Why Has CEO Pay Increased So Much?

Xavier Gabaix and Augustin Landier
MIT and NBER, NYU Stern

February 14, 2006*
Preliminary and incomplete – Please do not circulate

Abstract

This paper develops a simple competitive model of CEO pay. It appears to explain much of the rise in CEO compensation in the US economy, without assuming managerial entrenchment, mishandling of options, or theft. CEOs have observable managerial talent and are matched to assets in a competitive assignment model. The marginal impact of a CEO's talent is assumed to increase with the value of the assets under his control. Under very general assumptions, using results from extreme value theory, the model determines the level of CEO pay across firms and over time, and the pay-sensitivity relations. We predict that the level of CEO compensation should increase one for one with the average market capitalization of large firms in the economy. Therefore, the eight-fold increase of CEO pay between 1980 and 2000 can be fully attributed to the increase in market capitalization of large US companies. The model predicts the cross-section Cobb-Douglass relation between pay and firm size and can be used to study other large changes at the top of the income distribution, and offers a benchmark for calibratable corporate finance.

*xgabaix@mit.edu, alandier@stern.nyu.edu. For helpful comments, we thank David Yermack and seminar participants at MIT.

1 Introduction

This paper proposes a neoclassical model of equilibrium CEO compensation. It is simple, tractable, calibratable. CEOs have observable managerial talent and are matched to firms competitively. The marginal impact of a CEO's talent is assumed to increase with the value of the assets under his control. It makes definite predictions about CEO pay across firms, across countries, and across time. It also appears to explain, quantitatively, much of the rise in CEO compensation since the 1980s. In the model's view, this increase in pay is due to the rise in the market value of firms.

Our talent market is neoclassical and frictionless: Talent is observable, so that the equilibrium allocation is efficient. The best CEOs go to the bigger firms, which maximizes their impact. In the benchmark case, incentive considerations do not matter. The paper extends earlier work (e.g., Rosen 1982, 1992, Tervio 2003), by draws from extreme value theory to get general functional forms about the spacing of the distribution of talents. This allows to solve for everything in closed form without loss of generality, and get concrete predictions. In equilibrium, under very general conditions, the compensation of a CEO in firm i is:

$$\text{CEO compensation}_{it} = D \cdot S_* \cdot \left(\frac{S_{it}}{S_*}\right)^{\kappa} \tag{1}$$

where κ and D are positive constant, S_* is the size of a reference firm (e.g., the market capitalization of the median firm in the S&P 500), and S_{it} is the size of firm i. Hence, the model generates the well-established Cobb-Douglas relation between compensation and size (with $\kappa \simeq 0.3$ empirically).

The model also predicts that average compensation should move one for one with typical market capitalization S_* of firms. Figures 1 and 2 offer some suggestive evidence for this effect. Historically, in the U.S. at least, the rise of CEO compensation coincided with an increase in market capitalization of the largest firms. Between 1980 and 2000, the average market capitalization of S&P 500 firms has increased by a factor of 8 (i.e. a 700% increase).[1] The model predicts that CEO pay should increase by a factor 8. The result is just driven by the scarcity of CEOs, competitive forces, and the 8-fold increase in stock market valuations. Incentive concerns or managerial entrenchment play strictly no role in this model of CEO compensation. In our view, the question of the rise in CEO compensation is a simple mirror of the rise in

[1] The average market capitalization of firms in the S&P 500, was $2.9 billion in 1980, and $24 billion in 2000. The numbers are in real 2000 dollars. Source: CRSP.

the value of large US companies since the 80s. Our model also predicts that countries experiencing a lower rise in firm value than the US should also have experienced lower executive compensation growth, which resonates with the European experience.

The rise in executive compensation has triggered a large amount of public controversy and academic research. Our theory is to be compared with the three main types of economic arguments that have been put forward to explain this phenomenon.

The first explanation attributes the increase in CEO compensation to the widespread adoption of compensation packages with high-powered incentives since the late 80s. Holmstron and Kaplan (2001, 2003) link the rise of compensation value to the rise in stock-based compensation following the "LBO revolution" of the 80s. Both academics and shareholder activists have been pushing throughout the 90s for stronger and market-based managerial incentives (e.g. Jensen and Murphy 1990). According to Inderst and Muller (2005) and Dow and Raposo (2005), higher incentives have become optimal due to increased volatility in the business environment faced by firms. In the presence of limited liability and/or risk-aversion, increasing the performance sensitivity of a CEO's compensation, keeping his participation constraint unchanged, requires an increase in the dollar value of compensation. However, this link between the level and the "slope" of compensation has not been extensively calibrated: CEOs of large companies are typically very wealthy individuals. One can doubt that their level of risk-aversion and limited liability constraint represent quantitatively important economic frictions. For this reason, it remains unclear that increased incentives can explain the very large increase in CEO rents.

Following the wave of corporate scandals and the public focus on the limits of the US corporate governance system, a "skimming view" of CEO compensation has gained momentum. The tenants of the "skimming view" (e.g. Bebchuk et al. (2002)) explain the rise of CEO compensation simply by an increase in managerial entrenchment. "When changing circumstances create an opportunity to extract additional rents—either by changing outrage costs and constraints or by giving rise to a new means of camouflage—managers will seek to take full advantage of it and will push firms toward an equilibrium in which they can do so". Stock-option plans are viewed by these authors as a way to increase CEO compensation without attracting too much notice from the shareholders. According to them, "high-powered incentives" is just an excuse used by management to justify higher "rent-extraction". A milder form of the skimming view is expressed in Hall and Murphy (2003) and Jensen, Murphy and Wruck (2004). They attribute the explosion in the level of stock-option pay to an inability of boards to evaluate the true costs of this form of compensation. " Why has option compensation increased? Why has it increased with the market? (...) We believe the reason is that option grant decisions are made by board members and executives who believe (incorrectly) that options are a low-cost way to pay people and do not know or care that the value (and cost) of an option rises as the firm's share price rises"[2]. These forces have almost certainly been at work, but it is unclear how important they are in the main. The present paper offers a competitive benchmark, and indicates that forces of deception may not be a main determinant of the typical CEO compensation: according to our theory, the rise in US CEO compensation is an equilibrium consequence of the massive increase in firm size.

A third type of explanation relates the increase in CEO compensation to changes in the nature of the CEO job. Hermalin (2004) argues that the rise in CEO compensation reflects tighter corporate governance. To compensate CEOs for the increased likelihood of being fired, their pay must increase. Frydman (2006) and Murphy and Zabojnik (2005) provide evidence of a rise in "general skills" required on CEO jobs. As less firm-specific skills and more general skills become valuable, CEOs participate to a more competitive labor market and have higher outside options than before, leading to higher pay. However, a main difficulty with their proposal is quantitative. Changes in the talent composition appear small to moderate (Frydman 2005), while the level of CEO compensation increase by a factor of 5 to 10. It is hard to envision a calibrated model where moderate changes could explain very large changes in levels of compensation. Moreover, given the rise in the number of MBAs among executives and the spread of executive education, one can doubt that the scarcity of general skills is a plausible explanation for the rise in CEO compensation. In contrast, our model explains this increase readily. When stock market valuations are 5 times larger, CEO "productivity", which is proportional to firm value, increases by 5, and pay increases by 5 as firms compete to attract talent. In the economy we describe, total pay is independent of agency problems, and is just determined by the scarcity of CEO talent and the firms' demand for this talent. Agency considerations determine, in a second and subordinate step, the relative mix of average pay and incentives. This way, one derives a simple benchmark for the pay-sensitivity estimates that have caused much academic discussion (Jensen and Murphy 1990, Hall and Liebman 1998, Murphy 1999, Bebchuk and Fried 2003).

[2] Jensen, Murphy and Wruck[2004].

Finally, the model offers a calibration, which could be useful for future studies, a step that may be important to develop quantitative, calibratable corporate finance.

The core model is in section 2. Section 3 presents the model's predictions for pay to performance debate. Section 4 presents various extensions. In particular, it examines the equilibrium pay if some firm underestimate the true cost of stock options. Section 5 proposes a calibration of the quantities used in the model.

2 Basic model

There is a continuum of firms and potential managers, indexed by $x \in [0, X]$. Firm x has size $S(x)$, which is best thought of as earnings or market capitalization, while manager x has talent $T(x)$. Low x means a larger firm or a more talented manager: $S'(x) < 0$, $T'(x) < 0$. In equilibrium, a manager of talent T receives a compensation $\omega(T)$.

We consider a firm's problem. The firm starts with earnings a_0. [3] At $t = 0$, it hires a manager of talent T for one period. The manager's talent increases the firm's earnings as:

$$a_1 = a_0 (1 + CT) \tag{2}$$

for some $C > 0$. C represents how much a CEO talent increases this year's earnings.

First, suppose that the CEO's action at date 0 impacts earnings just in period 1. The firm's earnings are $(a_1, a_0, a_0, ...)$. The firm chooses the optimal talent CEO T, to maximizes current earnings, net of the CEO wage $\omega(T)$.

$$\max_T a_0 (1 + C \times T) - \omega(T) \tag{3}$$

Alternatively, suppose that the CEO's action at date 0 impacts earnings permanently. The firm's earnings are $(a_1, a_1, a_1, ...)$. The firm chooses the

[3] One can think of a_0 as being productivity, or a simple transform of productivity. CES models yield a direct mapping of productivity into earnings. For instance, suppose that the consumer's program is: $U = \sum Q_i^{1-1/\eta} / (1 - 1/\eta)^2 - \sum p_i Q_i$. Hence, his demand for good i satisfies $p_i = Q_i^{-1/\eta} / (1 - 1/\eta)$. We now analyze a typical firm, dropping the i. The firm produces Q with a linear technology: $Q = aL$, where the cost of labor is 1, and a is the firm's productivity. The profit is $\pi = pQ - Q/a = Q_i^{1-1/\eta} / (1 - 1/\eta) - Q/a$. So the profit-maximizing quantity is $Q = a^\eta$, the values of sales is $pQ = a^{\eta-1} / (1 - 1/\eta)$, and the realized profit is $\pi = a^{\eta-1} / (\eta - 1)$.

optimal talent CEO T, to maximizes the present value of earnings, net of the CEO wage $\omega(T)$.

$$\max_T \frac{a_0}{r}(1 + C \times T) - \omega(T) =: M \tag{4}$$

In both cases, there is a notion of "size" of a firm S, such that the firm's program is:

$$\max_T S \times C \times T - \omega(T) \tag{5}$$

If the CEO's action has a temporary impact, $S = a_0$, while if the impact is permanent, $S = a_0/r$.

If CEO talent does not matter very much (CT close to 0), then a_0 is close to the earning of the firm (the realized earnings are a_1), while a_0/r is close to the market capitalization M of the firm. Below, we mostly refer to "size" as the "market capitalization", but we are sympathetic to the interpretation of "size" as "earnings".[4]

We now turn to determination of the equilibrium wages. One needs to allocate one CEO to each firm. It is clear, given the absence of asymmetric information, that the equilibrium allocation will match the CEO indexed by x with the firm indexed by x. We call $w(x)$ the equilibrium compensation of a CEO with index x. Firm x, taking the compensation of each CEO as given, picks the potential manager y that maximizes performance net of salary:

$$\max_y CS(x)T(y) - w(y) \tag{6}$$

which gives: $CS(x)T'(y) - w'(y)$. The optimum should have $y = x$. Hence:

$$w'(x) = CS(x)T'(x) \tag{7}$$

The less talented CEO ($x = X$) has an opportunity wage, which for simplicity we normalize to 0.[5] So:

$$w(x) = -\int_x^X CS(u)T'(u)\,du \tag{8}$$

[4] The specification (2) can be generalized. For instance, the CEO impact could be assumed to be $a_1 = a_0 + Ca_0^\gamma T$+independent factors. If large firms are more difficult to change that small firms, then $\gamma < 1$. One replaces then S by S^γ in our formulas. The time-series prediction in Proposition 1 below may allow to determine γ empirically. We believe $\gamma = 1$ is the natural benchmark, and it appears to be consistent with the long term stylized facts.

[5] If the outside opportunity wage of the worse executive is w^*, all the wages are increased by w^*. This does not change the conclusions at the top of the distribution, as w^* is likely to be very small compared to the expressions derived in this paper.

As this stage, we need functional forms. The firm size distribution is taken to be Pareto with exponent $1/\alpha$:

$$S(x) = Ax^{-\alpha} \tag{9}$$

Empirically, the fit is quite good, and yields $\alpha \simeq 1$, a Zipf's law. Axtell 2001, Luttmer 2005, Gabaix 2006 for evidence and theory on Zipf's law for firms.

For the talent distribution, we use a result from extreme value theory. It shows that, for all "regular" continuous distributions, a large class that includes all usual distributions (including linear, Gaussian, exponential, log-normal, Weibull, Gumbel, Fréchet, Pareto), the following equation holds for the spacings in the talent distribution:

$$T'(x) = -Bx^{\beta-1} \tag{10}$$

in their upper tail, for some constant β and B, perhaps up to "a slowly varying" function.[6] Hence (10) should be considered a very innocuous functional form, satisfied, to a first degree of approximation, by any reasonable distribution of talent. In the language of extreme value theory (Resnick, 1987), $-\beta$ is the tail index of the distribution of talents, while α is the tail index of the distribution of firm sizes.[7]. Eq. (10) allows us to reach definite conclusion, at very low cost in generality.

The last three equations imply:

$$w(x) = -\int_x^X ABCu^{-\alpha+\beta-1}du = \frac{ABC}{\alpha-\beta}\left[x^{-(\alpha-\beta)} - X^{-(\alpha-\beta)}\right]$$

In what follows, we assume $\alpha > \beta$.

We consider the domain of very large firms, i.e. take the limit $x/X \to 0$, which gives:

$$w(x) = \frac{ABC}{\alpha-\beta}x^{-(\alpha-\beta)} \tag{11}$$

[6] $L(x)$ is said to be slowly varying at 0 (e.g., Embrechts, Kluppelberg, and Mikosch 1997, p.564) if for all $t > 0, \lim_{x\to 0} L(tx)/L(x) = 1$. Prototypical examples are $L = a$ and $L(x) = a \ln 1/x$ for a non-zero constant a. Hence, more precisely, for all usual distributions, there is a slowly varying function $L(x)$ such that $T'(x) = -Bx^{\beta-1}L(x)$. See Gabaix, Laibson and Li (2006), Appendix A and the first Lemma of Appendix B, for a formal statement of (10). If one replaces the constant B by a slowly varying function $L(x)$, all the theory belows remains, multiplying the expressions by slowly varying functions. Sometimes, by some abuse of language, $L(x)$ is called a "logarithmic correction" to the power function $Bx^{\beta-1}$.

[7] Gabaix, Laibson and Li (2006, Table 1) contains a tabulation of the tail indices of many usual distributions.

To interpret Eq. 11, we consider a reference firm, for instance firm number 250 – the median firm in the S&P500. Call its index x_*, and its size $S_* = S(x_*)$. S_* is the size of a "reference" or "typical" large firm, and can be used for time-series prediction. We get:

Proposition 1 *Call x_* the index of a reference firm – e.g., the median firm in a universe of large firms. In equilibrium, for large firms (small x), the manager of type x heads a firm of size $S(x)$, and is paid:*

$$w(x) = \frac{-Cx_*T'(x_*)}{\alpha - \beta} S_*^{\beta/\alpha} S(x)^{1-\beta/\alpha} \tag{12}$$

where S_* is the size of the reference firm. In particular, the compensation of the reference firm is

$$w(x_*) = \frac{-Cx_*T'(x_*)}{\alpha - \beta} S_* \tag{13}$$

- *Time-series prediction: compensation changes over time as the size of the median firm S_*.*
- *Cross-section prediction: compensation varies with firm size as $S^{1-\beta/\alpha}$.*
- *Cross-country prediction: for a given firm size S, CEO compensation will vary across countries, as the reference market capitalization $S_*^{\beta/\alpha}$*

It the talent distribution and the population do not change, $\frac{-Cx_*T'(x_*)}{\alpha-\beta}$ is just a positive constant.

Proof. $S_* = Ax_*^{-\alpha}$, $x_*T'(x_*) = Bx_*^{-\beta}$, so from Eq. 11,

$$(\alpha - \beta) w(x) = ABCx^{-(\alpha-\beta)} = ABCx_*^{-\alpha+\beta} \left(\frac{S}{S_*}\right)^{\frac{\alpha-\beta}{\alpha}} = Cx_*T'(x_*) S_* \left(\frac{S(x)}{S_*}\right)^{\frac{\alpha-\beta}{\alpha}}$$

■

Eq. 12 predicts, that, the average wage depends linearly on the size of the typical firm, S_*. For instance, in the U.S., between 1980 and 2000, the average market capitalization of S&P 500 firms has increased by a factor of 8 (i.e. a 700% increase). The model predicts that CEO pay should increase by a factor 8. This effect is very robust. Suppose all firm sizes S are multiplied by a factor λ. In Eq. 8, the right-hand side is multiplied by λ. Hence, the wages, in the left-hand side, are multiplied by λ.

Second, Eq. 12, first, predicts that the CEO compensation increases as a power function of the size of the firm, $S^{1-\beta/\alpha}$. This relation has been found empirically many times (Kostiuk 1990, Rosen 1992), namely that $w \sim S^{\kappa}$, with $\kappa \simeq 1/3$. In our framework, $\kappa = 1-\beta/\alpha$. Using the empirical estimates $\alpha \simeq 1$, and $\kappa \simeq 1/3$, we get $\beta \simeq 2/3$.[8]

Third, the model predicts that CEOs heading similar firms in different countries will earn different salaries. Suppose that the size S_* of the 250th Japanese firm is K times smaller than the size of 250th U.S. firm ($K = S_*^{\text{US}}/S_*^{\text{Japan}}$), and, to simplify, that the distribution of talents at the top is the same.[9] Consider two firms of equal size, one Japanese, one American. The salaries of their CEOs should not be equal. Indeed, according to Eq. 12, the salary of the US CEO should be $K^{\beta/\alpha}$ higher than the Japanese CEO.

We now offer a few remarks.

In the baseline model, there is no incentive problem. A CEO works, with no special incentives. Hence, the economist should not expect to see any "pay for performance" relation, and there is no scandal if there is no link between performance and pay.

A Rosen (1981) "superstar" effect holds. If $\beta > 0$, the talent distribution has an upper bound, but the distribution of wages has no upper bound, as the best managers are paired with very good firms. The reason is that the best managers' talent is paired up with a very large firms, which allows them to command a high compensation.

Finally, one can wonder how firms might know the spacings in the talent distribution and its impact on a firm – β, and BC. One way is by looking at the other firms. Proposition 1 implies than an "interpolation rule" is valid. If a firm of size S is between two firms S_1 and S_2, with $S = \sqrt{S_1 S_2}$, then, its CEO should be paid $w = \sqrt{w_1 w_2}$:

$$\text{Firm sizes satisfy } S = \sqrt{S_1 S_2} \Rightarrow \text{their CEO pays satisfy } w = \sqrt{w_1 w_2} \quad (14)$$

One could envision a dynamical process whereby a few firms increase their compensation, perhaps because they think that C is high, and this makes all firms follow in a dynamical version of (14). Working out such a dynamics would be interesting, and could be parsimoniously modelled as a belief

[8] Hence, we learn something from this back-of-the enveloppe calibration about the distribution of CEO talents: its tail-index is $-\beta = -2/3$. It would be interesting to compare it to other domains of human talent, such as athletes or artists. What is the spacing between the times of top runners? one expects that it spaces as 10. What is the spacing between the compensation of top artists?

[9] Section 4 discusses a possible impact of population size on the distribution of talents at the top.

about C. For simplicity, though, this paper assumes that the technological parameters are all common knowledge.

Eq. 12 offers a potential way to know if CEOs affect primarily earnings, or market capitalization – i.e., in the model at the beginning of this section, if CEO impact is temporary is permanent. One would run a regression of wages on earnings, sales, and market capitalization, and see which variables dominate. Technological change, or fashions, may change the relative strength of earnings or market capitalization in setting CEO pay.[10]

3 Extension with pay-performance sensitivity

The previous section determined total compensation without assuming any incentive problem. This section extends the model to allow to talk about stock options. The CEO's objective function is:

$$U = E\left[\frac{c}{1+\lambda e}\right] \tag{15}$$

where c is the compensation, $e \in \{-1, 0\}$ is the effort, $\lambda \in (0,1)$ is a disutility of effort. The CEO is risk neutral, subject to limited liability, $c \geq 0$. $e = 0$ is the high effort level, and $e = -1$ is "shirking". As always, the "effort" should be interpreted broadly (Holmstrom and Milgrom 1990).

Effort e and talent T increase the expected firm value in one year by the following enrichment of Eq. 5:

$$S(T, e) = S(0) + CST + DSe \tag{16}$$

The realized firm value is stochastic, $\widetilde{S} = S(T, e) + \epsilon$. The contract should ensure that a high e is chosen.[11] We study a particular compensation package, that will turn out to be first best.[12] The CEO's compensation c is the sum of a base pay b, and n options:

$$c = b + n\left(\widetilde{S} - S(T, e)\right)^{+} \tag{17}$$

[10] This leaves a free parameter that may be detrimental to scientific discipline, but may be relevant nonetheless. For instance, there was a stock market increase in the 1950s, but, in Frydman (2005)'s sample, CEO pay did not move much. It could be that this was because firms thought earnings, or sales, more relevant. Another possibility is that the phenomenon is confined to Frydman's particular sample.

[11] [Insert short proof for this]

[12] Given the manager is risk neutral (for $c \geq 0$), many compensation packages are optimal.

Normalizing the a share price to \$1, the options are worth $(r-\mu)^+ := \max(r-\mu, 0)$ at the end of the year.

At the optimum, effort is high ($e = 0$), and the CEO's utility is $E[c] = w$, the expected compensation. Hence, the market equilibrium is determined in two, independent steps. First,.the market equilibrium of the previous section determines $w(x)$, the average compensation of a manager. Then, an incentive scheme ensures high effort, while keeping the total salary / utility of $w(x)$.

We now determine the optimal amount of options.

Proposition 2 *Define*

$$\Delta = E\left[\eta^+ - (\eta - D)^+\right] \tag{18}$$

where $\eta = \epsilon/S$ is the noise around stock returns and D is the percentage decrease in firm value if the manager shirks.

The following compensation package offers the first best amount of incentives, and is a solution of the competitive equilibrium. A manager of talent $T(x)$ is paired with a firm of size $S(x)$, and receives an average compensation $w(x)$ determined in Proposition 1. He receives a fixed base pay f^* and n^* options, with:

$$n^* = w(x) \cdot \frac{\lambda}{\Delta} \tag{19}$$

$$f^* = w(x) \cdot \left(1 - \frac{\lambda}{\Delta} E\left[\eta^+\right]\right) \tag{20}$$

The average compensation satisfies $w(x) = f^* + n^* E\left[\left(\widetilde{S} - S(T,e)\right)^+\right]$.

Proof. The manager should get his market wage: $E[c \mid e = 0] = w(x)$. We calculate:

$$\begin{aligned} E[c \mid e = 0] &= f + nSE\left[\eta^+\right] = w(x) \\ E[c \mid e = -1] &= f + nSE\left[(\eta - D)^+\right] = w(x) - n\Delta \end{aligned}$$

The manager exerts a high effort $e = 0$ if:

$$\begin{aligned} E\left[\frac{c}{1+\lambda e} \mid e = 0\right] &\geq E\left[\frac{c}{1+\lambda e} \mid e = -1\right], \text{ i.e.} \\ w(x) &\geq \frac{w(x) - n\Delta}{1-\lambda}, \text{ i.e.} \\ n &\geq n^* := w(x)\frac{\lambda}{\Delta} \end{aligned}$$

If $b = w(x) - n^* E[\eta^+] \geq 0$, this is the solution to the problem. ■

In the world described by Proposition 2, options are not indexed on the market. Hence, an economist should not use the lack of option indexing as evidence of inefficiency. Also, CEOs will be rewarded for luck (Bertrand and Mullainathan 2001), but again, this does not violate efficiency. Both those features are consistent with a first best compensation scheme.[13]

Calling σ the volatility of the firm's stock price, the expected value from the option is: $E[\eta^+] = \frac{\sigma}{\sqrt{2\pi}}$.[14] So the ratio of average compensation coming from options to total compensation is:

$$\text{Option share} := \frac{\text{Compensation coming from options}}{\text{Total compensation}} = \frac{\lambda}{\Delta} \frac{\sigma}{\sqrt{2\pi}} \tag{21}$$

The share of compensation that is given in options increases with the volatility of the firm, but is independent of the size of a firm.

Jensen and Murphy (1998) and Hall and Liebman (2000), estimate empirical pay-performance measures. Those measures, b^I and b^{II}, are obtained by regressions of the type:

$$\begin{aligned} \Delta\$\text{Compensation} &= b^I \cdot \Delta\text{Value of the firm} + \text{controls} \\ \Delta \ln \text{Compensation} &= b^{II} \cdot \Delta \ln \text{Value of the firm} + \text{controls} \end{aligned}$$

Here, Δ lnValue of the firm is η, and ΔValue of the firm is $S\eta$. b^I is the absolute change in pay with the absolute change in the market valuation of the firm $S \cdot \delta r$. We define the theoretical counterparts of the OLS definitions:[15]

$$\begin{aligned} b^I &= \frac{1}{S} E\left[\frac{dc}{d\eta}\right] \\ b^{II} &= E\left[\frac{dc}{d\eta}\right] / E[c] \end{aligned}$$

The next Proposition derives predictions for those quantities.

Proposition 3 *The pay-performance sensitivities for a manager in firm i*

[13] The reason is that the manager is locally risk neutral, which may not be a bad approximation in the real world.

[14] We use that fact that if $R \sim N(0, \sigma^2)$, $E[R^+] = \sigma/\sqrt{2\pi}$. Note that this is the average expected payoff of the option, not the Black-Scholes value, which typically differs from it by a small amount.

[15] One can define the b's in terms of covariances, and one gets the same expressions if R has a symmetrical around 0.

are:

$$b_i^I = \frac{\Delta \$Compensation}{\Delta\, Value\ of\ the\ firm} = \frac{-x_* T'(x_*)}{\alpha - \beta} \left(\frac{S_i}{S_*}\right)^{-\beta/\alpha} \frac{\lambda}{2\Delta} \tag{22}$$

$$b_i^{II} = \frac{\Delta \ln Compensation}{\Delta \ln Value\ of\ the\ firm} = \frac{\lambda}{2\Delta} \tag{23}$$

Proof. We can calculate those values here, around $\eta = 0$. We observe that $d(\eta^+)/d\eta = 1\{\eta > 0\}$, so

$$E\frac{d(\eta^+)}{dr} = 1/2$$

The derivative of the option value with respect to the return is 1/2 on average. So: $E[dc/d\eta] = n^*/2$. If the manager has n options, compensation moves, on average, by a factor $n/2$ times the changes in returns. So in the incentive package of Proposition 2, the change in compensation is

$$E\frac{dc}{d\eta} = \frac{1}{2} w(x) \cdot \frac{\lambda}{\Delta}$$

writing $w_{it} = F_t \cdot S_{it}^\kappa$, we get:

$$\begin{aligned} E\frac{dc}{d\eta} &= \frac{\lambda F_t}{2\Delta} S^\kappa \\ b_i^I &= \frac{1}{S} E\frac{dc}{d\eta} = \frac{\lambda F_t}{2\Delta} S_i^{\kappa-1} \\ b_i^{II} &= \frac{1}{w} E\frac{dc}{d\eta} = \frac{1}{w(x)} \frac{1}{2} w(x) \cdot \frac{\lambda}{\Delta} = \frac{\lambda}{2\Delta} \end{aligned}$$

■

b^I is the Jensen-Murphy (1990) measure. The model predicts that it decreases with the size of a firm, with an elasticity equal to β/α, which the previous calibration assessed to be around 1/3. Indeed, Jensen and Murphy (1990), and the subsequent literature (Schaeffer 1998), have found that b^I decreases with firm size. It would be interesting to test the scaling $b_i^I \sim S_i^{-\beta/\alpha}$

b^{II} is predicted not to scale with firm size, at least in the baseline model, where λ and Δ do not depend on firm size.

4 Extensions

4.1 If other firms pay their CEO more, how much is a firm forced to follow?

If other firms increase their compensation, how much should a firm follow?

To answer this question, we observe that a shift in the "willingness to compensate" can be modeled as a shift in C. Hence, we extend the model to the case C's may differ across firms. Eq. 5 and 6 show that the "effective" size of a firm is $\widehat{S} = CS$, and the model applies directly. For concreteness, we say that firm i has a size S_i and a statistically independent "CEO impact" C_i, so its effective size is $\widehat{S}_i = C_i S_i$. We can now formulate the analogue of Proposition 1.

Proposition 4 *Suppose that firm i has a marginal impact of CEO talent C_i. The average impact C_* is defined as:*

$$C_* = E\left[C^{1/\alpha}\right]^{\alpha} \tag{24}$$

Call x_* the index of a representative talent – e.g., the median firm in a universe of large talent. In equilibrium, for large firms (small x), the manager of rank x heads a firm whose "effective size" CS is ranked x, and is paid:

$$w = \frac{-x_* T'(x_*)}{\alpha - \beta} (C_* S_*)^{\beta/\alpha} (CS)^{1-\beta/\alpha} \tag{25}$$

where S_* is the typical size of a firm. In particular, the representative compensation is

$$w(x_*) = \frac{-x_* T'(x_*)}{\alpha - \beta} C_* S_* \tag{26}$$

Proof. We need to calculate the analogue of (9) for the effective sizes $\widehat{S}_i = C_i S_i$. For convenience, we set x to the be normalized between $[0, 1]$. Then, by (9), $x = P(S > s) = A^{1/\alpha} s^{-1/\alpha}$. Hence:

$$\begin{aligned} P\left(\widehat{S} > s\right) &= P(CS > s) = P(S > s/C) = E[P(S > s/C \mid C)] = E\left[A^{1/\alpha} C^{1/\alpha} s^{-1/\alpha}\right] \\ &= A^{1/\alpha} E\left[C^{1/\alpha}\right] s^{-1/\alpha} = x \end{aligned}$$

so $\widehat{S}(x) = \widehat{A} x^{-\alpha}$ with $\widehat{A} = AE\left[C^{1/\alpha}\right]^{\alpha}$. Then, the proof of Proposition 4 applies. ■

The proposition is useful for the following thought experiment. Suppose that other firms believe that CEOs have become more productive, i.e. increase their C by a factor $\lambda > 1$, except for one firm, F, who does not believe that CEOs became more productive, and keep the original C. So $C'_* = \lambda C$ while a firm F keeps its own C. How much will the pay at firm F change?

First, if firm F wants to keep the same CEO, then it needs to increase his pay by a factor λ, i.e. "follow the herd" one for one. The reason is simply that firm F's CEO outside option is determined by the other firms (as per Eq. 8), and has been multiplied by λ.

Alternatively, firm F may want to re-optimize, and get a new CEO – with lower talent. Eq. 25 shows that the salary paid in firm F will still be higher than the previous salary, by a factor $\lambda^{\beta/\alpha}$. With $\beta/\alpha = 0.7$, if all firms increase their willingness to pay by 100% ($\lambda = 2$), firm F chooses to increase its CEO pay by $2^{0.7} - 1 = 60\%$. Such a high degree of "strategic complementarity" may make the market for CEO quite reactive to shocks, as initial shocks are not very dampened.

Another variant may be of interest. Suppose that a new sector, call it the "fund management" sector, competes for the same pool of people with the "corporate sector". For simplicity, say that the distribution of funds and firms is the same, and that talent affects a fund exactly as in Eq. 5, with the same C. Then, the aggregate demand for talent has been multiplied by 2. The pay of a given talent is multiplied by 2, while the pay at a corporate firm is multiplied by $2^{\beta/\alpha}$. Hence it is plausible that competition from new sectors, such as venture capital and the money management industry, have exerted a quantitatively large pressure on CEO pay.[16]

4.2 Misperception of the cost of compensation

Hall and Murphy (2003) and Jensen and Murphy (2004) have persuasively argued that at least some boards incorrectly perceived stock options to be inexpensive because options create no accounting charge and require no cash outlay. What is the impact on compensation?

To model this, say that the firm thinks that the pay, instead of costing w, costs w/M, where $M > 1$ is a misperception of the cost of compensation. Hence Eq. 6 for firm i becomes $\max_y CS_iT(y) - w(y)/M_i$ i.e.

$$\max_y CM_iS_iT(y) - w(y)$$

So, if the firm's willingness to pay is multiplied by M_i. The effective C is

[16] See Kaplan (xx) for this view.

now $C_i' = CM_i$. The analysis of section 4.1 applies: If all firms underestimate the cost of compensation by $\lambda = M$, then total compensation increases by λ, and even a "rational" firm that does not underestimate compensation, will increase its pay by $\lambda^{\beta/\alpha}$ if it is willing to change CEOs, and will increases its pay by λ if it does not want to change CEOs. Hence, other firms' misperceptions affect considerably a rational firm.

4.3 Executives below the CEO

One could generalize the model to the top H executives of each firm, assuming the enrichment of Eq. 2: $a_1/a_0 = 1 + \sum_{h=1}^{H} C_h T_h$. The $h-$th ranked executive improves firm productivity by his talent T_h and a sensitivity C_h, with $C_1 \geq ... \geq C_H$. A firm of size S wants to find the best H executives, net of costs:

$$\max_{T_1,...,T_H} \sum_{h=1}^{H} S \times C_h \times T_h - \sum_{h=1}^{H} \omega\left(T_h\right) \tag{27}$$

The next Proposition describes the equilibrium outcome.

Proposition 5 *(Extension of Proposition 1 to the top H executives). In a model where the top H executives increase firm value, the compensation for the $h-th$ executive h in firm i, is:*

$$w_{i,h} = \frac{-x_* T'\left(x_*\right)}{\alpha - \beta} \left(\sum_{k=1}^{H} C_k^{1/\alpha}\right)^{\beta} S_*^{\beta/\alpha} S\left(x\right)^{1-\beta/\alpha} C_h^{1-\beta/\alpha} \tag{28}$$

Hence, the predictions of Proposition 1 apply directly to the top H executives. From Eq. 28, one can estimate C_h from the salaries. In a given firm, the ratio between the CEO's pay and that of the h-th executive is $\left(C_1/C_h\right)^{1-\beta/\alpha}$.

The proof is very simple. As per Eq. 27, each firm behaves as H independent firms, with effective size $C_h S$, $h = 1...H$.

To be completed. . The proof follows the proof of Proposition 4. The average productivity is now: $C_* = \left(H^{-1} \sum_{k=1}^{H} C_k^{1/\alpha}\right)^{\alpha}$. So

$$w\left(x\right) = \frac{-x_* T'\left(x_*\right)}{\alpha - \beta} \left(\sum_{k=1}^{H} C_k^{1/\alpha}\right)^{\alpha} S_*^{\beta/\alpha} S\left(x\right)^{1-\beta/\alpha} \tag{29}$$

and the h-th executive in firm i earns (28). ■

5 A calibration

We propose a calibration for the model. We hope it is a useful step in the long-run goal of calibratable corporate finance.

5.1 Calibrating CEO talent

We present here indicative numbers, that will be made more exact in a future iteration of this paper. The empirical evidence (Axtell 2001, Luttmer 2005) on Zipf's law suggests:[17]

$$\alpha = 1$$

The evidence on the firm-size elasticity suggests $w \sim S^{1/3}$, which by Eq. 12 implies

$$\beta = 2/3$$

A value $\beta < 1$ implies that the distribution has an upper bound $T_{\max}$, and that in the upper tail of the talent density is (up to a slowly varying function of $T_{\max} - T$):

$$P(T > t) = B' (T_{\max} - t)^{1/\beta} \text{ for } t \text{ close to } T_{\max}$$

We index firms by rank, the largest firm having rank $x = 1$. Formally, if there are N firms, the fraction of firms larger than $S(x)$ is x/N: $P\left(\widetilde{S} > S(x)\right) = x/N$. The median firm in the S&P500 has rank $x_* = 250$.

Units are 2005 dollars. The CEO's compensation is about $w_* = 8 \cdot 10^6$. The market capitalization of firm $x_* = 250$ is: $S_* = 5 \cdot 10^9$. Using Proposition 1, we get: $w_* = \frac{BCx_*^{\beta}}{\alpha-\beta} S_*$, so $BC = (\alpha - \beta) \frac{w_* x_*^{-\beta}}{S_*}$, i.e.

$$BC = 1 \cdot 10^{-5} \tag{30}$$

It means that, the difference in marginal product between the top CEO and the 2nd CEO is $CT'(1) = BC = 10^{-5}$. The top CEO increases a firm's market value by only $10^{-3}\%$. This means that the spacings between talent can look very small. But this very small difference is big enough, in this neoclassical model, to generate large differences in compensation.

Tervio (2003), backs up talent differences in CEOs over a range. We answer his question in our framework. The difference of talent between the top CEO and the K-th CEO is:

$$T(1) - T(K) = \int_1^K T'(x)\, dx = \int_1^K Bx^{\beta-1} dx = \frac{B}{\beta}\left[K^{\beta} - 1\right]$$

[17] Here we use strict equalities, when we should be using approximate equalities.

For $K = 1000$, this differences yields: $C\left(T\left(1\right) - T\left(1000\right)\right) = 0.2\%$. If firm #1 replace its CEO #1 with CEO #1000, and kept that CEO for 10 years (a typical CEO tenure, see Denis and Denis 1995), its value would decrease by only 2%.[18]

In the "temporary impact" interpretation, where CEO affects earnings for just one year, one multiplies the estimate of talent by the price-earnings ratio. Taking an empirical P/E ratio of 15, replacing CEO #1000 by CEO #1 increases earnings by: $15 \times 0.2\% = 3\%$.

Such a small difference might be due to the difficulty of inferring talent. Suppose that the market gets only an noisy signal of talent: the value of the talent, plus firm-level noise, over 10 years: $T + \sigma\varepsilon$. As a first pass, consider the case where quantities are normally distributed. The posterior estimate of talent is a constant plus $\left(T + \sigma\varepsilon\right)/\left(1 + \sigma^2/var\left(T\right)\right)$. If firm level volatility is 30%, then the averaged noise over 10 years is has standard deviation $\sigma = 30\%/10^{1/2} \simeq 10\%$. If T has standard deviation 1%, then the standard deviation of the posterior will be $var\left(T\right)/\left(var\left(T\right) + \sigma^2\right)^{1/2} = 0.1\%$. The noise is so large, that the best estimates of talents have very small standard deviation.

Another way to put the finding (30) the following. If there is a paradox in CEO pay, it is that firms must think that CEOs do not have very superior talents (at least, that there is little difference between CEO #1 and CEO #1000), as they pay them so *little*.

5.2 Calibrating effort and incentives

We use $\Delta \sim D/2$ and Eq. 23, which gives $b^{II} = \lambda/D$. In the model, there is no way to know λ or D separately, but λ/D can be identified. What is a good value for D, which is the amount by how much a firm value decreases if the CEO slacks?We think that 30% is a plausible upper bound: $D \lesssim 0.3$. The empirical literature (Murphy 1999) finds $b^{II} \simeq 1/3$. So we conclude: $\lambda \leq 0.30 \times 1/3 = 0.10$. This means that utility that the CEO gets from shirks is around 10% of total pay, at most. CEOs receive relatively weak incentives, so firms must think that the shirking problem relatively minor.

[18] Event studies find that, when a founder dies, the market value goes up by about 3% (cite...). This means that the founder's talent is far from the talent in the "market for CEOs", while in the market for CEOs, estimates of talent are quite homogenous.

6 Conclusion

We provided a simple, calibratable competitive model of CEO compensation. Its structure is very simple, and it can be extended in many directions. In addition, it appears consistent with the main facts about CEO compensation, including with the recent rise in CEO . It generalizes readily to the top H executives in a firm. The model can be extended to analyze stock options, the impact of outside opportunities for CEO talent (such as the money management industry), and the impact of misperception of the price of options on the average compensation. Finally, the model allows us to propose a calibration of various quantities of interest in corporate finance and macroeconomics, the dispersion and impact of CEO talent, and the cost and impact CEO effort.

7 References

Aggarwal, R. and A. Samwick (1999), "The Other Side of the Tradeoff: The Impact of Risk on Executive Compensation," Journal of Political Economy 107(1): 65-105.

Axtell, Robert, "Zipf Distribution of U.S. Firm Sizes," *Science,* 293 (2001), 1818-20.

Bertrand, Marianne and Sendhil Mullainathan (2001): "Are CEOs Rewarded for Luck? The Ones Without Principles Are." Quarterly Journal of Economics, 116, pp. 901–932.

Baker, George P and Brian J Hall (2004): "CEO Incentives and Firm Size." Journal of Labor Economics.

Baker, George P and Michael C Jensen and Kevin J Murphy (1988): "Compensation and Incentives: practice vs Theory." Journal of Finance , pp. 593–616.

Bebchuk, Lucian and Jesse Fried, 2003, "Executive Compensation as an Agency Problem," Journal of Economic Perspectives, 17(3), 71-92.

Bebchuk, Lucian and Jesse Fried, 2004, *Pay without Performance: The Unfulfilled Promise of Executive Compensation*, Harvard University Press, November 2004

Bebchuk, Lucian, Jesse Fried and David Walker, 2002, "Managerial Power and Rent Extraction in the Design of Executive Compensation," University of Chicago Law Review 69, 751-846.

Bergman, Nittai and Dirk Jenter, "Employee Sentiment and Stock Option Compensation", WP.

Denis, David J & Denis, Diane K, JF 1995. "Performance Changes Following Top Management Dismissals"

Dow, James and Clara C. Raposo, 2005, "CEO Compensation, Change, and Corporate Strategy", forthcoming in Journal of Finance.

Brown, Charles and James Medoff (1989): "The Employer Size-Wage Effect." Journal of Political Economy, 97, pp. 1027–1059.

Frydman, Carola, 2005, "Rising Through the Ranks. The Evolution of the Market for Corporate Executives, 1936-2003", Harvard WP.

Frydman, Carola and Raven Saks, 2005, “Historical Trends in Executive Compensation, 1936-2003,” Harvard WP.

Gabaix, Xavier, “Zipf’s Law for Cities: An Explanation,” *Quarterly Journal of Economics*, 114 (1999), 739-67.

Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou and H. Eugene Stanley, “A Theory of Power Law Distributions in Financial Market Fluctuations,” *Nature*, 423 (2003), 267–230.

Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou and H. Eugene Stanley, “Institutional Investors and Stock Market Volatility,” *Quarterly Journal of Economics*, 121 (2006).

Gabaix, Xavier, and Yannis Ioannides, “The Evolution of the City Size Distributions,” in *Handbook of Regional and Urban Economics Vol. 4,* Vernon Henderson and Jacques-François Thisse, eds. Chapter 53 (Oxford: Elsevier Science, 2004) 2341-2378.

Gabaix, Xavier, David Laibson and Hongyi Li “Extreme Value Theory and the Effects of Competition on Profits”, WP.

Garicano, Luis and Esteban Rossi-Hansberg, ”Organization and Inequality in a Knowledge Economy”, NBER WP#11458.

Hall, Brian J.; Murphy, Kevin J., 2003, “The Trouble with Stock Options,” *The Journal of Economic Perspectives*, Volume 17, Number 3, 1 August 2003, pp. 49-70.

Hall, B. and Liebman, J. (1998). “Are CEOs Really Paid Like Bureaucrats?” *Quarterly Journal of Economics*, 113(3): 653-691.

Hermalin, Benjamin E., 2004, Trends in Corporate Governance, forthcoming in Journal of Finance.

Himmelberg, Charles P. and R. Glenn Hubbard (2000): “Incentive Pay and the Market for CEOs: An Analysis of Pay-for-Performance Sensitivity, ”Columbia Working Paper.

Holmström, Bengt (1992): “Contracts and the Market for Executives: Comment.” in *Contract Economics*, edited by Werin, Lars and Hans Wijkander, Blackwell, pp. 212–217.

Holmström, B. (1979), “Moral Hazard and Observability,” The Bell Journal of Economics, 10, 74-91.

Holmström, B. (1982), "Moral Hazard in Teams," The Bell Journal of Economics, 13(2), 324-40.

Holmström, B., and Steven Kaplan, (2003), "The State of U.S. Corporate Governance: What's Right and What's Wrong?", Journal of Applied Corporate Finance.

Holmström, B., and Steven Kaplan, (2001), "Corporate Governance and Merger Activity in the U.S", with Bengt Holmstrom. Journal of Economic Perspectives

Holmström, B. and P. Milgrom (1987), "Aggregation and Linearity in the Provision of Intertemporal Incentives" Econometrica, 55, 303-328.

Inderst, R. andH. Muller, "Keeping the Board in the Dark: CEO Compensation and Entrenchment," mimeo.

Jensen, M. and K. J. Murphy (1990), "Performance Pay and Top-Management Incentives," Journal of Political Economy, 98(2), 225-64.

Jensen, M., Murphy, K. and Eric Wruck (2004) "Remuneration: Where we've been, how we got to here, what are the problems, and how to fix them," WP.

Kostiuk, Peter F (1990): "Firm Size and Executive Compensation." Journal of Human Resources, 25, pp. 91–105.

Lazear, Edwin P and Sherwin Rosen (1981): "Rank-Order Tournaments as Optimum Labor Contracts." Journal of Political Economy, 89, pp. 841–864.

Lucas, Robert E Jr (1978): "On the Size Distribution of Business Firms." *Bell Journal of Economics*, 9, pp. 508–523.

Malmendier, Ulrike and Geoffrey Tate, "Superstar CEOs," Stanford WP.

Murphy, Kevin J., 1999, "Executive Compensation," in Orley Ashenfelter and David Card (eds.), Handbook of Labor Economics, Vol. 3b, Elsevier Science North Holland (1999), Chapter 38: 2485-2563.

Murphy, Kevin J. and Jan Zabojnik, "Managerial Capital and the Market for CEOs," USC WP.

Murphy, Kevin J. and Jan Zabojnik, “CEO Pay and Appointments: A Market-based explanation for recent trends" (with Jan Zabojnik), *American Economic Review Papers and Proceedings*, (May 2004).

Prendergast, Canice. "The Tenuous Trade-Off between Risk and Incentives." *Journal of Political Economy*, 2002, 110(5), pp. 1071-102.

Resnick, Sidney, *Extreme Values, Regular Variation, and Point Processes* (New York: Springer Verlag, 1987).

Rosen, Sherwin (1981): “The Economics of Superstars.” American Economic Review, 71, pp. 845–858.

Rosen, Sherwin (1982): “Authority, Control and the Distribution of Earnings.” Bell Journal of Economics, 13, pp. 311–323.

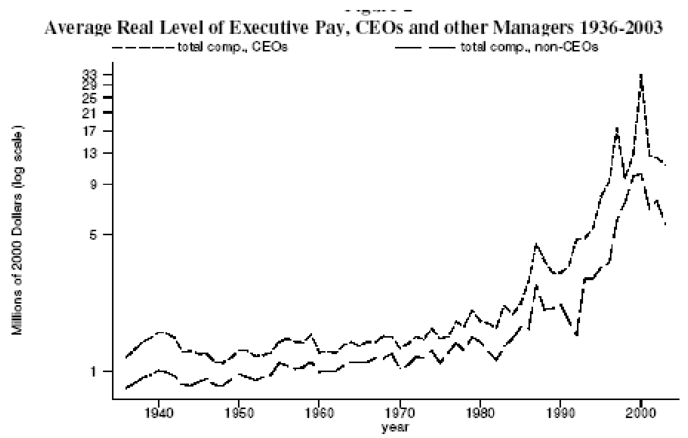
Rosen, Sherwin (1992): “Contracts and the Market for Executives.” in *Contract Economics*, edited by Werin, Lars and Hans Wijkander, Blackwell, pp. 181–211.

Sattinger (1993): “Assignment Models of the Distribution of Earnings.” Journal of Economic Literature, 31, pp. 831–880.

Schaefer, Scott (1998) “The Dependence of Pay-Performance Sensitivity on the Size of the Firm,” The Review of Economics and Statistics 80, 436-443.

Tervio, Marko (2003), “The Difference that CEOs Make: An Assignment Model Approach,” Berkeley WP.

Yermack, David (1995), “Do Corporations Award CEO Stock Options Effectively?” *Journal of Financial Economics*, 39(2-3), 237-69.



Note: Total compensation is the sum of salaries, bonuses, long term incentive payments, and the Black-Scholes value of options granted. The chief executive officer is identified as the president of the company in firms where the title "CEO" is not used. Based on the three highest-paid officers in the largest 50 firms in 1960.

Figure 1: Source: Frydman (2005).

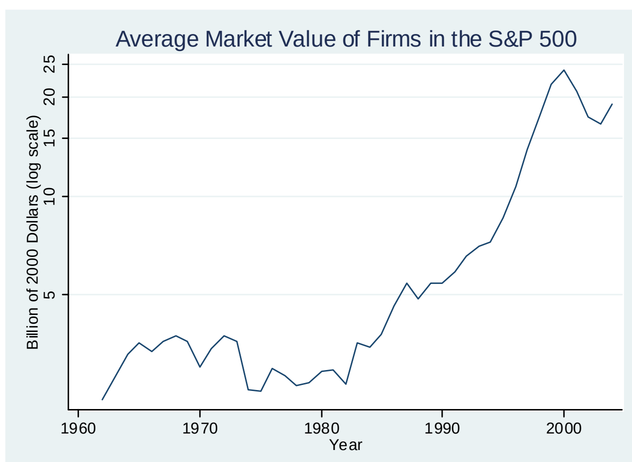


Figure 2: Average market capitalization of firms in the S&P 500, in constant 2000 dollars, from the Center for Research in Securities Prices. The model (Proposition 1) predicts that the average top CEO pay should be proportional to the market capitalization of the top firms (e.g., the firms in the S&P 500), hence proposes an explanation for the concomittance of the rise of CEO pay (Figure 1) and stock market valuations (this Figure 2).