Motelling: A Hotelling Model with Money[*]

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Abstract

We apply a mechanism design approach to a trading post environment where the household type space (tastes over variety) is continuous and it is costly to set up shops that trade differentiated goods. In this framework, we address Hotelling's [3] venerable question about where shops will endogenously locate in variety space across environments with and without money. Money has a role in our environment due to anonymity. Our specific question is whether monetary exchange leads to more product variety than an environment without money (i.e. a barter economy). We show that an efficient monetary mechanism does in fact lead to more product variety available to households provided the discount factor is sufficiently high, costs of operating shops are sufficiently low, and there is sufficient heterogeneity in tastes and abilities. We then show how this allocation can be implemented in a trading post economy with money. The paper is an attempt to integrate monetary theory and industrial organization.

1 Introduction

The question we address in this paper is whether monetary exchange can promote product variety. The framework we use to answer this question is a dynamic version of a trading post economy where agents have preferences over a continuum of varieties of goods. Since it is costly to set up trading posts, there is a finite set of active shops and where they decide to locate in the variety space is endogenously determined.

The paper is related to three different strands of literature. Shapley and Shubik [9] was one of the first papers to study exchange in a trading post economy. Recently, Howitt [4] extended this framework to one with possibilities of both barter and monetary trading posts and established conditions under which monetary exchange was preferred to barter. Our paper differs from his in two crucial aspects. First, our product space is not exogenously given. Instead the variety of goods produced is endogenously determined by firms' choosing

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where to locate in the product space as in Hotelling [3]. Second, we study the problem in a mechanism design framework as in Kocherlakota and Wallace [8] and then implement the allocation as a trading post economy.

Starting with the insights of Kiyotaki and Wright [6] about the essentiality of fiat money in bilateral matching environments with double coincidence problems, there is also a literature which studies agents' choices over which goods they specialize in production. Notably Camera, Reed, and Waller [1] show that the introduction of money leads to more specialization relative to barter and can increase welfare in a bilateral matching environment with divisible goods and Nash bargaining. There are many differences between our work and theirs, most notably that despite increased specialization with money, the entire exogenous set of varieties of goods are produced in their framework.

We proceed as follows. Section II outlines the environment. Section III states the general mechanism design problem. Section IV studies the allocation of a planner's problem where some of the constraints of the general mechanism design problem are relaxed. We then show that the allocation of the relaxed problem actually satisfies all the constraints of the general problem. In Section V we show how to implement the allocation of the relaxed problem as a trading post economy.

2 Environment

Time is discrete and the horizon is infinite. The economy is composed of a unit measure of infinitely-lived households indexed by $h \in [0,1]$ and a countably infinite number of infinitely-lived shopkeepers indexed by $k \in \mathbb{N}$. A household is composed of two members: a shopper denoted s and a worker denoted w.

Each good is indexed by its variety $\nu \in [0,1]$. We let good 0 denote fiat money. Fiat money is storable, divisible, and generates no direct utility to households or shopkeepers. Fiat money is in fixed supply and distributed uniformly, in per capita amount $\overline{m}$, at the beginning of time to households only. The quantity of money held by a household in period t is denoted $m(t)$ and the quantity held by shopkeeper k is denoted $M^k(t)$.

All goods other than money are nonstorable and consumable. Let $\mathcal{V} = (0,1]$ be the set of such consumable goods, which are also divisible. Households are heterogeneous with respect to their tastes in much the same way as Kiyotaki and Wright [6]. Specifically, they have a most preferred good to consume (denoted $\tau_c \in \mathcal{V}$) and most preferred good to produce ($\tau_p \in \mathcal{V}$). As in Kiyotaki and Wright [6], we make the assumption that a household cannot consume its own output. The household type space is denoted by $\Theta = \{(\tau_c, \tau_p) \in \mathcal{V} \times \mathcal{V}\}$. In any period, a household of type $\theta \in \Theta$ has preferences defined over consumption of c units of good v_c and production $\ell \in \{0,1\}$ of one unit of good v_p denoted by the utility function $U_\theta((c, v_c), (\ell, v_p))$. We further assume

$$U_\theta((c, v_c), (\ell, v_p)) = u(c) - f(|v_c - \tau_c|) - \ell \cdot f(|v_p - \tau_p|)$$

where f is a strictly increasing, convex function and u is a strictly increasing,

concave function of the quantity c of good v_c consumed which is bounded below.[1] Households discount the future at rate $\beta < 1$.

Each shopkeeper k has access to an exchange technology which allows him to set up a trading post in any pair of goods in $\mathcal{V} \cup \{0\}$ (specifically goods $(v_k, \widehat{v}_k) \in (\mathcal{V} \cup \{0\})^2$). Exchange at shopkeeper k's trading post is limited to its original designated pair (that is, after opening a trading post in one pair of goods, the shopkeeper cannot open a post in a different pair of goods in a subsequent period). Shopkeepers cannot produce goods and trading posts are in separate locations. All shopkeepers have identical linear preferences over all consumable goods and incur an initial fixed disutility cost $\overline{\kappa}$ if they set up shop. Let $C^k(t)$ denote shopkeeper k's consumption of good v_k, and $\widehat{C}^k(t)$ denote shopkeeper k's consumption of good $\widehat{v}_k$ in period t. Shopkeepers also discount the future at rate β.

We assume that all exchange must be voluntary (i.e. there is no technology to enforce exchange) and that there is free entry by shopkeepers.

In any period t, household identity, tastes, and money holdings (h, θ, and $m^h(t)$) are not observable. Shopkeeper identity, the varieties of goods at their shop, consumption, and cash $(k, (v_k, \widehat{v}_k), (C^k, \widehat{C}^k), M^k)$ are publicly observable. Households can send messages $(\widetilde{\theta}, \widetilde{m})$ about $(\theta, m(t))$ to the shopkeepers they visit at the trading post in any period t. This, coupled with the unobservability of h, is consistent with our assumption that households are anonymous. Our assumption about anonymity is consistent with shopkeepers keeping track of the distribution of household money holdings and tastes, but not specific identities as in Jovanovic and Rosenthal [5]. All agents have memory of their past actions and messages sent and received.

Shopkeepers allocate the two varieties of goods received at their trading post as follows. After receiving a message from a household, the shopkeeper offers a menu. A menu $a : \mathbb{R} \to \mathbb{R}$ is the set of all possible exchanges of quantities of goods $(v_k, \widehat{v}_k)$ where it is understood that a negative quantity means the good is received by the shopkeeper (having been produced by the worker). The household member(s) chooses a pair from the menu and exchange takes place. For example, a menu could be a rule where for $q > 0$ units of good v_k received by the shopkeeper, the household member receives αq units of good $\widehat{v}_k$ (i.e. $a = \{(-q, \alpha q) : q > 0\}$ for a given $\alpha > 0$).

The timing in any period is as follows. First, in any of a countably infinite number of rounds at the beginning of period t, shopkeepers choose whether to open a trading post (thereby incurring cost $\overline{\kappa}$). Second, if the trading post is opened, the shopkeeper makes a public announcement about the set of all possible menus offered conditional on household messages. Third, each household member decides whether to visit one (and only one) trading post or stay home. Fourth, if a household member visits a shop, it sends a message to the shopkeeper

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[1] Caplin and Nalebuff [2] posit a more general, yet separable form of a utility function where χ corresponds to v in our model, α corresponds to (τ_c, τ_p) in our model, and z is related to our good 0. We impose more symmetry on preferences in order to obtain a cleaner characterization of equilibrium.

after which the shopkeeper offers a menu (note that the public announcement and the actual offer may be different since there is no commitment). Fifth, in the event of a deviation of offers from the public announcement, a household member can announce to all other households at the trading post about the deviation. Sixth, production, exchange, and consumption decisions take place at each post. Finally, workers and shoppers return to their residence with any money accumulated.

Note that the environment implies a certain set of things that agents cannot do. First, after shopkeepers make public announcements, there can be no cross location communication between households and/or shopkeepers. That is, shopkeepers cannot

3 Mechanism Design Problem

A mechanism is a mapping from messages to an allocation both of which are consistent with the above environment. Household anonymity is a critical assumption in making the subsequent analysis simple. For any given period, after each household sends a message about its tastes and money holdings $(\theta, m) \in \Theta \times \mathbb{R}_+$, an allocation is a specification of the trading post to which each member of the household is directed (or to remain in autarky) and what trades they should make (which determines household production, consumption, and money holdings) as well as which types of trading posts should open and what trades should occur at those posts (which determines shopkeeper consumption). The mechanism makes proposals which prescribe actions to households and shopkeepers. Proposals can be accepted or rejected. In the case of rejection, the household or shopkeeper is precluded from trade for that period.

In the case of household proposals, the mechanism may direct the two members to the same location or different locations. The mechanism proposes two four-tuples, one to the worker and one to the shopper. A four-tuple to one of the members of the household is

$$(k, \nu, q, d) \in \Upsilon = (\mathbb{N} \cup \{0\}) \times \mathcal{V} \times \mathbb{R} \times \mathbb{R}$$

where k is the trading post to which the household member is sent (where we assume that $k = 0 \notin \mathbb{N}$ denotes that the household should stay home), ν is the variety of the good to be traded, q is the amount of the good the household member receives (if $q > 0$) or gives up (if $q < 0$), and d is the amount of money to be received (if $d > 0$) or given up (if $d < 0$). If the two proposals direct the household members to different trading posts, then given that trading posts are in different locations and there is no cross-location communication, these two proposals must be independent. If the household members are directed to the same trading post, the two proposals can be made conditional or independent of one another. Let $\iota = 0$ if the proposals are independent and $\iota = 1$ if they are conditioned on one another. The pure proposal to the household, γ_H, is a nine-tuple:

$$\gamma_H = (\iota, (k_p, \nu_p, -\ell, d_p), (k_c, \nu_c, c, d_c)) \in \Gamma_H = \{0, 1\} \times \Upsilon \times \Upsilon.$$

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Given our assumptions about anonymity, the pure proposals cannot be conditioned on h. Denote the mechanism allocation for any household as a function of the household's message in period $t \in T$ by $\mathcal{M}_H : T \times \Theta \times \mathbb{R}_+ \to \Gamma_H$.

In order to be consistent with the spatial, information, and enforcement assumptions of our environment, the household can reject one of the two four-tuples if they are directed to different trading posts. For instance, the mechanism may direct a household to two different monetary trading posts. If, for example, the household had a lot of accumulated money balances, it might choose to reject the first four-tuple of the proposal but accept the second four-tuple (i.e. consume but not produce). In contrast, if the mechanism directs both members of a household to a barter post, since both members are at the same location, it's possible for the mechanism to force compliance of both parts of the proposal (however not necessary).

In the case of shopkeepers, the mechanism proposes a five-tuple to any given shopkeeper k :

$$\gamma_K^k = \left(\nu^k, C^k, \hat{\nu}^k, \widehat{C}^k, D^k \right) \in \Gamma_K = \mathcal{V} \times \mathbb{R}_+ \times \mathcal{V} \times \mathbb{R}_+ \times \mathbb{R}$$

where C^k and $\widehat{C}^k$ are consumptions of goods ν^k and $\hat{\nu}^k$ and D^k denotes the change in the shopkeeper's money holding. Note that a shopkeeper who has positive money holdings can receive a proposal that directs him to open in order to diminish his money holdings and consume (accomplished by directing workers to his post). Denote the mechanism's allocation for a shopkeeper in period t by $\mathcal{M}_K(t) \in \Gamma_K$.

These allocations must be feasible with respect to the spatial, information, and commitment assumptions we have made in the environment. Here we list those constraints. First, conditioning of household proposals implies a cross location communication constraint:

$$k_p \neq k_c \implies \iota = 0. \tag{1}$$

Second, there must be certain consistency conditions imposed on the trading posts:

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\begin{aligned}
k_i &= k \Rightarrow \nu_i \in \left\{ \nu^k, \hat{\nu}^k \right\}, && i \in \{p, c\} \quad (2) \\
d_i &\neq 0 \text{ and } k_i = k \Rightarrow \hat{\nu}^k = 0, && i \in \{p, c\} \\
D^k &\neq 0 \Rightarrow \hat{\nu}^k = 0, \\
\hat{\nu}^k &= 0 \Rightarrow \widehat{C}^k = 0.
\end{aligned}
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Third, feasibility of goods exchange at any trading post requires that if agents

report $(\widetilde{\theta}, \widetilde{m})$ truthfully reveal their type and money holdings

$$C^k \leq \int \left[1_{\{k_p = k, \nu_p = \nu^k\}} \ell - 1_{\{k_c = k, \nu_c = \nu^k\}} c \right] d\mu \left(\widetilde{\theta}, \widetilde{m} \right) \tag{3}$$

$$\widehat{C}^k \leq \int \left[1_{\{k_p = k, \nu_p = \widehat{\nu}^k\}} \ell - 1_{\{k_c = k, \nu_c = \widehat{\nu}^k\}} \widehat{c} \right] d\mu \left(\widetilde{\theta}, \widetilde{m} \right)$$

$$0 = D^k + \int \left[1_{\{k_p = k, \nu_p = \widehat{\nu}^k\}} d_p + 1_{\{k_c = k, \nu_c = \widehat{\nu}^k\}} d_c \right] d\mu \left(\widetilde{\theta}, \widetilde{m} \right)$$

$$0 \leq \sum_{n=0}^{t} D^k(n)$$

where $\mu(\widetilde{\theta}, \widetilde{m})$ denotes the measure of households reporting $(\widetilde{\theta}, \widetilde{m})$. Fourth, the mechanism cannot direct households to make monetary transfers in excess of their reported money holdings.

$$\max\{0, -d_p\} + \max\{0, -d_c\} \leq \widetilde{m} \tag{4}$$

Note that both spatial and informational frictions play a role in this constraint. That is, the mechanism cannot propose that a household give up more money than it reports and money cannot be transferred between shopper and worker at two different locations.

We must introduce some notation before addressing the incentive compatibility and voluntary participation constraints. We define $V_{\mathcal{M}}^t(\theta, m)$ to be the present discounted utility of a household in state (θ, m) in period t after sending an optimal, though not necessarily truthful, message about its state when the proposals associated with mechanism $\mathcal{M}$ are implemented. Specifically,

$$V_{\mathcal{M}}^t(\theta, m) = \max_{\widetilde{\theta}, \widetilde{m}} W_{\mathcal{M}}^t \left((\theta, m), \left(\widetilde{\theta}, \widetilde{m} \right) \right)$$

where $W_{\mathcal{M}}^t \left((\theta, m), \left(\widetilde{\theta}, \widetilde{m} \right) \right)$ is the value of being in state (θ, m) in period t and choosing to reporting $(\widetilde{\theta}, \widetilde{m})$ to the mechanism, defined as:

$$W_{\mathcal{M}}^t \left((\theta, m), \left(\widetilde{\theta}, \widetilde{m} \right) \right) = \max_{\iota_c, \iota_p} Z_{\mathcal{M}}^t \left((\theta, m), \mathcal{M}_H^t \left(\widetilde{\theta}, \widetilde{m} \right), \iota_c, \iota_p \right)$$

where $Z_{\mathcal{M}}^t \left((\theta, m), \gamma_H, \iota_c, \iota_p \right)$ is the value of having type θ and money holding m in period t, with proposal γ_H, and rejecting or accepting the proposals (denoted by ι_c and $\iota_p \in \{0, 1\}$):

$$Z_{\mathcal{M}}^t \left((\theta, m), \gamma_H, \iota_c, \iota_p \right) = \left\{ \begin{array}{c} \iota_c \left(u(c) - f(\tau_c - \nu_c) \right) \\ + \iota_p \left(-\ell f(\tau_p - \nu_p) \right) \\ + \beta V_{\mathcal{M}}^{t+1}(\theta', m') \end{array} \right\}$$

such that:

$$\max\{0, -\iota_c d_c\} + \max\{0, -\iota_p d_p\} \leq m \tag{5}$$

$$m + \iota_c d_c + \iota_p d_p \leq m' \tag{6}$$

$$\iota = 1 \Rightarrow \iota_c = \iota_p \tag{7}$$

where γ_H is the mechanism's proposal:

$$\gamma_H = (\iota, (k_p, \nu_p, -\ell, d_p), (k_c, \nu_c, c, d_c)).$$

With this notation, we can write the individual rationality constraints. The fifth set of constraints assuring incentive compatibility by households can be written:

$$W_{\mathcal{M}}^t ((\theta, m), (\theta, m)) \geq W_{\mathcal{M}}^t \left((\theta, m), \left(\tilde{\theta}, \tilde{m}\right)\right) \quad \forall \tilde{\theta}, \tilde{m} \tag{8}$$

Sixth, the household participation constraint is given by

$$Z_{\mathcal{M}}^t \left((\theta, m), \mathcal{M}_H^t(\theta, m), 1, 1\right) \geq Z_{\mathcal{M}}^t \left((\theta, m), \mathcal{M}_H^t(\theta, m), \iota_c, \iota_p\right) \tag{9}$$

for all $\iota_c, \iota_p \in \{0, 1\}$. Since all information about shopkeepers characteristics are public information, we need only consider their participation constraint. Thus, if shopkeeper k opens a trading post in period t, the seventh set of constraints for shopkeeper participation can be written:

$$\overline{\kappa} \leq \sum_{n=t}^{\infty} \beta^{n-t} \left(C^k(n) + \widehat{C}^k(n)\right) \tag{10}$$

where $C^k(n)$ and $\widehat{C}^k(n)$ are consumption allocated to the broker k in period n by the mechanism.

Subject to constraints (1) through (10), the mechanism's problem can be stated:

$$\max_{\mathcal{M}} \int V_{\mathcal{M}}^0(\theta, \overline{m}) d\mu(\theta, \overline{m}). \tag{11}$$

4   A Relaxed Programming Problem

In this section, we solve a programming problem where certain incentive and resource conditions are relaxed which provides an upper bound on (11). Then we show that under certain conditions, the solution for this relaxed problem solves (11). First, we maintain resource feasibility only at the aggregate level rather than at the level of the trading post represented by (3). That is, the total quantity of all variety of goods consumed cannot exceed the total quantity of all variety of goods produced in any period. Second, for the household participation constraint (9), we only require that it is necessary for the shopper of a household to consume at a barter shop in order to induce the worker of that household to voluntarily produce at that barter shop. Third, we maintain the shopkeeper participation constraint (10). We neglect all other constraints. Thus, we neglect some constraints that must be satisfied in the full mechanism design problem of maximizing households' utility. This gives us an upper bound for household utility which may be attainable in certain regions of the parameter space.

In this case we prove the following results.

Lemma 1 (Relaxed Program) *If $\overline{\kappa}$ is sufficiently small and $u'(1)$ is sufficiently high, then the solution for the Relaxed Program is given by opening only monetary shops.*

Proposition 2 (Relaxed$\Longrightarrow$Complete) *Provided the assumptions of the previous lemma hold and that β is sufficiently high, the solution for the relaxed problem satisfies all the constraints of the mechanism design problem given by (11).*

Proposition 3 *(**Characterization of product variety**) Provided the assumptions of the previous proposition hold, a monetary economy has more product variety than an economy without money.*

5 Implementation

Proposition 4 (Implementation) *Provided the assumptions of the previous proposition hold, the solution to the mechanism design problem can be implemented as a subgame perfect Nash equilibrium.*

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