Making Financial Markets:
Contract Enforcement and the Emergence of Tradable Assets in Late Medieval Europe

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Abstract

The emergence of medieval markets has commonly been seen in the literature as hampered by lack of contract enforcement and archaic institutions like merchants' communal responsibility. Merchants traveling to a different marketplace to trade there could be arrested and held liable for any debts incurred by any merchant from their hometown. We revisit a traditional literature in legal history, which argued that market making and market privileges arose precisely in the context of communal responsibility and its regulation through trade treaties between cities. Communal responsibility was effective in enforcing credit contracts and enabled merchants to use bills of exchange in long distance trade even if reputation effects were absent. We implement communal responsibility in the Lagos and Wright (2005) matching model of money demand, assuming that preference shocks follow a two-state Markov chain. We derive conditions under which cash and credit in the anonymous matching market coexist. For fixed but sufficiently low cost of credit, agents will pay with cash in low-quality matches, and use cash and credit in high-quality matches. The use of credit in the communal responsibility equilibrium reduces the money holdup in the matching market and thus leads to Pareto improvements. We argue that historically, financial market making using communal responsibility was effective in overcoming the cash constraints in cross-city trade, while locking traders into a de-facto firm with their own merchant guild's members.

Keywords: Communal responsibility, matching, money demand, credit

JEL codes: N2, E41, D51

I. Introduction

How did financial instruments develop in late medieval Europe, and why? Financial contracts can only arise when trust or legal enforcement are strong enough to break the anonymous, one-shot character of barter and cash-in-advance transactions. Early medieval trader, dealing with their anonymous counterparts in an environment hardly protected by any enforcement, could not easily rely on repeated transactions giving rise to reputation effects. Nor could they hope for contract enforcement by local authorities, whose reach was ineffective and at best only local. Under those circumstances, transactions that were asynchronous across space and time were ruled out, or limited to ethnic and religious minorities small enough to make reputation effects work, see Greif (1993).

Nevertheless, starting in the late 13th century, precisely such transactions are being observed. Traders in Italy, as well as in Flanders, began to use credit letters to pay for deliveries in inter-city trade, or to remit money from one place to the other even if no underlying transaction of goods is being observed. Through the marketplaces of Northern France, Flanders, Germany along the Rhine, these methods gradually spread out to the north of Europe, and were adopted e.g. by the cities of the Hansa for their trade across the Baltic and the North Sea. Soon enough, there was also a growing practice of camouflaging credit between merchants of the same place as money remittances from one city to the other. A legal literature emerged that dealt with the admissibility of such purely financial transactions, which violated the ban on interest imposed by the Catholic Church. More importantly, a growing number of cases in dispute was being taken to the courts.

This is what interests us here. The present paper looks into the enforceability of credit transactions under the narrow legal constraints of the medieval marketplace – a city republic or often enough only a small market town, whose jurisdiction hardly extended beyond the city limits. Despite these restrictions, financial relations began to flourish everywhere across Europe.

The emergence of letters of credit and the financial instruments they involved into, like bills of exchange, money remittances, and on the British Isles, checks, has been studied intensely by legal and economic historians working in the tradition of the Historical School of the 19th and early 20th century. Their research looked into the possible legal sources of these instruments, as well as the interregional compatibility of town statutes and laws regulating their use. While there seems to have been no agreement among these scholars about the legalistic issues involved, these studies provide a rich sample of individual cases, as well as a broad meta-sample through its review of the pertinent legal literature itself, which goes back at least to the 16th century. The qualitative but rich evidence sampled in these studies can thus be considered representative of commercial and legal practice prevailing at the time.

Financial instruments, as far as their use was recorded, developed in the context of legal regulation by market and city authorities almost from the beginning. This is particularly manifest in the case of the big medieval and early modern fairs. These fairs mostly took place at regular frequencies, and developed a surprisingly rich market microstructure, ranging from centralized matching and brokering to detailed regulations about end-of-market clearing operations and the exclusion of aftermarket transactions from court enforcement – or even their outright ban. Fostered by their privileged status, the major fairs developed their own payment and clearing systems. This process would eventually culminate in annual European trans-market clearing fairs, dedicated exclusively to financial market transactions. However, while these fairs played a prominent role in shaping the financial map of Europe, a more complete picture is obtained by studying the less prominent everyday operations among merchants across cities. Our objective is to provide an economic rationale for the emergence of credit facilities in these environments.

The obvious hurdle to overcome for financial instruments under the institutional constraints of the Middle Ages and Early Modern Europe is that of lacking enforcement across space. If traders located in different places wanted to conclude financial agreements with one another, and if forces other (or stronger) than reputation were needed, instruments had to be devised that made it possible for merchants from City A, who had claims against merchants from City B, to seek and obtain enforcement at home, i.e. in the court of City A. The legal institution of communal responsibility indeed provided this: a city court would hold the merchants from any given other city collectively liable for the debts of their fellow citizens. In that way, their assets, to the extent they were within the court's reach, would serve as collateral. As a consequence, communal responsibility effectively bound all merchants of a given city together in some kind of forced mutual insurance, with obvious implications for the incentives of these traders to form guilds, restrict access, impose sanctions, and monitor each other's financial conduct closely.

Studying the evidence collected by lawyers and legal historians on credit and the emergence of financial instruments, we find that the emerging credit instruments were indeed tailored to exploit communal responsibility. We see the same structure arising from the very first credit letters down to the bill of exchange. In many cases, a merchant would draw on a fellow citizen living in the creditor's jurisdiction as an intermediary for payment of his debt. By communal responsibility, the intermediary would be held liable for the debts of his fellow citizen[1]. Hence, the creditor could sue the intermediary for payment in the creditor's court, while the intermediary could sue the debtor in the debtor's court. This way, communal responsibility played a favorable role in collateralizing credit across jurisdictions.

Communal responsibility also had important negative aspects. Traveling merchants arriving in a city might find their merchandise confiscated and themselves being taken prisoner, just in order to pay for claims against a fellow citizen, whose legitimacy might even be in dispute. This had potentially disruptive effects on trade relations wherever territorial states were absent, see Greif (2004). The creation of compact territories with unified jurisdictions was a major step forward towards easy enforceability of claims and of financial market integration, see North (1981). However, in Central Europe as well as in Italy, where territorial fragmentation was prevalent until the 19th century, collective responsibility remained the rule, to be broken only by trade and payments agreements between political authorities[2]. In the present paper, we will highlight the evolution of financial instruments under these conditions, in which terri-

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[1] Especially in Italy, there existed also a four-way bill of exchange, in which one trader made a cash payment to another in City A, to be redeemed by a third trader who would receive payment from a fourth trader in City B.

[2] In international law, the principle of collective responsibility survives to this day. As we write this paper, a Swiss oil broker attempted to confiscate pictures at an exhibition of Russian art in the Swiss city of Zurich, in order to be compensated for a failed oil deal. The same broker is reported to have previously obtained injunctions against his former Russian business partners in several countries, potentially threatening trade with seizures or sanctions. The Swiss government in the end resorted to national security concerns to override the claim, see *New York Times*, "Swiss Oil Broker Tries to Seize Russian Art to Collect Debt", November 16, 2005.

torial integration came far too late to provide a good explanation for financial market integration.

How collateralization may work to support the use of credit as a means of payment across jurisdictions is also an interesting issue from a theoretical standpoint. Monetary economists working in the tradition of Kiyotaki and Wright (1989) and Kiyotaki and Wright (1991) have studied the use of money in a matching framework, in which buyers and sellers meet randomly for anonymous, one-shot exchanges. He, Huang and Wright (2003) devise a matching model in which money and credit coexist. In their model, money is indivisible, giving rise to credit transactions. In what seems closest to the problem of interest to us here, Faig (2004) and Faig and Jerez (2005) interpret the matching setup as a village economy, in which cash is used transactions across villages, and credit for transactions within villages. In their model, however, all credit serves purely for financial and insurance motives; there is no trade credit involved.

In the present paper, we contribute to this research, drawing on the canonical setup of Lagos and Wright (2005). We investigate credit transactions, not within but across cities in a matching framework that we interpret as a village economy as in Faig (2004). The crucial issue in this context is collateralization and enforceability of credit transactions when the individual match is one-shot among anonymous trading partners. Communal responsibility in fact provides the required collateralization, thus supporting an equilibrium in which credit is employed.

The rest of this paper is organized as follows. Section II delves deeper into the history of credit instruments in Europe and puts their evolution in the context of communal responsibility. Section III adapts the Lagos and Wright (2005) framework and characterizes (restricted) social planner and decentralized market solutions with and without communal responsibility. Section IV discusses the results and places them in the context of modeling alternatives and their possible historical interpretations, tracking historical analogs to modeling devices that have become popular in the matching literature, such as the "big family" assumption of Shi (1997). Section VI concludes with implications for further research.

II. History

To remit money from one place to another, 13th century Italians had developed the business of "campsores", money changers who would receive cash payment in one location, and pay out the equivalent in a different location. The term for these money remittances, "cambium" (change), soon became synonymous for a wider class of payment transactions, in which credit was provided, see Freundt (1899). Along the Mediterranean, remittance and credit contracts would often be notarized, and two parallel letters of credit issued to be sent by separate mail, to make the presenter's claim more credible. In Western and Central Europe, court registers would assume the same task. These credit letters were themselves not yet a negotiable title that constituted an enforceable property right. Scholars in the tradition of the Historical School argued over the origins of the bill of exchange (see Goldschmidt (1891), Schaps (1892), Schaube (1898), Freundt (1899); for a brief summary of the argument and rich additional evidence see de Roover (1953)). The tendency of the discussion was to emphasize the continuity with Roman law, and to refute a claim by Goldschmidt (1891) that the observable "cambium" could be readily identified with the modern bill of exchange. Still, this older research agreed that money transfer and credit operations across cities occurred early, often, and were regularly brought to and enforced by the courts.

When these assets became tradable has continued to be a matter of dispute. Usher (1943), van der Wee (1963) argued for a rather late beginning, citing court cases from Flanders from the early 16th century. From then on, the holder of a bill of exchange was entitled to payment, no matter what the underlying original creditor/debtor relation was. de Roover (1953) finds tradability of bills of exchange in Italy to be a rare exception before the 16th century. Munro (1991), Munro (1994) argues that tradability of bills of exchange was the norm under the Law Merchant (albeit not under the Common Law) in medieval England already in the 14th century. However, bills of exchange were apparently used predominantly in foreign trade, especially with the Low Countries. For domestic transactions, easier, seemingly less sophisticated instruments were in use. Comparatively little attention has been paid to developments in Germany. This may partly be a consequence of van der Wee's (1963) hypothesis that Germany's financial markets were connected to the international market only in the 16th century. It is also partly due to large-scale loss of archival documents pertaining to major places like Cologne or Frankfurt.

Recent research by Rothmann (1998) has filled the gap for the city of Frankfurt, partly by consulting archival sources from Frankfurt's trade partners, often smaller cities throughout the German-speaking countries whose records survived World War II. Rothmann cites evidence for the use of bills of exchange, and their transfer from one party to the other, for as early as the late 14th century. In the first half of the 15th century, bills of exchange payable in Frankfurt regularly included the clause "*wer den* *brief innhat*" (who the bearer of the letter is) (Rothmann, p. 335, p. 484). This seems remarkably close to similar stipulations on English credit letters issued under the Law Merchant at the same time (see Munro, 1991).

Like elsewhere in Europe (on the prominent cases of the Campagne fairs, later Lyon, Genua, and Geneva, see Boyer-Xambeu, Deleplace and Gillard (1991)), Frankfurt's trade fairs soon became nationwide clearing institutions. For a sample of cities from Germany and Switzerland, Rothmann documents that among the bills of exchange, mortgaged rents, and other credit and payment instruments registered in the respective city courts, those payable in Frankfurt or at the time of the Frankfurt fairs are dominant. Thus, the fairs synchronized and integrated financial markets in all of Southern and Central Germany, even for payments that were not physically due in Frankfurt itself.

Transferring bills of exchange (or other letters of credit) was thus a common phenomenon on the Frankfurt market in the 14th century. As Frankfurt was strongly integrated with Venice (probably more so than with the Low Countries, where Cologne acted as an intermediary), the practice must have been less unusual in Northern Italy than de Roover (1953) would imply. However, von Stromer (1976) notes that the Frankfurt fairs as a financial market benefited from a trade and payments embargo by the Holy Roman Empire against Venice in 1412/13 and again from 1418-1433. This may have set off an isolated development in Germany, albeit only temporarily. Apparently, financial instruments were well developed in most areas of Europe in the 15th century, and agents were able to pay without cash, and to transform one form of financial asset into another both over time and space.

Transferable or not, credit letters and other financial instruments, like rents, critically depended for their practical use on enforceability. Notarization of credit letters was the norm in Southern Europe, see Freundt (1899), de Roover (1953). In central and Western Europe, registration in court or city registers prevailed, see Munro (1991). City courts convened to enforce payment, or to reach a binding settlement between both parties, and their rulings were apparently effective enough to secure the continuing use of financial instruments. The question is why this is.

The easy case is probably medieval England. Under English Common Law, merchants registering their claim in the courts of York, London, or Bristol would have the right to seize the debtors' assets in all of England in the case of default (Munro, 1991). The enforceability of claims across a large, unified jurisdiction was undoubtedly a major advantage for England's commercial development (Greif, 2004). Beginning in the mid-14th century, enforceability of European-style law merchant claims was assured by giving merchant courts applying these rules the right to seize assets. Again, this gave England an advantage, and Munro (1991, 1994) argues that the Law Merchant gradually crowded out the more expensive Common Law debt instruments among English merchants.

Merchants on the European continent had to cope with fragmented jurisdictions. Feuds, wars, and city bankruptcies inhibited the flow of goods and capital across locations. Feuds, wars, and city bankruptcies inhibited the flow of goods and capital across locations further. Still, financial flows were surprisingly undisturbed by such events, and the limitation of local jurisdictions hardly seems to have been a problem.

In 1410, the city of Cologne banned its citizens from attending the Frankfurt fairs in order to force Frankfurt to lower its fees. Only those having to settle their debts or collecting debt in Frankfurt were allowed to go (Rothmann, 1997, p. 422). While debt default by institutional debtors, such as cities or territories, was a problem, individual default was not, at least no more than within a given jurisdiction.

The reason for this was the threat of communal responsibility. Under this seemingly archaic institution, merchants would be held liable outside of their own city for the debts of their fellow citizens. Cities and marketplaces throughout Europe, particularly in Germany and Italy (but also in Flanders and France) adopted stringent rules and court practices on how to deal with bad debts by foreigners. The typical sanction was to take reprisals against the fellow citizens of a defaulting foreigner, be it through seizure of any goods or, if need be, outright imprisonment. The legal rules governing communal responsibility and the rights of reprisal were studied intensively by legal historians from the Historical School, see e.g. Wach (1868), Kisch (1914), Planitz (1919).

Medieval sources mention communal responsibility among merchants in the 12th century, usually as an obstacle to merchant activity, and a bad practice that had to be abdicated or at least regulated Planitz (1919, p.97, 171). Cases of wild seizure and violence were apparently frequent. Merchants would soon avoid places that were notorious for such wild reprisals. City charters regulated and restricted the application of communal responsibility, although the practice was not abdicated altogether, see Wach (1868). Mutual privileges aimed to provide safe passage for merchants, e.g. between Flanders and places in Germany in the late 12th century Planitz (1919, p. 168).

Similar agreements limiting reprisals existed between Lorraine and the Low Countries, and in numerous city charters and treaties among cities along the Rhine.

Legal historians concluded from examining hundreds of city charters that communal responsibility was ubiquitous on the European continent, and that its legal regulation was practically identical everywhere. (Kisch (1914), Planitz (1919)). Typically, evidence on the justification of the debt had to be procured, as well as of a failed previous attempt to enforce payment in the debtor's city court. Imperial law in Germany (whose enforceability was, however, notoriously low) included this clause as early as 1231, and city statutes everywhere repeated it. In Italy, cities maintained public blacklists of individual defaulters and of cities having failed to protect creditor rights.

The typical procedure included some amount of debtor rights. Although regulations often involved a right for the creditor to act on the spot before calling the judge, the rule was that the case had to be decided by the judge before dawn, otherwise the arrested merchant would be released Kisch (1914, p. 40). During major fairs and markets, judges would be walking the streets day and night to provide on-the-spot rulings, and all other court activity would be suspended while the fairs lasted.

Indeed, the major fairs enjoyed privileges of free access. Conditions on the Champagne fairs were special because of repeated interventions by the king (see Thomas (1977). Visitors to the less important Frankfurt fairs were exempt from communal responsibility for all debts owed elsewhere in the Holy Roman Empire (but of course not in Frankfurt itself), see Brübach (1994) with examples. This gave debts owed in Frankfurt senior rank over other debts, and certainly contributed to the privileged status of the Frankfurt fairs within the payment system evolving in Germany since the 14th century. Merchants drawing a bill of exchange on the place of a major fair thus delegated enforcement to the city authorities of that fair. With its large network of trade and payments relations, its capabilities of enforcement were superior to anyone else's. We also see that disputes about bills payable in Frankfurt were settled, not in Frankfurt but in the debtors' courts, as in the examples of Basel and Constance. This is in line with the typical way communal responsibility was regulated, as Frankfurt courts would only take a case that had not been settled there previously.

These networks were magnified by the indirect links that the smaller places provided. The trade partners of 14th and 15th century Frankfurt for which Rothmann (1998) studied the records in detail include, inter alia, Constance, Basle, Cologne and Antwerp. In each of these cities, the town records include numerous payment agreements between locals and foreign merchants, which were due in Frankfurt. The network extends quickly: other clearing places mentioned in Constance included Geneva, Venice, Ravensburg, Zurzach, Zurich and many more. The books of Basle mention Lyon, Strassbourg, Brugge, Mechelen and Antwerp as clearing places besides Frankfurt. The contract partners of Basle's merchants were from Berne, Cologne, Mecheln, Strassbourg, Augsburg, Ravensburg and again from Frankfurt. The foreign partners of merchants from Constance were from Strassbourg, Ulm, St. Gallen, Ravensburg, Nuremberg, Basle, and, again, Frankfurt. The liabilities were either trade credits or bill of exchanges. In such a multilateral network, towns cared about the enforcement of contracts, independently of where the trade took place and independently of whether the citizen was a creditor or a debtor.

Communal responsibility as a backstop when trade treaties and payment arrangements failed was surprisingly long-lived. In the following, we provide two cases from the late 16th and 17th century, in which the enforcement of liabilities still worked on a inter-town basis. We provide micro evidence of the town of Linz in Upper Austria and from St. Gallen in Switzerland. During the 17th century, both towns played an important role as inter-regional markets for goods as well as of bills of exchange.

The case of St. Gallen is a proposed unilateral change in the town's trade treaties from 1661 (see on the following Simon (1974)). It documents that reprisals between cities were still in use. Town officials sought to change statutes that guaranteed equal treatment of foreigners and locals in bankruptcy cases involving bills of exchange. The proposed new statutes would give preferential treatments to the locals of St. Gallen. These suggestions were intensively discussed and attacked by the local merchants. They argued that such rules would destroy the well-established position of St. Gallen as an international clearing place for bills of exchange. It would violate the trading agreements that existed with all major trading places like Augsburg, Nuremberg, Hamburg, Erfurt and Strassbourg, and the town would have to expect heavy reprisals. Merchants from St. Gallen would be discriminated not only in Germany, but also in France and Italy and wherever they tried to use bills of exchange. In this context, the merchants mention the statutes of the major European finance centers, Venice, Amsterdam, Hamburg, Barcelona, Rome, Genua, etc.

The fairs of Linz document the ongoing use of the laws of reprisal during the 17th century (on the following see Rausch (1969)). In the 16th century, Linz, situated in the heartlands of Habsburg's Austrian possessions, had become an important fair and financial market. The fair had its high time during the 17th century, when it was a clearing place for Central and Eastern Europe. During this period, the statutes regulating the use of reprisals were renewed and confirmed by the Austrian emperor several times. The arguments put forward by the town officials in favor of the reprisal option are illuminating. They argued that reprisals were absolutely necessary to guarantee that merchants pay their liabilities. As the application of the existing rules had become lax, debtors' willingness to pay had declined. The right to apply reprisal laws would guarantee the competitiveness of the Linz fairs in Europe. Support for the legislation came also from the merchants themselves. In 1649, the visitors to the fairs set up and signed a petition to the Emperor in support of the proposed measure. The list included traders from Augsburg, Nuremberg, Cologne, Vienna, Geneva, Salzburg and many other places. Apparently, communal responsibility supported an outcome that was more preferable to merchants than any feasible alternative.

III. Money and credit demand in a village economy with search

1. The basic setup

The following is a matching model of the demand for money and credit, adapted with minor changes from Lagos and Wright (2005). We assume a city [village] economy as in Faig (2004) and Faig and Jerez (2005), in which agents match with each other to trade city-specific goods. In our version of this model, agents may employ both money and/or bills of exchange as a medium of payment in inter-city trade. We are interested in characterizing equilibria under various assumptions on the enforceability of such payments, and want to seek conditions under which the use of credit and the coexistence of money and credit are supported.

Let there be a finite, countable number *N* of cities, each inhabited by a continuum *di* of agents on the unit interval, such that total population size is again *N*. For most of the paper we will adopt a sloppy notation and suppress the *di* index. Agents in each city produce a perishable city-specific good. While this good is not consumed in that city itself, it is a consumer good in all other *N* – 1 cities, and is a perfect substitute for the goods produced in all other cities. This setup is one of complete specialization and gives rise to the need for trade.

As in Lagos and Wright (2005), agents in these cities also produce and consume a general, non-tradable good, which is traded on a Walrasian market within the city and is perishable as well. The only store of value in the Lagos/Wright setup is money, which has no intrinsic value and carries no interest. We add interest-bearing bills of exchange, which are issues by the agents.

As to city-specific tradable goods, agents derive utility $u\ (q^b)$ from consuming quantity q^b. u is assumed twice differentiable, $u'>0,\ u''<0$, and satisfies $u(0)\to-\infty, u'(0)\to\infty, u(\infty)\to\infty, u'(\infty)\to 0$ [3]. Agents can produce this good according to a homogeneous, convex cost function $N(q^s)$ where $N'>0,\ \ N''\geq 0$.

Agents have utility $U(C)$ over the consumption *C* of the non-tradable general good, with the same properties as above and the additional regularity condition $U'(C^*)=1$ for some $C^*>0$. There is disutility from producing these general goods, which is as-

[3] We set $u(0)\to-\infty$ to rule out autarky equilibria, in which there is no consumption of special goods, and hence no need for trade.

sumed to be linear in their output *Y*. Without loss in generality, the cost or disutility of producing *Y* is assumed equal to *Y*, which implies that output is measured in (dis-) utils. This linearity assumption, introduced and discussed in detail in Lagos and Wright (2005), is crucial for the tractability of the problem, as it precludes wealth effects and ensures closed form solutions. A discussion of its meaning and of possible alternative is given further below.

We assume that the market for tradable goods is cleared first and the market for non-tradables afterwards. As in Faig and Jerez (2004), we interpret the market for special goods as a market for tradables. This market is characterized by an anonymous random matching process between sellers and buyers from different cities, with exogenous matching probabilities. Every morning, agents become buyers or sellers. Sellers set out to produce the tradable good according to cost function $N(q^s)$ and await the arrival of foreign buyers, while domestic buyers go out to another city to match with that city's sellers there, and derive utility from the quantity bought, $U(q^b)$.

We assume that the market for tradable goods is cleared first and the market for non-tradables afterwards. As in Faig and Jerez (2004), we interpret the market for special goods as a market for tradables. This market is characterized by a matching process between sellers and buyers from different cities, with exogenous matching probabilities. Every morning, agents become buyers or sellers. Sellers set out to produce the tradable good according to cost function $C(q^s)$ and await the arrival of foreign buyers, while domestic buyers go out to another city to match with that city's sellers there, and derive utility from the quantity bought, $U(q^b)$. The probability of becoming a seller and successfully being matched with a foreign buyer is π. The probability of becoming a buyer and being matched successfully with a foreign seller is, $1-\pi$. For simplicity, we set both probabilities equal to one half. Nothing depends on the equality of these probabilities, except for notational simplicity. We disregard the possibility unsuccessful matches (although their introduction would be straightforward)

Not all successful matches are good ones. Conditional on a buyer/seller match establishing itself, α denotes the probability that match quality is high. α may also be interpreted as a preference shock as in Faig and Jerez (2004). Associated with α is utility $U(\overline{q})$ from consuming $\overline{q}$. In contrast, a bad match occurs with conditional probability $1-\alpha$. It leads to lower utility $U(\underline{q}) < U(\overline{q})$ and hence to lower consumption $\underline{q} < \overline{q}$.

The matching setup in this city economy becomes interesting once money and a financial sector are introduced. We assume two assets, money *m* and one-period interest-bearing commercial bills *b*. As all goods in the economy are perishable, these two assets are the only stores of value. While we assume the stock of money to be given and constant, agents can create commercial bills with their signature, just like in everyday life when employing a checkbook. As money has no intrinsic value, agents would prefer to refrain from its use and resort to either barter or credit for their payments[4].

[4] Notice that this does not rule out gold or silver as monetary media. Adding utility of holding money (gold) would complicate the notation but not affect the results fundamentally.

Our specialization and matching setup across cities and agents rules out barter (technically speaking, we assume there is no double coincidence of wants). Agents wanting to consume tradables must leave their hometown to buy the goods. Agents producing stay at home and cannot consume the tradable home good. As a consequence, the only feasible matches in this market are between buyers and sellers from different cities. Nothing much depends on ruling out double coincidence of wants, as long as the complete specialization setup is retained.

We assume that there exist limits of enforcement across cities. In the extreme, enforcement may be absent altogether. Given the random one-shot matching of a continuum of anonymous agents, individual reputation effects that might help to ensure payment in cross-city trade are absent. However, agents are assumed to be tagged with their city identity. That is, although individual agents are anonymous, their city of origin is known. Therefore, reputation, tit-for-tat etc. equilibria may exist across cities if city authorities are allowed to behave strategically. We mention the possibility of such equilibria merely as a reference case and focus on the non-cooperative outcome instead.

Absent inter-city enforcement, agents in cross-city commerce would be reduced to using money in their transactions. The setup then generates precautionary money demand high enough to cover the needs from the highest expected preference shock. However, this is not the only non-cooperative equilibrium. The model also opens room for an enforcement technology, which we call city-specific reprisals. Such reprisals can be imposed unilaterally. The presence of city-specific reprisals leads to a welfare improvement.

Let $\tilde{b}$ be the amount that any individual wants to pay using bills of exchange. As the transaction volume in each match is dependent on the realization of the state (good, bad), the desired amount of payment by bills may be state dependent, too. Likewise, let $\tilde{m}$ be the amount of cash payments the same individual wants to make. Payment by cash carries the implicit cost $i \cdot \tilde{m}$, as cash bears no interest. Payment by bills of exchange is assumed to carry a fixed resource cost *T*. This cost is assumed to be lost to the economy, i.e. it does not accrue to sellers as revenue.

Thus, payments in cross-village trade may be effected in either cash or bills:

$$\tilde{m} + \tilde{b} \geq \tilde{q} \ , \ \ \tilde{q} \in \{\underline{q}, \overline{q}\}$$

As money and commercial paper are the only stores of value, each agent's initial wealth is equal to his or her initial money and bond holdings:

$$a = m + b$$

where b are the bond holdings inherited from the last period, before an agent goes out and makes financial decisions, e.g. by underwriting new bonds. Together, this leads to the following setup:

$$\begin{array}{lll} E\{V(a)\} = & \max\{\alpha\,\pi[U(\overline{q}) & + W(a - \overline{d}\,(a) - \overline{D} \cdot T + i\,(a - \overline{D}[\overline{m} - \overline{\tilde{m}}] - (1 - \overline{D})\,\overline{m} - \overline{D}\,\overline{b})] \\ \quad + & (1-\alpha)\,\pi[U(\underline{q}) & + W(a - \underline{d}\,(a) - \underline{D} \cdot T + i\,(b - \underline{D}[\underline{m} - \underline{\tilde{m}}] - (1 - \underline{D})\,\underline{m} - \underline{D}\,\underline{b})] \\ \quad + \alpha & (1-\pi)[-C(\overline{q}) & + W(a + \overline{d}\,(a) + i\,(b + \overline{D}\,\overline{b})] \\ \quad + (1-\alpha) & (1-\pi)[-C(\underline{q}) & + W(a + \underline{d}\,(a) + i\,(b + \underline{D}\,\underline{b}]\} \end{array}$$

subject to:

$$
\begin{array}{ccc}
a & = & m+b \\
\tilde{d} & \leq & \tilde{m}+\tilde{b} \\
\tilde{q} & = & \tilde{d} \\
\tilde{q} & \in & \{\underline{q},\overline{q}\}
\end{array}
$$

$$
\overline{D} \begin{cases} =1, & \tilde{q}=\overline{q}, \tilde{b}=\overline{b}>0 \\ =0, & \tilde{q}=\overline{q}, \tilde{b}=\overline{b}=0 \end{cases}
$$
$$
\underline{D} \begin{cases} =1, & \tilde{q}=\underline{q}, \tilde{b}=\underline{b}>0 \\ =0, & \tilde{q}=\underline{q}, \tilde{b}=\underline{b}=0 \end{cases}
$$

a nonnegativity constraint on money holdings:

$$m \geq 0$$

and a transversality condition on bond holdings to rule out bubbles and Ponzi schemes:

$$\lim_{T \to \infty} \beta^T E_t[b_{t+T}] = 0$$

Every agent maximizes the value function dependent on initial wealth, E$\{V(a)\}$. Buyers get matched with sellers at probability $\pi = .5$. Matches lead to high outcomes $\overline{q}$ with probability α and to low outcomes $\underline{q}$ with probability $1-\alpha$. A buyer takes out utility $U(\overline{q})$ and incurs expenditure $\overline{d}$ with probability α, or utility $U(\underline{q})$ and expenditure $\underline{d}$ with probability $1-\alpha$. If a buyer spends $\overline{d}$ paying partly or fully with credit (and if this is supported in equilibrium), the indicator variable is activated, $\overline{D}=1$, and the fixed cost of issuing bills of exchange, $T \geq 0$, is incurred. A buyer paying with bills of exchange in the morning will have to pay interest on these bills in the afternoon. A buyer's initial wealth a is reduced by the amount of resources paid, which is equal to $\overline{d} = \tilde{m} + \overline{b}$ in a good match. If the buyer makes parts or all of his payments in bills of exchange, the indicator variable is active, $\overline{D}=1$, and the interest a buyer earns on his initial bond holdings, $i \cdot b$, is reduced by the interest due on bills of exchange issued while buying, $i \cdot \bar{b}$. The same is true with $\underline{D}=1$ and interest payments $i \cdot \underline{b}$ if a buyer employs bills of exchange in a low-quality match.

Sellers get matched with buyers at probability $(1-\pi)=\pi=.5$. If match quality is high, which again happens with probability α, a seller gets to produce quantity $\bar{q}$ at cost $C(\bar{q})$, with revenues equal to $\bar{d}$. If a seller receives parts or all of the payment in bills of exchange, the indicator variable is active, $\bar{D}=1$, and the seller receives interest payments $i \cdot \bar{b}$ on these bills, which add to the earnings on the initial stock, $i \cdot b$. If a bad match is realized, the lower quantities $\underline{q}$ are produced, and revenue $\underline{d}$ is also lower. $\underline{D}=1$ if a seller received bills of exchange in a low-quality match, with $i \cdot \underline{b}$ as the associated interest payments on these bills. The continuation values $W(a-\underline{d}-...), W(a-\bar{d}-...)$, measuring the values in the afternoon, reflect the reduction of the initial endowment by the purchases made. In case part of all of the purchase has been paid with bills of exchange, indicated by the qualitative variables $\underline{D}, \bar{D},$ a buyer's initial endowment is further reduced by the fixed cost of using bills of exchange, *T*. This cost is assumed to be lost; it does not accrue to sellers as revenue.

A central feature of the Lagos/Wright setup is the afternoon market in general goods. According to the timing convention, the afternoon home market opens after all sellers and buyers return from inter-city trade, and after interest on initial bond holdings is paid. For buyers, these interest payments are further reduced through the amount of bills of exchange they issued in the morning market. For sellers, the interest payments are increased by any bills of exchange they receive as a means of payment[5]. The value of entering the centralized, Walrasian afternoon market with endowment *a* is:

$$\begin{aligned} W(a) = & \max_{X,Y,a'} U(C) - Y + \beta V(a') \\ \text{where:} & \\ C = & \qquad Y + a - a' \end{aligned}$$

where *C* is consumption and *Y* is production of the general good.

Maximizing with respect to *Y* and substituting for *Y* at the market clearing optimum yields:

$$W(a) = \quad U(C^*) - C^* + a + \max_{a'}\{-a' + \beta V(a')\}, \qquad (2)$$

$$\text{where:} \quad U(C^*) = 1$$

This quasilinear preference structure of the Lagos/Wright (2005) model gives the agents' continuation value, *W* (*a*), desirable properties: maximization of *V* (*a'*) with respect to *a'* is now independent of *a*, excluding wealth effects. In addition, *W* (*a*) can be written as a linear function in *a*:

$$W(a) = W(0) + a$$

This greatly simplifies the solution of the matching and bargaining setup. We follow the Nash bargaining setup of Lagos and Wright (2005), where θ is buyer *i*'s bargaining power. There are two Nash bargains to be considered, one in the case of a "good" match, the other in the case of a "bad" match. Except for the difference in transaction

---

[5] Discounting of the value functions *W* (.) could be introduced without substantially affecting the results.

volume d, however, both problems are analogous to each other. The threat points are given by the continuation values $W(a)$ (identical for unmatched buyers and sellers) that would apply in the case of no match at all:

$$[u(\tilde{q}) + W(a - \tilde{d}) - W(\underbrace{b + \tilde{m} + \tilde{b}}_{a})]^{\theta}[-n \cdot \tilde{q} + W(a + \tilde{d}) - W(a)]^{1-\theta}$$

where:

$$\{\tilde{u};\tilde{d}\} = \{\{\underline{u};\underline{d}\};\{\overline{u};\overline{d}\}\}$$

Thanks to the linearity of the continuation values, the bargaining problem simplifies to:

$$\max_{q,d}[\tilde{u}(\tilde{q}) - \tilde{d} - \tilde{D}(T + i\tilde{b})]^{\theta}[-N(\tilde{q}) + \tilde{d} + \tilde{D}\, i\, \tilde{b})]^{1-\theta}$$

where, again:

$$\{\tilde{u};\tilde{d}\} = \{\{\underline{u};\underline{d}\};\{\overline{u};\overline{d}\}\}$$

As can be seen, the bargaining problem and the size of the surplus vary not only with the state, but also with the payments regime $\tilde{D}$.

Lemma 1: *From the FONCs of this problem, we obtain* $\tilde{u}'(q^*) = N'(q^*)$. *The second-order conditions ensure that* $0 < q^* < \infty$.

Proof: see Appendix.

2. The demand for money and credit

Before finding equilibria, it remains to be determined under which conditions payment with cash or with bills of exchange could occur. Under the assumptions made above, cash and credit are perfect substitutes. For each agent, the match qualities define a Markov chain of transaction volumes $\tilde{d} \in \{\underline{d};\overline{d}\}$, and in each state, the cheaper medium of payment will be preferred.

(a) A creditless economy, all inter-city payments in cash

We first look into the case where no credit is used in any state, $\tilde{b} = 0$. Then, all transactions need to be carried out in cash: $\tilde{m} \geq \tilde{d} \in \{\underline{d};\overline{d}\}$. As money carries no interest, its user cost C_m is the interest burden on the transactions volume. So long as the utility of the highest possible transaction is higher than its interest burden, precautionary money demand in a creditless economy will be equal to the highest possible transaction:

$$U(\overline{q}) \geq i \cdot \overline{d}; \tilde{b} = 0, \Rightarrow m = \overline{m}, \quad \tilde{d} \in \{\underline{d};\overline{d}\}$$

In a pure cash economy, the agent's problem becomes:

$$\begin{array}{rlll} E\{V(a)\} = & & \max\{\alpha\,\pi[U(\overline{q}) & + W(a - \overline{d}(a) + i\,b)] \\ & + & (1-\alpha)\,\pi[U(\underline{q}) & + W(a - \underline{d}(a) + i\,b)] \\ & + \alpha & (1-\pi)[-C(\overline{q}) & + W(a + \overline{d}(a) + i\,b)] \\ & + (1-\alpha) & (1-\pi)[-C(\underline{q}) & + W(a + \underline{d}(a) + i\,b)]\} \end{array}$$

subject to:

$$\begin{array}{ccc} a & = & m + b \\ \tilde{d} & \leq & \tilde{m} \\ \tilde{q} & = & \tilde{d} \\ \tilde{q} & \in & \{\underline{q}, \overline{q}\} \end{array}$$

Consequently, the bargaining problem is:

$$[u(\tilde{q}) + W(a - \tilde{m}) - W(a)]^{\theta} [-n \cdot \tilde{q} + W(a + \tilde{m}) - W(a)]^{1-\theta}$$

which, due to the linearity of $W(.)$, becomes:

$$[u(\tilde{q}) - \tilde{m}]^{\theta} [-n \cdot \tilde{q} + \tilde{m}]^{1-\theta}$$

In this form, the bargaining problem is exactly as in Lagos and Wright (2005). From the first-order conditions of this problem, we obtain: $\tilde{u}'(q^*) = N'(q^*)$ as in Lemma 1.

(b) Coexistence of cash and credit: necessary conditions

Cash and credit may coexist in inter-city payments under two assumptions about cost (if such an equilibrium exists, which is yet to be determined). For bills of payment to be a candidate medium of payment, the expected cost of using bills in a "good" match must fall short of the expected interest savings from cash in advance left unused when a low-quality match is encountered:

Assumption 1: $\alpha T < (1-\alpha)\, i \cdot [\overline{m} - \underline{m}]$

The left-hand side of this condition is the expected fixed cost of using bills of exchange. The right hand side is the probability of a low-quality match, $1-\alpha$, multiplied with the interest cost of the cash $\overline{m} - \underline{m}$ that would be left unused if a buyer encounters a low-quality match.

Also, we want the fixed cost of using bills to be positive:

Assumption 2: $T > 0$

Proposition: *Under Assumptions 1 and 2, optimal money demand (if supported in equilibrium) is characterized by the following properties:*

(a) Money demand is positive at $\underline{m} = \underline{q}$*, irrespective of the realization of* $\tilde{q}$*.*

(b) Gross demand for bills of exchange equals $\alpha \tilde{b} = \alpha[\overline{q} - \underline{q}]$*.*

Proof: see Appendix.

The savings from using credit – if there are any in equilibrium, which is yet to be determined – come from the fact that credit is only used in a contingency, i.e. if match quality and transaction volumes are high. The use of bills of exchange allows agents to lower their money demand by $\overline{m} - \underline{m}$. Assumption 1 states that this reduction pays if the expected cost of using bills of exchange falls short of the expected cost of holding $\overline{m} - \underline{m}$. If using credit was costless, money would be driven out of circulation altogether. Assumption 2 precludes such a situation.

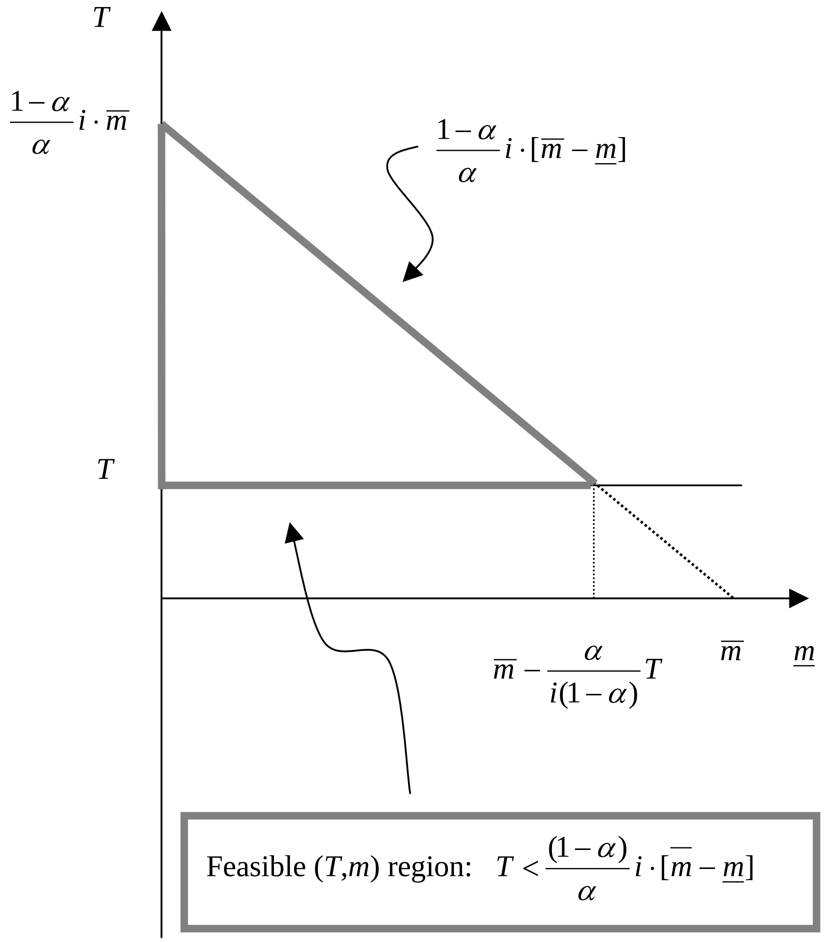

Figure 1: Coexistence of money and credit

Figure 1 represents the situation graphically. As long as the demand for money in a low-quality match is sufficiently small, there is a large wedge between the interest savings from not holding precautionary money demand for a good match and the fixed cost of paying with credit. If credit were costless, it would drive out money altogether; the feasible area would equal the whole triangle spanned by the interest saving condition. On the other hand, with growing fixed cost of paying with bills of exchange the feasible range gets smaller and smaller. It is zero if the expected fixed cost of paying with bills equals the expected interest cost of cash in advance for the best contingency.

The agent's problem in an economy characterized by coexistence of cash and credit becomes:

$$
\begin{array}{llll}
E\{V(a)\} = & \max\{\alpha\,\pi[U(\overline{q}) & + W(a - \overline{d}(a) - T + i\,(b - \overline{b}))] \\
+ & (1-\alpha)\,\pi[U(\underline{q}) & + W(a - \underline{d}\,(a) + i\,b)] \\
+\alpha & (1-\pi)[-N(\overline{q}) & + W(a + \overline{d}(a) + i\,(b + \overline{b}))] \\
+(1-\alpha) & (1-\pi)[-N(\underline{q}) & + W(a + \underline{d}\,(a) + i\,b)]\}
\end{array}
$$

$$
\begin{array}{rcc}
a & = & m + b > 0 \\
\tilde{d} & \leq & \underline{m} + \tilde{b} \\
\tilde{q} & = & \tilde{d} \\
\tilde{q} & \in & \{\underline{q}, \overline{q}\} \\
m & \geq & 0 \\
\lim\limits_{T\to\infty} \beta^T b_{t+T} & = & 0
\end{array}
$$

Consequently, the bargaining problem in a high-quality match becomes:

$$[u(\overline{q}) + W(a - [\underline{m} + \overline{b} + T] - i\,[b - \overline{b}]) - W(a)]^{\theta}[-n\cdot\overline{q} + W(a + \underline{m} + \overline{b}) + i\,[b + \overline{b}] - W(a)]^{1-\theta}$$

which simplifies to:

$$[u(\overline{q}) - (\underline{m} + \overline{b} + T) - i\,(b - \overline{b})]^{\theta}[-n\cdot\overline{q} + \underline{m} + \overline{b} + i\,(b + \overline{b})]^{1-\theta}$$

From the first-order conditions from this problem we obtain: $\overline{u}'(\overline{q}^*) = N'(\overline{q}^*)$ [the proof runs analogously to the proof of Lemma 1].

In a low-quality match, the bargaining problem is:

$$[u(\underline{q}) + W(a - \underline{m} - i\,b) - W(a)]^{\theta}[-n\cdot\underline{q} + W(a + \underline{m} + i\,b) - W(a)]^{1-\theta}\ .$$

From the first-order conditions from this problem we obtain: $\underline{u}'(\underline{q}^*) = N'(\underline{q}^*)$. Note that as $\underline{u}$ and $\overline{u}$ are different utility levels, the optimal output levels will also differ.

3. Equilibrium

(a) A Restricted Social Planner Problem

We proceed gradually towards characterizing equilibrium in this city economy. The first step is to study a social planner's solution, in which the incentive problems pertaining to the use of credit are ruled out. However, this is evidently a restricted problem, as the bargaining solutions in inter-city trade may not be Pareto-efficient. Our use of a central planner solution can be rationalized as restricted in the sense that the social planner is not free to impose a different price setting mechanism. Let $\lambda^j(i)\,di$ be the weight of agent *i* of city *j* in this restricted social planner's problem. Furthermore, let Assumptions 1 and 2 hold. Then, the social planner then considers the problem:

$$
\begin{aligned}
\lambda^j(i)\,di\,E\{V(a)\} = \quad \lambda^j(i)\,di\max\{ \quad & \alpha[U(\overline{q}) + W(a-(\underline{m}+\tilde{b}) - T + i[a-(\underline{m}+\tilde{b})])] \\
+ \quad & (1-\alpha)[U(\underline{q}) + W(a-\underline{m}+i[a-\underline{m}])] \\
+ \quad & \alpha[-C(\overline{q}) + W(a+\overline{d})] \\
+ \quad & (1-\alpha)[-C(\underline{q}) + W(a+\underline{d})]\}
\end{aligned}
$$

subject to:

$$
\begin{aligned}
a &= m + b > 0 \\
\tilde{d} &\leq \underline{m} + \tilde{b} \\
\tilde{q} &= \tilde{d} \\
\tilde{q} &\in \{\underline{q}, \overline{q}\} \\
m &\geq 0 \\
\lim_{T\to\infty} \beta^T b_{t+T} &= 0
\end{aligned}
$$

In this equilibrium, we have the following solutions for buyers' money and credit demand, as laid out in Proposition 1:

$$\tilde{m} = \underline{m}$$
$$\tilde{b} = \tilde{q} - \underline{m}, \ \ \tilde{q} \in \{\underline{q}, \overline{q}\}$$

Given the probabilities for high and low quality matches, α and $1-\alpha$, respectively, given also the continuity of agents, a law of large number applies, and proportion α of the agents matched is in a good match, while proportion $1-\alpha$ is in a bad match. Together, we find that total money demand equals:

$$\tilde{m}_{\text{aggr}} = \alpha \underline{q} + (1-\alpha)\underline{q} = \underline{q}$$

Supply and demand of bills of exchange is equal to:

$$\tilde{b}_{\text{aggr}} = -\alpha\,(\overline{q} - \underline{q}) + \alpha\,(\overline{q} - \underline{q}) = 0$$

i.e. bills of exchange (or inside money, which is what they are) are in zero net supply. The gross supply of bills of exchange is equal to the proportion of good matches in the population times the excess output under good conditions, i.e.:

$$\tilde{b}_{gross} = \alpha\,(\overline{q} - \underline{q})$$

Together, the demand for outside and inside money is:

$$\tilde{m} + \tilde{b}_{gross} = \underline{q} + \alpha\,\overline{q}$$

which equals total output in the economy.

(b) Decentralized market solution, no collective responsibility

Next we characterize equilibria in a decentralized economy, assuming for the moment that no collective responsibility exists, and hence no reprisals take place if debtors fail to honor their bills of exchange. As there is a continuum or anonymous individual agents, individual reputation effects or other strategies that could support a cooperative equilibrium are ruled out: the inter-city payment system collapses into a cash in advance economy. As a consequence,

$$m^b = \overline{q} > \underline{q}$$

$$C_m = i \cdot \overline{q} > i\,\underline{q} + T$$

This holds true if we assume that city governments cannot interact strategically, just as individuals in this setup cannot. As there is a countable, finite number of cities *N*, we cannot rule out strategic interactions like coalition formation, reputation, tit for tat strategies, and the like. To the extent that city governments behave strategically and engage in such behavior, the folk theorem points to infinitely many equilibria that may establish themselves, and that dominate the non-cooperative solution sketched here in terms of welfare. Indeed, European history offers rich accounts of city leagues that alternately privileged and fought each other, and that had fluctuating memberships. If *N* is high we would indeed expect such equilibria to be rather unstable. While we fully acknowledge the possible relevance of such equilibria, they are not our primary focus of interest. What we highlight here is the possibility of mimicking a fully cooperative outcome [or a restricted social planner solution] in the context of non-cooperative behavior.

(c) Collective Responsibility as an Enforcement Device

We now introduce collective responsibility into this framework. As mentioned earlier, collective responsibility means that all agents of a city can be held liable for one another in inter-city trade. This implies that if any bill of payment has been defaulted on, the creditor's city immediately sanctions this by seizing the resources of some other representative of this city[6]. Let $\tilde{b}$ be a defaulted bond presented to an agent while trading abroad. In our setup, an agent out of town is always a buyer[7]. With money holdings of this buyer equal to $\underline{m}$, this is also the amount to be seized from the agent in reprisal against his fellow citizen's default. If the agent had no way of recovering the payment from his fellow citizen, the buyer's value function of the agent under no enforcement, V_b^{NE}, is:

$$V_b^{NE} = U(0) + W(a - \underline{m}), \qquad \text{no enforcement in afternoon markets}$$

However, with full enforcement in local markets, upon returning home in the afternoon, the agent is able to enforce payment from the agent who defaulted in the first place. The buyer's value function under full enforcement at home, V_b^{NE}, then is:

[6] In the previous section we noted that city courts would typically require the plaintiff [i.e. the creditor] to prove that the claim had been impossible to enforce in the defendant's [i.e., the debtor's] court. Default in this sense would imply refusal of the debtor's home court to adequately deal with the case.
[7] This could easily be relaxed, allowing for matches with double coincidence of wants as in Lagos and Wright (2005).

$$V_b^{FE} = U(0) + W(a)\,, \qquad \text{full enforcement in afternoon markets}$$

which is identical to the value function of an unmatched agent. Evidently, this gives any agent who is presented with a defaulted bill, i.e. who is taken liable for his fellow citizen, an incentive to recover the payment after returning home. In other words, the agent becomes the *enforcement agent* of his host city upon returning home.

We have already seen that under collective responsibility, an agent held liable for the default of his fellow citizen has an incentive to recover his loss from the defaulter. As the defaulted sum is $\tilde{b} = (\overline{m} - \underline{m}) \neq \underline{m}$, it may take more than one act of reprisal for the creditor city to fully recover the defaulted credit. In each of this act, the loss to the victim of reprisal equals:

$$E\{\alpha[U(\overline{q}) + (1-\alpha)U(\underline{q})] + \min\{\underline{m}, \tilde{b}\}\} > \min\{\underline{m}, \tilde{b}\}$$

The first term is the agent's expected utility from getting matched and consuming. The second term is the money sum taken from the agent in reprisal. This sum may be equal to the original claim (if small enough), or fall short of it. A city government taking reprisals will seize assets (i.e. the money they brought into town) of more than one fellow citizens of a defaulter citizen until the full amount is met. Hence, any individual loss from taking reprisals will exceed the original claim. The same is true a fortiori of the aggregate loss:

$$s\,\pi\alpha\,[U(\overline{q}) + (1-\alpha)U(\underline{q})] + \tilde{b} > \tilde{b} \quad, s \geq 1$$

Here, s is a factor equal to or greater than one that depends on the number of [measure of] agents held liable to satisfy a single claim. Under full enforceability in the home market, all agents held liable abroad will reclaim their losses from the defaulter in the afternoon market.

In comparison, the gain from defaulting for an agent is:

$$\underbrace{\tilde{b}}_{\text{Agent's gain from default}} < \underbrace{\tilde{b} + s\,\pi\alpha\,[U(\bar{q}) + (1-\alpha)U(\underline{q})]}_{\text{Agent's loss from default}}$$

The LHS of this inequality is the agent's gain from defaulting on a bill of exchange signed when shopping out of town. The RHS of the expression is the same amount again plus the compensation payable to the victims of reprisal, which equals the deadweight loss from their being excluded from the outside market for that period. As can be seen, the net gain from defaulting is negative. Hence, under collective reprisal and collective responsibility, there is no incentive to default on bills of exchange, so that – within the narrow confines of our setup, which excludes bankruptcy for other reasons – default does not occur in equilibrium. As a consequence, the equilibrium under reprisal fully supports the use of bills of exchange, and precautionary money demand equals:

$$m_{\text{aggr}}^{CR} = \alpha\underline{q} + (1-\alpha)\underline{q} = \underline{q}$$

while the net supply of bonds is zero:

$$\tilde{b}_{\text{aggr}}^{CR} = -\alpha\,(\overline{q} - \underline{q}) + \alpha\,(\overline{q} - \underline{q}) = 0$$

and gross demand for bonds is:

$$\tilde{b}_{Gross}^{CR} = \alpha\,(\overline{q} - \underline{q})$$

In short, the equilibrium under collective responsibility replicates the (restricted) social planner equilibrium.

IV. Discussion

The setup of the previous section depends critically on the linearity of the value functions, which excludes wealth effects and permits the considerations underlying Proposition 1. There are several established ways of achieving linearity of $W(a)$. The one we followed in the previous section is the afternoon market for general (in our setup: non-traded) goods proposed by Lagos and Wright (2005); which is based on quasilinear preferences. Other mechanisms have been devised in the literature, and all could give rise to historically meaningful applications in our context. This section presents two of those alternative approaches briefly and provides a discussion of their possible interpretations.

One paradigm is the extended family setup introduced by Shi (1997). In this framework, each family consists a continuum of agents that pool their resources, and among which there exist no incentive problems. Under this risk sharing assumption and assuming the family is diversified over all economic activities, the extended family is fully insured against risk, and no aggregate wealth effects exist. In our setup, the extended family assumption would imply that the family as the basic decision making unit is pooled over sellers and buyers, so that wealth effects on the family as a whole from trading with other cities are absent. As a consequence, the device of a second, afternoon market becomes unnecessary. Under an extended family assumption, the basic value function maximization problem holds not just in expectation but, through the law of large numbers, as a deterministic relationship. Hence, under all realizations the continuation value of an extended family is constant.

Surprisingly, from an historian's perspective, the extended family setup is an empirically relevant concept and not just an artifact created for analytical convenience. Communal responsibility has itself been interpreted by legal historians as an institution originating from family feuds, see e.g. Planitz (1919). Family firms, frequent since the late Middle Ages, were indeed one way to overcome principal-agent problems, especially in long-distance trade – which is precisely what interests us here.

However, the connection between the extended family and the firm goes even further. Building on the work of legal historians on the origins on the firm, one of us, Boerner (2005), shows that the contractual relations characterizing early firms were indeed modeled on medieval family trusts, by which an indivisible family possession, such as a manor, would become the jointly owned asset of the heirs, forcing them to pool all their other assets and receiving a quasi-fixed rent in return.

Applying this logic to the problem of collective responsibility, it is obvious that both the extended family and the firm are stronger contractual relationships than mere collective responsibility towards a third party. In fact, as pointed out in Boerner (2005), collective responsibility itself is not a precursor of the firm. Instead, the firm originates in to solve what in the model of the previous sections is a problem with the linearity of the agents' continuation value.

A second, related concept in the money matching literature that helps to introduce linearity into the continuation values $W(.)$ is the idea of Faig (2004) that before entering a match, each agent can insure the state-dependent outcome within their home city. In our context where match quality follows a two-state Markov chain, insurance premium μ buys actuarily fair state-dependent payoffs such that:

$$\mu = \alpha\,\overline{q} + (1-\alpha)\,\underline{q}$$

Under this insurance scheme, agents' budget constraints become state-invariant; i.e. agents have a fixed income no matter what the outcome of their inter-city trade is. Again, under this assumption the concept of an afternoon market can be dispensed with. This insurance framework is very similar to invoking an extended family – or a firm. Agents trading across cities and representing firms would in fact face such a state-invariant budget constraint. Of course, the feasibility of such a scheme would inextricably depend on the tightness of the firm's internal contractual relationships to deal with the obvious incentive problems. Again, however, the insurance assumption is more than just an analytical device adopted out of convenience. There is evidence that early modern firm contracts indeed sought to implement fixed-income schemes with low-powered incentives but full liability of any agent.

However, setting up extended families or firms (or a combination of both) in the framework of the previous sections comes at a potential cost. As soon as we assume that families or firms have finite measure, strategic behavior and nontrivial dynamic equilibria are introduced. In an extended family, the family, not the individual, is the basic decision maker. Hence, strategic interactions between families will occur, be it within a city or (again assuming a finite number of cities) across cities. The same is true in the case of state-contingent insurance contracts for all agents of one city. This contract essentially makes the city collective the basic decision unit, and strategic interactions between cities should then be observed. Historical evidence for both phenomena is abundant. Our own approach to modeling inter-city trade deliberately abstracts from this, as our aim is only to show that even under the weaker assumption of collective responsibility, trade and payment with credit is supported.

V. Conclusion

Tradable assets emerged in Europe in the context of fragmented city jurisdictions, where the prospects for contract enforcement looked rather limited. This paper studies the effects of contract enforcement through the medieval institution of communal responsibility, which held all traders of a given city liable for each others' debts when trading abroad. We examined evidence collected by legal and economic historians on communal responsibility and its regulation in town statutes across Medieval and Early Modern Europe, and found that it served helped to collateralize debt quite effectively. Medieval cities acted as market makers, ensuring the safe use of credit instruments through the threat of applying communal responsibility. The emerging financial markets soon integrated regionally and across time, following the rhythm of the large national and European fairs. The fragmentation of jurisdictions did little to disturb this process, as communal responsibility gave every trader a strong individual incentive not to default.

Our paper also examines the effects of communal responsibility in a matching model of money demand à la Lagos and Wright (2003), interpreted as a village economy as in Faig (2004) and Faig and Jerez (2005), and makes a contribution to the literature on coexistence of money and credit in monetary search equilibrium. If there is a fixed but sufficiently low fixed cost of credit in trade across cities (such as notary and court register fees), cash and credit may coexist in equilibrium, provided the claims are enforceable.

The contribution of our paper is twofold. On the one hand, we find that the medieval and institution of communal responsibility, by which each city's merchant was held liable abroad for any claims on his fellow citizens, was highly effective in enforcing contracts across cities, and thus enabled financial markets to integrate even in the absence of territorial jurisdictions. On the other hand, we find that in the context of a random matching model of trade, communal responsibility is an enforcement device that supports the use of credit in equilibrium even if individual traders are anonymous, so long as they can be identified by their citizenship.

We see this research as a first step towards providing an integrating framework for the analysis of the demand for money and assets in a pre-modern economy, including also the issue of commodity money, gold inflation, and debasement. We also find that modeling devices that are common in the literature on monetary matching, like big families or other risk-sharing arrangements, are closer to historical reality than it might first seem. Also, the medieval practice of forming guilds can be interpreted as a reaction to communal responsibility, born out of the need to limit fee riding on other merchants within the same city. We see this as a promising field for further research.

References

Boerner, Lars, 2005, The Evolution of the Boundaries of the Firm, *manuscript*.

Boyer-Xambeu, Marie-Therèse, Ghislain Deleplace, and Lucien Gillard, 1991, Goldstandard, Währung und Finanz im 16. Jahrhundert, in Michael North, ed.: *Kredit im spätmittelalterlichen und frühneuzeitlichen Europa* (Böhlau, Vienna).

Brübach, Nils, 1994. *Die Reichsmessen von Frankfurt am Main, Leipzig und Braunschweig* (Franz Steiner, Stuttgart).

de Roover, Raymond, 1953. *L'Evolution de la Lettre de Change. XIV^{e}-XVIII^{e} siècles* (Librairie Armand Colin, Paris).

Faig, Miquel, 2004, Money and Banking in an Economy with Villages, *manuscript*.

Faig, Miquel, and Belen Jerez, 2005, Inflation, Prices, and Information in Competitive Search, *manuscript*.

Freundt, Carl, 1899. *Das Wechselrecht der Postglossatoren* (Duncker & Humblot, Leipzig).

Goldschmidt, Levin, 1891. *Universalgeschichte des Handelsrechts* (Stuttgart).

Greif, Avner, 1993, Contract Enforcement and Economic Institutions in Early Trade: The Maghribi Traders' Coalition, *American Economic Review* 83, 525-548.

Greif, Avner, 2004, Impersonal Exchange without Impartial Law: the Community Responsibility System, *Chicago Journal of International Law* 5, 109-138.

He, Ping, Lixin Huang, and Randall Wright, 2003, Money and Banking in Search Equilibrium, *manuscript*.

Kisch, Guido, 1914. *Der deutsche Arrestprozess* (Tempsky, Wien).

Kiyotaki, Nobuhiro, and Randall Wright, 1989, On Money as a Medium of Exchange, *Journal of Political Economy* 97, 927-954.

Kiyotaki, Nobuhiro, and Randall Wright, 1991, A Contribution to the Pure Theory of Money, *Journal of Economic Theory* 53, 215-235.

Lagos, Ricardo, and Randall Wright, 2005, A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy* 113, 463-484.

Munro, John, 1991, Die Anfänge der Übertragbarkeit: Einige Kreditinnovationen im englisch-flämischen Handel des Spätmittelalters (1360-1540), in Michael North, ed.: *Kredit im spätmittelalterlichen und frühneuzeitlichen Europa* (Böhlau, Vienna).

Munro, John, 1994, The International Law Merchant and the Evolution of Negotiable Credit in Late-Medieval England and the Low Countries, in John Munro, ed.: *Textiles, Towns, and Trade: Essays in the Economic History of Late-Medieval England and the Low Countries* (Ashgate, Aldershot).

North, Douglass, 1981. *Structure and Change in Economic History* (Norton, New York).

Planitz, Hans, 1919, Studien zur Geschichte des deutschen Arrestprozesses, *Zeitschrift der Savigny-Stiftung fuer Rechtsgeschichte, Germanistische Abteilung* 40, 87-198.

Rausch, Wilhelm, 1969. *Die Geschichte der Linzer Märkte im Mittelalter* (J. Wimmer, Linz).

Rothmann, Michael, 1998. *Die Frankfurter Messen im Mittelalter* (Franz Steiner, Stuttgart).

Schaps, Georg, 1892. *Zur Geschichte des Wechselindossaments* (Ferdinand Enke, Stuttgart).

Schaube, Adolf, 1898, Zur Geschichte und Natur des ältesten Cambium, *Jahrbücher für Nationalökonomie und Statistik* 70, 603-621.

Shi, Shouyong, 1997, A Divisible Search Model of Fiat Money, *Econometrica* 65, 75-102.

Simon, Volker, 1974. *Der Wechsel als Träger des internationalen Zahlungsverkehrs in den Finanzzentren Südwestdeutschlands und der Schweiz* (Müller & Gräff, Stuttgart).

Thomas, Heinz, 1977, Beitraege zur Geschichte der Champagne-Messen im 14. Jahrhundert, *Vierteljahrschrift fuer Sozial- und Wirtschaftsgeschichte* 64, 433-467.

Usher, A.P., 1943. *The Early History of Deposit Banking in Mediterranean Europe* (Harvard University Press, Cambridge, Mass.).

van der Wee, Herman, 1963. *The Growth of the Antwerp Market and the European Economy* (Den Haag).

von Stromer, Wolfgang, 1976, Die oberdeutschen Geld- und Kreditmärkte. Ihre Entwicklung vom Spätmittelalter bis zum Dreißigjährigen Krieg, *Scripta Mercaturae* 10, 23-51.

Wach, Adolf, 1868. *Der Arrestprozess in seiner geschichtlichen Entwicklung* (Leipzig).

Appendix

Proof of Lemma 1.

Let B be the maximizer of the Nash bargaining problem:

$$B = \max_{q,d} [\tilde{u}(\tilde{q}) - \tilde{d})]^{\theta} [-N(\tilde{q}) + \tilde{d})]^{1-\theta}$$

Take logarithmic derivatives to find the FONCs:

$$\frac{\partial \ln B}{\partial \tilde{q}} = \frac{\theta \tilde{u}'(\tilde{q})}{\tilde{u}(\tilde{q}) - \tilde{d}} - \frac{(1-\theta) N'(\tilde{q})}{-N(\tilde{q}) + \tilde{d}} \overset{!}{=} 0 \qquad \text{(I)}$$

$$\frac{\partial \ln B}{\partial \tilde{d}} = -\frac{\theta}{\tilde{u}(\tilde{q}) - \tilde{d}} + \frac{1-\theta}{-N(\tilde{q}) + \tilde{d}} \overset{!}{=} 0 \qquad \text{(II)}$$

Insert (II) into (I) to find $\tilde{u}'(\tilde{q}^*) = N'(\tilde{q}^*)$. The derivative properties of $\tilde{u}$ and N ensure that q^* and hence d^* (if they exist) must be positive. Verification of the second-order conditions is straightforward.

Remark: This is a special case of Lagos and Wright (2003), Lemma 2, for the case where the cash constraint on d is not binding. Our setup is simplified with respect to Lagos and Wright in the sense that bonds exist and Ponzi schemes are ruled out by a transversality condition. Then, the cash constraint never binds.

Proof of Proposition 1.

1. We first show that if and only if the fixed cost of payment with bills is positive, money demand will be at least high enough to pay for transactions in a low-quality match: $\tilde{m} \geq \underline{q} \Leftrightarrow T > 0$.

For this to be true, the expected value from paying with a mixture of cash and bonds $E\{W^{(mb)}(.)\}$ must be higher than the value from a pure credit arrangement $E\{W^{(b)}(.)\}$. Due to its linearity, $W(.)$ has the additivity property $W(A)+W(B)=W(A+B)$.

"$\Rightarrow$": We first observe that the trial solution $\tilde{m}=\underline{m}=\underline{q}$ is preferred to a cashless equilibrium $\tilde{m}=0$ if:

$$\begin{aligned}
E\{W^{(mb)}\} = \quad & W(0)+\alpha\cdot \quad [a-(\underline{m}+\tilde{b})-T \quad +i\,(a-(\underline{m}+\tilde{b}))] \\
& +\ (1-\alpha)\cdot \quad [a-\underline{m} \quad\quad\quad\quad\ +i\,(a-\underline{m})]
\end{aligned}$$

$$\begin{aligned}
> E\{W^{(b)}\} = \quad & W(0)+\alpha\cdot \quad [a-(\underline{m}+\tilde{b})-T \quad +i\,(a-(\underline{m}+\tilde{b}))] \\
& +\ (1-\alpha)\cdot \quad [a-\underline{m}-T \quad\quad\quad +i\,(a-\underline{m})]
\end{aligned}$$

$\Rightarrow 0>-(1-\alpha)T \Rightarrow T>0$.

Under the trial solution, if a high quality match is established, bills of exchange equal to $\tilde{b}=\bar{q}-\underline{q}$ will be issued in the morning. In a bad state, payments are by cash only. An agent paying exclusively by credit will face the same budget constraint in a good match. However, in a low-quality match, an agent paying only by credit now needs to bear the fixed cost of payment by credit as well. The value from doing so is lower than that of reserving cash for the purpose.

"$\Leftarrow$": Now assume $0<\tilde{m}<\underline{m}$. Then,

$$\begin{aligned}
E\{W^{(\tilde{m}<\underline{m})}\} = \quad & W(0)+\ \alpha\cdot \quad [a-(\overbrace{\tilde{m}+\tilde{\bar{b}}}^{\bar{q}})-T \quad +i(a-(\overbrace{\tilde{m}+\tilde{\bar{b}}}^{\bar{q}}))] \\
& +\ (1-\alpha)\cdot \quad [a-(\underbrace{\tilde{m}+\tilde{\underline{b}}}_{\underline{m}})-T \quad +i(a-(\underbrace{\tilde{m}+\tilde{\underline{b}}}_{\underline{m}}))] \\
& \quad\quad\quad\quad = W^{(b)} < W^{(mb)} \quad ,T>0
\end{aligned}$$

If the level of money demand $\tilde{m}$ is positive but below the threshold $\underline{m}$, the value associated with this level, $E\{W^{(\tilde{m}<\underline{m})}\}$, is lower than the value associated with $\underline{m}$.

Hence, if the transaction cost of paying with bills is positive, money demand is at least $\underline{m}$. This completes the first part of the proof.

2. We now establish that under Assumptions 1 and 2, money demand, which we know from the above to be positive and at least equal to $\underline{q}$, is actually no bigger than $\underline{q}$, i.e. $\tilde{m} \leq \underline{q} \Leftrightarrow \alpha\, T < (1-\alpha)\, i \cdot [\overline{m} - \underline{m}]$. We first have to show that under the assumptions, the value of holding $\tilde{m} = \underline{m}$ is higher than the value of a pure cash-in-advance arrangement $W^{(m)}(.)$. In a second step we show that money holdings higher than $\underline{m}$ will generate lower value than the one associated with $\underline{m}$.

"$\Rightarrow$" : We first observe that the trial solution $\tilde{m} = \underline{m} = \underline{q}$ is preferred to a pure cash-in-advance solution $\tilde{m} = \overline{m}$ if:

$$
\begin{aligned}
E\{W^{(mb)}\} = \quad & W(0) + \alpha \cdot [a - (\underline{m} + \tilde{b}) - T + i(a - (\underline{m} + \tilde{b}))] \\
& + (1-\alpha) \cdot [a - \underline{m} + i(a - \underline{m})]
\end{aligned}
$$

$$
\begin{aligned}
> E\{W^{(m)}\} = \quad & W(0) + \alpha \cdot [a - \overbrace{(\underline{m} + \tilde{b})}^{\substack{\text{cash payment} \\ \overline{m}}} + i(a - (\underline{m} + \tilde{b}))] \\
& + (1-\alpha) \cdot [a - \underline{m} + i(a - (\underline{m} + \tilde{b}))]
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad -\alpha T \quad & > \quad -(1-\alpha)\, i\, \tilde{b} \\
\underbrace{\alpha T}_{\substack{\text{Expected transaction} \\ \text{cost of using credit}}} \quad & < \quad \underbrace{(1-\alpha)\, i\, \tilde{b}}_{\substack{\text{Expected interest} \\ \text{savings from reducing} \\ \text{precautionary money demand}}}
\end{aligned}
$$

In the above expression for $E\{W^{(m)}\}$, the cash payment in the good match is $\overline{m}$, which is equivalent to expenditure $\underline{m} + \tilde{b}$ in the good match of mixed cash/credit case. The cash payment in the bad match is the lower amount $\underline{m}$. In the pure cash in advance economy of $E\{W^{(m)}\}$, the interest cost of holding cash for both contingencies is $i\,\overline{m} = i(\underline{m} + \tilde{b})$, while the cost of holding cash in the mixed cash/credit economy of $E\{W^{(mb)}\}$ is the lower amount $i\,\underline{m}$. Evaluation of the conditions leads right back to Assumption 2, which proves the "if" part: under the assumption, a combination of cash and credit is preferred to cash only.

$"\Leftarrow"$: Now assume $\underline{m} < \tilde{m} < \overline{m}$. Then,

$$\begin{aligned}
E\{W^{(\underline{m}<\tilde{m}<\overline{m})}\} = \quad & W(0) + \alpha \cdot \quad [a - (\overbrace{\tilde{m} + \tilde{b}}^{\bar{q}}) - T \quad + i\,(a - (\overbrace{\tilde{m} + \tilde{b}}^{\bar{q}}))] \\
& + (1-\alpha) \cdot \quad [a - \underline{m} \quad + i\,(a - \underbrace{\tilde{m}}_{>\underline{m}})] \\
= \quad & E\{W^{(mb)}\} \quad - i\,(\underbrace{\tilde{m} - \underline{m}}_{>0})
\end{aligned}$$

If cash holdings exceed the value associated with purchases in the low match but fall short of a pure cash-in-advance condition, i.e. for $\underline{m} < \tilde{m} < \overline{m}$, the associated value is lower than for $\underline{m}$. The reason is that if a bad match is realized, agents holding $\underline{m} < \tilde{m} < \overline{m}$ have unmatched cash holdings $\tilde{m} - \underline{m}$ whose interest cost is $i\,(\tilde{m} - \underline{m})$. By reducing cash holdings to $\underline{m}$, the interest costs on the excess cash are reduced to zero. Thus, $\tilde{m} \leq \underline{m}$, which completes the second part of the proof.

Taking both parts of the proof together, under assumptions 1 and 2, $\underline{m} \leq \tilde{m} \leq \underline{m}$. □

Proof of Lemma 2:

We need to prove that money holdings $\tilde{m}$ are always bounded. From the proof of Proposition 1, money demand is finite and bounded away from zero under Assumptions 1 and 2: $\tilde{m} = \underline{m} = \underline{q} > 0$. Hence we only need to examine what happens if Assumptions 1 and 2 do not hold. As to Assumption 1, $T > 0$, it follows immediately from the first part of the proof of Proposition 1 that $T = 0 \Leftrightarrow \tilde{m} = 0$: there is no demand for money if the use of credit is costless. This establishes the lower bound on money demand.

As to Assumption 2, $\alpha T < (1-\alpha)\, i \cdot [\overline{m} - \underline{m}]$, the proof of Proposition 1 implies that if the fixed cost of credit is high, $\alpha T > (1-\alpha)\, i \cdot [\overline{m} - \underline{m}] \Rightarrow \tilde{m} \geq \bar{q}$. To show that the strict equality holds, consider the value of holding cash at level $\tilde{m} = \overline{m} = \overline{q}$, denoted before as $E\{W^{(m)}\}$, and compare it to the value of holding a higher amount of cash $\tilde{m} = \overline{q} + m^{+}$, to be denoted at $E\{W^{(m+)}\}$. Then:

$$
\begin{aligned}
E\{W^{(m)}\} \; = \; & W(0) + \quad \alpha \quad \cdot \; [a - \overline{m} - \quad + i\,(a - \overline{m})] \\
& \quad + (1-\alpha) \cdot \; [a - \underline{m} \qquad + i\,(a - \overline{m})]
\end{aligned}
$$

$$
\begin{aligned}
E\{W^{(m+)}\} = \; & W(0) + \quad \alpha \quad \cdot \; [a - \overline{m} \qquad + i\,(a - (\overline{m} + m^{+}))] \\
& \quad + (1-\alpha) \cdot \; [a - \underline{m} \qquad + i\,(a - (\overline{m} + m^{+}))]
\end{aligned}
$$

Agents holding $\overline{m}$ will face interest cost on their cash holdings $i\,(a - \overline{m})$. Agents holding cash in excess of that level, $\tilde{m} = \overline{q} + m^{+}$, will face higher interest cost. The difference in values of holding just $\overline{m}$ or $\tilde{m} = \overline{q} + m^{+}$ is positive:

$$E\{W^{(m)}\} - E\{W^{(m+)}\} = i\,m^{+} > 0$$

which implies that $\overline{m} = \overline{q}$ is indeed the highest upper bound for money demand. ☐

Proof of Proposition 2:

(preliminary)

We first obtain the value function at the optimized levels, and then check its boundedness conditions. To obtain the value function, we insert the bargaining outcomes and obtain:

$$V(a) = \max_{a'} E\{v(a) + a - a' + \beta V(a')\}$$

where

$$v(a) = v_0 + u[q^*(a)] - d[q^*(a)]$$

with $E\{v_0\} = E\{d[q^*(a)] - N[q^*(a)]\} + U(X^*) - X^*$

Due to the concavity of u and convexity of N, $v(a)$ is strictly concave in a (keeping in mind that d is linear). However, it is unbounded above. It is also unbounded below, as $u(0) = U(0) = -\infty$. Moreover, $V(a)$ is unbounded above because of its linearity in a. We must therefore provide upper bounds. The strategy will be to use the constraints on the endowment process and insert these into $v(a)$ and $V(a)$, respectively.

The boundedness of $v(a)$ follows directly from the solution of the bargaining problem in Lemma 1: As $0 < q^*(a) < \infty$ and as u and N are continuous in q, they are bounded from above and below, $0 < u(q^*), N(q^*) < \infty$. For the general good, $0 < (U(X^*), X^*) < \infty$ follows from (2) in the text and from the regularity condition $U'(C^*) = 1$ for some $C^* > 0$.

To find an upper bound for $V(a)$, note that Proposition 2 implied that under Assumptions 1 and 2, an upper bound for money holdings is $\underline{m} = \underline{q} > 0$. By Lemma 2, if Assumptions 1 and 2 do not hold, the upper bound for money holdings is $\overline{m} = \overline{q} > 0$. As a consequence, the transversality condition on bond holdings translates into a transversality condition on total assets:

$$\lim_{t\to\infty} \beta^t a_t = \lim_{t\to\infty} \beta^t \tilde{m} + \lim_{t\to\infty} \beta^t b_t = 0$$

where $\tilde{m}$ assumes the values $\underline{m} = \underline{q} > 0$ or $\overline{m} = \overline{q} > 0$, depending on whether Assumptions 1 and 2 hold or not. Assuming a finite initial value $a_0 > 0$, $V(a)$ must therefore be bounded. To see this, write $V(a_o)$ as:

$$\begin{aligned}
V(a_o) &= E_o\{v_0 + u(q[a_o]) - d(q[a_0]) + a_0 - a_1 + \beta V(a_1)\} \\
&= E_0\{v_0 + u(q[a_o]) - d(q[a_0]) + a_0 - a_1 + \\
&\quad + \beta[v_1 + u(q[a_1]) - d(q[a_1]) + a_1 - a_2 \ ] + \beta^2 V(a_2) \\
&\ \ \vdots \qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad \vdots \quad \} \\
&= \sum_{t=0}^{T-1} E_0\{\beta^t [v(a_t) + u(q[a_t]) - d(q[a_t]) + a_t - a_{t+1}] + \beta^T V(a_T)\}
\end{aligned}$$

Taking limits (which is equivalent to continuing the above substitution process indefinitely):

$$V(a_o) = \lim_{T\to\infty}\sum_{t=0}^{T} E_0\{\beta^t[v(a_t)+u(q[a_t])-d(q[a_t])+a_t - a_{t+1}]\} \quad + \lim_{T\to\infty} E_0\{\beta^T V(a_T)\}$$

The second term on the right hand side of this term is zero. In the first term, $v(a), u(a)$ and $d(a)$ are bounded. It remains to evaluate the linear terms in a_t. As the sequence $\beta^t a_t$ converges to zero from above, the series generated by the partial sums of this sequence converges to a positive finite value: $\lim_{T\to\infty}\sum_{t=0}^{T}\beta^t E_0 a_t = A^* > 0$.

Hence, by substituting into the

$$\sum_{t=0}^{\infty}\beta^t E_0\{a_t - a_{t+1}\} = \sum_{t=0}^{\infty}\beta^t E_0 a_t + \beta^{-1}a_0 - \beta^{-1}\sum_{t=0}^{\infty}\beta^t E_0 a_t = \frac{a_0}{\beta} + \frac{\beta - 1}{\beta} A^* < \infty$$

Thus, $V(a_o)$ is bounded from above. The remainder of the proof is a special case of Lemma 7 in Lagos and Wright (2005).