House Price Fluctuations and Residential Sorting

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Abstract

Empirical evidence indicates local jurisdictions are internally more heterogenous than standard sorting models predict. We develop a dynamic multi-region model, with fluctuating regional house prices, where an owner-occupying household's location choice depends on its current wealth and its current "match" and involves both consumption and investment considerations. The relative strength of the consumption motive and the investment motive in location choice determines the equilibrium pattern of residential sorting, with a strong investment (consumption) motive implying sorting according to the match (wealth). The model predicts a negative relation between house price fluctuations and residential sorting in the match dimension. Also, movers should be more sorted than stayers. Using age, education and income as proxies for the match, we test these predictions with US data, and find support for our theory.

Keywords: Residential sorting, House prices, Incomplete markets, Owner-occupation, Household mobility

JEL classification: D31, D52, R13, R21, R23

1 Introduction

A central theme in regional and urban economics has been to examine how households sort themselves into neighborhoods and communities according various socioeconomic characteristics, such as income, household size or education. Roughly speaking, the sorting approach predicts that local jurisdictions should be internally more homogenous than the larger geographical or economic unit of which they are a part. Also, the jurisdictions should differ from each other.

However, recent empirical evidence reveals that there is considerable heterogeneity within municipalities and local neighborhoods. Epple and Sieg (1999) find that 89% of the total variance of income in the Boston metropolitan area was accounted for by within-community variance in 1980. Davidoff (2005) reports that only 6% of the variation of household income within US metropolitan areas could be explained by differences across jurisdictions in 1990. According to Ioannides (2004), a typical American neighborhood is composed by highly different households, residing in largely similar homes. The correlation coefficient between incomes of a randomly chosen individual and her closest neighbors is only 0.3. The degree of sorting is equally low with respect to education, and still lower in terms of age. By contrast, local property values tend to be more highly correlated. These findings are put into a historical perspective by Rhode and Strumpf (2003), who report that heterogeneity across US municipalities and counties, measured with respect to income and (so called) public good preference proxies (including education, age categories, race, nativity, party vote shares in presidential elections, owner-occupation rate, and religion) did not increase over the period 1850-1990, although migration costs fell, which should have made sorting easier.[1]

In this paper we develop a dynamic general equilibrium model of residential sorting, which provides one possible explanation as to why local jurisdictions may be internally rather heterogenous, and not very distinct from each other. We also derive a number of empirical predictions about the degree of sorting under different circumstances, and test

[1] Rhode and Strumpf derive this theoretical prediction from an augmented Tiebout model, which includes migration costs.

these hypotheses using data from US metropolitan areas.

Our approach is based on the following main elements: (i) For owner-occupying households, housing is both a consumption good and an investment, and location choices involve both consumption and investment considerations. (ii) Regional house prices fluctuate, and the capital gains and losses made in the housing market play a major role in determining how a household's wealth evolves over time. (iii) Borrowing constraints may prevent a household from moving from a currently cheap location to a currently expensive location.

There are two locations in the economy, both having enough housing capacity for half of the households. In each period, the utility that a household derives from residing in different locations depends on the household's current "match". Empirically, the "match" can be interpreted as reflecting various demographic and socioeconomic characteristics of the household (other than wealth) such as education, the age of the household head(s), or the number of children etc., which affect housing and location decisions. In each period, one of the locations is considered to be "desirable" while the other location is "less desirable". The basic allocation problem arises from the fact that more than one half of the households (and possibly all of them) derive more utility from residing in the (currently) "desirable" location, and thus housing is in short supply there.

In equilibrium a pattern of two-dimensional residential sorting emerges, and location choices depend on both wealth and the "match". It may be instructive to first have a brief look at simple polar cases. If residential sorting takes place mainly in the wealth dimension, the wealthiest households tend to live in the currently popular, and expensive, location, and those who reside in the unpopular location do so because they cannot afford a more expensive home; they are borrowing constrained. According to our theory, a household is currently wealthy (impecunious), because it has been fortunate (unlucky) in the timing of its housing market transactions; the current wealth position largely reflects past fortunes in the housing market, rather than some inherent characteristics of the household. Thus the theory predicts that under wealthwise sorting the locations should be internally rather heterogenous in the match dimension. Within the same neighborhood there may live households which have little in common, apart from value of their home.

In the opposite case, where sorting takes place in the match dimension, those who have the best current match with the desirable location also live there. Given the empirical interpretation we attribute to the match, households living within the same jurisdiction should then resemble each other with respect to various socioeconomic characteristics (such as household size, the age of the household head(s), or education). Also different jurisdictions should differ from each other with respect to the distribution of these observable characteristics.

A major objective of the paper is to study under what circumstances sorting happens primarily in the wealth dimension, and when location choices are made principally according to the match. Residential sorting in the match dimension requires that the basic allocation problem in the economy is not mainly solved through the borrowing constraint, but rather by self-selection. Here self-selection essentially means that some households, which receive a higher current utility stream from the desirable location, voluntarily choose the less desirable area. The incentives to make such a choice arise from the basic elements (i), (ii) and (iii), underlying our approach, and essentially reflect a trade-off between the consumption motive and the investment motive of housing.

To be more specific, we assume that the relative ranking of the locations is not set once and for all, but with a certain probability there may be a regional shock, so that the ranking is reversed, and also regional house prices change. The regional shock may reflect e.g. altering labor market conditions, changes in the supply of public goods and services, or the evolution in the tastes and the needs of the population. Alternatively, the house price dynamics may be interpreted as resulting from the interaction between housing demand and supply. According to this interpretation, an area is currently expensive, because housing supply has not yet increased to absorb a positive demand shock.[2]

This pattern of house price fluctuations implies that choosing the currently popular and expensive location today involves the risk of incurring capital losses, if regional house prices fall, and then facing the borrowing constraint in the future, when the match with the (then) expensive location may be better than today. By contrast, buying a property

[2] When the housing supply adjusts, and the demand shock is absorbed, the prices will fall. However, since construction takes time, and may also require changes in local public policies, such as rezoning, this may only happen after a long, and random, delay. See e.g. Capozza et al. (2004), Evenson (2003) or Mayer and Somerville (2000).

in the currently cheap area provides the opportunity to realize capital gains. Thus while the currently "desirable" and expensive location is, for most households, more attractive from the consumption point of view, the currently cheap location offers better investment opportunities.

The pattern of residential sorting that emerges in equilibrium essentially reflects the relative strength of the consumption motive and the investment motive of housing. If regional shocks are large and/or persistent, the consumption motive dominates. The households make their location choices mainly by comparing current benefit streams. Since only a small group voluntarily chooses the less desirable location based on the consumption motive only, the regional allocation of households basically boils down to differences in wealth. Also, regional house price differences, as well as capital gains and losses realized in the housing market, are large, compared to typical household wealth.

When regional shocks are small and/or transient, the investment motive is stronger (in relative terms, compared to the consumption motive). Caring about their future prospects, many households, which would receive a larger immediate welfare stream from the desirable location, voluntarily choose the less desirable location, in the hope of making capital gains. Typically, a household resides in the desirable location, only if its current match with that location is truly good. The regional allocation of households then happens mainly through self-selection, according to the match, rather than based on wealth differences and borrowing constraints. The fact that many households voluntarily choose the "less desirable" location is also reflected in house prices. In equilibrium, regional house price differences, and capital gains and losses, are small, compared to typical household wealth.

The model produces two main empirical predictions. First, the size of house price fluctuations should be negatively correlated with the degree of residential sorting in the match dimension. Second, movers should be more sorted than stayers. We confront these predictions of the model with data from US metropolitan areas. Consistent with the predictions, we find that those metropolitan areas, where house price fluctuations have been large, also tend to have a more diverse mix of different educational and age groups, whereas metropolitan areas where prices have been less volatile tend to have a less diverse population, with certain age or educational categories under- or overrepresented, compared to the national average.

We also have data concerning the distribution of age, education and income, at the municipality level, and thus we can examine residential sorting within metropolitan areas. We find that metropolitan areas (such as Seattle) where house prices have been volatile tend to display lower degrees of residential sorting (so that there is heterogeneity within, rather than between, municipalities) than metropolitan areas (such as Atlanta), where price swings have been small.

In the second part of our empirical analysis, we establish that, among owner-occupying households, movers are more sorted (with respect to age, education and income) than stayers. Based on our theory, we interpret these differences as resulting from housing market related wealth shocks. For example, some stayer households whose characteristics are ill-suited for their present location, may be unable to move, since, due to a depreciation in local property values, they have negative housing equity. To further check if this wealth shock based mechanism might be (a part of) the explanation, we also look at renters (who are not subject to wealth shocks in the housing market) in our data. Interestingly, we find that among renters, stayers are more sorted than movers.

As a final result, the model predicts a non-linear relation between wealth and mobility: households with intermediate wealth levels should be more mobile than poor households and wealthy households. While we do not address this issue in the empirical part of the study, Henley (1998) has documented that this humpshaped relationship holds for British households.

Generally speaking, our theoretical framework combines themes, which are typically dealt with in two separate branches of literature. (i) Most papers in the literature on residential sorting use static general equilibrium models. Earlier sorting models[3] routinely assumed that households differ with respect to one characteristic only (typically income), and predicted perfect stratification along that dimension, a prediction clearly contradicted by the empirical evidence, cited above. The more recent two-dimensional sorting models

[3]Examples include the early seminal paper by Ellickson (1971), and more recent contributions by Epple and Romer (1989, 1991), Henderson (1991) and Wheaton (1993).

by Epple and Platt (1998), Epple and Sieg (1999) and Epple, Sieg and Romer (2001) are more successful in explaining the data. In these models households differ both with respect to income and with respect to tastes, and there is imperfect sorting along both dimensions. An alternative approach to account for the observed diversity of households within jurisdictions is based on the heterogeneity of the housing stock (e.g. Nechyba (2000)). In contrast to the present paper, the atemporal nature of these models means that housing and location choices do not involve investment considerations, and there is no feed-back from house price fluctuations to household wealth.[4]

(ii) The second branch of literature analyzes housing wealth as an important component of a household's asset portfolio. While the double nature of housing, as a consumption good and as an investment, and house price fluctuations play an important role here, this literature essentially focuses on the optimization problem of an individual household, and the implications for residential sorting are not examined.[5]

A few recent papers take up a similar mix of issues as we do here. Ortalo-Magné and Rady (2006) examine income heterogeneity in booming cities, where house prices rise, and home-owners, who make capital gains, may choose to stay put, even when newcomers typically earn higher incomes. The model has two periods and, with a certain probability, the newcomers enter the city in the second period. The first-period income/wealth distribution and population structure in the city are taken as given, whereas in our model expected future price movements affect today's location choices, through the investment motive.

Glaeser and Gyourko (2005) study the joint process of falling house prices and neighborhood change in declining cities. Due to the durability of housing, a negative shock leads to a sharp fall in housing prices, but only a slow and gradual decline in city size. Low housing costs in a city attract low-income households. In the model households are

[4] A few papers (e.g. Benabou (1996, Fernandez and Rogerson 1996) analyze sorting in a dynamic contex. Even in these models, however, the households are typically assumed to be renters, and they are also assumed to choose their location once and for all (in the first period), so that realized capital gains and losses do not shape the equilibrium pattern of residential sorting.

[5] Theoretical papers include Ranney (1982), Ioannides and Henderson (1983), Poterba (1984), Bruenecker (1997) and Sinai and Souleles (2005). Empirical research has been conducted by e.g. Ioannides and Henderson (1987) and Flavin and Yamashita (2001).

assumed to rent their homes, and the role of capital losses and borrowing constraints is not addressed.

The plan of the paper is as follows. Section 2 presents some stylized facts, which underlie our modelling approach. The model is developed in Section 3. Section 4, which contains our main theoretical results, establishes a relationship between the equilibrium pattern of residential sorting and house price fluctuations, and compares the degree of sorting among movers and stayers. In Section 5 the empirical predictions of the model are tested using data from US metropolitan areas. Finally Section 6 concludes.

2 Stylized facts

The modelling approach we adopt in this paper is consistent with the following stylized facts:

(1) In most developed countries, owner-occupied housing is the single most important investment for a typical household. For example in the late 1990's, single family owner-occupied housing composed 2/3 of household wealth in the UK, 1/3 of household wealth in the US, and 2/3 of the assets of a US household with median wealth.[6]

(2) House prices are often highly volatile, and in different regions property values tend to rise and fall asynchronously, so that also relative regional prices may vary a lot. Figures 1 and 2 illustrate this finding with price data from four UK regions and US metropolitan areas; in the figures the country average is normalized to 1.[7] Other OECD countries with large regional price fluctuations include e.g. Denmark, Finland, Ireland, and Spain. Relative prices may vary quite a lot even at a more local level, e.g. between London boroughs. For example in 1995 average house prices were 3% lower in Hackney than in Greenwich, but in 2001 Hackney was 20% more expensive than Greenwich[8]; see

[6] Banks et al. (2002), *Federal Reserve's 2001 Survey of Consumer Finances.*

[7] According to Shiller (1993, Ch 5 p. 79) real estate booms and busts in US cities have been regionally asyncronized and prize movements often dramatic. XX (2004) finds that, with the exception of the current housing boom, US house price dynamics have been mainly driven by local or regional, rather than national, shocks. For further evidence on US prices, see also e.g. Case and Shiller (1989), Malpezzi (1999), Case, Quigley and Shiller (2005), or Himmelberg, Mayer and Sinai (2005). For UK evidence, see e.g. Muellbauer and Murphy (1997), or Cook (2003).

[8] Source: Land Registry, http://www.landreg.gov.uk.

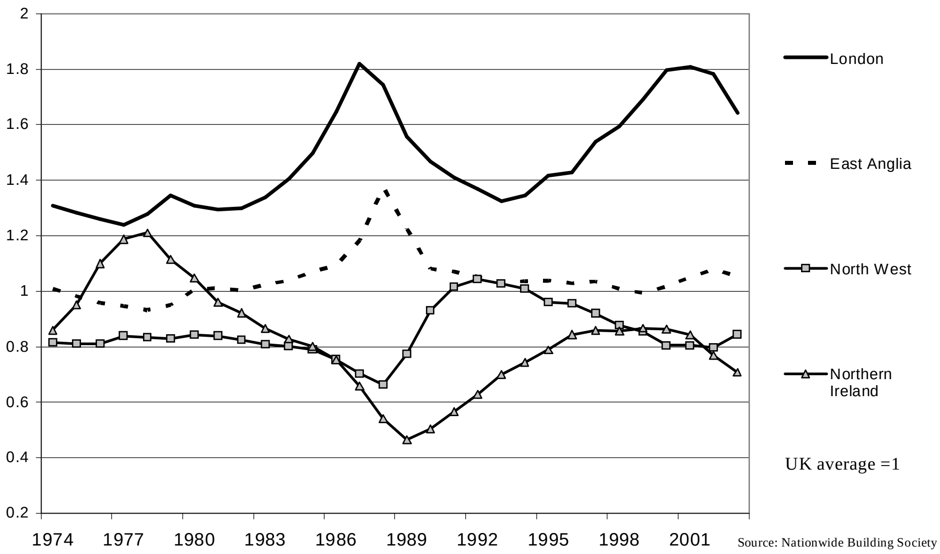

Figure 1: Relative house prices in the UK

also Iacoviello and Ortalo-Magné (2003). For similar findings on the Boston metropolitan area, see Case and Mayer (1996).

(3) While (relative) house prices may vary a lot in the short and medium run, in many cases there seems to be a long-run equilibrium relationship between house prices in different areas[9], or between local house prices and local economic fundamentals, such as average income or constructions costs[10]. If prices are currently below (above) the long-run equilibrium level, they are likely to rise (fall) some time in the future.[11]

(4) Capital gains and losses made in the housing market can be remarkably large compared to typical household incomes and savings. To illustrate this point, Table 1 shows maximum and minimum house-price-to-income ratios in four major US cities over

[9] That is, regional house prices are cointegrated. For evidence from UK regions, see e.g. MacDonald and Taylor (1993), Alexander and Barrow (1994) or Cook (2003, 2004). For evidence from US census regions, as well as for a comparison between the US and the UK, see Meen (2002). Also, Pollakowski and Ray (1997) find that in the Greater New York metropolitan area, the evolution of local house prices in a municipality can be predicted using lagged price changes in neighboring jurisdictions. Tirtiroglu (1992) reports similar findings from Hartford CT.

[10] For US evidence, see e.g. Capozza et al. (2004), Malpezzi (1999), Abraham and Hendershott (1996) or DiPasquale and Wheaton (1994). For UK evidence, see e.g. Ashworth and Parker (1997).

[11] The adjustment may take a long time. Factors that slow down the process include high population density, the scarcity of land, and strict public regulations. See e.g. Capozza et al. (2004) or Evenson (2003).

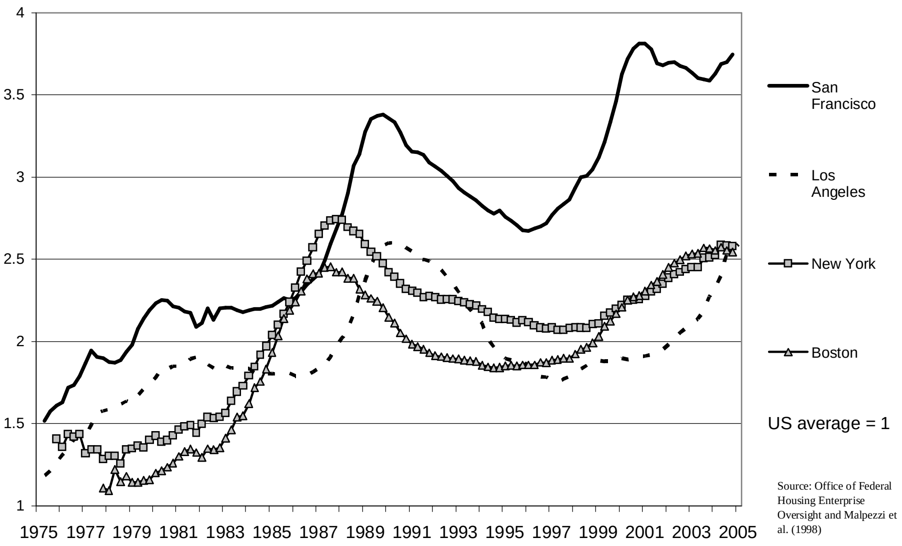

Figure 2: Relative house prices in the US

the period 1979-1996. In the UK, the average annual capital gain in the London market between 1983 and 1988 corresponded to 72% of the mean annual disposable household income in the UK over that period, and exceeded by the factor of 7.8 average yearly household savings. Between 1989 and 1992, the annual capital loss of a typical London homeowner was equivalent to 77% of average disposable household income, and 8.4 times average household savings.

Table 1. Maximum and minimum house-price-to-income ratios, 1979-1996

	House-price-to-income ratio	
	min	max
Boston	5.4	12.0
New York	5.3	12.0
Los Angeles	6.7	11.1
San Francisco	6.4	11.4

Source: Malpezzi, 1999

(5) Empirical studies of household mobility reveal that capital losses made in the housing market may seriously limit a household's ability to move from one location to another. This is the case especially if a household has negative equity, i.e. the value of the mortgage exceeds the value of the house.[12]

(6) As a general rule, housing market risks are uninsurable. For example Shiller (1993, 2003) lists home equity insurance as one of the key financial markets currently missing. Shiller (1993), and Shiller and Weiss (1998) discuss the potential problems, both economic and psychological, involved in providing hedging against house price swings, as well as ways to overcome these problems.[13]

3 The model

3.1 The basics of the economy

The economy has two locations, or neighborhoods. Each location has an equal, fixed, stock of identical houses. Each house is occupied by a single household and no one household is ever homeless. For convenience, assume that the stock of houses and the mass of households each comprises a continuum of size unity.

There are infinite discrete time periods indexed by $t = 0, 1, ...$ In each period, one of the locations is deemed to be "desirable" while the other one is "less desirable". When a period changes, the relative ranking of the locations is reversed with probability $\pi \in (0, 1]$.

[12] See e.g. Henderson and Ioannides (1989), Chan (1996, 2001), or Henley (1998).

[13] In the UK, real estate futures were traded in the London Futures and Options Exchange (London Fox) in 1991, from May through October. Trading volume was low, and the market was closed when it was reported that the exchange had allegedly attempted to create a the false impression of high trading value by false trades (Shiller (1993, Ch 1)). In the early 2000's house price index derivatives were (re)introduced by two spread betting firms IG Index and City Index (Iacovello and Ortalo Magne (2003)). The interest shown by the British public has been minimal, and IG Index has already withdrawn the products from the market

In the US, there are a few local experiments with home equity insurance. The Oak Park Experiment has been running since 1977, and the South-West Home Equity Assurance Program was initiated in 1988. Both of these programs are in Chicago, and insure homeowners against price declines caused by neighborhood change. More recently, the Yale/Neighborhood Reinvestment Corporation Home Equity Guarantee Project has developed home equity insurance products, to be initially used in Syracuse, New York. See Shiller (2003, Ch 8).

We also consider a small region interpretation of the model, with a continuum of locations. Then in each period, one half of the locations are "desirable" while the remaining locations are "less desirable", and when a period changes, a measure π of the locations is hit by a regional shock. The long-run equilibrium of the model is essentially identical under both interpretations.

The households are heterogenous in the quality of their match, or their type, θ. The aggregate heterogeneity of households is unchanged over time, and θ has a stationary distribution, with a cumulative distribution function $G(\theta)$, on some support $[\theta_L, \theta_H]$. Without loss of generality, we assume that the median match $\theta_m = 0$, i.e. $G(0) = \frac{1}{2}$.

The households live forever and discount future utilities by a common factor $\beta \in (0, 1)$. The per period utility of a household is conditional on the neighborhood where it resides. A household with current match θ receives utility $\frac{1}{2}\varepsilon + \theta$, when living in a currently desirable location. The per period utility of anyone household living in a less desirable location is $-\frac{1}{2}\varepsilon$. Here the parameter $\varepsilon > 0$ gauges regional welfare differences, as well as the size of regional shocks. Given these assumptions, all households with a current match $\theta > -\varepsilon$ receive a higher welfare stream when residing in a desirable location. The measure of these households is $1 - G(-\varepsilon) > \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ and $G(-\varepsilon) = 0$, all households would rather live in the popular area. Since the measure of houses in the desirable locations is one half, housing is in short supply in the popular neighborhoods.

A household's match may change over time. First, if the neighborhood or jurisdiction where the household resides is hit by a regional shock, the match between the household and the location is broken, and a new type is independently drawn from the distribution function $G(\theta)$.[14] Second, even if the overall popularity of the jurisdiction remains unaltered, between periods the match may change for some idiosyncratic, or household specific, reason, with probability $\lambda \in [0, 1]$, and the new match is drawn from the distribution $G(\theta)$. In Section 5 we drop the assumption that the new match is independently drawn, and the match is allowed to follow a general Markov process.

[14] An underlying premise is that a location which was popular (unpopular) in period t and another location which is popular (unpopular) in period $t+1$ are likely to be "desirable" ("undesirable") in different ways; thus it is plausible to assume that the match that the household had with the period-t desirable (undesirable) location does not carry over to the period-$(t+1)$ desirable (undesirable) neighborhood.

In any period, the aggregate welfare is maximized, if all households with $\theta > \theta_m = 0$ are allocated to the (currently) desirable locations, those with $\theta < 0$ live in the less desirable locations, and the group (always of measure zero, if G is continuous) with $\theta = 0$ is divided between the locations so that capacity constraints in housing are not violated. In other words, there is perfect residential sorting according to the match. If this allocation rule is followed, the aggregate utility in any period is $w^* = \frac{1}{2}E[\theta \mid \theta \geq 0]$.

3.2 The household's problem

In the market outcome, the location choice depends on not only the match, but also on house prices and wealth. We fix the house price in (currently) desirable locations to 1, and normalize the price of housing in (currently) less desirable regions to 0. These normalizations can be made without loss of generality, since in this model the only choice available to the households is whether to own a house in a popular neighborhood or in an unpopular neighborhood; a household cannot sell a house without buying another one. The normalizations adopted here also mean that capital gains and losses are of size unity.

Consistent with empirical evidence, we assume that capital gains and losses made in the housing market (as well as idiosyncratic shocks affecting the match θ) are uninsurable. The incomplete markets setting we consider here is the simplest possible. In addition to owning a home, the households can carry wealth to the future by holding a single non-interest bearing financial asset, which can be interpreted as outside money. The fixed nominal supply of money is M. If the price of money, in terms of housing (in good locations), is $1/p$, the real supply is M/p. We could also easily introduce pure credit, or inside money, and allow the households to borrow up to a certain limit, without changing any of the results. Since the households have no income sources outside the housing market, the steady state interest rate cannot be positive; with a positive interest rate, a household with negative initial asset holdings[15] exceeds any finite debt limit with a positive probability. Then in steady state the interest rate is zero, so that inside and outside money

[15] The (non-degenerate) asset market equilibrium of a pure credit economy (see e.g. Hugget 1993) necessarily involves some households having negative positions.

are perfect substitutes.[16] These simplifying assumptions (no income sources outside the housing market, and, by implication, zero interest rate) are adopted so as to focus on the role of capital gains and losses in wealth dynamics.

Denote financial asset holdings by a and let h be housing. h is equal to 1, if the household owns a house in a desirable location, and equal to 0, if the house is in an undesirable location. We also define a household's total wealth (n), which consists of both financial wealth (money) and housing wealth

$$n_t = a_t + h_t. \tag{1}$$

Given our simple wealth dynamics, a household's wealth position changes if and only if the household makes a capital gain or suffers a capital loss in the housing market[17]:

$$n_{t+1} = n_t + s_t (1 - 2h_t) \tag{2}$$

where s_t is an indicator function which is equal to 1 if there is a regional shock at date t and 0 otherwise. If, prior to the shock, the household owned a property in a then undesirable location, $(h_t = 0)$ the household makes a capital gain and climbs one rung in the wealth ladder; if the house was in a good neighborhood $(h_t = 1)$ before the change of fortunes, the household suffers a loss and falls one rung down.

There is a lower limit for financial asset holdings $a_{\min}$, that a household is not allowed to exceed. A simple and fairly natural normalization is adopted here by fixing the minimum balance to be zero, $a_{\min} = 0$, but allowing a negative minimum balance, say $-b$, would just involve a change of origin, without altering the analysis or any of the results[18]. With the origin fixed to zero, and households making capital gains and losses of size unity, we can now assume, without loss of generality, that wealth only takes non-negative integer values $n = 0, 1, 2, ...$ At wealth levels $n \geq 1$, a household may freely choose its housing

[16] See e.g. Ljungqvist and Sargent (2004, Ch 17.10).

[17] In any given period t, the household's budget constraint is $h_t + a_t = a_{t-1} + (1 - s_{t-1})h_{t-1} + s_{t-1}(1 - h_{t-1})$. Combining the budget constraint and the definition of total wealth (1) yields (2).

[18] See e.g. Aiyagari (1994) or Ljungqvist and Sargent (2004, Ch. 17.10).

location, and its wealth portfolio may consist of n units of real money balances and a cheap house ($h = 0$), or $n - 1$ units of financial assets and an expensive home ($h = 1$). If $n = 0$, the household owns a house in an undesirable location, $h = 0$, and since it has no money, $a = a_{\min} = 0$, it cannot afford a house in a desirable location: choosing $h = 1$ would imply $a = -1 < a_{\min}$, and this is not allowed. The liquidity (or borrowing) constraint that limits a household's location choices can be expressed as follows:

$$h_t = 0 \text{ if } n_t = 0. \tag{3}$$

Now consider the optimization problem of any one household. At each moment of time t it chooses its location $h_t \in \{0, 1\}$ so as to maximize the expected discounted utility stream

$$E_\theta \sum_{t=0}^{\infty} \beta^t \left[h_t \left(\frac{1}{2}\varepsilon + \theta_t \right) - (1 - h_t) \frac{1}{2}\varepsilon \right],$$

subject to (2) and (3). The problem can be conveniently presented in a recursive form. Let $V(\theta, n)$ be the (ex post) value function of a household with current type θ and current wealth n, and denote by $V^g(\theta, n)$ and $V^b(\theta, n)$ the household's expected prospects, if it chooses the good/bad location in the current period (and also makes the same choice in future periods, as long as its type θ and its wealth n remain unaltered). Obviously for a maximizing household $V(\theta, n) = \max\{V^b(\theta, n), V^g(\theta, n)\}$. Also define the household's ex ante value function $V(n) = E_\theta[V(\theta, n)]$, which describes the household's expected prospects when the household faces a shock (idiosyncratic or regional) and does not yet know its new match. The value functions $V^b(\theta, n)$ and $V^g(\theta, n)$ satisfy the following recursive equations:

$$\begin{aligned} V^b(\theta, n) &= -\tfrac{1}{2}\varepsilon + \beta \left[(1-\pi)\{(1-\lambda) V^b(\theta, n) + \lambda V(n)\} + \pi V(n+1) \right] \\ V^g(\theta, n) &= \tfrac{1}{2}\varepsilon + \theta + \beta \left[(1-\pi)\{(1-\lambda) V^g(\theta, n) + \lambda V(n)\} + \pi V(n-1) \right] \end{aligned} \tag{4}$$

In the current period, the household's utility is $-\frac{1}{2}\varepsilon$ or $\frac{1}{2}\varepsilon + \theta$, depending on its location choice. Its prospects for the next period are discounted by β, and are given inside the square brackets. With probability $(1-\pi)(1-\lambda)$ the household faces no shocks, and it will face the same value function $\left(V^g(\theta, n) \text{ or } V^b(\theta, n)\right)$ as today. With the complementary probability $[1-(1-\pi)(1-\lambda)]$ the match is broken, and the household's prospects are captured by the ex ante value function. If the match changes for household specific reasons, the wealth of the household remains unaltered, and future welfare is given by $V(n)$. If there is a regional shock, not only the match changes, but also house prices rise or fall, and depending on housing location, the household makes a capital gain or suffers a capital loss, resulting in expected future welfare $V(n+1)$ or $V(n-1)$.

In all unconstrained wealth classes, a household chooses a desirable location if and only if $V^g(\theta, n) \geq V^b(\theta, n)$. Solving the equations (4) for $V^g(\theta, n)$ and $V^b(\theta, n)$ reveals that at each unconstrained wealth level $n \geq 1$, the household chooses a desirable neighborhood if and only if

$$\theta + \varepsilon > \pi\beta\left[V(n+1) - V(n-1)\right] \tag{5}$$

The condition (5) involves a useful decomposition of the decision problem into the consumption motive, figuring on the left-hand side, and the investment motive, visible on the right-hand side. The strength of the consumption motive depends only on the current match θ, and the measure of regional disparities ε. If there were no need to care about the future, all households with $\theta > -\varepsilon$ would choose the currently desirable region, while only those with $\theta < -\varepsilon$ would (voluntarily) live in the less popular area. The downside of choosing a currently popular and expensive location is that a household may suffer capital losses, if regional house prices fall, and it may then be borrowing constrained in the future, when the match θ with an expensive location can be better than today. By contrast, opting for a currently less popular and less expensive area entails the chance of making capital gains. These considerations are captured by the investment motive. Due to the investment motive, even some households with $\theta > -\varepsilon$, i.e. households whose immediate benefits are higher in the desirable location, voluntarily choose the unpopular area.

At each wealth level n, there is then a critical value of the match

$$\theta_n^* = \begin{cases} \theta_H & \text{if } n = 0 \\ -\varepsilon + \pi\beta\left[V(n+1) - V(n-1)\right] & \text{if } n \geq 1 \end{cases} \tag{6}$$

and the household's location choice rule assumes a simple threshold form:

$$h(\theta, n) = \begin{cases} 1 & \text{if } \theta > \theta_n^* \\ 0 & \text{if } \theta \leq \theta_n^* \end{cases} \tag{7}$$

Given the threshold rule (7), $V(n) = E_\theta\left[(1 - h(\theta, n))\, v^b(\theta, n) + h(\theta, n)\, v^g(\theta, n)\right] = \int_{\theta_L}^{\theta_n^*} v^b(\theta, n)\, dG(\theta) + \int_{\theta_n^*}^{\theta_H} v^g(\theta, n)\, dG(\theta)$, and, with the help of (4), we get a recursive equation for the ex ante value function $V(n)$

$$\begin{aligned} V(n) &= -G(\theta_n^*)\tfrac{1}{2}\varepsilon + (1 - G(\theta_n^*))\left(\tfrac{1}{2}\varepsilon + E_\theta[\theta \mid \theta \geq \theta_n^*]\right) \\ &+\beta\left[(1-\pi)V(n) + \pi\{G(\theta_n^*)V(n+1) + (1 - G(\theta_n^*))V(n-1)\}\right]. \end{aligned} \tag{8}$$

The equation (8) can be interpreted as follows: From the ex ante perspective, i.e. before the match is drawn, a household with n units of wealth chooses the undesirable location with probability $\Pr(\theta < \theta_n^*) = G(\theta_n^*)$, and the desirable location with probability $\Pr(\theta > \theta_n^*) = (1 - G(\theta_n^*))$. Then the expected utility stream is $-G(\theta_n^*)\frac{1}{2}\varepsilon + (1 - G(\theta_n^*))\left(\frac{1}{2}\varepsilon + E_\theta[\theta \mid \theta \geq \theta_n^*]\right)$. With probability π, there is a regional shock, and depending on its location choice the household makes a capital gain or suffers a loss. The household's location choice essentially boils down to equations (6) and (8).

Figure 3 shows the critical match θ_n^* with different values of n when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$. Clearly, θ_n^* decreases with n, and wealthier households are ready to choose the desirable location even with a more modest match. This is a general property of θ_n^*, and it stems from the fact that the value function is concave. (Concavity is proved in the appendix.) Also, this finding has a natural interpretation. Assets are valued since they provide the option to make unconstrained choices in the future. But if a household is wealthy, additional assets are of less value: the more assets the household has, the more distant is the prospect of being borrowing constrained at some point in the future. To put it differently, the investment motive is more important for poor households than for wealthy households.

Essentially, Figure 3 reveals a pattern of two-dimensional residential sorting, which emerges in equilibrium:

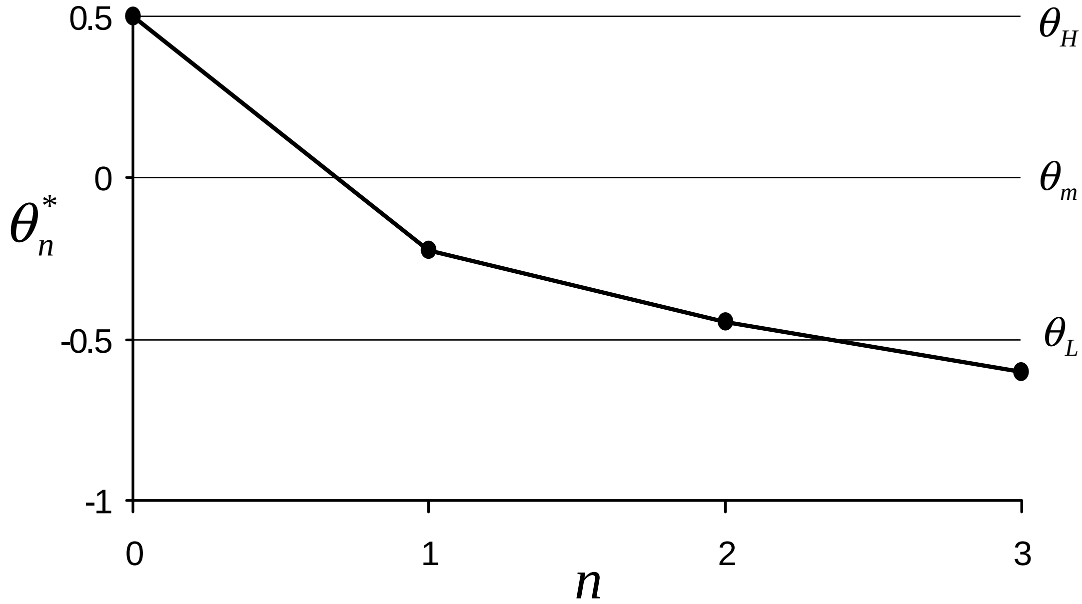

Figure 3: The θ_n^*-curve when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$.

Proposition 1 *Residential sorting takes place both according to the match and according to wealth. Wealthy households and households with a good match tend to choose a popular neighborhood, while less wealthy households and households with a poorer match live in the less desirable locations.*

Next we show that the relative strength of the consumption motive and the investment motive, as well as location choices, depend on the size and frequency (persistence) of regional shocks.[19]

[19]Notice that the parameter λ does not appear in the equations (6) and (8), and a household with current wealth n and current match θ makes the same location choice, no matter how frequent (λ close to 1) or how rare (λ close to 0) idiosyncratic shocks are. To understand this finding notice first, that the household's expected future prospects, if its match changes for idiosyncratic reasons, are unaffected by its housing location (see equations (4) which both contain the term $(1-\pi)\lambda V(n)$). Second, remember that the investment motive, affecting location choice, derives from the prospect of capital gains and losses made in the housing market. Now, although the expected time that the household has a given match (current or future), $1/[1-(1-\pi)(1-\lambda)]$ periods, depends on λ, every time a capital gain or loss is realized, the household's match is destroyed by a regional shock. Finally notice that the parameter λ does, however, affect the agents' *ex post* prospects: differentiating (4) reveals that the more permanent the types are, the more the ex post value function $v(\theta, n)$ depends on the current match.

The parameter ε provides a measure of how different the desirable and the undesirable locations are from each other. It also gauges the size of regional shocks: if a location is hit by a shock, the utility stream it offers to the (median) household changes from $\frac{1}{2}\varepsilon$ to $-\frac{1}{2}\varepsilon$, or vice versa. An increase in regional differences, so that ε rises, strengthens the households' incentives to choose a desirable location in the current period (consumption motive). On the other hand, it also reinforces the incentives to accumulate assets (investment motive), since a household stands to loose more if it faces the borrowing constraint at some point in the future. However, since the future losses are discounted, and only occur with a certain probability, while the higher welfare stream is available today, the effect on the consumption motive dominates. The larger the regional differences, and the regional shocks, the more likely it is, ex ante (i.e. before the realization of the match), that a household with a given level of disposable wealth n chooses a currently desirable location. The following lemma states these findings more formally.

Lemma 1 *The θ_n^*-schedule shifts down when ε increases. That is, for all $n \geq 1$, $\frac{d\theta_n^*}{d\varepsilon} < 0$.*

Proof See the appendix. ■

The parameter π measures the frequency of regional shocks. As the parameter π does not affect the utility streams available in different locations, an increase in π leaves the consumption motive unaffected. By contrast, the investment motive becomes stronger, encouraging the households to choose a currently less popular and less expensive location. This is because households living in a popular neighborhood are increasingly likely to suffer capital losses, while capital gains are an increasingly likely prospect if the household buys a property in a currently less desirable area. Viewing the household's dilemma from a somewhat different angle, when transitions up and down the wealth ladder take place with a higher probability, the prospect of facing the borrowing constraint at some point in the future becomes an increasingly powerful deterrent. Then in any unconstrained wealth class, a household needs a better match before it chooses to live in a currently popular neighborhood.

Lemma 2 *The θ_n^*-schedule shifts up when π increases. That is, for all $n \geq 1$, $\frac{d\theta_n^*}{d\pi} > 0$.*

Proof See the appendix. ■

3.3 Equilibrium

The previous section showed, how a household chooses its location based on its current wealth and its current match. On the other hand, a household's current wealth depends on its past fortunes in the housing market, and its past location choices. Then the long-run wealth distribution arises as a result of households' moving policies. Location choices and the stationary wealth distribution together determine the long run equilibrium of the model.

Denote by f_n the size of wealth class n, and let $f_n^b = G(\theta_n^*) f_n$, and $f_n^g = (1 - G(\theta_n^*)) f_n$ be the frequency of households with disposable wealth n who live in bad (or undesirable) locations and good (or desirable) locations, respectively. Notice also that

$$\frac{f_n^b}{f_n^g} = \frac{G(\theta_n^*)}{1 - G(\theta_n^*)} \equiv \gamma_n \tag{9}$$

The "odds ratio" γ_n decreases with n as wealthy households are more likely to choose a good neighborhood.

Consider first the two-region interpretation of the model. If there is no regional shock in a given period $(s = 0)$, the wealth distribution remains unaltered. If there is a shock $(s = 1)$, all f_n households who were previously in wealth class n either climb one step up or fall one step down, depending on their house location. They are replaced by f_{n-1}^b class $n-1$ households who have made a capital gain, and f_{n+1}^g class $n+1$ households who have suffered a capital loss. The distribution is stationary if and only if

$$f_n = (1 - s) f_n + s \left(f_{n-1}^b + f_{n+1}^g\right) \tag{10}$$

for all n. Next, if there is a continuum of atomistic regions, in every period, the fraction π of the locations is hit by a regional shock, and the wealth distribution is stationary if and only if

$$f_n = (1 - \pi) f_n + \pi \left(f_{n-1}^b + f_{n+1}^g\right) \tag{11}$$

for all n. It is easy to see that the equations (10) and (11) both boil down to the same stationarity condition

$$f_n \equiv f_n^b + f_n^g = f_{n-1}^b + f_{n+1}^g \tag{12}$$

As a consequence, both model variants have the same long-run wealth distribution, and the same long-run equilibrium.

There are no wealth classes below the borrowing constrained class 0, and at wealth level 0 the borrowing constrained households can only choose an unpopular location. Observing that $f_{-1} = f_0^g = 0$, equation (12) implies $f_0 \equiv f_0^b = f_1^g$; that is, the group of households with minimum financial asset holdings, but a house in a good location (f_1^g) must be of the same size as the borrowing constrained group (f_0^b). But then $f_1 \equiv f_1^b + f_1^g = f_0^b + f_2^g$ implies $f_1^b = f_2^g$, and iterating forward leaves us with the sequence of equations

$$f_n^b = f_{n+1}^g \tag{13}$$

for all $n \geq 0$. In words, each two groups with the same financial asset holdings, but different housing location, must be of equal size. Summing over all wealth classes yields $\sum_n f_n^b = \sum_n f_n^g$ and given that the aggregate mass of households is unity $\sum_n f_n = \sum_n \left(f_n^b + f_n^g\right) = 1$, it follows that

$$\sum_n f_n^i = \frac{1}{2}, \quad i \in \{b, g\} \tag{14}$$

But these equations indicate that the demand for housing, on the left-hand side, is equal to the supply of housing $\left(\frac{1}{2}\right)$, in both location types. Thus housing markets clear.

To obtain an explicit characterization of the wealth distribution, we next combine (9) and (13). What we get is a simple first order difference equation for both f_n^g and f_n^b.

$$f_{n+1}^g = \gamma_n f_n^g \text{ and } f_{n+1}^b = \gamma_{n+1} f_n^b \tag{15}$$

and the frequency of any node can be linked to the size of the liquidity constrained group f_0 : $f_n^b = f_{n+1}^g = f_0 \prod_{i=0}^{n} \gamma_i \quad n \geq 0$, where $\gamma_0 \equiv 1$. These equations, combined with the housing market equilibrium (14) lead to the formulae[20]

$$f_{n+1}^{g}=f_{n}^{b}=\frac{\prod_{i=0}^{n}\gamma_i}{\sum_{k=0}^{\infty}\prod_{i=1}^{k}\gamma_i}\quad \text{for } n=0,1,...$$

which, together with the equalities $f_n = f_n^b + f_n^g$, determine the stationary distribution. Since the "odds ratio" γ_n is decreasing in n, the equations (15) imply that the wealth distribution is single-peaked, with wealth classes in the middle having more mass than those on the tails, a property which is consistent with observed empirical wealth distributions. Intuitively, households with fewer assets are likely to choose the less popular location and make capital gains. For wealthy households, capital losses are more probable. Thus transitions in the wealth distribution tend to happen towards the middle.

Next we study what happens to the wealth distribution, when the size or the frequency of regional shocks changes. To do so, let us define the cumulative distribution function

$$F(n;\varepsilon,\pi)=\sum_{i=0}^{n} f_i.$$

Lemma 3 *When ε decreases or π increases, the wealth distribution shifts to the right, in the sense of first-order stochastic dominance. That is $\frac{\partial F(n;\varepsilon,\pi)}{\partial\varepsilon}\geq 0$ and $\frac{\partial F(n;\varepsilon,\pi)}{\partial\pi}\leq 0$. In particular, the size of the borrowing constrained group (f_0) decreases, when ε decreases or π increases.*

Proof By Lemmas 1 and 2, the θ_n^*-schedule shifts up when π grows or ε decreases. This then increases the "odds ratio" γ_n, and equations (15) imply that the ratio f_{n+1}^i/f_n^i, $i\in\{b,g\}$ goes up. But then it follows immediately that for each n, $\frac{\partial F(n;\varepsilon,\pi)}{\partial\varepsilon}\geq 0$ and $\frac{\partial F(n;\varepsilon,\pi)}{\partial\pi}\leq 0$. ■

Intuitively, when capital gains and losses become more probable (π increases), the households adopt increasingly cautious strategies, and are more likely to choose one of the less desirable locations, in the hope of capital gains. As a result a larger proportion of the households reach higher wealth levels, and fewer of them face the borrowing constraint.

[20] Notice that $\lim_{i\to\infty}\gamma_i=\frac{G(\omega)}{1-G(\omega)}<1$. Thus the sum $\sum_{k=0}^{\infty}\prod_{i=1}^{k}\gamma_i$ always converges.

Likewise smaller regional differences (smaller values of ε) enhance the popularity of the less desirable locations, and make upward transitions in the wealth ladder more probable.

The aggregate long-run equilibrium of the economy is defined by the stationary wealth distribution, combined with the households' location choices. In this section we have essentially studied how location choices induce the distribution. It is however also useful to move the other way round. The wealth distribution, and especially Lemma 3, allows us to further characterize the environment that the households face, assess how wealthy they are, and to reinterpret the location choices they make.

First, the average wealth level in the economy[21], $E[n]$, or the quantiles of the wealth distribution, can be used as yardsticks, against which the size of capital gains and losses, normalized to 1, can be measured.

Remark 1 *The larger, or the less frequent (and more unexpected), the regional shocks, the larger the capital gains and losses are, compared to household wealth, measured by e.g. average wealth, median wealth, or any other quantile of the wealth distribution.*

Proof The size of capital gains and losses is normalized to 1. Lemma 3 implies that average wealth $E[n]$ increases (decreases) together with π (ε); also any quantile n_q of the wealth distribution, where $n_q = \max\{n\}$ such that $F(n) \leq q$, $q \in [0,1]$, increases (or remains constant) when π increases or ε decreases. ■

The next remark points out how the households' wealth can be assessed, and reinterprets Lemmas 1 and 2, in revealing a relation between a household's relative position in the wealth distribution, and its location choice.

Remark 2 *Consider a household with disposable wealth n. The smaller (larger) the value of π (ε), (i) the wealthier (poorer) the household is, relative to other households and (ii) the more likely the household is to choose a currently desirable (undesirable) location.*

Proof The result follows from Lemmas 1-3. ■

[21] Notice that $\lim_{n\to\infty} \frac{n+1}{n}\frac{f_{n+1}}{f_n} = \lim_{n\to\infty} \frac{n+1}{n}\frac{f^b_{n+1}+f^g_{n+1}}{f^b_n+f^g_n} = \lim_{n\to\infty} \frac{n+1}{n}\frac{1+\gamma_{n+1}}{1+\frac{1}{\gamma_n}} = \lim_{n\to\infty}\gamma_n = \frac{G(-\varepsilon)}{1+G(-\varepsilon)} < 1$. Thus the sum $E[n] \equiv \sum_{n=0}^{\infty} n f_n$ converges, and $E[n]$ is always finite.

Finally, for the sake of completeness, we characterize the asset market equilibrium.[22] The market clearing condition is

$$E[a] = \frac{M}{p} \tag{16}$$

where the left-hand side of (16) is the aggregate demand for financial assets, and the right-hand side is the net supply, equal to real outside money. Using (1), and the housing market equilibrium condition $E[h] = \frac{1}{2}$, (16) can be rewritten as $E[n] = \frac{1}{2} + \frac{M}{p}$, and

$$p = \frac{M}{E[n] - \frac{1}{2}}.$$

In light of Lemma 3, it is evident that p, the monetary size of capital gains and losses, increases when regional shocks become larger, or less frequent.

4 Residential sorting

4.1 House price fluctuations and residential sorting

This section studies the aggregate behavior of the economy. We begin by analyzing social welfare. Addressing this normative issue will eventually allow us to characterize the form of residential sorting emerging in equilibrium (i.e., sorting according to the match, or according to wealth), since in the present model high social welfare is essentially associated with location choices based on the match, rather than on wealth.

Consider a household with n units of wealth. In any period it chooses a desirable neighborhood if its match $\theta \geq \theta_n^*$ and otherwise goes to a less desirable neighborhood. Given this strategy, the expected utility (before the draw of θ) of the household, or alternatively the average realized utility of all households in class n, is

$$u_n = -G(\theta_n^*) \frac{1}{2}\varepsilon + (1 - G(\theta_n^*)) \left(\frac{1}{2}\varepsilon + E[\theta \mid \theta \geq \theta_n^*]\right)$$

[22] The equilibrium we establish here essentially resembles the equilibrium of the simple Bewley-type model considered by Ljungqvist and Sargent (2004, Ch 17.10.4), in which outside money and inside money (credit) are perfect substitutes, and the interest rate is zero.

Aggregation then involves summing over all wealth classes. Notice that $\sum_n f_n G(\theta_n^*) = \sum_n f_n^b = \frac{1}{2}$ (by the housing market equilibrium (14)), and overall welfare in any given period is

$$w = \sum_{n=0}^{\infty} f_n u_n = \sum_{n=0}^{\infty} f_n^g E[\theta \mid \theta \geq \theta_n^*] = \frac{1}{2} E[\theta \mid h = 1] \tag{17}$$

An alternative way to approach social welfare is to imagine that a new household enters the economy. The entrant is assigned to wealth class n with probability f_n, and its expected intertemporal prospects are then given by the value function $V(n)$. The household's prospects ex ante, i.e. before it knows its wealth, are

$$W = \sum_{n=0}^{\infty} f_n V(n) \tag{18}$$

The appendix shows that, up to a constant multiplier, the per period utility and value function based measures of welfare are equivalent:

$$w = (1 - \beta) W \tag{19}$$

These equalities are also needed in the proof of the following proposition.

Proposition 2 *Social welfare increases, when (i) the size of regional shocks (ε) decreases, or (ii) regional shocks become more frequent (π increases).*

Proof See the appendix. ■

Combing Proposition 2 and Remark 1 reveals that large (small) house price fluctuations tend to be associated with low (high) levels of social welfare.

The normative Proposition 2 is next used as a building block, as we characterize residential sorting in the match dimension, and the relation between sorting and house price fluctuations.

Proposition 3 *When (i) the size of regional shocks (ε) decreases or (ii) the regional shocks become more frequent (π increases), the degree of residential sorting in the match dimension increases in the following sense. (a) In each location $h \in \{0, 1\}$, the average* *match $E[\theta \mid h]$ becomes more distinct from the economywide average $E[\theta]$. (b) The locations become more distinct from each other, and the between-locations variance of the match increases. (c) The locations become internally more homogenous, in the sense that the within-location variance of the match decreases.*

Proof When conditions (i) and/or (ii) hold, it follows from Proposition 2 that $E[\theta \mid h=1]$ increases. (a) Then, since $\frac{1}{2}E[\theta \mid h=1]+\frac{1}{2}E[\theta \mid h=0]=E[\theta]$, and $E[\theta]$ is a constant, it follows that $E[\theta \mid h=0]$ decreases. Thus the difference $|E[\theta \mid h]-E[\theta]|$ increases for $h \in \{0,1\}$. (b) Item (a) implies that the between-locations variance $Var(E[\theta \mid h]) = \frac{1}{2}(E[\theta \mid h=0]-E[\theta])^2+\frac{1}{2}(E[\theta \mid h=1]-E[\theta])^2$ increases. (c) The economywide variance of the match $Var(\theta)$ can be decomposed $Var(\theta)=Var(E[\theta \mid h])+E[Var(\theta \mid h)]$. Since $Var(\theta)$ is a constant, it follows from item (b) that the within-locations component $E[Var(\theta \mid h)]$ must decrease. ■

Corollary 1 *The smaller, or the more frequent the regional shocks are, (i) the smaller are house price fluctuations, and (ii) the more residential sorting there is in the match dimension.*

Proof The result follows from Proposition 3 and Remarks 1. ■

Proposition 3, together with Corollary 1, motivates a large part of our empirical work, which we discuss in Section 5.

Now we proceed to characterizing the degree of residential sorting in the wealth dimension. The households posses both housing wealth and financial capital. Then, in principle the municipalities could differ in terms of both wealth categories. However, equations (13) indicate that in the long-run equilibrium the distribution of financial assets is identical in both location types. Then given that $E[a \mid h=1]=E[a \mid h=0]$, interregional wealth differences derive entirely from different house values

$$E[n \mid h=1]-E[n \mid h=0]=E[h \mid h=1]-E[h \mid h=0]=1$$

and proportioning regional wealth differences to the average wealth gives

$$\frac{E\left[n \mid h=1\right]-E\left[n \mid h=0\right]}{E\left[n\right]}=\frac{1}{E\left[n\right]} \tag{20}$$

Also, since $E\left[n \mid h=1\right]-E\left[n\right]=\frac{1}{2}$ and $E\left[n \mid h=0\right]-E\left[n\right]=-\frac{1}{2}$, it is easy to conclude that the between-locations variance of wealth is $Var\left(E\left[n \mid h\right]\right)=\frac{1}{4}$, and the coefficient of variance is then given by

$$CV(E\left[n \mid h\right])=\frac{Var\left(E\left[n \mid h\right]\right)}{E\left[n\right]}=\frac{1}{4}\frac{1}{E\left[n\right]} \tag{21}$$

Proposition 4 *(i) The larger the regional shocks (ε) are, or (ii) the more infrequent/persistent the shocks are (the smaller π is) the more the locations differ from each other in terms of wealth.*

Proof The result follows by combining Lemma 3 and equation (20), or equation (21). ■

The following proposition is about idealtypes.

Proposition 5 *(a) There is perfect sorting in the match dimension (and no sorting in the wealth dimension), when $\varepsilon \to 0$, or when $\delta \to 1$, where $\delta \equiv \frac{\pi\beta}{1-\beta(1-\pi)} \in [0,1)$. (b) There is perfect sorting in the wealth dimension (and no sorting in the match dimension) if $\theta_L+\varepsilon > \pi\beta\frac{E[\theta]-\theta_L}{1-\beta}$.*

Proof See the appendix. ■

The equilibrium pattern of residential sorting, with different values of ε, is illustrated in Figure 4. In each panel, the cumulative wealth distribution is measured on the horizontal axis, and the cumulative match distribution on the vertical axis. Then area has a simple frequency mass interpretation (with one quarter of the unit square's area corresponding to one quarter of the households etc.). The figure shows a clear pattern, with the degree of residential sorting in the match dimension decreasing, and the degree of wealthwise sorting increasing, as the size of the regional shocks grows. Also the magnitude of house price fluctuations, measured by $P \equiv \frac{1}{2}\frac{1}{E[n]}$, $P \in [0,1]$, grows together with the size of the shocks (see Remark 1). Panels a (no shocks) and d (large shocks) correspond to idealtypes, with

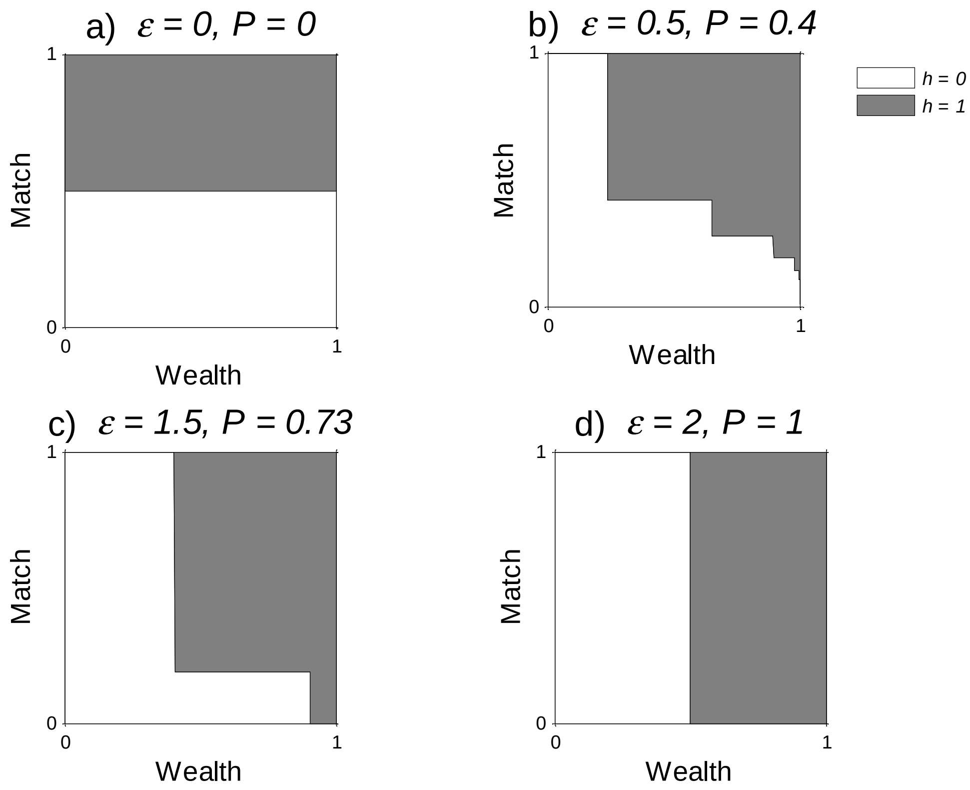

Figure 4: Equilibrium pattern of residential sorting with different values of ε. The measure of house price fluctuations reported in the figures is $P = \frac{1}{2}\frac{1}{E[n]}, \ P \in [0, 1]$.

perfect sorting in the match dimension and in the wealth dimension, respectively (and no sorting in the complementary dimension). Panels b and c are intermediate cases, with shocks of intermediate size, and imperfect sorting along both dimensions. A similar sequence of figures could be also presented with respect to the parameter π.

The pattern of residential sorting that emerges in equilibrium essentially reflects the relative strength of the consumption motive and the investment motive of housing. If regional shocks are large and/or persistent, the consumption motive dominates. The households make their location choices mainly by comparing current benefit streams. Since only a "small" group (always less than one half of the households, sometimes no household) voluntarily chooses the less desirable location based on consumption motive only, the regional allocation of households basically boils down to differences in wealth.

The wealthiest households live in the desirable location, while the less wealthy (borrowing constrained) households, which cannot afford an expensive home, reside in the less desirable location.

When regional shocks are small and/or transient, the investment motive is stronger. Caring about their future prospects, many households, which would receive a larger immediate welfare stream from the desirable location, voluntarily choose the less desirable location, in the hope of making capital gains. Typically, a household lives in the desirable location, only if its current match with that location is truly good. The regional allocation of households then happens mainly through self-selection, according to the match, rather than based on wealth differences and borrowing constraints.

Empirically, the theory predicts that residential sorting according to the match and according to wealth should produce different neighborhoods and regions. Remember that a household's match may be interpreted as reflecting various socioeconomic characteristics of the household (such as household size, education, age structure, and perhaps income). Thus sorting in the match direction tends to create residential areas, where neighbors resemble each other. By contrast, according to our theory, a household's current wealth depends on capital gains and losses realized in the housing market. These are random events, and otherwise similar (different) households may end up at different (similar) wealth levels. Then under wealthwise sorting neighbors may have little in common, apart from the value of their home.

4.2 Movers and stayers

The US metropolitan area data that we shall examine below, in Section 5, contain information on household mobility. Also, the empirical mobility literature (Henley (1998)) has found an interesting non-linear (humpshaped) relationship between wealth and mobility, and to the best of our knowledge, there have been no theoretical attempts to explain this empirical regularity. Thus we find it useful to briefly examine the degree of household mobility at different wealth levels, and compare the degree of residential sorting between movers and stayers.

We begin by demonstrating a simple non-linear connection between wealth and mobility. Take any given wealth class n. In the steady state, the portion $1 - G(\theta_n^*)$ of agents with wealth level n live in the desirable location. In any period $(1-s)\lambda + s$ households are hit by a shock, which breaks their match. Then the share $G(\theta_n^*)$ of the households, which are in the popular neighborhood at the beginning of the period, get a realization $\theta < \theta_n^*$ and move to the unpopular neighborhood. Therefore, mobility from the desirable to the undesirable location in wealth class n is equal to $((1-s)\lambda + s)\, G(\theta_n^*)[1 - G(\theta_n^*)]$. Likewise, it is easy to conclude that mobility from the undesirable to the desirable location equals the same measure. Then overall mobility in wealth class n is

$$\mu(n) = ((1-s)\lambda + s)\, 2G(\theta_n^*)[1 - G(\theta_n^*)]. \tag{22}$$

Clearly, there is more mobility in those periods, when the economy is hit by a regional shock and $s = 1$. Under the atomistic locations interpretation, in any given period mobility at wealth level n is $\overline{\mu}(n) = ((1-\pi)\lambda + \pi)\, 2G(\theta_n^*)[1 - G(\theta_n^*)]$. Notice also that in the two-region case, $\overline{\mu}(n)$ is the long-run average mobility at wealth level n.

Essentially, $\mu(n)$, or $\overline{\mu}(n)$, defines a humpshaped relation between wealth and mobility:

Proposition 6 *Mobility is increasing in wealth at low wealth levels, and decreasing in wealth at high wealth levels. Then households with intermediate levels of wealth are more mobile than rich households and poor households.*

Proof Equation (22) implies that the measure of mobility $\mu(n)$ (or $\overline{\mu}(n)$) is a downward opening parabola, with its peak at $G(\theta_m) = \frac{1}{2}$. Also $\mu(n) = 0$ at the extreme points $G = 0$ and $G = 1$. According to Proposition 1, θ_n^*, and thus $G(\theta_n^*)$, is decreasing in n. Also, $G(\theta_n^*) > \frac{1}{2}$ at low values of n, with $G(\theta_0^*) = 1$. On the other hand $G(\theta_n^*) < \frac{1}{2}$ at high levels of n, since $\lim_{n\to\infty} \theta_n^* = -\varepsilon$ and $G(-\varepsilon) < \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ we have $G(\theta_n^*) = 0$ for all $n \geq \overline{n}$, where $\overline{n} < \infty$. ■

This pattern of mobility essentially reflects the varying strength of the investment motive at different wealth levels. Rich households, with a weak investment motive, want to live in the popular location most of the time, and only rarely find it optimal to move. Poor households typically stay in the unpopular location; for the liquidity constrained this is obviously the only alternative. At intermediate levels of wealth, the investment motive is neither extremely strong nor very weak; when choosing their location, these households are more sensitive to changes in the match, and they move more often.

Remarkably, the relationship between wealth and mobility established in Proposition 6 is essentially the same as empirically documented by Henley (1998) for the UK; see especially Figure 2 in Henley (1998). According to Henley (1998, p.425), "levels of housing wealth are an important factor in explaining mobility, and the relationship between the two is not linear." British households with large negative housing equity are virtually immobile. Also very wealthy households tend to move relatively little. Households with intermediate levels of wealth are the most mobile.

Next we proceed to comparing the degree of residential sorting between movers and stayers. We define cumulative distribution functions $G(\theta \mid h, m)$ separately for four groups, conditioning on the households present location ($h \in \{0,1\}$), and on whether the household has moved in the present period ($m = 1$, if the household has moved, and $m = 0$, if the household has not moved). So, for example $G(\theta \mid h = 0, m = 1)$ is the distribution function for those households, which moved at the beginning of the period (from an expensive location) and currently live in a cheap location. We also define the functions $DG(\theta \mid h) \equiv G(\theta \mid h, m = 1) - G(\theta \mid h, m = 0)$, for $h \in \{0,1\}$, which allow us to compare (in the sense of first order stochastic dominance) the distributions of newcomers and old residents, who live in the same location (0 or 1).

To address the degree of residential sorting among movers and stayers, we further define

$$DG(\theta \mid m) \equiv G(\theta \mid h = 1, m) - G(\theta \mid h = 0, m), \ m \in \{0,1\} \tag{23}$$

Then $DG(\theta \mid m = 1)$ tells how the distribution of households which have moved from a cheap location to an expensive location differs from the distribution of those households which have moved the other way round; also $DG(\theta \mid m = 0)$ allows us to compare the distributions of immobile households living in different locations. Finally, to compare the degree of residential sorting between movers and stayer, we define the function

$$RS_{m/s}(\theta) \equiv DG(\theta \mid m=1) - DG(\theta \mid m=0)$$

It is clear than both among movers and among stayers, those who live in the desirable location typically have a higher value of θ than those who reside in the less desirable location, that is $DG(\theta \mid m) \leq 0$ for all θ and for $m \in \{0,1\}$. Now we use the function $RS_{m/s}(\theta)$ to address the question: among which group (movers or stayers) are the households residing in different locations more distinct from each other. In particular, if $RS_{m/s}(\theta) \leq 0$ for all θ (as we shall find), movers are more sorted in this sense.

Proposition 7 *(a) Movers have a better match with their (new) housing location than stayers. That is $DG(\theta \mid h=1) \leq 0$ and $DG(\theta \mid h=0) \geq 0$, for all θ. (b) Movers are more sorted than stayers. That is $RS_{m/s}(\theta) \leq 0$ for all θ.*

Proof Item (a) is proved in the appendix. (b) $RS_{m/s}(\theta) = DG(\theta \mid m=1) - DG(\theta \mid m=0)$ $= G(\theta \mid h=1, m=1) - G(\theta \mid h=0, m=1) - [G(\theta \mid h=1, m=0) - G(\theta \mid h=0, m=0)]$ $= DG(\theta \mid h=1) - DG(\theta \mid h=0)$. Then the result follows from item (a). ■

When interpreting item (a) of the proposition, remember that a good match with the cheap location means that a household has a low realization of θ.

Item (a) reflects the fact that those who move from one location to another tend to have rather strong match-related reasons to make that choice, while those who stay put may do so largely because they have been lucky or unlucky in the housing market. For example, households which move from the desirable location to the less desirable location, choose a cheap jurisdiction, although they could afford a more expensive house (their former home). By contrast, at least a part of the old residents live in the cheap area because they have suffered capital losses in the housing market, and are now borrowing constrained. Item (b) is a rather straightforward corollary of item (a), and it essentially reflects the same logic. The empirical work reported in Section 6 is based on item (b).

5 Empirical evidence

The empirical hypotheses that we have derived from the model concern the relation between house price fluctuations and the degree of residential sorting (Proposition 3, Corollary 1), as well as different sorting patterns among movers and stayers (Proposition 7). We begin by describing our data sources.

5.1 Data sources

House prices. The Office of Federal Housing Enterprise Oversight (OFHEO) provides quarterly house price indices for US metropolitan statistical areas (MSAs), as well as for the nation as a whole. Using these data we first compute the relative house prices $p_{it} = \log(I_{it}/I_t)$, where I_{it} is the house price index of MSA i in quarter t and I_t is the US house price index for the same period. Our measure of house price fluctuations in each MSA i is then the standard deviation of p_{it} over the period 1985-2000.[23] Our house price volatility data cover about 250 MSAs, for most of which we can also find measures of residential sorting.

Residential sorting. We use two main data sources. The first data set is from the Inter-University Consortium for Political and Social Research (ICPSR), at the University of Michigan. Based on the 1990 Census, these data[24] provide information on local distributions of a large number of socioeconomic variables, including education, age and income (the variables we use here). This information is available for various geographic levels, including municipalities or the Census minor civic divisions (MCDs) and MSAs.

The second data set is from the Integrated Public Use Microdata Series (IPUMS), at the Minnesota Population Center; we use 1% micro sample from the 1990 Census. Geographical units include the (so called) Public Use Microdata Areas (PUMAs), each having a population of approximately 100 000, and MSAs. Unlike the ICPSR data (which

[23] The house price indexes provided by OFHEO relate the house price in a given MSA i in quarter t to the house price in the same MSA in the base quarter (Q1 1995). Then the (log) price level in MSA i in quarter t, defleated by the US price index, is $\widehat{p}_{it} = \log(I_{it}/I_t) + \widehat{p}_{i0}$, where $\widehat{p}_{i0}$ is the (log) price level in the base quarter. Then $SD(\widehat{p}_{it}) = SD(\log(I_{it}/I_t)) = SD(p_{it})$.

[24] ICPSR Study No 2889.

does not contain local distributions of the variables for movers and stayers), the IPUMS data set allows us to compare the patterns of residential sorting among movers and stayers.

5.2 House price fluctuations and residential sorting

The first hypothesis we study is based on Proposition 3, and Corollary 1. These results suggest that a metropolitan area where house prices are volatile should also have a diverse population, with the shares of different demographic groups by and large corresponding to the national population structure. On the other hand, metropolitan areas where prices have been less volatile should have a less diverse population, with certain age or educational categories under- or overrepresented, compared to the national average. (Here the interpretation is adopted, that a location in the model economy can be though of as a metropolitan area, while the other location(s) can be thought of as the rest of the US.)

As the indicator of how much a given MSA deviates from the US average we use the measure

$$DV = \sum_{m=1}^{M} \frac{(S_m - S_m^{US})^2}{S_m^{US}}, \tag{24}$$

where S_m is the share of group m in the metropolitan area and S_m^{US} is the corresponding share at the national level. Notice that the larger the value of DV the more the MSA groups depart from those of the US as a whole.

Table 2 presents OLS regression estimates of the DV measure in (24) for education, age and income on house price volatility. In each of the regressions, the coefficient estimate of the house price volatility is negative, as expected. The estimated effect is clearly statistically significant for education, but not clearly so for the age and income groups. Thus, the results can be taken as initial evidence that metropolitan areas with high house price volatility tend to have similar demographic groupings (especially in terms of education) as the US average, while MSAs with little house price variation may depart more from the national average.

Table 2. OLS regression estimates of the effect of house price volatility on MSA's deviation from the US average education, age and income groupings

Independent variable	$DV_{Education}$	DV_{Age}	DV_{Income}
House price volatility	-0.20	-0.12	-0.23
	(0.09)	(0.15)	(0.18)
Intercept	0.07	0.04	0.13
	(0.01)	(0.02)	(0.02)

Notes: Dependent variable varies by column. Robust standard errors are given in parentheses. The sample consists of 243 MSA level observations. $DV_{Education}$, DV_{Age} and DV_{Income} indicate the measures in (24) computed for education (with three groups), age (with five groups) and income (with 25 groups), respectively. Precise definitions of the groups are given in the appendix.

In addition to metropolitan area level information, we also have data on education, age and income distributions in local municipalities. By analyzing heterogeneity both within and across municipalities, we get a clearer view of the pattern of residential sorting within metropolitan areas.

As a measure of residential sorting we use the Gini coefficient[25], defined by

$$GC = \frac{1}{2} \sum_{m} \sum_{k} \sum_{j} \frac{N_k N_j}{N^2 S_m (1 - S_m)} |S_{mk} - S_{mj}| \tag{25}$$

where N is the population of the metropolitan area, N_i is the population of municipality i, S_m is the share of group m in the metropolitan area and S_{mi} is the share of group m in municipality i. Small values of the Gini coefficient are associated with low degrees of residential sorting and large values with high degrees of sorting. The minimum value 0 (”no sorting”) is attained if and only if the relative proportions of different educational, age or income groups are identical in all municipalities. The maximum value 1 corresponds to ”perfect sorting”, a situation, with no overlap in the population structure of any two municipalities.

Proposition 3, especially items b and c, combined with Corollary 1, suggests that there should be negative correlation between house price fluctuations and the degree of

[25] See e.g. Rhode and Strumpf (2003).

residential sorting (in the match dimension), measured by the Gini coefficient. That is, in a metropolitan area where house price swings are small (large), local municipalities should be internally rather homogenous (heterogenous), and clearly (not very) distinct from each other. (Here an interpretation of the model is adopted, where a location corresponds to a municipality, while the entire economy is the metropolitan area.)

Unfortunately we do not have information on house prices at the municipality level, and thus the best we can do is to use the metropolitan area measures of price volatility. The results are summarized in Table 3. For all three match proxies (education, age, income), the coefficient of price variation is of the expected sign (negative), and statistically significant.

Table 3. OLS regression estimates of the effect of house price volatility on residential sorting

Independent variable	$GC_{Education}$	GC_{Age}	GC_{Income}
Price volatility	-0.28	-0.13	-0.31
	(0.11)	(0.06)	(0.09)
Intercept	0.19	0.11	0.20
	(20.3)	(0.01)	(0.01)
Sample size	242	242	238

Notes: Dependent variables vary by column. Robust standard errors are given in parentheses. Sample size indicates the number of MSA level observations. $GC_{Education}$, GC_{Age} and GC_{Income} refer to the values of the Gini coefficients defined in (25) for education, age and income, respectively. Precise definitions of the groups in each case are given in the appendix.

5.3 Movers and stayers

According to Proposition 7, movers are more sorted than stayers. That is, if two mobile households have chosen the same jurisdiction, we expect that these newcomers share some common characteristics. We also conjecture that they are somehow distinct from other mobile households, which have chosen a different location. By contrast, stayers living within the same jurisdiction should typically have less in common with each other, as their decision not to move is (according to our model) motivated by housing market related wealth shocks. At least some of these old residents stay put, because they have suffered capital losses and cannot move; alternatively they may have made capital gains, as there housing location has become more popular.

To test this hypothesis, we calculate Gini coefficients separately for movers and stayers. Here we classify as a mover a person, who has lived less than five years in its current home. For each characteristic (education, age, income), a single Gini coefficient is calculated for the whole US, so that N in (25) now stands for the US population, S_m is the share of group m in the US, and S_{mi} is the share of group m in PUMA i. The results for owner-occupyers are reported in the first two columns of Table 4. The empirical findings are consistent with Proposition 7: Gini coefficients for education, age and income are larger for movers than for stayers, indicating a higher degree of sorting in the former group.

Table 4. Gini coefficients for movers and stayers

	Owners		Renters	
	Movers	Stayers	Movers	Stayers
Education	0.28	0.24	0.29	0.31
Age	0.17	0.12	0.17	0.21
Income	0.37	0.28	0.31	0.41

Notes: The entries of the table refer to Gini coefficients computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups in each of the cases (education, age, income) are given in the appendix.

The explanation we offer for the different sorting patterns of owner-occupying movers and stayers is based on housing market related wealth shocks. Then one expects that the constellation should be somehow different for renters, who do not incur these wealth shocks. And indeed, for renters the results are reversed: stayers are more sorted than movers, with respect to all three variables we consider; see Table 4.

Our theory also predicts that households, which move from a (positively) correlated housing market (so that the prices of the old and the new home move up or down in tandem), should be less sorted than households, which move from an uncorrelated market. To test this hypothesis, we compute separate Gini coefficients for "short distance movers", i.e. households which have moved within the same metropolitan area, and "long distance movers", i.e. households, which have moved from another metropolitan area.[26] These computations are carried out for owner-occupiers, only. The results, reported in Table 5, lend support to our theory: "long distance movers" are more sorted, according to all three criteria.

Table 5. Gini coefficients for long and short distance movers

	Long distance	Short distance
Education	0.33	0.30
Age	0.26	0.17
Income	0.52	0.41

Notes: The entries of the table refer to Gini coefficients computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups for each characteristic (education, age, income) are given in the appendix. "Short distance movers" refer to households which have moved within the same MSA, while "long distance movers" have migrated from another MSA. See appendix for more detailed definitions.

To further test whether movers are more sorted than stayers, we compare them in terms of how their education, age and income vary across regions. If movers are more sorted than stayers, then we expect that educational attainment, age and income are more dispersed across regions among movers than among stayers. Table 6 reports standard deviations

[26] We also use data on persons that have moved *from* or *to* a non-MSA region. If a person has moved from an MSA region to a non-MSA region, or vice versa, he or she is recorded as "long distance mover", while a person that has moved between two non-MSA regions is recorded as "long distance mover" only, if his or her current state of residence is different from that five years ago. (See appendix for more details.)

over PUMA regions of the share of home-owners with a high school degree, separately for movers and stayers. Clearly, the share of moving high school graduates varies more across regions than that of stayers. This difference is also statistically significant, as shown by the p-values of the Levene (1960) and the Brown-Forsythe (1974) tests for equal variance. Similar observations apply to the share of people with at least a college degree. Table 6 also compares owner-occupying movers and stayers in terms of their age and income. In line with the above evidence on education, movers' age and income vary more across PUMA areas than those of stayers.

Like earlier with Gini coefficients, we can make a robustness check by comparing movers and stayers that live in rental housing. Because renters do not face similar housing market related wealth shocks as owners, there is no reason for moving renters to be more sorted than staying renters. Table 6 shows that the variance over PUMA regions of the share of moving renters with a high school degree is statistically even smaller than that of moving renters, while for tenants with at least a college degree it is the other way round. Furthermore, Table 6 shows that moving renters vary (significantly) less in age over PUMA regions than staying renters. Finally, there is no statistically significant difference in the income variance between moving and staying renters. Thus, most of the evidence indicates that among renters movers are no more sorted than stayers, which is consistent with our theoretical considerations.

Table 6. Comparing movers and stayers

	Mean		Std. deviation		Tests for equal variance	
	Movers	Stayers	Movers	Stayers	Levene	Brown-F.
Owners						
High School degree, %	0.45	0.46	0.12	0.07	0.00	0.00
College degree, %	0.38	0.26	0.17	0.13	0.00	0.00
Age	42.3	56.2	3.3	3.0	0.00	0.02
Income	46039	42021	15503	13113	0.00	0.00
Renters						
High School degree, %	0.48	0.45	0.09	0.11	0.00	0.00
College degree, %	0.27	0.17	0.13	0.10	0.00	0.00
Age	37.6	53.0	2.5	4.8	0.00	0.00
Income	23405	21760	6924	6839	0.28	0.47

Notes: The entries of the table are computed using PUMA level observations (total 1726). Each PUMA observation is obtained by averaging relevant observations (household heads) in the corresponding PUMA sample (from the 1990 Census). A household head is classified as a mover (a stayer), if he or she did not live (lived) in his or her current house five years ago. "High school degree, %" refers to the share of persons with a high school degree but not a college degree, "College degree, %" refers to the share of persons with at least a college degree, "Age" refers to the average age in years, while "Income" refers to the average annual income of household heads. (See the appendix for more detailed description of the variables.) "Levene" and "Brown-F.", respectively, refer to the p-values of the Levene (1960) and Browne and Forsythe (1974) tests for the equality of variances.

6 Conclusions

Recent empirical evidence indicates that local jurisdictions are internally more heterogenous, and less distinct from each other, than standard economic models of residential sorting predict. Motivated by these findings, this paper developed a dynamic model of two-dimensional sorting, with the following main properties. (i) A household's location choice depends both on its current wealth, and on its current "match", where the "match" may reflect various socioeconomic characteristics of the household. (ii) For an owner-occupying household, a house is both a consumption good and an investment, and location choice involves both aspects. (iii) Regional house prices fluctuate, and the resulting capital gains and losses affect household wealth. (iv) After suffering capital losses a household may face a borrowing constraint, which prevents mobility from a cheap housing location to an expensive location.

The pattern of residential sorting that emerges in equilibrium depends on the relative strength of the consumption motive and the investment motive of housing. The potency of these two motives is shown to depend on the size of and frequency (or persistence) of regional shocks. (When regional shocks, which affect the benefit streams available in the different locations, are large, or when these shocks are persistent, the consumption motive is strong compared to the investment motive). When the consumption motive is strong, compared to the investment motive, the households essentially care about today, rather than worry about tomorrow. Then residential sorting takes place primarily in the wealth dimension. In each period, the wealthiest households live in the currently desirable, and expensive locations, while those who reside in the less popular areas do so because they cannot afford a more expensive home. With this pattern of wealthwise sorting, neighborhoods or local jurisdictions are internally heterogenous with respect to socioeconomic characteristics other than wealth: neighbors may have little in common apart from the value of their home.

When the investment motive is strong, compared to the consumption motive, there is sorting according to the match. Households, which care about their future prospects, voluntarily choose a location which is currently unpopular and cheap, but where property values may rise in the future, and only live in the currently popular area, when their match with that location is truly good. Then, given the empirical interpretation of the match, neighbors should be alike.

The model produces two main empirical predictions. First, the size of house price fluctuations should be negatively correlated with the degree residential sorting in the match dimension. Second, movers should be more sorted than stayers. Using income, age and education as proxies for the "match", we tested these hypotheses with data from US metropolitan areas, and found support for our theory.

Mathematical Appendix

Proof of Proposition 1, and Lemmas 1 and 2

In this appendix we show that the value function is concave. We also demonstrate that the θ_n^*-schedule shifts down (up) when ε (π) increases. To establish these result, first notice that a household's strategy, telling how it chooses its location in each wealth class, essentially involves finding an optimal threshold value θ_n^* for each $n \geq 1$. As there is a one-to-one mapping between the threshold θ_n^*, and the corresponding value of the cumulative distribution function $G(\theta_n^*)$, also $x_n \equiv G(\theta_n^*)$ can be equivalently used as the choice variable. Given this reinterpretation of the problem, and using matrix notation, the Bellman equation for the ex ante value function (8) can be reexpressed in the following form

$$V = \max_{\{x_n\}} u + \beta\left[(1-\pi)I + \pi P\right]V \tag{26}$$

for $n \geq 1$ (and $x_0 = 1$) where V is the value function, stacked as a column vector, u is the column vector of expected immediate utility, with elements

$$u_n(x_n) \equiv \left(\frac{1}{2} - x_n\right)\varepsilon + \int_{x_n}^{1} G^{-1}(x)\,dx$$

and P is a transition matrix, with elements

$$P_{i,j} = \begin{cases} 1 - x_i & \text{if } j = i-1 \\ x_i & \text{if } j = i+1 \qquad i,j \in \{0,...,\overline{n}\} \\ 0 & \text{otherwise} \end{cases}$$

Proof of Proposition 1. Notice that $\frac{d^2 u_n}{dx_n^2} = -\frac{1}{G'(\theta_n^*)} < 0$. Thus (26) defines a maximization problem with a concave objective function and linear constraints. As a consequence the value function $V(n)$ is concave.$\square$

Also the first order-conditions can be rephrased using matrix notation

$$\theta^* = -\varepsilon \mathbf{1} + \pi\beta D V \tag{27}$$

where $\theta^* = (\theta_1^*, ..., \theta_{\overline{n}}^*)'$ is the vector of threshold values[27], $\mathbf{1} = (1, ..., 1)'$ and D is the difference matrix, with elements

$$D_{i,j} = \begin{cases} 1 \text{ if } j = i+1 \\ -1 \text{ if } j = i-1 & i \in \{1, ..., \overline{n}\},\ j \in \{0, ..., \overline{n}\} \\ 0 \text{ otherwise} \end{cases}$$

Proof of Lemma 1. The derivative of the right-hand side of (27) with respect to ε is $-\mathbf{1} + \pi\beta D \frac{dV}{d\varepsilon}$, and differentiating the Bellman equation (26) with respect to ε yields $\frac{dV}{d\varepsilon} = \frac{\delta}{\pi\beta}(I - \delta P)^{-1}\left(\frac{1}{2}\mathbf{1} - x\right)$, where $x \equiv (x_0, ..., x_{\overline{n}})$ and $\delta \equiv \frac{\pi\beta}{1-\beta(1-\pi)}$, $\delta \in [0,1)$. Then $\frac{d\theta^*}{d\varepsilon} = -\mathbf{1} + \pi\beta D \frac{dV}{d\varepsilon} = -\mathbf{1} + \delta D (I - \delta P)^{-1}\left(\frac{1}{2}\mathbf{1} - x\right)$. As x is a decreasing sequence, the elements of the vector $\delta D (I - \delta P)^{-1}\left(\frac{1}{2}\mathbf{1} - x\right)$ are positive. Next we want to establish that these terms are smaller than one. To do so, we adopt the notation $q \equiv D (I - \delta P)^{-1}\left(\frac{1}{2}\mathbf{1} - x\right)$. The elements of the vector q satisfy the recursive equations

$$q(n) = x_{n-1} - x_{n+1} + \delta\left[x_{n+1} q(n+1) + (1 - x_{n-1}) q(n-1)\right] \tag{28}$$

Assume that the maximum value of q is attained at some $n = \widetilde{n}$, that is $q(\widetilde{n}) = \max_n q$. Then in particular, $q(\widetilde{n}+1), q(\widetilde{n}-1) \leq k(n)$, and from (28) it is immediately clear that $q(\widetilde{n}) \leq x_{\widetilde{n}-1} - x_{\widetilde{n}+1} + \delta\left[x_{\widetilde{n}+1} q(\widetilde{n}) + (1 - x_{\widetilde{n}-1}) q(\widetilde{n})\right]$ or $q(\widetilde{n}) \leq q_{\max}(\Delta x) \equiv \frac{\Delta x}{1-\delta(1-\Delta x)}$, where $\Delta x \equiv x_{\widetilde{n}-1} - x_{\widetilde{n}+1}$. As x is a non-increasing sequence, Δx can take values over the interval $[0,1]$. Differentiating $q_{\min}$ with respect to Δx yields $\frac{dq_{\min}}{d\Delta x} = \frac{1-\delta}{(1-\delta(1-\Delta x))^2} > 0$. Thus $q_{\max}(\Delta x) \leq q_{\max}(1) = 1$, and $\max_n q \leq q_{\max}(\Delta x) \leq 1$. As a consequence we can

[27] Notice that θ_0^* cannot be freely chosen, as the agents are liquidity constrained.

conclude that

$$\frac{d\theta^*}{d\varepsilon} = -\mathbf{1} + \delta D (I - \delta P)^{-1} \left(\frac{1}{2}\mathbf{1} - x \right) \leq -(1-\delta)\,\mathbf{1} < \mathbf{0}$$

□

Proof of Lemma 2. Differentiating the right-hand side of (27) with respect to π yields $\frac{d(\pi\beta DV)}{d\pi} = \beta D \left(V + \pi \frac{dV}{d\pi}\right)$. Differentiating the Bellman equation (26) with respect to π, and using the envelope theorem, gives $\frac{dV}{d\pi} = \frac{\partial V}{\partial \pi} = \frac{\delta}{\pi}(I - \delta P)^{-1}(P - I)V$. Then

$$\frac{d\theta^*}{d\pi} = \beta D \left(V + \pi \frac{dV}{d\pi} \right) = \beta(1-\delta)D(I - \delta P)^{-1}V > \mathbf{0}$$

In signing the expression, the following facts have been used: (i) The value function V is increasing n. (ii) Then also $(I - \delta P)^{-1}V = \sum_{i=0}^{\infty}(\delta P)^i V$ is increasing in n. To see this, notice that $(1 - \delta P)^{-1}V$ is the value of a Markov process, with transition matrix P and immediate gain in state n given by $V(n)$. As this immediate gain increases with n, the expected present value of the program also increases. (iii) When we premultiply an increasing vector by the difference matrix D, the result is positive.[28]□

Proof of Proposition 2

We begin by deriving equation (19), which is used in the proof of the proposition. Premultiplying both sides of the Bellman equation (26) by f' yields

$$f'V = f'u + f'\beta \left[(1-\pi)\, I + \pi P\right] V. \tag{29}$$

Since the stationary distribution f is induced by the transition matrix P, it satisfies the equation

$$f'P = f'. \tag{30}$$

[28] If $\theta_L < -\varepsilon$, the state space n is infinite. Then we define a "meta value function" $Y(n)$, which satisfies the recursive equation $Y(n) = V(n) + \delta\left[x_n Y(n+1) + (1-x_n)\, Y(n-1)\right]$. It is straightforward to show that $Y(n)$ is increasing in n, and for each $n \geq 1$, $Y(n+1) - Y(n-1) > 0$.

Equations (29) and (30) imply that

$$w = f'u = (1-\beta) f'V = (1-\beta) W.$$

As proving the proposition with respect to π and ε, involves the same steps, we introduce a generic parameter ρ, where $\rho \in \{\pi, \varepsilon\}$. Now totally differentiating (18) yields

$$\frac{dW}{d\rho} = \frac{df'}{d\rho}V + f'\frac{dV}{d\rho} = \frac{df'}{d\rho}V + f'\frac{\partial V}{\partial \rho},$$

where the second equality is obtained by using the envelope theorem: as the threshold θ_n^* is chosen optimally in every wealth class $n \geq 1$, the indirect effect on the value function can be ignored. Next we use the identity (19) to show that also the weighted sum of direct effects $f'\frac{\partial V}{\partial \rho}$ vanishes:

$$f'\frac{\partial V}{\partial \rho} = \frac{\partial W}{\partial \rho} = (1-\beta)^{-1}\frac{\partial w}{\partial \rho} = 0$$

The first equality exploits the fact that the stationary distribution f depends on the parameters π, ε only indirectly, through the choice of policy and the second equality uses (19). The final equality follows from the observation that expected utility in a given period (u) does not depend directly on π and ε, and thus w does not depend directly on these parameters.

Thus only the effect through the stationary wealth distribution remains. By Lemma 3 we know that the distribution shifts to the right, towards higher wealth classes, when π increases or ε decreases. As the value function V is increasing in n, this shift in the stationary distribution translates into higher aggregate net welfare W:

$$\frac{dW}{d\pi} = \frac{df'}{d\pi}V \geq 0, \quad \frac{dW}{d\varepsilon} = \frac{df'}{d\varepsilon}V \leq 0$$

Finally, equations (17) and (19) imply that the average match in the desirable location improves, when regional shocks become more frequent, and it deteriorates when the shocks get larger

$$\frac{dE\left[\theta \mid h=1\right]}{d\pi} \geq 0, \quad \frac{dE\left[\theta \mid h=1\right]}{d\varepsilon} \leq 0$$

□

Proof of Proposition 5

(a) Match dimension. The household chooses $\{x_n\}$, so as to maximize the value function V, where V satisfies the recursive equation

$$V = \delta P V + (1-\delta)\frac{u}{1-\beta} \tag{31}$$

Iterating the equation (31) forward, we get

$$V = (1-\delta)\sum_{t=0}^{\infty}(\delta P)^t \frac{u}{1-\beta}$$

Next notice that $\lim_{t\to\infty} P^t = \mathbf{1}\nwarrow\!\!\searrow f'$ (where $\nwarrow\!\!\searrow$ is Kronecker product). Thus when $\pi \to 1$ and $\beta \to 1$, so that $\delta \to 1$, maximizing V becomes essentially equivalent to maximizing $f'u = w = \frac{1}{2}E[\theta \mid h=1]$. The objective function $w = \frac{1}{2}E[\theta \mid h=1]$ is maximized iff there is perfect sorting in the match dimension.

(b) Sorting in the wealth dimension. The putative equilibrium strategy is of the following form: $h(0,\theta) = 0$ for all θ (due to the borrowing constraint), $h(n,\theta) = 1$ for all θ and $n \geq 1$. Then in equilibrium $f_0 = f_1 = \frac{1}{2}$ and $f_n = 0$ for all $n \geq 2$.

Given this strategy, it is easy to calculate the ex ante values of the program $V(n)$ at different wealth levels $n \geq 0$. In particular, one can show that $V(2) - V(0) = (1-\delta)\frac{\varepsilon + E[\theta]}{1-\beta}$. Given the optimal location choice rule (5), the putative strategy is optimal for the household iff it always prefers the desirable location at wealth level $n = 1$, i.e. iff

$$\theta + \varepsilon > \pi\beta[V(2) - V(0)] = \pi\beta(1-\delta)\frac{\varepsilon + E[\theta]}{1-\beta} \text{ for all } \theta. \tag{32}$$

In particular, the condition (32) must hold for the lowest possible realization of the match θ_L. Inserting $\theta = \theta_L$, and slightly manipulating (32), yields the condition for residential sorting in the wealth dimension:

$$\theta_L + \varepsilon > \pi\beta \frac{E[\theta] - \theta_L}{1 - \beta}$$

□

Proof of Proposition 6

To prove the proposition, we need to construct the cumulative distribution functions $G(\theta \mid h, m)$, $h, m \in \{0, 1\}$.

(i) As a first step we characterize the match distributions of households living in the desirable and in the undesirable location, conditional on wealth class n. Given the threshold location choice rule (7), the distribution in the desirable location $G(\theta \mid h = 1, n) = G(\theta \mid \theta \geq \theta_n^*)$ is left-truncated, with truncation point θ_n^*, while the distribution in the undesirable location $G(\theta \mid h = 0, n) = G(\theta \mid \theta < \theta_n^*)$ is right-truncated with the same truncation point θ_n^*. The cumulative distribution functions are given by

$$\begin{aligned} G(\theta \mid h = 1, n) = G(\theta \mid \theta \geq \theta_n^*) &= \begin{cases} 0 & \text{when } \theta < \theta_n^* \\ \frac{G(\theta) - G(\theta_n^*)}{1 - G(\theta_n^*)} & \text{when } \theta \geq > \theta_n^* \end{cases} \\ G(\theta \mid h = 0, n) = G(\theta \mid \theta \leq \theta_n^*) &= \begin{cases} \frac{G(\theta)}{G(\theta_n^*)} & \text{when } \theta \leq \theta_n^* \\ 1 & \text{when } \theta > \theta_n^* \end{cases} \end{aligned} \tag{33}$$

Using the definitions (33), it is easy to show that $\frac{\partial G(\theta \mid \theta \geq \theta_n^*)}{\partial \theta_n^*} \leq 0$ and $\frac{\partial G(\theta \mid \theta \leq \theta_n^*)}{\partial \theta_n^*} \leq 0$ for all θ. This property means that if we compare two wealth levels n_1 and n_2, such that $n_1 < n_2$, and consequently $\theta_{n_1}^* > \theta_{n_2}^*$, the higher threshold $\theta_{n_1}^*$ in group n_1 implies that the distribution $G(\theta \mid h, n_1)$ first-order stochastically dominates the distribution $G(\theta \mid h, n_2)$ for $h \in \{0, 1\}$. More formally

$$G(\theta \mid h, n_1) \leq G(\theta \mid h, n_2) \text{ for all } \theta, \text{ when } n_1 < n_2 \text{ and } h \in \{0, 1\}. \tag{34}$$

(ii) As a second step, we need to study the conditional wealth distributions, contingent on housing location and mobility. The main objective is to establish a first-order stochastic dominance relation between movers and stayers in each location.

Denote the mass of households with wealth n, and group (h,m), by $\varphi_n(h,m)$. Now

$$\begin{aligned}\varphi_n(0,0) &= f_n^b \psi(\theta_n^*) \equiv f_n^b \{(1-s)[(1-\lambda)+\lambda G(\theta_n^*)] + sG(\theta_n^*)\} \\ \varphi_n(1,0) &= f_n^g \widehat{\psi}(\theta_n^*) \equiv f_n^g \{(1-s)[(1-\lambda)+\lambda(1-G(\theta_n^*))] + s(1-G(\theta_n^*))\} \\ \varphi_n(0,1) &= \varphi_n(1,1) = f_n^b(1-G(\theta_n^*))[(1-s)\lambda+s] = f_n^g G(\theta_n^*)[(1-s)\lambda+s]\end{aligned} \tag{35}$$

Also let $\widehat{\varphi}_n(h,m) \equiv \varphi_n(h,m)/\sum_i \varphi_i(h,m)$ be the relative share of wealth class n in group (h,m). Next, to compare the wealth distributions, we need the size ratios of adjacent wealth classes in different groups. Denote $\widehat{\gamma}_n(h,m) \equiv \widehat{\varphi}_{n+1}(h,m)/\widehat{\varphi}_n(h,m) = \varphi_{n+1}(h,m)/\varphi_n(h,m)$. Now using the equations (35) we get

$$\widehat{\gamma}_n(0,0)/\widehat{\gamma}_n(0,1) = \frac{\psi\left(\theta_{n+1}^*\right)}{\psi(\theta_n^*)} \frac{1-G(\theta_n^*)}{1-G\left(\theta_{n+1}^*\right)} \leq 1 \tag{36}$$

$$\widehat{\gamma}_n(1,0)/\widehat{\gamma}_n(1,1) = \frac{\widehat{\psi}\left(\theta_{n+1}^*\right)}{\widehat{\psi}(\theta_n^*)} \frac{G(\theta_n^*)}{G\left(\theta_{n+1}^*\right)} \geq 1 \tag{37}$$

These inequalities hold, since clearly $\psi\left(\theta_{n+1}^*\right)/\psi(\theta_n^*) \leq 1$, $(1-G(\theta_n^*))/\left(1-G\left(\theta_{n+1}^*\right)\right) \leq 1$, $\widehat{\psi}\left(\theta_{n+1}^*\right)/\widehat{\psi}(\theta_n^*) \geq 1$ and $G(\theta_n^*)/G\left(\theta_{n+1}^*\right) \geq 1$. The inequality (36) allows us to compare the wealth distributions of mover and stayer households, which currently reside in a cheap location. The inequality tells that, for any adjacent wealth classes $(n+1)$ and n, the ratio $\widehat{\varphi}_{n+1}/\widehat{\varphi}_n$ is larger for movers than for stayers. But this means that in the cheap location newcomers are wealthier than the old residents, in the sense of first-order stochastic dominance. The inequality (37) then implies that in the expensive location the opposite is true, and old residents are wealthier than newcomers, in the sense of first-order stochastic dominance.

(iii) As a final step, we combine the results of steps (i) and (ii), and construct the conditional match distribution functions

$$G(\theta \mid h,m) = \sum_n \widehat{\varphi}_n(h,m)\, G(\theta \mid h,n), \text{ for } h,m \in \{0,1\}. \tag{38}$$

That is, the conditional match distributions $G(\theta \mid h, m)$ are convex combinations of the location-contingent distributions $G(\theta \mid h, n)$ at different wealth levels n. In each group (h, m), the weight assigned to the distribution function $G(\theta \mid h, n)$ corresponds to the relative size of wealth class n in the group, $\widehat{\varphi}_n(h, m)$.

Using (23) and (38), we get

$$\begin{aligned} DG(\theta \mid h=0) &= \textstyle\sum_n \left[\widehat{\varphi}_n(0,1) - \widehat{\varphi}_n(0,0)\right] G(\theta \mid h=0, n) \geq 0, \\ DG(\theta \mid h=1) &= \textstyle\sum_n \left[\widehat{\varphi}_n(1,1) - \widehat{\varphi}_n(1,0)\right] G(\theta \mid h=1, n) \leq 0 \end{aligned} \tag{39}$$

for all θ. The signs follow from stochastic dominance, results (34), (36) and (37). The expressions (39) mean that in a currently cheap location, the match distribution of old residents stochastically dominates the match distribution of newcomers, while the in a currently expensive location the opposite is true. Thus we have proved that in both areas movers (with $m = 1$) tend to have a better match with the location than stayers ($m = 0$). (Remember, that a low (negative) realization of θ implies a good match with a currently unpopular location).$\square$

Data Appendix

Sorting measures computed from ICPSR data

The sorting measures applied in Tables 2 and 3 are computed from an extraction of data from the 1990 decennial Census. Detailed description of these data and the associated variables are given in the ICPSR study 2889 (1990). Tables 2 and 3 apply the data set 2 (DS2) where each variable is aggregated to the municipality (MCD) level. Because MCDs are geographically comprehensive, our MSA level observations are formed by summing up all relevant MCD level data.

The groups of types that we use in computing the DV measures in Table 2 and the Gini coefficients in Table 3 are as follows. We use five categories for age: (1) “children” (those of 0-15 years old), (2) “youth” (16-24 years old), (3) “adults, early career” (25-44 years old), (4) “adults, late career” (45-64 years old), and (5) “seniors” (those at least 65 years old). For education, we have three groups: (1) less than a high school degree, (2) at least a high school degree but not a college degree, and (3) a college degree or more. The Census defines the education groups for only those who are at least 25 years old. This age category is used to normalize the education groups within each region. Finally, for income we apply all the 25 income groups available in the ICPSR study 2889. In each of the cases, the US level groups are obtained by a population weighted average of the MSA level groups. The education and income categories applied here are similar to those of the Gini coefficients considered by Rhode and Strumpf (2003, p. 1660) (see also their Data Appendix at www.unc.edu/~cigar/ or www.unc.edu/~prhode/).

The samples of observations applied in Tables 2 and 3 derive from all those MSA level matches that we find between the house price volatility measure and the sorting measures in each case.

Sorting measures computed from IPUMS data

The sorting measures applied in Tables 4, 5 and 6 are computed from the Census data provided at www.ipums.org. The web site provides detailed definitions for each variable. For each observation unit (i.e., person) in the 1% sample from the 1990 Census, we downloaded household id (SERIAL), age (AGE), educational attainment (EDUC99), household income (FTOTINC), tenure (OWNERSHP), migration information (MIGRATE5, MIGMET5, MIGPLAC5) and location indicators (PUMA, STATEFIP, METAREA). These data include observations on $2,479,568$ persons from 1760 different PUMAs. The actual number of people in each PUMA is also obtained from www.ipums.org.

To compute the Gini coefficients in Table 4, we first classify each sample person into four categories depending on whether the person is an owner (OWNERSHP = 10) or a renter (OWNERSHP = 20) and whether the person is a mover (MIGRATE5 = 2) or a stayer (MIGRATE5 = 1). Persons with missing observations on OWNERSHP or MIGRATE5 are excluded from the calculations. To compute the Gini coefficients for age, education and income we apply similar categories as in Table 3. For computing the Gini coefficient for age, we estimate the shares of "children", "youth", etc. in each PUMA by computing the relative shares of the sample persons belonging to the relevant age category (for "children" the share of those 0-15 years old, etc.). For computing the education Gini coefficient, we restrict the sample to those at least 25 years old. The three education groups (consistent with those in Table 3) are formed by (1) EDUC99 ≤ 9, (2) $10 \leq$ EDUC99 ≤ 11, and (3) $12 \leq$ EDUC99. Finally, to compute the Gini coefficient for income, we first restrict the sample to household heads only (SERIAL = 1). Then we employ FTOTINC to classify each household into one of the 25 income ranges used in the ICPSR data, and compute the corresponding relative shares in each PUMA. In all cases (age, education and income), the US level shares are obtained as a population weighted average of the PUMA shares.

To compute the Gini coefficients in Table 5 we first restrict the sample to persons that own their house (OWNERSHP = 10) and that have moved recently (MIGRATE5 = 2). Within this subsample, we classify a person as a "short distance mover", if his current MSA is the same as five years ago, i.e., if METAREA and MIGMET5 match; otherwise the person is classified as a "long distance mover". In addition to data on persons that have moved from one MSA region to another, we also use data on persons that have moved *from* or *to* a non-MSA region. If a person has moved from an MSA region to a non-MSA region, or vice versa, he or she is recorded as a "long distance movers", while a person that has moved between two non-MSA regions is recorded as a "long distance mover" only, if his or her current state of residence (STATEFIP) is different from that five years ago (MIGPLAC5). The Gini coefficients for age, education and income are formed by applying the same convention of groupings as in Table 4.

The PUMA observations of the variables considered in Table 6 are computed for household heads only, while the applied groupings ("Owners", "Renters", "Movers", "Stayers") are defined in the same way as in Table 4. "High school degree, %" is the relative share of household heads at least 25 years old that have $10 \leq$ EDUC99 ≤ 11, "College degree, %" is the corresponding share of those that have $12 \leq$ EDUC99 ≤ 17. Finally, "Age" and "Income", respectively, refer to the average age (AGE) and income (FTOTINC) over the relevant households in each case.

References

Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Alexander, C., and M. Barrow (1994): "Seasonality and cointegration of regional house prices in the UK," *Urban Studies*, 31, 1667–1689.

Banks, J., R. Blundell, Z. Oldfield, and J. P. Smith (2004): "Housing wealth over the life cycle in the presence of house price volatility," University College London Discussion Paper 04-09.

Banks, J., R. Blundell, and J. P. Smith (2002): "Wealth portfolios in the UK and the US," NBER Working Paper 9128.

Bénaboy, R. (1996): "Equity and efficiency in human capital investment: the local connection," *Review of Economic Studies*, 63(2), 237–264.

Bewley, T. (1983): "A difficulty with the optimum quantity of money," *Econometrica*, 51, 1485–1504.

Brown, M. B., and A. B. Forsythe (1974): "Robust tests for the equality of variances," *Journal of the American Statistical Association*, 69, 364–367.

Brueckner, J. K. (1997): "Consumption and investment motives and the portfolio choices of homeowners," *Journal of Real Estate Finance and Economics*, 15(2), 159–180.

Case, K. E., and C. J. Mayer (1996): "Housing price dynamics within a metropolitan area," *Regional Science and Urban Economics*, 26, 387–407.

Case, K. E., J. M. Quigley, and R. J. Shiller (2005): "Comparing wealth effects: The stock market versus the housing market," *Advances in Macroeconomics*, 5(1), 1–32.

Case, K. E., and R. J. Shiller (1989): "The Efficiency of the Market for Single-Family Homes," *American Economic Review*, 79(1), 125–137.

CHAN, S. (1996): "Residential mobility and mortgages," *Regional Science and Urban Economics*, 26, 287–311.

CHO, M. (1996): "House price dynamics: A survey of theoretical and empirical issues," *Journal of Housing Research*, 7, 145–172.

COCCO, J. (2005): "Portfolio choice in the presence of housing," *Review of Financial Studies, forthcoming.*

COOK, S. (2003): "The convergence of regional house prices in the UK," *Urban Studies*, 40, 2285–2294.

DAVIDOFF, T. (2005): "Income Sorting: Measurement and Decomposition," *Journal of Urban Economics*, 58, 289–303.

ELLICKSON, B. (1971): "Jurisdictional fragmentation and residential choice," *Papers and Proceeding of the American Economic Association*, 61, 334–339.

EPPLE, D., R. FILIMON, AND T. ROMER (1984): "Equilibrium among local jurisdictions: Toward an integrated treatment of voting and residential choice," *Journal of Public Economics*, 24, 281–308.

EPPLE, D., AND G. J. PLATT (1998): "Equilibrium and local redistribution in an urban economy when households differ by preferences and income," *Journal of Urban Economics*, 43, 23–51.

EPPLE, D., AND T. ROMER (1989): "On flexible municipal boundaries," *Journal of Urban Economics*, 26, 307–319.

——— (1991): "Mobility and redistribution," *Journal of Political Economy*, 99, 828–859.

EPPLE, D., T. ROMER, AND H. SIEG (2001): "Interjurisdictional sorting and majority rule: An empirical analysis," *Econometrica*, 69(6), 1437–1465.

EPPLE, D., AND H. SIEG (1999): "Estimating equilibrium models of local jurisdictions," *Journal of Political Economy*, 107(4), 645–681.

FERNANDEZ, R., AND R. ROGERSON (1996): "Income distribution, communities, and the quality of public education," *Quarterly Journal of Education*, 111, 135–164.

FLAVIN, M., AND T. YAMASHITA (2002): "Owner-Occupied housing and the composition of the household portfolios," *American Economic Review*, 92(1), 345–362.

GLAESER, E., AND J. GYOURKO (2005): "Urban decline and durable housing," *Journal of Political Economy*, 113, 345–375.

GRUBER, J., AND R. F. MARTIN (2003): "Precautionary savings and the wealth distribution with illiquid durables," Board of Governors of the Federal Reserve System, International Finance Discussion Papers 773.

GYOURKO, J., C. MAYER, AND T. SINAI (2005): "Superstar cities," mimeo, University of Pennsylvania.

HENDERSON, J. V. (1991): "Separating Tiebout equilibrium," *Journal of Urban Economics*, 29, 128–152.

HENDERSON, J. V., AND Y. M. IOANNIDES (1983): "A model of housing tenure choice," *American Economic Review*, 73, 98–113.

——— (1987): "Owner occupancy: Investment vs consumption demand," *Journal of Urban Economics*, 21, 228–241.

HENLEY, A. (1998): "Residential mobility, housing equity and the labour market," *Economic Journal*, 108, 414–427.

HIMMELBERG, C., C. MAYER, AND T. SINAI (2005): "Assessing high house prices: Bubbles, fundamentals and misperceptions," *Journal of Economic Perspectives*, 19, 67–92.

HUGGETT, M. (1993): "The risk free rate in heterogenous-agent, incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

Iacoviello, M., and F. Ortalo-Magné (2003): "Hedging housing risk in London," *Journal of Real Estate Finance and Economics*, 27, 191–209.

Ioannides, Y. M. (2004): "Neighborhood Income Distributions," *Journal of Urban Economics*, 56, 435–457.

Levene, H. (1960): "Robust tests for the equality of variances," In Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling (eds.) Stanford University Press, pp. 278–292.

Ljungqvist, L., and T. J. Sargent (2004): *Recursive macroeconomic theory, Second Edition.* The MIT Press, Cambridge, Massachusetts.

MacDonald, R., and M. Taylor (1993): "Regional house prices in Britain: Long-run relationships and shurt-run dynamics," *Scottish Journal of Political Economy*, 40, 43–55.

Malpezzi, S. (1999): "A simple error correction model of house prices," *Journal of Housing Economics*, 8, 27–62.

Muellbauer, J., and A. Murphy (1997): "Booms and Busts in the UK Housing Market," *Economic Journal*, 107, 1701–1727.

Ortalo-Magné, F., and S. Rady (1999): "Boom in, bust out: Young households and the housing price cycle," *European Economic Review*, 43, 755–766.

——— (2001): "Housing market dynamics: on the contribution of income shocks and credit constraints," CESifo Working Paper No. 470.

——— (2002a): "Low household mobility, volatile housing prices, high income dispersion," CESifo Working Paper No. 823.

——— (2002b): "Tenure choice and the riskiness of non-housing consumption," *Journal of Housing Economics*, 11, 266–279.

——— (2006): "Heterogeneity within communities: A stochastic model with tenure choice," Mimeo, University of Munich and University of Wisconsin-Madison.

Poterba, J. (1984): "Tax subsidies in owner-occupied housing: an asset-market approach," *Quarterly Journal of Economics*, 99, 729–752.

Ranney, S. (1981): "The future house prices, mortgage market conditions, and the returns to homeownership," *American Economic Review*, 71, 323–333.

Rhode, P. W., and K. S. Strumpf (2003): "Assessing the importance of Tiebout sorting: local heterogeneity from 1850 to 1990," *American Economic Review*, 93(5), 1648–1677.

Schwab, R. (1982): "Inflation expectations and the demand for housing," *American Economic Review*, 72, 143–153.

Shiller, R. J. (1993): *Macro markets: Creating institutions for managing society's largest economic risks*. Oxford University Press, United Kingdom.

——— (2003): *The new financial order: Risk in the 21st century*. Princeton University Press, USA.

Shiller, R. J., and A. Weiss (1999): "Home equity insurance," *Journal of Real Estate Finance and Economics*, 19(1), 21–47.

Wheaton, W. C. (1993): "Land capitalization,Tiebout mobility, and the role of zoning regulations," *Journal of Urban Economics*, 34, 102–117.