Liquidity and the Market for Ideas*

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December 14, 2005

Abstract

We study markets where innovators can sell ideas to entrepreneurs, who may be better at implementing them. These markets are decentralized, with random matching and bargaining. Entrepreneurs hold liquid assets lest potentially profitable opportunities may be lost. We extend existing models of the demand for liquidity along several dimensions, including allowing agents to put deals on hold while they try to raise funds. We determine which ideas get traded in equilibrium, compare this to the efficient outcome, and discuss policy implications. We also discuss several special aspects of ideas, as opposed to generic consumption goods: e.g. they are intermediate inputs; they are indivisible; and they are at least partially public (nonrivalous) goods.

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1 Introduction

We take it for granted that people understand that the development and implementation of new ideas is one of the major factors underlying economic performance.[1] In this vein, the concept of *technology transfer* is important to innovators and entrepreneurs looking to come up with and commercialize new technologies, and also to policy makers seeking to spur economic development. The issue is this: When innovators come up with new inventions (or ideas or projects), should they try to implement them themselves, say through start-up firms? Or should they try to sell them, perhaps to established firms, or more generally to entrepreneurs who are better at implementing these ideas?

If agents are heterogeneous in their abilities to come up with ideas and to extract their returns, one can imagine that some will specialize in innovation while others will specialize in implementation or commercialization. A superior allocation of resources will generally emerge when those who have the ideas are not necessarily those who implement them. Scholars in the "knowledge transactions field" share the view that the transfer of ideas from innovators to entrepreneurs leads to a more efficient use of resources, making all parties better off and increasing the incentives for investments

[1] Both the inputs to and outputs of this process are important. On the input side, research and development expenditures account for 3% of US GDP, and according to a survey by the Association of University Technology Managers, the licensing of innovations just by universities, hospitals, research institutions, and patent management firms added more than $40 billion to the economy in 1999 and supported 270,000 jobs. On the output side, it is obvious that new ideas and technologies are essential to production and growth, and going back to Schumpeter (1934) it is often said that the creation of new firms is a significant mechanism through which new technologies are implemented.

in research. As Katz and Shapiro (1986) put it, "Inventor-founded startups are often second-best, as innovators do not have the entrepreneurial skills to commercialize new ideas or products."[2]

Obviously, however, this requires some mechanism – say, some *market* – for the exchange of ideas, and the details of how this mechanism works could in principle have a big impact on outcomes. This is the subject of the current study.

Our analysis is related to the well-known work of Holmes and Schmitz (1990, 1995), although we also deviate considerably from their approach. What we share with them is, in their words, the following: "The model has two key features. The first crucial assumption is that opportunities for developing new products repeatedly arise through time... The second key feature is that we assume that individuals differ in their abilities to develop emerging opportunities." Hence, "There are two tasks in the economy, developing products and producing products previously developed" (Holmes and Schmitz 1990, p. 266-7). Where we differ is the way we envision the market where ideas or projects get traded. While they model it as a centralized

[2] As reported in a recent special feature in The Economist (Oct. 2005) on ideas, patents and related topics, "as the patent system has evolved, it ... leads to a degree of specialization that makes business more efficient. Patents are transferable assets, and by the early 20th century they had made it possible to separate the person who makes an invention from the on who commercializes it. This recognized the fact that *someone who is good at coming up with ideas is not necessarily the best person to bring these ideas to market*" (p.6, emphasis added). And they quote Henry Chesbrough as saying "You see people innovating and creating new ideas and technologies, but not taking them all the way through to the market. They carry it to a certain stage and then hand the baton on to others who bring it on to commercialization" (p.14). Of course, one could imagine innovators trying to buy implementation expertise from entrepreneurs, but the usual view is that such expertise is largely tacit and difficult to measure, so it seems more natural for the ideas to be sold to implementers. See also Teece et al. (1997), Pisano and Mang (1993), and Shane (2002).

market in competitive equilibrium, we take seriously the notion that there are considerable frictions in this market.

We think it is clear that there is in reality no centralized market for ideas. Innovators do not simply choose a quantity of ideas to supply to maximize profit taking as given the competitive price, and entrepreneurs do not simply choose how many new ideas to buy at a given price. The idea market is in our view much more *decentralized*. Hence, we model it using search theory, with random matching and bilateral bargaining between innovators and entrepreneurs. Also, in accordance with a large literature regarding entrepreneurs and liquidity constraints (discussed below), we consider that the *availability of liquid assets* may be important to close the deal: when there are imperfect markets for the exchange of ideas, it is not only relevant who you meet and what they know, there is also the issue of how to pay for it. The fact that you are better at implementing a project may means little if you have nothing to offer in exchange.

This is especially important in highly decentralized markets, where it is easy to imagine reasons why I might be reluctant to give you my idea for a promise of future payment (e.g. once I give you the information, you might decide not to pay, and it may be hard to take the idea back). Hence, it is easy to imagine that quid pro quo may well be the order of the day: "You want my idea? Show me the money." Given this, entrepreneurs may choose to keep liquid assets, cash being the purest example, in case they come across a potentially profitable opportunity that could be lost if there is not a quick agreement. Naturally, how much liquidity they hold depends on the cost, e.g. interest rates, which are at least in part determined by policy.

Our view that liquidity broadly speaking matters in this context is by no means new. Evans and Jovanovic (1989) e.g. find that the decision to become an entrepreneur depends positively on wealth, and interpret this as evidence of financial constraints. They conclude that the "liquidity constraint is binding for virtually all the individuals who are likely to start a business." They predict that if such constraints were removed, the probability of becoming an entrepreneur would increase by 34%. Others come to similar conclusions.[3] To be fair, Lusardi and Hurst (2004) provide a dissenting opinion: while they also find a positive correlation between wealth and the probability a household subsequently owns a business, they suggest it is at least partly due to differences between business owners and non-owners in abilities, preferences and background, rather than liquidity.[4]

At one level, this discussion is not pivotal for what we want to say, because ideas are not only inputs to *new* businesses but also existing businesses, and liquidity constraints may impinge on the scale of operations as

[3] See also Evans and Leighton (1989), Holtz-Eakin et al. (1994), Fairlie (1999), Quadrini (1999, 2000), Gentry and Hubbard (2000), Paulson and Townsend (2000), Guiso, Sapienza and Zingales (2001), and Lel and Udell (2002).

[4] They conclude "Our results do not imply that any given household wanting to start a small business has unlimited access to credit at reasonable borrowing rates. Given optimal lender behavior, and common sense, such results would be implausible. We do conclude, however, that even if some households that want to start small businesses are currently constrained in their borrowing, such constraints are not empirically important in deterring the majority of small business formation in the United States. This may simply reflect the fact that the starting capital required for most businesses is sufficiently small. ... Alternatively, even if the required starting capital for some small businesses is high, existing institutions and lending markets in the United States appear to work sufficiently well at funnelling funds to households with worthy entrepreneurial projects."

well as the probability of starting up. But in any case, we want to remain agnostic and construct a model where, by varying parameters, we cover the case where liquidity is critical, where it is irrelevant, and anything in between. Moreover, the way we model liquidity is quite different than previous work on entrepreneurship where various credit market imperfections are imposed in sometimes rather ad hoc ways.[5] We also emphasize that liquidity is *endogenous* in our framework – entrepreneurs choose their liquid assets, depending on various factors, including interest rates, market frictions, etc.

In this sense our model is related to some work in the search-based monetary literature, and particular we follow Lagos and Wright (2005) by assuming agents sometimes trade in centralized markets and sometimes in decentralized markets. But while we adopt that feature, we also extend existing versions of that model in several important ways. First, to the extent that ideas are intermediate inputs, we stress that operation of the idea market can spill over to other markets and especially to wages and employment. Second, we take seriously the notion that there may be a public good aspect at play: the fact that I tell you my idea does not mean that I cannot also use it (although it may be somewhat less valuable to me if you also know it).

Third, partly because of the previous points, we suggest that monetary policy may be more potent than is commonly understood from models where

[5]Some people simply assume there is no credit (Lloyd-Ellis and Bernhardt 2000 and Buera 2005), some assume credit is exogenously limited to a fixed multiple of wealth (Evans and Jovanovic 1989), some try to model it using moral hazard (Aghion and Bolton 1996), and some using asymmetric information (Fazzari et al. 1988, 2000).

liquidity is only relevant for trading consumption goods. Fourth, since ideas are indivisible and have random valuations, and since trades may be liquidity constrained, our bargaining problem may be nonconvex and hence we consider the interesting possibility of randomized trade (lotteries). Finally, in what is perhaps the most interesting innovation, agents with insufficient liquidity can attempt to put deals on hold and raise additional funds in the next centralized market; the probability this attempt fails is what parameterizes the extent of the liquidity problem. All of this goes well beyond existing work in the related monetary literature.

The rest of the paper is organized as follows. Section 2 lays out our basic assumptions. Section 3 discusses the centralized market, and Section 4 discusses the decentralized market where ideas are traded. Section 5 puts things together to characterize equilibrium. Section 6 takes up various extensions. Section 7 concludes. Some technical results are relegated to the Appendix.[6]

[6] We mention some other related work. Several studies consider the transfer of ideas as a strategic action among firms, including Katz and Shapiro (1986), Gallini and Winter (1985), and Shepard (1994). Baccara and Razin (2004) consider strategic behavior among agents forming a team to implement an idea. Anton and Yao (1994, 2002) study markets where buyers do not know the value of an idea, and sellers are reluctant to reveal it because buyers may not pay afterwards. Others focus on licensing contracts in terms of incentives, including Aghion and Tirole (1994) and Arora (1995). There is a literature that focuses on university inventions, including Lowe (2003), Shane (2002), and Jensen and Thursby (2001). Chari, Golosov and Tsyvinski (2004) study the effects of taxation, den Haan, Ramey and Watson (2003) study matching between entrpreneurs and lenders, and Serrano (2005) studies empirically the market for patent transfers.

2 Basic Assumptions

Time is discrete and continues forever. As in Lagos and Wright (2005), alternating over time there are two types of markets: a centralized market, denoted CM, where agents perform the usual activities of working, consuming and adjusting their assets; and a decentralized market, denoted DM, where agents meet bilaterally and, in our model, may trade ideas. Agents have discount factor β between one DM and the next CM, and discount factor δ between the CM and the next DM, where $\delta\beta < 1$. There are two types of agents: *innovators*, denoted i, who are relatively good at coming up with ideas, and *entrepreneurs*, denoted e, who may be better at implementing them. For now the numbers of each type, N_i and N_e, are exogenous.

Every time the DM opens, innovator i gets some idea (for now for free) that has value $R_i \geq 0$ if he implements it himself, where R_i is drawn from CDF $F_i(\cdot)$. This return is realized in the next CM. To keep things simple, if not implemented in one CM, an idea's value in the next period is an i.i.d. draw. Hence, if i finds himself in the CM with an idea, he will always implement it, since he gets a new draw in any event. Entrepreneur e does not get ideas on his own, but if i with an idea worth R_i to him meets e in the DM, it has value $R_e \geq 0$ to e, where R_e is drawn from $F_e(\cdot|R_i)$. When convenient we sometimes assume the densities $F_i'(R_i)$ and $F_e'(R_e|R_i)$ exist and are continuous, but this is not really necessary.[7]

[7] As a special case, if R_i and R_e are independent, we can say what matters is only the match between the idea and the agent. Another special case discussed in Section 6.1 is the one with $R_i = \bar{R}_i$ with probability 1, including $\bar{R}_i = 0$, where i is are purely an "idea man" who cannot implement anything. We could easily allow entrepreneurs to come up

One may well ask, *what exactly is an idea*? One view is that an idea I is an intermediate input into some production process that can be implemented by agent j with technology $f_j(\mathbf{h}, I)$, where $\mathbf{h}$ is a vector of inputs, including labor. Given I, j solves

$$R_j(I) = \max_h \{f_j(\mathbf{h}, I) - \mathbf{wh}\}, \tag{1}$$

where $\mathbf{w}$ is a vector of factor prices, including wages. This is important because it shows the allocation of ideas can affect employment, wages, and other variables in general equilibrium, and having the wrong agent implementing I can have a big impact on economic aggregates. However, to ease the presentation we begin with the case where $R_j = f_j(I)$ does not require additional inputs, and return to the general specification in Section 6.

We assume ideas are indivisible: either I tell you or I don't. We assume there is no private information: in a meeting, both agents know (R_i, R_e), even though e cannot implement the idea without i giving him the details. For example, if my idea is for a restaurant with some new cuisine, I can let you taste a sample without necessarily giving you the recipe. We abstract from informational frictions here not because they are uninteresting, but because we want to focus on different issues (several papers mentioned in the Introduction consider private information).

We do take seriously the notion that there may be a public good aspect to ideas: the fact that I tell you my idea does not mean that I cannot also

with some of their own ideas, or even reduce the model a single type – all agents get ideas from $F(R)$, but an idea worth R to you is worth $\hat{R}$ to me, drawn from $\hat{F}(\hat{R}|R)$. The reason for having two types is that it will be interesting to endogenize their numbers.

use it. One way to capture this is to assume that if agent j is the only one to implement the idea he gets R_j, while if another agent also implements it then j only gets $\lambda_j R_j$. If $\lambda_j = 1$, e.g., ideas are pure public (nonrival) goods. In general, if i sells his idea to e and both implement, they get $\lambda_i R_i$ and $\lambda_e R_e$; if i keeps it for himself, they get R_i and 0 respectively. To simplify the presentation we begin with the case $\lambda_i = 0$ and $\lambda_e = 1$, which can be interpreted as saying that i does not also implement an idea once he sells it – say, because there is only room for one new restaurant in town, or more generally, stepping outside the model, perhaps because of some exclusive licensing agreement. We return to the general case in Section 6.

If i and e meet in the CM and there are gains from trade, they bargain over the price of the idea. The price is in terms of *money* – by which we do not necessarily mean cash, but relatively liquid assets generally, where a liquid asset is one that can be readily accessed on short notice.[8] If the price p at which they would trade in the absence of liquidity considerations is greater than the assets e happens to have available in the DM, several things could happen: i could keep the idea for himself; he could settle for a lower price; or he could suggest they try to meet again in the next CM, where e can always raise funds. However, if they try to meet again, with probability $1 - \gamma$ they fail. Rather than go into details, we simply label this

[8] He, Huang and Wright (2005) introduce banks and checking accounts explicitly into a search model of monetary exchange, and something similar can be done here. To focus the discussion, we avoid these details and frame the formal analysis in terms of money, but hopefully it is understood that the point applies to liquid assets more broadly. Obviously, in the real world different assets can have marginal differences in liquidity, but for simplicity we assume it is either 0 or 1.

event an exogenous breakdown, or say that *the deal falls through.*

This possibility is what generates a demand for liquidity. Clearly, we need imperfect credit enforcement for this to work; so i does not give up his idea for a promise of future payments, since e can simply renege.[9] Also, we understand that there are many ways in the real world for innovators to get people who are good at implementation involved in their projects: hiring them, forming partnerships, licensing, etc. We consider only the case where they sell the idea. While this is not the only possibility it is surely an interesting one. Among other reasons, it captures the notion that innovators prefer not to be involved in actual operations so that they can focus on coming up with new ideas.[10]

3 The CM

Let $W_j(m, R)$ be the the value functions for type $j = i, e$ agents entering the CM, with m dollars and a project in hand with value R (for i this would be his own idea if he did not sell it in the previous DM, and for e this would be an idea that he purchased). We use $R = 0$ to indicate either a project with 0 return or no project (for i this would be because he sold his idea, and for e this would be because he failed to buy one). Let $V_j(m)$ be the value function for agents entering the DM with m dollars before the random

[9] It is important for this that we cannot use reputation to enforce payment. A standard way in monetary theory to rule out reputation is to assume some form of anonymity; see e.g. Kocherlakota (1998), Wallace (2001) or Corbae et al. (2003).

[10] Again referring to The Economist (Oct. 2005), "licensing usually works only alongside a basket of products or services. For IBM, for example, the majority of its intellectual-property revenue comes from the sale of knowledge, not patent licenses alone. In essence, the difference is that between the recipe for a dish and a list of ingredients."

values of the ideas are drawn.

Then for $j = i, e$, the CM problem is

$$\begin{aligned} W_j\left(m, R\right) &= \max_{X,H,\hat{m}} \left\{U(X) - h + \delta V_j(\hat{m})\right\} \\ \text{s.t. } X &= X_0 + wh + \phi(m - \hat{m} + \pi M) + R, \end{aligned} \tag{2}$$

where X is consumption, h labor supply, $\hat{m}$ money taken out of the CM, X_0 an endowment, w the real wage, and ϕ the value of money (i.e., $1/\phi$ is the nominal price level). The term πM is a lump sum transfer, with M the aggregate money stock when the CM opens, which evolves over time according to $M' = (1 + \pi)M$. Formally it looks like we are taking liquidity literally to be money here, although as discussed above the theory is meant to apply to liquid assets more generally. If one interprets m as fiat money, π will be the equilibrium inflation rate, and the model can be used to discuss monetary policy; one does not have to buy into this, however, and m can be interpreted as any asset that can be accessed from the DM.

Assume for now there is a representative firm with a linear technology, so the equilibrium wage is pinned down and can be normalized without loss of generality to $w = 1$. Then, using the budget equation, rewrite (2) as

$$\begin{aligned} W_j\left(m, R\right) &= X_0 + \phi m + \phi \pi M + R + \max_X \left\{U(X) - X\right\} \\ &\quad + \max_{\hat{m}} \{-\phi \hat{m} + \delta V_j(\hat{m})\}. \end{aligned} \tag{3}$$

$>$From (3) the following results are immediate.[11]

[11] These results assume an interior solution. One can guarantee this for X by assuming $U'(0) = \infty$; for h, one can adapt the assumptions in Lagos and Wright (2005). The results also assume the strict concavity of V_j, which we verify below.

Lemma 1 *(i) W_j is linear in (m, R), with $\partial W_j / \partial m = \phi$ and $\partial W_j / \partial R = 1$; (ii) X is given by the solution to $\partial U(X)/\partial X = 1$; (iii) $\hat{m}$ is given by the solution to*

$$-\phi + \delta \frac{\partial V_j(\hat{m})}{\partial \hat{m}} \leq 0, = 0 \text{ if } \hat{m} > 0, \tag{4}$$

and in particular, all agents of a given type j take the same $\hat{m}_j$ out of the CM, regardless of the (m, R) with which they enter.

4 The DM

Let α_j be the DM arrival rate (probability of a meeting) for $j = i, e$. Normalizing $N_e = 1$, the only restriction on arrival rates is $\alpha_e = \alpha_i N_i$, so we can take α_j to be exogenous, for now. If an e does not meet anyone, he enters the next CM with his money but no project, $(\hat{m}_e, 0)$. Similarly, if i does not meet anyone, he enters the next CM with $(\hat{m}_i, R_i)$. If an e and i do happen to meet, several things can happen. If $R_e \leq R_i$ there are no gains from trade; if $R_e > R_i$ there are, and two cases need to be considered.

On the one hand, suppose $\hat{m}_e \geq p$, where p is the price they would agree to if there were no issues of liquidity – e.g. if e had access to the funds he will have available in the next CM from his endowment and his labor supply. Then they can – an in equilibrium they will – trade immediately at price p.

On the other hand, suppose $\hat{m}_e < p$. In this case the bargaining problem is nonconvex, and in principle they may want to trade using lotteries. However, for simplicity we assume lotteries are not available for now, and revisit the issue in Section 6, where we show that the main economic results are very similar. Hence, for now they can either settle for $\hat{m}_e$, or put the deal on hold and try to meet again in the next CM, where e can always raise the funds. If they do meet again they can renegotiate the price to p', but we will see that $p' = p$. In any case, meeting again in the next CM only happens with probability γ; otherwise the deal falls through. The big decision for i is: should he settle for $\hat{m}_e$ and close the deal now, or put the deal on hold for a chance at p'?

We now analyze the bargaining problems in more detail. We use the generalized Nash solution, where threat points are given by continuation values and θ denotes the bargaining power of e. To begin, consider what happens if they put the deal on hold and meet again in the next CM. Given the value function next period W_j', the bargaining solution is:

$$\max_{p'}[W_e'(\hat{m}_e - p', R_e) - W_e'(\hat{m}_e, 0)]^\theta [W_i'(\hat{m}_i + p', 0) - W_i'(\hat{m}_i, R_i)]^{1-\theta}$$

By Lemma 1, $W_e'(\hat{m}_e - p', R_e) - W_e'(\hat{m}_e, 0) = R_e - \phi' p'$ and $W_i'(\hat{m}_i + p', 0) - W_i'(\hat{m}_i, R_i) = \phi' p' - R_i$. Hence the problem reduces to:

$$\max_{p'} \left(R_e - \phi' p'\right)^\theta \left(\phi' p' - R_i\right)^{1-\theta}$$

This immediately yields

$$p' = \frac{\theta R_i + (1-\theta) R_e}{\phi'}. \tag{5}$$

Now, back up to this period and consider what happens in the DM. One difference from CM bargaining is that the threat points are given by the expected values of putting the deal on hold,

$$\begin{aligned}\overline{W}'_e &= \gamma W'_e(\hat{m}_e - p', R_e) + (1-\gamma)W'_e(\hat{m}_e, 0)\\ \overline{W}'_i &= \gamma W'_i(\hat{m}_i + p', 0) + (1-\gamma)W'_i(\hat{m}_i, R_i).\end{aligned}$$

A second difference is that we have a constraint $p \leq \hat{m}_e$, since e can only pay out of liquid assets in the DM (by definition). The problem becomes:

$$\max_{p \leq \hat{m}_e} [-\phi' p + \gamma \phi' p' + (1-\gamma)R_e]^{\theta} [\phi' p - \gamma \phi' p' - (1-\gamma)R_i]^{1-\theta}$$

Suppose first that the constraint does not bind. Then it is simple to show $p = p'$, the same as the solution in the CM next period. In this case the agents settle immediately. Suppose now that $\hat{m}_e < p'$, which is equivalent to $R_e > B(R_i) \equiv \frac{\phi' \hat{m}_e - \theta R_i}{1-\theta}$ (the label B stands for the fact that the liquidity constraint just *binds*). In this case e wants to pay $\hat{m}_e$ and close the deal now, but i prefers to put the deal on hold iff $W'_i(\hat{m}_i + \hat{m}_e, 0) < \overline{W}'_i$, which simplifies to $R_e > H(R_i) \equiv \frac{\phi' \hat{m}_e - R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)}$ (the label H stands for the fact that he is just willing to putting the deal on *hold*).

Given $R_e > R_i$, so that there are gains from trade, we can summarize the outcome as follows, where the proof follows directly from the discussion in the text.

Lemma 2 *In the DM, if $R_e \leq B(R_i)$ they trade now at $p = p'$, given by (5); if $B(R_i) < R_e \leq H(R_i)$ they trade now at $p = \hat{m}_e$; and if $R_e > H(R_i)$ the deal is put on hold. In the CM, they trade at p'.*

The results are illustrated in Figure 1 in the space of realizations for a DM meeting, taking $z = \phi' \hat{m}_e$ as given (it will be determined in equilibrium below). We want to emphasize is that it is potential deals for the *best* ideas that are put on hold, and hence may fall through. Intuitively, when R_i and R_e are high p' is high, so i has a big incentive to take a chance; and when R_i is high, there is also less downside risk. Hence, if this market functions poorly, for whatever reason, there can potentially be serious consequences.

FIGURE 1 ABOUT HERE.

Given the bargaining solution, we now proceed to the DM value function. For e, this is given by

$$
\begin{aligned}
V_e(\hat{m}) &= (1-\alpha_e)\beta W_e'(\hat{m},0) + \alpha_e\beta \int\limits_{A_0} W_e'(\hat{m},0) \\
&\quad + \alpha_e\beta \int\limits_{A_1} W_e'(\hat{m}-p,R_e) + \alpha_e\beta \int\limits_{A_2} W_e'(0,R_e) + \alpha_e\beta \int\limits_{A_3\cup A_4} \overline{W}_e'
\end{aligned} \tag{6}
$$

where $\int_{A_j}(\cdot)$ is the integral over region A_j in Figure 1; e.g.

$$
\int\limits_{A_1}(\cdot) = \int\limits_0^{\phi'\hat{m}} \int\limits_{R_i}^{B(R_i)} (\cdot) dF_e(R_e|R_i)dF_i(R_i).
$$

In words, the first term in (6) is the payoff to no meeting; the second is the payoff to a meeting with no trade; the third is the payoff to trading at $p = [\theta R_i + (1-\theta)R_e]/\phi'$; the fourth is the payoff to trading at $p = \hat{m}$; and the fifth is the payoff to a deal on hold.[12] Using the linearity of W_e' and

[12] We distinguish between A_3 and A_4, even though in both regions the outcome is the same (the deal is put on hold) for two reasons. First, it is useful for some technical results in the Appendix. Second, the economic interpretation is different. In A_4, even if e were to give i all his money, i is better off keeping the idea since $R_i > \hat{m}_e\phi'$. In A_3, because $R_i < \hat{m}_e\phi'$, i prefers trading for $\hat{m}_e$ to implementing the idea himself, but he still prefers putting the deal on hold for a chance at p'.

inserting p, we can simplify (6) to

$$V_e(\hat{m}) = \beta W_e'(\hat{m},0) + \alpha_e\beta\theta \int\limits_{A_1} (R_e - R_i) + \alpha_e\beta \int\limits_{A_2} (R_e - \phi'\hat{m}) + \gamma\alpha_e\beta\theta \int\limits_{A_3\cup A_4} (R_e - R_i). \tag{7}$$

A similar exercise can be performed for i, and it turns out that

$$V_i(\hat{m}) = \beta\phi'\hat{m} + v, \tag{8}$$

where v does not depend on $\hat{m}$. Intuitively, for i, neither the probability of trade nor the terms of trade depend on his own liquidity (they depend on liquidity on the other side of the market). So any $\hat{m}$ he brings to the DM, he simply takes back to the next CM.

We also need the derivatives. For i, this is trivially $\partial V_i/\partial\hat{m} = \beta\phi'$. For e, we establish in Appendix A the following result, which is somewhat complicated because we have to break things into several cases to avoid dividing by 0.

Lemma 3 $\partial V_e/\partial\hat{m} = \beta\phi'\left[1 + \ell(\phi'\hat{m})\right]$, *where for any* z, $\ell(z)$ *is defined as follows: (i) if* $\gamma > 0$ *and* $\theta < 1$ *then*

$$\ell(z) = \frac{\alpha_e(1-\gamma)}{\gamma^2(1-\theta)^2} \int\limits_0^z (z - R_i)\, F_e'\left[H(R_i)|R_i\right] dF_i(R_i) - \alpha_e \int\limits_0^z \left\{F_e\left[H(R_i)|R_i\right] - F_e\left[B(R_i)|R_i\right]\right\} dF_i(R_i); \tag{9}$$

(ii) if $\gamma = 0$ and $\theta < 1$ then

$$\begin{aligned} \ell(z) &= \alpha_e F_i'(z) \int_z^\infty (R_e - z) dF_e(R_e|z) \\ &\quad - \alpha_e \int_0^z \{1 - F_e[B(R_i)|R_i]\} dF_i(R_i); \end{aligned} \tag{10}$$

(iii) if $\theta = 1$ then

$$\ell(z) = (1-\gamma)\alpha_e F_i'(z) \int_z^\infty (R_e - z) dF_e(R_e|z). \tag{11}$$

Note that $\ell(z)$ is the net marginal benefit of liquidity. Consider for example case (ii), with $\gamma = 0$. The first term in (10) is the probability of meeting i with $R_i = z$, which is $\alpha_e F_i'(z)$, times the net gain for e from buying the idea, $R_e - z$, integrated over R_e. The second term is the probability of $(R_i, R_e) \in A_2$ times -1, since in A_2 the constraint binds and the marginal dollar is simply taken by i. Notice also that, comparing the marginal benefits for e and i, we have $\partial V_e/\partial \hat{m} = [1 + \ell(z)] \partial V_i/\partial \hat{m}$, which says that for e the return on $\hat{m}$ includes a liquidity component that is not there for i.[13]

5 Equilibrium

We now combine the DM and CM and define equilibrium. The key condition from the CM is the FOC for $\hat{m}$, given by (4). All we need to do is insert the derivative of the DM value function V_j to determine the choice of $\hat{m}_j$. For $j = i$ this is easy: by (8), $\partial V_i/\partial \hat{m} = \beta\phi'$, so (4) becomes

$$-\phi + \delta\beta\phi' \leq 0, = 0 \text{ if } \hat{m} > 0.$$

[13] See Lagos (2005) for an discussion of similar equations in a related model of liquidity applied to asset pricing.

As is standard in monetary economics (see e.g. Lagos and Wright 2005), we only consider equilibria where $\delta\beta\phi' < \phi$, and hence we conclude that innovators have no demand for liquidity: $\hat{m}_i = 0$.

To be clear, the reason it is standard to only consider equilibria satisfying $\delta\beta\phi' < \phi$ is this: when $\delta\beta\phi' > \phi$ no equilibrium exists, and when $\delta\beta\phi' = \phi$ equilibrium is indeterminate. One way to understand this is to use the Fisher equation: $1 + i_n = (1 + i_r)\phi/\phi'$, where i_n is the nominal interest rate, i_r the interest real rate, and ϕ/ϕ' the inflation rate between two meetings of the CM. In this model, $1 + i_r = 1/\beta\delta$, so $\delta\beta\phi' < \phi$ reduces to $i_n > 0$. To restate what we said above in terms of the interest rate, $i_n < 0$ is inconsistent with equilibrium and $i_n = 0$ implies equilibrium is indeterminate. Hence, our assumption is simply $i_n > 0$, although we do consider the limiting case $i_n \to 0$, which is called the Friedman rule. In other words, we assume liquidity is costly, but also take the limit where it is not.

A similar exercise for $j = e$ is less simple. By Lemma 3, we have

$$-\phi + \delta\beta\phi' \left[1 + \ell(\phi'\hat{m})\right] \leq 0, = 0 \text{ if } \hat{m} > 0. \tag{12}$$

Given any path for M, equilibrium could be defined in terms of a path for ϕ satisfying (12), plus some side conditions, but to simplify the discussion we focus on steady state equilibria where $z = \phi M$ is constant. Since $\hat{m}_e = M'$ (entrepreneurs hold all the money), again using the Fisher equation, in steady state (12) can be simplified to

$$\ell(z) \leq i_n, \; = i_n \text{ if } z > 0. \tag{13}$$

Note that in this condition i_n is exogenous – it is a policy variable.[14]

Consider first the generic case $\gamma \in (0,1)$ and $\theta \in (0,1)$. Since $\ell(0) = 0$, a steady state with $z = 0$ always exists. From now on we concentrate on what we call a monetary steady state, which is a $z > 0$ such that $\ell(z) = i_n$ and, in addition, $\ell'(z) = \partial^2 V_e / \partial \hat{m}^2 \leq 0$, which is necessary because otherwise the SOC for $\hat{m}_e$ in the CM problem is violated. We formalize this as:

Definition 1 *A monetary steady state equilibrium is a $z > 0$ such that $\ell(z) = i_n$ and $\ell'(z) \leq 0$.*

Continuing with the case $\gamma \in (0,1)$ and $\theta \in (0,1)$, in Appendix B we verify that $\lim_{z\to\infty} \ell(z) = 0$. We also verify in Appendix B that one can impose simple conditions, such as F_j' continuous, to guarantee that ℓ is continuous, and $\ell(z) > 0$ for some $z > 0$. Hence there exist a solution to $\ell(z) = i_n$ iff i_n is not too big, and these solutions generically come in pairs. For each pair of solutions, the higher z constitutes a monetary equilibrium, while the lower one does not because it violates $\ell'(z) \leq 0$. Figures 2 and 3 show two examples that illustrate the idea. But the general result, the proof of which follows directly from Appendix B and the above discussion, is this:

Proposition 1 *Given $\gamma \in (0,1)$, $\theta \in (0,1)$ and ℓ continuous, there exists a monetary equilibrium $z > 0$ iff i_n is not too big.*

FIGURES 2 AND 3 ABOUT HERE.

[14] A central bank can either set i_n directly and let the money growth rate π adjust, or they can fix π and $\phi/\phi' = 1 + \pi$ will pin down i_n through the Fisher equation.

For completeness, we discuss what happens in extreme cases. When $\theta = 1$ or $\gamma = 0$, the results are the same but for minor details; e.g. we could have $\ell(0) > 0$, but this is irrelevant for the economics.[15] If $\gamma = 1$, however, things are quite different, because then $\ell(z) = 0$ for all z and the only equilibrium is $z = 0$. This is because, when $\gamma = 1$, e has no demand for liquidity as he can always raise funds in the CM without fear that a deal will fall through. Also, perhaps surprisingly, when $\theta = 0$ a monetary equilibrium can still exist, contrary to the typical search and bargaining model. Usually when $\theta = 0$ money cannot be valued because the buyer gets 0 surplus from trade. Here e still gets positive surplus in region A_2, where the constraint $p \leq \hat{m}$ is binding, because the idea is indivisible.[16]

In any case, given $R_e > R_i$, i and e want to trade, but if $\gamma < 1$ then not every deal can get done in the next CM. If a deal is put on hold, with probability $1 - \gamma$ it falls through. If z is bigger, it is more likely that deals get closed in the DM and less likely that they fall through. The following obvious result then says that the higher is i_n the more likely it is that deals fall through, and therefore the best outcome obtains in the limit when i_n is as small as possible. If we interpret m as central bank money, this says that the optimal central bank policy is the Friedman rule, $i_n \to 0$. This is not really too surprising, perhaps, as this policy minimizes the cost of liquidity which maximizes the amount of liquidity in equilibrium which maximizes

[15] If $\ell(0) > 0$, we may lose the first solution to $\ell(z) = i_n$, but this is irrelevant because this solution is not an equilibrium anyway since it violates $\ell'(z) \leq 0$.

[16] This result (the possible existence of equilibrium with $z > 0$ even if $\theta = 0$) is not true when we introduce lotteries in Section 6.

the amount of trade.

Proposition 2 *$\partial z/\partial i_n \leq 0$, so z is maximized and the number of deals that fall through is minimized at $i_n = 0$.*

We think this points to a new and potentially important channel through which monetary policy might matter. It is potentially important because it highlights the idea that money (liquidity) may not only be needed for small purchases like cigarettes and taxi rides, but perhaps also for in the market for ideas. If interest rates go up and people start to economize on liquidity, this can not only lead to less smoking and more walking, but to less efficient technology transfer, and due to spill-over effects like those discussed in the next section this can have a big impact on economic aggregates. We do not want to push the policy line *too* hard, because we do not want to be forced to say that liquidity means only central bank money, but the general implication that it is good to have liquidity relatively inexpensive seems sound.

Even though the limiting case of $i_n = 0$ is optimal, it does *not* generally entail full efficiency. The efficient outcome is for e to have sufficient liquidity to close the deal in the DM with probability 1 whenever $R_e > R_i$. If $i_n = 0$, we minimize the probability that a deal fall through, but for efficiency generally we also need $\theta = 1$. >From Figure 2 it is clear that when $\theta < 1$ the equilibrium z generally does not allow e to to close profitable deals with probability 1, and from (11) it is clear that $\theta = 1$ does yield full efficiency at $i_n = 0$. This is a classic *holdup problem*. When e chooses $\hat{m}$, he is making an investment, but as long as $\theta < 1$ he does not get the full return on his investment in the DM. This causes him to underinvest in liquidity, and the rest is obvious.

Proposition 3 *Equilibrium is efficient iff* $i_n = 0$ *and* $\theta = 1$.

6 Extensions

6.1 One-Sided Uncertainty

Consider a version of the model where $R_i = \bar{R}$ with probability 1, so that all ideas are valued the same to i (which could be $\bar{R} = 0$), but the value to any e is still random. Because there is somewhat less algebra in this version, it could be useful in applications. There are two cases to consider for possible equilibria: $z < \bar{R}$ and $z > \bar{R}$.

In the former case $z < \bar{R}$, we are either in A_0 or A_4 in any meeting, and therefore

$$V_e(\hat{m}) = \beta W_e'(\hat{m}, 0) + \alpha_e \beta \gamma \theta \int_{\bar{R}}^{\infty} (R_e - \bar{R}) dF_e(R_e).$$

In the latter case $z < \bar{R}$, we can be in A_0, A_1, A_2 or A_3, and

$$\begin{aligned} V_e(\hat{m}) &= \beta W_e'(\hat{m}, 0) + \alpha_e \beta \theta \int_{\bar{R}}^{B(\bar{R})} (R_e - \bar{R}) dF_e(R_e) \\ &\quad + \alpha_e \beta \int_{B(\bar{R})}^{H(\bar{R})} (R_e - \phi' \hat{m}) dF_e(R_e) + \alpha_e \beta \gamma \theta \int_{H(\bar{R})}^{\infty} (R_e - \bar{R}) dF_e(R_e). \end{aligned}$$

As always, $V_e' = \beta \phi' [1 + \ell(z)]$, where after a little calculus one finds the following: if $z < \bar{R}$ then $\ell(z) = 0$, and if $z > \bar{R}$ then

$$\ell(z) = \frac{\alpha_e(1-\gamma)(z-\bar{R})}{\gamma^2(1-\theta)^2} F_e' \left[H(\bar{R})\right] - \alpha_e \left\{F_e \left[H(\bar{R})\right] - F_e \left[B(\bar{R})\right]\right\}.$$

Suppose for example that R_e is uniform on $[0,1]$. Straightforward algebra yields

$$\ell(z) = \begin{cases} 0 & z < \bar{R} \\ \frac{\alpha_e(1-\gamma)(1-\gamma+\gamma\theta)(z-\bar{R})}{\gamma^2(1-\theta)^2} & \bar{R} < z < z_H \\ \frac{\alpha_e(z-\theta\bar{R}-1+\theta)}{1-\theta} & z_H < \bar{R} < z_B \\ 0 & \bar{R} > z_B \end{cases}$$

where $z_H = \gamma(1-\theta)+(1-\gamma+\gamma\theta)\bar{R}$ and $z_B = 1-\theta+\theta\bar{R}$ solve $H(z_H) = 1$ and $H(z_B) = 1$. As seen in Figure 4, $\ell(z)$ is piece-wise linear with a discontinuity at z_H, and for any $i_n < \hat{\imath}_n$ there is a unique monetary equilibrium at $z = z_H$.[17] The upper bound on i_n is

$$\hat{\imath}_n = \frac{\alpha_e(1-\gamma)(1-\theta+\theta\gamma)(1-\bar{R})}{\gamma(1-\theta)}.$$

This example is nice because we can solve for everything explicitly, but it does have the property that the equilibrium z is (locally) insensitive to i_n. Figure 5 shows an example with R_e log-normal, which does not yield a closed-form solution, but it is easy to see that $\ell(z)$ is continuous and z smoothly decreases with i_n.

FIGURES 4 AND 5 ABOUT HERE.

[17] The discontinuity is no problem for existence: at $z = z_H - \varepsilon$ the marginal value of additional liquidity exceeds i_n, and at $z = z_H + \varepsilon$ marginal value is actually negative, so e chooses $z = z_H$.

6.2 Lotteries

When ideas are indivisible, one might think agents would want to trade using lotteries.[18] Appendix C shows that lotteries are never be used in the CM, because even if ideas are indivisible, when there is no liquidity constraint the bargaining problem is convex; but they may be used in the DM when the constraint $p \leq \hat{m}_e$ binds. Appendix C also shows that deals are not put directly on hold when we allow lotteries: when the constraint $p \leq \hat{m}_e$ binds, first there is a deal where e gives i all his money in exchange for a probability $\mu \in (0, 1)$ of transferring the idea. If e does not win this lottery he does not get the idea, but they still might meet in the next CM, where as always e gets it for p'.

Thus, e potentially pays twice: once for the lottery in the DM, and if that does not pan out, also in the CM if they manage to meet again. Whether or not this is "realistic" we want to know for completeness what happens when lotteries are allowed. To this end, the payoff for e is

$$\mu W_e'(\hat{m}_e - p, R_e) + (1 - \mu)[\gamma W_e'(\hat{m}_e - p - p', R_e) + (1 - \gamma) W_e'(\hat{m}_e - p, 0)],$$

and his threat point is $\gamma W_e'(\hat{m}_e - p', R_e) + (1 - \gamma) W_e'(\hat{m}_e, 0)$. Hence his surplus is $-\phi' p + \mu(1 - \gamma) R_e + \mu \gamma \phi' p'$. Similarly, the surplus for i is $\phi' p - \mu(1 - \gamma) R_i - \mu \gamma \phi' p'$, and the bargaining problem reduces to:[19]

$$\max_{p \leq \hat{m}_e, \mu \leq 1} \left[-\phi' p + \mu(1 - \gamma) R_e + \mu \gamma \phi' p' \right]^{\theta} \left[\phi' p - \mu(1 - \gamma) R_i - \mu \gamma \phi' p' \right]^{1-\theta}$$

[18] One might think this is based on some previous work in monetary theory. The analysis here follows Berentsen, Molico and Wright (2002), although that paper only considers the very special case where agents are restricted to $\hat{m} \in \{0, 1\}$.

[19] We ignore nonnegativity constraints since they will not bind as long as $R_e > R_i$.

Absent the constraints $p \leq \hat{m}_e$ and $\mu \leq 1$, FOC wrt p and μ are

$$\begin{aligned} 0 &= \theta\left[\phi' p - \mu(1-\gamma)R_i - \mu\gamma\phi' p'\right] \\ &\quad -(1-\theta)\left[-\phi' p + \mu(1-\gamma)R_e + \mu\gamma\phi' p'\right] \end{aligned} \tag{14}$$

$$\begin{aligned} 0 &= \theta\left[\phi' p - \mu(1-\gamma)R_i - \mu\gamma\phi' p'\right]\left[(1-\gamma)R_e + \gamma\phi' p'\right] \\ &\quad -(1-\theta)\left[-\phi' p + \mu(1-\gamma)R_e + \mu\gamma\phi' p'\right]\left[(1-\gamma)R_e + \gamma\phi' p'\right] \end{aligned} \tag{15}$$

These cannot both hold when $R_e > R_i$; hence we cannot have $p < \hat{m}_e$ and $\mu < 1$. If $\mu = 1$ and $p < \hat{m}_e$ then (14) implies $p = p'$. If $p = \hat{m}_e$ and $\mu < 1$ then (15) implies $\mu = \Omega\phi'\hat{m}_e$, where

$$\Omega = \frac{(\theta+\gamma-2\gamma\theta)R_e + (1-\theta-\gamma+2\gamma\theta)R_i}{[(1-\theta\gamma)R_e + \gamma\theta R_i][\gamma(1-\theta)R_e + (1-\gamma+\theta\gamma)R_i]}.$$

Appendix C verifies that $\partial\Omega/\partial R_i < 0$ and $\partial\Omega/\partial R_e < 0$, and that $\mu = \Omega\phi'\hat{m}_e < 1$ iff $R_e > B(R_i)$, where B is the same as in the model without lotteries. Appendix C also shows that $\partial\mu/\partial\theta > 0$ and $\partial\mu/\partial\gamma < 0$.

All of this implies that the outcome is as depicted in Figure 6, which shows the bargaining solution (p, μ) as a function of R_e for a given R_i (the Figure is drawn assuming $R_i < \phi'\hat{m}_e$). As R_e increases, p increases while μ stays at 1 until p hits $\hat{m}_e$, after which μ decreases while p stays at $\hat{m}_e$. The main impact of introducing lotteries is to allow immediate trade to potentially occur in what was region $A_3 \cup A_4$, where the deal previously was put on hold. However, the lottery only allows the idea to be transferred with probability μ in the DM. Also, note that it is still the best deals that have the greatest risk of falling through; indeed, $\mu \to 0$ as $R_e \to \infty$.[20]

[20] One can derive the liquidity function as above with lotteries; e.g. if $\gamma > 0$ and $\theta < 1$

FIGURE 6 ABOUT HERE.

6.3 Nonrival Ideas

Earlier we said that, in general, if i transfers his idea to e their payoffs are $\lambda_i R_i$ and $\lambda_e R_e$, but to this point we assumed $\lambda_i = 0$ and $\lambda_e = 1$. Here we consider the general case. In terms of interpretation, suppose i gives e an idea for a good restaurant, e.g., and they both open for business. Then instead of receiving the full return either would get if he were a monopolist, they receive only a fraction since they are now monopolistic competitors. One immediate difference from the baseline model is that there are now gains from trade as long as $\lambda_e R_e > (1 - \lambda_i) R_i$ – i.e. as long as the return to e exceeds the loss to i.

The CM bargaining problem is now

$$\max_{p'} [\lambda_e R_e - \phi p]^\theta [\phi p' - (1 - \lambda_i) R_i]^{1-\theta}$$

which implies $\phi p' = \theta(1 - \lambda_i) R_i + (1 - \theta) \lambda_e R_e$. The DM problem is

$$\max_{p \le m_e} [-\phi p + \gamma \phi p' + (1 - \gamma) \lambda_e R_e]^\theta [\phi p - \gamma \phi p' - (1 - \gamma)(1 - \lambda_i) R_i]^{1-\theta}$$

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then

$$\begin{aligned}
\ell(z) \quad \equiv \quad & \tfrac{\alpha_e \theta(1-\gamma)}{(1-\theta)^2} \int_0^z \tfrac{[z\gamma + R_i(1-\gamma) - 1](z - R_i)}{z\gamma + R_i(1-\gamma)} F_e' \left[B(R_i) | R_i \right] dF_i(R_i) \\
& + \alpha_e \theta(1-\gamma) \int_0^z \int_{B(R_i)}^\infty \tfrac{R_e - R_i}{(1-\theta)\gamma R_e + (1 - \gamma + \theta\gamma) R_i} dF_e(R_e | R_i) \; dF_i(R_i) \\
& + \alpha_e \theta(1-\gamma) \int_z^\infty \int_{R_i}^\infty \tfrac{R_e - R_i}{\gamma(1-\theta) R_e + (1 - \gamma + \theta\gamma) R_i} dF_e(R_e | R_i) \; dF_i(R_i).
\end{aligned}$$

and, as in the baseline model, if the constraint does not bind the solution is $p = p'$. The constraint binds iff $R_e > B(R_i) = \frac{\phi m - \theta(1-\lambda_i)R_i}{(1-\theta)\lambda_e}$, and the deal is put on hold iff $R_e > H(R_i) = \frac{\phi m - R_i(1-\lambda_i)(1-\gamma+\gamma\theta)}{\gamma(1-\theta)\lambda_e}$, generalizing what we had with $\lambda_i = 0$ and $\lambda_e = 1$. The outcome is shown in Figure 7, which looks a lot like Figure 1 except the slopes of the lines are modified.[21]

FIGURE 7 ABOUT HERE.

What is interesting about this extension is probably its quantitative impact. In the baseline model, if a deal falls through the social loss is $R_e - R_i$, but here it is $\lambda_e R_e + \lambda_i R_i - R_i$. At the extreme, if an idea is a pure nonrival good ($\lambda_i = \lambda_e = 1$) the social loss is simply R_e. Thus the potential benefit from having the idea market function relatively smoothly – say, because γ is high or because i_n is low e.g. – is bigger when there is a public good component to ideas. To the extent that policy has an effect on the cost of liquidity, the cost of bad policy is therefore bigger than one might think based on theories where liquidity is important only for certain private consumption goods; i.e. the cost is *even bigger* than the discussion following Proposition 2 suggests.

[21] In the limiting case where $\lambda_i = 1$, e.g., A_0 vanishes since all trade is profitable and B and H become horizontal. Again one can derive the liquidity function; e.g. if $\gamma > 0$ and $\theta < 1$ then

$$\begin{aligned}\ell(z) \equiv\ & \frac{(1-\gamma)\alpha_e}{\gamma^2(1-\theta)^2\lambda_e} \int_0^{\frac{z}{1-\lambda_i}} [z - (1-\lambda_i)R_i]\, F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \\ & -\alpha_e \int_0^{\frac{z}{1-\lambda_i}} \left\{F_e\left[H(R_i)|R_i\right] - F_e\left[B(R_i)|R_i\right]\right\} dF_i(R_i).\end{aligned}$$

6.4 Ideas as Intermediate Inputs

Here we return to the specification in (1), $R_j(I) = \max_{\mathbf{h}} \{f_j(\mathbf{h}, I) - \mathbf{wh}\}$ where $\mathbf{h}$ is a vector of inputs and $\mathbf{w}$ a vector of prices. Since it suffices to make the point, we assume that the only input other than the idea is labor, so we can write $\mathbf{h} = h$ and $\mathbf{w} = w$. >From the FOC $f_j'(h, I) = w$, as long as $f_j'(h, I)$ is increasing in I, it is clear that having a better match between I and j increases the maximizing choice of h. Hence, anything that improves the functioning of the idea market – including a higher γ or lower i_n – increases labor demand.

Since we have a general equilibrium model we also need to consider labor supply. From the CM budget equation, for j in state (m, R), this is

$$h_j(m, R) = \frac{1}{w}\left[X - X_0 - \phi(m - \hat{m}_j + \pi M) - R\right].$$

Hence, individual labor supply depends on (m, R) as well as w. Aggregating across agents and using money market clearing $\int_j \hat{m}_j dj = M'$, market labor supply is

$$H(w) = \int_j h_j(m, R) = \frac{1}{w}\left[X(w) - X_0 - ER\right],$$

where $X(w)$ solves $U'(X) = 1/w$. Notice $X'(w) = -1/w^2 U'' > 0$, and so

$$H'(w) = \frac{wX'(w) - X(w)}{w^2} \simeq -1 - XU''/U',$$

where $\simeq$ means "is equal in sign to." Thus, $H'(w) > 0$ iff the coefficient of relative risk aversion exceeds 1. Let us assume $H' > 0$, for the sake of discussion.

Observe that H is decreasing in ER, since higher ER means agents do not have to work as hard to finance X (and we do not have to worry about effects on $\hat{m}$ since they aggregate to 0). Therefore, anything that makes the allocation in the DM better – higher γ or lower i_n e.g. – reduces labor supply at the same time it increases demand. The net effect is to unambiguously increase the equilibrium wage w and consumption $X(w)$, while employment could go up or down. In any case, anything that affects the market for ideas can have important general equilibrium effects. This is further evidence that the cost of a bad policy, to the extent that policy can affect the cost of liquidity, may be higher than one might think.[22]

6.5 Endogenous γ

Suppose e can choose his γ in the CM, at the same time as he chooses $\hat{m}$. On the one hand, having γ big is desirable because then fewer profitable deals will fall through – e.g., if $\gamma = 1$ then e can set $\hat{m} = 0$ and still close all deals in the next CM. On the other hand, having γ small has an advantage, because it makes i reluctant to put deals on hold, and hence e may get the idea for $\hat{m}$ rather than the CM price p'. So we look for symmetric Nash equilibrium in the game where e chooses $(\hat{\gamma}, \hat{m})$. Note that we assume e can

[22] Actually, an alternative version of the model (that is equivalent for most purposes but simpler for this extension) is to assume utility is linear in X rather than h, say $U = X - v(h)$, with $v' > 0$ and $v'' > 0$. Then the CM problem is

$$\begin{aligned} W_j(m, R) &= \max_{X,h,\hat{m}} \{X - v(h) + \delta V_j(\hat{m})\} \\ s.t.\ X &= X_0 + wh + \phi(m - \hat{m} + \pi M) + R. \end{aligned}$$

This implies the same FOC for $\hat{m}$, but now $H(w)$ solves $w = v'(h)$ and hence $H'(w) = 1/v'' > 0$. Labor supply is unambiguously increasing in w and independent of wealth. So any improvement in the DM unambiguously increases employment, as well as w and X.

choose any $\hat{\gamma} \in [0,1]$ here *for free*, although it would also be interesting to assume there is a cost to choosing higher $\hat{\gamma}$.

First, given e can have any $\hat{\gamma}$ for free, we claim there is always an equilibrium with $(\hat{\gamma}, \hat{m}) = (1, 0)$. Suppose $\gamma = 1$ for all e. From the discussion following Proposition 1, equilibrium must entail $z = 0$, and m is not valued. Then the only way for e to trade is in the CM, and it is easy to check that this implies the best choice is $\hat{\gamma} = 1$. Therefore $\gamma = 1$ is always an equilibrium. We do not think this is especially interesting, however, for a couple of reasons. For one thing, in a more general model, m could still be valued even if it is not used in the idea market; for another, in a more general model there would be some cost to having higher γ. What we want to emphasize is not equilibrium with $\gamma = 1$, but the possibility of equilibrium with $\gamma < 1$, even though e can have $\gamma = 1$ for free.

In Appendix D we show that under the weak condition $F_e'' \geq 0$, V_e is a convex function $\hat{\gamma}$. Hence, any best response and therefore any Nash equilibrium must have $\gamma \in \{0, 1\}$. We already know $\gamma = 1$ is an equilibrium, so consider a candidate equilibrium with $\gamma = 0$. Let z_0 be the solution to $\ell(z_0) = i_n$, which gives us the equilibrium value of money $\phi_0 = z_0/M$ when $\gamma = 0$. Suppose e in the CM contemplates a one-shot deviation to $(\hat{\gamma}, \hat{m})$.[23] Since V_e is convex in $\hat{\gamma}$, if such a deviation is to be profitable, we may as well consider the best such deviation, which is to $(\hat{\gamma}, \hat{m}) = (1, 0)$ (it is clear that given $\hat{\gamma} = 1$ the demand for $\hat{m}$ is 0).

[23] Restricting attention to one-shot deviations here is without loss of generality, by the unimprovability principle.

Let $V_e(\hat{\gamma}, \hat{m})$ denote the DM payoff when e chooses $(\hat{\gamma}, \hat{m})$, given the candidate equilibrium where all others choose $\gamma = 0$ and the value of money is given by z_0. Then in the CM, e gains from the contemplated deviation iff $\Delta = z_0 + \delta V_e(1, 0) - \delta V_e(0, z_0) > 0$, since he saves on acquiring money but also must weigh the consequences for DM trade. After some algebra,

$$\begin{aligned}\Delta \quad \simeq \quad & z_0 i_n + \alpha_e \theta \int_{z_0}^{\infty} \int_{R_i}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ & -\alpha_e \int_0^{z_0} \int_{B_0(R_i)}^{\infty} [(1-\theta)R_e + \theta R_i - z_0] dF_e(R_e|R_i) dF_i(R_i)\end{aligned} \tag{16}$$

where $B_0(R_i) = \frac{z_0 - \theta R_i}{1-\theta}$. Intuitively, a deviation to $(\hat{\gamma}, \hat{m}) = (1, 0)$ saves e the interest cost $z_0 i_n$ and allows trade at p' in region A_4 (the first integral), but also leads to trade a higher price in A_2 (the last integral).

It is clear that when i_n is high, $\Delta > 0$ and a deviation to $(\hat{\gamma}, \hat{m}) = (1, 0)$ is profitable. Suppose $i_n \approx 0$, and therefore we can ignore this effect. Then Δ depends on θ. Clearly if θ is sufficiently low then $\Delta < 0$ and the deviation is not profitable, since when $\theta \approx 0$, e gets very no surplus from trade at the CM (unconstrained) price p', so he may as well take a shot trading at the (constrained) price in the DM. Hence when θ is low, there is an equilibrium with $\gamma = 0$. When $\theta = 1$, the last term vanishes, since $B_0 \to \infty$ and region A_2 disappears. If $i_n > 0$ then typically we have $\Delta > 0$, since z_0 will not be sufficient to close all deals in the DM, and the deviation is always profitable. As $i_n \to 0$, however, with $\theta = 1$, z_0 will be big enough to close all deals in the DM by Proposition 3. Hence, in case $i_n \approx 0$ and $\theta = 1$, we have $\Delta = 0$, and e is indifferent between $\hat{\gamma} = 0$ and $\hat{\gamma} = 1$.

Figure 8 About Here

Figure 8 shows examples for R_e and R_i independent log-normal (with the same mean but $\sigma_e = 0.5$ and $\sigma_i = 1$). Notice that for a given i_n, a deviation is more likely to be profitable when θ is bigger; and for a given θ a deviation is more likely to be profitable when i_n is bigger. We summarize the results as follows.

Proposition 4 *There is always a Nash equilibrium with $\gamma = 1$ and $z = 0$. There is a Nash equilibrium with $\gamma = 0$ and $z > 0$ iff i_n and θ are not too big.*

While this result is perhaps special due to the simplicity of the model, it highlights the notion that liquidity constraints can be endogenized, and it is not obvious that individuals want them relaxed – especially when liquidity costs are low and the pricing implications are significant.

7 Conclusion

We developed a model of the market for ideas characterized by several frictions, including matching, bargaining and liquidity. We have fairly strong results about the outcome in any bilateral meeting, and about the existence of equilibrium where the amount of liquidity z is determined endogenously. We also have strong results concerning the impact of interest rates and bargaining power on efficiency. We considered some technical extensions, such as allowing lotteries in nonconvex bargaining games. We also allowed ideas to be at least partially nonrival goods, or intermediate inputs, to show how outcomes in the idea market can spill over to other markets, wages and employment. We also allowed entrepreneurs to choose the extent to which liquidity is important, and found an equilibrium where they choose $\gamma = 0$.

Our emphasis on bargaining seems realistic, and highlights certain holdup problems that may well be important in this context in the real world. The way we focus on liquidity in this market is perhaps especially novel, although it is also consistent with some of the literature discussed in the Introduction; in any case the importance of liquidity can varied by adjusting γ. One interesting aspect of the way we model liquidity is that it may generate some different implications from existing theories. For instance, if the problem in the market is simply borrowing constraints, high interest rates may help by increasing savings, while our approach suggests high interest rates may make things worse by raising the cost of maintaining liquidity. This remains to be studied carefully.

Of the many other possible extensions, several come to mind immediately. It would be good to endogenize the number of agents on one side of the market, say innovators, by a free entry condition a la Pissarides (2000), and thus determine arrival rates endogenously. A related extension is to make innovators pay something ex ante to come up with ideas. Both of these extensions would introduce two-sided holdup problems in the bargaining. Another extension is to give ideas returns that are not i.i.d. across periods when they are not implemented. This would introduce speculative issues for an innovator deciding whether to sell an idea to an entrepreneur with a moderately high R_e, or hold out in case he meets one later with an even higher R_e. Similar issues would arise when deciding whether to implement an idea or take it to the market.

Another extension is to introduce alternative ways to trade that help get around the liquidity problem. For example, one can explicitly introduce banks into the model, where entrepreneurs who happen across ideas with very high R_e may be able to borrow from funds deposited by others; Chiu and Meh (2005) have already made some progress on this. Finally, although perhaps it is obvious, an explicit endogenous growth version of the model seems worth pursuing. After all, one of the ways we began motivating the project was to mention the importance of technology transfer for growth. It seems a natural application, and it may also be useful to further extend growth theory to incorporate frictions – including matching, bargaining and liquidity – the way we do in here.

Appendix A: Derivation of ℓ

Inserting the correct limits for the various regions, we can write (7) explicitly as

$$\begin{aligned}
V_e(\hat{m}) &= \beta W_e'(\hat{m},0) + \alpha_e\beta\theta \int_0^{\phi'\hat{m}} \int_{R_i}^{B(R_i)} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\
&+\alpha_e\beta \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{H(R_i)} (R_e - \phi'\hat{m}) dF_e(R_e|R_i) dF_i(R_i) \\
&+\alpha_e\beta\gamma\theta \int_0^{\phi'\hat{m}} \int_{H(R_i)}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\
&+\alpha_e\beta\gamma\theta \int_{\phi'\hat{m}}^{\infty} \int_{R_i}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i).
\end{aligned}$$

We now show how to differentiate this to get $\ell(\cdot)$ in the various cases.

(i) $\gamma > 0$ and $\theta < 1$. The derivative of the first term wrt $\hat{m}$ is $\beta\phi'$. By Leibniz Rule, the derivatives of the four integral terms are:

$$D_1 = \phi' \int_0^{\phi'\hat{m}} \frac{(\phi'\hat{m} - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i)$$

$$\begin{aligned}
D_2 &= \phi' \int_0^{\phi'\hat{m}} \frac{(\phi'\hat{m} - R_i)(1-\gamma+\theta\gamma)}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i) \\
&-\phi' \int_0^{\phi'\hat{m}} \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i) \\
&-\phi' \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{H(R_i)} dF_e(R_e|R_i) dF_i(R_i)
\end{aligned}$$

$$D_3 = \phi' F_i'(\phi'\hat{m}) \int_{\phi'\hat{m}}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m})$$

$$-\phi' \int_0^{\phi'\hat{m}} \frac{\phi'\hat{m} - R_i}{\gamma^2(1-\theta)^2} F_e'[H(R_i)|R_i] dF_i(R_i)$$

$$D_4 = -\phi' F_i'(\phi'\hat{m}) \int_{\phi'\hat{m}}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m})$$

Summing these and simplifying yields (9).

(ii) $\gamma = 0$ and $\theta < 1$. The results similar except for two things. First, $H(R_i) = \infty$ becomes vertical at $R_i = z$, so region A_3 vanishes and we can ignore D_3. Second, the derivative D_2 is not correct since we divided by $\gamma = 0$. The correct derivative in this case over region A_2 is

$$D_2 = \phi' F_i'(\phi'\hat{m}) \int_{B(R_i)}^{\infty} (R_e - \phi'\hat{m}) dF_e(R_e|\phi'\hat{m})$$

$$- \int_0^{\phi'\hat{m}} \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} F_e'[B(R_i)|R_i] dF_i(R_i)$$

$$-\phi' \frac{\theta(\phi'\hat{m} - R_i)}{(1-\theta)^2} \int_0^{\phi'\hat{m}} \int_{B(R_i)}^{\infty} dF_e(R_e|R_i) dF_i(R_i).$$

Summing now leads to (10).

(iii) $\theta = 1$. In this case $B(R_i) = H(R_i) = \infty$ both become vertical at z, and A_2 as well as A_3 vanish. Also, in this case the correct derivatives over regions A_1 and A_4 are

$$D_1 = \phi' F_i'(R_i) \int_0^{\infty} \left(R_e - \phi'\hat{m}\right) dF_e(R_e|\phi'\hat{m})$$

$$D_4 = -\phi' F_i'(R_i) \int_0^{\infty} \left(R_e - \phi'\hat{m}\right) dF_e(R_e|\phi'\hat{m})$$

Summing now leads to (11). ■

Appendix B: Existence

Here we derive some properties of $\ell(z)$ and use them to verify the existence result in Proposition 1, assuming for simplicity a continuous joint density for (R_i, R_e) with $ER_j < \infty$. We claim first that $\lim_{z\to\infty} \ell(z) = 0$. Consider the generic case $\gamma > 0$ and $\theta < 1$, and begin by rewriting (9) as $\ell(z) = \alpha_e \sum_{j=1}^4 I_j(z)$, where

$$\begin{aligned}
I_1(z) &\equiv \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^z z F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \\
I_2(z) &\equiv -\tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int_0^z R_i F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \\
I_3(z) &\equiv -\int_0^z F_e\left[H(R_i)|R_i\right] dF_i(R_i) \\
I_4(z) &\equiv \int_0^z F_e\left[B(R_i)|R_i\right] dF_i(R_i).
\end{aligned}$$

We claim each $I_j(z) \to 0$ as $z \to \infty$.

Consider $I_1(z)$, and suppose that $\int_0^\infty z F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \nrightarrow 0$ as $z \to \infty$. Making a change of variable using $R_e = \frac{z - R_i(1-\gamma+\theta\gamma)}{\gamma(1-\theta)} = H(R_i) = H$, this is equivalent to

$$\int_0^\infty \left[\gamma(1-\theta)H + R_i(1-\gamma+\gamma\theta)\right] F_e'(H|R_i) dF_i(R_i) \nrightarrow 0 \text{ as } H \to \infty.$$

Integrating with respect to H over $(0, \infty)$, this implies

$$\begin{aligned}
\infty &= \int_0^\infty \int_0^\infty [\gamma(1-\theta)H + R_i(1-\gamma+\gamma\theta)]\, F_e'(H|R_i) dF_i(R_i) dH \\
&= \gamma(1-\theta) \int_0^\infty \int_0^\infty H F_e'(H|R_i) dF_i(R_i) dH \\
&\quad +(1-\gamma+\gamma\theta) \int_0^\infty \int_0^\infty R_i F_e'(H|R_i) dF_i(R_i) dH
\end{aligned}$$

But this implies either $ER_e = \infty$ or $ER_i = \infty$, a contradiction. Hence $I_1(z) \to 0$ as $z \to \infty$. Similar arguments can be used to show $I_j(z) \to 0$ as $z \to \infty$, $j = 2, ...4$. Hence, as we claimed, $\ell(z) \to 0$ as $z \to \infty$ in the case where $\gamma > 0$ and $\theta < 1$.

The same basic approach works when $\gamma = 0$ and $\theta < 1$. Rewrite (10) as $\ell(z) = \sum_j I_j(z)$ where now $I_1(z) \equiv F_i'(z) \int_z^\infty R_e dF_e(R_e|z)$ and so on. For example, consider $I_1(z)$, and suppose $F_i'(z) \int_0^\infty R_e dF_e(R_e|z) \nrightarrow 0$. Integrating with respect to z, this implies the contradiction

$$ER_e = \int_0^\infty F_i'(z) \int_0^\infty R_e dF_e(R_e|z) dz = \infty.$$

Similar arguments show $I_j(z) \to 0$ as $z \to \infty$, $j = 2, ...4$. Hence $\ell(z) \to 0$ as $z \to \infty$ when $\gamma = 0$. The same basic approach works for $\theta = 1$. However, to ease the presentation somewhat, for the rest of the discussion we focus on the generic case $\gamma > 0$ and $\theta < 1$ and leave other cases as exercises.

The next thing we prove is that, in the generic case, $\ell(\underline{R}) = 0$ and $\ell(z) > 0$ for some z in the neighborhood of $\underline{R}$, where $\underline{R} = \inf\{R | F_i'(R) F_e'(R|R) >$

$0\}$, and we assume $\underline{R} < \infty$. For the first result, notice that

$$\begin{aligned}
\ell(\underline{R}) &= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int\limits_0^{\underline{R}} (\underline{R} - R_i) F_e'[H(R_i)|R_i] dF_i(R_i) \\
&\quad - \int\limits_0^{\underline{R}} \left\{ F_e[H(R_i)|R_i] - F_e[B(R_i)|R_i] \right\} dF_i(R_i) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} (\underline{R} - \underline{R}) F_e'[H(\underline{R})|\underline{R}] F_i'(\underline{R}) \\
&\quad - \left\{ F_e[H(\underline{R})|\underline{R}] - F_e[B(\underline{R})|\underline{R}] \right\} F_i'(\underline{R}) = 0,
\end{aligned}$$

because $H(\underline{R}) = B(\underline{R}) = \underline{R}$ when $z = \underline{R}$.

Now consider

$$\begin{aligned}
\ell'(\underline{R}) &= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \int\limits_0^{\underline{R}} \left\{ F_e'[H(R_i)|R_i] + \tfrac{\underline{R}-R_i}{\gamma(1-\theta)} F_e''[H(R_i)|R_i] \right\} dF_i(R_i) \\
&\quad - \tfrac{1}{\gamma(1-\theta)} \int\limits_0^{\underline{R}} \left\{ F_e'[H(R_i)|R_i] - \gamma F_e'[B(R_i)|R_i] \right\} dF_i(R_i) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} \left\{ F_e'(\underline{R}|\underline{R}) + \tfrac{\underline{R}-\underline{R}}{\gamma(1-\theta)} F_e''(\underline{R}|\underline{R}) \right\} F_i'(\underline{R}) \\
&\quad - \tfrac{1}{\gamma(1-\theta)} \left\{ F_e'(\underline{R}|\underline{R}) - \gamma F_e'(\underline{R}|\underline{R}) \right\} F_i'(\underline{R}) \\
&= \tfrac{1-\gamma}{\gamma^2(1-\theta)^2} F_e'(\underline{R}|\underline{R}) F_i'(\underline{R}) \left[1 - \gamma(1-\theta) \right].
\end{aligned}$$

By definition of $\underline{R}$, $\ell'(\underline{R} + \varepsilon) > 0$ for some $\varepsilon > 0$. Hence, $\ell(z) > 0$ for some z near $\underline{R}$. The combination of the results in this Appendix, $\ell(z) > 0$ for z near $\underline{R}$ and $\lim_{z \to \infty} \ell(z) = 0$, tells us that for small i_n there always exist solutions to $\ell(z) = i_n$, and for big i_n there does not. Obviously when there do exist solutions they come in pairs, and generically every alternative solution satisfies $\ell'(z) < 0$. This is all we need to establish Proposition 1, the existence of equilibrium $z > 0$ for small i_n.

Appendix C: Lotteries

First, we verify that agents never use lotteries in the CM. Assume e pays p' to i in exchange for a lottery that gives e the idea with probability μ' (the possibility that e pays a random amount is easily ruled out as in Berentsen et al. 2002). The payoff to e is $\mu' W_e'(\hat{m}_e - p', R_e) + (1-\mu')W_e'(\hat{m}_e - p', 0)$ and the payoff to i is $\mu' W_i'(p', 0) + (1-\mu')W_i'(p', R_i)$, while the threat points are as before. Using the linearity of W_j', the bargaining problem can be written:

$$\max_{p',\mu'} \left(\mu' R_e - \phi' p'\right)^{\theta} \left(\phi' p' - \mu' R_i\right)^{1-\theta}$$

Maximizing wrt p', we get $\phi' p' = \mu' \left[\theta R_i + (1-\theta) R_e\right]$. Using this, we can reduce the derivative wrt μ' to $(1-\theta)(R_e - R_i)(\mu' R_e - \phi' p')$. As long as $R_e > R_i$ and $\mu' R_e > \phi' p'$, both of which are necessary for trade, this is strictly positive for all $\mu' > 0$. Hence, for a maximum $\mu' = 1$.

Returning to the DM, the next claim to verify is that profitable deals are never directly put on hold when we have lotteries. The usual calculation indicates that i puts the deal on hold iff $R_e > H(R_i)$, except that with lotteries we have $H(R_i) = \frac{\phi' \hat{m} - R_i \mu(1-\gamma+\theta\gamma)}{\mu\gamma(1-\theta)}$. Substituting μ from the bargaining solution into H, it is easy to show $R_e > R_i$ implies $R_e < H(R_i)$, establishing the claim.

Next we verify $\partial\Omega/\partial R_j < 0$, $j = i, e$. Considering i (the other case is symmetric), straightforward algebra yields $\partial\Omega/\partial R_i \simeq -c_1 R_e^2 - c_2 R_i R_e - c_3 R_i^2$, where $\simeq$ means "equal in sign" and c_1, c_2 and c_3 are functions of

(θ, γ). One can show c_1, c_2 and c_3 are positive, the only tricky case being c_1, which is a complicated polynomial in θ and γ. Consider minimizing c_1 over (θ, γ). First we checked that $c_1 > 0$ on the boundary of $[0,1]^2$, then we checked that it is positive at every possible critical point in $[0,1]^2$. So $c_1 > 0$ for all (θ, γ), which establishes the claim $\partial\Omega/\partial R_i < 0$.

Next we verify $\mu < 1$ iff $R_e > B(R_i)$. This actually follows easily from inspection of the Figure showing the outcome of the bargain. Suppose we fix R_i and increase R_e starting at $R_e = R_i$. Then we switch from $\mu = 1$ to $\mu < 1$ at some point, say $\tilde{R}_e = \tilde{R}_e(R_i)$. Since this is the same point at which switch from $p = [\theta R_i + (1-\theta)R_e]/\phi' < \hat{m}$ to $[\theta R_i + (1-\theta)R_e]/\phi' > \hat{m}$, we conclude that this point is $\tilde{R}_e(R_i) = \frac{\phi\hat{m} - \theta R_i}{1-\theta}$, which tells us that $\tilde{R}_e = B(R_i)$. This completes the argument.

Finally, we verify that $\partial\mu/\partial\theta > 0$ and $\partial\mu/\partial\gamma < 0$. The first derivative is simple, the second less so. A straightforward calculation yields $\partial\mu/\partial\gamma \simeq \Upsilon$, where

$$\begin{aligned}\Upsilon \;\equiv\; & -(1-\gamma+2\gamma\theta)R_i^3 + (1-3\gamma+6\gamma\theta)R_e R_i^2 \\ & +(1+3\gamma-6\gamma\theta)R_e^2 R_i - (1-\gamma+2\gamma\theta)R_e^3.\end{aligned}$$

Notice that $\gamma = 0$ implies $\Upsilon < 0$. Can Υ ever be positive? Suppose we try to maximize it. Since $\partial\Upsilon/\partial\theta = 2\gamma(R_e - R_i)^3 > 0$, this means, as long as $\gamma > 0$ which is must be if we are to have any hope of $\Upsilon > 0$, we must set $\theta = 1$. Then $\partial\Upsilon/\partial\gamma = (R_e - R_i)^3(2\theta - 1)$, which is also positive given $\theta = 1$, and we must also set $\gamma = 1$. Hence, the unique maximum occurs at $\gamma = \theta = 1$, where $\Upsilon = -2R_i(R_e - R_i)^2 < 0$. This completes the argument. ■

Appendix D: V_e Convex in γ

The first partial of V_e wrt γ is

$$\begin{aligned}\frac{\partial V_e}{\partial \gamma} &= -\alpha_e \beta \int_0^z \tfrac{(R_i-z)^2(1-\gamma)}{\gamma^3(1-\theta)^2}\; F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \\ &\quad +\alpha_e\beta\theta \int_0^z \int_{H(R_i)}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i) \\ &\quad +\alpha_e\beta\theta \int_z^{\infty} \int_{R_i}^{\infty} (R_e - R_i) dF_e(R_e|R_i) dF_i(R_i).\end{aligned}$$

Hence the second derivative satisfies

$$\begin{aligned}\frac{\partial^2 V_e}{\partial \gamma^2} &\simeq \frac{(1-\gamma)\left[\gamma(1-\gamma)+3(1-\theta)^2\right]}{\gamma^3(1-\theta)^3} \int_0^z (R_i - z)^2 F_e'\left[H(R_i)|R_i\right] dF_i(R_i) \\ &\quad -\frac{(1-\gamma)}{3\gamma^5(1-\theta)} \int_0^z (R_i - z)^3\, F_e''(H(R_i)|R_i) dF_i(R_i) \\ &\quad +\frac{\theta}{\gamma^3(1-\theta)^2} \int_0^z (z - R_i)^2\, dF_i(R_i)\end{aligned}$$

The first and third terms are unambiguously positive. As long as $F_e'' \geq 0$, the middle term is also positive. ■

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Graphs

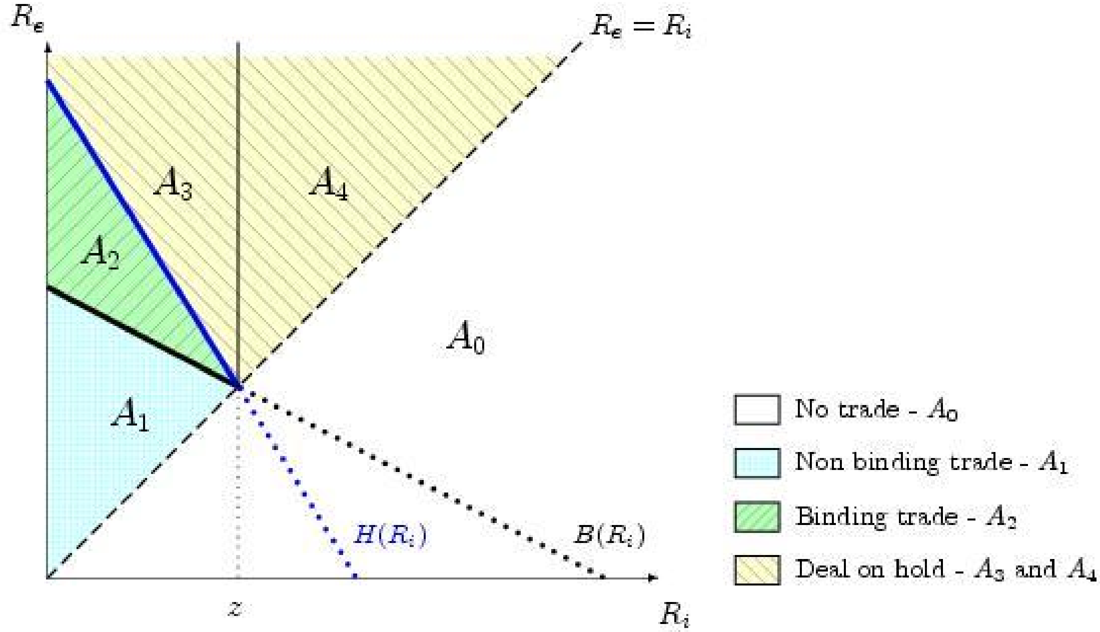

Figure 1: Meeting outcomes for (Ri,Re).

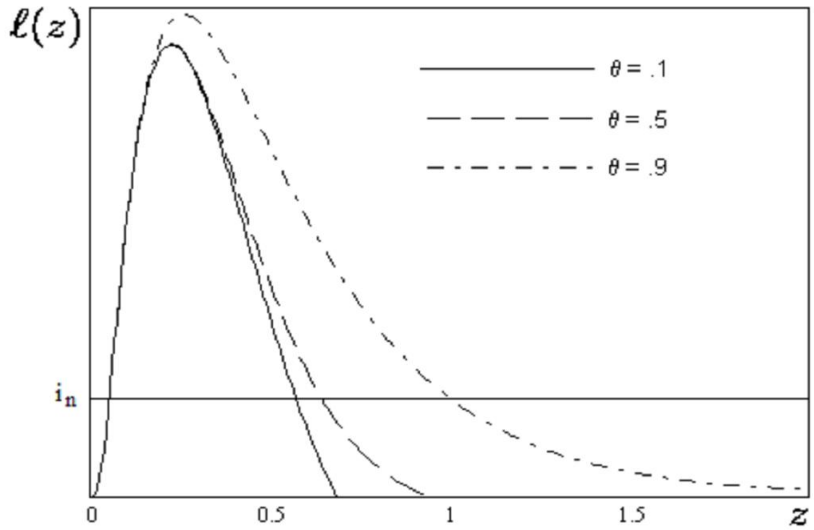

Figure 2: Plot for independent Lognormal distribution.

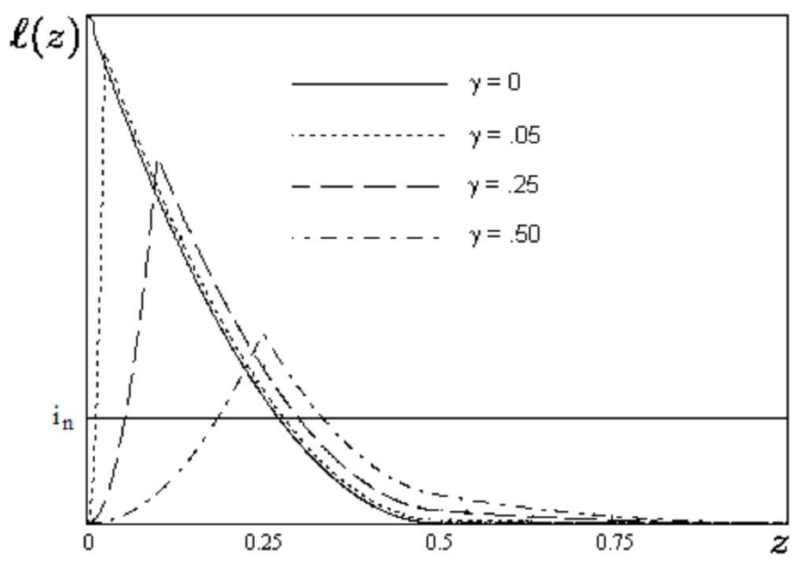

Figure 3: Plot for independent Uniform distribution.

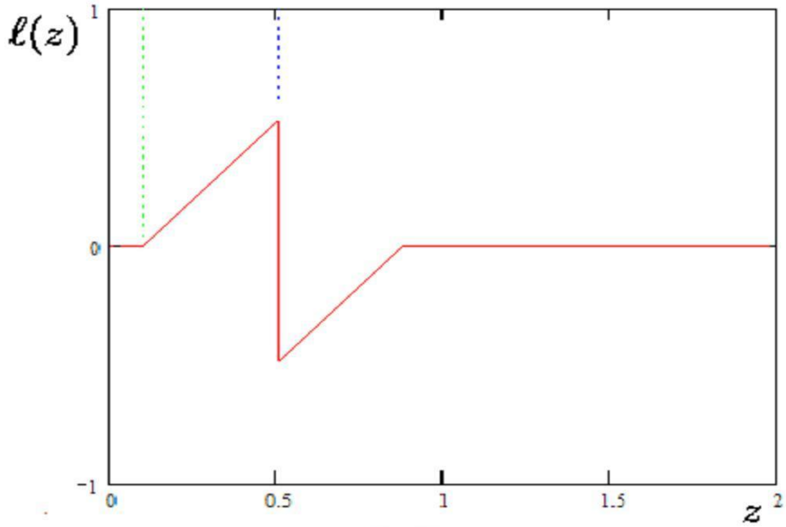

Figure 4: Plots for Re uniform and Ri degenerate.

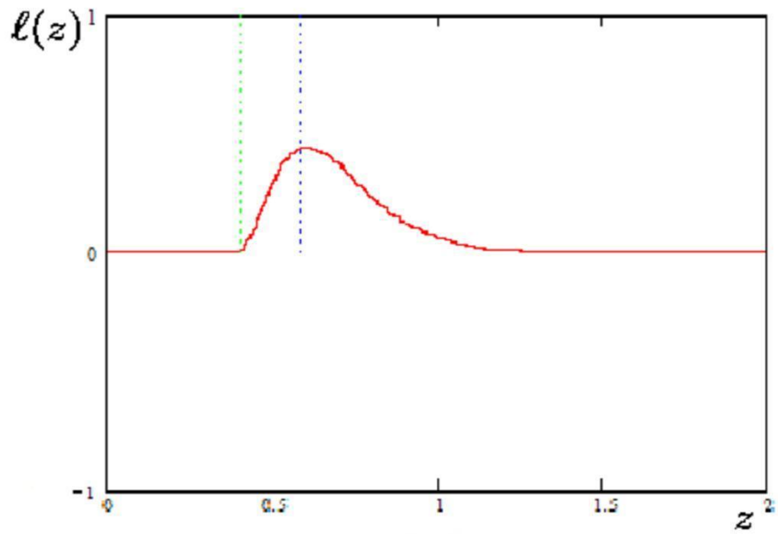

Figure 5: Plot for Re lognorm and Ri degenerate.

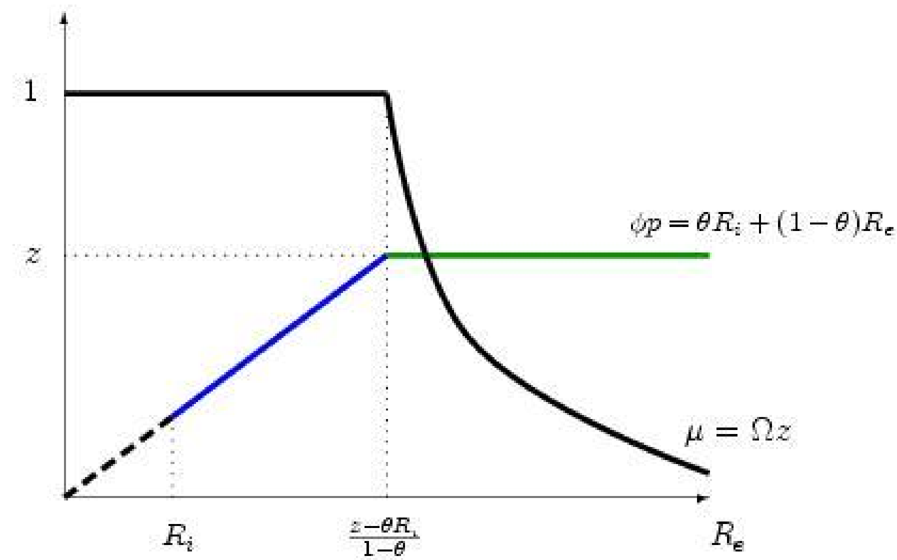

Figure 6: Lottery outcomes given z and Ri.

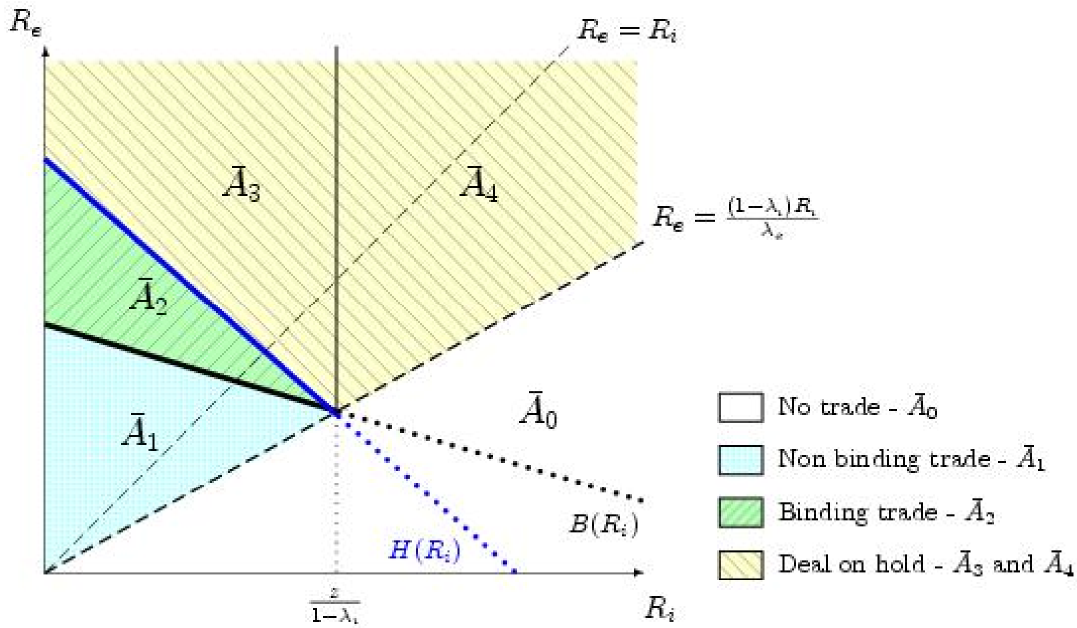

Figure 7: Meeting outcome for Non-rivalrous ideas where $1 - \lambda_i < \lambda_e$.