Optimal monetary policy with imperfect unemployment insurance

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Abstract

We consider an efficiency-wage model with the Calvo-type sticky prices and analyze optimal monetary policy when unemployment insurance is not perfect. With imperfect risk sharing, strict zero-inflation policy is no longer optimal even if the zero-inflation steady-state equilibrium is assumed to be (conditionally) efficient. Quantitative result depends on how idiosyncratic earning losses, measured by the (inverse of the) relative income of the unemployed to the employed, vary over business cycles. If idiosyncratic income losses are acyclical, optimal policy differs very little from the zero-inflation policy. However, if they vary countercyclically, as evidence suggests, the deviation of optimal policy from complete price stabilization becomes quantitatively significant. Furthermore, optimal policy in such a case involves stabilization of output to a much larger extent.

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1 Introduction

There is a growing literature on optimal monetary policy based on stochastic dynamic general equilibrium framework with imperfect competition and staggered price-setting. Its simplest version has two types of distortions: relative-price distortions due to staggered price-setting and distortions associated with imperfect competition (market power). As discussed by Goodfriend and King (1997), Rotemberg and Woodford (1997) and Woodford (2003), if fiscal policy is used to offset the distortions caused by market power, then optimal monetary policy is characterized as complete stabilization of the price level. Intuition is very simple: without distortions due to market power, the flexible-price equilibrium becomes efficient, which, in turn, can be attained by zero-inflation policy.[1] It is the price level that has to be stabilized, but not the level of output.[2] As long as the inflation rate is kept at zero, any fluctuations in output would be efficient.

The basic model has been extended in several directions. For instance, Benigno and Woodford (2003, 2005) and Khan, King and Wolman (2003) consider the case where distortions due to market power are present, and illustrate that the complete price stabilization is not optimal in general. Schmitt-Grohé and Uribe (2005) extend the analysis further, by studying a even richer model, based on Christiano, Eichenbaum and Evans (2005). The existing research on this literature, however, has restricted attention to complete-markets (representative-agent) models. In this paper we are interested to see the extent to which the nature of optimal monetary policy is affected by the presence of unemployment when unemployment insurance is not perfect. In particular, we'd like to examine whether or not the existence of the imperfectly insured unemployed calls for more output stabilization.

For this purpose, we bring unemployment into the basic sticky-price model, building on the efficiency-wage model of Alexopoulos (2004). The model has a representative household with a continuum of individual members. In each period, each member is either employed or unemployed. An employed worker may or may not shirk. A detected shirker will be punished by an exogenous reduction in the wage payment.[3] Firms determine the wage rate so that no workers would shirk in equilibrium. It is assumed that all the savings-related decisions are made by the household rather than by individual members, so that, even though the level of consumption differs between the employed and the unemployed, we can still use the representative household framework. The rest of the model is similar to the basic sticky-price model of Woodford (2003).

[1] Note that this argument assumes that initial price dispersion is nil (or "small" if we are interested in a first-order approximation of optimal monetary policy). See Yun (2005) on this point.

[2] What is stabilized is the "output gap," which is defined as the difference between the actual level of output and the efficient level of output.

[3] A relation with the model of Shapiro and Stiglitz (1984) is discussed in Appendix.

We analyze optimal monetary policy using the linear-quadratic approach developed by Rotemberg and Woodford (1997), Woodford (2003), and Benigno and Woodford (2003, 2005). To focus on the effect of imperfect unemployment insurance on stabilization policy, we assume that fiscal policy is used to make the zero-inflation steady-state conditionally efficient. It follows that with perfect insurance the flexible-price equilibrium would be efficient so that complete price-level stabilization would be the optimal policy. This is not true with imperfect insurance, so that optimal policy would involve some fluctuations in the inflation rate. Our qualitative analysis shows that a government-purchase shock is a negative cost-push shock, while a productivity shock is a positive one. That is, optimal policy should generate some deflation (inflation) when there is an exogenous increase in government purchases (productivity).

But, quantitatively, how large is the deviation of optimal policy from the complete price stabilization? The answer crucially depends on how idiosyncratic income shocks vary over business cycles. Specifically, what matters is how the relative income of the unemployed to that of the employed varies over business cycle. We say that idiosyncratic income losses are acyclical if the relative income of the unemployed is constant over business cycles and countercyclical if it varies procyclically. We begin with the case where the relative income of the unemployed is constant over business cycles. In this case, although complete price stabilization is not exactly optimal with imperfect insurance, optimal policy differs very little from it. Thus, as long as idiosyncratic income losses are acyclical, optimal policy essentially takes the form of complete price stabilization. This is so even though the unemployment rate goes up in a recession.

Evidence seems to suggest, however, that idiosyncratic shocks are countercycal. In particular, earning losses of unemployed or displaced workers are found to be countercyclical (e.g., Jacobson, LaLonde and Sullivan, 1993). To take it into account, our second numerical exercise assumes that the relative income of the unemployed varies procyclically over business cycles. In this case, the deviation of optimal policy from zero-inflation policy becomes much larger. Furthermore, optimal policy under countercyclical idiosyncratic income losses involves stabilization of the level of output, much more so compared to the case where idiosyncratic income losses are acyclical. Intuition is simple: if a bad shock to the economy worsens uninsured idiosyncratic shocks and makes the unemployed more miserable, policy should respond to reduce the number of unemployment, which is to increase the level of output.

Our numerical exercise suggests that the mere existence of the imperfectly insured unemployed may not justify output stabilization, for which there need to be systematic variation of idiosyncratic risk over business cycles. An important limitation of our model is that idiosyncratic shocks are purely transitory. Evidence such as Storesletten, Telmer and Yaron (2004) suggests, however, that idiosyncratic shocks are highly persistent as well as countercyclical. Based on a non-monetary

growth model, Krebs (2005) demonstrates that the welfare cost of business cycles can be sizable with such idiosyncratic shocks. Analyzing optimal policy with persistent idiosyncratic shocks is left for future research.

This paper is organized as follows. In Section 2 the model economy is described. In Section 3 the efficient allocation and the flexible-price equilibrium are discussed. In Section 4 a linear-quadratic approximation of the model is derived. In Section 5 optimal monetary policy is examined in the case where the degree of risk sharing is constant over business cycles. Section 6 considers the case where the degree of risk sharing is procyclical. Concluding remarks are in Section 7.

2 The model economy

In this section we describe our model economy. Its key features are staggered price setting and unemployment. Our model builds on Woodford (2003) for the former and the efficiency-wage model of Alexopoulos (2004) for the latter. Alexopoulos's model differs from the well known model of Shapiro and Stiglitz (1984) in that a detected shirker is punished by a reduction in the wage rate, rather than by getting fired. Nevertheless, as discussed in Appendix, it becomes observationally equivalent to the Shapiro-Stiglitz model with a particular unemployment insurance program. Indeed, we find it very convenient that Alexopoulos's model can be made observationally equivalent to the standard indivisible-labor model of Hansen (1985) and Rogerson (1988), or to the Shapiro-Stiglitz model, depending on the assumed unemployment insurance program.

2.1 Households

There is a representative household which has a continuum of individual members of unit measure. In each period, randomly selected N_t individuals receive job offers. The rest, $1 - N_t$, are unemployed.[4] All employed workers work for a fixed length of hours, h. An employed worker, however, may or may not shirk. A shirker is a worker whose effort level is different from that required by her employer, e_t.[5]

The utility flow of an employed individual who consumes C and exerts an effort level e is given by

$$\mathcal{U}(C, e) = \ln C + \omega \ln(\mathcal{H} - he), \tag{1}$$

[4]We assume that whether or not each individual receives a job offer is observable and that a person who turns down the job offer loses the eligibility for unemployment benefits. Then as long as the unemployment-insurance fee is not too large, no one would turn down a job offer.

[5]As we shall see, the required level of effort will be the same for all firms.

where $\omega, \mathcal{H} > 0$ are constant parameters, and C is the Dixit-Stiglitz aggregate of differentiated consumption goods, $c(i)$, $i \in [0,1]$:

$$C = \left[\int_0^1 c(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}}.$$

Given the prices of differentiated products, $p(i)$, $i \in [0,1]$, the standard cost-minimization argument yields the price index, P:

$$P = \left[\int_0^1 p(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}},$$

and derived demand:

$$c(i) = C \left[\frac{p(i)}{P} \right]^{-\theta}, \qquad i \in [0,1].$$

The utility flow of an unemployed individual is given by $\mathcal{U}(C, 0)$.

Individual members of a household do not participate in the asset market. Instead, it is the household that trades state-contingent claims, $\mathcal{A}_{t+1}$; receives (nominal) dividends from the firms, $\Pi_t(i)$, $i \in [0,1]$; and pays (nominal) lump sum taxes to the government, T_t. The flow budget constraint of the household is then given by

$$\mathcal{I}_t + E_t[Q_{t,t+1}\mathcal{A}_{t+1}] = \mathcal{A}_t + \int_0^1 \Pi_t(i) \, di - T_t, \tag{2}$$

where $\mathcal{I}_t$ is the "income" distributed equally across the household members, and $Q_{t,t+1}$ is the stochastic discount factor used to evaluate state-contingent claims, $\mathcal{A}_{t+1}$. We assume the natural debt limit to prevent from the Ponzi scheme:

$$\mathcal{A}_{t+1} \geq -E_{t+1} \sum_{j=0}^{\infty} Q_{t+1,t+1+j} \left\{ \int_0^1 \Pi_{t+1+j}(i) \, di - T_{t+1+j} \right\}. \tag{3}$$

Here, $Q_{t,t+j}$ is the stochastic discount factor used to evaluate date-$t+j$ nominal income at date t, which is defined recursively as

$$Q_{t,t+j} = Q_{t,t+j-1} Q_{t+j-1,t+j}, \qquad j \geq 1,$$

with $Q_{t,t} \equiv 1$.

With lump-sum transfer $\mathcal{I}_t$ from the household, the date-t consumption of an employed individual who is not detected shirking, $C_{e,t}$, is given by

$$P_t C_{e,t} = \mathcal{I}_t + hW_t - \mathrm{UI}_t^f, \tag{4}$$

where W_t is the nominal wage rate, and UI_t^f is the unemployment-insurance fee. A shirker is caught with probability $d \in (0,1)$. A detected shirker receives only a fraction $s \in [0,1)$ of the

wage. Both s and d are constant, exogenous parameters. The date-t consumption of a detected shirker, $C_{s,t}$, becomes

$$P_t C_{s,t} = \mathcal{I}_t + shW_t - \mathrm{UI}_t^f. \tag{5}$$

Given this, a shirker would always choose $e = 0$. Finally, the level of consumption of an unemployed is given as

$$P_t C_{u,t} = \mathcal{I}_t + \mathrm{UI}_t^b, \tag{6}$$

where UI_t^b denotes unemployment benefits.

The objective of the household is to maximize the average utility of its members. As we shall see, firms set the wage rate, W_t, and the required level of effort, e_t, so that employed workers never shirk. Hence, the objective function of the household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[N_t \mathcal{U}(C_{e,t}, e_t) + (1 - N_t)\mathcal{U}(C_{u,t}, 0) \Big] \tag{7}$$

Taking as given $\mathcal{A}_0$ and $\{N_t, e_t, P_t, Q_{t,t+1}, T_t, \mathrm{UI}_t^f, \mathrm{UI}_t^b, W_t, \Pi_t(i); i \in [0,1], t \geq 0\}$, the household chooses $\{\mathcal{I}_t, \mathcal{A}_{t+1}; t \geq 0\}$ so as to maximize the average utility (7) subject to (2), (3), (4), (6).

The first-order conditions imply that

$$Q_{t,t+1} \frac{P_{t+1}}{P_t} = \beta \frac{N_{t+1}\mathcal{U}_C(C_{e,t+1}, e_{t+1}) + (1 - N_{t+1})\mathcal{U}_C(C_{u,t+1}, 0)}{N_t \mathcal{U}_C(C_{e,t}, e_t) + (1 - N_t)\mathcal{U}_C(C_{u,t}, 0)}$$

Notice that the marginal rate of substitution involves the average marginal utilities. The transversality condition takes the standard form:

$$\lim_{j \to \infty} E_t Q_{t,t+j} \mathcal{A}_{t+j} = 0.$$

2.2 Firms

2.2.1 No shirking condition

Each differentiated product is produced by a single supplier. Each producer has the same production technology:

$$\begin{aligned} y_t &= A_t f\big[e_t h(n_t - n_t^s)\big], \\ &\equiv A_t \big[e_t h(n_t - n_t^s)\big]^{\frac{1}{\phi}}, \end{aligned}$$

where $\phi \geq 1$, A_t is the economy-wide productivity shock, e_t is the level of effort required by the firm, n_t and n_t^s are the numbers of employed and of shirkers, respectively. Given this production technology, having shirkers would never be profitable for firms. Each firm offers an employment

contract, $\{e_t, W_t\}$, to its employed. As the following argument shows, all firms offer the same contract, so that the index of firms, i, is omitted here.

Because a shirker is detected with probability d, no workers in a given firm would shirk if

$$\mathcal{U}(C_{e,t}, e_t) \geq (1-d)\mathcal{U}(C_{e,t}, 0) + d\mathcal{U}(C_{s,t}, 0).$$

Given that $C_{e,t}$ and $C_{s,t}$ are determined as in (4) and (5), the incentive-compatible level of effort must satisfy

$$e_t \leq e(W_t) \equiv \frac{\mathcal{H}}{h} - \frac{\mathcal{H}}{h}\left(\frac{shW_t + \mathcal{I}_t - \mathrm{UI}_t^f}{hW_t + \mathcal{I}_t - \mathrm{UI}_t^f}\right)^{\frac{d}{\omega}},$$

where the firm take $\mathcal{I}_t$, UI_t^f as given.

The cost minimization problem of the firm is then given by

$$\min_{W_t, n_t} W_t n_t \qquad \text{s.t.} \quad A_t f(e_t h n_t) \geq y_t, \quad \text{and} \quad e_t \leq e(W_t). \tag{8}$$

The solution to this problem is given by

$$\begin{aligned} e_t &= e, \\ \frac{W_t}{P_t} &= \frac{\chi_w}{h}\frac{1}{\mathcal{U}(C_{e,t}, e)} \end{aligned} \tag{9}$$

where e and $0 < \chi_w < 1$ are constants defined in Appendix. As we shall discuss below, the equilibrium wage rate in (9) is inefficient unless unemployment insurance is perfect.

2.2.2 Calvo pricing

The producer of product i faces the demand function:

$$y_t(i) = Y_t\left[\frac{p_t(i)}{P_t}\right]^{-\theta}, \tag{10}$$

where

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{11}$$

Let τ be the tax rate on firms' revenue. The profit flow of firm i is then given by

$$\begin{aligned} \Pi_t\big[p_t(i)\big] &= (1-\tau)p_t(i)y_t(i) - hn_t(i)W_t \\ &= (1-\tau)Y_t P_t^{\theta} p_t(i)^{1-\theta} - \frac{W_t}{e} f^{-1}\left(\frac{Y_t P_t^{\theta} p_t(i)^{-\theta}}{A_t}\right) \end{aligned}$$

The real marginal cost, $s_t(i)$, is defined by

$$s_t(i) = \frac{W_t}{eA_t P_t}\frac{1}{f'\big(f^{-1}\big[y_t(i)/A_t\big]\big)} \tag{12}$$

Following Calvo (1983), we assume that only a fraction $(1-\alpha)$ of randomly selected firms can reset their prices in each period. The rest of firms simply charge the same prices as in the previous period. Thus, if firm i receives the opportunity of resetting its product price in period t, it chooses $p_t(i)$ so as to maximize

$$\max E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_T \left[p_t(i) \right]$$

In this model, all firms which reset prices in the same period choose the same price.[6] Let p_t^* denote the price chosen by all firms resetting their prices in period t. It satisfies the first-order condition:

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} Y_T P_T^{\theta} \left\{ p_t^* - \frac{1}{1-\Phi} P_T s_{t,T} \right\} = 0, \tag{13}$$

where $s_{t,T}$ is the real marginal cost in period T of those firms that reset their prices in period t, and

$$\Phi \equiv 1 - (1-\tau)\frac{\theta - 1}{\theta}.$$

2.3 Government

The government conducts monetary and fiscal policy. The flow budget constraint for the government is

$$T_t + \tau P_t Y_t + N_t \mathrm{UI}_t^f + E_t[Q_{t,t+1} \mathcal{A}_{t+1}] = \mathcal{A}_t + P_t G_t + (1 - N_t)\mathrm{UI}_t^b,$$

where $\mathcal{A}_{t+1}$ denotes the state-contingent debt issued by the government and $\mathcal{A}_0$ is given.

We assume a very simple form of fiscal policy. The government takes as given τ, UI_t^f, UI_t^b, G_t, as well as P_t, N_t, and Y_t. Fiscal policy sets T_t in the "Ricardian" way (Woodford, 1995) so that we do not need specify the details of the conduct of fiscal policy. Monetary policy is formulated as in Woodford (2003, Chapter 7), Benigno and Woodford (2003, 2005), among others. Thus, optimal monetary policy is implicitly defined as the solution to the (adequately modified version of) Ramsey problem. With a linear-quadratic approximation, in particular, monetary policy is to set a state-contingent path of inflation rates.

2.4 Exogenous variables

The unemployment-insurance fee, UI_t^f, is assumed to remain small enough that no worker with a job offer would turn it down. Specifically, given that $\mathcal{U}(C_e, e) = \mathcal{U}(C_s, 0)$ in equilibrium and that

[6] An implicit assumption here is that each firm possesses the same, constant amount of firm-specific capital. If we allow for accumulation of such capital, the price chosen by a firm would depend on the amount of capital it holds. See Woodford (2005) for such a model.

a worker who turns down a job offer is not eligible for unemployment benefits, a job offer would never be rejected if $P_t C_{s,t} \geq \mathcal{I}_t$, that is, if

$$\mathrm{UI}_t^f \leq shW_t,$$

which is assumed to hold throughout this paper.

Let B_t denote the ratio of the level of consumption of the unemployed to that of the employed:

$$B_t \equiv \frac{C_{u,t}}{C_{e,t}} = \frac{\mathcal{I}_t + \mathrm{UI}_t^b}{hW_t + \mathcal{I}_t - \mathrm{UI}_t^f}.$$

If unemployment insurance is perfect, $B_t = 1$; otherwise, $B_t < 1$. Let C_t be the aggregate level of consumption:

$$C_t \equiv N_t C_{e,t} + (1 - N_t) C_{u,t}.$$

The goods-market equilibrium condition is given by

$$Y_t = C_t + G_t, \tag{14}$$

where G_t is government purchases. The levels of consumption of the employed and the unemployed are expressed respectively as

$$C_{e,t} = \frac{1}{N_t + (1 - N_t) B_t} C_t, \tag{15}$$

$$C_{u,t} = \frac{B_t}{N_t + (1 - N_t) B_t} C_t. \tag{16}$$

The unemployment insurance program is run with balanced budget: $N_t \mathrm{UI}_t^f = (1 - N_t)\mathrm{UI}_t^b$. Note that here unemployment insurance affects equilibrium only through its effect on B_t. In our benchmark analysis, we assume for simplicity that the unemployment benefits (and fees) in each period are determined so that this ratio remains constant:

$$B_t = \bar{B} \in (0, 1], \qquad \text{for all } t.$$

We later relax this assumption in Section 6 and let this ratio, B_t, fluctuate procyclically over time.

In the benchmark case, there are two stochastic shocks: the government-purchase shock, G_t, and the productivity shock, A_t. Assume that they take the form:

$$G_t = s_G \bar{Y} e^{\xi_{G,t}}, \qquad \text{and} \qquad A_t = \bar{A} e^{\xi_{A,t}},$$

where $s_G \in (0, 1)$, $\bar{Y}$ is the steady-state level of output, and $\{\xi_{G,t},\ \xi_{A,t}\}$ follows a stationary stochastic process with unconditional mean of zero. Let ξ_t denote the vector of these exogenous disturbances:

$$\xi_t = (\xi_{G,t}, \xi_{A,t}).$$

When B_t is allowed to fluctuate, we let $B_t = \bar{B} e^{\xi_{B,t}}$, and $\xi_t = (\xi_{G,t}, \xi_{A,t}, \xi_{B,t})$.

3 Efficient allocation and flexible-price equilibrium

In this section we first rewrite the household's utility in terms of aggregate output and a measure of output dispersion across firms. A key finding is that the less risk sharing is, the less concave the household's utility is in aggregate output. Then we consider the efficient allocation given the exogenous shocks: G_t and A_t. Here, efficiency is defined conditional on that the level of effort equals the equilibrium level, e, and that unemployment insurance is limited by $\bar{B}$. We shall also derive the flexible-price equilibrium. It provides a useful benchmark, because, to a first-order approximation, the level of output in the flexible-price equilibrium coincides with that in a sticky-price equilibrium with zero inflation.

3.1 Utility flow of the household

Using (14)-(16), the flow utility of the household (i.e., the average utility flow of its members) is given by

$$\begin{aligned}
\mathcal{W}_t &\equiv N_t\mathcal{U}(C_{e,t},e_t)+(1-N_t)\mathcal{U}(C_{u,t},0),\\
&= N_t\ln\left[\frac{1}{N_t+(1-N_t)B_t}C_t\right]+(1-N_t)\ln\left[\frac{B_t}{N_t+(1-N_t)B_t}C_t\right],\\
&\quad -\omega\big[\ln(\mathcal{H})-\ln(\mathcal{H}-he)\big]N_t+\ln(\mathcal{H}),\\
&= \ln(Y_t-G_t)+z(N_t;\bar{B})-\omega\big[\ln(\mathcal{H})-\ln(\mathcal{H}-he)\big]N_t+\ln(\mathcal{H}), \qquad (17)
\end{aligned}$$

where

$$z(N;B)\equiv(1-N)\ln B-\ln\big[N+(1-N)B\big].$$

The function $z(N;B)$ represents the inefficiency caused by imperfect risk sharing, B. If $B=1$, $z(N;1)=0$ for all N, so that the flow utility of the household takes the same form as in the indivisible labor model of Hansen (1985) and Rogerson (1988):

$$\mathcal{W}_t=\ln(Y_t-G_t)-\omega\big[\ln(\mathcal{H})-\ln(\mathcal{H}-he)\big]N_t+\ln(\mathcal{H}).$$

When $B<1$, $z(N;B)$ has a minimum at $N=\underline{N}(B)$, where

$$\underline{N}(B)\equiv\frac{1-B+B\ln(B)}{-(1-B)\ln(B)}<\frac{1}{2},$$

and is increasing in N for $N>\underline{N}(B)$ and decreasing in N for $N<\underline{N}(B)$. In what follows, we focus on the case where $N_t>1/2$ holds almost surely for all t. Note also that the function $z(N;B)$ is convex in N. Therefore, imperfect risk sharing makes the household's objective function less concave.

The aggregate employment, N_t, is expressed as

$$\begin{aligned} N_t &= \int_0^1 n_t(i)\,di = \int_0^1 \frac{1}{eh}\left[\frac{y_t(i)}{A_t}\right]^{\phi} di, \\ &= \frac{1}{eh}\left(\frac{Y_t}{A_t}\right)^{\phi}\Delta_t, \\ &\equiv N(Y_t, \Delta_t; A_t), \end{aligned} \tag{18}$$

where Δ_t is the output (or price) dispersion measure defined as

$$\Delta_t \equiv \int_0^1 \left[\frac{y_t(i)}{Y_t}\right]^{\phi} di = \int_0^1 \left[\frac{p_t(i)}{P_t}\right]^{-\theta\phi} di \geq 1. \tag{19}$$

where the inequality follows from Jensen's inequality.

Using this, the flow utility of the household can be expressed as a function of Y_t, Δ_t, and exogenous disturbances:

$$\mathcal{W}(Y_t, \Delta_t; \xi_t) = U(Y_t; G_t) + Z(Y_t, \Delta; A_t, \bar{B}) - V(Y_t, \Delta_t; A_t) + \ln(\mathcal{H}), \tag{20}$$

where

$$U(Y; G) \equiv \ln(Y - G), \tag{21}$$

$$Z(Y, \Delta; A, B) \equiv z\big[N(Y, \Delta; A); B\big], \tag{22}$$

$$V(Y, \Delta; A) = \omega\big[\ln(\mathcal{H}) - \ln(\mathcal{H} - he)\big]N(Y, \Delta; A) \tag{23}$$

Since $N(Y, \Delta; A)$ is convex in Y, so is $Z(Y, \Delta; A, B)$. Hence imperfect unemployment insurance, $\bar{B} < 1$, makes the objective function of the household less concave relative to the case of perfect insurance. That is, ceteris paribus, the household tends to be willing to accept larger fluctuations in output when risk sharing is not perfect. This property plays an important role in determining the character of optimal monetary policy in our model. Throughout this paper we assume that $Z(Y, \Delta; A, B)$ is not so convex that $\mathcal{W}(Y, \Delta; \xi)$ is strictly concave in Y and Δ for each ξ.

Assumption 1. *For each ξ, $\mathcal{W}(Y, \Delta; \xi)$ is strictly concave in Y and Δ.*

3.2 Efficient rate of output

The efficient allocation is the feasible allocation that maximizes the expected discounted sum of the household's average utility flows, $\{\mathcal{W}_t\}$, in (20). This Pareto problem has no predetermined variables and can be solved state by state in a static fashion. For each ξ_t, the efficient allocation, $\{y_t^*(i) : i \in [0, 1]\}$, is the solution to

$$\max_{\{y_t(i)\}} \mathcal{W}(Y_t, \Delta_t; \xi_t)$$

where Y_t is given by (11). Under our assumption, it is straightforward to see that there is no output dispersion in the efficient allocation:

$$y_t^*(i) = Y_t^*, \qquad \text{and} \qquad \Delta_t^* = 1,$$

and that the efficient level of aggregate output satisfies the first-order condition:

$$U_Y(Y_t^*; G_t) + Z_Y(Y_t^*, 1; A_t, \bar{B}) = V_Y(Y_t^*, 1; A_t). \tag{24}$$

As shown in Appendix, the efficient level of output is decreasing in the level of risk sharing, $\bar{B}$:

$$\frac{\partial Y_t^*}{\partial \bar{B}} \leq 0. \tag{25}$$

Thus lower risk sharing (lower $\bar{B}$) raises the efficient level of output. This is because less risk sharing makes unemployment more costly, and hence the efficient level of unemployment is lower, which implies that the efficient level of output is higher.

3.3 Flexible price equilibrium

Here we consider the flexible-price equilibrium, in which each firm can change its product price freely in every period. The flexible-price equilibrium defines the "natural rates" of endogenous variables, which are denoted by superscript n.

With flexible prices, each firm $i \in [0, 1]$ chooses $p_t(i)$ so that

$$\frac{p_t(i)}{P_t} = \frac{1}{1-\Phi} s_t(i)$$

In the symmetric equilibrium, all firms charge the same price, $p_t(i) = P_t$, which yields

$$s_t(i) = 1 - \Phi, \qquad \forall i \in [0, 1]. \tag{26}$$

In the flexible-price equilibrium, consumption of the employed can be written as

$$C_{e,t}^n = D(Y_t^n; A_t, \bar{B})(Y_t^n - G_t),$$

where

$$D(Y; A, B) \equiv \frac{1}{N(Y, 1; A) + \big[1 - N(Y, 1; A)\big] B}.$$

Using (9), (21) and (23), condition (26) can be expressed as

$$\chi(1-\Phi) U_Y(Y_t^n; G_t) D(Y_t^n; A_t, \bar{B})^{-1} = V_Y(Y_t^n, 1; A_t), \tag{27}$$

where χ is the constant defined by

$$\chi \equiv \frac{\omega[\ln(\mathcal{H}) - \ln(\mathcal{H} - he)]}{\chi_w}$$

The natural rate of output, Y_t^n, is defined implicitly in (27).

As shown in Appendix, in contrast with the case of the efficient rate of output (25), the natural rate of output increases with the level of risk sharing:

$$\frac{\partial Y_t^n}{\partial \bar{B}} \geq 0. \tag{28}$$

This is because, other things being equal, an increase in risk sharing tends to reduce the amount of consumption of the employed due to a rise in the unemployment-insurance fee. As shown in equation (9), a decline in consumption of the employed, in turn, lowers the wage rate and hence increases production.

4 Linear-quadratic approximation

We wish to characterize the optimal monetary policy using the linear-quadratic approach developed by Woodford (2003) and Benigno and Woodford (2003, 2005). In that approach, the monetary authority maximizes a quadratic approximation of the utility of the representative household subject to a log-linear approximation of the aggregate supply relation. Each approximation is taken around the zero-inflation steady state.

With the Calvo pricing, the price index, P_t, evolves as

$$P_t = \left[(1-\alpha){p_t^*}^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{29}$$

where p_t^* is the newly set price in period t, defined in (13). It follows that

$$\frac{p_t^*}{P_t} = \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}}, \tag{30}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross rate of inflation in period t. Similarly, the evolution of the price dispersion measure, Δ_t, is given by

$$\begin{aligned}\Delta_t &= \int_0^1 \left[\frac{p_t(i)}{P_t}\right]^{-\theta\phi} di \\ &= (1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{-\theta\phi} + \alpha\Pi_t^{\theta\phi}\Delta_{t-1}\end{aligned}$$

Using (30), we obtain

$$\Delta_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}} + \alpha\Pi_t^{\theta\phi}\Delta_{t-1} \tag{31}$$

Consider the zero-inflation steady state, that is, the equilibrium in which $\xi_t = 0$ and $\Pi_t = 1$, for all t. In what follows, the value of each variable at the zero-inflation steady state is denoted

by a bar. Equation (31) implies that $\Delta_t = 1$, all t. The first-order condition (13) reduces to $s_t(i) = 1 - \Phi$, for all i, which implies that the level of output at the zero-inflation steady state, $\bar{Y}$, is the solution to

$$\chi(1-\Phi)U_Y(\bar{Y};\bar{G})D(\bar{Y};\bar{A},\bar{B})^{-1} = V_Y(\bar{Y},1;\bar{A})$$

We assume that the zero-inflation steady-state equilibrium is (conditionally) efficient.

Assumption 2. *The tax rate on monopoly revenue, τ, is set so that the level of output in the zero-inflation steady state is efficient:*

$$\bar{Y} = \bar{Y}^*$$

Whether or not unemployment insurance is perfect, imperfect competition would cause inefficiency at the steady state. How such inefficiency affects the optimal equilibrium path has been analyzed, for instance, by Khan, King and Wolman (2003) and Benigno and Woodford (2003, 2005). With Assumption 2, we can focus on the inefficiency that imperfect unemployment insurance introduces *outside* the steady state.

As shown in Appendix, a log-linear approximation of first-order condition (13) for p_t^* is given by

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t. \tag{32}$$

Here x_t is the (welfare-relevant) output gap:

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^*,$$

u_t is the "cost-push shock," defined by

$$u_t \equiv \kappa(\hat{Y}_t^* - \hat{Y}_t^n),$$

and κ is the constant defined by

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}-\delta+\phi-1}{1+(\phi-1)\theta},$$

where σ^{-1} and δ are the elasticities of U_Y and D^{-1} with respect to Y evaluated at the zero-inflation steady state:

$$\sigma^{-1} \equiv -\frac{U_{YY}\bar{Y}}{U_Y} = \frac{1}{1-s_G} > 1, \qquad \delta \equiv -\frac{D_Y\bar{Y}}{D} = \frac{(1-\bar{B})\bar{N}}{\bar{N}+(1-\bar{N})\bar{B}} \geq 0.$$

Note that $\delta = 0$ with perfect insurance. It immediately follows that imperfect insurance makes κ smaller. In other words, the real effect of a nominal shock is larger with imperfect insurance.

Proposition 1. *Imperfect insurance makes the coefficient κ in the AS relation (32) smaller:*

$$\kappa|_{\bar{B}<1} < \kappa|_{\bar{B}=1}.$$

Also as shown in Appendix, a quadratic approximation of the household's utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t = -\bar{Y} V_Y E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}\Big[q_\pi \pi_t^2 + q_y x_t^2\Big], \tag{33}$$

where

$$q_\pi \equiv \frac{\alpha\theta\big[1+(\phi-1)\theta\big]}{(1-\alpha)(1-\alpha\beta)}(1-\Gamma),$$
$$q_y \equiv \sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1.$$

Here, ζ and Γ are constants defined by

$$\zeta \equiv \frac{Z_{YY}\bar{Y}}{Z_Y} \geq 0, \qquad \Gamma \equiv \frac{Z_Y}{U_Y + Z_Y} \in [0,1],$$

where all derivatives are evaluated at the zero-inflation steady state. From (32) and (33), it follows that the exogenous shocks relevant for the optimal policy problem are summarized into a single composite variable, u_t.

5 Optimal policy with constant risk sharing

In the traditional (Ramsey) approach, the optimal policy problem, say at date t_0, is to choose a state-contingent path, $\{\pi_t, x_t\}_{t\geq t_0}$, so as to maximize the household's utility (33) subject to the aggregate-supply relation (32) for $t \geq t_0$. As is well known, this type of optimization fails to be time consistent: if the planner is allowed to reoptimize at a future date, it will choose a different path of inflation and output gap. Concerning this issue, Woodford (2003) and Benigno and Woodford (2003, 2005) have shown that the optimal policy problem can be modified into a recursive form with an additional constraint, which is to allow the planner to make a commitment for one period. The solution to such a constrained policy problem is called optimal policy from a timeless perspective. Specifically, in the linear-quadratic problem here, the modified policy problem at any date t_0 is to choose a state-contingent path, $\{\pi_t, x_t\}_{t\geq t_0}$, so as to maximize the household's utility subject to the aggregate-supply relation as well as to the commitment from the previous period of the form:

$$\pi_{t_0} = \bar{\pi}_{t_0}.$$

Following Woodford (2003) and others, we shall consider the policy problem constrained in this fashion. Note, however, that it yields the same impulse responses to exogenous disturbances as the traditional, unconstrained policy problem (Woodford, 2003, Proposition 7.9).

Letting φ_t be the Lagrange multiplier for (32), the first-order conditions yield

$$\pi_t = \frac{1}{q_\pi}(\varphi_{t-1} - \varphi_t), \tag{34}$$

$$x_t = \frac{\kappa}{q_y}\varphi_t. \tag{35}$$

Substituting into (32), we obtain the second-order difference equation in φ_t:

$$\beta q_y E_t\varphi_{t+1} - \big[(1+\beta)q_y + \kappa^2 q_\pi\big]\varphi_t + q_y\varphi_{t-1} = q_\pi q_y u_t. \tag{36}$$

Its characteristic equation,

$$\beta q_y \mu^2 - \big[(1+\beta)q_y + \kappa^2 q_\pi\big]\mu + q_y = 0,$$

has a solution pair, $\mu \in (0,1)$ and $1/(\beta\mu) > 1$. It follows that a bounded solution to (36) takes the form of

$$\varphi_t = \mu\varphi_{t-1} - q_\pi \sum_{j=0}^{\infty} \beta^j \mu^{j+1} E_t u_{t+j} \tag{37}$$

where φ_{t_0-1} satisfies the initial condition: $\varphi_{t_0-1} - \varphi_{t_0} = q_\pi \bar{\pi}_{t_0}$. Given $\{\varphi_t\}$, the optimal state-contingent evolution of π_t and x_t are derived using (34)-(35).

Equations (34), (35) and (37) tell us how the optimal state-contingent paths of π_t and x_t depend on the composite shock, $u_t = \kappa(\hat{Y}_t^* - \hat{Y}_t^n)$. For example, consider impulse responses to a cost-push shock in period t. To be specific, suppose that u_t follows an AR(1) process given by $u_t = \rho_u u_{t-1} + \epsilon_{u,t}$ where $\rho_u \in (-1,1)$ and $\epsilon_{u,t}$ is i.i.d. with zero mean. Equation (37) implies that

$$\varphi_{t+j} = \mu\varphi_{t+j-1} + \phi_u u_{t+j},$$

where $\phi_u \equiv -\mu q_\pi/(1-\beta\mu\rho_u)$. It follows that impulse responses at dates $t+j$, $j = 0, 1, \ldots$, become

$$E_t\varphi_{t+j} - E_{t-1}\varphi_{t+j} = \frac{\mu^{j+1} - \rho_u^{j+1}}{\mu - \rho_u}\phi_u\epsilon_{u,t}$$

$$E_t x_{t+j} - E_{t-1}x_{t+j} = \frac{\kappa}{q_y}\frac{\mu^{j+1} - \rho_u^{j+1}}{\mu - \rho_u}\phi_u\epsilon_{u,t}$$

$$E_t p_{t+j} - E_{t-1}p_{t+j} = -\frac{1}{q_\pi}\frac{\mu^{j+1} - \rho_u^{j+1}}{\mu - \rho_u}\phi_u\epsilon_{u,t}$$

and

$$E_t\pi_{t+j} - E_{t-1}\pi_{t+j} = \begin{cases} -\dfrac{1}{q_\pi}\phi_u\epsilon_{u,t}, & \text{for } j = 0 \\ \dfrac{1}{q_\pi}\dfrac{\mu^j(1-\mu) - \rho_u^j(1-\rho_u)}{\mu - \rho_u}\phi_u\epsilon_{u,t}, & \text{for } j \geq 1 \end{cases}$$

To see now how u_t depends on the fundamental shocks, log-linearize the first-order conditions (24) and (27) around the zero-inflation steady state:

$$\hat{Y}_t^* = c_A^* \xi_{A,t} + c_G^* \xi_{G,t} \tag{38}$$

$$\hat{Y}_t^n = c_A^n \xi_{A,t} + c_G^n \xi_{G,t} \tag{39}$$

where[7]

$$c_A^* \equiv \frac{\phi - \Gamma(\zeta + 1)}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} \tag{40}$$

$$c_G^* \equiv \frac{\sigma^{-1}(1-\Gamma)s_G}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} > 0 \tag{41}$$

$$c_A^n \equiv \frac{\phi - \delta}{\sigma^{-1} - \delta + \phi - 1} > 0 \tag{42}$$

$$c_G^n \equiv \frac{\sigma^{-1} s_G}{\sigma^{-1} - \delta + \phi - 1} > 0 \tag{43}$$

Given this, we can express the cost push shock as

$$u_t = c_A^u \xi_{A,t} + c_G^u \xi_{G,t},$$

where $c_s^u \equiv \kappa(c_s^* - c_s^n)$, for $s = A, G$.

5.1 Effects of imperfect insurance: Theoretical results

Optimal policy involves strict price stability (zero inflation), if the flexible price equilibrium is optimal so that $\hat{Y}_t^n = \hat{Y}_t^*$ and $u_t = 0$. It is obviously the case when unemployment insurance is perfect: $\bar{B} = 1$. It is also the case when the technology shock, A_t, is the only shock to the economy, $s_G = 0$. This is due to our homothetic preferences, as is discussed in Benigno and Woodford (2005). The following proposition summarizes.

Proposition 2. *(a) If $\bar{B} = 1$, then $c_A^* = c_A^n$ and $c_G^* = c_G^n$. (b) If $s_G = 0$, then $c_A^* = c_A^n$.*

In general, the flexible-price equilibrium is not efficient outside the steady state, $Y_t^n \neq Y_t^*$, in spite of Assumption 2. Given the first-order conditions (24) and (27), the elasticities of $U_Y + Z_Y$ and $U_Y D^{-1}$ with respect to Y are important in determining the nature of optimal monetary policy. At the zero-inflation steady state, those elasticities are given by

$$-\frac{U_{YY}Y + Z_{YY}Y}{U_Y + Z_Y} = \sigma^{-1}(1-\Gamma) - \zeta\Gamma \leq \sigma^{-1}$$

$$-\frac{U_{YY}Y}{U_Y} + \frac{D_Y Y}{D} = \sigma^{-1} - \delta \leq \sigma^{-1}$$

[7] If inequality (44) below holds, $c_A^* > 0$.

With $\bar{B} = 1$, they are both equal to σ^{-1} since $\delta = \Gamma = 0$. Thus, imperfect insurance makes both $U_Y + Z_Y$ and $U_Y D^{-1}$ less elastic with respect to Y. The former follows from the fact that imperfect insurance makes the aggregate utility less concave. The latter follows from the fact that an increase in Y raises C_e less than C, because it reduces unemployment (this effect is reflected in the term D^{-1}). As the next proposition states, this property implies that the response of Y_t^* and Y_t^n to an exogenous shift in G_t is larger with imperfect insurance than with perfect insurance.

Proposition 3. *Assume that $s_G > 0$. The responses of Y_t^* and Y_t^n to G_t are larger with imperfect insurance than with perfect insurance:*

$$c_G^*|_{\bar{B}=1} < c_G^*|_{\bar{B}<1},$$
$$c_G^n|_{\bar{B}=1} < c_G^n|_{\bar{B}<1}.$$

In other words, imperfect insurance makes the efficient and natural rates of output more volatile in response to a "demand shock." The opposite is true for the response to a "supply shock," A_t.

Proposition 4. *Assume that $s_G > 0$. The responses of Y_t^* and Y_t^n to A_t are smaller with imperfect insurance than with perfect insurance:*

$$c_A^*|_{\bar{B}<1} < c_A^*|_{\bar{B}=1},$$
$$c_A^n|_{\bar{B}<1} < c_A^n|_{\bar{B}=1}.$$

With perfect insurance, the efficient (and the natural) rate of output is determined by the equation $U_Y = V_Y$, where the left-hand side expresses the marginal benefit of increasing Y and the right-hand side its marginal cost. An increase in productivity, A, lowers the marginal cost but does not affect the marginal benefit, and hence raises the efficient rate of output. With imperfect insurance, this effect is partially offset because A lowers Z_Y and D^{-1}.

Whether G and A are positive or negative cost push shocks depends on the elasticities of $U_Y + Z_Y$ and $U_Y D^{-1}$. The following lemma provides a necessary and sufficient condition that the former is greater than the latter.

Lemma 1.

$$\sigma^{-1}(1-\Gamma) - \zeta\Gamma > \sigma^{-1} - \delta > 0 \tag{44}$$

if and only if $\sigma^{-1} - \delta > 0$ and

$$(\sigma^{-1} - \delta)\big[2\delta + \ln(\bar{B})N\phi\big] > (\phi - 1)\big[-\ln(\bar{B})N\phi - \delta\big]$$

Condition (44) holds if $\phi = 1$ and $\bar{B} \in (0.21, 1)$. Indeed, it is satisfied for all the numerical exercises we have considered, and hence, we shall restrict our attention to such a case.

Proposition 5. *Assume that $s_G > 0$, $\bar{B} < 1$ and (44) holds. Then the government-purchase shock, G, is a negative cost-push shock and the productivity shock, A, is a positive cost-push shock:*

$$c^u_G < 0, \qquad \text{and} \qquad c^u_A > 0.$$

The following proposition shows how imperfect insurance affects the persistence parameter μ of optimal policy.

Proposition 6. *Under condition (44), imperfect insurance makes the persistence parameter μ in (37) larger:*

$$\mu|_{\bar{B}=1} < \mu|_{\bar{B}<1}.$$

5.2 Effects of imperfect insurance: Quantitative results

We have seen that exact price stability is not optimal if unemployment insurance is not perfect. Here we examine quantitatively how different optimal policy is from complete price stabilization. Assume that the exogenous disturbances, $\xi_{A,t}$ and $\xi_{G,t}$, follow the AR(1) process given by $\xi_{A,t} = \rho_A \xi_{A,t-1} + \epsilon_{A,t}$ and $\xi_{G,t} = \rho_G \xi_{G,t-1} + \epsilon_{G,t}$, where $\epsilon_{A,t}$ and $\epsilon_{G,t}$ are i.i.d. random variables with mean zero. In the numerical exercise below, we set $\alpha = 0.66$, $\beta = 0.99$ (the time unit is a quarter), $\phi = 1.47$, $\theta = 10$, which are in accordance with the parameter values assumed in Woodford (2003, Table 5.1). In addition we assume $s_G = 0.2$ and $\bar{N} = 0.94$. Different values are examined for $\bar{B}$, ρ_A and ρ_G.

Figures 1-4 plot optimal responses of π_t, $x_t \equiv \hat{Y}_t - \hat{Y}^*_t$, and $\hat{Y}_t$ to the productivity and government-purchase shocks, for different values of $\bar{B}$, ρ_A, and ρ_G.[8] We set the size of the initial innovation to the two shocks as $\epsilon_{A,0} = -2.34\%$ and $\epsilon_{G,0} = -13.76\%$, both of which reduce the efficient level of output by 2 percent, $\hat{Y}^*_0 = -2\%$, in the case of $\bar{B} = 1$ and $\rho_A = \rho_G = 0$. In Figures 1-2, shocks are serially uncorrelated, $\rho_A = \rho_G = 0$, and different degrees of risk sharing are considered: $\bar{B} = 0.5, 0.75, 1.0$. Consistent with the theoretical results above, exact price stabilization is optimal in the case of perfect insurance ($\bar{B} = 1$), and the less risk sharing is (the lower $\bar{B}$ is), the more optimal policy differs from the complete price stabilization. Consistent with Propositions 3-4, less insurance makes optimal responses of output to the government-purchase shock (the productivity shock) larger (smaller). In Figures 3-4, $\bar{B} = 0.75$ and $\rho_A, \rho_G = 0, 0.5, 0.9$. As the persistence of a shock becomes greater, the optimal responses to it involve larger fluctuations in inflation and the output gap. Those figures show, however, that, regardless of the values of $\bar{B}$, ρ_A, and ρ_G, deviations of optimal policy from the complete price stabilization is quantitatively very small (note that the inflation rate is expressed in percent per year). We thus conclude that, as far

[8] Specifically, those figures plot $E_0\pi_t - E_{-1}\pi_t$ etc. for $t = 0, 1, \ldots, 8$.

as the degree of risk sharing is constant, imperfect risk sharing does not have quantitatively big impact on optimal policy so that optimal policy is essentially characterized by price stabilization.

6 Optimal policy with countercyclical idiosyncratic shocks

We have so far focused on the case where the degree of risk sharing is constant, $B_t = \bar{B}$. However, evidence seems to suggest that idiosyncratic risk is countercyclical. In particular, what is relevant for this paper is that earning losses of displaced workers are countercyclical.[9] In this section we shall see that optimal policy would involve much larger fluctuations in inflation if idiosyncratic earning losses are countercyclical, that is, if B_t fluctuate procyclically.

With time-varying $B_t = \bar{B}\exp(\xi_{B,t})$, the efficient and the natural rates of output are given, respectively, as

$$\hat{Y}_t^* = c_A^* \xi_{A,t} + c_G^* \xi_{G,t} + c_B^* \xi_{B,t}$$
$$\hat{Y}_t^n = c_A^n \xi_{A,t} + c_G^n \xi_{G,t} + c_B^n \xi_{B,t}$$

where c_A^*, c_G^*, c_A^n and c_G^n are as given in (40)-(43), and

$$c_B^* \equiv \frac{1}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} \frac{\phi(1-\bar{B})\bar{N}[(1-\bar{N})^2\bar{B} - \bar{N}^2]}{[\sigma^{-1} - \ln(\bar{B})\bar{N}\phi - \delta][(1-\bar{B})\bar{N} + \bar{B}]^2}$$
$$c_B^n \equiv \frac{1}{\sigma^{-1} - \delta + \phi - 1} \frac{(1-\bar{N})\bar{B}}{\bar{N} + (1-\bar{N})\bar{B}}$$

It follows from equations (25) and (28) that $c_B^n > 0$ and $c_B^* < 0$. Hence B_t is a negative cost-push shock.

Proposition 7. *The insurance shock, B_t, is a negative cost-push shock:*

$$c_B^u < 0.$$

Now let us examine quantitatively how countercyclical idiosyncratic earning losses affects optimal policy. Assume the same size of the initial innovations to the productivity and government-purchase shock as in the previous figures: $\epsilon_{A,0} = -2.34\%$ and $\epsilon_{G,0} = -13.76\%$. Let also the steady-state level of risk sharing is given by $\bar{B} = 0.75$ and shocks are serially uncorrelated: $\rho_A = \rho_G = 0$. Furthermore, assume that those negative shocks arrive with a temporary decline in the degree of risk sharing. We consider three values for $B_0 = 0.65, 0.7, 0.75$. It returns to the steady state level after one period: $B_t = \bar{B}$ for $t \geq 1$. Note that even the largest decline in B we consider (from $\bar{B} = 0.75$ to $B_0 = 0.65$) seems to be an empirically plausible value. For instance, based on various

[9] See, for instance, Storesletten, Telmer and Yaron (2004) and Jacobson, LaLonde and Sullivan (1993).

empirical work, Krebs (2005) assumes that the difference in the earnings losses of displaced workers between booms and recessions is 12 percent in his numerical analysis.

Figures 5-6 plot the impulse response functions for those composite shocks. As we have already seen, with constant risk sharing, optimal policy is essentially characterized as complete price stabilization. For instance, when $B_t \equiv 0.75$, $\epsilon_{A,0} = -2.34\%$ leads to $\pi_0 = -0.0063\%$. As we know from Figure 1, even with $B_t \equiv 0.5$, $\pi_0 = -0.011\%$. However, if B_0 moves together with $\epsilon_{A,0}$, then optimal policy involves much larger responses of the inflation rate: when $B_0 = 0.7 = \bar{B} - 0.05$, $\pi_0 = 0.12\%$; when $B_0 = 0.65 = \bar{B} - 0.1$, $\pi_0 = 0.25\%$. Similarly, such countercyclical idiosyncratic income losses imply much larger responses of the output gap, $x_0 = \hat{Y}_0 - \hat{Y}_0^*$ ($x_0 = 0.013\%, -0.25\%, -0.53\%$ for $B_0 = \bar{B}, \bar{B} - 0.05, \bar{B} - 0.1$, respectively). It is also noteworthy that countercyclical idiosyncratic income shock calls for more stabilization of the actual level of output, $\hat{Y}_t$: $\hat{Y}_0 = -1.966\%, -1.159\%, -0.29\%$ for $B_0 = \bar{B}, \bar{B} - 0.05, \bar{B} - 0.1$, respectively. Figure 6 illustrates that optimal responses to the government-purchase shock share similar characters.

We find it interesting that the actual level of output, $\hat{Y}_t$, is stabilized quite strongly under optimal policy when idiosyncratic earning losses are countercyclical. In the case where B_0 declines to 0.65, the optimal responses of π_0 and $\hat{Y}_0$ are in similar magnitude. There are two reasons for this. First, although negative shocks $\epsilon_{A,0}$ and $\epsilon_{G,0}$ tend to reduce the efficient level of output, Y_0^*, the deterioration in risk sharing calls for stimulation of the economy and hence tends to raise the efficient level of output. These two forces offset each other so that $\hat{Y}_0^*$ is close to zero and the equilibrium level of output is stabilized under optimal policy. Second, fluctuations in the inflation rate and the output gap are larger with countercyclical idiosyncratic shock because u_0 is larger, which, in turn, is the result that a shock to risk sharing affects the efficient and natural levels of output in the opposite directions (recall that $c_B^n > 0$ and $c_B^* < 0$).

7 Concluding remarks

In this paper, we have considered an efficiency-wage model with the Calvo-type sticky prices and examined optimal monetary policy when unemployment insurance is not perfect. In the standard sticky-price model, the strict zero-inflation policy becomes optimal if the zero-inflation steady state is efficient. This is because relative-price distortions would be the only distortion in that case and such distortions would vanish under the strict zero-inflation policy. We have seen, however, that with imperfect unemployment insurance, the strict zero-inflation policy is no longer optimal even if the zero-inflation steady-state equilibrium is (conditionally) efficient. Quantitatively, though, if the level of risk sharing is constant over business cycles, the difference between optimal policy and strict zero-inflation policy is minimal. We have also shown, however, that if the level of risk sharing

is procyclical, that is, if idiosyncratic shocks are countercyclical, as evidence suggests, the difference becomes substantial. Indeed, in such a case, output must be stabilized much more compared to the case with perfect insurance.

One important limitation of our model is that in order to keep the representative-household framework idiosyncratic shocks are purely temporary. Evidence suggests that idiosyncratic shocks are highly persistent as well as countercyclical.[10] Krebs (2005) argues that persistence as well as countercyclicality of idiosyncratic shocks matter a lot concerning the welfare cost of business cycles. Incorporating persistent idiosyncratic shocks is left for future research.

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Appendix

Cost minimization problem of a firm

The first-order conditions for the cost minimization problem (8) are

$$\frac{e'(W)W}{e} = 1,$$
$$Af(ehn) = y.$$

The first equation implies that $C_s/C_e = \tilde{s} \in [s, 1]$, where $\tilde{s}$ is defined as the solution to

$$d(\chi - s)(1 - \tilde{s}) = \omega(1 - s)\tilde{s}(\tilde{s}^{-\frac{d}{\omega}} - 1).$$

Then the cost-minimizing level of effort is given by

$$e = \frac{\mathcal{H}}{h} - \frac{\mathcal{H}}{h}\tilde{s}^{\frac{d}{\omega}}. \tag{45}$$

The real wage rate is

$$\frac{W_t}{P_t} = \frac{\chi}{h}C_{e,t}, \qquad \text{where} \quad \chi \equiv \frac{1 - \tilde{s}}{1 - s}.$$

Equivalence with a version of Shapiro and Stiglitz's (1984) model

Consider the following version of Shapiro and Stiglitz's (1984) model: if a shirker gets caught she is immediately fired and receives no wages; there are two levels of effort $e_t \in \{0, \bar{e}\}$. The rest is the same as our model in text. Then the incentive compatibility constraint becomes

$$\mathcal{U}(C_{e,t}, \bar{e}) \geq (1 - d)\mathcal{U}(C_{e,t}, 0) + d\mathcal{U}(C_{u,t}, 0),$$

where $C_{e,t}$ and $C_{u,t}$ are as given in (4) and (6), respectively. This model and our model become essentially identical if (i) $\bar{e}$ is at the level given by (45) and (ii) the unemployment insurance program is given by

$$\text{UI}_t^f = (1 - N_t)shW_t, \qquad \text{and} \qquad \text{UI}_t^b = N_t shW_t.$$

This is because this insurance program implies $C_{s,t} = C_{u,t}$ in our original model.

Derivation of (25) and (28)

To derive inequality (25), note that

$$\frac{\partial Y^*}{\partial \bar{B}} = -\frac{Z_{YB}}{U_{YY} + Z_{YY} - V_{YY}}$$

The numeraire is negative, $U_{YY} + Z_{YY} - V_{YY} < 0$, because of Assumption 1. The denominator is also negative:

$$\begin{aligned} Z_{YB} &= -\frac{N_Y}{B} + \frac{N_Y}{N + (1 - N)B} + \frac{(1 - B)(1 - N)N_Y}{[N + (1 - N)B]^2} \\ &= \frac{(1 - B)N_Y}{B[N + (1 - N)B]} \frac{B(1 - N)^2 - N^2}{N + (1 - N)B} \leq 0, \end{aligned}$$

where the last inequality follows from the assumption that $N > 1/2$.

For (28), $d \ln Y_t^n / d \ln \bar{B}$ is easier to compute:

$$\frac{\partial \ln Y^n}{\partial \ln \bar{B}} = \frac{1}{\sigma^{-1} - \delta + \phi - 1} \frac{(1 - N)\bar{B}}{N + (1 - N)\bar{B}} \geq 0.$$

Here, note that $\sigma^{-1} \geq 1$ and $\phi \geq \delta$.

Derivation of the aggregate-supply relation (32)

A log-linear approximation of the first-order condition for p_t^*, (13), is given by

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \hat{p}_t^* - \hat{s}_{t,T} - \sum_{\tau=t+1}^{T} \pi_\tau \right\} = 0, \tag{46}$$

where $\hat{p}_t^* \equiv \ln p_t^* - \ln P_t$.

The real marginal cost of firm i is written as

$$\hat{s}_t(i) = (\phi - 1)\hat{y}_t(i) + (\sigma^{-1} - \delta)\hat{Y}_t - (\sigma^{-1} - \delta + \phi - 1)\hat{Y}_t^n$$

Taking the average over $i \in [0, 1]$, the average real marginal cost in period t is

$$\hat{s}_t = (\sigma^{-1} - \delta + \phi - 1)(\hat{Y}_t - \hat{Y}_t^n)$$

Log-linearizing the demand function (10) yields

$$\hat{y}_t(i) = \hat{Y}_t - \theta\big[\ln p_t(i) - \ln P_t\big]$$

It follows that

$$\begin{aligned} \hat{s}_{t,T} &= \hat{s}_T + (\phi - 1)\big(\hat{y}_{t,T} - \hat{Y}_T\big) \\ &= \hat{s}_T - (\phi - 1)\theta\hat{p}_t^* + (\phi - 1)\theta \sum_{\tau=t+1}^{T} \pi_\tau \end{aligned}$$

Substituting this into (46) yields

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \big[1 + (\phi - 1)\theta\big]\hat{p}_t^* - \hat{s}_T + \big[1 + (\phi - 1)\theta\big] \sum_{\tau=t+1}^{T} \pi_\tau \right\} = 0$$

Solving for $\hat{p}_t^*$ and writing it in a recursive form, we obtain

$$\hat{p}_t^* = \frac{1 - \alpha\beta}{1 + (\phi - 1)\theta}\hat{s}_t + \alpha\beta E_t \pi_{t+1} + \alpha\beta E_t \hat{p}_{t+1}^* \tag{47}$$

Log-linearizing the evolution of P_t, (29), leads to

$$\pi_t = \frac{1 - \alpha}{\alpha}\hat{p}_t^*$$

Using this, (47) is rewritten as

$$\begin{aligned} \pi_t &= \frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + (\phi - 1)\theta}\hat{s}_t + \beta E_t \pi_{t+1} \\ &= \frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + (\phi - 1)\theta}(\sigma^{-1} - \delta + \phi - 1)(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t \pi_{t+1} \end{aligned}$$

which is equation (32) in the main text.

Derivation of the welfare approximation (33)

Remember that the household's flow utility is given by

$$\mathcal{W}(Y_t, \Delta_t; \xi_t) = U(Y_t; G_t) + Z(Y_t, \Delta; A_t, B_t) - V(Y_t, \Delta_t; A_t) + \ln(\mathcal{H}),$$

where U, Z, and V are as defined in (21)-(23). We follow Woodford (2003), and Benigno and Woodford (2003, 2005) to obtain a quadratic approximation of the household welfare.

We denote by Ξ the vector of expansion parameters: $\Xi = (\hat{Y}, \xi, \Delta_{-1}^{1/2})$. First, $U(Y_t; G_t)$ is approximated as

$$\begin{aligned} U(Y_t; G_t) &= \bar{U} + U_Y \tilde{Y}_t - U_Y \tilde{G}_t + \frac{1}{2} U_{YY} \tilde{Y}_t^2 - U_{YY} \tilde{Y}_t \tilde{G}_t + \frac{1}{2} U_{YY} \tilde{G}_t^2 + \mathcal{O}(\|\Xi\|^3) \\ &= U_Y \bar{Y} \left(\hat{Y}_t + \frac{1}{2} \hat{Y}_t \right) + \frac{1}{2} U_{YY} \bar{Y}^2 \hat{Y}_t^2 - U_{YY} \bar{Y} \bar{G} \xi G, t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \\ &= U_Y \bar{Y} \hat{Y}_t + \frac{1}{2} \big(U_Y \bar{Y} + U_{YY} \bar{Y}^2 \big) \hat{Y}_t^2 - U_{YY} \bar{Y}^2 g_t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \end{aligned}$$

where g_t measures the change in Y_t required to keep U_Y constant:

$$g_t \equiv -\frac{U_{YG} \bar{G}}{U_{YY} \bar{Y}} = s_G \xi_{G,t}$$

Next, note that the evolution of Δ_t, (31), implies that

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1} + \frac{\alpha}{1-\alpha} \theta \phi \big[1 + (\phi - 1)\theta \big] \frac{\pi_t^2}{2} + \mathcal{O}(\|\Xi\|^3)$$

It follows that

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\alpha \theta \phi \big[1 + (\phi - 1)\theta \big]}{(1-\alpha)(1-\alpha\beta)} \sum_{t=0}^{\infty} \beta^t \frac{\pi_t^2}{2} + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \tag{48}$$

Then $Z(Y_t, \Delta_t; A_t, B_t)$ and $V(Y_t, \Delta_t; A_t)$ are approximated as

$$\begin{aligned} Z(Y_t, \Delta_t; A_t, B_t) &= \frac{Z_Y \bar{Y}}{\phi} \hat{\Delta}_t + Z_Y \bar{Y} \hat{Y}_t + \frac{1}{2} \big(Z_Y \bar{Y} + Z_{YY} \bar{Y}^2 \big) \hat{Y}_t^2 - Z_{YY} \bar{Y}^2 k_t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \\ V(Y_t, \Delta_t; A_t) &= \frac{V_Y \bar{Y}}{\phi} \hat{\Delta}_t + V_Y \bar{Y} \hat{Y}_t + \frac{1}{2} \big(V_Y \bar{Y} + V_{YY} \bar{Y}^2 \big) \hat{Y}_t^2 - V_{YY} \bar{Y}^2 q_t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \end{aligned}$$

where k_t and q_t are the change in Y_t required to keep Z_Y and V_Y constant, respectively:

$$\begin{aligned} k_t &\equiv \frac{Z_{YA} \bar{A}}{Z_{YY} \bar{Y}} \xi_{A,t} - \frac{Z_{YB} \bar{B}}{Z_{YY} \bar{Y}} \xi_{B,t} \\ q_t &\equiv -\frac{V_{YA} \bar{A}}{V_{YY} \bar{Y}} \xi_{A,t} \end{aligned}$$

Since the zero-inflation steady-state is conditionally efficient,

$$U_Y + Z_Y - V_Y = 0$$

Note also that

$$\hat{Y}_t^* = \frac{1}{U_{YY} + Z_{YY} - V_{YY}}\big(U_{YY} g_t + Z_{YY} k_t - V_{YY} q_t\big)$$

It follows that

$$\begin{aligned}\mathcal{W}(Y_t, \Delta_t; \xi_t) &= -\frac{U_Y \bar{Y}}{\phi}\hat{\Delta}_t + \frac{1}{2}\bar{Y}^2(U_{YY} + Z_{YY} - V_{YY})(\hat{Y}_t - \hat{Y}_t^*)^2 + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3) \\ &= -V_Y \bar{Y}\left\{\frac{1-\Gamma}{\phi}\hat{\Delta}_t + \frac{1}{2}\big[\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1\big]\big(\hat{Y}_t - \hat{Y}_t^*\big)^2\right\} \\ &\quad + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3)\end{aligned}$$

where Γ is defined by

$$\Gamma \equiv \frac{Z_Y \bar{Y}}{U_Y \bar{Y} + Z_Y \bar{Y}}$$

Finally, using (48), we obtain

$$\begin{aligned}E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t = -\bar{Y} V_Y E_0 \sum_{t=0}^{\infty} \frac{1}{2}\beta^t &\\ \times \left\{\frac{\alpha\theta[1 + (\phi-1)\theta]}{(1-\alpha)(1-\alpha\beta)}(1-\Gamma)\pi_t^2 + \big[\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1\big]\big(\hat{Y}_t - \hat{Y}_t^*\big)^2\right\} &\\ + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3)&\end{aligned}$$

which is (33) in the main text.

Proof of Proposition 3

For the first part,

$$\begin{aligned}c_G^*|_{B<1} - c_G^*|_{B=1} &= \frac{\sigma^{-1}(1-\Gamma)s_G}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} - \frac{\sigma^{-1}s_G}{\sigma^{-1} + \phi - 1} \\ &= \frac{\sigma^{-1}s_G\Gamma(\zeta + 1 - \phi)}{[\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1][\sigma^{-1} + \phi - 1]} > 0\end{aligned}$$

because

$$\zeta + 1 - \phi = \frac{\delta^2}{Z_Y \bar{Y}} > 0.$$

For the second part, note that

$$c_G^n|_{B<1} - c_G^n|_{B=1} = \frac{\sigma^{-1}s_G}{\sigma^{-1} - \delta + \phi - 1} - \frac{\sigma^{-1}s_G}{\sigma^{-1} + \phi - 1} > 0$$

because $\delta > 0$.

Proof of Proposition 4

For the first part, note that

$$\begin{aligned} c_A^*|_{B<1} - c_A^*|_{B=1} &= \frac{\phi - \Gamma(\zeta+1)}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} - \frac{\phi}{\sigma^{-1} + \phi - 1} \\ &= -\frac{(\sigma^{-1}-1)(\zeta+1)\Gamma}{[\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1][\sigma^{-1} + \phi - 1]} < 0 \end{aligned}$$

because $\sigma^{-1} \equiv 1/(1-s_G) > 1$ as long as $s_G > 0$. The second part follows from:

$$\begin{aligned} c_A^n|_{B<1} - c_A^n|_{B=1} &= \frac{\phi - \delta}{\sigma^{-1} - \delta + \phi - 1} - \frac{\phi}{\sigma^{-1} + \phi - 1} \\ &= -\frac{(\sigma^{-1}-1)\delta}{[\sigma^{-1} - \delta + \phi - 1][\sigma^{-1} + \phi - 1]} < 0, \end{aligned}$$

again, because $\sigma^{-1} > 1$.

Proof of Lemma 1

Lemma 1 follows from

$$\begin{aligned} \sigma^{-1}(1-\Gamma) + \zeta\Gamma - (\sigma^{-1} - \delta) &= \delta - \Gamma(\sigma^{-1} + \zeta) \\ &= \delta - \frac{Z_Y Y}{Z_Y Y + \sigma^{-1}} \left\{ \sigma^{-1} + \frac{\delta^2}{Z_Y Y} + \phi - 1 \right\} \\ &= \frac{1}{Z_Y Y + \sigma^{-1}} \Big\{ (\sigma^{-1} - \delta)\big[2\delta + \ln(B) N\phi\big] - (\phi - 1)\big[-\ln(B) N\phi - \delta\big] \Big\} \end{aligned}$$

Proof of Proposition 5

That $c_G^u < 0$ follows from

$$\begin{aligned} c_G^u &= c_G^* - c_G^n \\ &= \frac{\sigma^{-1}(1-\Gamma)s_G}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} - \frac{\sigma^{-1} s_G}{\sigma^{-1} - \delta + \phi - 1} < 0 \end{aligned}$$

because $\Gamma > 0$ and $\sigma^{-1}(1-\Gamma) - \zeta\Gamma > \sigma^{-1} - \delta$ under condition (44). The second inequality, $c_A^u > 0$, is derived as:

$$\begin{aligned} c_A^u &= c_A^* - c_A^n \\ &= \frac{\phi - \Gamma\zeta - \Gamma}{\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1} - \frac{\phi - \delta}{\sigma^{-1} - \delta + \phi - 1} \\ &= \frac{(\sigma^{-1} - 1)\big[(1-\Gamma)\delta + \Gamma(\phi - 1 - \zeta)\big]}{\big[\sigma^{-1}(1-\Gamma) - \zeta\Gamma + \phi - 1\big]\big[\sigma^{-1} - \delta + \phi - 1\big]} \end{aligned}$$

Remember that

$$\zeta = \phi - 1 + \frac{\delta^2}{Z_Y Y}, \qquad \text{and} \qquad \Gamma = \frac{Z_Y Y}{Z_Y Y + \sigma^{-1}}.$$

Thus

$$(1-\Gamma)\delta+\Gamma(\phi-1-\zeta)=\frac{\delta}{Z_Y Y}(\sigma^{-1}-\delta)$$

We finally obtain

$$c_A^u=\frac{(\sigma^{-1}-1)(\sigma^{-1}-\delta)\delta}{\left[\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1\right]\left[\sigma^{-1}-\delta+\phi-1\right]Z_Y Y}>0$$

because $\sigma^{-1}>1$ because $s_G>0$ and $\sigma^{-1}>\delta$ because of (44).

Proof of Proposition 6

Define the quadratic function $f(\mu)$ by

$$f(m)\equiv\beta m^2-\left(1+\beta+\kappa^2\frac{q_\pi}{q_y}\right)m+1$$

Then $f(m)=0$ has two roots: $\mu\in(0,1)$ and $(\beta\mu)^{-1}>1$. Remember that

$$\begin{aligned}\kappa&=\frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\theta(\phi-1)}(\sigma^{-1}-\delta+\phi-1)\\ q_\pi&=\frac{\alpha\theta[1+\theta(\phi-1)]}{(1-\alpha)(1-\alpha\beta)}(1-\Gamma)\\ q_y&=\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1\end{aligned}$$

It follows that

$$\kappa^2\frac{q_\pi}{q_y}=\theta\frac{(1-\alpha)(1-\alpha\beta)}{\alpha[1+\theta(\phi-1)]}\frac{(1-\Gamma)(\sigma^{-1}-\delta+\phi-1)^2}{\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1}$$

For $\mu|_{\bar{B}<1}>\mu|_{\bar{B}=1}$, it suffices to show that

$$\frac{(1-\Gamma)(\sigma^{-1}-\delta+\phi-1)^2}{\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1}<\sigma^{-1}+\phi-1$$

Under our assumption,

$$\sigma^{-1}-\delta+\phi-1<\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1$$

It then follows that

$$\begin{aligned}\frac{(1-\Gamma)(\sigma^{-1}-\delta+\phi-1)^2}{\sigma^{-1}(1-\Gamma)-\zeta\Gamma+\phi-1}&<\sigma^{-1}-\delta+\phi-1\\ &<\sigma^{-1}+\phi-1\end{aligned}$$

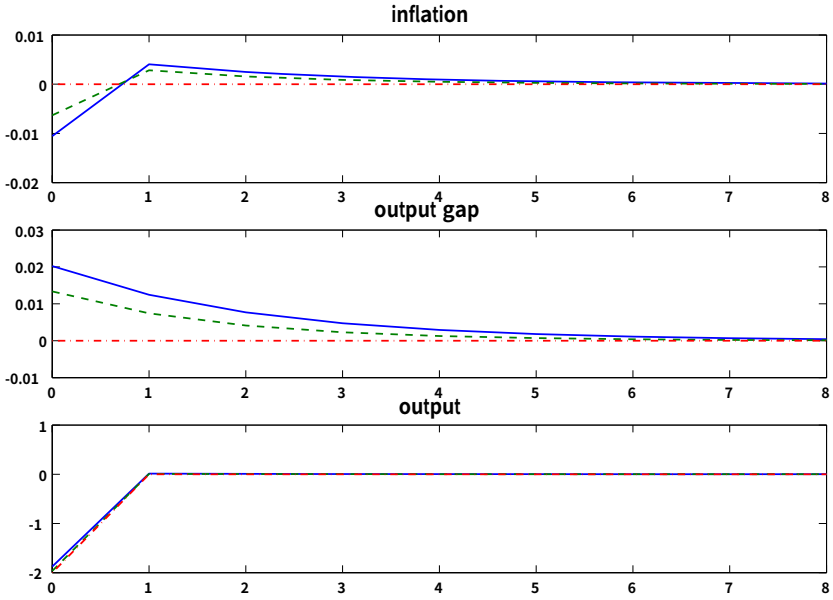

Figure 1: Optimal responses to a productivity shock for different degrees of risk sharing. In each panel, the solid, dashed, and dash-dotted lines correspond to $\bar{B} = 0.5, 0.75, 1.0$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.

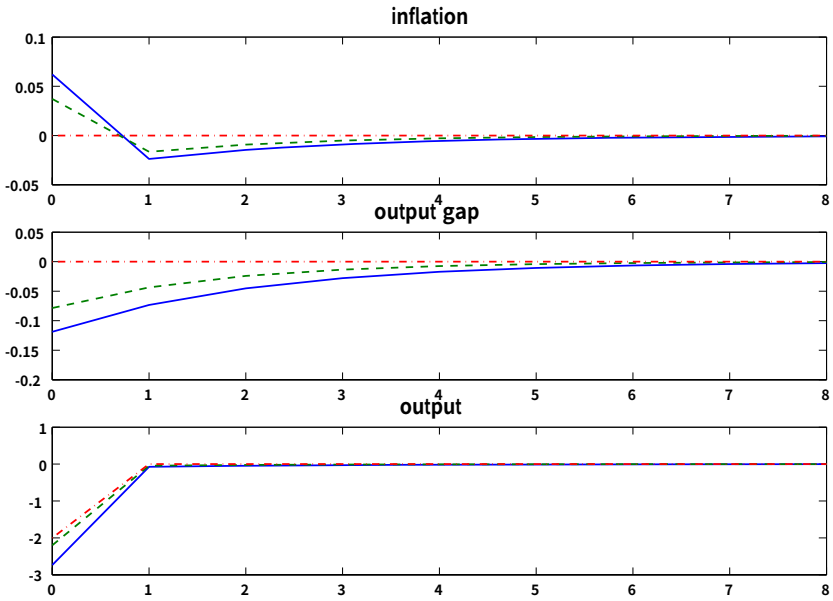

Figure 2: Optimal responses to a government-purchase shock for different degrees of risk sharing. In each panel, the solid, dashed, and dash-dotted lines correspond to $\bar{B} = 0.5, 0.75, 1.0$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.

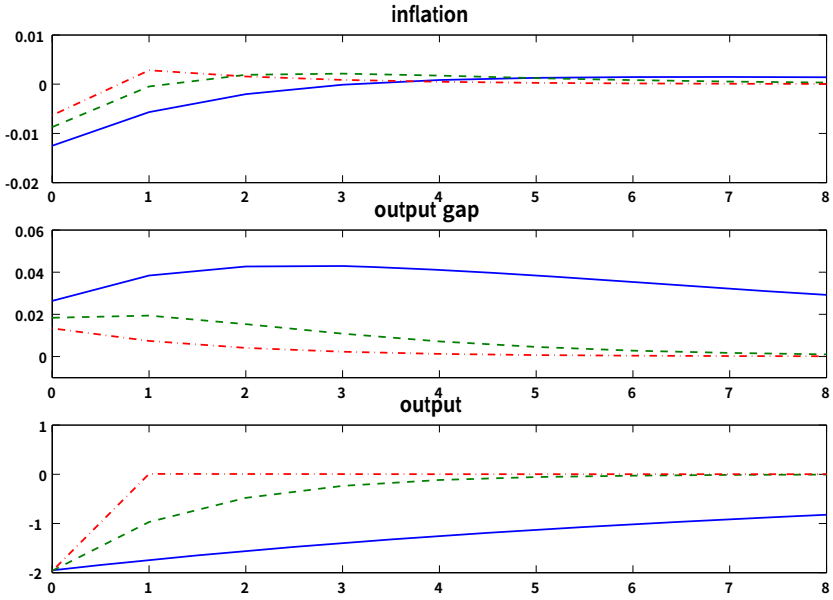

Figure 3: Optimal responses to a productivity shock for different auto-correlation coefficients. In each panel, the solid, dashed, and dash-dotted lines correspond to $\rho_A = 0.9, 0.5, 0$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.

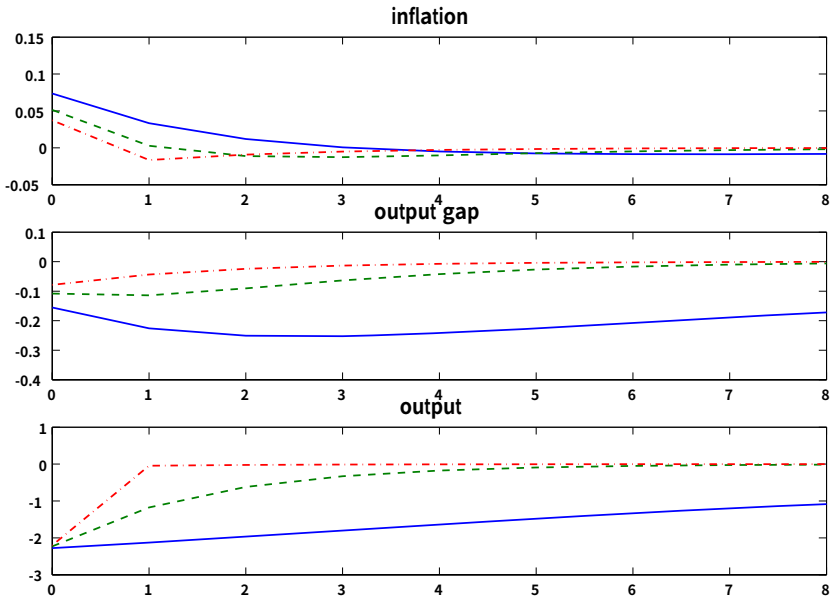

Figure 4: Optimal responses to a government-purchase shock for different auto-correlation coefficients. In each panel, the solid, dashed, and dash-dotted lines correspond to $\rho_G = 0.9, 0.5, 0$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.

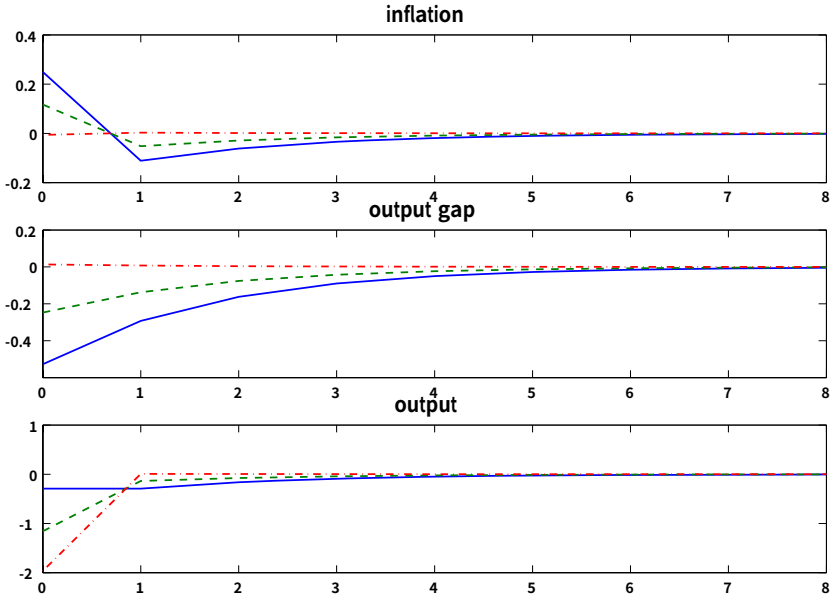

Figure 5: Optimal responses to a productivity shock with countercyclical risk sharing. In each panel, the solid, dashed, and dash-dotted lines correspond to $B_0 = 0.65, 0.7, 0.75$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.

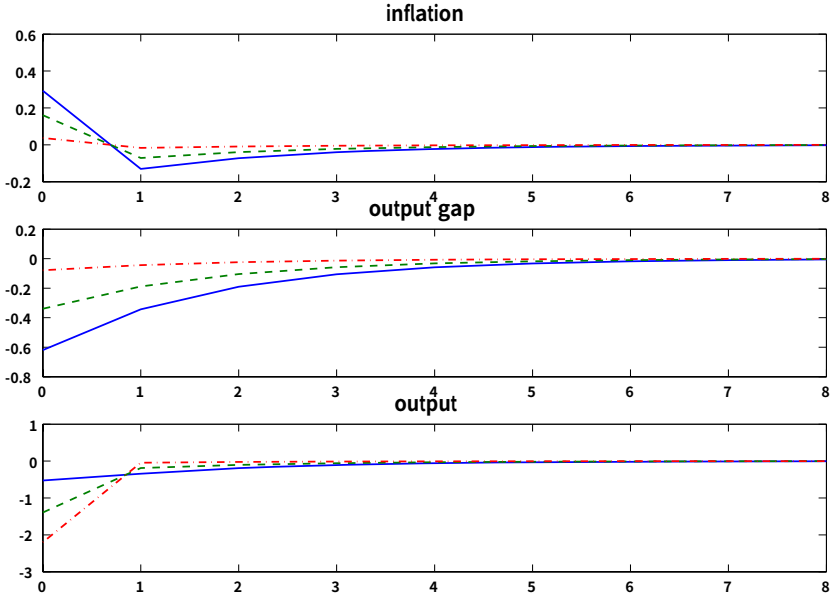

Figure 6: Optimal responses to a government-purchase shock with countercyclical risk sharing. In each panel, the solid, dashed, and dash-dotted lines correspond to $B_0 = 0.65, 0.7, 0.75$, respectively. The inflation rate is expressed in percent per year. The output gap and the level of output are expressed in percentage deviations from their respective steady-state values.