cost) of the enforcement constraints $\left[\mu_1 + \mu_2 + \cdots + \mu_t + \mu_{t+1}\right]$. Third, this externality cost of capital investment is normalized by the period t price of consumption $\beta^t C_t^{-\gamma} h_t^{\gamma}$. Therefore, this is the cost at period t that the planner should pay if he wants to increase capital stock by one unit at period $t+1$. In order for the planner to keep agents from defaulting, he needs to compensate the agents more when he increases the capital stock.

Proposition 2.1 *Externality cost of capital investment* χ_{t+1} *is positive unless the full risk-sharing is feasible from initial period.*

Proof. *See the Appendix.* ■

Proposition 2.2 *Externality cost of capital investment* χ_{t+1} *is zero if the value of autarky does not depend on the capital investment.*

Proof. *See the Appendix.* ■

3 Characterization of Steady State Allocation

We open this section with a definition of the steady state. We define the steady state to be a state where all aggregate variables and the distribution of agents stay constant. We assume that the economy converges asymptotically to the steady state. It is important to note that even though the aggregate state will be stationary in steady state, each agent's consumption will still fluctuate over time.

In this section, we also explain how we compute the steady state allocations. To summarize, consumption allocations and shadow prices are computed for a given capital level, $\overline{K}$; we then pin down the optimal steady state capital stock, given steady state individual consumptions, $c\left(\nu_0, s^t, \overline{K}\right)$, shadow price $\overline{R}$, and the invariant distribution, Φ.

3.1 Steady State Consumption Allocation

In this subsection, we discuss the individual household's steady state consumption allocations. Throughout this subsection, we take the steady state capital stock $\overline{K}$ as given and then compute the steady state consumptions, a shadow price and an invariant distribution. In the next subsection, we will discuss how to decide the steady state capital stock $\bar{K}^*$.

For a given capital stock $\bar{K}$, the aggregate steady state consumption $C\left(\bar{K}\right)$ is:

$$C\left(\bar{K}\right) = F\left(\bar{K}, \bar{L}\right) - \delta\bar{K}$$

and we know this aggregate consumption should be allocated across the agents.

$$C\left(\bar{K}\right) = \sum_{s^t} \int c_t\left(\nu_0, s^t, \bar{K}\right) \pi\left(s^t \middle| s^0\right) d\Phi_0$$

We use consumption weights as state variables instead of cumulative multipliers because we want stationary state variables. The consumption share of a household $\left(\nu_0, s^t, \bar{K}\right)$ is defined as:

$$\omega_t\left(\nu_0, s^t, \bar{K}\right) \equiv \frac{\xi_t^{1/\gamma}\left(\nu_0, s^t, \bar{K}\right)}{h_t\left(\bar{K}\right)}$$

Notice that the individual's consumption share is history dependent. When the agent (ν_0, s^t) does not switch to a state with a binding constraint, its consumption share next period drifts downwards to:

$$\begin{aligned}
\omega_{t+1}\left(\nu_0, s^t, \bar{K}\right) &= \omega_t\left(\nu_0, s^t, \bar{K}\right) \frac{h_t\left(\bar{K}\right)}{h_{t+1}\left(\bar{K}\right)} \\
&= \omega_t\left(\nu_0, s^t, \bar{K}\right) \frac{1}{g_t\left(\bar{K}\right)}
\end{aligned}$$

When the agent (ν_0, η^t) does switch to a state with a binding constraint, its consumption share in next period is

$$\omega_{t+1}\left(\nu_0, s^t, \bar{K}\right) = \omega_t\left(\nu_0, s^t, \bar{K}\right) \frac{h_t\left(\bar{K}\right)}{h_{t+1}\left(\bar{K}\right)} \frac{\xi_{t+1}^{1/\gamma}\left(\nu_0, s^t, \bar{K}\right)}{\xi_t^{1/\gamma}\left(\nu_0, s^t, \bar{K}\right)}$$

Proposition 3.1 *(Lustig (2004)) When the agent (ν_0, s^t) switches to a state with a binding constraint, its consumption share equals to some cutoff level that does not depend on the history s^t, if the labor supply shock is first-order Markov.*

Proof. See the Appendix. ■

3.2 Cutoff rule

For a given weight growth $\hat{g}\left(\bar{K}\right)$, a cutoff value, $\underline{\omega}\left(s,\bar{K}\right)$ is determined such that the enforcement constraint binds exactly. Define the continuation value, $V\left(\omega,s,\bar{K}\right)$:

$$V\left(\omega,s,\bar{K}\right)=\left(\frac{\omega}{\hat{g}}C\left(\bar{K}\right)\right)^{1-\gamma}\frac{1}{1-\gamma}+\beta\sum_{s'}\pi\left(s'|\,s\right)V\left(\omega',s',\bar{K}\right)$$

and then we can choose the cutoffs $\underline{\omega}\left(s,\bar{K}\right)$ such that

$$V\left(\underline{\omega},s,\bar{K}\right)=V_{aut}\left(s,\bar{K}\right),$$

where $V_{aut}\left(s,\bar{K}\right)=\sum_{\tau=t}^{\infty}\sum_{s^{\tau}|s^{t}}\ \beta^{\tau-t}\ \pi\left(s^{\tau}|\,s^{t}\right)\ u\left(F_{l}\left(\bar{K},\bar{L}\right)\cdot\bar{L}s_{\tau}\right)$.

The cutoffs are the minimum compensation (in terms of consumption share) that the planner needs to give to the agent who switches to a state with a binding constraint in order to prevent the agent from defaulting.

For a given weight growth $\hat{g}$, the cutoff rule for the consumption weight is given by:

$$\omega'=\begin{cases}\ \omega_{t-1}\text{ if }\omega_{t-1}>\underline{\omega}\left(s\right)\\ \ \underline{\omega}\left(\eta\right)\ \text{ if }\omega_{t-1}\leq\underline{\omega}\left(s\right)\end{cases}$$

and the actual individual consumption is given by:

$$c_{t}=\frac{\omega'}{\hat{g}}\cdot\bar{C}$$

Therefore, cutoff rule implies the following. As long as an agent's consumption share is bigger than the cutoff value, then the agent's actual consumption drifts down at rate $\hat{g}$. It keeps shrinking until the agent has a good enough shock realization, whose corresponding cutoff is greater than the previous period's consumption share. If this happens, then the agent's consumption share equals to the cutoff divided by the growth rate of $h\left(\bar{K}\right)$.

Lemma 3.1 *(Lustig (2004), Lemma 14) If the transition matrix satisfies monotonicity, then the cutoff rules can be ranked:*

$$\underline{\omega}\left(s_{n},\bar{K}\right)\geq\underline{\omega}\left(s_{n-1},\bar{K}\right)\geq\underline{\omega}\left(s_{n-2},\bar{K}\right)\geq...\geq\underline{\omega}\left(s_{1},\bar{K}\right)$$

Proof. See the Appendix. ■

A wealthy household that starts off with an initial weight above the highest cutoff will end up hitting that bound in finite time, unless there is perfect risk-sharing. After some finite τ, all of this household's consumption shares $\omega\left(\nu_{0},s^{t},\bar{K}\right)$ fluctuate between the highest and the lowest cutoff weights.

3.3 Invariant Probability Measure

Given the monotonicity assumptions we must impose on π, we know that the consumption weight ω stays within a closed domain W because we know that $\omega \in \left[\underline{\omega}\left(s_1, \bar{K}\right), \underline{\omega}\left(s_n, \bar{K}\right)\right]$ since g is bounded. If some agent starts with an initial weight $a_0 \geq \underline{\omega}\left(s_n, \bar{K}\right)$ his consumption weight drops below $\underline{\omega}\left(s_n, \bar{K}\right)$ after a finite number of steps unless there is perfect risk sharing.

Let $W = \left[\underline{\omega}\left(s_1, \bar{K}\right), \underline{\omega}\left(s_n, \bar{K}\right)\right]$ and $B(W)$, $P(S)$ be the set of Borel sets of W and the power set of S respectively. The cutoff rule together with the transition function π for the labor shock process jointly defines a Markov transition function on shock realizations and consumption weight: Q: $(W \times S) \times (B(W) \times P(S)) \to [0,1]$ where

$$Q(\omega, \eta, W, S) = \sum_{s' \in S} \begin{cases} \pi(s') & \text{if } \omega' \in W \\ 0 & \text{else} \end{cases}$$

Given this transition function, we define an operator T^* on the space of probability measures $\Lambda((W \times S), (B(W) \times P(S)))$ as

$$\begin{aligned} (T^*\Phi)(W, S) &= \int Q(\omega, s, W, S)\, d\Phi \\ &= \sum_{s' \in S} \pi(s') \int_{\omega \in W | \omega' \in W} d\Phi \end{aligned}$$

for all $(W, S) \in B(W) \times P(S)$. Note that T^* maps Λ into itself.[6] A fixed point of this operator is an invariant probability measure. Let Φ^* denote the invariant measure over the space $((W \times S), (B(W) \times P(S)))$ that satisfies invariance:

$$T^*\Phi^*(W, S) = \Phi^*$$

In this section, we address the question of whether such a probability measure exists and is unique. Intuitively, this invariant measure describes the long-run cross-sectional distribution of the agent's consumption shares implied by the planner's social welfare maximizing policies.

Proposition 3.2 *For a steady state capital stock $\bar{K}$, there exists a unique invariant probability measure, $\bar{\Phi}$.*

Proof. See the Appendix ■

[6] See Stockey et. al. (1989), Theorem 8.2

3.4 Determination of the shadow price

Proposition 3.3 *For a given steady state capital stock $\bar{K}$, if there is a unique invariant distribution Φ^* with no aggregate uncertainty, then there is a stationary equilibrium in which shadow interest rate $\bar{R}^*$ is unique and constant.*

Proof. See the Appendix. ■

3.5 Steady State Capital Allocation

The optimal capital $\bar{K}^*$ is pinned down such that the steady state euler equation (8) is satisfied.

$$1 = \frac{1}{\bar{R}} \left[F_K(\bar{K}^*) + (1-\delta) - \bar{R}\chi'(\bar{K}^*) \right] \tag{8}$$

where

$$\chi' \equiv \sum_{s'} \int \beta g^{\gamma} \left[\omega'^{\gamma} - \frac{a_0}{h'^{\gamma}} \right] \bar{C}^{\gamma} \frac{du\left(F_{\bar{L}} \bar{L} s'\right)}{dK} \pi\left(s'\right) d\Phi$$

$\omega' \left(\equiv \frac{\xi'^{1/\gamma}}{h'} \right)$ is an individual agent's consumption share tomorrow. This records how the degree to which one agent has been constrained by the enforcement constraints over his history since it contains all previous multipliers in it including the initial Pareto weight. We subtract the initial consumption share since we do not have a enforcement constraint at time 0. The initial consumption share part $\frac{a_0}{h'^{\gamma}}$ will approach zero as time approaches infinity since h is a non-decreasing sequence. Then the aggregate steady state consumption to the power of γ multiplied by an agent's consumption share tomorrow to the power of γ is the inverse of the agent's marginal utility of consumption at $t+1$. Finally, we convert the marginal utility of labor income with respect to capital investment, $\frac{du\left(F_{\bar{L}} \bar{L} s'\right)}{dK}$, into time $t+1$ units of consumption by multiplying by $\omega'^{\gamma} \bar{C}^{\gamma}$.

4 Decentralization with Solvency constraints and Capital Income Taxes

Consider now decentralizing the constrained efficient allocations as a competitive equilibrium with capital taxes and solvency constraints. With these two instruments, the government can mimic the distorted first order conditions that define the constrained efficient

allocations. The role of solvency constraints is the same here as in Alvarez and Jermann (2000, 2001). The role of capital taxes is to make the households internalize the externality cost that capital investment creates.

There are two assets available. We have a complete set of contingent claims $b_{t+1}\left(s_{t+1}; W_0, \eta^t\right)$ at price q_t. This is a security that pays one unit of consumption good at $t+1$ if s_{t+1} is realized at $t+1$. The other asset is capital asset K_{t+1}, which yields the return of r_{t+1}.

Instead of using the initial promised utility ν_0 to label the agents, we will use the initial wealth W_0. So each household will be indexed by a pair of (W_0, s_0). We will show how to construct the initial wealth below.

Firms

Firms rent labor from households and physical capital from the intermediaries to maximize period profits. Firms solve a static profit-maximizing problem,

$$\max_{K_t, L_t} F\left(K_t, L_t\right) - w_t L_t - r_t K_t$$

which implies the following two first order conditions.

$$\begin{aligned} r_t &= F_K\left(K_t, L_t\right) \\ w_t &= F_L\left(K_t, L_t\right), \end{aligned}$$

where K_t is market supply of capital and L_t is market supply of labor. r_t and w_t denote the wage rate and the rental rate of capital, respectively.

Households

A household of type (W_0, s_0) chooses a sequence of consumption $\{c_t\left(W_0, s^t\right)\}_{t=0}^{\infty}$ and a sequence of contingent bonds $\{b_{t+1}\left(W_0, s^t\right)\}_{t=0}^{\infty}$ to maximize his expected lifetime utility:

$$V\left(W_0, s_0, K_0\right) \equiv \max_{\{c_t\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t > s_0} \beta^t \pi\left(s^t \middle| s_0\right) \frac{c_t\left(W_0, s^t\right)^{1-\gamma}}{1-\gamma}$$

subject to the usual budget constraint:

$$\begin{aligned} c_t + \sum_{s_{t+1}} q_t b_{t+1} \pi\left(s_{t+1} \middle| s^t\right) &= W_t \\ W_{t+1} &= w_{t+1} s_{t+1} \overline{L} + b_{t+1} + T_{t+1} \end{aligned}$$

and a solvency constraint, one for each state:

$$b_{t+1}\left(s_{t+1}; W_0, s^t\right) \geq \underline{B}_{t+1}\left(s_{t+1}; W_0, s^t\right),$$

given a sequence of prices and policies $\{w_t, r_t, q_t, \tau_{k,t}, T_t\}_{t=0}^{\infty}$

Financial Intermediaries

We introduce financial intermediary sector as in Carceles-Poveda and Abraham (2004). The financial intermediaries rent capital K_{t+1} to the firms, earning an after-tax rental revenue of $(1+\bar{r}_{t+1})K_{t+1}$ in the following period after paying the tax, where $\bar{r}_{t+1} = (r_{t+1}-\delta)(1-\tau_{K,t+1})$ denotes after-tax return on capital investment. To finance the capital investment, the intermediaries sell the future consumption goods in the spot market for one period ahead contingent claims. Given this, the zero profit condition implied by the presence of perfect competition in the financial intermediary sector requires that:

$$1 = q_t\left[1+\bar{r}_{t+1}\right]$$

Further, in order for the state contingent debt issued by the intermediaries to match the demand from the households, it must be the case that:

$$\sum_{s^t}\int b_t\left(W_0, s^t\right)\pi\left(s^t\middle|\, s_0\right)d\Phi_0 = \left[1+\bar{r}_t\right]K_t$$

Government

The government collects tax revenue from financial intermediaries and transfers it to households in a lump-sum fashion such that her budget constraint is the following:

$$T_t = \tau_{K,t}\left(r_t-\delta\right)K_t$$

Notice that we don't have any government spending in this model.

4.1 Competitive Equilibrium

Definition 4.1 ***A competitive equilibrium with capital income tax*** $\{\tau_{K\,t}\}$ ***and solvency constraints*** $\left\{\underline{B}_{t+1}\right\}$ *for initial distribution* Φ_0 *over* (W_0, s_0) *and capital stock* K_0 *consists of a set of allocations,*$\{c_t\left(W_0, s^t\right)\}$, $\{b_t\left(W_0, s^t\right)\}$, *and* $\{K_t\}$, *a set of prices,* $\{r_t\}$,$\{w_t\}$ *and* $\{q_t\}$ *and policies* $\{\tau_{K,t}, T_t\}$ *such that* (1) *Given the set of prices and policies, the allocations solve the household's problem,*(2) *Given the set of prices, the allocations solve the firm's problem,* (3) *the government budget constraint holds,* (4) *the resource constraints hold and* (5) *the markets clear;*

1.
$$\sum_{s'} \int b_t \left(W_0, s^t\right) \pi \left(s^t \middle| s^0\right) d\Phi_0 = \left[1 + \bar{r}_t\right] K_t, \ \textit{for } \forall t$$

2.
$$\sum_{s^t} \int c_t \left(W_0, s^t\right) \pi \left(s^t \middle| s^0\right) d\Phi_0 = K_t^{\alpha} \bar{L}^{1-\alpha} - K_{t+1} + (1-\delta) K_t, \ \textit{for } \forall t$$

3.
$$L_t = \overline{L}, \ \textit{for } \forall t$$

Definition 4.2 *Borrowing constraints are not too tight if they satisfy*

$$V\left(\underline{B}_t, s^t, K_t\right) = V_{aut}\left(s_t, K_t\right) \ \forall s^t$$

The link between enforcement constraints in the planner's problem and solvency constraints in the household's problem is the following. When an enforcement constraint in the planner's problem binds, the corresponding solvency constraint in that state will bind. This condition guarantees that the borrowing constraints prevent default by not letting the agent accumulate more debt than they are willing to pay back.

Definition 4.3 *The price of the contingent claims are not too high if the infinite sums of the form are finite for all equilibrium object* x_{t+j}

$$\sum_{j=1}^{\infty} q_{t,t+j} x_{t+j} < \infty$$

This condition guarantees that in a decentralized equilibrium, the present value of any allocation is finite. We use it to show that the value of the constructed assets is finite and that the household's transversality condition holds.

Proposition 4.1 *Given allocations* $\{c_t\left(W_0, s^t\right)\}$ *and* $\{K_t\}$ *that satisfies*

1. *the feasibility condition at any period,*
2. *the enforcement constraints at any period and any state,*
3. *that the implied price of contingent claims are not too high and*

4. that the marginal utility of consumption stays finite:

$$\lim_{t\to\infty} E_0 u\left(c_t\right) < \infty$$

then there exist processes $\{b_t\left(W_0, s^t\right), B_t, r_t, w_t, q_t, T_t\}$ *such that sequences* $\{c_t\left(W_0, s^t\right)\}$, $\{b_{t+1}\left(W_0, s^t\right)\}$ *and* $\{K_t\}$ *compose a competitive equilibrium given the prices* $\{r_t, w_t, q_t\}$, *the solvency constraints* $\left\{\underline{B}_{t+1}\right\}$ *and the taxes on capital income* $\{\tau_{K\,t}\}$. *In addition, the borrowing constraints are not too tight.*

Proof. *See the Appendix.* ■

4.2 Steady State Capital Tax

This subsection explains how we compute the steady state capital tax rate. Equation (9) is the planner's Euler equation, upon which the planner bases his capital investment decision and Equation (10) is the financial intermediaries' no arbitrage condition in the competitive equilibrium. We choose the steady state tax rate such that these two equations are consistent with each other.

$$\begin{aligned} 1 &= \bar{q}\left[F_K(\bar{K}^*) + (1-\delta) - \frac{\chi(\bar{K}^*)}{\bar{q}}\right] && (9) \\ &\equiv \bar{q}\left[\left(1-\bar{\tau}_K^*\right)\left(F_K - \delta\right) + 1\right] && (10) \end{aligned}$$

In order for the second welfare theorem to hold, two Euler equations should be equivalent and consistent and the capital tax rate can be backed out of the following equation:

$$\bar{\tau}_K^* = \frac{\chi(\bar{K}^*)}{\bar{q}\left(F_K - \delta\right)} \tag{11}$$

Proposition 4.2 *If* $\chi(\bar{K}^*) = 0$, *then* $\bar{\tau}_K^* = 0$.
Proof. *See the Appendix.* ■

5 Quantitative Analysis

In order to generate the simulated data, we first parameterize the idiosyncratic labor shock process and preferences. Our calibration for the idiosyncratic shock process is mainly

based on Krueger (1999). He uses estimation for the idiosyncratic endowment shock process based on Storesletten, Telmer and Yaron (2004). The latter authors use labor market earnings to calibrate the process for the labor efficiency units from the PSID (1969 − 1992). Their idiosyncratic process includes the effect of government programs such as unemployment insurance that are devised to share risk. Since we are interested in income risk that must be insured by private arrangements, net of those risks already insured by the government, their idiosyncratic process is actually more appropriate for our study than for Krueger (1999).

Let z_{it} be the logarithm of individual income. They assume that idiosyncratic income has a persistent and transitory component and estimate

$$\begin{aligned} z_{it} &= \alpha_i + u_{it} + \varepsilon_{it}, \quad \alpha_{it} \sim Niid\left(0, \sigma_\alpha^2\right) \ and \ \varepsilon_{it} \sim Niid\left(0, \sigma_\varepsilon^2\right) \\ u_{it} &= \rho u_{it-1} + \eta_{it}, \quad \eta_{it} \sim Niid\left(0, \sigma_\eta^2\right) \end{aligned}$$

They find estimates of $\left(\rho, \sigma_\alpha^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) = (0.98, 0.326, 0.005, 0.019)$. Note that this idiosyncratic process displays a high degree of persistence. Given their estimation, Krueger (1999) ignores the individual-specific fixed effects α_i and approximates the continuous AR(1) process by a 5 state Markov chain, using the procedure described by Tauchen and Hussey (1992). The individual labor shocks are then normalized so that the aggregate labor shock equals 1. We use the baseline parameterization for the idiosyncratic process that Krueger (1999) constructs as in the following table. A set of parameters is also summarized in the following table:

Parameters		Value
β	Discount Factor	0.75
γ	Risk Aversion	2.00
δ	Depreciation Rate	0.10
α	Capital Income Share	0.20
$\overline{L}$	Aggregate Labor Supply	0.3271

Parameter		Value
ρ	Persistence	0.98
σ_ε^2	Std. Dev. of Transitory Shocks	0.005
σ_η^2	Std. Dev of Permanent Shocks	0.019
S	Markov Chain States	$\begin{Bmatrix} 0.63 & 0.79 & 0.96 & 1.17 & 1.47 \end{Bmatrix}$
Π	Stationary Distribution	$\begin{bmatrix} 0.183 & 0.212 & 0.210 & 0.212 & 0.183 \end{bmatrix}$
π	Transition Probability	$\begin{bmatrix} 0.71 & 0.26 & 0.02 & 0.01 & 0.00 \\ 0.23 & 0.51 & 0.24 & 0.02 & 0.00 \\ 0.02 & 0.24 & 0.48 & 0.24 & 0.02 \\ 0.00 & 0.02 & 0.24 & 0.51 & 0.23 \\ 0.00 & 0.01 & 0.02 & 0.26 & 0.71 \end{bmatrix}$

Our annual discount factor β is set constant at 0.75[7] and we explores the quantitative effects of the discount rate β below. The perfectly inelastic aggregate labor supply is set at 0.3271 from Krusell and Smith (1997).

We set the capital income share α at 0.2 which implies we set the labor income share at 0.8. Lustig (2004) argues that the average labor income share $(1-\alpha)$ of national income in the US between 1946 and 1999 is 70 percent (source, NIPA) and the additional 11 percent is proprietor's income derived from farms and partnerships, mainly doctors and lawyers and it should be treated as labor income for the purpose of this exercise. This brings the total labor income share to 81 percent.

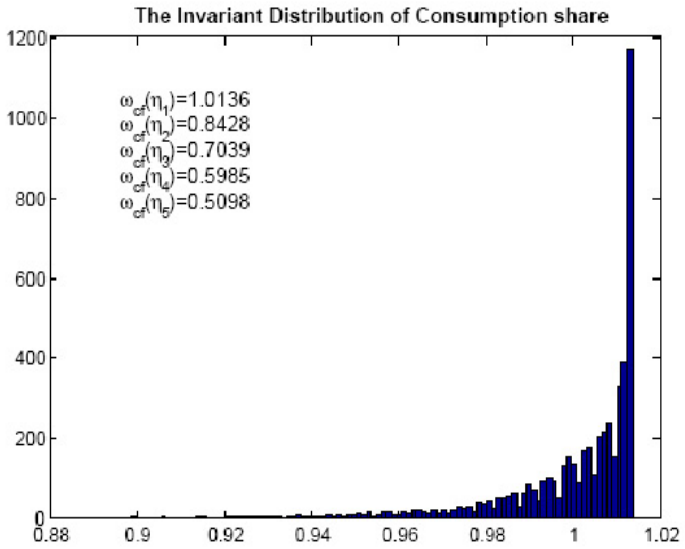
5.1 Results

This section briefly describes the numerical results with the parameter values from the previous section. We compute the steady state constrained efficient allocations, prices and tax rate:

[7] Alvarez and Jermann (2001) choose β equal to .65 in their two-agent model with permanent exclusion from trading.

Variable	Values
$\bar{K}^*$	0.020068
K_{FR}	0.12443
$\frac{\bar{K}^*}{K_{FR}}$	0.16127
$\bar{C}^*$	0.18517
$\bar{\tau}_K^*$	0.88098
$\bar{R}$	1.3183
$\bar{g}$	1.0006

The steady state capital stock is only about 16% of the capital stock that would be obtained if full risk-sharing were feasible and the steady state tax rate on capital income is approximately 88.1%. In the Appendix, we show that we can also compute the lower bound of the tax rate for a set of parameters. We obtain a lower bound of approximately 60% $(\underline{\tau}_K = 0.60)$.



Invariant Distribution of Consumption Shares in the Steady State

5.2 Comparative Statics

This section provides an overview of how the constrained efficient allocations, prices and tax rates depends on the time discount factor β and the capital income share α.

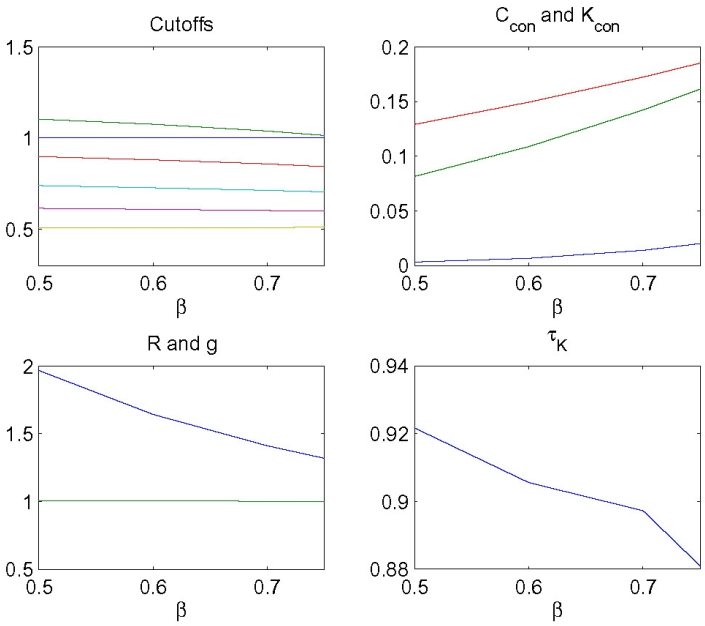


Figure 1: β-Comparative Statics

5.2.1 β-Comparative Statics

For the results we reported in the prior section, we have kept the annual discount factor β constant at 0.75. Discount values higher than 0.75 allow for the possibility of full risk sharing because our assumptions about autarky values are more severe than usual. Figure 2 explores the different values of the time discount factor. Lowering the discount rate puts more weight on today's income realization and makes autarky more tempting so that the borrowing constraints become more binding as a result. The more agents are constrained as the discount rate lowers, the more they are compensated. This will increase the cutoff values, the externality cost as well as the capital tax rate.

5.2.2 α-Comparative Statics

As the labor income share $1 - \alpha$ increases, the autarky value becomes more attractive since the only source of income in autarky is labor income and agents are more constrained as a result. Therefore cutoff values should increase and the tax rate also should increase.

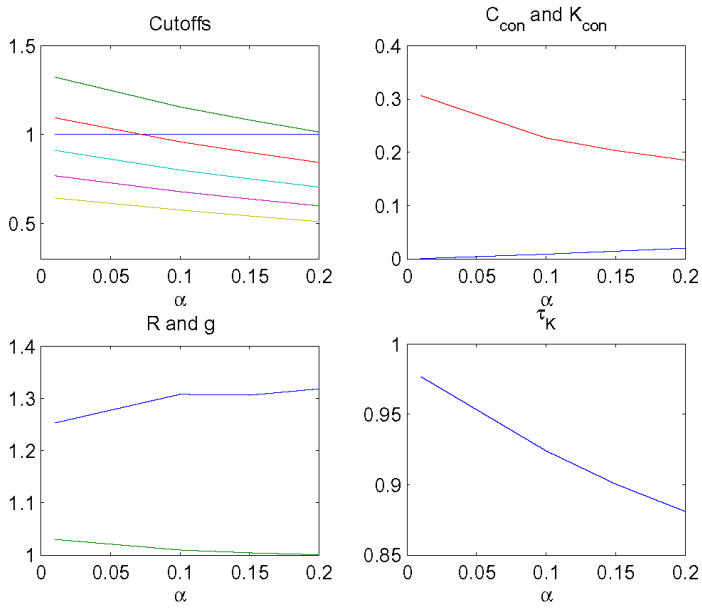


Figure 2: α-Comparative Statics

6 Concluding Remark

We study optimal capital taxation under limited commitment. We prove that the optimal tax rate on capital income should be positive in steady state and should be increasing over time provided that full risk-sharing is not feasible.

A one unit increase of capital investment by an agent increases all individuals' autarky values in the economy and generates externality costs in the economy. This externality cost provides a rationale for positive capital taxation even in the absence of government expenditure. Moreover, we show that both this externality cost of capital investment and the optimal tax rate are potentially much bigger than one might expect.

To the best of our knowledge, this is the first paper to study optimal capital taxation in a limited commitment environment. Furthermore, this paper relates a model of risk-sharing to the fiscal policy literature and suggests that risk-sharing has important consequences in designing the optimal fiscal policy and should not be overlooked.

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Appendix 1: Proofs

Proof of Proposition 2.1

Proof is straight-foward from equation (7). Full risk-sharing means that no agent is constrained and this implies that $\mu_t(\nu_0, s^t) = 0, \ \forall t$ and $\forall(\nu_0, s^t)$ by Khun-Tucker.

Proof of Proposition 2.2

Proof is straight-foward from equation (7).If $\frac{du\left(F_{\bar{L},t+1}\bar{L}\eta_{t+1}\right)}{dK_{t+1}} = 0$, then χ should be zero.

Proof of Proposition 3.1

When the agent is constrained, the participation constraint is satisfied with equality;

$$\sum_{\tau=t}^{\infty}\sum_{s^\tau|s^t}\beta^{\tau-t}\pi\left(s^\tau|\,s^t\right)u\left(c_\tau\left(\nu_0,s^\tau,\bar{K}\right)\right)=\sum_{\tau=t}^{\infty}\sum_{s^\tau|s^t}\beta^{\tau-t}\pi\left(s^\tau|\,s^t\right)u\left(F_l\left(\bar{K},\bar{L}\right)\cdot\bar{L}s_\tau\right),$$

Now, if the labor supply shock s is first-order Markov, then the value of autarky in the right hand side of the enforcement constraint does depends on the current realization of the shock s_t. This implies that $\{c_\tau\left(\nu_0,s^\tau,\bar{K}\right)\}_{\tau=t}^{\infty}$ cannot depend on s^t, but on only s_t.

Proof of Lemma 3.1

First, we define monotonicity. A transition matrix Π is monotone if for any non-decreasing function f on H, Πf is also non-decreasing. If this condition is satisfied, then We can rank the all of the cutoff weights. Assume that the transition matrix $\pi(s'|\,s)$ satisfies this condition. Then our claim is that value of the outside option can be ranked such that:

$$V_{aut}\left(s_n,\bar{K}\right)\geq V_{aut}\left(s_{n-1},\bar{K}\right)\geq ...\geq V_{aut}\left(s_1,\bar{K}\right)$$

where $V_{aut}\left(s,\bar{K}\right)=\sum_{\tau=t}^{\infty}\sum_{s^\tau|s^t}\ \beta^{\tau-t}\ \pi\left(s^\tau|\,s\right)\ u\left(F_l\left(\bar{K},\bar{L}\right)\cdot\bar{L}s_\tau\right)$. Now we need to show that this implies a similar ranking for the cutoff weights, by definition, the following holds:

$$V\left(\omega,s,\bar{K}\right)=\left(\frac{\omega}{\hat{g}}C\left(\bar{K}\right)\right)^{1-\gamma}\frac{1}{1-\gamma}+\beta\sum_{s'}\pi\left(s'|\,s\right)V\left(\omega',s',\bar{K}\right)$$

Since $V\left(\omega,s,\bar{K}\right)$ is monotonically increasing in ω, we can conclude the following ranking of the cutoff weights:

$$\underline{\omega}\left(s_n,\bar{K}\right)\geq\underline{\omega}\left(s_{n-1},\bar{K}\right)\geq\underline{\omega}\left(s_{n-2},\bar{K}\right)\geq ...\geq\underline{\omega}\left(s_1,\bar{K}\right)$$

Proof of Proposition 3.2

This proof follows Lustig (2004) [8]. We define an operator on the space of probability measures $\Lambda\left(\left(W\times S\right),\left(B\left(W\right)\times P\left(S\right)\right)\right)$ as:

$$T^{*}\lambda\left(W,S\right)=\int Q\left(\left(\omega,s\right),\left(W,S\right)\right)d\lambda$$

A fixed point of this operator is defined to be an invariant probability measure. To show there exists a unique fixed point of this operator, We check condition M in (Stokey,Lucas and Prescott (1989) p.348). If this condition is satisfied, we can use Theorem 11.12 in Stokey, Lucas, and Prescott (1989) p.350. To be perfectly general, let $W=[\underline{\omega}\left(s_{1},\bar{K}\right),\omega_{max}]$. There has to be an $\epsilon>0$ and an $N\geq1$ such that for all sets W,S

$$Q^{N}\left(\left(\omega,s\right),\left(W,S\right)\right)\geq\epsilon\text{ and }Q^{N}\left(\left(\omega,s\right),\left(W,S\right)^{c}\right)\geq\epsilon$$

It is sufficient to show that there exists an $\epsilon>0$ and an $N\geq1$ such that for all $(\omega,s)\in(W,S)$: $Q^{N}\left(\left(\omega,s\right),\left(\omega_{max},s_{n}\right)\right)\geq\epsilon$, but we know that $Q\left(\left(\omega,s\right),\left(\omega_{max},s_{n}\right)\right)\geq\pi\left(s_{n}|\,s\right)$. If $\omega_{max}\geq\underline{\omega}\left(s_{n},\bar{K}\right)$ then define

$$N=min\{n\geq0:\frac{\omega_{max}}{g^{n}}\leq\underline{\omega}\left(s_{n},\bar{K}\right)\}$$

, where N is finite unless there is perfect risk sharing. Then we know the $Q^{N}\left(\left(\omega,\eta\right),\left(\omega_{max},\eta_{n}\right)\right)\geq\epsilon$ where

$$\epsilon=\pi\left(s_{n}|\,s\right)\cdot\left(\pi\left(s_{n}|\,s\right)\right)^{N-1}.$$

If $\omega_{max}\leq\underline{\omega}\left(s_{n},\bar{K}\right)$, the proof is immediate by setting $\epsilon=\pi\left(s_{n}|\,s\right)$. This establishes the existence of a unique, cross-sectional distribution.

Proof of Proposition 3.3

Again we follow Lustig (2004)[9]. If there is a unique Φ^{*}, then It is clear that there is a unique growth rate:

$$g^{*}=\int\sum_{s'}\pi\left(s'|\,s\right)\omega'\left(\omega,s',\bar{K}\right)d\Phi^{*}$$

$$Tg\left(\Phi^{*}\right)=\sum_{s'}\int_{\underline{\omega}(s')}\pi\left(s'|\,s\right)\omega d\Phi^{*}+\sum_{s'}\underline{\omega}\left(s'\right)\int^{\omega(s')}\pi\left(s'|\,s\right)\omega d\Phi^{*}$$

[8] See also Atkeson and Lucas (1995).

See also Krueger (1999): Lemma 14, Corollary 14 and Theorem 15.

[9] See also Atkeson and Lucas (1995).

See also Krueger (1999): Lemma 16, 17, 19 and 21, and Theorem 18 and 21

and then this implies that there exists a unique constant shadow price R^* that clear the markets.

Proof of Proposition 4.1

By construction, First construct the equilibrium prices and transfer as follows

$$\begin{aligned} r_t &= F_K(K_t, L_t) \\ w_t &= F_L(K_t, L_t) \\ q_{t,t+1} &= \max\left\{\beta\frac{u_{c,t+1}}{u_{c,t}}\right\} \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{h_{t+1}}{h_t}\right)^{\gamma} \\ Q_{t,t+j} &= \prod_{i=t}^{t+j-1} q_{i,i+1} \\ T_t &= \tau_{K,t}(r_t - \delta)K_t \end{aligned}$$

Next, construct the initial wealth and the asset holdings as follows:

$$W_0 = E_0\left[\sum_{t=0}^{\infty} Q_{0,t}(c_t - w_t s_t L - T_t)\right]$$

and

$$b_t = E_t\left[\sum_{j=0}^{\infty} Q_{t,t+j}(c_{t+j} - w_{t+j}s_{t+j}L - T_{t+j})\right]$$

Under condition 4, these sums are well-defined. Finally, the construction of the solvency constraints (borrowing limits) is identical to that in Alvarez and Jermann (2000, 2001). If

$$q_{t,t+1}\cdot u_{c,t} > \beta u_{c,t+1}$$

then set $\underline{B}_{t+1}\left(\eta_{t+1}; W_0, \eta^t\right) = b_{t+1}$. otherwise set $B_{t+1} = -E_t\left[Q_{t,t+1}(w_t\eta_t L + T_t)\right]$. For $t > 0$, the tax on capital income is backed out from the financial intermediaries' no arbitrage condition

$$1 = q_{t,t+1}\left[1 + (1 - \tau_{K,t+1})(r_{t+1} - \delta)\right]$$

so that $R_{t+1} = 1 + (1 - \tau_{K,t+1})(r_{t+1} - \delta)$ is set equal to $\frac{1}{q_{t,t+1}} = \frac{1}{\beta}\left(\frac{C_t}{C_{t+1}}\right)^{-\gamma}\left(\frac{h_t}{h_{t+1}}\right)^{\gamma}$, for $t = 0$, we set $R_0 = 1$. To check the constructed assets are budget feasible and that the transversality conditions for the household are satisfied. we use the budget constraint to

construct asset holdings at each time and state so allocations are budget feasible. budget constraints together with government budget constraint and market clearing condition guarantee that the allocations are also resource feasible. It is easy to show that the transversality condition for the bond holds,

$$\lim_{t\to\infty} E_0\beta^t u_{c,t}\left[b_t - \underline{B}_t\right] = 0$$

is satisfied assuming that

$$\lim_{t\to\infty} E_0\beta^t u_{c,t} = 0$$

which is satisfied by condition 4.

Proof of Proposition 4.2

Proof is straight-foward from equation (11).

7 Appendix 2: Lower bound of capital tax rate

When $\delta = 0$, we can get a nice form of τ_K

$$\tau_K = (1-\alpha)^{1-\gamma} \sum_{s'} \int s'^{1-\gamma} \omega'^{\gamma} \pi(s' \mid s) d\Phi$$

Let $\underline{\omega}_i$ be the cutoff level of consumption share when agent receives shock s_i. And ω'_{ij} be the equilibrium consumption share of agent j if shock i realized next period. We know that

$$\begin{aligned}
\omega'_{ij} &\geq \underline{\omega}_i \text{ for all } i,j \\
\omega'^{\gamma}_{ij} &\geq \underline{\omega}^{\gamma}_i \text{ for all } i,j \text{ (Suppose } \gamma > 1) \\
s'^{1-\gamma} \omega'^{\gamma}_{ij} &\geq s'^{1-\gamma} \underline{\omega}^{\gamma}_i \text{ for all } i,j \\
\sum_{s'} \int s'^{1-\gamma} \omega'^{\gamma} \pi(s' \mid s) d\Phi &\geq \sum_{s'} \int s'^{1-\gamma} \underline{\omega}'^{\gamma} \pi(s' \mid s) d\Phi
\end{aligned}$$

- Cutoff computation. Since right now $\delta = 0$, $Y = C$.

  1. Given the Invariant distribution of s', we can compute the

$$\begin{aligned}
V_{aut}(s) &= u(MP_L \overline{L} s) + \sum_{s'} \pi(s' \mid s) V_{aut}(s') \\
&= u((1-\alpha) s Y) + \sum_{s'} \pi(s' \mid s) V_{aut}(s')
\end{aligned}$$

$$V_{aut} = (I - \beta \Pi)^{-1} u((1-\alpha) s Y)$$

  2. The cutoffs is solved by the following equation

$$\begin{aligned}
V_{aut}(s) &= u(\underline{\omega} Y) + \beta \sum_{s'} \pi(s' \mid s) V_{aut}(s') + \beta \sum_{s'} \pi(s' \mid s) V(\frac{\omega}{g}, s') \\
u(\underline{\omega} Y) &= V_{aut}(s) - \beta \sum_{s'} \pi(s' \mid s) V_{aut}(s') - \beta \sum_{s'} \pi(s' \mid s) V(\frac{\omega}{g}, s') \\
&\geq V_{aut}(s) - \beta \sum_{s'} \pi(s' \mid s) V_{aut}(s') - \beta \sum_{s'} \pi(s' \mid s) V_{aut}(s) \\
&\equiv u(\underline{\omega}_L Y)
\end{aligned}$$

  3. Then we can compute the $\underline{\omega}_L$

Therefore,

$$\begin{aligned}
\tau_K &= (1-\alpha)^{1-\gamma} \sum_{s'} \int s'^{1-\gamma} \omega'^{\gamma} \pi(s' \mid s) d\Phi \\
&\geq (1-\alpha)^{1-\gamma} \sum_{s'} \int s'^{1-\gamma} \underline{\omega}'^{\gamma} \pi(s' \mid s) d\Phi \\
&\geq (1-\alpha) \sum_{s'} \int s'^{1-\gamma} \underline{\omega}_L^{\gamma} \pi(s' \mid s) d\Phi
\end{aligned}$$

- In our example (given the set of parameters), $\tau_K \geq 63.37\%$