Understanding Wage Inequality: Ben-Porath Meets Skill-Biased Technical Change*

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Abstract

In this paper we present a tractable general equilibrium overlapping-generations model of human capital accumulation which is consistent with several features of the evolution of the U.S. wage distribution from 1970 to 2000. The key feature of the model, and the only source of heterogeneity, is that individuals differ in their ability to accumulate human capital. To highlight the working of the model, we abstract from all kinds of idiosyncratic uncertainty, and thus, wage inequality results only from differences in human capital accumulation. We examine the response of this model to skill-biased technical change (SBTC) both theoretically and quantitatively. First, we theoretically show that in response to SBTC, the model generates behavior consistent with the U.S. data including (i) a rise in total wage inequality, (ii) an initial fall in the education (skill) premium followed by a strong recovery, leading to a higher premium in the long-run, (iii) the fact that most of this fall and rise takes place among younger workers, (iv) a rise in within-group inequality, (v) an increase in educational attainment, (vi) stagnation in median wage growth (and a slowdown in aggregate labor productivity), and (vii) a rise in consumption inequality that is much smaller than the rise in wage inequality. We then calibrate the model to the U.S. data before 1970,

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and find that the evolutions of these variables are quantitatively consistent with their empirical counterparts during SBTC (from 1970 on). These results suggest that the heterogeneity in the ability to accumulate human capital is an important feature for understanding the effects of SBTC and interpreting the transformation that the US economy has gone through since the 1970's.

1 Introduction

The U.S. economy has gone through dramatic changes since the early 1970's. Among the most notable of these changes were the following seven trends:[1]

(i) There was a substantial rise in overall wage inequality throughout this period.

(ii) The college premium (the average wage of individuals with a college degree relative to the wage of less-educated individuals) fell during the 1970's, but rose strongly in the subsequent two decades.

(iii) Most of the fall and rise in the college premium took place among younger workers.

(iv) The rise in wage inequality also happened within narrowly defined groups, and was spread across every percentile of the wage distribution.

(v) There was a significant increase in the supply of college educated labor.

(vi) Average wages were stagnant (and there was a parallel slowdown in aggregate labor productivity) which started with a sharp fall in 1973 and persisted until mid 1990's.

(vii) While consumption inequality also increased, it arguably has not kept pace with the rise in wage inequality.

In this paper we present an analytically tractable general equilibrium overlapping-generations model of human capital accumulation which is consistent with these facts, as well as some others that have been observed during this period.

Among the trends mentioned above, perhaps the most puzzling has been the joint behavior of overall inequality and the college premium (i and ii), and in particular, their movement in opposite directions during the 1970's. In an influential paper, Juhn, Murphy and Pierce (1993) have documented these patterns and stated: "The rise in within-group inequality preceded the increase in returns to observables by over a decade. On the basis of this difference in timing, it seems clear to us that there are at least two unique dimensions of skill (education and skill differences within an education group) that receive unique prices in the labor market (p. 429)." They then added: "Our conclusion is that the general rise in

[1] For extensive documentation of these trends, see Bound and Johnson (1992), Katz and Murphy (1992), Murphy and Welch (1992), Juhn, Murphy and Pierce (1993), Card and Lemieux (2001), Acemoglu (2002), Krueger and Perri (2004), Attanasio, Battistin and Ichimura (2004) and Autor, Katz and Kearney (2005).

inequality and the rise in education premium are actually distinct economic phenomena (p. 412)." This widely accepted conclusion has then led the subsequent literature to search for separate driving forces and mechanisms to explain each of these phenomena.[2] In contrast, we propose a single mechanism that simultaneously generates a monotonic rise in overall (and within-group) inequality and a non-monotonic change in the college premium.

Here are the basic features of the model. Individuals begin life with a fixed endowment of "raw labor" (i.e., strength, health, etc.), and are able to accumulate "human capital" (skills, knowledge, etc.) over the life-cycle. Raw labor and human capital earn separate wages in the labor market and each individual supplies both of these factors of production at competitively determined wage rates. Following the standard interpretation of the Ben-Porath (1967) model we assume that investment in human capital takes place on-the-job unless it equals 100 percent of an individual's time, in which case it is interpreted as "schooling." We assume that skills are general (i.e., not firm specific) and labor markets are competitive. As a result, the cost of human capital investment will be completely borne by the workers, and the firm will adjust the hourly wage rate downward by the fraction of time invested on the job (Becker (1965)). Thus, the cost of human capital investment is the foregone earnings while individuals are learning new skills. Except for the fact that we distinguish between raw labor and human capital, the model described so far is essentially the same as the standard Ben-Porath framework.

We introduce two key features into this framework. First we assume that individuals differ in their ability to accumulate human capital, which is the only source of heterogeneity in the model. As a result, individuals differ systematically in the amount of investment they undertake, and consequently, in the growth rate of their wages over the life-cycle. This assumption is consistent with, and motivated by, recent empirical evidence from panel data on individual wages; see for example Lillard and Weiss (1979), Baker (1997), Guvenen (2005) and Huggett, Ventura and Yaron (2005). Thus, wage inequality in the model only results from this systematic fanning out of the wage profiles as individuals get older. In particular,

[2] Notable exceptions exist, such as Acemoglu (1998) who considers an extension of his baseline model that can be used to study both between and within-group inequality, and Galor and Moav (2000) who discuss some implications of their model for within-group inequality.

we completely abstract from idiosyncratic income risk to isolate the role of differential human capital investment for the observed trends mentioned above.

The demand side of the model consists of an aggregate production technology of the CES form which takes raw labor and human capital as its inputs. The second element in the model and the driving force behind the non-stationary changes during this period is skill-biased technical change (SBTC) that occurs starting in the early 1970's. A key difference of our model is that we do not equate "skill" to education as is commonly done in previous studies. Instead, we interpret skill more broadly as human capital, and view SBTC as a change that raises the price of human capital relative to that of raw labor. This seemingly small difference in perspective has important consequences. To see this, note that in this model all workers have some amount of human capital (which varies by ability and age) and raw labor (which is the same for all). Therefore, SBTC not only changes wages across education groups (because of differences in average human capital levels), but also affects individuals within each group differently depending on their ability and age. In this framework education is merely a noisy indicator of one's ability to learn, which in turn is an indicator of his human capital level and of how strongly he responds to SBTC. In this sense, the model allows us to study both between-group and within-group inequality simultaneously.

If raw labor and human capital are assumed to be perfect substitutes in the production function, the model described so far can be solved in closed form. But apart from analytical convenience, this assumption has another important advantage: it eliminates the feedback from the relative supply of skilled labor to the college premium, which has been suggested as an explanation for the behavior of the college premium by several authors (see the papers cited in footnote 1). Thus, we make this assumption throughout the paper to show that our results—and especially the non-monotonic behavior of the college premium—are not driven by this channel.[3] In the quantitative robustness analysis we find that our main results

[3] In addition, this assumption essentially reduces the present framework to a one-skill model, because the price of the two factors move perfectly proportionately to each other. Yet this version of the model is still consistent with the *joint* behavior of overall inequality and college premium contrary to the assertion made by Juhn, et al (1993) quoted above. As will become clear later, the reason we still have two factors (skills), despite having perfectly correlated prices, is that it allows us to sensibly talk about SBTC as a change in the *relative* price of these two skills without necessarily implying anything about TFP growth.

continue to hold when we allow for a certain degree of imperfect substitution.

We begin with a theoretical analysis, and show that all of the seven facts noted in the first paragraph emerge as robust implications of this model. Only facts (ii) and (vi) require a restriction on parameter values, which basically ensures that the immediate response of investment to SBTC is sufficiently large. This happens when (i) individuals' time discount factor is not too low (or interest rates are not too high), and/or (ii) the human capital accumulation function does not feature strong diminishing marginal returns (i.e., is not too concave). This condition is satisfied for a range of empirically plausible values (figure 2).

The mechanism behind the productivity slowdown (and the stagnation of average wages) can be explained as follows. SBTC increases the returns to human capital, which leads to a rise in investment rates. While this higher investment results in an immediate increase in costs (in the form of foregone earnings) its benefits are realized only gradually as the total stock of human capital slowly increases. Thus after SBTC begins, observed wages immediately fall due to increased investment on the job, and inherits the sluggish growth of the human capital stock thereafter.

A closely related mechanism is behind the non-monotonic behavior of the college premium during SBTC. Because college graduates have higher learning ability than those with lower education, their investment increases more in response to SBTC, which causes an initial fall in their relative wages. In the long-run, however, this higher investment yields a larger increase in their human capital stock, leading to a higher college premium. Finally, it is also easy to see that this mechanism will affect younger workers—who have a longer horizon and thus expect larger benefits from investing—more than older ones, resulting in a more pronounced decline in the college premium among younger workers consistent with fact (iii). Moreover, in the quantitative analysis we show that the increase in on-the-job investment necessary to explain facts (i) to (iii) are not implausibly large. The intuition for this result is explained in Section 4.

Another surprising implication of our model is that the rise in life-time income inequality in response to SBTC is much smaller than the rise in wage inequality. To the extent that consumption is linked to life-time income, this implies a small rise in consumption inequality as well. There are two reasons for this small rise. First, because wage inequality rises due to an increased dispersion in the *growth rate* of wages, life-time inequality—which is the variance calculated *after* averaging wages over the life-cycle—increases by less. This would not be the case if the increase in dispersion was in the *levels* of wages, as has commonly been modeled in the previous literature (among others, Moffitt and Gottschalk (1994), and Meghir and Pistaferri (2002)). In the latter case, lifetime inequality would increase one for one with wage inequality. Second, and furthermore, the rise in the wages of high ability individuals later in life come at an increased cost in the form of larger investment and lower wages early on, driving down the lifetime gain from human capital investment (see Kuruscu (2005)). Our model thus offers a mechanism which is consistent with a large increase in wage inequality but a small change in consumption inequality.

Finally we calibrate the model to the U.S. data before SBTC, which is assumed to start in 1970 and continue until 1995. We then analyze the behavior of several variables in the model during SBTC, including the seven trends noted above, and find that they are consistent with the corresponding empirical patterns quantitatively. We then present sensitivity analyses and extensions, such as allowing for adaptive learning instead of perfect foresight, allowing for imperfect substitutability in the production function, among others.

The paper is organized as follows. The next section describes the model. Section 3 analyzes the model theoretically and proves the results described above. Section 4 contains the quantitative analysis. Extensions and robustness are discussed in Section 5; Section 6 concludes.

1.1 Related Literature [Incomplete]

There is a vast literature on the empirical trends that motivate this paper. A short list of these papers are mentioned in footnote 1; for excellent surveys of the literature see Katz and Autor (1999) and Acemoglu (2002). A notable precursor to our paper is the seminal work of Heckman, Lochner and Taber (1998), who build an overlapping generations model of human capital accumulation and quantitatively examine some of the trends mentioned above. Our model shares some important similarities with theirs, such as our emphasis on general equilibrium and on the response of human capital accumulation decisions to SBTC. As a result of the latter, observed wages differ from skill prices, which is a key insight that we take from that paper. Our model is also different in several important respects however. First, a central thesis of our paper is that individuals differ *significantly* in their ability to accumulate human capital, which is motivated by recent empirical evidence (see Baker (1997), Guvenen (2005), and Huggett, Ventura and Yaron (2005)).[4] This feature generates substantial differences in cross-sectional investment behavior in response to SBTC, and is crucial for many of our results (especially for (ii), (iii), (iv) and (vii) above). Instead Heckman et al. equate this "learning ability" to the AFQT (Armed Forces Qualification Test) score, which results in much smaller differences in cross-sectional investment behavior. Second, we abstract from several features considered in their paper, such as differences in skill prices and human capital production technology by education groups, physical capital accumulation, retirement, taxes, and so on. This simplification allows us to solve the model in closed-form, derive explicit expressions for the moments of the wage and consumption distributions, and prove many of our results theoretically. Third, we assume that individuals face the same skill prices regardless of their schooling choices. As a result, our model has essentially one type of skill, whereas theirs has two. Fourth, while in our model all measures of inequality increase in the long-run after SBTC, this is not the case in that paper. Finally, we study additional empirical facts not examined in their paper.

In some interesting recent work, Krueger and Perri (2005), and Heathcote, Storesletten and Violante (2005) have constructed models which can also generate a smaller increase in consumption inequality despite a large rise in wage inequality. In both of these papers individual wage processes are exogenous but feature idiosyncratic shocks, and changes in the insurability of these shocks over time mitigate the rise in consumption inequality. Compared to these papers, our model lacks several features that are likely to be important for a detailed study of consumption behavior. Nevertheless, our model highlights an alternative channel which suggests that even when the rise in wage inequality is entirely systematic (and sub-

[4] In particular, Huggett, Ventura and Yaron (2005) show that a substantial fraction (between half and three-quarters) of wage variation across individuals is generated by heterogeneity in learning ability (in the context of the Ben-Porath model) and no more than 30 percent is explained by idiosyncratic income shocks.

stantial), life-time income inequality may not change much. The two channels are probably complementary to each other. Perhaps the main contribution of the present paper is to offer a mechanism that is simultaneously consistent with a broad set of facts.

2 A Baseline Model

2.1 Human Capital Accumulation Decision

The economy consists of overlapping generations of individuals who live for S years. Individuals begin life with an endowment of "raw labor" (i.e., strength, health, etc.) which is the same across individuals and constant over the life-cycle, and are able to accumulate "human capital" (skills, knowledge, etc.) over the life-cycle, which is the only skill that can be accumulated in this economy. There is a continuum of individuals in every cohort, indexed by $j \in [0, 1]$, that differ in their ability to accumulate human capital, denoted by A^j (also referred to as their "type"). This is the only source of heterogeneity in the model.

Each individual has one unit of time endowment in each period that can be allocated between producing output and accumulating human capital. Let l denote raw labor and $h_{j,s}$ denote the human capital of an s-year-old individual of type j. We assume that raw labor and human capital earn separate wages in the labor market and each individual supplies both of these factors of production at competitively determined wage rates. Then the "potential income" of an individual is given by $P_{L,t}l + P_{H,t}h_{j,s}$ where $P_{L,t}$ and $P_{H,t}$ are the rental prices of raw labor and human capital respectively. The "potential income" is the income an individual would earn if he spent all his time producing for his employer.

Following the standard interpretation of the Ben-Porath (1967) model, we assume that investment in human capital takes place on-the-job unless it equals 100 percent of an individual's time, which is then interpreted as "schooling." We assume that skills are general (i.e., not firm specific) and labor markets are competitive. As a result, the cost of human capital investment will be completely borne by workers, and the firm will adjust the hourly wage rate downward by the fraction of time invested on the job (Becker (1965)). Then, the observed wage income of the individual is given by[5]

$$w_{j,s} = \underbrace{[P_{L,t}l + P_{H,t}h_{j,s}]}_{x_{j,s}(t)}(1 - i_{j,s}) = \underbrace{x_{j,s}(t)}_{\text{Potential earnings}} - \underbrace{x_{j,s}(t)\, i_{j,s}}_{\text{Cost of investment}}$$

where $i_{j,s}$ is the fraction of time spent on human capital investment, henceforth referred to as "investment time." Thus, wage income can be written as the potential earnings minus the "cost of investment," which is simply the foregone earnings while individuals are learning new skills.

Individuals begin their life with zero human capital, $h_{j,0} = 0$, and accumulate human capital according to the following technology:

$$h_{j,s+1} = h_{j,s} + Q_{j,s}, \tag{1}$$

where $Q_{j,s}$ is the newly produced human capital which will be referred to as "investment" in the rest of the paper which should not be confused with investment time ($i_{j,s}$). Let $\widetilde{A}_j$ denote the learning ability of an individual, then investment is given by

$$Q_{j,s} = \widetilde{A}_j((\lambda_{L,t}l + \lambda_{H,t}h_{j,s})i_{j,s})^{\alpha}. \tag{2}$$

According to this formulation new human capital is produced by combining the existing stocks of raw labor and human capital with the available investment time.[6] The key parameter in this specification is $\widetilde{A}_j$, which determines the productivity of learning. Due to the heterogeneity in $\widetilde{A}_j$, individuals will differ systematically in the amount of investment they undertake, and consequently, in the growth rate of their wages over the life-cycle. Another important parameter is $\alpha \in [0, 1]$, which determines the degree of diminishing marginal returns in the human capital production function. A low value of α implies higher diminishing returns, in which case it is optimal to spread out investment over time. In contrast, when α is high, the marginal return on investment does not fall quickly, and investment becomes

[5] Since labor supply is inelastic, this is also the individual's observed "wage rate".

[6] The dependence of aggregate factor prices and weights in the human capital production function on t is to stress that these could be time-varying.

bunched over time. In the extreme case when $\alpha = 1$, individuals either spend all their time on investment ($i_{j,s} = 1$) or none at all in a given period.

The main difference between the Ben-Porath (1967) model and the formulation in (2) is the introduction of raw labor as an additional factor into our model. When $\lambda_{L,t} = 0$, $\lambda_{H,t} = 1$, and $P_{L,t} = 0$, this model reduces to the standard Ben-Porath model. As will be clear in the analysis below, the reason for our deviation from the standard Ben-Porath model is because it is difficult to sensibly think about SBTC when there is a single skill type.

2.2 Individual's Dynamic Problem

A standard result in the literature is that the consumption-savings and income maximization decisions can be disentangled from each other under complete markets without borrowing constraints. Therefore, for the purposes of analyzing human capital investment we concentrate on the lifetime income maximization problem. Letting $\Gamma^j_{s,t}(h_{j,s})$ denote lifetime income, an individual's income maximization problem can be written as

$$\Gamma^j_{s,t}(h_{j,s}) = \max_{i_{j,s}} \left[x_{j,s}(t)(1 - i_{j,s}) + \frac{1}{1+r} \Gamma^j_{s+1,t+1}(h_{j,s+1}) \right]$$

subject to (1), (2), and $h_{j,0} = 0$. It should be stressed again that this formulation does not rest on the assumption of risk-neutrality; it only requires markets to be complete (which in this context requires that individuals can borrow and lend at a constant interest rate).

2.3 Aggregate Production Technology

Let the aggregate factors used in production at a point in time be defined as

$$\begin{aligned} L^{net} &= \int_{j,s} l(1 - i_{j,s})\,\mu(s)\,dj\,ds, \quad \text{and} \\ H^{net} &= \int_{j,s} h_{j,s}(1 - i_{j,s})\,\mu(s)\,dj\,ds, \end{aligned}$$

where $\mu(s)$ is the (discrete) measure of s-year-old individuals, and the integrals are thus taken over the distribution of individuals of all types and ages.[7] The superscript "*net*" indicates that these variables measure the *actual* amounts of each factor used in production (that is, net of the time allocated to human capital investment on the job) to distinguish them from the "total stocks" of these factors defined later below. The aggregate firm uses these two inputs to produce a single good, denoted by Y, according to the familiar CES production function:

$$Y = Z\left(\left[\theta_L L^{net}\right]^{\rho} + \left[\theta_H H^{net}\right]^{\rho}\right)^{1/\rho},$$

where $\rho \leq 1$, and Z is the total factor productivity (TFP). For simplicity we assume that capital is not used in production. The firm solves a static profit maximization problem by hiring factors from households to maximize $Y - P_L L^{net} - P_H H^{net}$. The factor prices corresponding to human capital and raw labor are:

$$P_H = \frac{\partial Y}{\partial H^{net}} = Z\theta_H^{\rho}\left(\theta_L^{\rho}\left[H^{net}/L^{net}\right]^{-\rho} + \theta_H^{\rho}\right)^{\frac{1-\rho}{\rho}}, \quad \text{and}$$

$$P_L = \frac{\partial Y}{\partial L^{net}} = Z\theta_L^{\rho}\left(\theta_H^{\rho}\left[H^{net}/L^{net}\right]^{\rho} + \theta_L^{\rho}\right)^{\frac{1-\rho}{\rho}}.$$

The price of human capital *relative* to raw labor has a simple expression:

$$\frac{P_H}{P_L} = \left(\frac{\theta_H}{\theta_L}\right)^{\rho}\left(\frac{H^{net}}{L^{net}}\right)^{\rho-1}. \tag{3}$$

While the aggregate production function has the same CES form as commonly used in the literature, its inputs are different than what is typically assumed. In most previous work H^{net} and L^{net} denote the labor supplied by workers with college and high school education respectively.[8] Therefore, a change in the price of H^{net} relative to L^{net} has the same effect on all individuals within an education group. As a result, the college premium is simply equal to P_H/P_L and satisfies the relationship in (3). A key implication of this equation is that

[7] For the population structure assumed so far, $\mu(s) = 1/S$.

[8] A notable exception is Beaudry and Green (2003), who has a similar formulation to ours. However, the focus of their paper is different.

a rise in the relative supply of high-skill workers will reduce the college premium. Several authors have emphasized this link to argue that the fall in the college premium during the 1970's resulted from the rapid increase in the supply of college-educated workers (c.f., Katz and Murphy (1992), Juhn, Murphy and Pierce (1993)).

In contrast, in the present model, all workers have some endowment of $h_{j,s}$ (which varies by ability and age) and l (which is the same for all), and every worker contributes to both factors of production. Therefore, a change in the price of H^{net} relative to L^{net} affects all individuals differently depending on their ability level as well as their age, which gives rise to rich dynamics in wage inequality. Moreover, as we show below, the college premium is now very different than P_H/P_L.

An important special case arises when $\rho = 1$. In this case, human capital and raw labor become perfectly substitutable (which in turn implies that any two workers are also perfectly substitutable) and the relative wage in equation (3) reduces to $P_H/P_L = \theta_H/\theta_L$. Therefore, this assumption eliminates the link between the relative supply of high-skill labor and the college premium, which has received a lot of attention in the existing literature. To isolate and highlight the role of the mechanism proposed in this paper for the college premium, we thus make this assumption.

A second implication of $\rho = 1$ is that it essentially reduces the present framework to a one-skill model, because in this case the price of raw labor and human capital move perfectly proportionately to each other. As noted in the introduction, a model with one type of skill has been viewed as inconsistent with facts about the *joint* behavior of total inequality and the college premium (Juhn, Murphy and Pierce (1993)). Therefore, we make this assumption throughout the paper to show that our results—and especially the monotonic rise in overall wage inequality and the non-monotonic behavior of the college premium—are not driven by the feedback from the relative supply of human capital. In the quantitative robustness analysis we show that our main results continue to hold when we allow for a certain degree of imperfect substitution.

2.4 Analyzing the Individual's Problem

Several variants of this basic framework can be obtained that are distinguished by the specification of $(\lambda_{L,t}, \lambda_{H,t})$, which determines how raw labor and human capital enter the production of human capital. As it turns out, several of these different specifications deliver similar implications for the broad questions we are interested in. Therefore, we consider one special case, which is analytically very tractable, in most of the paper. In particular, we assume that (i) $\rho = 1$ which implies that $P_{L,t} = \theta_L(t)$ and $P_{H,t} = \theta_H(t)$ (we normalize $Z = 1$ for simplicity), and (ii) SBTC affects the human capital production and the aggregate production technology in the same manner which implies the following accumulation technology for human capital:

$$h_{j,s+1} = h_{j,s} + \widetilde{A}_j((\theta_L(t)l + \theta_H(t)h_{j,s})i_{j,s})^{\alpha}.$$

Our analysis shows that this simplified framework captures many salient features of wage inequality and its response to SBTC. We next rewrite the problem to simplify the exposition. Using equation (2) the opportunity cost of investing an amount $Q_{j,s}$ can be written as:

$$C_j(Q_{j,s}) \equiv (\theta_L(t)l + \theta_H(t)h_{j,s})i_{j,s} = \left(\frac{Q_{j,s}}{\widetilde{A}_j}\right)^{1/\alpha}.$$

Using this transformation, the problem of an individual can be written as

$$\Gamma^j_{s,t}(h_{j,s}) = \max_{Q_{j,s}} \left[\theta_L(t)l + \theta_H(t)h_{j,s} - C_j(Q_{j,s}) + \frac{1}{1+r}\Gamma^j_{s+1,t+1}(h_{j,s+1})\right]$$

subject to

$$h_{j,s+1} = h_{j,s} + Q_{j,s}, \quad \text{with } h_{j,0} = 0.$$

The optimality condition which determines the amount of investment at time t is:

$$C'_j(Q_{j,s}) \quad = \quad \frac{1}{1+r}\left\{\theta_H(t+1) + \frac{\theta_H(t+2)}{1+r} + ... + \frac{\theta_H(t+S-s)}{(1+r)^{S-s}}\right\}. \tag{4}$$

The left hand side of this equation is the marginal cost, and the right hand side is the marginal benefit (MB) of increasing an individual's human capital stock. The latter is the presented discounted value of the future stream of wages that is earned by an additional unit of human capital. An important implication of this optimality condition (4) is that an expected increase in the future price of skill (the sequence $\theta_H(t)$) will immediately affect current investment decision because of the forward looking nature of this equation.[9] Notice also that the current level of human capital $h_{j,s}$ does not appear anywhere in this condition, so the optimal choice of $Q_{j,s}$ is independent of it.[10] However, $Q_{j,s}$ does depend—and in particular, is increasing in—an individual's ability level through the cost function.

Using the functional form for the cost function, the optimal investment choice can be solved for explicitly:

$$Q_{j,s} = \widetilde{A}_j^{1/(1-\alpha)} \left[\alpha MB\right]^{\alpha/(1-\alpha)} .$$

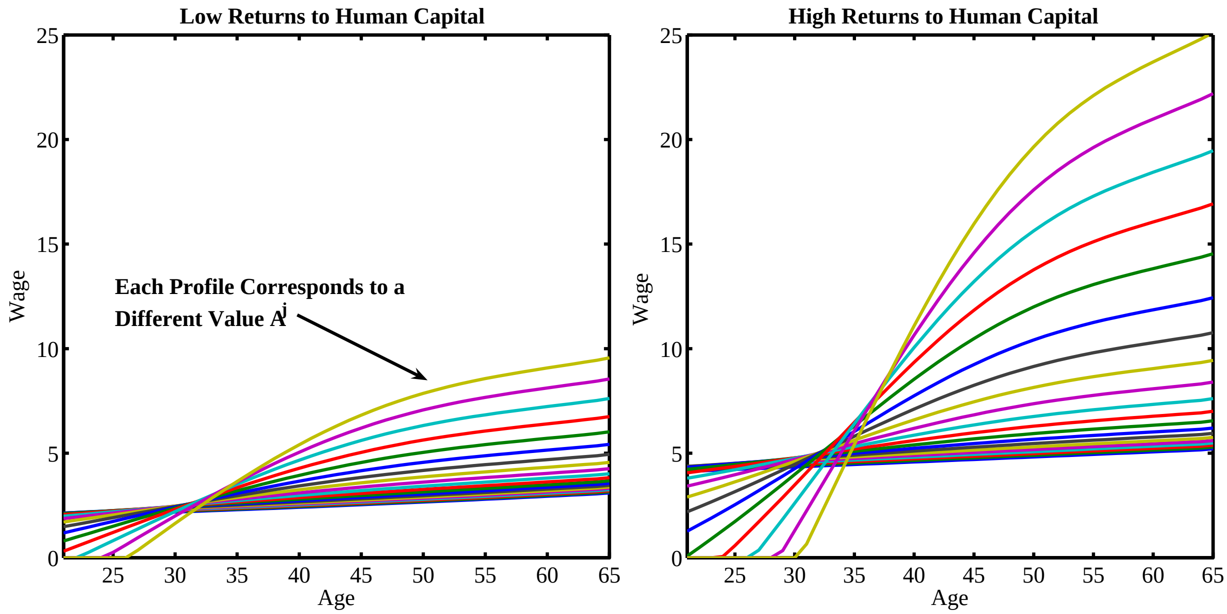
In the rest of the paper we let $Q_{j,s}$ refer to the "optimal" level of investment with a slight abuse of notation. The last expression shows that: (i) individuals with higher learning ability invest more in human capital; and (ii) the response of investment to a change in MB, (either due to an increase in the $\theta_H(t)$ sequence or a fall in interest rates) is increasing in an individual's ability level.

To illustrate how the model works, consider two economies that only differ in the price of human capital, θ_H and θ'_H with $\theta'_H > \theta_H$. Figure 1 compares the wage profiles of individuals with different ability levels in these two worlds. First, note the features common to both cases: workers with high ability invest more than others, accepting lower wages early on in

[9] The optimality condition in the Ben-Porath model would be $C'_j(Q_{j,s}) = (1+r)^{-1}\left\{1+(1+r)^{-1}+...+(1+r)^{-S+s}\right\}$. As should be evident from this condition, the rental price of human capital has no effect in individual's investment choice. The reason is that an increase in price human capital increases both the cost and the future benefit of investment at the same rate. Although it is possible to generate a rise in inequality by increasing the growth rate of $W_{H,t}$, there is no evidence of increased rate of TFP growth after 1970's; in fact there is ample evidence to the contrary. A steady decline in interest rates or increase in learning ability would also generate an increase in inequality. We are not aware of any clear empirical evidence supporting these mechanisms either. Instead, in our specification, SBTC takes the form of an increase in $W_{H,t}$ relative to $W_{L,t}$. Such a change increases the benefit of human capital investment relative to the cost of investment, and therefore increases the incentives to invest in human capital, without necessarily implying anything about TFP growth.

[10] Although, this feature is not crucial for any of our results, it simplifies the analysis siginificantly.

Figure 1: The Effect of the Returns to Human Capital on Life-Cycle Wage Profiles



return for higher wages later in life. As a result, wage inequality increases over the life-cycle due to the systematic fanning out of the wage profiles. Workers with ability level above a certain threshold invest full time early in life (they attend college).

A comparison of these two economies reveals a number of important points that are key to understanding the results in the paper. First, a higher price of human capital induces higher investment (and consequently, a higher college enrollment rate), where the strength of this response increases with ability. As a result, cross-sectional wage inequality increases due to the fanning out of income growth rates. Notice however that cross-sectional wage inequality increases more than lifetime income inequality because the latter is a discounted average over the life-cycle, and early on the change in wage inequality is small. Second, the latter increases even less because those with high wages later on are exactly those who invest more and thus have low wages early in the life-cycle.

3 Theoretical Analysis

In this section we consider a simplified demographic structure that allows us to prove some of our main results theoretically. In particular we specialize to the "perpetual youth" version of the overlapping generations model as in Blanchard (1985): individuals can potentially live forever ($S = \infty$) but face a constant probability of death $(1 - \delta)$ every period. Under this assumption, s is no longer a state variable in individuals' problem, simplifying the analysis substantially. We normalize the population size to one, and assume that each period a cohort of measure $(1 - \delta)$ is born to replace the individuals who die. Therefore, the measure of an s-year-old cohort is given by $\mu(s) = (1 - \delta)\delta^{s-1}$. In the rest of the analysis, we restrict our attention to an interior solution, hence we assume that $w_{j,s} \geq 0$ for all j. This provides us analytical tractability.

3.1 Characterizing the Steady State Before SBTC

To examine the effect of SBTC, we assume that the economy is in steady state in the period preceding the shock, and characterize how investment, wages and consumption are determined. In this initial steady state, let $\theta_H(t) = \theta_H$ and $\theta_L(t) = \theta_L$ for all t.

The assumption of constant survival probability simplifies the structure of the model in many ways. First, the optimality condition for investment choice (4) reduces to:

$$C_j'(Q_j) = \frac{\theta_H \beta\delta}{1 - \beta\delta},$$

where the marginal benefit of investment is a constant since the expected life span is now independent of age. Using the functional form for the cost function we get[11]

$$Q_j = A_j \left(\frac{\alpha\delta\beta}{1 - \beta\delta} \theta_H \right)^{\alpha/(1-\alpha)}, \tag{5}$$

where $A_j \equiv \widetilde{A}_j^{1/1-\alpha}$. The fact that Q_j is independent of age implies that the human capital

[11] In all the analysis here we focus on interior solutions.

stock at age s is simply $h_{j,s} = Q_j(s-1)$. Furthermore, this optimal investment choice satisfies the following equalities, which will be useful in our derivations below:

$$C_j(Q_j) = \alpha C_j'(Q_j)Q_j = \left(\frac{\alpha\delta\beta}{1-\delta\beta}\theta_H\right)Q_j.$$

This expression makes clear that the cost of investment evaluated at the optimal investment level depends on j only through Q_j, implying that the subscript j can be dropped from the cost function: $C_j(Q_j) = C(Q_j)$. The optimal amount of time investment $i_{j,s}$ is given by the total cost of investment divided by potential earnings:

$$i_{j,s} = \frac{C(Q_j)}{\theta_L l + \theta_H h_{j,s}} = \frac{\frac{\alpha\delta\beta}{1-\delta\beta}\theta_H Q_j}{\theta_L l + \theta_H Q_j(s-1)}. \tag{6}$$

A few intuitive results can be seen from these expressions. First, equation (5) implies that individuals with higher ability make larger investments: $dQ_j/dA_j > 0$. Second, even though individuals increase their human capital stock by a constant amount Q_j every period, investment time falls with age: $di_{j,s}/ds < 0$. Third, equations (5) and (6) can be combined to show that $di_{j,s}/dA_j > 0$: conditional on age, individuals with higher ability also devote a larger fraction of their time investing in human capital. Finally, the increase in investment time in response to SBTC is larger for individuals with higher ability, i.e. $d^2 i_{j,s}/d\theta_H dA_j > 0$. These results play a central role for the results that we prove below.

The wage of individual j at age s is given by:

$$w_{j,s} = \theta_L l + \underbrace{\theta_H}_{\text{Price effect}} \times \underbrace{Q_j(s-1)}_{\text{Quantity effect}} - \underbrace{C(Q_j)}_{\text{Investment effect}}. \tag{7}$$

It is useful to discuss how a change in the price of human capital (SBTC) would affect this wage rate. As can be seen from this expression, an increase in θ_H affects the average wage via three channels. First, for a given stock of human capital $h_{j,s}$ $(= Q_j(s-1))$, an increase in θ_H increases the average wage ("price effect"). Second, a higher θ_H induces more investment which dampens the average wage by increasing the foregone earnings due to investment ("investment effect"). Third, higher investment increases the stock of human capital which in turn increases the average wage ("quantity effect"). Notice that while the price effect is proportional to the stock of human capital, the investment effect is independent of existing human capital. Consequently, for a given ability level, the investment effect is the same for all individuals regardless of age, whereas the price effect increases with age.

Next we derive an expression for the average wage rate in the economy. In order to express the average wage in an easily interpretable form, it is convenient to introduce some new variables. Define the "average investment" in the economy:

$$\overline{Q} \equiv \int_{j,s} Q_j \mu(s)\, dj ds = \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} E_j(A_j), \tag{8}$$

the corresponding "average cost of investment":

$$C(\overline{Q}) \equiv \int_{j,s} C(Q_j)\, \mu(s)\, dj ds = \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H \overline{Q},$$

the "average human capital stock":

$$H\left(\overline{Q}\right) \equiv \int_{j,s} h_{j,s}\mu(s)\, dj ds = \int_j Q_j dj \times \int_s (s-1)\mu(s)\, ds = \frac{\delta}{1-\delta}\overline{Q}, \tag{9}$$

and, finally, the "average raw labor endowment" in the economy:

$$L \equiv \int_{j,s} l\mu(s)\, dj ds = l. \tag{10}$$

Notice that $H\left(\overline{Q}\right)$ and L measure the aggregate human capital stock and raw labor, *inclusive* of on-the-job investment activities, which should not be confused with H^{net} and L^{net} defined earlier.

At a given point in time, $\overline{Q}$ and $C(\overline{Q})$ only depend on the current value of θ_H, whereas the *stock* of human capital also depends on past levels of investment, which in turn is determined by the history of θ_H's. Thus, the definition of $H\left(\overline{Q}\right)$ in equation (9) is only valid in steady state when all past returns to human capital equal θ_H. Moreover, $\overline{Q}$ and $C(\overline{Q})$ will adjust immediately in response to a change in θ_H such as SBTC (making them "jump variables"), whereas the total human capital stock will adjust only gradually (making it a "stock variable"). This distinction will play a crucial role in the analysis below. Now, using the definition of an individual's wage in equation (7), the *average wage rate* in the economy can be calculated as

$$\overline{w} = \int_{j,s} w_{j,s}\mu(s)\,djds = \left[\theta_L l + \theta_H H\left(\overline{Q}\right)\right] - C(\overline{Q}).$$

By substituting the expressions for $H\left(\overline{Q}\right)$ and $C(\overline{Q})$ we obtain:

$$\overline{w} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}. \tag{11}$$

Optimal consumption.—We assume that individuals can borrow and save at a constant exogenous interest rate $r = 1/(\delta\beta) - 1$, which implies that the optimal consumption path is constant and equals $(1-\beta\delta)\Gamma_j$. Then the *average consumption* in the economy is

$$\overline{c} = \left[\theta_L l + \frac{\delta\beta}{1-\delta\beta}\theta_H\overline{Q}\right] - C(\overline{Q}) = \theta_L l + \left((1-\alpha)\frac{\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}. \tag{12}$$

Comparing the last two formulas, it is easy to see that average consumption is less than average wage ($\overline{c} < \overline{w}$) whenever $\beta < 1$. The reason can be explained as follows. Given that the interest rate equals the reciprocal of the effective discount rate $(\delta\beta)$, individuals would like to maintain a constant consumption over their lifetime. But because all individuals have upward sloping wage profiles, they need to borrow against their future income to maintain a constant consumption. As long as the interest rate is positive they pay interest on the amount they borrow (from the rest of the world). Hence, average consumption is less than the average wage.

3.2 Characterizing the Behavior after SBTC

In this section, we consider a one-time increase in the degree of skill bias at time t^*, which is modeled as a one-time increase in the price of human capital from θ_H to $\boldsymbol{\theta}'_H$ while the price of raw labor, θ_L, remains constant.[12] We analyze the behavior of average wages (and labor productivity) and college premium both in the short-run and in the long-run. For the short-run analysis, we focus our attention to the period immediately after the shock occurs. Analyzing the economy in this time period captures the fact that, in the short-run, the human capital stock does not fully adjust yet, but investment jumps to its new level immediately. Later in the quantitative analysis, we model SBTC as a gradual change in the price of human capital that takes place over several years; we find that the main conclusions drawn here remain valid.

3.2.1 Slowdown in Labor Productivity (and Stagnation of Average Wages)

The average wage in the initial steady state is given by

$$\overline{w}_I = \theta_L l + \underbrace{\theta_H}_{\text{(Jump variable)}} \times \underbrace{H\left(\overline{Q}\right)}_{\text{(Stock variable)}} - \underbrace{C(\overline{Q})}_{\text{(Jump variable)}} .$$

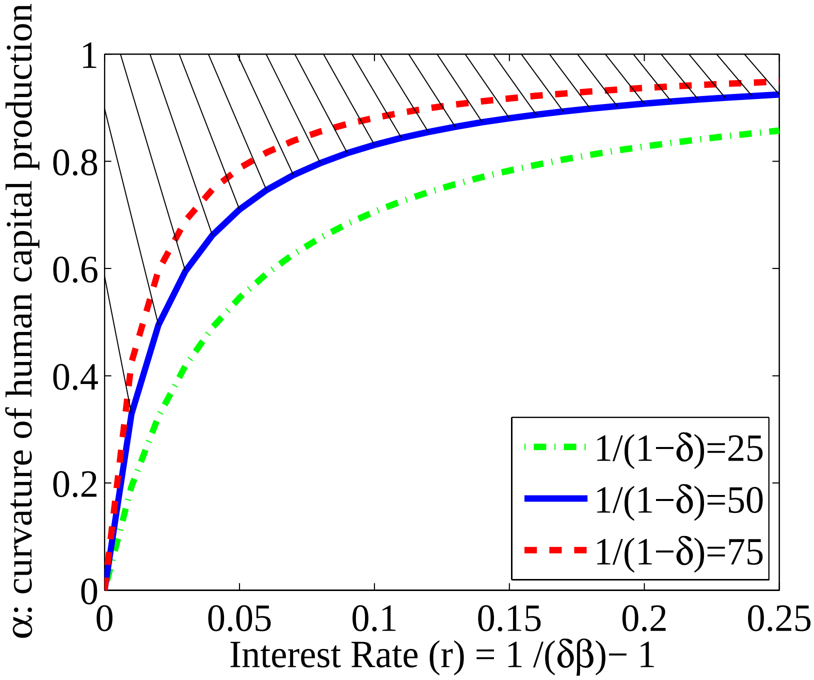
As noted before, a key point to observe is that θ_H and $C(\overline{Q})$ are "jump variables," that is, they increase immediately after SBTC, whereas $H\left(\overline{Q}\right)$ is a "stock variable" and hence adjusts only gradually. Therefore, the average wage immediately after SBTC (in the short-run) is given by

$$\overline{w}_{SR} = \overline{w}\big|_{t=t^*+\varepsilon} = \left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{Q}\right)\right] - C(\overline{\mathbf{Q}}').$$

where bold letters denote the values of variables after the shock. As is evident from this expression, the price and investment effects are key for the short-run behavior of average wages. During the transition to the new steady state after SBTC, the stock of human capital increases from its initial steady state value, $H\left(\overline{Q}\right)$, to its new steady state value $H\left(\overline{\mathbf{Q}}'\right)$. Hence, the average wage in the new steady state is given by

[12] An alternative and common way of modeling SBTC is to assume that the rise in θ_H is accompanied by a fall in θ_L. Under this assumption, the average wage would increase in the long-run as long as the stock of human capital is large relative to the stock of raw labor. Almost all our other results carry on to this case, and some of them become stronger. For example, the decline in average wage in the short-run would be larger in this case. We do not want this result to be driven by a decline in θ_L, hence we assume that θ_L is fixed and θ_H increases.

Figure 2: The Parameter Combinations (α, r) in the Shaded Region Imply Condition 1.



$$\overline{w}_{LR} = \overline{w}|_{t\to\infty} = \left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{\mathbf{Q}}'\right)\right] - C(\overline{\mathbf{Q}}').$$

We now characterize the behavior of the average wage in the short-run and in the long-run. For this purpose, we have the following condition.

Condition 1 $\frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} > \frac{\delta}{1-\delta}$.

Figure 2 illustrates the range of (α, r) combinations that satisfy condition 1. Basically these are the parameter values such that the investment effect dominates the price effect in the short-run. This condition is key in the behavior of the average wage and college premium in the short-run. It would be satisfied if either the stock of human capital is low so that an increase in its price does not have a significant effect on wages, or the response of investment is high. The response of investment is, in turn, determined by three variables. First, the immediate response of investment to SBTC is larger when α is high. This is because, as noted earlier, a higher α implies less diminishing marginal returns in human capital production. Consequently, there is little benefit from spreading out investment over time (as would be the case if α were low.) Second, for a given α and a survival probability δ, a higher β makes the present discounted value of future wages larger, implying a higher benefit from a given increase in the price of human capital. Thus, the response of investment to SBTC increases with β (and the corresponding low interest rate). Third, the stock of human capital is increasing in the survival probability, so the price effect is more likely to dominate the investment effect when δ is large. The combination of these three effects gives rise to the the region of admissible parameters shown in Figure 2. This region contains a wide range of plausible parameter combinations. For example, assuming an expected working life of 50 years and interest rate of 5 percent, any curvature value above 0.70 satisfies condition 1. Most estimates of this parameter in the literature are around 0.8 and higher (see for example, Heckman (1976), and more recently, Heckman, Lochner and Taber (1998)). The following proposition characterizes the behavior of average wages.

Proposition 1 (Productivity Slowdown) *In response to SBTC, labor productivity (alternatively, the average wage)*

1. *increases in the long-run, i.e.* $\overline{w}_{LR} > \overline{w}_I$.

2. *falls in the short-run, i.e.* $\overline{w}_{SR} < \overline{w}_I$, *if condition 1 holds.*

Proof. *See the appendix for all the proofs and omitted derivations.* ■

It should be emphasized that for a *marginal* increase in θ_H, condition 1 is not only sufficient but also necessary for labor productivity to decline in the short-run. However, if the increase in price of human capital is larger, labor productivity would decline under a weaker condition, making condition 1 sufficient but not necessary in general. Moreover, after SBTC the stock of human capital increases monotonically over time from its initial steady state level $H\left(\overline{Q}\right)$ to its new steady state level $H\left(\overline{\mathbf{Q}}'\right)$. Hence, the average wage increases monotonically over time after the initial decline.

Corollary 1 *After the initial decline, the average wage increases monotonically to its new steady state level.*

The following corollary states that the initial fall in average wages is due to the response of investment to SBTC.

Corollary 2 *If individuals' investment behavior did not respond to SBTC (i.e.,* $Q_{j,s} = \mathbf{Q}'_{j,s}$ *for all* j, s*) the average wage would monotonically increase from its initial steady state value to final steady state value.*

3.2.2 College Premium

In this section, we characterize the behavior of the college premium. We show that under condition 1, an increase in the price of human capital leads to a fall in the college premium in the short-run while increasing the premium in the long-run.

Consistent with the standard interpretation of the Ben-Porath model, the perspective adopted in this paper is that educational labels merely represent some threshold level for the human capital investment completed. Thus, a "college graduate" is defined as an individual who has invested above a certain threshold in a specified number of periods.[13] Since there is a one-to-one relationship between investment time and ability at every age, there is a corresponding threshold ability level, $\overline{A}^*$, above which all individuals become college graduates. To simplify the analysis, we abstract from changes in $\overline{A}^*$ in response to SBTC. In other words, we ignore the effect of compositional changes on the college premium in this theoretical analysis. Later in the quantitative section, we allow for compositional changes and show that the mechanism highlighted by this simple model is still valid in that more general setting.

Let $\overline{Q}_c$ and $E_c[A]$ denote the average investment and average ability of college graduates, respectively. We define $\overline{Q}_{nc}$, and $E_{nc}[A]$ in an analogous fashion for individuals without a college degree ("nc" stands for non-college). From the discussion above, it is clear that $E_c[A] > E_{nc}[A]$, which also implies $\overline{Q_c} > \overline{Q}_{nc}$ from equation (8). Finally, let $\overline{w}_c$ ($\overline{w}_{nc}$) be the

[13] More formally, the condition can be stated as $\sum_{s=1}^{\widetilde{s}} 1\{i_{j,s} > i^*\} \geq S_c$, where i^* is the investment time threshold, $\widetilde{s}$ is the individual's current age and S_c is the number of years of schooling required to qualify as a college graduate.

average wage of college (high school) graduates. Then, the college premium before SBTC (which happens at time t^*) is:

$$\omega_I^* = \left.\frac{\overline{w}_c}{\overline{w}_{nc}}\right|_{t<t^*} = \frac{\theta_L l + \theta_H H\left(\overline{Q}_c\right) - C(\overline{Q}_c)}{\theta_L l + \theta_H H\left(\overline{Q}_{nc}\right) - C(\overline{Q}_{nc})}.$$

The college premium in the *short-run* (that is, immediately after SBTC, at time $t^* + \varepsilon$) is given by

$$\omega_{SR}^* = \left.\frac{\overline{w}_c}{\overline{w}_{nc}}\right|_{t=t^*+\varepsilon} = \frac{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{Q}_c\right)\right] - C(\overline{\mathbf{Q}}'_c)}{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{Q}_{nc}\right)\right] - C(\overline{\mathbf{Q}}'_{nc})}.$$

In the long-run, the premium is given by

$$\omega_{LR}^* = \left.\frac{\overline{w}_c}{\overline{w}_{nc}}\right|_{t\to\infty} = \frac{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{\mathbf{Q}}'_c\right)\right] - C(\overline{\mathbf{Q}}'_c)}{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{\mathbf{Q}}'_{nc}\right)\right] - C(\overline{\mathbf{Q}}'_{nc})}.$$

The following proposition characterizes the behavior of college premium.

Proposition 2 (Behavior of College Premium) *In response to SBTC the college premium:*

(i) rises in the long-run, i.e., $\omega_{LR}^* > \omega_I^*$,

(ii) falls in the short-run, i.e. $\omega_{SR}^* < \omega_I^*$, *if condition 1 holds.*

Despite the similarities between the statements of propositions 1 and 2, there is an important difference between the two. While the fall in average wages only requires the endogenous response of human capital investment to SBTC (i.e., that $C(Q)$ increases after the shock), the fall in the college premium requires, in addition, that this response be different across education groups. In other words, if heterogeneity in ability was eliminated from the previous model, average wages would still stagnate after SBTC, but the college premium would not fall in the short-run.

Since college graduates accumulate skills faster than high school graduates, the college premium increases monotonically towards the new steady state value after the initial fall.

Moreover, it can be easily shown that the response of the college premium is proportional to the ability differential between college and high school graduates.

Corollary 3 *In response to SBTC, the decline (increase) in the college premium in the short-run (long-run) is larger, when the ability differential between college graduates and high school graduates,* $E_c[A]/E_{nc}[A]$, *is larger.*

To understand the behavior of the college premium further, an intuitive discussion is helpful. To this end, assume that there are no differences in ability within each education group, and the investment levels are denoted by Q_c and Q_{nc} for college and non-college groups respectively. However, investment time (i) will be different within each education group due to differences in age and hence in potential earnings. Using the expression for investment time in (6), we can re-write the college premium as

$$\omega^* = \frac{\theta_L l + \theta_H H(Q_c)}{\theta_L l + \theta_H H(Q_{nc})} \times \frac{1-\overline{i_c}}{1-\overline{i_{nc}}} = \underbrace{\frac{l + (\theta_H/\theta_L) H(Q_c)}{l + (\theta_H/\theta_L) H(Q_{nc})}}_{G_1} \times \underbrace{\frac{1-\overline{i_c}}{1-\overline{i_{nc}}}}_{G_2}, \tag{13}$$

where all the variables that appear in this expression are defined as before, but the averages are now taken with respect to the group indicated by the subscript.[14]

The first term in the decomposition, G_1, captures the price and quantity effects of changes in θ_H. Both of these effects are larger for college graduates because they have a larger human capital stock, and moreover, their human capital stock increases more after SBTC (though the latter happens only gradually). The key point to note that there is no reason for G_1 to behave in any way other than increase monotonically after SBTC. If there was no investment response in the model, G_2 would be constant over time and the college premium would be equal to G_1 and would also increase monotonically. The *differential* investment response captured by G_2 is thus crucial for the initial decline in the college premium. There are two reasons for the initial decline in G_2. First, after SBTC college graduates increase their

[14] $\overline{i}_c$ is the average investment time of college graduates which is calculated as the ratio of each individual's potential earnings $(\theta_L l + \theta_H Q_c(s-1))$ to average potential earnings of that group $(\theta_L l + \theta_H \frac{\delta}{1-\delta}\overline{Q}_c)$ as weights. $\overline{i}_c$ is defined analoguously.

investment time more than high school graduates. In the long-run, this follows from the fact that $d^2 i_{j,s}/d\theta_H dA_j > 0$ shown above. The same can be shown to be true in the short-run.[15] A second and reinforcing effect responsible for the fall in G_2 is that the initial level of investment time is larger for college graduates. As a result, even the same amount of increase in investment time would cause a decline in $\left(1-\bar{i}_c\right)/\left(1-\bar{i}_{nc}\right)$. Overall then, the college premium falls initially because G_2 (which depends on the jump variables, $\bar{i}_c$ and $\bar{i}_{nc}$) falls quickly, but then recovers as G_1 gradually increases over time.

3.2.3 College Premium within Age Groups

Several authors have documented that the decline in the college premium until 1980 was mainly driven by the decline in the college premium among younger individuals (see Katz and Murphy (1992), Murphy and Welch (1992) and Author, Katz, and Kearney (2005) for the US data, and Card and Lemieux (2001) for additional evidence from UK and Canada). To examine this issue, we now look at the college premium among individuals in the same age category. The college premium before SBTC among individuals with s years of age as

$$\omega_I^*(s) = \frac{\theta_L l + \theta_H \overline{Q}_c(s-1) - C(\overline{Q}_c)}{\theta_L l + \theta_H \overline{Q}_{nc}(s-1) - C(\overline{Q}_{nc})}.$$

The college premium in the *short-run* is

$$\omega_{SR}^*(s) = \frac{\theta_L l + \boldsymbol{\theta}'_H \overline{Q}_c(s-1) - C(\overline{\mathbf{Q}}'_c)}{\theta_L l + \boldsymbol{\theta}'_H \overline{Q}_{nc}(s-1) - C(\overline{\mathbf{Q}}'_{nc})}.$$

The college premium in the long-run is given by

$$\omega_{LR}^*(s) = \frac{\theta_L l + \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_c(s-1) - C(\overline{\mathbf{Q}}'_c)}{\theta_L l + \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_{nc}(s-1) - C(\overline{\mathbf{Q}}'_{nc})}.$$

The following proposition characterizes the behavior of the college premium among indi-

[15] More formally, we evaluate how the increase in investment time changes with A: we calculate $d^2 i/dAd\boldsymbol{\theta}'_H$ which equals a positive constant times $\theta_L l + (2\alpha - 1)\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H \overline{Q}$. It is clear that $\alpha > 0.5$ is a sufficient condition for $d^2 i/dAd\boldsymbol{\theta}'_H > 0$ or when $\alpha < 0.5$, $d^2 i/dAd\boldsymbol{\theta}'_H > 0$ if $\theta_L l/(\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H \overline{Q})$ is large enough.

viduals within the same age groups.

Proposition 3 (Behavior of College Premium Within Age Groups) *Define* $\underline{s} = 1 + \frac{\alpha\delta\beta}{1-\delta\beta}$ *and* $\overline{s} = 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}$*. Then in response to SBTC, the college premium among s-year-old individuals*

(i) decreases in the short-run, $\omega^*_{SR}(s) < \omega^*_I(s)$*, if and only if* $s < \overline{s}$*,*[16]

(ii) increases in the long-run, $\omega^*_{LR}(s) > \omega^*_I(s)$*, if and only if* $s > \underline{s}$*.*

An important difference of this proposition from the previous one is that here the fall in the college premium for young individuals does not require condition 1, and therefore, holds under more general conditions than proposition 2. Moreover, given that the college premium falls in the short-run—unconditionally—for young individuals, it is easy to conjecture that whether this also holds for the *average* college premium depends on whether there are sufficiently many young individuals in the population. In fact, this is exactly what condition 1 ensures: the condition that the average age in the population, $s = 1/(1-\delta)$, be less than $\overline{s}$ is exactly the same as condition 1.

We summarize proposition 3 in Table 1. The intuition for the results can be understood by considering the relative sizes of price, quantity, and investment effects on wages of college and high school graduates. First, notice that all of these effects are larger for higher ability individuals, which implies that the price and quantity effect on college premium is positive while the investment effect is negative. However, within the same ability group, the price and quantity effects are larger for older individuals while the investment effect is the same regardless of age. Hence, among very young individuals ($s < \underline{s}$), the investment effect dominates the price and quantity effects. Consequently, the college premium declines both in the short-run and in the long-run. For relatively older agents ($\underline{s} < s < \overline{s}$), the investment effect dominates the price effect but is dominated by the price and quantity effect together.

[16] *The statement "if and only if" applies if we increase* $\boldsymbol{\theta}'_H$ *marginally above* θ_H*. For larger increases in* $\boldsymbol{\theta}'_H$*, the condition* $s < \overline{s}$ *is a sufficient condition to have decline in the college premium in the short-run, not a necessary condition.*

Table 1: Evolution of College Premium Within Age Groups

If s satisfies:	$s \leq \underline{s}$	$\underline{s} < s < \overline{s}$	$\overline{s} \leq s$
	College premium within s-year old individuals:		
Short-run:	*Declines*	*Declines*	Increases
Long-run:	*Declines*	Increases	Increases

Notes: See proposition 3 for the definitions of $\underline{s}$ and $\overline{s}$.

Hence, the ability premium declines in the short-run but increases in the long-run. Among agents who are older than $\overline{s}$, the price effect is large enough to dominate the investment effect. Hence, the ability premium increases both in the short-run and in the long-run.

It should be pointed out that among the very young ($s \leq \underline{s}$), individuals who have higher ability earn less than those who have lower ability. This is consistent with the evidence from panel data sets such as PSID, where individual income at the labor market entry and income growth rates over the life-cycle are negatively correlated (c.f., Lillard and Weiss (1979), Baker (1997), and Guvenen (2005) among others).

3.2.4 College Enrollment

In this section, we show that educational attainment rises in response to SBTC. In the previous section we identified individuals above a certain ability threshold, $\overline{A}^*$, as college graduates. To study college enrollment we proceed along similar lines. Basically, an individual is considered to be currently *enrolled in college* if his investment time exceeds i^* in the current period. As before, there is a corresponding threshold ability level at each age above which all individuals enroll in college. In the initial *steady state*, this age-dependent threshold level, which we denote $A_I^*(s)$, can be solved (using equation 6) as:

$$A_I^*(s) = \frac{\eta_0}{\theta_H^{1/(1-\alpha)}} \left[\frac{\theta_L l}{\eta_1 / i^* - \eta_2 (s-1)} \right]$$

where η_0, η_1 and η_2 are positive constants. As could be expected, the threshold is increasing with s implying that college enrollment falls with age. Replacing θ_H in this expression with $\boldsymbol{\theta}'_H$ yields the threshold in the long-run after SBTC, which is denoted by $A^*_{LR}(s)$. The expression shows that steady state college enrollment at a given age increases with the price of human capital. Similarly, the threshold ability level in the short-run after SBTC can be shown to be:

$$A^*_{SR}(s) = \frac{\eta_0}{\boldsymbol{\theta}'^{1/(1-\alpha)}_H} \left[\frac{\theta_L l}{\eta_1 / i^* - \eta_2 (s-1) \left(\theta_H / \boldsymbol{\theta}'_H \right)^{1/(1-\alpha)}} \right],$$

again, using equation 6. It is easily noted that this threshold is lower than both $A^*_I(s)$ and $A^*_{LR}(s)$. This result follows from the fact that the opportunity cost of investing, which is determined by current potential earnings, does not change immediately after SBTC while the potential benefits (determined by $\boldsymbol{\theta}'_H$) increases. As a result, more individuals enroll in college in the short-run. Over time, as the price of human capital rises, investment becomes more costly and college enrollment falls to its new steady state level which is still higher than the initial level. The following proposition summarizes these results.

Proposition 4 (College Enrollment) $A^*_{SR}(s) < A^*_{LR}(s) < A^*_I(s)$ *for all* s*. Therefore, in response to SBTC the fraction of population enrolled in college increases in the long-run, but increases even more in the short-run.*

3.2.5 Within-group Inequality

A well-known empirical fact, first documented by Juhn, Murphy and Pierce (1993), is that the wage growth in a given percentile of the wage distribution during SBTC has been monotonically related to the ranking of that percentile before SBTC. In particular, wages in the higher percentiles in 1963 also experienced high growth from 1963 to 1989, while the opposite happened at lower percentiles. This resulted in an increase in the wage inequality that was spread to all parts of the wage distribution. The following proposition states that the same outcome happens in the present model.

Proposition 5 (Within-group Inequality) *Let* $w_I(\Omega)$ *be the average wage at the* Ω^{th} *percentile of the wage distribution before SBTC and* $w_{LR}(\Omega)$ *be the average wage at the* Ω^{th} *percentile of the wage distribution in the new steady state after SBTC. Then,* $w_{LR}(\Omega)/w_I(\Omega)$ *is increasing in* Ω*.*

Juhn et al (1993, figure 5) also show that the same fanning out of the wage distribution is obtained when one conditions on a given age group. This is also true in the present model. The intuition is simple, and could already be anticipated from figure 1, which plots the wage profiles in two environments that only differ in the returns to human capital. As can be seen there, the increase in the returns to human capital results in a fanning out of the wage distribution at every age (above a threshold) without a change in the relative ranking of individuals. Thus, individuals who earn high wages before SBTC, also experience a larger increase in their wages after SBTC. The next corollary states this.

Corollary 4 *Let $w_I(\Omega|s)$ and $w_{LR}(\Omega|s)$ be the average wage at the Ω^{th} percentile of the wage distribution conditional on age before SBTC and in the new steady state after SBTC respectively. Then, $w_{LR}(\Omega|s)/w_I(\Omega|s)$ is increasing in Ω when $s > \underline{s}$.*

3.2.6 Wage Inequality and Consumption Inequality

In this section we analyze the effect of SBTC on wage and consumption inequality. We first focus on steady state comparisons and then discuss short-run behavior. An attractive feature of the present framework is that it allows us to obtain explicit formulas for wage and consumption inequality. Therefore, in addition to determining what happens to inequality after SBTC qualitatively, these formulas also allow us to make quantitative statements about how much wage and consumption inequality will change.[17] First, using the expressions for wage and consumption, the variance of wage and consumption can derived as[18]:

$$Var(w) = \left\{n_1 Var(A) + n_2 E\left[A\right]^2\right\} \theta_h^{2/(1-\alpha)}, \tag{14}$$

and

$$Var(c) = n_3 Var(A) \theta_h^{2/(1-\alpha)}. \tag{15}$$

[17] The appendix contains the derivations.

[18] $n_1 \equiv \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}\right)\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0$, $n_2 \equiv \frac{\delta}{(1-\delta)^2}\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0$ and $n_3 \equiv \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta}\right]^2\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0$.

Unless otherwise stated, the coefficients n_i's are all positive in the rest of the text. The expressions above reveal that the variance of wages is driven by two sources: heterogeneity in learning ability (which is captured by $Var(A)$) and the difference in wages due to life-cycle effects, which would arise even without heterogeneity in learning ability. The latter one is captured by the square of average investment $E[A]^2$. If average investment, hence average wage growth, is higher holding the variance of ability constant, then the variance of wages would be larger. This effect would not be present in the variance of consumption because individuals within the same ability group would consume the same amount regardless of their age since they have the same lifetime income. Therefore, the variance of consumption is mainly driven by heterogeneity in learning ability which is the source of permanent differences.[19]

An increase in θ_H increases wage inequality because it increases the wage dispersion across individuals with different ability and makes wage profiles steeper which increases wage inequality within the same ability group. On the other hand, the variance of consumption increases only because differences in lifetime income increases. Hence, the variance of wages increases more than variance of consumption.

Proposition 6 *In response to SBTC, the cross-sectional variances of wages and consumption increase:* $Var_{LR}(w) > Var_I(w)$, *and* $Var_{LR}(c) > Var_I(c)$. *Moreover, the* ***difference*** *between the variances of wages and consumption also increases:* $(Var_{LR}(w) - Var_{LR}(c)) > (Var_I(w) - Var_I(c))$.

To eliminate the effect of levels on measures of inequality, we also look at the square of coefficient of variation. Let $CV(w)$ denote the coefficient of variation of the wage distribution.

[19] Consumption is a discounted average of wages over the lifecycle. This is reflected in the fact that $E[A]$ does not appear in consumption inequality. Moreover, individuals who have higher wages later in life pay for it by accepting lower wages early on. This is why $n_3 < n_1$.

Then it is given by[20]:

$$CV(w) \equiv \frac{Std(w)}{\overline{w}} = \frac{\left[n_1 Var(A) + n_2 E\left[A\right]^2\right]^{1/2}}{\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E\left[A\right]}.$$

This expression shows that $CV(w)$ is larger if $Var(A)$ is larger. This is because the variance of wages increases with $Var(A)$ but the average wage is unaffected. On the other hand, the effect of average ability on wage inequality is ambiguous. Higher average ability makes wage profiles steeper increasing the variance of wages. But at the same time, it increases the average wage. The former effect would dominate and hence wage inequality would increase if $Var(A)$ is small relative to $E\left[A\right]$.

The preceding formula measures wage inequality in steady state. As shown in the appendix, it is possible to proceed along similar lines to derive an expression for the coefficient of variation in the short-run (immediately after SBTC) and prove the following result.

Proposition 7 (Rise in Wage Inequality) *In response to SBTC, for all* $\boldsymbol{\theta}'_H > \theta_H$, *wage inequality (as measured by the square of the coefficient of variation):*

1. *increases in the long-run, i.e.,* $CV_{LR}(w) > CV_I(w)$,

2. *increases in the short-run, i.e.,* $CV_{SR}(w) > CV_I(w)$, *if* $\beta = 1$.

The expression for the coefficient of variation above shows that it increases with the heterogeneity in learning ability. Although wage inequality would increase even in the absence of ability heterogeneity (due to the second term in the curly bracket), this case has counterfactual implications. First, since all individuals within the same age group would have the same wage, one would have to assume an unrealistic rate of wage growth over the life-cycle to match the level of wage inequality. Second, our quantitative experiments suggest that the behavior of wage inequality is completely at odds with the data if we abstract from heterogeneity in learning ability.

[20] $n_4 \equiv \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} > 0$

The condition $\beta = 1$ is sufficient for an increase in wage inequality in the short-run, but it is far from being necessary. When $\beta < 1$, there is still a wide range of parameters resulting in an increase in wage inequality in the short-run, though we have not been able to find a simple sufficient condition in that case.

When the discount rate is zero, it is possible to show that SBTC increases $CV(w)^2$ more than $CV(c)^2$ regardless of other parameter values. The next proposition states that.

Proposition 8 (Rise in Wage and Consumption Inequality) *Assume that* $\beta = 1$. *In response to SBTC, wage inequality rises more than consumption inequality in the long-run for all* $\boldsymbol{\theta}'_H > \theta_H$, *i.e.,* $\left(CV_{LR}(w)^2 - CV_{LR}(c)^2\right) > \left(CV_I(w)^2 - CV_I(c)^2\right)$.

The proof of this proposition is straightforward. If $\beta = 1$ then $\overline{c} = \overline{w}$. Then using the expression for average wage in (11), and for the variances of wages and consumption in (14) and (15), we have:

$$CV(w)^2 - CV(c)^2 = \frac{n_2\left(Var(A) + E\left[A\right]^2\right)}{\left\{\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E\left[A\right]\right\}^2}.$$

It is easy to see that this expression increases with θ_H. Moreover, it increases more with an increase in θ_H if heterogeneity in learning ability is larger. Notice that if we were to think of SBTC as involving a simultaneous fall in θ_L then the difference between wage and consumption inequality would increase even further after SBTC. Although we have not been able to extend this result to the more general case with $\beta < 1$, in the quantitative analysis we have always found wage inequality to increase (substantially) more than consumption inequality for a wide range or parameter values. We discuss this issue further in the next section.

4 Quantitative Analysis

In this section we calibrate our baseline model to the U.S. data under the assumption that the U.S. economy was in steady state before skill-biased technical change took effect in 1970. We then examine the behavior of several variables from 1970 to 2000.

To carry out this exercise we relax some of the assumptions made for analytical convenience in the previous section as described below. However, in the baseline version of the quantitative model, we maintain the perfect substitutability between human capital and raw labor, $\rho = 1$, because this eliminates the feedback from the supply of skills to the college premium, which has been emphasized as an important channel in the previous literature (c.f., Katz and Murphy (1992)). Later in Section 5 we analyze the case with $\rho < 1$ as an extension.

4.1 Calibration

Aggregate Production Function.—The growth rate of neutral technology level, Z, is set equal to 1.5 percent per year. As noted before and will become clear below, *measured* TFP growth will be different than this number when the amount of investment on-the-job changes over time.

In the baseline calibration, we take the curvature of the aggregate production technology to be unity, implying that labor and human capital are perfect substitutes. As noted in the theoretical analysis, this provides a stark benchmark where workers with different education levels (in fact, all workers) become perfectly substitutable, which has been suggested to be incompatible with facts on wage inequality. In the robustness analysis later below we relax this assumption and consider the imperfectly substitutable case. Finally, since θ_L and θ_H always appear multiplicatively with labor and human capital, the initial values of these parameters serve only as a normalization (given that H and L are going to be calibrated below). So we normalize $\theta_{L,t} = \theta_{H,t} = 0.5$ for all $t < 1970$, and denote $\theta_{H,t} = \theta_t$ and $\theta_{L,t} = 1 - \theta_t$ in the rest of the paper.

Human Capital Accumulation.—The model introduced in Section 3 inherits one feature of the Ben-Porath framework which—although analytically very convenient—is unrealistic and makes a direct calibration to data difficult. This is basically the assumption that individuals can invest any fraction of their time while working on the job. With a continuum of ability levels, there will be some individuals who invest slightly less than 100 percent of their time, appearing as employed while earning a wage income very close to zero. Because many of the statistics we analyze below involve the logarithm of wage rates as well as the variances of these logarithms, even a small number of such individuals can easily wreak havoc with the quantitative exercise.

To circumvent this difficulty, we modify the human capital accumulation problem used in the theoretical section by imposing an upper bound on the choice of i_s while on the job. In particular, the choice set for investment is now $i_s \in [0, \chi] \cup \{1\}$, where $\chi < 1$ denotes the maximum on-the-job human capital investment possible. This upper bound could arise, for example, if the firm incurs fixed costs for employing each worker (administrative burden, the cost of office space, supplies, etc.), or due to minimum wage laws.[21] We calibrate χ to 0.50, which implies that in the initial steady state before skill-biased technical change, the lowest wage earned in the economy (which happens to be the starting salary of highest ability individuals) is about 70 percent of the average wage. This choice of χ is somewhat lower than what would be implied by the minimum wage interpretation, considering that the legal minimum wage averaged about 50 percent of average wage from 1950 to 1970 in the U.S. data. We choose this more conservative figure in the baseline calibration and discuss the results with higher values of χ (which generally makes it easier for the model to generate certain empirical phenomena) later.

We now set the other parameters of the individual's problem. We assume that individuals enter the labor market at age 20 and retire at 65 ($T = 45$). The interest rate is set equal to 0.05, and the subjective time discount rate is set to $\beta = 1/R$, implying that individuals will choose a constant consumption path over their life-cycle (given the absence of uncertainty and borrowing constraints). The curvature of the human capital accumulation function, α, is set to 0.8, which is close to the estimate of 0.81 reported by Heckman (1976) and is also consistent with the range of estimates obtained by Heckman, Lochner and Taber (1998). Considering values between 0.75 and 0.90 had a qualitatively small effect.

Accounting for Idiosyncratic Shocks.—The remaining parameters of the model will be chosen to match some empirical moments of the U.S. wage distribution. To this end, it is important to first account for the fact that the model abstracts from all types of shocks to

[21] The model with this extension can still be solved in closed form.

the wage process, including idiosyncratic ones, which are clearly present in the data. Thus, we assume that the observed wage in the data of individual i at age s in year t can be written as

$$\widetilde{w}^i_{s,t} = \widehat{w}^i_{s,t} + \upsilon^i_{s,t} + \varepsilon^i_{s,t}, \tag{16}$$

where $\widehat{w}^i_t$ denotes the systematic (or deterministic) component of wages, and is given by our baseline model; $\upsilon^i_{s,t}$ and $\varepsilon^i_{s,t}$ represent an AR(1) and an *i.i.d* shock process respectively.[22] The key assumption we make is that the variances of these idiosyncratic shocks—denoted σ^2_υ and σ^2_ε—have been *stationary* during the period under study.[23] Under this assumption, and letting $var_{i,s}(\cdot)$ denote the cross-sectional variance of a variable (taken over individuals of all types and ages), we then have: $var_{i,s}\left(\widetilde{w}^i_{s,t}\right) = var_{i,s}\left(\widehat{w}^i_{s,t}\right) + \sigma^2_\upsilon + \sigma^2_\varepsilon$. Two points are easily noted from this expression. First, the *level* of the variance of wages in the model should be adjusted by $(\sigma^2_\upsilon + \sigma^2_\varepsilon)$ before it can be compared to the data. Second, the *change* over time in the variance of observed wages will mirror that in the deterministic component $(\Delta var_{i,s}\left(\widetilde{w}^i_{s,t}\right) = \Delta var_{i,s}\left(\widehat{w}^i_{s,t}\right))$ which allows a direct comparison of the *trend* in the model variances to its empirical counterpart.

Similarly, the implications of the specification in (16) for the first moment of wages can also be seen easily. The average of observed wages will equal that of the systematic component, $E\left(\widetilde{w}^i_{s,t}|I\right) = E\left(\widehat{w}^i_{s,t}|I\right)$, where I denotes a set of individuals, for example, those in the same age or education group. Thus, both the level and the trend in the first moments of wages in the model can be directly compared to the data.

We are now ready to calibrate the remaining parameters of the model, which determine

[22] Notice a caveat of this specification: Because idiosyncratic shocks are additive with the logarithm of the deterministic component of wages, they are in fact multiplicative with the level. Thus, it can easily be shown that if individuals take the existence of these shocks into account when they make human capital accumulation decisions, this would lead to a different optimal choice than the present one. Although such a modification is possible and the model could still be solved numerically, we do not tackle this potential complication here.

[23] Notice that several empirical studies have found the variances of both transitory and persistent shocks to have increased during the period that we study (among others, Moffitt and Gottschalk 1994, Meghir and Pistaferri 2002). One point to note is that these studies do not account for the possibility that the dispersion of income growth rates could have increased during this time, which is the main thesis of the present paper. Moreover, given the goal of this paper, it seems natural to abstract from other sources of rise in wage inequality, such as the increasing variances of shocks, to see how much mileage one can get by the mechanism emphasized in this paper alone. This is the approach we pursue in this paper.

the extent of heterogeneity in the population. First, learning ability, $\widetilde{A}_j$, is assumed to be uniformly distributed in the population with the same parameter for every cohort. Second, the present model is interpreted as applying to human capital accumulation after secondary school. But then, the assumption we made in the theoretical model—that individuals start out with the same human capital level—may be too restrictive because it seems likely that they would have accumulated different amounts of human capital by the time they make the college enrollment decision. A simple way to model this heterogeneity is by assuming that the amount of raw labor, l, has a non-degenerate distribution in the population.[24] We also assume l to have a uniform distribution that is the same for all cohorts. Each distribution is fully characterized by two parameters, giving us a total of four parameters to be calibrated.[25]

Since these parameters are model-specific and are not directly observable we choose them so that the model matches some key moments of the data in the first steady state. First, the mean value of raw labor, $E_j[l_j]$, is a scaling parameter and is normalized to one. The remaining three parameters: (i) the cross-sectional variance of raw labor, $\sigma(l_j)$, (ii) the mean learning ability, $E\left[\widetilde{A}_j\right]$, and (iii) the dispersion in the ability to learn, $\sigma\left(\widetilde{A}_j\right)$, are chosen to match the following moments of the data in 1969:

1. the cross-sectional wage inequality,

2. the level of the log college premium, and

3. average wage growth over the life-cycle.

As discussed above, we need an estimate of the variances σ_υ^2 and σ_ε^2 to obtain the target value for the cross-sectional wage inequality. These estimates can be obtained from empirical studies, but for consistency, they need to be based on an econometric specification that allows

[24] Alternatively, initial heterogeneity could be introduced through differences in h_0 which we assumed to equal zero in the baseline model. This turns out to make almost no difference.

[25] Of course, we also need to calibrate the cross-sectional correlation of l and $\widetilde{A}$. Since we interpret the heterogeneity in l as arising from investments been made prior to college, and high ability individuals are likely to have invested more even before college it seems plausible to have a positive correlation between $\widetilde{A}_j$ and l_j. For simplicity we assume perfect correlation between $\widetilde{A}_j$ and l_j. It will become clear later that the heterogeneity in l_j plays little role in this model implying that the choice of perfect correlation is also innocuous.

for heterogeneity in income growth rates as implied by the human capital model we study. Guvenen (2005) estimates such a specification and reports σ_{ε}^2 to be 0.047. Similarly, σ_v^2 can be calculated to be 0.088 using the estimates in that paper (Table 1, row 2). Finally, the cross-sectional variance of observed wages in 1969 is 0.242, implying a target value for $var_{i,s}\left(\widehat{w}_{s,t}^i\right)$ of 0.107. Second, the college premium was 0.398 in our data set in 1969, which is another empirical target we choose.[26] Finally, the last target is the average wage growth over the life-cycle before SBTC. Studies that estimate life-cycle wage and income profiles from panel data sets such as the PSID report total life-cycle wage growth rates somewhere between 40 to 70 percent (c.f., Gourinchas and Parker (2002), Davis, Kubler and Willen (2002), Guvenen (2005)). One caveat is that these studies have to rely on wage data from the period coinciding with SBTC. But, Kambourov and Manovskii (2005) find some evidence for a flattening of the life-cycle profiles for successive cohorts during this period. With this in mind, we choose a target value of 65 percent, closer to the upper end of values reported in the papers cited above.

The last three rows of Table 2 displays the implied values for the distributions of $\widetilde{A}_j$ and l_j. Notice that the coefficient of variation of ability is about three and a half times that of raw labor. In fact, as it turns out ignoring cross-sectional heterogeneity in l_j altogether has quantitatively little, and qualitatively no effect on the results presented in the following section.

Skill-Biased Technical Change.—The driving force behind the non-stationary changes in the model is a sustained increase in relative productivity of human capital relative to raw labor, measured by θ_H/θ_L. We assume that this ratio starts to grow at a constant rate from 1970 until 1995. The latter year is chosen to be roughly consistent with the observation that the rise in wage inequality seems to have slowed down by the mid-1990s. However, this choice turns out not to be critical: assuming that SBTC continues until 2010 had very similar implications for the behavior of the model during the 70's and 80's.

The main quantitative experiment we conduct is the following. After calibrating the model as above, we choose the total increase in the skill bias of technology between 1970 and

[26] This is the series called clp_hsg in Autor et al.'s (2005) data set.

Table 2: Baseline Parameterization

Parameter		Value
R	Interest rate	0.05
β	Time discount rate	$1/R$
α	Curvature of human capital function	0.80
T	Years spent in the labor market	45
ρ	Curvature of Aggregate prod. function	1.00
ΔZ	Growth rate of neutral technology	0.015
$E[l_j]$	Average labor endowment (scaling)	1.0
Parameters calibrated to match 1969 targets:		
$E\left[\widetilde{A}_j\right]$	Average Ability	.0705
$\sigma(l_j)/E[l_j]$	Coefficient of variation of Labor endowment	.075
$\sigma\left[\widetilde{A}_j\right]/E\left[\widetilde{A}_j\right]$	Coefficient of variation of Ability	.245

1995, $\theta_{1995}/\theta_{1970}$, to match the total rise in wage inequality in the U.S. data between these two years, which is equal to 13 log points. Although in principle it is possible to choose the entire path of θs during this period to match the path of wage inequality, we do not pursue this approach. Rather we choose the simplest path—a constant increase in skill bias per year during the transition phase. Table 2 summarizes our baseline parameter choices.

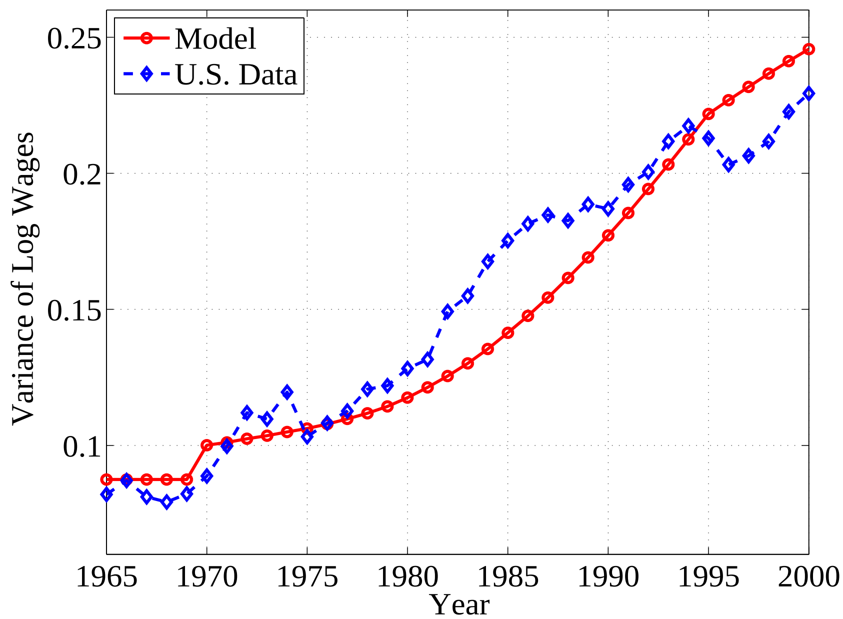
4.2 Evolution of Wage Inequality

4.2.1 Overall inequality

We begin by analyzing the implications of the model for the evolution of total wage inequality during this period. Figure 3 plots the variance of log wages generated by the model together with its empirical counterpart. Remember that the change in the skill bias was chosen to match the levels of wage inequality in 1969 and 1995, and not the evolution between these end points. Yet, the model seems to nicely capture the broad pattern during this period, with a slow increase in the 1970's that accelerates over time.

This pattern deserves some attention as it is intimately related to the behavior of the college premium that we discuss later below. To understand the evolution of overall inequality two separate effects, which sometimes work in opposite directions, should be noted. First,

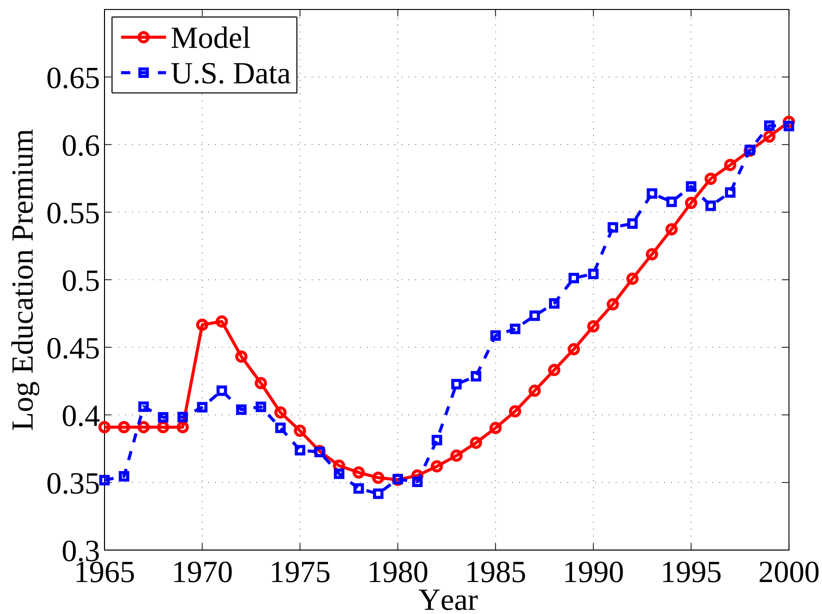
Figure 3: The Evolution of Wage Inequality in the Model and in the U.S. Data: 1965—2000.



the price of human capital relative to raw labor (P_H/P_L) increases at a roughly linear rate as can be seen in the left panel of figure 5. If there was no change in investment rates in response to SBTC (and thus the distribution of human capital remain unchanged over this period) this price effect would increase wage inequality at the same constant rate as the relative price change. However, the investment rate *does* respond to SBTC, which is a key feature of this model. This effect works to offset the price effect early on, because individuals whose investment responds more to SBTC are exactly those with a higher ability and thus who have relatively more human capital already. As a result the rise in wage inequality is depressed early on. Over time however the differential investment response leads to an even larger dispersion in human capital levels, which reinforces the price effect and lead to an accelerating rise in wage inequality.

One notable divergence however occurs during the 1980's when inequality rises faster in the data compared to the model. Some authors have emphasized the role played by the erosion of the legal minimum wage due to high inflation in the late 70's, which resulted in the fall of wages in the lower tail of the distribution thereby increasing inequality (c.f., Card and

Figure 4: The College-High School Premium: Model versus Data, 1965-2000.



Dinardo (2002)). This factor is not present in the model which might explain the divergence from the data during the 80's.

Another point to observe is that inequality continues to increase after 1995 when P_H/P_L stops increasing. This is due to the fact that older cohorts with lower dispersion in human capital levels retire from the economy, and are replaced by younger cohorts with higher dispersion. As a result, wage inequality in this model will continue to rise until year 2040, when the population is composed only of individuals born after 1995.

4.2.2 The College Premium

Figure 4 illustrates the behavior of the college premium implied by the model along with its empirical counterpart. Recall that the only data point in this graph targeted in the calibration was the level of the premium in 1969. In the model, the college premium falls throughout the 1970's followed by a robust increase in the next two decades, showing an overall pattern that is both qualitatively and quantitatively consistent with the data.[27]

To gain a better understanding of the behavior of the college premium, we use a decomposition that is a more general version of the one used in equation (13):

$$\omega^* = \frac{\left(P_H H_c^{net} + P_L L_c^{net}\right)/N_c}{\left(P_H H_{nc}^{net} + P_L L_{nc}^{net}\right)/N_{nc}} = \frac{\left[P_H\left(H_c^{net}/L_c^{net}\right) + P_L\right]}{\left[P_H\left(H_{nc}^{net}/L_{nc}^{net}\right) + P_L\right]}\frac{\left(L_{nc}^{net}/N_c\right)}{\left(L_{nc}^{net}/N_{nc}\right)}$$

where $H_c^{net} = \int_{j,s\in C} h_{j,s}\left(1 - i_{j,s}\right)\mu\left(s\right)djds$, that is, the human capital supplied to the market by college graduates, and $L_c^{net} = \int_{j,s\in C} l_j\left(1 - i_{j,s}\right)\mu\left(s\right)djds$. Other aggregates are defined analogously and the subscript "nc" denotes high-school graduates. Note that we divide both the numerator and denominator by the number of people in that group who are currently active in the labor market: $N_c = \int_{j,s\in C} 1\left\{i_{j,s} < 1\right\}\mu\left(s\right)djds$ to get average wages for each type. Note that $\left(L_c^{net}/N_c\right)$ is equal to the average hours devoted to the labor market (that is, the average hours not spent on training) by college graduates. Let $H_c^{net}/L_c^{net} = k_c^{net}$, and $H_{nc}^{net}/L_{nc}^{net} = k_{nc}^{net}$. Divide and multiply the previous equation by P_L to get:

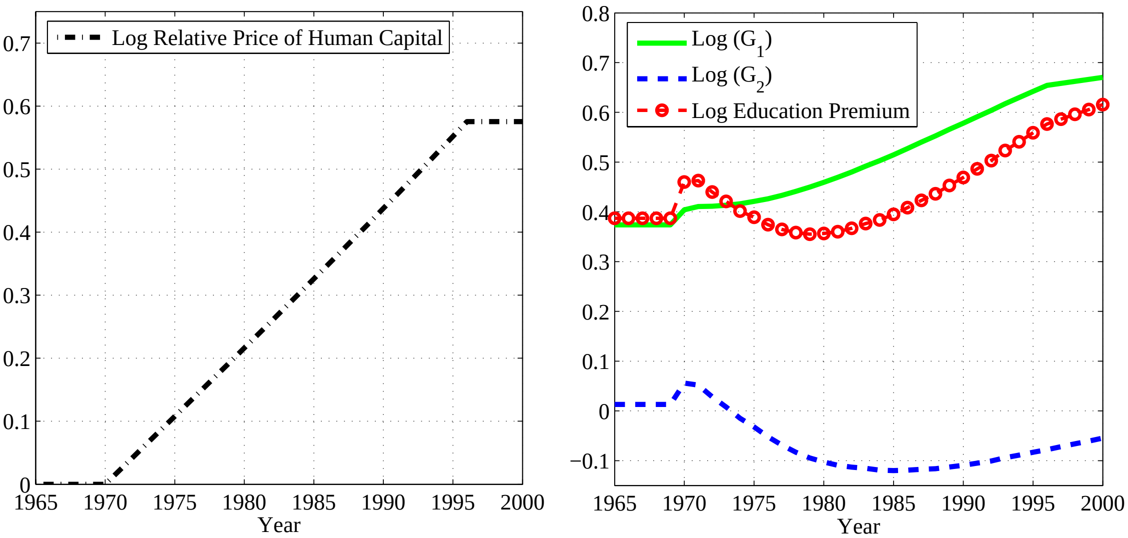
$$\omega^* = \underbrace{\frac{(P_H/P_L)k_c^{net} + 1}{(P_H/P_L)k_{nc}^{net} + 1}}_{G_1}\underbrace{\frac{\left(L_c^{net}/N_c\right)}{\left(L_{nc}^{net}/N_{nc}\right)}}_{G_2} = G_1 \times G_2.$$

The right panel of figure 5 plots the evolution of the logarithms of G_{1t} and G_{2t}. The term G_{1t} depends on variables that adjust slowly and grow monotonically over time. On the other hand, in response to SBTC college graduates increase their time investment more (that is, both Q and i increases more for this group) causing a steep decline in G_{2t} especially early on. Thus, the log education premium (line with circles) initially goes down with G_{2t}, and over time it bounces back when the increase in G_{1t} begins to dominate.[28]

[27] A natural concern could be whether the falling premium during the 70's is driven by a small number of college graduates who are investing close to 100 percent and thus receiving wages close to zero. This is not the case, since the fraction of time devoted to investment on the job is bounded from above by $\chi = 0.50$.

[28] The initial jump in the premium is driven by the change in composition of the labor force in 1970 after the skill-biased technical change. The jump is caused by the fact that the future evolution of skill premium becomes known upon impact. But the opportunity cost of investing today, which is the current wage rate, goes up by less than the present discounted value since the skill premium is upward trending. Moreover, because investment and college enrollment are forward looking decisions, new cohorts and younger cohorts respond strongly causing the jump. Those who respond most are the young individuals who have higher ability. Therefore, a large fraction of the young college graduates return back to college and drop out of

Figure 5: Decomposing the College-High School Premium



4.2.3 The College Premium within Experience Groups

A well-documented fact is that the behavior of the college premium in the U.S. during this period has been different for different experience groups (Katz and Murphy (1992), Murphy and Welch (1992), Autor, Katz and Kearney (2005)). In particular, these authors show that the fall and rise in the overall college premium discussed in the previous section was largely attributable to this behavior among individuals with less experience. In contrast, the fall and rise in the premium among more experienced individuals has been very much muted. Similarly, Card and Lemieux (2001) focus on age-groups (rather than experience), and examine data from U.K. and Canada in addition to the U.S. They find the same pattern to emerge in these countries as well.

In figure 6 we plot the college premium for two different experience groups (1-15 and 30-45) implied by the model. The college premium is higher among more experienced individuals before SBTC, which is consistent with the data. After SBTC, the initial decline and the strong rise in college premium is apparent among younger workers, but there is no fall among

our sample. Since, they have lower wages among all college graduates, the relative wages of college gradutes increase when they drop out of our sample.

more experienced workers and the rise is slightly smaller as well. As a result, the gap between the older and younger workers widens initially (from 0.35 to 0.49) and then narrows (to 0.28). This result is largely due to the fact that young individuals, with a longer horizon and hence marginal benefit from investing, respond to SBTC more than old individuals. In contrast, the increase in the college premium among old is mainly driven by price effects: the 30-45 experience group in 1995 had 10 to 25 years of experience in 1975. Especially the older ones in this group do not respond to SBTC much. Therefore, in the short-run the investment effect (and in the long-run, the quantity effect) has less negative (positive) impact on the college premium among older workers.

The increase in the college premium among older individuals is more pronounced than in the U.S. data. One factor that could explain this discrepancy is the fact that our model assumes that all SBTC has been "disembodied." As a result an older individual who accumulated human capital before 1970 stood to gain as much as newer cohorts. Instead, if some part of SBTC was embodied in new types of human capital (such as the skills to use computer software, etc.) then older cohorts would have benefitted less resulting in a smaller increase in the college premium. Of course, such a modeling would have other implications as well. But the idea of analyzing human capital decision in an environment with embodied SBTC seems interesting and deserves further examination.

4.2.4 The Rise in College Enrollment

Another prominent trend during this period has been the significant rise in educational attainment and the consequent increase in the relative supply of labor hours worked by college-educated individuals. This trend can be seen in figure 7 where the dashed line (marked with triangles) plots the total hours worked by individuals with a college-equivalent degree or more, relative to those with lower educational levels. This measure more than doubles from 1970 to 2000 in the U.S. data. Several studies have emphasized exogenous driving forces behind this rising supply, and viewed the evolution of the college premium as resulting from the interaction of this supply growth together with the demand change due to SBTC (c.f., Freeman (1976), Katz and Murphy (1992), Acemoglu (1998)).

Figure 6: The College Premium By Experience Level in the Model

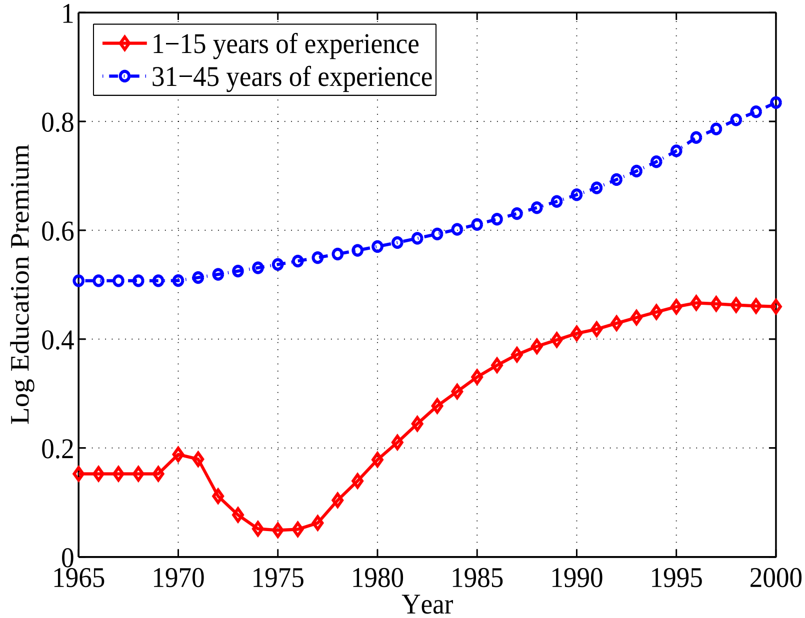
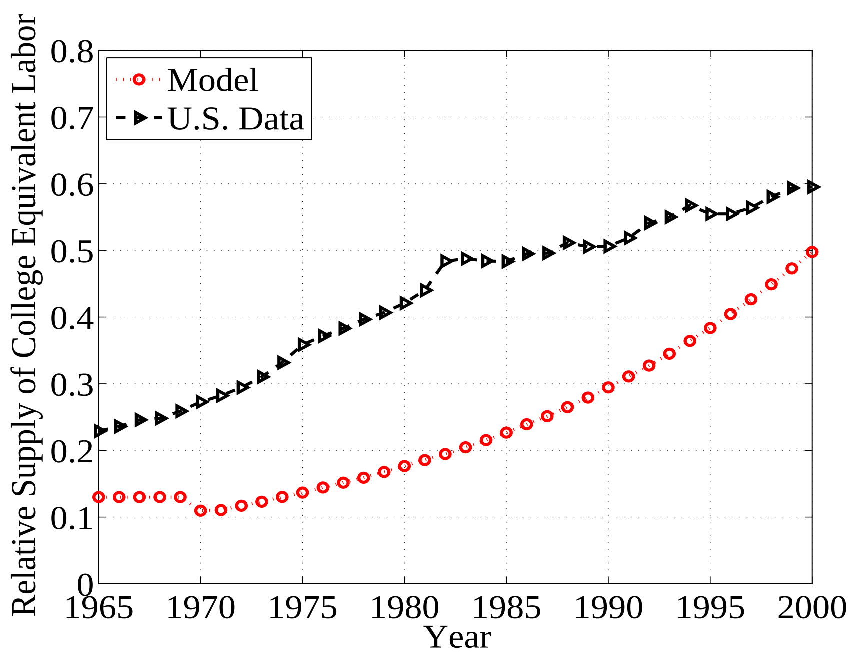


Figure 7: The Relative Supply of College Equivalent Labor, 1965—2000: Model versus U.S. Data



In the present model, instead, college enrollment is determined endogenously, and in particular, it responds to the change in the price of skills resulting from SBTC. The line with circles in figure 7 shows the relative supply of college-equivalent labor in the model.[29] The first point to observe is that the model understates the level of relative supply prior to SBTC. Perhaps this does not come as a surprise, since no attempt was made to match any aspect of relative supply in the calibration. However, this supply grows significantly during SBTC, because human capital investment (and hence educational attainment) is increasing for every subsequent cohort, and over time these younger cohorts are replacing older ones with lower educational attainment. As a result, relative supply increases by a total of 0.36 in the model, which compares well with the rise of 0.35 in the data. This similarity is surprising given that in the model educational attainment is modeled merely as a by-product—depending on whether investment exceeds a certain threshold or not—and many potentially important aspects of education have been left out, such as tuition costs, tuition subsidies, changes in the quality of education, changes in cohort sizes, etc. This analysis suggests that changes in the price of skills might have played a more important role than these factors in determining the overall rise in educational attainment.

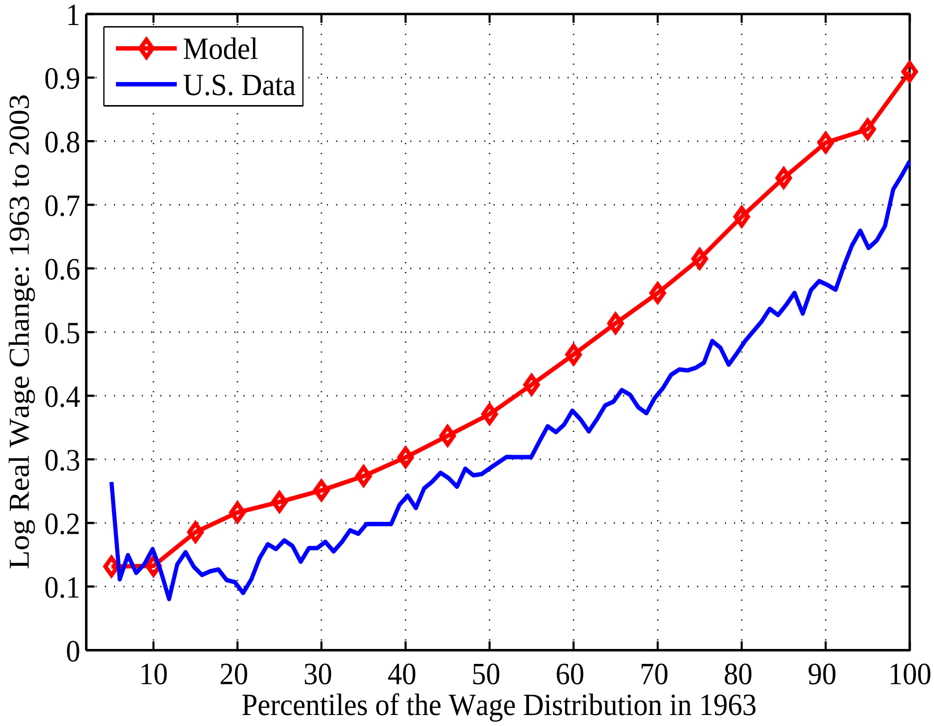
One aspect of the data not explained well by the model is the slowdown in the growth of supply starting in the 1980's. This discrepancy could be due to some of the factors omitted in the model noted above. In reality learning requires more than time: it requires school buildings, equipment, teachers, and so on. Many of these inputs could have inelastic supply in the short-run and even in the intermediate-run. The rise in college tuitions and the relative wages of educators in the last several decades could be indicative of demand pressures on inputs whose supply may have limited elasticity.

4.2.5 Within-group Inequality

The analysis so far has focused on the evolution of some key moments of the wage distribution. However, a distribution typically contains much more information than can be

[29] Relative supply is defined as the ratio of working individuals ($i < \chi$) who have completed more than two years of full-time investment ($i = 1$), to those who have had less investment. This is analogous to the definition adopted by Autor et al (2005) when constructing the empirical counterpart.

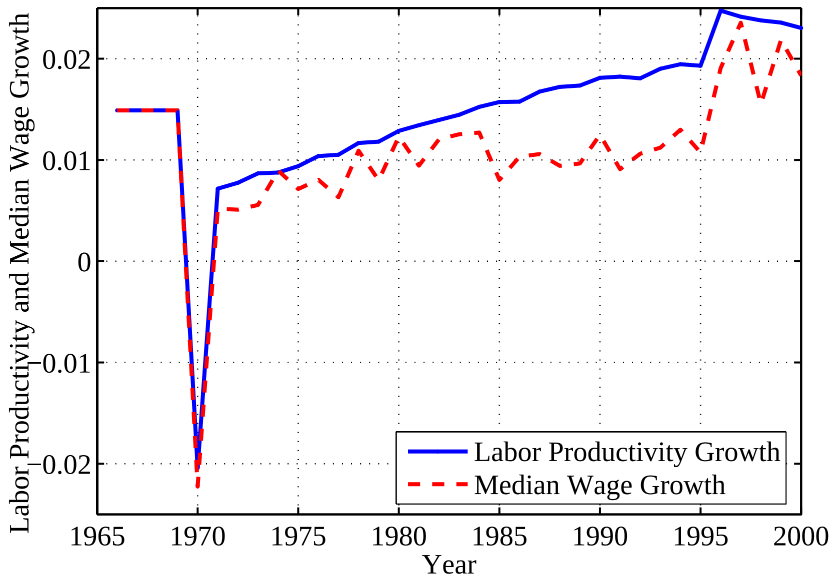
Figure 8: L



summarized by a few moments, and it is possible for a model to be consistent with some summary statistics, but generate patterns inconsistent with the data at a more disaggregated level. Juhn, Murphy and Pierce (1993) have documented a striking empirical regularity at a very disaggregated level that presents such a challenge. In figure 8 we report the same finding using our data set which covers a longer time span (solid line). The graph plots how each percentile of the wage distribution in 1963 (horizontal axis) has changed between 1963 and 2003 (vertical axis). The first point to note is that wage growth over this period has been systematically different for *every percentile* of the distribution. This shows that there is more to the rise in overall inequality than can be explained by differences in education alone.[30] Second, the relationship between a given percentile in 1963 and the wage growth in the subsequent 40 years is almost linear, except at the very low end of the distribution. This implies that wage inequality has increased by a fanning out of the entire distribution, leaving the relative ranking of each percentile largely unchanged over time.

[30] Juhn, Murphy and Pierce (1993) also find the same pattern when they examine the wage distribution for each education group and each age group, making this point even stronger. We have generated corresponding graphs from our model that qualitatively look similar to the data. We do not discuss them for brevity; they are available upon request.

Figure 9: The Growth Rate of Labor Productivity and Median Wages in the Model: 1965—2000.



The model counterpart is also plotted in figure 8 (line marked with diamonds). It shows the same general pattern of widening inequality which is spread evenly across the wage distribution as observed in the data. The mechanism behind this result should be clear from earlier discussions. Wage inequality arises entirely from differences in human capital accumulation rates, which in turn arises from differences in ability. Because individuals' investment response to SBTC is monotonically increasing in their ability, those with high ability have both higher wages in 1963, and a higher wage growth in the subsequent 40 years. In our view the existence of this same pattern in the data provides support that this mechanism could be an important channel behind the rise in within-group inequality. Finally, one notable discrepancy between the model and the data is the higher average wage growth in the model, which is discussed next.

4.3 Evolution of Average Wages

4.3.1 The Productivity Slowdown and Stagnation of Median Wages

Labor economists and macroeconomists have documented two closely related trends during this period: the stagnation of median wages and the slowdown in labor productivity, which both started with a sharp fall in 1973 and persisted until about 1995. For example, Juhn, Murphy and Pierce (1993) document that the median real wage has increased by 2.2 percent per year between 1963 and 1973, but actually *fell* by about 0.3 percent per year between 1973 and 1989. Similarly, labor productivity (measured as the non-farm business output per hour) has grown by 2.6 percent per year from 1955 to 1973, but only by 1.45 percent per year from 1973 to 1995.[31]

Figure 9 plots the growth rates of median wages and labor productivity implied by the model.[32] First, both series fall sharply immediately after SBTC starts in 1970. Thus, the model is able to generate the sharp initial slowdown, but this happens three years earlier than in the data. This may suggest that our timing of the start of SBTC could be off by three years, or alternatively, that it took some time for individuals to fully realize its advent. We explore the latter possibility by allowing for adaptive learning in the robustness analysis.

After the initial fall, the median wage continues to stagnate: it grows at 0.41 percent per year from 1970 to 1980, and averages 0.81 percent overall until 1995, representing a significant slowdown compared to the 1.5 percent growth during the period before 1970. Similarly, labor productivity grows by only 0.6 percent per year during the 1970's, but recovers somewhat faster and averages 1.24 percent per year until 1995. Overall then, while the magnitude of slowdown is somewhat smaller than in the data, the model correctly predicts the qualitative aspects of this evolution, including the sharp initial fall, the sluggish nature of the subsequent recovery, and the fact that the slowdown was larger for median wages than it was for labor productivity.

The basic intuition for the slowdown in wage growth has been discussed earlier in the

[31] Authors' calculation from Bureau of Labor Statistics data.

[32] Notice that since there is no capital in the model total output equals total wages, implying that labor productivity (output per hour) equals the mean wage rate in the economy.

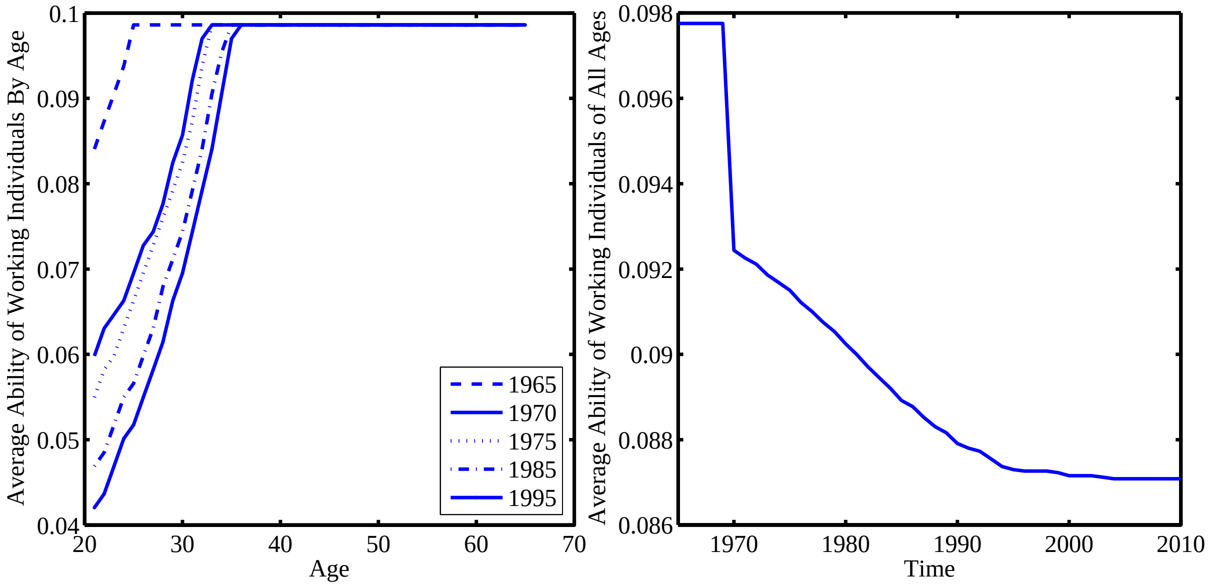
context of proposition 1. However, in the more general quantitative model here there is an additional channel which plays an important role that is useful to discuss. To see this, recall that the increase in investment after SBTC can take one of two forms. First, it will increase both the fraction of individuals who invest full time (i.e., enroll in college) and lengthen the duration for those already planning to go to college. Since this change takes place at the upper tail of the ability distribution, the average ability of individuals who remain in the labor market continually falls during SBTC, as can be seen in the right panel of figure 10. Because on average lower ability individuals also have low human capital, this "selection effect" reduces average wages and productivity after SBTC. The magnitude of this selection effect seems empirically plausible as evidenced by the fact that the model matches the total growth in college enrollment rate during this period (figure 7).

Second, those who remain in the labor market also respond to SBTC by increasing their on-the-job investment. This is shown in the right panel of figure 11. The fraction of time invested before SBTC is 7.2 percent (or 2.9 hours in a 40-hour work week) and increases to reach 13.1 percent in 1995 (or 5.2 hours a week). Neither the initial investment level, nor the increase during SBTC appears substantial, especially considering that what matters for average wages is the change in $(1 - i)$ which goes from 93 percent down to 87 percent. One reason is that the investment response is concentrated among younger individuals (left panel of figure 11) and thus the change in average investment is small. Another reason is that on-the-job investment is bounded from above by $\chi = 0.50$. Still, this "on-the-job investment effect" works in the same direction as the "selection effect" to further reduce wage growth after SBTC.

To sum up, during this period, the labor market is dominated by individuals who have lower ability than before, but who also invest more than before, resulting in slow wage and productivity growth. Over time, the increase in the total human capital stock due to both types of investment begins to dominate, resulting in a recovery in average wages.

An important point to take away from this discussion is that the choice of χ does not play a critical role here. A higher value of χ would allow for a larger increase in "on-the-job investment," but this would be offset by a smaller change in the ability composition because

Figure 10: Evolution of the Average Ability of *Working* Individuals Over Time

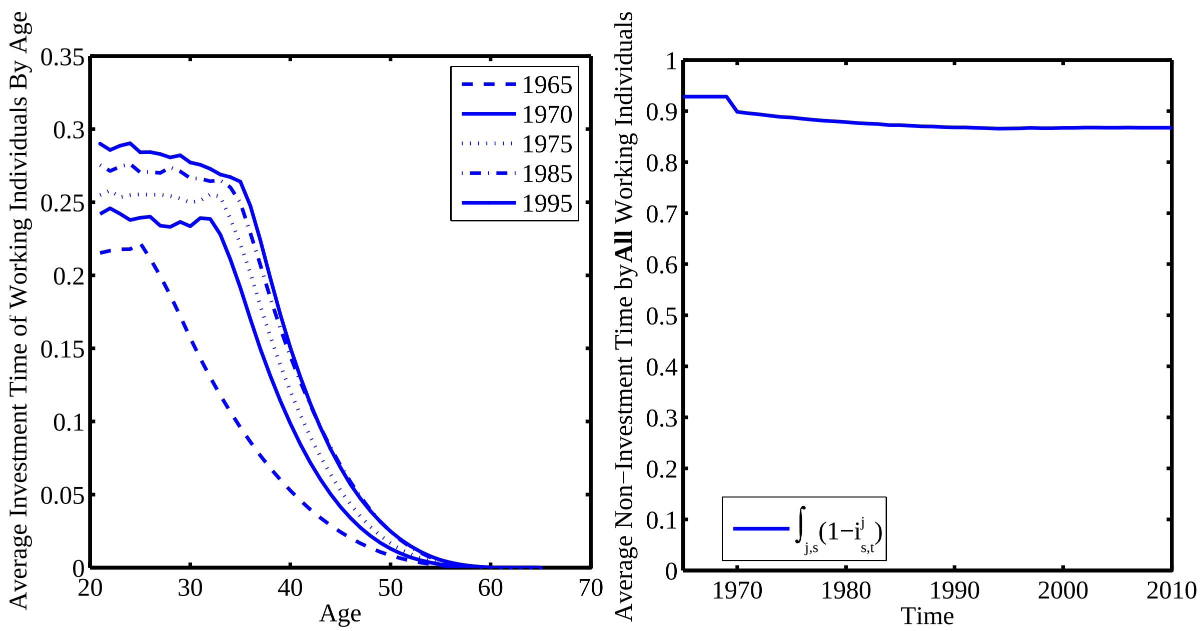


fewer individuals would now leave the labor market for full-time education. For example, setting $\chi = 0.80$ results in a median wage growth of 0.77 percent (compared to 0.82 above), and a productivity growth of 1.23 percent (compared to 1.24 above). To sum up, the model predicts that the stagnation in average wages results from the response of total investment to SBTC. What fraction of this investment happens on-the-job or at school has a small effect on the broad picture.

4.3.2 Cross-sectional Wage Profiles by Education and Experience

Another set of well-documented trends during this period concern cross-sectional wage profiles. To discuss these facts, in Table 3 we reproduce the relevant figures from Katz and Murphy (1992, table 1). The table reports the average wage growth for different education-experience groups over time. Perhaps the most striking fact—noted by several authors—that emerges from this table happens between 1979 and 1987 (last column). First, among high-school graduates, the average wage of workers with few years of experience plummet by 19.8 percent while older workers see only a small fall of 2.8 percent. As a result, the cross-sectional wage profiles of high-school graduates significantly steepen during this period. Remarkably,

Figure 11: Evolution of the Average Investment of *Working* Individuals Over Time



the opposite happens among college graduates: young workers see a wage growth of 10.8 percent, whereas older ones only experience a small increase of 1.8 percent. Consequently, the cross-sectional wage profile flattens for this group.[33] See also Bound and Johnson (1992, Figure 1).

We construct the model counterparts of the same statistics with one difference. As discussed in the previous section the model does not fully capture the magnitude of the slowdown in average wage growth. Given that our focus here is on the *relative* wage changes across education-experience groups, we normalize the data with the mean wage in a given year before calculating the statistics. This allows us to isolate the relative changes without being distracted by the overstated wage growth for all individuals. The model seems to capture the changes for each education-experience group, not only during the 1980's but also going back to the 1970's, rather well. For example, among high-school graduates there is little difference in wage growth by experience levels during the 1970's, whereas for college

[33] Clearly these facts are closely related to the evolution of the college premium within age groups discussed above. However, notice that the college premium is only informative about the *relative* wages of these two groups, whereas the current facts relate separately to the evolution of the *levels* of each group's wages.

Table 3: Real Wage Changes By Education and Experience Groups, 1971-1987

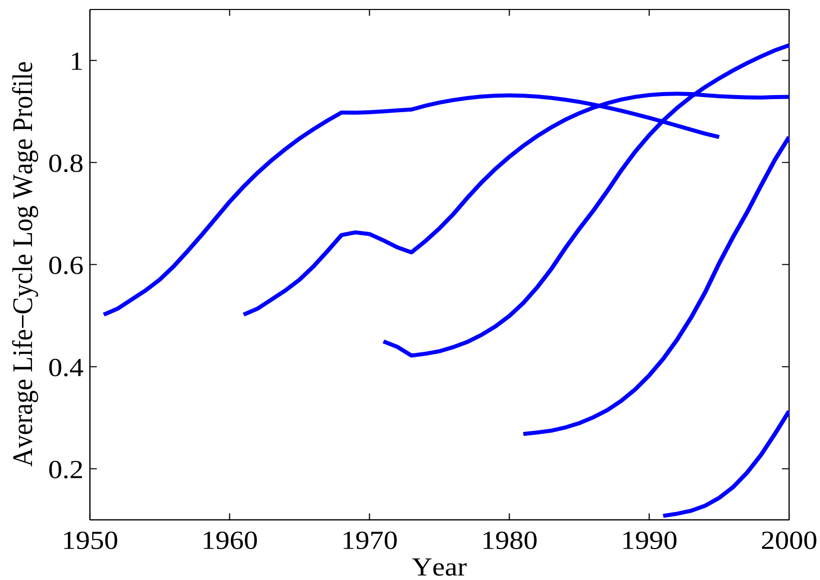
Group			Change in Log Average Real Wage (multiplied by 100)	
Education	Experience	Sample	1971-79	1979-87
12	Low	Data	**0.8**	**−19.8**
12	Low	Model	−2.4	−9.2
12	High	Data	**3.2**	**−2.8**
12	High	Model	−1.1	−3.7
16+	Low	Data	**−11.3**	**10.8**
16+	Low	Model	−7.8	13.7
16+	High	Data	**−4.0**	**1.8**
16+	High	Model	3.6	2.5

Notes: The empirical statistics reported are taken from Katz and Murphy (1992, Table 1). The low (high) experience group is defined as workers with 1 to 5 years of experience (26-35 years of experience) in Katz and Murphy (1992) and those with 1 to 15 years of experience (30-45 years of experience) in our model.

graduates there is a larger fall for younger individuals than for older ones. More importantly, the model is also consistent with the wage changes of all four education-experience groups from 1979 to 1987 noted above. As a result, the cross-sectional profiles steepen for individuals with low education and flatten for those with high education during this time.

There are three effects that should be taken into account to understand the wage changes of high-school graduates. First, young high school graduates also respond to SBTC—even if it is not to the same extent as high ability individuals—by increasing their investment. But since these individuals are below the threshold for going to college, all this investment takes place on-the-job, which reduces their observed wages. Second, there is selection: in response to SBTC, more able high school graduates go to college, so the average ability pool of those remaining falls, further reducing their wages. Neither one of these channel are a problem for older high school graduates: since they have a much shorter horizon they do not increase their on-the-job investment by much, nor do they decide to go back to college to create any compositional change. There is also a third effect: young workers have very little human capital, so the main factor they supply is raw labor. Therefore, they suffer from the lower returns to raw labor, but do not benefit from the higher returns to human capital. In contrast, older high-school graduates do have some human capital, so they are able to benefit

Figure 12: The Evolution of Life-Cycle Wage Profiles: 1950—2000.



from SBTC which partly offset their loss on their raw labor endowment. A combination of these three factors, which work in opposite directions for the young and old, explain why the former group experienced a large wage loss while the latter saw no significant change during the 1980's. Notice also that even though SBTC begins in 1970, the three mentioned effects strengthen gradually (as θ_H/θ_L rises) over time, and has little impact on the wages of the young until much later (1980's).

The argument for college graduates is similar, but the existence of an upper bound on on-the-job training also plays a role. This is because, after SBTC high ability individuals who want to increase their investment significantly have to stay in college longer due to the upper limit on investment on the job. As a result, new college graduates accumulate significant amounts of human capital before entering the labor market. Since SBTC raises the value of this human capital their wages do not fall at lower experience levels similar to that of high school graduates.

4.3.3 The Flattening (or Steepening?) of Wage Profiles

What happened to *life-cycle* wage profiles during this period? Kambourov and Manovskii (2005) report some evidence indicating that wage profiles have become flatter for each successive cohort. Moreover, they find that the starting wage has fallen for successive cohorts. Figure 12 plots the wage profile of cohorts entering the labor market ten years apart starting in 1950 in the model.[34] The fall in starting salaries for newer cohorts is easily noticed. As for the slope of the profiles, the model generates a non-monotonic pattern: In the early part of the life-cycle wage profiles become flatter while in the latter part they become steeper. Table 4 displays the wage growth rates of different cohorts. The bold letters (roughly) correspond to observations for which wage data is available in PSID (1968-97) or CPS (1962-2004). The wage growth over the life-cycle of a cohort before SBTC can be seen by looking at the cohort who enters the job market in 1920 and retires in 1965. This cohort observes 32% wage growth in the first 15 years of experience while the wage growth of cohorts who enter in 1960, 1970, and 1980 are smaller.

The reason for flattening early on in life is closely related to the stagnation of average wages discussed earlier. The two effects described in Section 4.3.1 both work in the same direction to reduce wage growth early on. In particular, since the returns to human capital investment increases continuously until 1995, newly entering cohorts invest more each year relative to previous ones. In addition, early cohorts are dominated by low ability individuals since those with high ability are in college. As a result wages grow more slowly early on, but faster later in life when both types of investments pay off in the form of higher human capital and thus higher wages.

4.4 The Rise in Consumption Inequality—Or the Lack Thereof

A somewhat surprising empirical finding from this period is that the rise in consumption inequality has been muted compared to the rise in wage inequality. Although uncertainty remains about the exact magnitude of change in consumption inequality mainly due to

[34] We have divided all wages in a given year by the average wage in that year to control time effects. If we did not do this, wage profiles for younger cohorts would look even flatter.

Table 4: Wage Growth Over the Life-Cycle for Different Cohorts

Cohort's entering year:	Log Wage Change Between Ages: 1–15	16—30	31—45
1920*	0.32	0.14	0.03
1950	**0.32**	**0.11**	**0.01**
1960	**0.15**	**0.24**	−0.00
1970	**0.18**	**0.32**	0.04
1980	**0.28**	0.43	0.04
1990	0.20	0.48	0.17

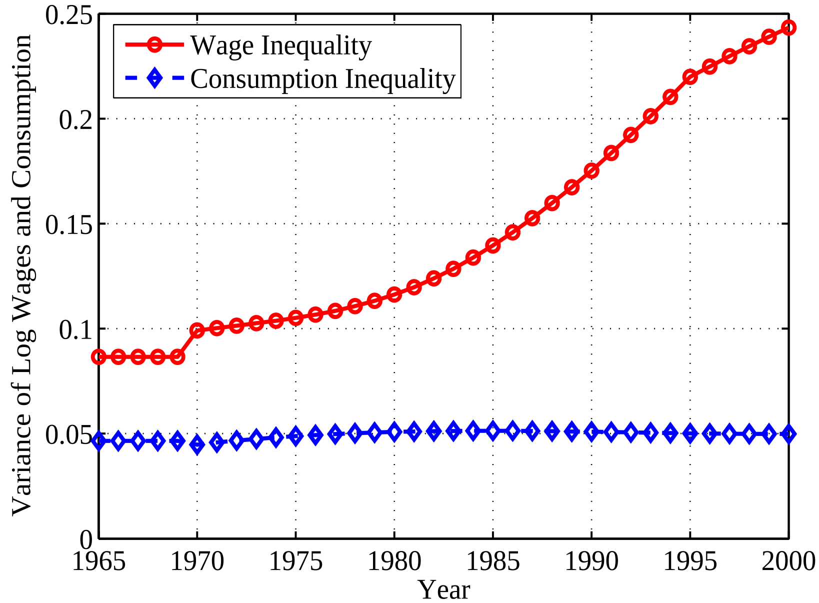
Notes: * 1920 represents a typical cohort which retires without encountering the effect of SBTC..

data problems, several authors report findings broadly supporting this conclusion (see for example Krueger and Perri (2005) and Attanasio, Battistin and Ichimura (2004)). Moreover, the change between the 90th and 50th percentiles of the consumption distribution does not seem to have tracked the large rise in the 90-50 percentile wage inequality. Autor, Katz and Kearney (2004) document this fact and call it puzzling.

The present model abstracts from many features that would be important for a detailed analysis of consumption inequality (such as incomplete markets, retirement savings, demographic changes, etc.). But the model still addresses a simple but fundamental question: Has the substantial rise in cross-sectional wage inequality during this period resulted in a parallel rise in *life-time* income inequality? Figure 13 plots the evolution of life-time income (which equals consumption in the model) inequality, which shows a very small increase of 0.4 log points during SBTC.

At first blush, it seems quite surprising that wage inequality could rise in such a systematic fashion without a significant change in life-time incomes. The mechanism can be anticipated from the earlier discussion of figure 1. First, because wage inequality increases by an increased dispersion in *growth rates*, life-time inequality—which is the variance calculated after averaging wages over the life-cycle—increases by less. This would not be the case if the increase in dispersion was in the levels of the wage profiles, in which case, consumption inequality would increase one for one with wage inequality. Second, and furthermore, the higher wages of high ability individuals later in life come at a large cost in the form of high

Figure 13: The Evolution of Wage and Consumption Inequality in the Model: 1965—2000.



investment and low wages early on, driving down the lifetime gain from human capital investment. As shown in Kuruscu (2005), for a range of plausible parameter values similar to those used here, the gain in lifetime income due to human capital investment is surprisingly small—about 1 percent. This is because the costs of foregone earnings during the high investment early in life is close to the future discounted benefits. Our model thus offers a mechanism which is consistent with a large increase in wage inequality but a small change in consumption inequality.

The model does imply larger changes in the latter under alternative parameterizations. However, for parameter values broadly consistent with facts about the wage distribution, the largest rise we obtained was about 4-5 log points. Introducing depreciation in human capital also works to generate a larger increase in consumption inequality. A fuller investigation of this model for consumption facts is left for future work.

5 Robustness

5.1 The Importance of Investment

In the previous sections, we stated in several contexts that the response of investment to SBTC is key to the results of this paper. To further substantiate this point, we now examine the implications of the model with fixed investment behavior. Specifically, the model is the same before 1970, but after SBTC we assume that the only change is in the price of skills *without* any change in investment behavior. Most studies in the literature (Juhn et al (1993) being the prime example) envisioned this scenario when they concluded that one-skill models cannot explain the joint behavior of total wage inequality and the college premium.

We first decompose total wage inequality into the component explained by: (i) the increase in the price of human capital, which would imply that even without any change in the distribution of human capital inequality would rise (price effect), and (ii) the dispersion in human capital changes (quantity effect). The question is, suppose we fix the distribution of human capital at its value in 1970 and only allow skill prices to change. How much would total wage and consumption inequality change? What would happen to the college premium? It is important to understand how much the heterogeneous investment response explains of each of these statistics.

Table 5 displays the increase in total wage and consumption inequality in the benchmark model and when the investment is fixed at its 1969 level. It turns out that 36 percent of the increase in wage inequality is due to price effect while 64 percent is due to the changes in investment. On the other hand, most of the increase in consumption inequality is due to price effects (62% of the increase). The effect of investment on consumption inequality is relatively small. The different patterns we observe for wage and consumption inequality is due to the fact that those who benefit from SBTC are the ones who pay for the human capital investment by accepting lower wages early in life.

Figure 14 displays several statistics that we are interested in. As can be seen in the figure, one-third of the increase in wage inequality is due to changes in prices. The increase in the college premium is similar to the benchmark case. The main difference is that it increases

Table 5: EVOLUTION OF INEQUALITY WITHOUT INVESTMENT RESPONSE TO SBTC

	Benchmark		Fixed Investment	
Year	$Var(log(w))$	$Var(log(c))$	$Var(log(w))$	$Var(log(c))$
69	0.0875	0.0486	0.0875	0.0486
95	0.2218	0.0528	0.1358	0.0516
Difference	0.1343	0.0042	0.0483	0.0030

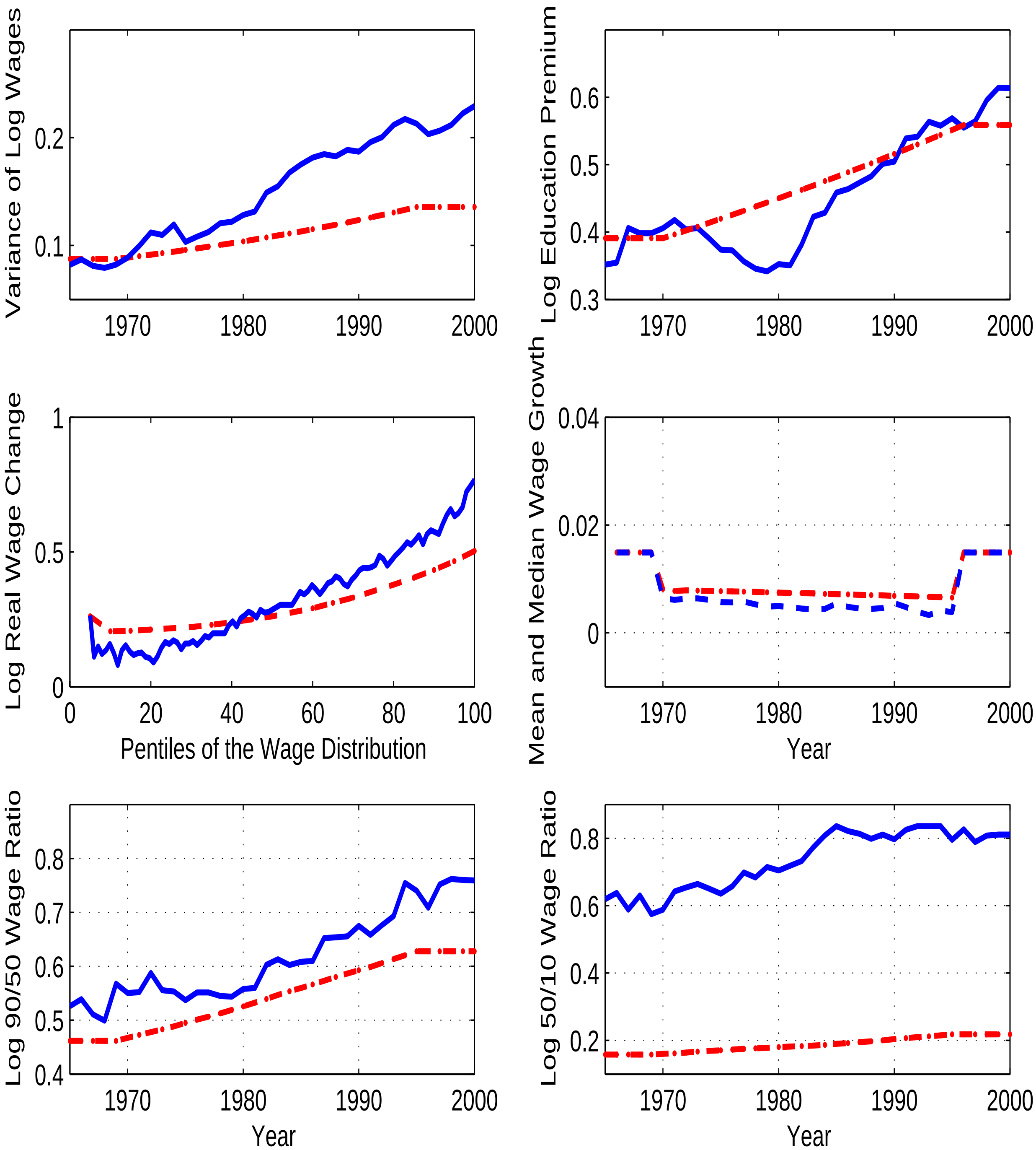
monotonically. This shows the importance of investment response in explaining the initial decline in the college premium. Wage changes at different percentiles of the distribution reveals that investment response is important for the growth of wages at the upper end of the distribution. The growth of median and mean wages are smaller if investment was fixed. The increase in human capital investment in the model, offsets the decline in mean wages to some extent. The evolution of 90-50 and 50-10 differential seems to mimic the data even without the investment response. From this figure, we see that the most important difference is the behavior of the college premium.

5.2 Adaptive Learning

The previous sections assume that individuals perfectly forecast the future evolution of skill prices P_H and P_L. In this section, we evaluate what happens if we relax this assumption. For this purpose, we incorporate adaptive learning into the model. Individuals have belief about what the growth rate of skill prices are going to be in the future.[35] This belief is based on past experiences of growth rate of skill prices and is updated with the arrival of new information each period. In particular, we assume that individuals discount past information at rate γ: the growth rate of skill prices s periods earlier receive $(1-\gamma)\gamma^s$ weight in the current belief. Given this assumption, if we denote the belief about the future growth rate of skill prices at time t by $\widehat{g}_{P_H}(t)$ (or $\widehat{g}_{P_L}(t)$ similarly), the evolution of belief is given by

[35] We have also tried a specification where individuals have beliefs about the growth rate of skill bias parameter θ. Results are not quantitatively significant.

Figure 14: Evolution of Key Variables When Investment Does Not Respond to SBTC



$$\widehat{g}_{P_H}(t) = (1-\gamma)\sum_{s=0}^{\infty}\gamma^s g_{P_H}(t-s)$$

where $g_{P_H}(t) = P_H(t)/P_H(t-1)$. The belief about P_L is defined similarly. Individuals at time t, believe that skill prices will grow at rates $\widehat{g}_{P_H}(t)$ and $\widehat{g}_{P_L}(t)$ indefinitely. As $\gamma \to 1$, the updating of beliefs become extremely slow because individuals assign an enormous weight to past observations. In this case, beliefs adjust extremely sluggishly, and they act as if they are surprised by the growth rate of skill prices each period. In this case, they only update the level of the skill prices but their belief about the growth rate of skill prices remain the same. Although this case is different than the fixed investment case considered above (because here investment does respond to SBTC since the *level* of P_H relative to P_L increases over time), quantitatively the results are very similar, and are omitted for brevity.

Next, we consider the case with $\gamma = 0.7$. Now individuals do update their beliefs about the growth rate of skill prices over time. Since, they do not perfectly anticipate the changes in skill prices upon impact of the shock, the response of investment is delayed. As the following figure illustrates, delayed investment response mainly affects the college premium where the decline in premium is delayed for about 5 years. The other statistics, however, remain largely similar to the benchmark case. Overall, the relatively fast updating of expectations is essential mainly for the success of the model in explaining the college premium.

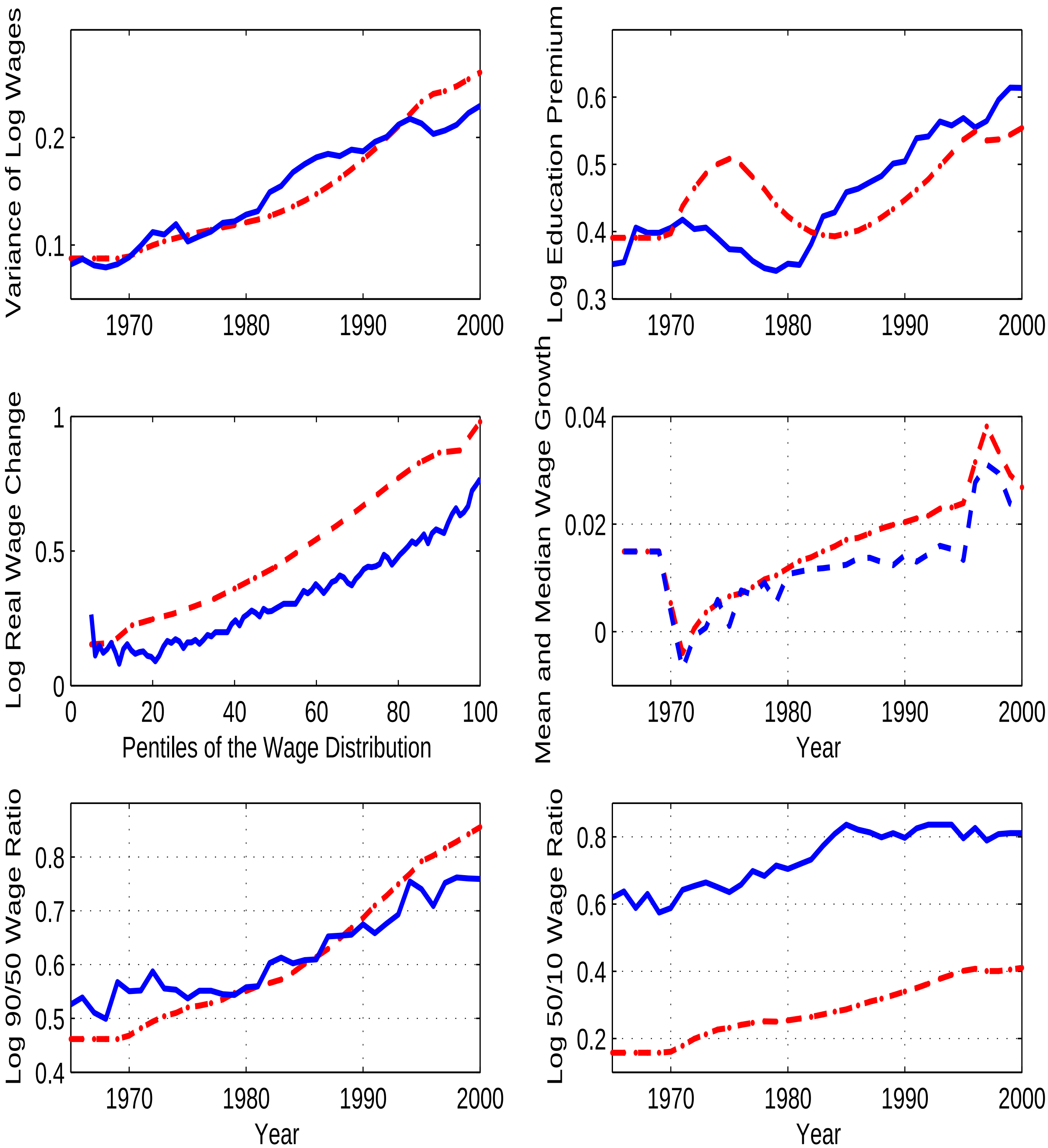
5.3 Allowing for Imperfect Substitution: $\rho < 1$ [To be written]

5.4 Positive Depreciation of Human Capital [To be written]

6 Conclusion

In this paper we have examined the implications of a tractable general equilibrium model of human capital accumulation for several economic trends observed since early 1970's. The key element in the model is the interaction between skill-biased technical change and heterogeneity in the ability to accumulate human capital. Because of the latter heterogeneity, the response of different individuals to SBTC turns out to be systematically and dramatically

Figure 15: Evolution of Key Variables When Individuals Learn Adaptively with $\gamma = 0.7$



different than each other. As a result, the model generates rich behavior in the relative wages of individuals depending on their age and ability. Since the latter is not observable in the data but is crudely approximated by education, the rise in wage inequality among individuals with different ability levels appears as an increase in residual inequality. The model is tractable enough that it can be extended in several directions. Overall, we view this model as a promising framework for analyzing the consequences of SBTC for the macroeconomy.

A Appendix: Derivations and Proofs of Propositions

Proof of Proposition 1: Substituting the optimal investment level leads to following expressions for the average wage before the shock and in the short-run.

$$\overline{w}_{SR} = \theta_L l + \left(\frac{\delta}{1-\delta}\theta'_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \right)^{\alpha/(1-\alpha)} - \frac{\alpha\delta\beta}{1-\delta\beta}\theta'_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H \right)^{\alpha/(1-\alpha)} \right) E\left[A\right].$$

$$\overline{w}_{I} = \theta_L l + \left(\frac{\delta}{1-\delta}\theta_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \right)^{\alpha/(1-\alpha)} - \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \right)^{\alpha/(1-\alpha)} \right) E\left[A\right].$$

Then $w_{SR} < w_I$ iff

$$\frac{\theta'_H}{\theta_H} \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \left(\frac{\theta'_H}{\theta_H} \right)^{\alpha/(1-\alpha)} \right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

To see under what conditions this inequality is satisfied, consider the function

$$f(x) = x \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} x^{\alpha/(1-\alpha)} \right).$$

Notice that a skill biased technical change is equivalent to increasing $x = \frac{\theta'_H}{\theta_H}$ above 1. Therefore, if $f'(1) < 0$, $f'(x) < 0$ for $x > 1$, then the inequality above is satisfied and $w_{SR} < w_I$.

$$f'(1) = \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)},$$

therefore $f'(1) < 0$ and $f'(x) < 0$ for $x > 1$, iff $\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0$. ■

Proof of Proposition 2: Let

$$\phi = \frac{\overline{Q_c}}{\overline{Q}_{nc}} = \frac{\overline{\mathbf{Q}_c}'}{\overline{\mathbf{Q}}'_{nc}} = \frac{E_c\left(A\right)}{E_{nc}\left(A\right)}.$$

Substitute $\overline{Q_c} = \phi\overline{Q}_{nc}$ and $C(\overline{Q}_{nc}) = \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H\overline{Q}_{nc}$ in the college premium to get

$$\omega_I^* = \frac{\theta_L l + \phi\theta_H\overline{Q}_{nc}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}{\theta_L l + \theta_H\overline{Q}_{nc}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}$$

and

$$\omega_{LR}^* = \frac{\theta_L l + \phi\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}{\theta_L l + \boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)}$$

Since $\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc} > \theta_H\overline{Q}_{nc}$, if the function $g(x) = \frac{\theta_L l + \phi x}{\theta_L l + x}$ is increasing in x, then $\omega^*_{LR} > \omega^*_I$.

$$g'(x) = \frac{\phi\theta_L l - \theta_L l}{(\theta_L l + x)^2}$$

$g'(x)$ is positive iff $\phi > 1$. Then $\omega^*_{LR} > \omega^*_I$ iff $E_c(A) > E_{nc}(A)$.

The premium in the short-run can be written as

$$\omega^*_{SR} = \frac{\theta_L l + \phi\left(\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\overline{Q}_{nc} - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}\right)}{\theta_L l + \frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\overline{Q}_{nc} - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}} = \frac{\theta_L l + \phi x_{SR}}{\theta_L l + x_{SR}},$$

where

$$x_{SR} = \frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\overline{Q}_{nc} - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}.$$

Let

$$x_I = \frac{\delta}{1-\delta}\theta_H\overline{Q}_{nc} - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\overline{Q}_{nc}.$$

If $x_{SR} < x_I$, then $\omega^*_{SR} < \omega^*_I$. Therefore we will characterize the condition under which $x_{SR} < x_I$. Plugging optimal investment choices we can show that $x_{SR} < x_I$ iff

$$\frac{\boldsymbol{\theta}'_H}{\theta_H}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

This is the same condition as in proposition 1, therefore $\omega^*_{SR} < \omega^*_I$ for all $\boldsymbol{\theta}'_H > \theta_H$ if $\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0$. ■

Proof of Proposition 3: The proof is very similar to the proof of proposition **2.** Let

$$\phi = \frac{\overline{Q_c}}{\overline{Q}_{nc}} = \frac{\overline{\mathbf{Q}_c}'}{\overline{\mathbf{Q}}'_{nc}} = \frac{E_c(A)}{E_{nc}(A)}.$$

The premium in the short-run can be written as

$$\omega^*_{SR}(s) = \frac{\theta_L l + \phi\left(\boldsymbol{\theta}'_H\overline{Q}_{nc}(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}\right)}{\theta_L l + \boldsymbol{\theta}'_H\overline{Q}_{nc}(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}} = \frac{\theta_L l + \phi x_{SR}}{\theta_L l + x_{SR}}$$

where $x_{SR} = \boldsymbol{\theta}'_H\overline{Q}_{nc}(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}$. Let $x_I = \theta_H\overline{Q}_{nc}(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\overline{Q}_{nc}$.

The education premium declines in the short-run iff $x_{SR} < x_I$.

$$x_{SR} < x_I \iff \frac{\boldsymbol{\theta}'_H}{\theta_H}\left(s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

Define the function

$$f_s(x) = x\left(s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}x^{\alpha/(1-\alpha)}\right).$$

Notice that a skill biased technical change is equivalent to increasing $x = \frac{\theta'_H}{\theta_H}$ above 1. Therefore, if $f'_s(1) < 0$ then $\omega^*_{SR}(s) < \omega^*_I(s)$.

$$f'_s(1) < 0 \iff s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}.$$

Therefore, $\omega^*_{SR}(s) < \omega^*_I(s)$ if $s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}$.

The college premium in the long-run is given by

$$\omega^*_{LR}(s) = \frac{\theta_L l + \phi\left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}}{\theta_L l + \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc}}.$$

Since $\boldsymbol{\theta}'_H\overline{\mathbf{Q}}'_{nc} > \theta_H\overline{Q}_{nc}$ and $\phi > 1$, the college premium would increase in the long-run if $s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta} > 0$. ■

Derivation of the Variances of Wages and Consumption:

The wage of an s years-old individual of type-j who is $w_{j,s} = \theta_L l + \theta_H Q_j(s-1) - C_j(Q_j)$. We rewrite it as $w_{j,s} = m_j + p_j(s-1)$ where $m_j = \theta_L l - C_j(Q_j)$ and $p_j = \theta_H Q_j$.

The average wage is given by

$$\overline{w} = \sum_{s=1}^{\infty}(1-\delta)\delta^{s-1}\int_j w_{j,s} = \sum_{s=1}^{\infty}(1-\delta)\delta^{s-1}\int_j\left[m_j + p_j(s-1)\right].$$

With some algebra we get

$$\overline{w} = \overline{m} + \frac{\delta}{1-\delta}\overline{p} = \theta_L l - C(\overline{Q}) + \frac{\delta}{1-\delta}\theta_H\overline{Q} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}$$

where

$$\overline{m} = \theta_L l - \int_j C_j(Q_j) = \theta_L l - C(\overline{Q}) = \theta_L l - \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H\overline{Q}$$

and $\bar{p} = \theta_H\int_j Q_j = \theta_H\overline{Q}$. Using the expression for Q_j we get

$$\overline{Q} = \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} E\left[A\right].$$

The consumption of type- j individual is $c_j = m_j + \frac{\beta\delta}{1-\beta\delta}p_j$. Then the average consumption is

$$\overline{c} = \overline{m} + \frac{\delta\beta}{1-\delta\beta}\overline{p} = \theta_L l + \left(\frac{\delta\beta}{1-\delta\beta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}$$

Variance of wages is given by

$$
\begin{aligned}
Var(w) &= \int_j \sum_{s=1}^{\infty} (1-\delta)\delta^{s-1} \left[m_j + p_j(s-1) - \overline{m} - \frac{\delta}{1-\delta}\bar{p} \right]^2 \\
&= Var(m) + \frac{2\delta}{1-\delta} Cov(m,p) + \frac{\delta(1+\delta)}{(1-\delta)^2} Var(p) + \frac{\delta}{(1-\delta)^2}\bar{p}^2
\end{aligned}
$$

and variance of consumption is

$$
\begin{aligned}
Var(c) &= \int_j \left[m_j + \frac{\beta\delta}{1-\beta\delta} p_j - \overline{m} - \frac{\beta\delta}{1-\beta\delta}\overline{p} \right]^2 \\
&= Var(m) + \frac{2\delta\beta}{1-\delta\beta} Cov(m,p) + \frac{(\delta\beta)^2}{(1-\delta\beta)^2} Var(p).
\end{aligned}
$$

$$
\begin{aligned}
Var(m) &= \left(\frac{\alpha\delta\beta}{1-\delta\beta} \right)^2 \theta_H^2 Var(Q) \\
Var(p) &= \theta_H^2 Var(Q) \\
Cov(m,p) &= -\frac{\alpha\delta\beta}{1-\delta\beta} \theta_H^2 Var(Q) \\
Var(Q) &= \left(\frac{\alpha\delta\beta\theta_H}{1-\delta\beta} \right)^{2\alpha/(1-\alpha)} Var(A)
\end{aligned}
$$

Plugging the expressions above we get

$$
\begin{aligned}
Var(w) &= \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} \right) \theta_h^2 Var(Q) + \frac{\delta}{(1-\delta)^2} \theta_h^2 \overline{Q}^2 \\
&= \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} \right) \left(\frac{\alpha\delta\beta}{1-\delta\beta} \right)^{2\alpha/(1-\alpha)} \theta_h^{2/(1-\alpha)} Var(A) \\
&\quad + \frac{\delta}{(1-\delta)^2} \left(\frac{\alpha\delta\beta}{1-\delta\beta} \right)^{2\alpha/(1-\alpha)} \theta_H^{2/(1-\alpha)} E\left[A\right]^2
\end{aligned}
$$

and

$$
\begin{aligned}
Var(c) &= \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta} \right]^2 \theta_h^2 Var(Q) \\
&= \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta} \right]^2 \left(\frac{\alpha\delta\beta}{1-\delta\beta} \right)^{2\alpha/(1-\alpha)} \theta_h^{2/(1-\alpha)} Var(A).
\end{aligned}
$$

Proof of Proposition 5: Remember that $w_{s,j} = \theta_L l + \theta_H Q_j(s-1) - C(Q_j)$.

Plugging the optimal investment, we can write $w_{s,j} = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$, where $n_5 = \left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)}$ and $y = \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right) A$. It is clear that $w_{s,j}$ is increasing in y. Hence, one's relative position Ω in the wage distribution is positively related to y.

The wage of an agent with y before the shock is given by $w_I(y) = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$. The corresponding wage in the long-run is $w_{LR}(y) = \theta_L l + n_5 \boldsymbol{\theta}_H'^{1/(1-\alpha)} y$. It is then easy to show that $w_{LR}(y)/w_I(y)$ is increasing in y. ■

Proof of Proposition 6: The proof directly follows from the fact that

$$\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} = \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)^2 + \frac{\delta}{(1-\delta)^2} > 0$$

and that

$$\left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}\right) - \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta}\right]^2 > 0. \quad ■$$

Proof of Proposition 7:

a. **Long-run:**

$$\begin{aligned}
& CV(w)^2\big|_{\theta_H'} - CV(w)^2\big|_{\theta_H} \\
= \; & \frac{n_1 Var(A) + n_2 E\left[A\right]^2}{\overline{w}'^2} \theta_h'^{2/(1-\alpha)} - \frac{n_1 Var(A) + n_2 E\left[A\right]^2}{\overline{w}^2} \theta_h^{2/(1-\alpha)} \\
= \; & \left(n_1 Var(A) + n_2 E\left[A\right]^2\right) \left(\frac{\theta_h'^{2/(1-\alpha)}}{\overline{w}'^2} - \frac{\theta_h^{2/(1-\alpha)}}{\overline{w}^2}\right)
\end{aligned}$$

where $\overline{w}$ and $\overline{w}'$ are the average wages in the old and new steady. Plugging

$$\overline{w} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right) \left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} E\left[A\right]$$

and

$$\overline{w}' = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right) \left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} \theta_H'^{1/(1-\alpha)} E\left[A\right]$$

we get

$$\begin{aligned}
& CV(w)^2\big|_{\theta_H'} - CV(w)^2\big|_{\theta_H} \\
= \; & \theta_L l \left\{n_1 Var(A) + n_2 E\left[A\right]^2\right\} \left[\theta_H'^{1/(1-\alpha)} \overline{w} + \theta_H^{1/(1-\alpha)} \overline{w}'\right] \times \frac{\theta_H'^{1/(1-\alpha)} - \theta_H^{1/(1-\alpha)}}{\overline{w}^2 \overline{w}'^2}
\end{aligned}$$

Since n_1 and n_2 are positive, and $\theta'_H > \theta_H$,

$$CV(w)^2\big|_{\theta'_H} - CV(w)^2\big|_{\theta_H} > 0$$

An alternative way is to look at the derivative of $CV(w)^2\big|_{\theta_H}$ with respect to θ_H. It is easy to see that in fact $CV(w)^2\big|_{\theta_H}$ increases with θ_H.

b. **Short-run:** Remember that the wage in the short-run is

$$w^{SR}_{j,s} = \underbrace{\theta_L l - C_j(\mathbf{Q}'_j)}_{m'_j} + \underbrace{\boldsymbol{\theta}'_H Q_j}_{p'_j}(s-1).$$

Notice that the difference between $w_{j,s}$ and $w^{SR}_{j,s}$ is that we have replaced m_j and p_j with m'_j and p'_j. Hence the average wage in the short-run is $\overline{w}_{SR} = \overline{m}' + \frac{\delta}{1-\delta}\overline{p}'$ and the variance of wages in the short-run is

$$Var_{SR}(w) = Var(m') + \frac{2\delta}{1-\delta}Cov(m',p') + \frac{\delta(1+\delta)}{(1-\delta)^2}Var(p') + \frac{\delta}{(1-\delta)^2}\overline{p}'^2.$$

$$\begin{aligned}
Var(m') &= \left(\frac{\alpha\delta\beta}{1-\delta\beta}\boldsymbol{\theta}'_H\right)^2 \times \underbrace{Var(\mathbf{Q}')} \\
&= \left(\frac{\alpha\delta\beta}{1-\delta\beta}\boldsymbol{\theta}'_H\right)^2 \left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{2\alpha/(1-\alpha)} Var(Q) \\
Var(p') &= \theta'^2_H Var(Q) \\
Cov(m',p') &= -\frac{\alpha\delta\beta}{1-\delta\beta}\theta'^2_H Cov(Q,\mathbf{Q}') \\
Cov(Q,\mathbf{Q}') &= \left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)} Var(Q).
\end{aligned}$$

Then we can write the variance in the short-run as

$$Var_{SR}(w) = \left\{\left(\frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}x^{2/(1-\alpha)} - \frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)}x^{(2-\alpha)/(1-\alpha)} + \frac{\delta(1+\delta)}{(1-\delta)^2}x^2\right)Var(Q) + \frac{\delta}{(1-\delta)^2}\overline{Q}^2 x^2\right\}\theta_h^2$$

where $x = \boldsymbol{\theta}'_H/\theta_H$. Similarly the average wage in the short-run can be written as

$$\overline{w}_{SR} = \theta_L l + \left(\frac{\delta}{1-\delta}x - \frac{\alpha\delta\beta}{1-\delta\beta}x^{1/(1-\alpha)}\right)\theta_H\overline{Q}.$$

We look at what happens to $CV^2_{SR}(w)$ if x is increased marginally above one, or equivalently

$\boldsymbol{\theta}'_H$ is increased marginally above θ_H. Hence we compute

$$\frac{d}{dx}CV^2_{SR}(w) = 2\frac{\frac{\overline{w}_{SR}}{2}\frac{d}{dx}Var_{SR}(w) - Var_{SR}(w)\frac{d}{dx}\overline{w}_{SR}}{\overline{w}^3_{SR}}.$$

If $\frac{d}{dx}CV^2_{SR}(w) > 0$ then we conclude that inequality increases in the short-run with an increase in price of human capital. Since wage is positive, $\frac{d}{dx}CV^2_{SR}(w)$ would be positive if $\frac{\overline{w}_{SR}}{2}\frac{d}{dx}Var_{SR}(w) - Var_{SR}(w)\frac{d}{dx}\overline{w}_{SR} > 0$. Then we look at this expression at $x = 1$ which is equal to

$$\begin{aligned}
& \frac{\overline{w}_{SR}}{2}\frac{d}{dx}Var_{SR}(w) - Var_{SR}(w)\frac{d}{dx}\overline{w}_{SR} \\
= \quad & \left\{\left[\frac{1}{1-\alpha}\frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} - \frac{2-\alpha}{1-\alpha}\frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)} + \frac{\delta(1+\delta)}{(1-\delta)^2}\right]Var(Q) + \frac{\delta}{(1-\delta)^2}\overline{Q}^2\right\} \\
& \times\left\{\theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}\right\} \\
& -\theta_H\overline{Q}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}\right) \\
& \times\left\{\left(\frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} - \frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)} + \frac{\delta(1+\delta)}{(1-\delta)^2}\right)Var(Q) + \frac{\delta}{(1-\delta)^2}\overline{Q}^2\right\}.
\end{aligned}$$

Evaluating this expression at $\beta = 1$ gives

$$\frac{\delta}{(1-\delta)^2}\left\{(1+\delta-2\alpha\delta)Var(Q)\theta_L l + \theta_L l\overline{Q}^2 + \frac{\alpha\delta^2}{1-\delta}\theta_H\overline{Q}^3 + (1-\alpha^2+\alpha^2\delta)\frac{\alpha\delta^2}{(1-\delta)(1-\alpha)}\theta_H\overline{Q}Var(Q)\right\}.$$

Notice that $1+\delta-2\alpha\delta > 0$ and $1-\alpha^2+\alpha^2\delta > 0$ for all $\alpha \leq 1$. Therefore, $\frac{d}{dx}CV^2_{SR}(w)\big|_{x=1} > 0$ when $\beta = 1$. ■

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