Ramsey Meets Hosios:
The Optimal Capital Tax and Labor Market Efficiency

David M. Arseneau *
Federal Reserve Board

Sanjay K. Chugh†
Federal Reserve Board

First Draft: February 2006
This Draft: June 28, 2006

Abstract

Heterogeneity between unemployed and employed individuals matters for optimal fiscal policy. This paper considers the consequences of welfare heterogeneity between these two groups for the determination of optimal capital and labor income taxes in a model with matching frictions in the labor market. In line with a recent finding in the literature, we find that the optimal capital tax is typically non-zero because it is used to indirectly mitigate an externality along the extensive labor margin that arises from search and matching frictions. However, the consideration of heterogeneity makes our result differ in an important way: even for a well-known parameter configuration (the Hosios condition) that typically eliminates this externality, we show that the optimal capital income tax is still non-zero. We also show that labor adjustment along the intensive margin has an important effect on efficiency at the extensive margin, and hence on the optimal capital tax, independent of welfare heterogeneity. Taken together, our results show that these two empirically-relevant features of the labor market can have a quantitatively-important effect on the optimal capital tax.

JEL Classification: E24, E62, H21, J64

Keywords: Optimal fiscal policy, labor search, Hosios condition

* E-mail address: david.m.arseneau@frb.gov.

† E-mail address: sanjay.k.chugh@frb.gov. We thank Shigeru Fujita, Sylvain Leduc, Robert Martin, and Thomas Tallarini for helpful discussions, and participants at the Board's International Finance Workshop, the Spring 2006 Federal Reserve System Macroeconomics Meetings, the 2006 Midwest Macroeconomics Meetings, the 2006 Econometric Society Summer Meetings, and the 2006 Society of Computational Economics Meetings. The views expressed here are solely those of the authors and should not be interpreted as reflecting the views of the Federal Reserve System.

1 Introduction

Heterogeneity between unemployed and employed individuals matters for optimal fiscal policy. This paper considers the consequences of welfare heterogeneity between unemployed and employed individuals for the determination of optimal capital and labor income taxes in a model with matching frictions in the labor market. In line with a recent finding in the literature, we find that the optimal capital tax is typically non-zero because it is used to indirectly mitigate an externality that arises from search and matching frictions, one that cannot be corrected by the labor tax. However, the consideration of heterogeneity makes our result differ in an important way: even for a well-known parameter configuration that typically eliminates this externality, we continue to find a non-zero optimal capital tax. This difference stems from the heterogeneity in welfare between the employed and the unemployed. We also show that labor adjustment along the hours margin has an important effect on efficiency at the extensive margin, and hence on the optimal capital tax, independent of welfare heterogeneity.

Models featuring labor search and matching have two central features. First, because matches are costly to form, existing matches generate a surplus to be split between the worker and the firm. Second, matching models exhibit an externality due to the relative number of agents on each side of the labor market. This "market-tightness" externality arises from the fact that one additional job-seeker in the market increases the probability that a firm will match with a worker but decreases the probability that job-seekers already in the market will match with a firm.[1] Hosios (1990) shows these externalities are balanced when a worker's share of the match surplus equals his contribution to the formation of the match, yielding the socially-optimal market tightness. In a model with no taxes, the Hosios sharing rule typically boils down to a simple parameter restriction, specifically that the bargaining power of workers equals the elasticity of workers' input into the matching technology.

Much less is known about efficiency in this class of models in the presence of proportional taxes. As interest grows in using such models to answer policy questions, it seems important to understand the implications of policy for the nature of efficiency in frictional labor markets, and vice-versa.[2] We show that proportional labor income taxes disrupt the usual Hosios efficiency rule for two distinct reasons: if unemployed and employed individuals do not experience the same level of welfare and if labor adjustment occurs not only at the extensive margin (number of people working) but also at the intensive margin (hours worked per person). With the Hosios condition disrupted for either (or

[1] Equivalently, the externality can be thought of as arising from the fact that one additional firm with a vacancy increases the probability that a job-seeker will match with a firm but decreases the probability that firms already in the market will match with a job-seeker.

[2] Trigari (2003), Krause and Lubik (2004), and Walsh (2005) are just a few recent examples of the emerging use of labor search models to address policy questions.

both) of these reasons, a non-zero optimal capital tax can be used as an indirect instrument to steer the labor market towards efficiency along the extensive margin. This result is an instance of using the capital income tax to substitute for a missing tax instrument.[3] Our results show that each of these channels — welfare heterogeneity and intensive labor adjustment — has a quantitatively important effect on the optimal capital income tax rate.

In each case, the link between the Hosios condition and a zero capital tax is severed through the Nash wage bargaining process, which is the typical assumption about how wages are set in this class of models, an assumption we maintain. The wage is the critical endogenous variable that affects efficiency along the extensive labor margin. In stylized form, the wage that emerges from Nash bargaining can be expressed as

$$w = f\left(\frac{A - v}{1 - \tau^n}, h(\tau^n), Hosios\right), \tag{1}$$

where $A - v$ is the difference in welfare between employed and unemployed individuals, τ^n is a proportional labor income tax, h is an endogenous choice of hours worked that depends on τ^n, and $Hosios$ denotes the degree of departure from the Hosios sharing rule. If $A = v$ and if h does not vary with τ^n, then the Nash wage only depends on the degree of departure from the Hosios rule. In this case, if the Hosios rule is satisfied, then the bargained wage is efficient. On the other hand, even if there is no departure from the Hosios condition, differences between A and v or dependence of h on τ^n affect the wage and hence the extensive labor margin. We find these latter effects to be quantitatively important in terms of their implications for the optimal capital tax.

A study closely related to ours is Domeij (2005). In an important contribution to the literature, he also studies optimal labor and capital income taxation in a model with labor matching frictions. Building on the model of Shi and Wen (1999), Domeij (2005) finds that when the Hosios parameter restriction is satisfied, the optimal capital tax is always zero. An attractive feature of their model is that it allows for an analytical solution. Part of the analytic tractability of Domeij's (2005) setup comes from the lack of an intensive margin and the assumption that individuals are indifferent to whether or not they work. In terms of the above stylized wage equation, Domeij's (2005) setup can be interpreted as featuring $A = v$ and a h independent of τ^n. However, for applied analysis, it may be important to allow for ex-post heterogeneity and an intensive margin, two realistic features a model of the labor market should capture. Our quantitative results show that each of these features makes the coincidence between the Hosios parameterization and a zero optimal capital tax disappear.

In terms of our specific findings, three are noteworthy of mention at the outset. First, with neither welfare heterogeneity nor endogenous choice of hours, Ramsey allocations are socially-

[3] In this case, the missing instrument is one that directly affects market tightness. See Ljungqvist and Sargent (2004, p. 478) for more discussion on using the capital tax as part of an incomplete tax system.

optimal when the Hosios condition is satisfied. That is, even though the Ramsey planner levies a proportional labor income tax in order to finance government spending, this tax turns out to be non-distortionary. The optimal labor tax here is thus somewhat akin to a levy on initial capital. To our knowledge, this is the first instance in the optimal taxation literature in which Ramsey allocations achieve social efficiency. Second, both of the channels we study — welfare heterogeneity and intensive labor adjustment — imply that optimal capital tax rates should be *positive*, which stands in contrast to the findings of, among others, Judd (2002), Schmitt-Grohe and Uribe (2005), and Chugh (2006) that *negative* optimal capital tax rates should be used to indirectly affect some types of market frictions.[4] Third, in terms of welfare, following the standard prescription of a zero capital income tax rate instead of the positive tax rates we find entails a welfare loss of about 0.7 percent of steady-state consumption, which seems large enough to be worthy of attention.

The rest of the paper is organized as follows. In Section 2, we lay out our basic model without an intensive margin. Section 3 presents the Ramsey problem, and Section 4 describes how we parameterize our model. Quantitative results are presented in Section 5, along with a detailed analysis of how welfare heterogeneity between employed and unemployed individuals affects the extensive labor margin and the optimal capital income tax. In Section 6, we extend the basic model to include an intensive labor margin and show how it exerts a distorting effect on the extensive margin, one that can be corrected by a capital tax, independent of welfare heterogeneity. Section 7 computes the welfare losses that stem from implementing a zero capital income tax instead of the optimal non-zero capital taxes in our models. Section 8 concludes.

2 Baseline Model

The model embeds the Pissarides (2000) textbook search model with capital into a general equilibrium framework. The crucial feature of the model in this section is that individuals who are employed may experience total utility different from individuals who are unemployed, and these utility differences stem from individuals' labor-status subutility functions. Bargaining occurs between an individual worker and a firm. The structure we employ ensures perfect consumption insurance between employed and unemployed individuals. While this simplification shuts down what may be an important source of heterogeneity between the unemployed and the employed, it allows us to focus on the implications for fiscal policy stemming from welfare heterogeneity due solely to the utility of work versus non-work.

[4] In these papers, a capital subsidy boosts output, which is inefficiently low due to the presence of a monopolistic distortion, closer to its efficient level by encouraging capital accumulation.

2.1 Households

There is a representative household in the economy. Each household consists of a continuum of measure one of family members. Each member of the household either works during a given time period or is unemployed and searching for a job. There is a measure n_t of employed individuals in the household and a measure $1 - n_t$ of unemployed individuals. We assume that total household income is divided evenly amongst all individuals, so each individual has the same consumption.[5]

The household's discounted lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \int_0^{n_t} v^i di + \int_{n_t}^1 A^i di \right], \tag{2}$$

where $u(c)$ is each family member's utility from consumption, v^i is the disutility individual i suffers from working, and A^i is the utility enjoyed by individual i from leisure. The function u satisfies $u'(c) > 0$ and $u''(c) < 0$. We assume symmetry in the disutility of work amongst the employed, so that $v^i = v$, as well as symmetry in the utility of leisure amongst the unemployed, so that $A^i = A$. Thus, household lifetime utility can be expressed as

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + n_t v + (1 - n_t) A \right]. \tag{3}$$

The household does not choose how many family members work. As described below, the number of people who work is determined by a random matching process. We also assume that each employed individual works a fixed number of hours $\bar{h} < 1$. This assumption facilitates comparison of our baseline model without intensive labor adjustment to our full model in Section 6 with intensive adjustment. The household chooses sequences of consumption, real government bond holdings, and capital $\{c_t, b_t, K_{t+1}\}$, to maximize lifetime utility subject to the flow budget constraint

$$c_t + K_{t+1} + b_t = (1 - \tau_t^n) w_t n_t \bar{h} + R_{b,t} b_{t-1} + \left[1 + \left(1 - \tau_t^k \right) (r_t - \delta) \right] K_t + d_t, \tag{4}$$

where b_t denotes the household's bond holdings at the end of period t, each unit of which pays a gross real return $R_{b,t+1}$ at the beginning of $t + 1$, K_t denotes the household's capital holdings at the start of period t, and δ is the depreciation rate of capital. Each worker's wage is w_t, and the capital rental rate is r_t. Household labor income is taxed at the rate τ_t^n, and capital income net of depreciation is taxed at the rate τ_t^k. Finally, d_t denotes profit income received from firms, which the household takes as given.

Denote by ϕ_t the time-t Lagrange multiplier on the flow budget constraint. The first order conditions with respect to c_t, b_t, and K_{t+1} are

$$u'(c_t) - \phi_t = 0, \tag{5}$$

[5] Thus, we follow Merz (1995), Andolfatto (1996), and much of the subsequent literature in this regard by assuming full consumption insurance between employed and unemployed individuals.

$$-\phi_t + \beta R_{b,t+1}\phi_{t+1} = 0, \tag{6}$$

$$-\phi_t + \beta\phi_{t+1}\left[1 + \left(1 - \tau^k_{t+1}\right)(r_{t+1} - \delta)\right] = 0. \tag{7}$$

These first-order conditions can be combined to yield a standard Euler equation

$$u'(c_t) = \beta R_{b,t+1} u'(c_{t+1}) \tag{8}$$

and the no-arbitrage condition between government bonds and capital

$$R_{b,t+1} = \left[1 + \left(1 - \tau^k_{t+1}\right)(r_{t+1} - \delta)\right]. \tag{9}$$

We again point out that although all individuals are perfectly insured against consumption risk, there is welfare heterogeneity, measured by $A - v$, between unemployed and employed because of differences in their work status.

2.2 Firms

There is a representative firm that produces and sells a homogenous final good in a perfectly competitive product market. The firm must engage in costly search for a worker to fill each of its job openings. In each job j that will produce output, the worker and firm bargain simultaneously over the pre-tax real wage w_{jt} paid in that position. Output of job j is given by $y_{jt} = g(k_{jt}, \bar{h})$, where $g(k, \bar{h})$ exhibits diminishing returns in capital. Note again that we allow here for a fixed $\bar{h} < 1$ number of hours worked in each job. The capital k_{jt} used in production is specific to a particular job and is rented by the firm in a spot capital market. Any two jobs i and j at the firm are identical, so from here on we suppress the first subscript and denote by k_t the capital used in any job, by w_t the wage in any job, and so on. Total output thus depends on the production technology and the measure of matches n_t that produce,

$$y_t = n_t g(k_t, \bar{h}). \tag{10}$$

The total wage paid by the firm in any given job is $w_t\bar{h}$, and capital rental payments for any given job are $r_t k_t$. The total wage bill of the firm is the sum of wages paid at all of its positions, $n_t w_t \bar{h}$, and the total capital rental bill is $n_t r_t k_t$.

The firm begins period t with employment stock n_t. Its future employment stock depends on its current choices as well as the random matching process. With probability $q(\theta)$, taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is $\theta \equiv v/u$, and matching probabilities depends only on tightness given the Cobb-Douglas matching function we will assume. The firm rents capital k_t for use in each job and chooses vacancies to post v_t and future employment n_{t+1} to maximize

$$\Pi_t = \sum_{t=0}^{\infty} \frac{\beta^t \phi_t}{\phi_0}\left[n_t g(k_t, \bar{h}) - n_t w_t \bar{h} - n_t r_t k_t - \gamma v_t\right] \tag{11}$$

subject to the law of motion for employment

$$n_{t+1} = (1-\rho^x)(n_t + v_t q(\theta_t)). \tag{12}$$

Firms incur the cost γ for each vacancy created. Job separation occurs with exogenous fixed probability ρ^x. Firms discount profits using the household's pricing kernel, $\beta\phi_t/\phi_0$, derived in the household problem above.

Associate the multiplier μ_t with the employment constraint. The first-order conditions with respect to n_{t+1}, v_t, and k_t are, respectively,

$$\mu_t = \left[\left(\frac{\beta\phi_{t+1}}{\phi_t}\right)(g(k_{t+1},\bar{h}) - w_{t+1}\bar{h} - r_{t+1}k_{t+1} + (1-\rho^x)\mu_{t+1})\right], \tag{13}$$

$$\frac{\gamma}{q(\theta_t)} = (1-\rho^x)\mu_t, \tag{14}$$

$$r_t = g_k(k_t,\bar{h}). \tag{15}$$

Combining the optimality conditions (13) and (14) yields the job-creation condition,

$$\frac{\gamma}{q(\theta_t)} = \left[\left(\frac{\beta\phi_{t+1}}{\phi_t}\right)(1-\rho^x)\left(g(k_{t+1},\bar{h}) - w_{t+1}\bar{h} - r_{t+1}k_{t+1} + \frac{\gamma}{q(\theta_{t+1})}\right)\right], \tag{16}$$

which states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from the match. Profits from a match take into account both the wage cost of that match as well as the capital rental cost for that match. This condition is a free-entry condition in the creation of vacancies and is one of the critical equilibrium conditions of the model.

2.3 Government

The government has a stream of purchases $\{g_t\}$ to finance using labor and capital income taxes and real debt. The flow budget constraint of the government is

$$\tau_t^n n_t w_t \bar{h} + \tau_t^k (r_t - \delta) n_t k_t + b_t = g_t + R_t b_{t-1} \tag{17}$$

Note again that capital taxation is net of depreciation.

2.4 Nash Bargaining

As is standard in the literature, we assume that the wage paid in any given job is determined in a Nash bargain between the matched worker and firm. Details of the solution are given in Appendix A. Here we present only the outcome of the Nash bargain. Bargaining over the wage payment yields

$$w_t\bar{h} = \eta\left[g(k_t,\bar{h}) - r_t k_t\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A-v}{u'(c_t)}\right] + \frac{\eta\gamma}{q(\theta_t)}\left[\frac{\tau_{t+1}^n - \tau_t^n + \theta_t q(\theta_t)(1-\tau_{t+1}^n)}{1-\tau_t^n}\right], \tag{18}$$

where η is the bargaining power of the worker and $1-\eta$ is the bargaining power of the firm. The first term in square brackets on the right-hand-side is the firm's contemporaneous surplus (excluding wage payments) from consummating the match and is equal to output net of capital rental payments. The second term in square brackets is the worker's threat point in bargaining. If the worker walks away from the match, he would suffer the disutility value A of not working and avoid the disutility v from working (divided by $u'(c)$ to express this in terms of goods). The third term in square brackets represents the saving on hiring costs that the firm enjoys when a job is created. In a model with either no distortionary taxation or a constant labor tax rate over time, this term reduces to $\eta\gamma\theta_t$. A time-varying tax rate thus drives an extra wedge into the bargaining process. This dynamic effect of taxes on bargaining is one that, to our knowledge, has not been noted in the literature and may be interesting to study in future work.

Here, we limit our attention to steady-states. In steady-state, the bargained wage payment reduces to

$$w\bar{h} = \eta\left[g(k,\bar{h}) - rk + \gamma\theta\right] + (1-\eta)\left[\frac{1}{1-\tau^n}\frac{A-v}{u'(c)}\right]. \tag{19}$$

In steady-state, the wedge created by τ^n works through its effect on the worker's threat point. For a non-zero threat point, a rise in τ^n changes the wage payment, thus altering firms' incentives to create vacancies. This simple mechanism is well-known in this class of models (see, for example, Pissarides p. 210-211) and is the key to our results. We provide more intuition, especially with regard to its implication for optimal capital taxation, in Section 5. The wage equation is one of the critical equilibrium conditions of the model; together, (19) and the job-creation condition (16) provide most of the intuition for the results we describe below.

2.5 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a matching technology, $m(u_t, v_t)$, where u_t is the number of searching individuals and v_t is the number of posted vacancies. A match formed in period t will produce in period $t+1$ provided it survives exogenous separation at the beginning of period $t+1$. The evolution of total employment is thus given by

$$n_{t+1} = (1-\rho^x)(n_t + m(u_t, v_t)). \tag{20}$$

2.6 Equilibrium

The household optimality conditions are summarized by (4), (8), and (9). We condense (4) and (8) along with the firm's optimality condition with respect to capital, (15), into the single present-value expression,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t-\left(g(k_t,\bar{h})-\tau_t^n w_t\bar{h}-g_k(k_t,\bar{h})k_t\right)n_t+\gamma v_t(1-\tau_t^s)\right]= \tag{21}$$
$$u'(c_0)\left[R_{b,0}b_{-1}+\left(1+\left(g_k(k_0,\bar{h})-\delta\right)\left(1-\tau_0^k\right)\right)K_0\right],$$

the derivation of which is presented in Appendix B. This present-value condition is convenient for the formulation of the Ramsey problem in the next section. Along with this present-value restriction, equilibrium is described by the no-arbitrage condition between government bonds and capital (9); the job-creation condition (16); the Nash solution for wage payments (18); the law of motion for employment (20); an identity restricting the size of the labor force to one,

$$n_t+u_t=1; \tag{22}$$

and the resource constraint

$$c_t+n_{t+1}k_{t+1}-(1-\delta)n_t k_t+g_t+\gamma u_t\theta_t=n_t g(k_t,\bar{h}). \tag{23}$$

Note that total costs of posting vacancies $\gamma u_t\theta_t$ are a resource cost for the economy, and we have made the substitution $v_t=u_t\theta_t$. Also note that we have made the substitution that aggregate capital K_t is related to match-specific capital k_t by $K_t=n_t k_t$. The unknown processes are $\{c_t,n_{t+1},k_{t+1},u_t,\theta_t,w_t\}$ for given processes $\{g_t,\tau_t^n\}$.

3 Ramsey Problem

The problem of the Ramsey planner is to raise revenue for the government through labor and capital income taxes in such a way that maximizes the welfare of the representative household, subject to the equilibrium conditions of the economy. In period zero, the Ramsey planner commits to a policy rule. Unlike the household, the Ramsey planner does choose n_{t+1} (subject to the matching frictions and bargaining outcomes in the decentralized economy), and the planner's explicit consideration of the aggregate welfare of employed individuals versus unemployed individuals when making these choices is important for our results.

The Ramsey problem is thus to choose $\{c_t,n_{t+1},k_{t+1},u_t,\theta_t,w_t,\tau_t^n\}$ to maximize (3) subject to (15), (16), (18), (20), (21), (22), and (23) and taking as given $\{g_t\}$. We leave the labor tax rate as a choice of the Ramsey planner, rather than eliminating it in order to cast the problem in terms of only allocations. Thus, we do not adopt the strict primal approach and instead solve a hybrid Ramsey problem that mixes allocation and policy variables as Ramsey choices.[6] We could eliminate

[6] See Ljungqvist and Sargent (2004, Chapter 15) for more discussion about alternative setups of Ramsey problems.

τ^n using the wage equation, but to make the structure of the Ramsey problem comparable to that of our full model in Section 6 we choose not to make this substitution.

Because we are concerned only with steady-states, we consider only the Ramsey first-order conditions for $t > 0$. We assume that the time-zero Ramsey allocation is the same as the asymptotic steady-state Ramsey allocation, thus endogenizing the initial condition of the economy.[7] This assumption does not affect whether or not a zero capital income tax rate is optimal in the asymptotic steady-state. Throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. Once we have the Ramsey allocation, we back out the capital tax rate that supports the allocation using (9).

4 Model Parameterization

We characterize the Ramsey steady-state of our model numerically. Before turning to our results, we describe how we parameterize the model. We assume that the instantaneous utility function over consumption is

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}. \tag{24}$$

The time unit of the model is meant to be a quarter, so we set the subjective discount factor to $\beta = 0.99$, yielding an annual real interest rate of about four percent. We set the curvature parameter with respect to consumption to $\sigma = 1$, consistent with many macro models.

Our timing assumptions are such that production in a period occurs after the realization of separations. Following the convention in the literature, we suppose that the unemployment rate is computed *before* the realization of separations. We set the quarterly probability of separation at $\rho^x = 0.10$, consistent with Shimer (2005). Thus, letting n denote the steady-state level of employment, $n(1-\rho^x)^{-1}$ is the employment rate, and $1 - n(1-\rho^x)^{-1}$ is the steady-state unemployment rate.

The production function displays diminishing returns in capital,

$$g(k_t) = k_t^{1-\alpha}\bar{h}^{\alpha}, \tag{25}$$

and we set capital's share to $1-\alpha = 0.3$. The quarterly depreciation rate of capital is $\delta = 0.02$. In the production function, we allow for a fixed number of hours $\bar{h} < 1$ in each job, making our baseline model readily comparable to the richer model in Section 6. In the richer model, we allow intensive

[7] This adoption of the "timeless" perspective is innocuous here since we focus on only the steady-state rather than on transitional dynamics. We should also point out that, as is well-known in this literature, nothing guarantees a solution to the Ramsey problem, nor, if a solution exists, convergence to a steady-state. We do, however, numerically find a steady-state.

β	σ	α	δ	ξ_u	ρ^x	η
0.99	1	0.70	0.02	0.40	0.10	0.40

Table 1: Fixed parameters in models I and II.

labor adjustment and calibrate utility parameters such that steady-state hours are $h = 0.35$. Thus, we set $\bar{h} = 0.35$ here.

As in much of the literature, the matching technology is Cobb-Douglas,

$$m(u_t, v_t) = \psi u_t^{\xi_u} v_t^{1-\xi_u}, \tag{26}$$

with the elasticity of matches with respect to the number of unemployed set to $\xi_u = 0.40$, following Blanchard and Diamond (1989), and ψ a calibrating parameter that can be interpreted as a measure of matching efficiency.

We normalize the disutility of working to $v = 0$. With this normalization, there are two natural cases to consider regarding the calibration of A, the utility value of not working. The first case is $A = v = 0$, so that there is no difference at all in the realized welfare of employed versus unemployed individuals. We label this case model I. Although model I may not be an accurate description of the relative welfare between unemployed and employed individuals, it serves as a very useful benchmark for our main results.

In the second case, we introduce ex-post heterogeneity between employed and unemployed individuals by allowing A to differ from v. Here, our choice of a specific value of A is guided by Shimer (2005), who calibrates his model so that unemployed individuals receive, in the form of unemployment benefits, about 40 percent of the wages of employed individuals. With his linear utility assumption, unemployed individuals are therefore 40 percent as well off as employed persons. Our model differs from Shimer's (2005) primarily in that we assume full consumption insurance, but also in that we have capital formation and curvature in both utility and production. Thus, when we allow for welfare heterogeneity, we interpret Shimer's (2005) calibration to mean that unemployed individuals must receive 2.5 times more consumption (in steady-state) than employed individuals in order for the total utility of the two types of individuals to be equalized. That is, we set A such that in steady-state

$$u\left(2.5\bar{c}\right) + A = u\left(\bar{c}\right) + v, \tag{27}$$

where $\bar{c}$ denotes steady-state consumption. We label this case model II. We point out that our qualitative results do not depend on the exact value of A in model II. As we discuss in Section 5, all that is important is that A is different from v.

With A selected in these two different ways, we calibrate γ and ψ in model I to hit the firm and worker matching rates $q(\theta) = 0.70$ and $\theta q(\theta) = 0.60$. The resulting values are $\psi = 0.66$ and

$\bar{g}$	τ^k	τ^n	n	θ	w	$q(\theta)$	$\theta q(\theta)$	c	K	y	K/y	c/y	$\gamma v/y$	$\bar{g}/y$
							Ramsey allocations							
0	0	0	0.844	0.857	1.495	0.700	0.600	0.532	7.885	0.791	9.966	0.672	0.129	0
0.142	0	0.323	0.844	0.857	1.495	0.700	0.600	0.389	7.885	0.791	9.966	0.492	0.129	0.180
							Socially-efficient allocations							
0	—	—	0.844	0.857	1.495	0.700	0.600	0.532	7.885	0.791	9.966	0.672	0.129	0
0.142	—	—	0.844	0.857	1.495	0.700	0.600	0.389	7.885	0.791	9.966	0.492	0.129	0.180

Table 2: Steady-state Ramsey and socially-efficient allocations in model I at the Hosios condition, $\eta = \xi_u$.

$\gamma = 0.75$, which we then hold fixed when we move to model II, as well as when we move to the models in Section 6. We thus view model I as the reference point for all of our analysis.

In each model, we focus on the case where the Nash bargaining power of workers is set to $\eta = \xi_u = 0.40$, so that the usual Hosios condition for search efficiency is satisfied. The implications of welfare heterogeneity (and, in Section 6, the intensive margin) for the optimal capital income tax *when the Hosios condition is satisfied* is the focus of our study. We choose steady-state government purchases $\bar{g}$ so that they constitute 18 percent of total output. The same value of $\bar{g}$ ($\bar{g} = 0.14$) delivers a government share of output very close to 18 percent in both models (as well as the models in Section 6). Finally, the steady-state value of government debt is assumed to be $b = 0$. While clearly not realistic, the assumption of zero debt facilitates the welfare comparisons between the Ramsey policy and alternative sub-optimal policies we conduct in Section 7. Our quantitative results are not very sensitive to b. Table 1 summarizes the parameter values that are common across all versions of our models.

5 Quantitative Results

After obtaining the dynamic first-order conditions of the Ramsey problem described in Section 3, we impose steady-state and numerically solve the resulting non-linear system. The capital income tax rate τ^k is then determined from (9). With the exception of Section 5.3, the usual Hosios condition holds throughout this section, so that $\eta = \xi_u$. The Hosios condition typically achieves efficiency along the extensive labor margin; we show that with proportional labor income taxation and welfare heterogeneity between employed and unemployed individuals, this is no longer true.

5.1 Model I: Homogenous Welfare and the Hosios Condition

We begin by presenting results from model I, in which employed and unemployed individuals are identical in welfare terms. The upper panel of Table 2 presents the key steady-state allocation and policy variables under the Ramsey plan for our calibrated value of $\bar{g}$ as well as $\bar{g} = 0$. The lower panel of Table 2 presents the social planner's allocations for these two cases. By social efficiency here, we mean those allocations that are subject to the technological constraints imposed by production and search and matching but which are not necessarily implementable as a decentralized equilibrium with proportional taxes, a requirement which of course is imposed on the Ramsey planner.[8] Thus, socially-optimal allocations are the solution of a planning problem that maximizes (3) subject to (20), (22), and (23), taking as given $\{g_t\}$.

Immediately striking in Table 2 is that, with the exception of consumption, the allocation is invariant both to government spending and to whether or not it is required to be implementable as a decentralized equilibrium. With no government spending to finance, the Ramsey planner has no need to impose any taxes as long as the Hosios condition holds, as the first row of Table 2 shows; in section 5.3, we show that away from the Hosios condition, the Ramsey planner does levy taxes to try to mimic the efficient sharing delivered by the Hosios rule. Once $\bar{g} > 0$, the Ramsey planner of course cannot set all taxes to zero. As shown in the second row, the optimal capital income tax rate is zero, and all government spending is financed through the labor income tax. This echoes the hallmark Chamley (1986) zero-capital-taxation result that arises in a model with homogenous (representative) consumers. It is also consistent with the results of Domeij (2005) for a model with no welfare heterogeneity between unemployed and employed individuals. The only effect of government spending is to crowd out private consumption one-for-one; despite a positive zero labor tax, the Ramsey allocation remains identical to the socially-efficient allocation.

At first, it seems quite surprising that the Ramsey planner is able to implement the first-best allocation because $\bar{g} > 0$ requires levying a proportional wage tax. The mechanism by which a Ramsey policy typically transmits into a Ramsey allocation is that it affects prices — post-tax prices are different from pre-tax prices and hence agents' decisions are affected. In the canonical Ramsey fiscal policy problem with Walrasian labor markets — as in, say, Lucas and Stokey (1983) — the two prices to be manipulated are the real return on capital and the real wage. With a zero optimal capital tax, the return on capital is clearly not altered. The issue then is the reaction of the wage to the proportional labor income tax in our model. In the simple environment of model I, the labor tax does not affect the pre-tax wage (compare the first two rows of Table 2). The only decision in model I made according to wages is by firms when choosing how many vacancies to post

[8]By the Second Welfare Theorem, the socially-efficient allocation can be implemented as an equilibrium with an appropriate set of lump-sum tax instruments.

— future wages appear in the job-creation condition (16). Neither the household nor individuals make any decisions regarding their labor market status. Hence allocations turn out to be unaffected despite $\tau^n > 0$; in other words, the proportional labor income tax is not distortionary.

It is apparent from the Nash wage equation (19), reproduced here for convenience,

$$w\bar{h} = \eta \left[g(k, \bar{h}) - rk + \gamma\theta \right] + (1 - \eta) \left[\frac{1}{1 - \tau^n} \frac{A - v}{u'(c)} \right], \tag{28}$$

why τ^n does not affect w in model I. The terms in brackets can be thought of as the bargaining threat points of the firm and the worker. Because it is only households that pay labor taxes, the tax rate appears only in the worker threat point. If $A = v$, the worker threat point, and hence the labor tax, is irrelevant for the Nash wage and thus in turn for the outcome in the labor market. If individuals are indifferent to whether or not they work *and* if the Hosios condition is satisfied, labor taxes do not affect wages and the result of the "second-best" Ramsey problem coincides with the first-best outcome.

We think this result is quite interesting on its own, as we know of no Ramsey model in which the socially-efficient allocation is attained. The key to our result, of course, is that with wages unaffected by taxes, the proportional labor tax acts as a lump-sum "labor levy." It is also clear from the Nash wage equation that this labor-levy feature of the Ramsey policy will not hold in the empirically-relevant case $A \neq v$ because then labor taxes will affect wages. This is the case to which we turn in the next section. Also, to preview the richer model we develop in Section 6, this lump-sum nature of labor taxes will not hold when there are both extensive and intensive labor margins. With both extensive and intensive labor adjustment, the effect of taxes on hours worked will exert independent influence on the wage. Nonetheless, with the majority of empirically-observed labor adjustment occurring at the extensive margin, it is interesting to understand the benchmark results we have developed in this section.

To summarize these results so far: when the Hosios condition is satisfied and labor adjustment occurs at only the extensive margin, welfare homogeneity implies the optimal capital tax is zero, confirming the results of Domeij (2005). In this environment, a proportional labor tax ends up acting as a lump-sum tax instrument, allowing Ramsey allocations to be first-best instead of only second-best. The rest of our paper demonstrates that this result depends critically on three key assumptions — welfare homogeneity ($A = v$), the Hosios condition ($\eta = \xi_u$), and the lack of an intensive margin ($h = \bar{h}$). Relaxing any of these assumptions has an important effect on the Ramsey policy prescription — in particular, on the optimal capital income tax rate — and hence the coincidence of the Ramsey solution and the social planning solution.

$\bar{g}$	τ^k	τ^n	n	θ	w	$q(\theta)$	$\theta q(\theta)$	c	K	y	K/y	c/y	$\gamma v/y$	$\bar{g}/y$
							Ramsey allocations							
0	0	0	0.890	1.676	1.379	0.535	0.897	0.528	8.315	0.834	9.966	0.632	0.168	0
0.142	0.044	0.327	0.892	1.733	1.360	0.528	0.915	0.383	8.152	0.831	9.814	0.461	0.172	0.171
							Socially-efficient allocations							
0	—	—	0.890	1.676	1.379	0.535	0.897	0.528	8.315	0.834	9.966	0.632	0.168	0
0.142	—	—	0.881	1.455	1.406	0.567	0.824	0.388	8.234	0.826	9.966	0.469	0.159	0.172

Table 3: Steady-state Ramsey and socially-efficient allocations in model II at the Hosios condition, $\eta = \xi_u$.

5.2 Model II: Ex-Post Welfare Heterogeneity and the Optimal Capital Tax

We now turn to the case in which individuals differ in their realized levels of utility according to their labor market status. As described in Section 4, we model this ex-post heterogeneity between unemployed and employed individuals by choosing a value of A different from v in a way consistent with Shimer (2005). The value of A needed to achieve this calibration is $A = -0.9163$; recalling that the labor-status subutility of employed individuals is normalized to $v = 0$, this means unemployed individuals are worse off than employed individuals. The upper panel of Table 3 presents the key steady-state allocation and policy variables under the Ramsey plan both with and without government spending for model II. The lower panel of the table presents the corresponding socially-efficient allocations.

In order to understand how the introduction of welfare heterogeneity alters the results in Section 5.1, it is useful to begin by comparing the steady states of model I when $\bar{g} = 0$ (the first row of Table 2) and model II when $\bar{g} = 0$ (the first row of Table 3). Welfare heterogeneity dampens the Nash wage because with A lower than v, individuals have a preference for being employed over being unemployed, rather than being indifferent as was the case in Section 5.1. In the interest of preserving jobs, individuals are willing to accept lower wages. Lower wages in turn raises the number of people working in the economy. The wage equation (28) shows how the first part of this transmission mechanism works: $A < v$ erodes the bargaining position of individuals, lowering the Nash wage ($\partial w/\partial A > 0$). The firm's vacancy-creation condition (16) shows how the second part of this transmission mechanism works: a lower wage encourages firms to post more vacancies because each job is now more profitable. The ensuing tighter labor market (a higher $\theta = v/u$) in equilibrium leads to more individuals working because it is easier for a given unemployed individual to make contact with a firm. Thus, unemployment falls and output rises relative to the model I economy in which individuals are indifferent to whether or not they work. On the other hand, because vacancy posting is costly, it also crowds out private consumption through the resource constraint. Thus, heterogeneity by itself, without any government financing considerations, lowers wages, raises employment, and crowds out consumption.

Next, turn to the the solution of the Ramsey problem with $\bar{g} > 0$, presented in the second row of Table 3. The optimal capital income tax rate is no longer zero as in model I; this is due to an inefficiently high level of vacancy creation by firms. Once again, understanding the mechanism requires inspecting the wage equation (28) and the vacancy-creation condition (16). The wage equation shows that with $A < v$, a positive labor tax erodes workers' bargaining position and therefore reduces the Nash wage.[9] That is, $\partial w/\partial \tau^n < 0$ when $A < v$. This dependence of the wage on the labor tax rate is in contrast to model I, in which, because individuals were indifferent between working and not working ($A = v$), the bargained wage was invariant to the labor tax rate.

With the wages firms pay dampened by the positive labor income tax, firms' incentives to post vacancies would rise above the efficient level. That is, the extensive labor margin becomes inefficient even though the Hosios condition holds. The Ramsey planner, interested in balancing efficiency with the need to finance government spending, tries to influence this outcome with the policy instruments at his disposal. Thus far, we have limited the Ramsey planner to using only labor and capital income taxes. The positive capital income tax here balances two efficiency considerations: the desire to not disrupt the capital accumulation margin against indirectly promoting efficiency along the extensive labor margin. A positive capital income tax, by dampening capital accumulation, lowers the output $g(k, \bar{h})$ of a match, making vacancy-posting less profitable for firms. Thus, lacking an instrument that directly affects the extensive labor margin, the Ramsey planner uses the capital tax as a substitute instrument.[10]

This effect of the capital tax on firms' vacancy-posting incentives was first described by Domeij (2005). In Domeij's (2005) model, however, the capital tax needs to be used as an indirect instrument only when the Hosios condition is not satisfied — in his model, when the Hosios parameterization is in place, there is never a need to influence the extensive margin via any (direct or indirect) instruments. As we mentioned in the introduction, Domeij's (2005) setup can be interpreted as one in which all individuals, whether employed or not, experience the same level of welfare. Our results show that when unemployed individuals are worse off than employed individuals, a positive capital tax acts to promote efficient job-creation even though the Hosios condition is satisfied. It is this

[9]On the other hand, if $A > v$, the labor tax raises workers' bargaining power and thus raises the bargained wage. The intuition for this case is that individuals prefer to be unemployed in an economy where $A > v$. A rise in τ^n in this case increases the opportunity cost of foregoing leisure and working instead. The bargained wage must then compensate workers with a higher wage in order to entice them to join a match.

[10]The ability of the capital tax to act as a substitute instrument in an incomplete tax system is well-known — see Ljungqvist and Sargent (2004, p. 478).

consideration of heterogeneity that most sets our work and results apart from Domeij's (2005).

The indirect use of the capital tax to correct a labor market distortion suggests that other fiscal instruments might be able to more directly steer the economy towards efficiency along the extensive margin. We investigate this further in Section 5.4 by allowing for a vacancy tax. Before considering a vacancy tax, we briefly consider the behavior of the optimal capital tax away from the Hosios condition.

5.3 Away from the Hosios Condition: Taxes and Bargaining Power

All of our analysis so far has been conducted at the Hosios rule, $\eta = \xi_u$, which is known to deliver efficient sharing of the match surplus between firms and workers in the absence of proportional taxes. Here we consider how tax policy can be used to steer the economy towards the efficient allocation when the Hosios rule is not satisfied. We concentrate on the use of the capital tax as the indirect means of affecting the labor market; the general lessons developed in this section readily apply to the use of a vacancy tax in Section 5.4. To focus on the inefficiency stemming from non-Hosios sharing, we assume $\bar{g} = 0$. Doing so removes the issue of optimal financing of government spending from the analysis.

In Figure 1, we plot the optimal steady-state capital tax rates as a function of the worker bargaining share η for models I and II. Regardless of the assumed degree of heterogeneity between unemployed and employed individuals, τ^k is strictly decreasing in η. The intuition is as we discussed in Section 5.2: the capital tax indirectly affects firms' incentives to post vacancies by lowering the output of a match. Here, when workers' Nash bargaining power is low ($\eta < \xi_u$), firms' incentives to create vacancies are inefficiently high because they expect to receive a share of the match surplus disproportionately large relative to their contribution to match formation. This incentive is inefficiently high because Nash wages are inefficiently low — that is, $\partial w/\partial \eta > 0$. Thus, efficiency requires that firms post fewer vacancies, which the Ramsey planner attempts to replicate by levying a positive capital tax that lowers the output of a match, dampening firms' incentives to post vacancies. The opposite occurs when workers' bargaining power is high ($\eta > \xi_u$): firms receive a disproportionately small share of the match surplus because wages are high, so the incentive to post vacancies is inefficiently low. In this case, a capital subsidy boosts match output, partially restoring the incentive to post vacancies.

We truncate Figure 1 at $\eta = 0.80$ because for larger values of η the optimal capital tax rates (subsidies) become implausibly large. Note again that the results in Figure 1 are for $\bar{g} = 0$. The results for our benchmark $\bar{g} = 0.14$ are very similar.

We see at least two interesting issues our findings here raise. We know from our results thus

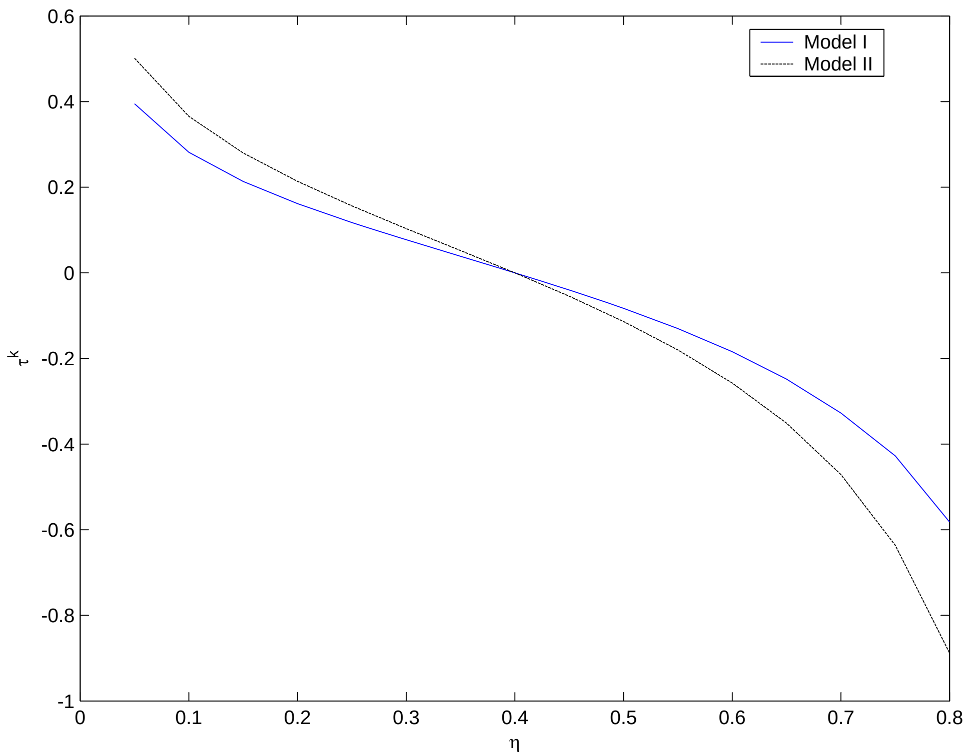

Figure 1: Optimal steady-state capital tax rate as a function of η in models I and II, with $\bar{g} = 0$.

far that efficiency in the labor market is associated with a zero optimal capital income tax.[11] A Nash share η that varied appropriately with τ^n would achieve this. That is, one could imagine generalizing the notion of the efficient sharing rule to depend on taxes, so that efficient sharing would be described not by $\eta = \xi_u$ but instead by a function $\eta(\xi_u, \tau^n)$. For efficiency in the labor market to come about solely through efficient bargaining when $\tau^n > 0$, workers' share of the match surplus would need to be raised so long as unemployed workers are worse off than employed workers — that is, efficiency would require $\eta > \xi_u$ so long as $A < v$. An efficient mix of fiscal policy then would need to consider the response of η to τ^n, perhaps allowing the Ramsey planner to achieve efficiency in the long run.

A second issue our findings raise is one of the effects of time-varying taxes and wage-bargaining protocols. The bargaining share η, which is descriptive of an institutional feature of the economy, may not be something we think could move around at business cycle frequencies. If this is the case, then it seems that it is impossible for governments to deliver efficient, or even Ramsey, allocations over the business cycle without having access to variable vacancy taxes or using the capital tax to correct labor market inefficiencies at business cycle frequencies. We leave consideration of these interesting issues for future work.

5.4 A Vacancy Tax

The nonzero capital tax in our model is used to indirectly mitigate an inefficiency that the government does not have an instrument to address directly. By itself, this is not a very surprising result in the Ramsey literature. For example, Judd (2002), Schmitt-Grohe and Uribe (2005), and Chugh (2006) show that a capital subsidy can be used as part of the Ramsey policy to offset monopoly power by producers when there is no direct way of doing so. Here, we demonstrate that the capital income tax in our model is substituting for a more direct labor market instrument.

The two natural candidates for a direct labor market instrument are a vacancy tax and an unemployment benefit. As Domeij (2005) shows, the two instruments are perfect substitutes for each other, as might be expected because all that matters for matching efficiency is market tightness θ, so that affecting either v or u can do the job. Thus, we will only consider a vacancy tax, which we introduce into our model in a straightforward way. We replace γ with $\gamma(1 + \tau_t^s)$ in the firm's profit function and the resulting job-creation condition, and we introduce $\tau_t^s \gamma v_t$ as a revenue item in the government budget constraint, where τ_t^s is the vacancy tax rate. If $\tau_t^s > 0$, the firm must pay a tax for each vacancy it creates, while if $\tau_t^s < 0$, the firm receives a subsidy for each vacancy. Note that the total vacancy tax adds to government revenues and is now part of the optimal financing problem.

[11] Again, assuming no vacancy tax exists.

τ^s	τ^k	τ^n	n	θ	w	$q(\theta)$	$\theta q(\theta)$	c	K	y	K/y	c/y	$\gamma v/y$	$\bar{g}/y$	Welfare Gain
							Model I								
0	0	0.323	0.844	0.857	1.495	0.700	0.600	0.389	7.884	0.791	9.966	0.492	0.129	0.180	0
							Model II								
0.149	0	0.298	0.881	1.455	1.336	0.567	0.824	0.388	8.234	0.826	9.966	0.469	0.159	0.172	0.374

Table 4: Steady-state Ramsey allocations in models I and II when a vacancy tax is available, the Hosios condition, $\eta = \xi_u$, is satisfied, and $\bar{g} = 0.142$. Last column shows welfare gain, measured as percentage of steady-state consumption, of using the vacancy tax instead of the capital tax.

With this vacancy tax available, we again compute the Ramsey allocations and policies. To isolate the effect of having a vacancy tax available, we keep γ at its calibrated value above. Results are presented in Table 4. The optimal capital tax is now always zero as the vacancy tax, being the more direct instrument to target the inefficiency in vacancy-creation stemming from welfare heterogeneity, assumes the role previously played by the capital tax. Domeij (2005) shows that using such a direct labor market instrument is more efficient than using the capital tax, as would be expected. Such a welfare gain of using a vacancy tax arises in our model, as well. The steady-state welfare gain, in terms of percentage of consumption, of using the vacancy tax is documented in the last column on Table 4. In model I, there is no welfare gain because the Ramsey allocation without the vacancy tax already achieves socially efficiency. In model II, the welfare gain of having access to the vacancy tax is about 0.4 percent of steady-state consumption. Note that having access to the vacancy tax allows the Ramsey planner to achieve the socially-optimal allocation in model II (compare the second row of Table 4 to the last row of Table 3), so this is another instance of the Ramsey allocation achieving Pareto optimality — that is, the proportional labor income tax is not distortionary in the presence of welfare heterogeneity even though the Hosios condition does not hold. In effect, the vacancy tax acts to restore the Hosios sharing rule.

Having demonstrated that a capital tax is an imperfect proxy for a vacancy tax in our model, we continue our analysis by omitting the vacancy tax. Some justification for this might be that, given how much attention is usually paid to *promoting* job-creation, an explicit vacancy tax may be a politically infeasible issue.

6 Intensive Labor Adjustment

For applied quantitative work, it may be of interest to model variations in the intensive labor margin as well as the extensive margin. In this section, we show that, even if the elasticity of hours supply is small as most micro evidence suggests, the presence of adjustment at the intensive margin has important consequences for the optimal capital tax. Specifically, the intensive margin introduces another channel through which the link between the Hosios condition and a zero optimal capital tax is broken. This channel is a standard-looking condition linking the marginal rate of substitution between consumption and hours and the marginal product of labor, one that is distorted by a proportional labor income tax. Thus, in this full version of our model, the proportional labor income tax is distortionary, in contrast to the model with only extensive adjustment in Section 2. We proceed by highlighting how the basic model we have been using up to now is modified to incorporate the intensive margin and then present quantitative results.

6.1 Households

We enrich the household structure in the following way. Each individual i that works in a time period spends h^i hours working and suffers disutility $v(h^i)$ from working, with $h^i \in (0,1)$. With symmetry across working individuals, the household lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + n_t v(h_t) + (1-n_t) A \right]. \tag{29}$$

Hours are no longer fixed as in Section 2. Rather, the number of hours worked by an individual is determined in Nash bargaining between the worker and the firm to which it is attached — neither the worker nor the household unit directly chooses hours. The Nash solution for hours is presented below. The budget constraint of the household is modified in the obvious way to account for the fact that household labor earnings now depend on the total variable hours worked by family members,

$$c_t + K_{t+1} + b_t = \left(1-\tau_t^n\right) w_t h_t n_t + R_{b,t} b_{t-1} + \left[1 + \left(1-\tau_t^k\right)(r_t - \delta)\right] K_t + d_t. \tag{30}$$

Because the household does not choose hours for individuals that work, the household optimality conditions remains those in Section 2.1.

6.2 Firms

Output of job j is now produced according to $y_{jt} = g(k_{jt}, h_{jt})$, where $g(k,h)$ has constant returns to scale. Once again, jobs are symmetric, thus h and k are common across matches. Total output is thus

$$y_t = n_t g(k_t, h_t), \tag{31}$$

and the total wage bill of the firm is $n_t w_t h_t$. We assume a Cobb-Douglas technology,

$$g(k,h) = k^{1-\alpha} h^{\alpha}, \tag{32}$$

and continue to use $\alpha = 0.70$.

The firm's dynamic profit function is

$$\Pi_t = \sum_{t=0}^{\infty} \frac{\beta^t \phi_t}{\phi_0} \left[n_t g(k_t, h_t) - n_t w_t h_t - n_t r_t k_t - \gamma v_t \right], \tag{33}$$

and firm optimization is identical to that in the baseline model. The job-creation condition generalizes to account for variable hours,

$$\frac{\gamma}{q(\theta_t)} = \left[\left(\frac{\beta \phi_{t+1}}{\phi_t} \right) (1 - \rho^x) \left(g(k_{t+1}, h_{t+1}) - w_{t+1} h_{t+1} - r_{t+1} k_{t+1} + \frac{\gamma}{q(\theta_{t+1})} \right) \right], \tag{34}$$

which has the same interpretation as in the baseline model.

6.3 Government

In the flow budget constraint of the government, total labor tax revenue now depends on hours worked by each individual,

$$\tau_t^n n_t w_t h_t + \tau_t^k (r_t - \delta) n_t k_t + b_t = g_t + R_t b_{t-1}. \tag{35}$$

6.4 Nash Bargaining

We assume that hours worked and the wage paid in any given job are simultaneously determined in Nash bargaining between the matched worker and firm. More detail is provided in Appendix A. A useful way to think about the outcome of this simultaneous bargaining is that the choice of hours maximizes the joint surplus of the match, while the bargained wage payment divides the surplus between the worker and the firm.

The choice of hours that maximizes the joint surplus from the match satisfies a labor optimality condition that looks standard in a neoclassical labor market,

$$\frac{-v'(h_t)}{u'(c_t)} = (1 - \tau_t^n) g_h(k_t, h_t), \tag{36}$$

which states the firm's marginal revenue product must equal the household's marginal rate of substitution between consumption and leisure taking into account the distortionary labor tax. Thus, given any τ_t^n, the choice of hours is privately efficient because it maximizes the total surplus inside a match.

Bargaining over the wage payment yields a total wage payment $w_t h_t$,

$$w_t h_t = \eta \left[g(k_t, h_t) - r_t k_t \right] + \frac{1 - \eta}{1 - \tau_t^n} \left[\frac{A - v(h_t)}{u'(c_t)} \right] + \frac{\eta \gamma}{q(\theta_t)} \left[\frac{\tau_{t+1}^n - \tau_t^n + \theta_t q(\theta_t)(1 - \tau_{t+1}^n)}{1 - \tau_t^n} \right], \tag{37}$$

where h_t is determined by (36) and, as before, η is the bargaining power of the worker and $1 - \eta$ is the bargaining power of the firm. The worker's threat point in bargaining, the second term in square brackets on the right-hand-side, now depends on hours worked. In steady-state, the bargained wage payment is

$$wh = \eta \left[g(k,h) - rk + \gamma\theta \right] + (1-\eta) \left[\frac{1}{1-\tau^n} \frac{A - v(h)}{u'(c)} \right]. \tag{38}$$

Note from (36) that bargained hours depend on τ^n. To emphasize this dependence, we somtimes write $h(\tau^n)$. It is clear then that the bargained wage payment depends on τ^n not only through the worker threat point but also through h. That is, in (38), even if $A = v(h)$, meaning there is no welfare heterogeneity between employed and unemployed persons, w is still a function of τ^n because hours depend on τ^n, and the production function g now depends on a variable h. This is how labor adjustment along the intensive margin opens up a second channel through which a nonzero labor tax affects wages and thus firms' incentives to create vacancies.

6.5 Equilibrium and Ramsey Problem

The equilibrium conditions remain those in Section 2.6 appropriately modified to include h_t and now also include (36). We use (36) to eliminate τ^n from the implementability constraint facing the Ramsey planner; the implementability constraint in this version of our model is thus

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - n_t h_t \left(g_h\left(k_t, h_t\right) - w_t - \frac{w_t}{g_h(h_t, k_t)} \frac{v'(h_t)}{u'(c_t)} \right) + \gamma v_t \right] = \\ u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(1 - \tau_0^k \right) \left(g_k(k_0, h_0) - \delta \right) \right) K_0 \right]. \tag{39}$$

A complete derivation is presented in Appendix B.

The Ramsey problem is a straightforward extension of that described in Section 3. To the list of Ramsey choice variables, add $\{h_t\}$ and remove $\{\tau_t^n\}$ (because of our substitution using (36)). Thus, the unknown processes here are $\{c_t, n_{t+1}, h_t, k_{t+1}, u_t, \theta_t, w_t\}$. The constraints facing the Ramsey planner are the same as those in Section 3 with appropriate inclusion of h_t inside the production function and factor prices. We continue to assume commitment to a policy at time-zero and concern ourselves with only the $t > 0$ Ramsey first-order conditions. With the Ramsey allocation we compute the steady-state capital and labor income tax rates from (9) and (36), respectively.

6.6 Parameterization

We describe here the new elements of the parameterization. For those parameters we do not discuss here, our choices remain those in Section 4.

For the labor subutility function, we assume the functional form

$$v\left(h\right) = -\frac{\nu h^{1+\upsilon}}{1+\upsilon} \tag{40}$$

and set the curvature parameter with respect to hours to $\upsilon = 5$, which yields a labor supply elasticity along the intensive margin of 1/5. This value for υ is more in line with micro-estimates of labor supply elasticity than the typically lower values for υ used in macro models without an extensive margin. In macro models without an explicit extensive margin, lower values for υ (and thus higher values for the intertemporal elasticity of labor supply) are a stand-in for the unmodelled variations in the extensive margin. Because in this full version of our model we have both an extensive and an intensive margin, we follow micro evidence in choosing υ. The parameter ν is then calibrated so that an employed individual works $h = 0.35$ hours under the Ramsey plan in model III with $\bar{g} = 0$, described below.

Regarding A, we define analogs of our models I and II from Section 4. In model III, meant to parallel model I, we calibrate A so that $A = v\left(\bar{h}\right)$, so there is no realized welfare heterogeneity between employed and unemployed persons. In model IV, we calibrate A so that under the Ramsey plan

$$u\left(2.5\bar{c}\right) + A = u\left(\bar{c}\right) + v\left(\bar{h}\right), \tag{41}$$

which mimics model II above. We perform the calibration of A in model III by setting $\bar{g} = 0$ so that the Ramsey and socially-efficient allocations of model III match those of model I, facilitating comparability across models. To be consistent, our calibration of A in model IV is also with $\bar{g} = 0$. The resulting values are $A = -0.2058$ in model III and $A = -1.1236$ in model IV. Finally, we keep the fixed cost of posting a vacancy at $\gamma = 0.75$ from models I and II in order to isolate the effects of an intensive labor adjustment margin on our results.

6.7 Quantitative Results

As before, after obtaining the dynamic first-order conditions of the Ramsey problem, we impose steady-state, numerically solve the resulting non-linear system, and then determine τ^k and τ^n residually from the equilibrium conditions. Table 5 presents the Ramsey and socially-optimal steady-states in models III and IV for $\bar{g} = 0$ and our benchmark calibration for $\bar{g} > 0$ from models I and II.

The Ramsey allocation in model III when $\bar{g} = 0$ (the first row of Table 5) is identical to the (Ramsey and socially-efficient) allocation in model I (Table 2). This is by construction — we calibrate A in model III so that these results are the same, making any new findings here due only to adjustment at the intensive labor margin. The Ramsey allocation with $\bar{g} = 0$ here is again of course identical to the socially-efficient allocation (the third row of Table 5).

To understand the effect of the presence of adjustment at the hours margin, compare the first two rows of Table 5. In the second row, with $\bar{g} > 0$, the Ramsey planner requires a positive labor income tax. The tax reduces hours worked through the hours bargaining solution (36), from 0.350

$\bar{g}$	τ^k	τ^n	h	n	θ	w	$q(\theta)$	$\theta q(\theta)$	c	K	y	K/y	c/y	$\gamma v/y$	$\bar{g}/y$
Model III (no heterogeneity between unemployed and employed)															
Ramsey allocations															
0	0	0	0.350	0.844	0.857	1.495	0.700	0.600	0.532	7.884	0.791	9.966	0.672	0.129	0
0.142	0.028	0.324	0.346	0.843	0.853	1.484	0.701	0.598	0.380	7.681	0.778	9.872	0.489	0.131	0.183
Socially-efficient allocations															
0	—	—	0.350	0.844	0.857	1.495	0.700	0.600	0.532	7.884	0.791	9.966	0.672	0.129	0
0.142	—	—	0.367	0.844	0.863	1.512	0.698	0.602	0.420	8.270	0.830	9.966	0.506	0.123	0.172
Model IV (heterogeneity between unemployed and employed)															
Ramsey allocations															
0	0	0	0.351	0.890	1.678	1.379	0.535	0.898	0.528	8.326	0.835	9.966	0.632	0.168	0
0.142	0.073	0.327	0.346	0.891	1.711	1.348	0.531	0.908	0.374	7.933	0.817	9.711	0.458	0.174	0.174
Socially-efficient allocations															
0	—	—	0.350	0.844	0.857	1.495	0.700	0.600	0.532	7.884	0.791	9.966	0.672	0.129	0
0.142	—	—	0.367	0.884	1.512	1.421	0.558	0.843	0.419	8.658	0.869	9.966	0.483	0.154	0.164

Table 5: Steady-state Ramsey and socially-efficient allocations in model III and model IV at the Hosios condition, $\eta = \xi_u$.

to 0.346. Fewer hours worked per job, *ceteris paribus*, causes output per job, $g(k, h(\tau^n))$, to fall. The bargained wage thus also falls, as (38) shows. This effect on the wage arises solely through $h(\tau^n)$ because the second term on the right-hand-side of (38) is zero in model III since we calibrate $A = v(h)$. A lower wage in turn raises the incentive for firms to post more vacancies despite the Hosios rule being in effect. The Ramsey planner would like to prevent this outcome because job-postings would then be too high from the point of view of social efficiency. As in Section 5, a positive capital income tax indirectly dampens the job-posting incentive and steers the economy closer towards efficiency along the extensive margin. The second row of Table 5 show that a capital income tax rate of $\tau^k = 0.028$ is optimal when there is no heterogeneity between employed and unemployed individuals but labor adjustment occurs at the intensive margin.

The upper half of Table 5 shows that when $\bar{g} > 0$, the Ramsey allocation no longer coincides with the socially-efficient allocation as it did in model I. Model III, like model I, displays no welfare heterogeneity between employed and unemployed individuals. The divergence of the Ramsey allocation from the socially-efficient allocation thus comes through the hours channel. The nonzero labor income tax rate distorts the hours margin, just like in a standard neoclassical model of the labor market. The hours distortion then affects wage bargaining as described above. Thus, the proportional labor income tax is distortionary in model III (and model IV), unlike in model I, preventing the Ramsey allocation from achieving full efficiency. In other words, the labor-levy feature of the Ramsey policy described in Section 5.1 vanishes with the neoclassical intensive margin.

The lower half of Table 5 considers the joint effects of welfare heterogeneity and intensive adjustment. In model IV, a labor income tax distorts wage bargaining and hence the extensive employment margin through two distinct channels: through $h(\tau^n)$ in the first term on the right-hand-side of (38) and through $A \neq v(h)$ in the second term of (38). The optimal capital income tax rate here is $\tau^k = 0.073$, which is essentially just the sum of the optimal capital tax rates that arise due to just the hours distortion ($\tau^k = 0.028$ in the second row of Table 5) and just welfare heterogeneity ($\tau^k = 0.044$ in the second row of Table 3). Thus, it seems heterogeneity and the hours distortion have somewhat additive implications for the optimal capital income tax. To the extent that labor is elastic at the intensive margin, it is important to know that these intensive variations — however small, which we capture by calibrating the elasticity of intensive labor supply to be very low — can have what seem to be an important quantitative effect on the optimal capital tax. Because our optimal capital taxation results are driven by inefficiencies in the extensive labor margin, our findings more fundamentally demonstrate that the presence of an intensive margin when taxes are proportional can have important consequences for efficiency along the extensive labor margin, adding to our understanding of the nature of efficiency of this class of models.

7 Welfare Loss of Following a Zero Capital Tax

The largest of our optimal capital income tax rates is around seven percent, in our model IV. One may think that it does not make much difference whether the capital tax rate is set to the seemingly small positive values we find in our models or simply set exactly to $\tau^k = 0$, the standard prescription in the optimal capital taxation literature. We show here that in fact the welfare consequences, while small, are not negligible. We quantify the welfare loss of following a zero capital tax versus following the optimal policy in our models.

We address this issue in the following way. Holding the level of government spending fixed at our benchmark $\bar{g} = 0.142$ and keeping intact the Hosios condition $\eta = 0.40$, we compute steady-state welfare under the Ramsey policy in each of our models according to

$$(1-\beta)V^* = u(c^*) + n^* v(h^*) + (1-n^*)A, \tag{42}$$

where asterisks denote the steady-state Ramsey allocation and V^* denotes the resulting household welfare. We then solve a restricted version of the Ramsey problem in which τ^k is forced to be zero rather than left as a free policy variable. Requiring $\tau^k = 0$ means we must modify the Ramsey problem in the following way. To prevent any wedge between the intertemporal marginal rate of transformation and the intertemporal marginal rate of substitution (which is what a non-zero capital tax introduces), we add as a constraint on the Ramsey problem

$$\frac{u'(c_t)}{u'(c_{t+1})} - \beta(1 - \delta + g_k(k_{t+1}, h_{t+1})) = 0, \tag{43}$$

which says that the gross return on bonds equals the gross return on capital with no tax wedge.[12] This restriction is simply the no-arbitrage condition (9) with $\tau_{t+1}^k = 0$ imposed. In models I and II, in which h is fixed, we of course set $\bar{h} = 0.35$ as above.

Denote the allocation resulting from this restricted Ramsey problem with a hat over variables. We follow the convention of computing welfare by assuming it is only consumption compensation that is required, so we compute the welfare loss from the restricted Ramsey policy as the percentage increase 100ξ in consumption that the household requires in order to be just as well off as in the unrestricted Ramsey allocation — that is, we find the ξ such that

$$u(\hat{c}(1+\xi)) + \hat{n}v(\hat{h}) + (1-\hat{n})A = (1-\beta)V^*. \tag{44}$$

In models I and II, we set $v(\bar{h}) = v = 0$, again as above. All other parameter settings are as described in Sections 4 and 6.

Table 6 compiles the welfare losses of following a zero capital income tax versus the optimal capital income tax in our four models. In model I, the optimal capital income tax rate is zero, thus welfare is identical under the unrestricted and restricted Ramsey solutions. In model IV, with both welfare heterogeneity and the distortion due to endogenous hours, the welfare loss is about 0.7 percent of steady-state consumption. This loss decomposes roughly additively into welfare losses due to each channel — losses of about 0.5 percent of consumption due to welfare heterogeneity by itself (model II) and about 0.25 percent of consumption due to the hours distortion by itself (model III). This welfare decomposition seems consistent with our finding in Section 6.7 that the effects on the optimal capital tax themselves are roughly additive. While seemingly not huge, steady-state welfare losses of this magnitude are not negligible. For example, given that consumption constitutes around 70 percent of output, a welfare loss of 0.7 percent of consumption translates into roughly \$50 billion for the U.S. economy.

8 Conclusion

This paper shows that heterogeneity between unemployed and employed individuals matters for optimal fiscal policy. Our main result is that when unemployed individuals are worse off than employed individuals, making them willing to accept lower wages in order to secure employment, a positive capital tax can be used as an indirect way of preventing firms from creating an inefficiently

[12] See Chari and Kehoe (1999, p. 1679-1680) for more discussion on this type of restriction in a Ramsey problem.

Model	Welfare Heterogeneity	Intensive Margin	Optimal τ^k	Welfare Loss of $\tau^k = 0$
I	no	no	0	0
II	yes	no	0.044	0.479
III	no	yes	0.028	0.250
IV	yes	yes	0.073	0.679

Table 6: Welfare loss, measured in percentage (100ξ) of consumption, in models II, III, and IV of following a zero capital tax instead of the optimal (non-zero) capital tax when the Hosios condition, $\eta = \xi_u$ is in place. Government spending is $\bar{g} = 0.142$.

high number of jobs in response to lower wages. In contrast to Domeij (2005), we find that this result holds even at the well-known Hosios parameter configuration that typically sets the wage at its socially-optimal level. In addition, labor adjustment at the hours margin also implies a non-zero optimal capital tax even though the Hosios parameterization is satisfied. We find that the welfare loss of not taking into account welfare heterogeneity and intensive labor adjustment when setting capital income taxes, and instead just following the usual prescription of a zero tax rate, can be as high as 0.7 percent of steady-state consumption.

There are at least two directions in which this model could be extended in future research. First, heterogeneity enters our model through differences in the utility of employed versus unemployed individuals. While our results show that even this simple source of heterogeneity matters for optimal policy, it no doubt also is important to consider alternative sources of differences between employed and unemployed individuals. For example, we could move away from the consumption insurance assumed in this model, allowing individual consumption and capital holdings to differ across labor states. Moving in this direction, however, adds to the complexity of the problem by increasing the state space of the model.

Second, our results consider optimal steady state taxation and thus ignore dynamics. However, the solution to the wage bargaining problem shows that the dynamic effects of the labor tax on wage bargaining may be an important consideration for the dynamically optimal Ramsey policy. In particular, equation (18) shows that a time-varying tax rate drives an extra wedge into the division of the surplus created by a match. The way in which this wedge influences the optimal choice of fiscal policy may be interesting to study in future work.

A Derivation of Nash Bargaining Solution

Here we derive the Nash-bargaining solution between an individual worker and the firm for both the total wage payment and the number of hours worked. At the end of the derivations, we show how the bargaining outcome simplifies when there is no intensive labor margin.

The value of working for an individual is

$$\mathbf{W_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)((1-\rho^x)\mathbf{W_{t+1}} + \rho^x \mathbf{U_{t+1}})\right]. \tag{45}$$

The value of not working is

$$\mathbf{U_t} = \frac{A}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(\theta_t q(\theta_t)(1-\rho^x)\mathbf{W_{t+1}} + (1-\theta_t q(\theta_t)(1-\rho^x))\mathbf{U_{t+1}})\right], \tag{46}$$

where $\theta_t q(\theta_t) = m(u_t, v_t)/u_t$ is the probability that an unemployed individual finds a match. Note that because A is measured in utils, we divide by the marginal utility of consumption to express the utility of leisure in terms of goods.

The value of a filled job is

$$\mathbf{J_t} = g(k_t, h_t) - w_t h_t - r_t k_t + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)\mathbf{J_{t+1}}\right], \tag{47}$$

and we have, from the job-creation condition,

$$\frac{\gamma}{q(\theta_t)} = \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)\mathbf{J_{t+1}}\right]. \tag{48}$$

The firm and worker choose w_t and h_t to maximize the Nash product

$$(\mathbf{W_t} - \mathbf{U_t})^\eta\, \mathbf{J_t}^{1-\eta}, \tag{49}$$

with η the bargaining power of the worker. The first-order condition with respect to w_t is

$$\eta\,(\mathbf{W_t} - \mathbf{U_t})^{\eta-1}\left(\frac{\partial \mathbf{W_t}}{\partial w_t} - \frac{\partial \mathbf{U_t}}{\partial w_t}\right) J_t^{1-\eta} + (1-\eta)\,(\mathbf{W_t} - \mathbf{U_t})^\eta\, J_t^{-\eta}\frac{\partial \mathbf{J_t}}{\partial w_t} = 0. \tag{50}$$

With $\frac{\partial \mathbf{W_t}}{\partial w_t} = (1-\tau_t^n)h_t$, $\frac{\partial \mathbf{U_t}}{\partial w_t} = 0$, and $\frac{\partial \mathbf{J_t}}{\partial w_t} = -h_t$, the first-order condition gives the Nash sharing rule

$$\frac{\mathbf{W_t} - \mathbf{U_t}}{1-\tau_t^n} = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{51}$$

The labor tax drives a wedge in the Nash sharing rule, so net-of-taxes the worker receives a smaller share of the surplus than he would absent the tax.

Using the definitions of $\mathbf{W_t}$ and $\mathbf{U_t}$,

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)((1-\rho^x)(1-\theta_t q(\theta_t))\mathbf{W_{t+1}} - (1-\rho^x)(1-\theta_t q(\theta_t))\mathbf{U_{t+1}})\right]. \tag{52}$$

Combine terms on the right-hand-side to get

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\theta_t q(\theta_t))(\mathbf{W_{t+1}} - \mathbf{U_{t+1}})\right]. \tag{53}$$

Using the sharing rule $\mathbf{W_t} - \mathbf{U_t} = \frac{(1-\tau_t^n)\eta}{1-\eta}\mathbf{J_t}$,

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)} + (1-\theta_t q(\theta_t))\left(\frac{\eta}{1-\eta}\right)\beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right], \tag{54}$$

or

$$\frac{\mathbf{W_t} - \mathbf{U_t}}{1-\tau_t^n} = w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta E_t\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right]. \tag{55}$$

Next, insert this in the sharing rule to get

$$w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right] = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{56}$$

Using $\mathbf{J_t} = g(k_t, h_t) - w_t h_t - r_t k_t + \frac{\gamma}{q(\theta_t)}$ on the right-hand-side,

$$w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right] = \frac{\eta}{1-\eta}\left(g(k_t, h_t) - w_t h_t - r_t k_t + \frac{\gamma}{q(\theta_t)}\right). \tag{57}$$

Solving for $w_t h_t$,

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t + \frac{\gamma}{q(\theta_t)}\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] - \eta\left(\frac{1-\theta_t q(\theta_t)}{1-\tau_t^n}\right)\beta\left[\frac{u'(c_{t+1})}{u'(c_t)}(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right]. \tag{58}$$

Make the substitution $\mathbf{J_{t+1}} = g(k_{t+1}, h_{t+1}) - w_{t+1}h_{t+1} - r_{t+1}k_{t+1} + \frac{\gamma}{q(\theta_{t+1})}$ to write

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t + \frac{\gamma}{q(\theta_t)}\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] - \eta\left(\frac{1-\theta_t q(\theta_t)}{1-\tau_t^n}\right)\beta\left[\frac{u'(c_{t+1})}{u'(c_t)}(1-\rho^x)(1-\tau_{t+1}^n)\left(g(k_{t+1}, h_{t+1}) - w_{t+1}h_{t+1} - r_{t+1}k_{t+1} + \frac{\gamma}{q(\theta_{t+} \tag{59}$$

Next, combine the term involving $\gamma/q(\theta_t)$ in the first expression in square brackets with the third term on the right-hand-side. Rearranging, the wage payment can be written as

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] + \frac{\eta\gamma}{q(\theta_t)}\left[\frac{\tau_{t+1}^n - \tau_t^n + \theta_t q(\theta_t)(1-\tau_{t+1}^n)}{1-\tau_t^n}\right], \tag{60}$$

which is expression (37) in the text.

Turning to the determination of hours worked in a match, the first-order-condition of the Nash product with respect to h_t is

$$\eta\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta-1}\left(\frac{\partial \mathbf{W_t}}{\partial h_t} - \frac{\partial \mathbf{U_t}}{\partial h_t}\right)J_t^{1-\eta} + (1-\eta)\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta}J_t^{-\eta}\frac{\partial \mathbf{J_t}}{\partial h_t} = 0. \tag{61}$$

With $\frac{\partial \mathbf{W_t}}{\partial h_t} = (1-\tau_t^n)w_t + \frac{v'(h_t)}{u'(c_t)}$, $\frac{\partial \mathbf{U_t}}{\partial h_t} = 0$, and $\frac{\partial \mathbf{J_t}}{\partial h_t} = g_h(k_t, h_t) - w_t$, the first-order-condition can be written

$$\frac{\eta}{1-\eta}\mathbf{J_t}\left[(1-\tau_t^n)w_t + \frac{v'(h_t)}{u'(c_t)}\right] = \left(\mathbf{W_t} - \mathbf{U_t}\right)\left[w_t - g_h(k_t, h_t)\right]. \tag{62}$$

Substituing $\frac{\eta}{1-\eta}\mathbf{J_t} = \frac{\mathbf{W_t}-\mathbf{U_t}}{1-\tau_t^n}$, we have that hours are determined according to

$$\frac{-v'(h_t)}{u'(c_t)} = (1-\tau_t^n) g_h(k_t, h_t), \tag{63}$$

which is expression (36) in the text. Note this condition is unaffected by the wage. The labor tax rate can be expressed in terms of allocations as

$$\tau_t^n = 1 + \frac{1}{g_h(k_t, h_t)} \frac{v'(h_t)}{u'(c_t)}. \tag{64}$$

In our simpler model without the intensive margin, the worker and firm asset values **W** and **J** simplify to

$$\mathbf{W_t} = (1-\tau_t^n) w_t \bar{h} + \frac{v}{u'(c_t)} + \beta \left[\left(\frac{u'(c_{t+1})}{u'(c_t)} \right) ((1-\rho^x)\mathbf{W_{t+1}} + \rho^x \mathbf{U_{t+1}}) \right] \tag{65}$$

and

$$\mathbf{J_t} = g(k_t, \bar{h}) - w_t \bar{h} - r_t k_t + \beta \left[\left(\frac{u'(c_{t+1})}{u'(c_t)} \right) (1-\rho^x) \mathbf{J_{t+1}} \right]. \tag{66}$$

That is, hours worked in a job are fixed at $\bar{h}$. Maximization of the Nash product is now with respect to only w_t. Proceeding similarly as above, the dynamic wage equation in the model without the intensive margin is

$$w_t \bar{h} = \eta \left[g(k_t, \bar{h}) - r_t k_t \right] + \frac{1-\eta}{1-\tau_t^n} \left[\frac{A-v}{u'(c_t)} \right] + \frac{\eta\gamma}{q(\theta_t)} \left[\frac{\tau_{t+1}^n - \tau_t^n + (1-\tau_{t+1}^n)\theta_t q(\theta_t)}{1-\tau_t^n} \right], \tag{67}$$

which is expression (18) in the text.

B Derivation of Implementability Constraint

The derivation of the implementability constraint follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). We present the derivation first for the full model with intensive adjustment and then show how the implementability constraint is modified in the model without intensive adjustment.

Start with the household flow budget constraint in equilibrium

$$c_t + K_{t+1} + b_t = (1-\tau_t^n) w_t h_t n_t + R_{b,t} b_{t-1} + \left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + d_t. \tag{68}$$

Multiply by $\beta^t u'(c_t)$ and sum over dates,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^t u'(c_t)b_t \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_{b,t}b_{t-1} \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t.\end{aligned}$$

Use the household's Euler equation, $u'(c_t) = \beta[R_{b,t+1}u'(c_{t+1})]$, to substitute for $u'(c_t)$ in the term on the left-hand-side involving b_t,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^{t+1} R_{b,t+1}u'(c_{t+1})b_t \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_{b,t}b_{t-1} \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + \sum_{t=0}^{\infty}\beta^t u'(\end{aligned}$$

Canceling terms in the second summation on the left-hand-side with the second summation on the right-hand-side leaves only the time-zero bond position,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t + u'(c_0)R_{b,0}b_{-1}.\end{aligned}$$

Again use $u'(c_t) = \beta[R_{b,t+1}u'(c_{t+1})]$ along with the no-arbitrage condition between government bonds and capital, $R_{b,t+1} = \left(1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right)$, to substitute in for the second term on the left hand side involving K_{t+1}. Doing so leaves only the time-zero capital stock,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t + u'(c_0)\left[R_{b,0}b_{-1} + \left(1 + (r_0 - \delta)\left(1 - \tau_0^k\right)\right)K_0\right]. \tag{69}$$

Now insert equilibrium profits, given by $d_t = n_t g\left(k_t, h_t\right) - w_t h_t n_t - r_t k_t n_t - \gamma v_t \left(1 - \tau_t^s\right)$ on the right hand side to yield

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t - n_t g\left(k_t, h_t\right) + \tau_t^n w_t h_t n_t + r_t k_t n_t + \gamma v_t\left(1 - \tau_t^s\right)\right] = u'(c_0)\left[R_{b,0}b_{-1} + \left(1 + (r_0 - \delta)\left(1 - \tau_0^k\right)\right)K_0,\right] \tag{70}$$

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - \left(g\left(k_t, h_t\right) - \tau_t^n w_t h_t - g_k\left(k_t, h_t\right) k_t\right) n_t + \gamma v_t \left(1 - \tau_t^s\right)\right] = u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(g_k(k_0, h_0) - \delta\right) \left(1 - \tau_0^k\right)\right) K_0\right] \tag{71}$$

where in the last expression we use the condition $r_t = g_k\left(k_t, h_t\right)$. Because g is homogeneous of degree one we have that

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - \left(g_h\left(k_t, h_t\right) - \tau_t^n w_t\right) n_t h_t + \gamma v_t \left(1 - \tau_t^s\right)\right] = u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(g_k(k_0, h_0) - \delta\right) \left(1 - \tau_0^k\right)\right) K_0\right]. \tag{72}$$

Using the Nash solution for hours $\frac{-v'(h_t)}{u'(c_t)} = (1 - \tau_t^n) g_h(k_t, h_t)$ we can write $g_h\left(k_t, h_t\right) - \tau_t^n w_t = g_h\left(k_t, h_t\right) - w_t - \frac{w_t}{g_h(k_t, h_t)} \frac{v'(h_t)}{u'(c_t)}$. Inserting this into the previous expression, we arrive at our form of the present value implementability constraint,

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - n_t h_t \left(g_h\left(k_t, h_t\right) - w_t - \frac{w_t}{g_h(k_t, h_t)} \frac{v'(h_t)}{u'(c_t)}\right) + \gamma v_t \left(1 - \tau_t^s\right)\right] =$$
$$u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(g_k(k_0, h_0) - \delta\right) \left(1 - \tau_0^k\right)\right) K_0\right],$$

presented in Section 3. Note that setting $\gamma = 0$ (no vacancy posting cost), $n_t = 1$ (no extensive margin), and using the result that in a Walrasian labor market $w_t = g_h(k_t, h_t)$, the implementabilty constraint becomes

$$\sum_{t=0}^{\infty} \beta^t \left[u'(c_t) c_t + v'(h_t) h_t\right] = u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(g_k(k_0, h_0) - \delta\right) \left(1 - \tau_0^k\right)\right) K_0\right], \tag{73}$$

which is identical to the household present-value budget constraint in a baseline (i.e., Lucas and Stokey (1983) or Chari and Kehoe (1999)) Ramsey model.

In the model without intensive adjustment, the manipulations following expression (71) are unavailable because the production function $g(k)$ is not constant returns to scale. We also cannot eliminate the labor tax rate τ^n as we did above because there is no intensive margin with which to eliminate it. Thus, in deriving the implementability constraint in the model without intensive adjustment, we must stop at the analog of expression (71),

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - \left(g(k_t, \bar{h}) - \tau_t^n w_t \bar{h} - g_k(k_t, \bar{h}) k_t\right) n_t + \gamma v_t (1 - \tau_t^s)\right] = \tag{74}$$
$$u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(g_k(k_0, \bar{h}) - \delta\right) \left(1 - \tau_0^k\right)\right) K_0\right],$$

which is equation (21) in the text.

References

Andolfatto, David. 1996. "Business Cycles and Labor-Market Search." *American Economic Review,* Vol. 86, pp. 112-132.

Blanchard, Olivier and Peter Diamond. 1989. "The Beveridge Curve." *Brookings Papers on Economic Activity,* Vol. 1, pp. 1-76.

Chamley, Christophe. 1986. "The Welfare Cost of Capital Income Taxation in a Growing Economy." *Econometrica,* Vol. 54, pp. 607-622.

Chari V. V., and Patrick J. Kehoe. 1999. "Optimal Fiscal and Monetary Policy. In *Handbook of Macroeconomics,* edited by John B. Taylor and Michael Woodford, Vol. 1C. Elsevier.

Chugh, Sanjay K. 2006. "Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices." *Review of Economic Dynamics.* Forthcoming.

Domeij, David. 2005. "Optimal Capital Taxation and Labor Market Search." *Review of Economic Dynamics,* Vol. 8, pp. 623-650.

Hagedorn, Marcus and Iourii Manovskii. 2006. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." University of Pennsylvania.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies,* Vol. 57, pp. 279-298.

Judd, Kenneth L. 2002. "Capital Income Taxation with Imperfect Competition." *American Economic Review,* Vol. 92, pp. 417-421.

Krause, Michael U. and Thomas A. Lubik. 2004. "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions." Johns Hopkins University.

Ljungqvist, Lars and Thomas J. Sargent. 2004. *Recursive Macroeconomic Theory.* MIT Press.

Lucas, Robert E. Jr. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics,* Vol. 12, pp. 55-93.

Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics,* Vol. 36, pp. 269-300.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* MIT Press.

Rogerson, Richard, Robert Shimer, and Randall Wright. 2005. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature,* Vol. 43, pp. 959-988.

Schmitt-Grohe, Stephanie and Martin Uribe. 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Macroeconomics Annual 2005.

Shi, Shouyong and Quan Wen. 1999. "Labor Market Search and the Dynamic Effects of Taxes and Subsidies." *Journal of Monetary Economics,* Vol. 43, pp. 457-495.

Trigari, Antonella. 2003. "Equilibrium Unemployment, Job Flows and Inflation Dynamics." Bocconi University.

Walsh, Carl E. 2005. "Labor Market Search, Sticky Prices, and Interest Rate Policies." *Review of Economic Dynamics,* Vol. 8, pp. 829-849.