From Busts to Booms, in Babies and Goodies

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Abstract

After the fall in fertility during the Demographic Transition, many developed countries experienced a baby bust, followed by the Baby Boom and subsequently a return to low fertility. Received wisdom from the Demography literature links these large fluctuations in fertility to the series of Economics 'shocks' that occurred with similar timing – the Great Depression, WWII, the economic expansion that followed and then the productivity slow down of the 1970's. To economists, this line of argument suggests a more general link between fluctuations in output and fertility decisions, of which the Baby-Bust-Boom-Bust event (BBB) is a particularly stark example. This paper is an attempt to formalize the conventional wisdom in simple versions of stochastic growth models with endogenous fertility. First, we develop initial tools to address the effects of "temporary" shocks to productivity on fertility choices. Second, we analyze calibrated versions of these models. We can then answer several qualitative and quantitative questions: Under what conditions is fertility pro- or countercyclical? How large are these effects and how is this related to the 'persistence' of the shocks? How much of the BBB can be accounted for by the kinds of medium run productivity fluctuations described as computed from the data? Preliminary results show that under reasonable parameter values fertility is procyclical, that the elasticity of fertility to shocks lays between 1 and 1.7 and, finally, that in our models, productivity shocks capture between 1/3 and 2/3 of the US baby bust and between $\frac{1}{4}$and $\frac{1}{2}$of the US baby boom.

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1 Introduction

The Demographic Transition is almost invariably rended as a monotone, if nonlinear, process along which countries move from high to low mortality and fertility levels with, possibly, an intermediate phase during which, because fertility is still high but mortality has already dropped, a substantial increase in population takes place. When looking at data through these spectacles, almost the entire 20th century appears to constitute, for the United States and for most European countries, a period of unusual deviations: first, during 1900-1940, the Baby Bust, an acceleration of the decrease in fertility, followed by an upswing – the Baby Boom during 1940-1965 – and subsequently a second slump to even lower fertility levels, 1965-1985. Received wisdom from the demography literature links these large fluctuations in fertility to the series of Economics 'shocks' that occurred with similar timing – the Great Depression, WWII and the economic expansion that followed, and then the productivity slow down of the 1970's.[1] To economists, this line of argument suggests a more general link between fluctuations in output and fertility decisions, of which the Baby Bust-Boom-Bust event (BBB) is a particularly stark example.

One hypothesis in the demography literature is that the baby boom was a 'catching up' phenomenon from a period of relatively low fertility during the Great Depression. (See Whelpton (1954), Freedman, Whelpton and Campbell (1959), Goldberg, Sharp and Freedman (1959), and Whelpton, Campbell and Patterson (1966)). This can be interpreted as current fertility being a stable function of productivity levels (or trends) and the current stock of children/people in the economy. That is, unusual (relative to trend) drops in income cause fertility to fall, other things equal; vice versa, unusual (relative to trend) increases in income cause fertility to rise. If the increase in income follows a period of below trend growth, the boost to fertility may be even larger because the current stock of people is low compared to the long run 'target' level. Roughly speaking, in this view fertility is a function of the current income shock and of the stock of population, with either long run average income or long run 'trend' growth rates determining the target level of population, or of family size.

Among recent ones, perhaps the best known conjecture relating cyclical movements in income and fertility is the one advanced by Easterlin (2000). He argues that fertility decisions are based on expected lifetime income relative to material aspirations which are formed in childhood. When aspirations are low (high) relative to expected income, fertility is high (low). Since the baby boom mothers grew up in 'bad' times and therefore had low material aspirations while they were making fertility decisions during 'good' times, i.e. expected lifetime income was high, they had many children. Operationally, low income for today's generation implies high fertility

[1]Pushing this line of reasoning to our days, one may link the small, but significant, increase in the US fertility rate since 1985 as a reflection of the improvement in GDP growth rates during the same period.

for the next generation, especially if its expected lifetime income is particularly high.[2]

It is not hard to see that this version of the relationship between (relative) income and fertility decisions, as well as the previous one linking above (below) average growth with high (low) fertility are, in fact, 'dynamic variations' on the Malthusian hypothesis, both in spirit and in their substantive predictions. Recall that, in the traditional Malthusian view, the long run population level is determined by a fixed natural ratio between available economic resources and population size. When income per capita increases above this natural level, fertility also increases until the long run ratio is re-established, viceversa for periods of economic crisis. Hence the prediction that periods of unusually high mortality, during which population is depleted while economic resources remain unchanged, should be followed by years of above average fertility, and viceversa. In this view, periods of very harsh economic conditions, in which per capita income decreases below the natural level are also periods in which fertility decreases. Cipolla (1962), Simon (1977), and Boserup (1981) are some of the best historical renditions of such a 'generalized' Malthusian model, and to them we refer for the many details we must by force omit in our brief historical overview below.

From the perspective of a theorist of economic growth, the Malthusian view is cast in terms of a 'stationary' model, one in which there is no persistent growth in income per capita but there is an essential fixed factor, e.g, land. In such a setting, summarized as $Y_t = F(L, N_t)$ where L is the time invariant stock of land and N_t is the time-varying population, standard neoclassical properties imply that when N_t is below (above) it stable long run level, N^*, the wage rate $w_t = F_2(L, N_t)$ increases above (below) its 'natural' level $w^* = F_2(L, N^*)$. Also, from a growth-theoretical perspective, the view exemplified by the work of Easterlin describes a 'growth' model $Y_t = F(X_t, N_t)$, in which labor productivity $w_t = F_2(X_t, N_t)$ has a trend $\gamma_t = \gamma_{t-1} + \varepsilon_t$, with ε_t a possibly random disturbance, and the 'other inputs' X_t are either not essential in production or fully reproducible. In general, this can either be an exogenous growth model, in which the growth rate γ_t is determined outside the model, or an endogenous one, in which γ_t is in fact a function of the rate at which the other inputs X_t are accumulated. Either way, oscillations are mean reverting, to the long run income *level* in the first case and to the long run income *growth rate* in the second. Again, in both cases the crucial steps when taking the theory to the data consist in making additional operational assumptions about (a) what the long run growth rate (level) of income is, and (b) how economic agents make predictions about it, and the degree of time persistence in the deviations from the supposedly stable long run value. Our study makes no exception to this rule, hence the substantial attention we dedicate to both (a) and (b) in teh discussion below, and the sensitivity that the quantitative predictions of the formal model display to variations in either (a) or (b).

[2] One version of this would be to assume that there is habit formation in consumption. With this, much of the existing literature on asset pricing could be used as a foundation. See *****

While one can see from the above that the theory being considered here dates back to the very origins of modern economic demography, surprisingly little has been done to formally address the link between productivity shocks[3] and fertility in a stochastic model of optimal fertility choice. This paper aims at filling this gap by investigating the theoretical and quantitative implications of such link in an accordingly modified version of the traditional 'dynastic model' of endogenous fertility. The formal idea we exploit is that, in this type of model, the size of a dynasty in period t, N_t, behaves like the capital stock in a more standard growth model. This analogy is imperfect since, for example, in the Barro-Becker model[4], N_t also enters the utility function of the dynastic planner. Thus, in truth it has features that are a mixture of capital and consumption in the standard model.

Given that proviso, recall the simple intuition from the single sector growth model with productivity shocks. There is a fundamental desire to smooth consumption due to the concavity of the utility function. Because of this, in a period when the shock is lower than average (and, as a result, output is correspondingly low), agents lower investment to smooth consumption. When the shock is high, the opposite occurs. Thus, the growth rate of K is high when the shock is high and low when it is low. In the case where the analogy to N_t in the endogenous fertility models holds, this implies that the growth rate of N_t, i.e., the fertility rate, is high when the shock is high, and low when the shock is low. This 'first order deviations' induced by variations in current productivity, can be either damped or magnified by the particular type of production function one adopts. As a benchmark, take the simplest case, in which output is just equal to labor times productivity. Now, when the production function assumes that some other (essential) input is used in production together with labor, the final effect depends on the additional factor being fixed or accumulable. If the additional input is fixed, it tends to dampen the impact of a productivity shock on fertility because, everything else the same, an increase (decrease) in N_t decreases (increases) labor productivity in this case. To the contrary, when the additional input is easily accumulable, its presence may magnify the variation in fertility as the productivity of N_t receives a second boost, over and above the one coming from the original productivity shock, from the increase in the second input.

To implement the above, we start with a stochastic version of a Barro-Becker type model where, for now, we abstract from all other inputs besides pure labor (such as physical and human capital or land). First, we derive homogeneity properties of the model and find that, due to these, fertility depends on the current productivity shock

[3]Although it seems unlikely that fertility decisions are affected by quarter to quarter fluctuations in productivity (as addressed in the Business Cycle literature), longer fluctuations, such as the Great Depression where productivity was below trend for about 10 years, the post-war period where it was above trend for about the same number of years, and the extended productivity slow-down in the seventies, are likely to affect parents outlook on their children's future well being, and therefore their current fertility decisions. In the present context, temporary shocks should therefore be understood as extended swings around a very long term trend.

[4]Becker and Barro (1988) and Barro and Becker (1989).

only, and not on the size of the current stock, N_t. This implies that, while there is a link between productivity shocks and fertility, this type of model does not exhibit 'catching up' fertility due to low fertility in the past. Second, we show that by reinterpreting the discount factor, this model is equivalent to one in which productivity follows an exogenous exponential growth process. As long as costs of children grow at that same rate, the population growth rate is stable, while consumption per capita grows at the exogenous rate.

Next, we analyze a version of the model in which population does not enter the dynastic planner's utility function. This requires a particular configuration of parameters in the B&B type preferences but has the addded simplicity of reducing the model to one which is perfectly analoguous to a stochastic Ak model. Under the additional assumption that the shocks to productivity are *i.i.d.*, we give analytical characterizations of the model for two particular cases: first, the case in which all child care costs are goods costs; second, the case in which children only cost time.

In the first case, we show analytically that fertility is, indeed, procyclical – a high shock relative to trend is associated with higher than average fertility. In fact, it is a linear function of the shock, with the sign of the slope independent of preference parameters. Thus, in this case the link between productivity and fertility is qualitatively consistent with the BBB episode. The magnitude of the effect, however, depends on the length of the period which affects the depreciation or, more appropriately, the mortality rate in the economy. In general, the magnitude of the effect on fertility is decreasing in the survival rate or, in other words, in the length of the shock. This makes sense in the light of the earlier intuition and the homogeneity properties of the model, as a very long lasting deviation from the trend is much closer to establishing a new trend than a short lasting deviation.

Finally, we perform quantitative experiments on this version of the model. To do this, we use actual magnitudes of productivity shocks from 1910 to 1970 and compare the predictions of the model in terms of fertility rates to the data. We consider two extreme cases. In the first, survival is zero. That is, the dynasty has to build a new stock of people every period. In this case, the elasticity of fertility to the current shock is 1. Second, we consider a case in which survival of working age people over a 10 year period is 80 percent, which roughly corresponds to an expected working life of 50 years. In this case, the elasticity of fertility to productivity shocks increases to 1.7. Since during the Great Depression productivity was about 10 percent below trend, we get about one third to one half of the 30 percent downward deviation in fertility in the first case and about 2/3 in the second. For the Baby Boom, the model does less well. There was a take-off in productivity after WWII, but it was more pronounced in the 1960's than in the 1950's. Thus, the model predictions are smaller and happen later than what is seen in the data, capturing about 25 to 40 percent of the Baby Boom itself. All in all however, given the simplicity of the model, we view this as quite a success.[5]

[5]All of these statements are contingent on a particular way of identifying the 'shocks' to produc-

When all child care costs are in terms of time, matters become more complicated. Since shocks to productivity affect wages, they also affect the cost of children. In this case, we again give analytical results for the case of *i.i.d.* shocks. We find that whether fertility is pro or counter cyclical depends on the parameters of preferences. With low curvature as is typically assumed in B&B models, fertility is counter-cyclical, but at the limiting case of log utility it is acyclic. When the curvature of utility with respect to per capita consumption is high, fertility is procyclical.

The original demographic literature on the BBB was mostly qualitative in nature. Similar versions of the 'Catching Up' can be found in examples such as Whelpton (1954), Freedman, Whelpton and Campbell (1959), Goldberg, Sharp and Freedman (1959), and Whelpton, Campbell and Patterson (1966). More recently, there have been two interesting studies on the Baby Boom using Dynamic General Equilibrium techniques. These are the papers by Greenwood, Seshadri, and Vandenbroucke, (2005) and Doepke, Hazan and Maoz (2005). Greenwood, Seshadri, and Vandenbroucke analyze a DGE model of fertility choice where the precipitating event is a reduction in the cost of raising children in the post WWII period due to an acceleration in the rate of productivity growth in the home. They find that much of the upswing in the Baby Boom can be accounted for by this mechanism in their model. Doepke, Hazan and Maoz also use DGE techniques but focus on the entry of women in the labor force during WWII and the long lasting effect this had for that cohort due to the extra-normal human capital that they accumulated from this experience. They also have significant successes in matching the quantitative magnitudes of the Boom. In contrast, we focus on these same events as responses to medium run shocks to the income process of a dynasty. Thus, the approaches are quite complementary.

One issue that we have not been able to address to this point is that both demographers and economists, have argued that one of the difficulties with the 'catching up' hypothesis is that, in the data, this 'catching up' clearly does not take place for a given woman (see Greenwood, et. al., and section 2 below for documentation). That is, completed fertility was low for both the women immediately preceding, and immediately following the Baby Boom mothers. So, if there is 'catching up', it is in a dynastic (aggregate) sense, not at the individual level. That is, in a dynastic model, it is quite possible that the dynasty purposefully 'catches up,' although this need not be observed for any particular generation of women. This is a distinction that we hope to address in detail in subsequent versions of this work.

The paper is organized as follows. Section 2 describes the data. In Section 3, we lay out a stochastic version of the Barro-Becker model without capital, its homogeneity properties and equivalence to a model with exogenous growth. In Section 4 we describe the special case in which the model reduces to an Ak model, calibrates and simulates several versions of the latter. In Section 5, we discuss several extensions we plan to address in the future. Section 6 concludes with some related empirical

tivity in the TFP time series. And the results will certainly be affected by how this issue is treated. We are currently studying alternatives along this line.

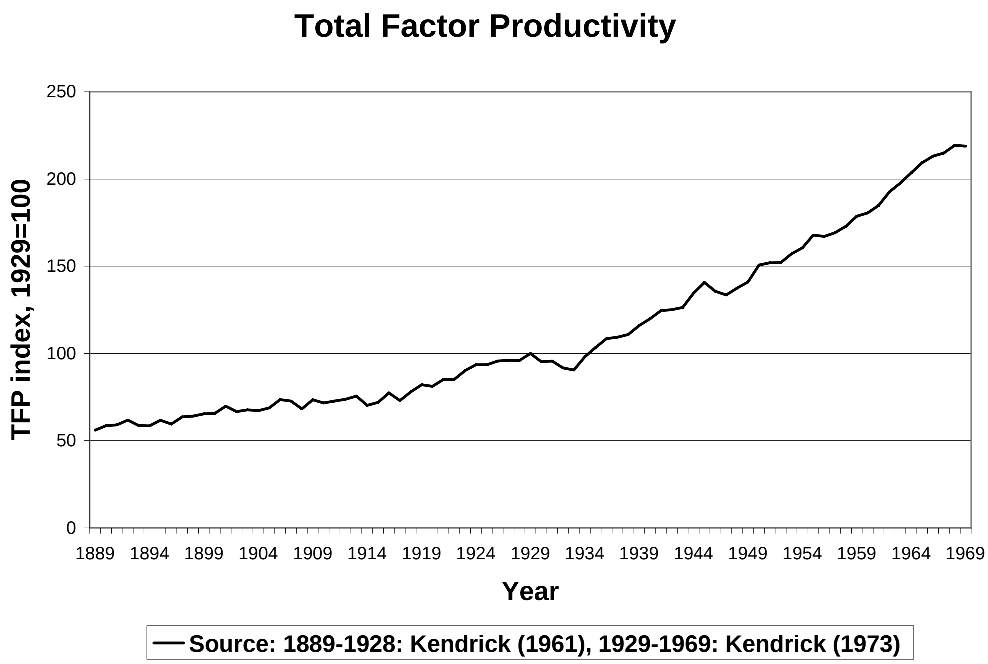


Figure 1:

questions.

2 Data

2.1 The U.S. Experience, 1900-2000

In this section, we lay out the basic facts surrounding the time paths of TFP and fertility in the U.S. over the 20th century. We begin with the facts pertaining to the growth in productivity as laid out in Kendrick (1961) and Kendrick (1973). As most economists know, this period is one of more or less continued growth in productivity with a few significant breaks. The most significant of these is the Great Depression. Figure 1 shows Kendrick's compilation of TFP for the U.S. over the period from 1889 to 1969.

The obvious facts about TFP over this period are:

1. The continual upward trend,

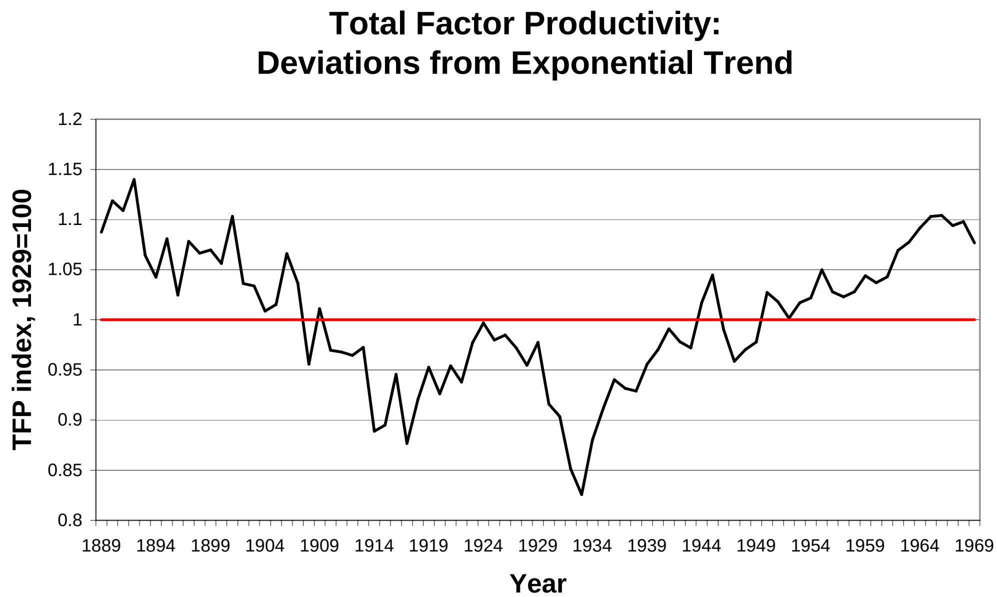


Figure 2:

2. The marked decline below trend that took place in the 1930's,

3. The return to trend following WWII.

The exact sizes of these features of the data depend on how one treats the 'trend growth' in productivity over the period. For example: Was there a common, exogenous growth rate in TFP over the entire period with higher frequency fluctuations (albeit highly autocorrelated fluctuations) around this trend? Or, were there two 'regimes' of growth, one prior to WWII and one after with higher growth in the second regime? These questions will have an impact on the analysis we present below and because of this, we will try several alternatives. (See more discussion on this in Section 5 below.)

After fitting an exponential trend to this series, we obtain the following 'shocks' to productivity over this period. (See Figure 2.)

As can be seen in the figure, there was a fairly long and deep fall in productivity from trend that took place from 1910 to 1940. In the deepest part of the Depression, productivity was about 17% below trend. Since that time there was a steady increase up until the late 1960's when productivity was about 10% above trend.

This timing of the movements of productivity around trend fits well with the movements in fertility seen in the data. Figure 3 shows the time path of the Total

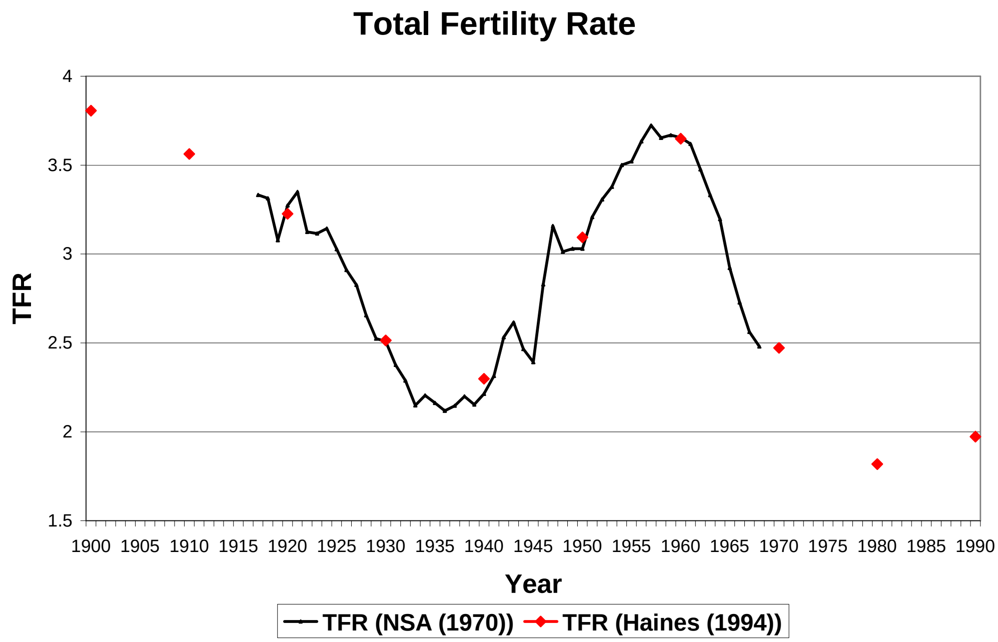


Figure 3:

Fertility Rate (TFR) over the period from 1890 to 1968. The Figure plots two time series for TFR. The first is the one prepared by Haines (1994) using Census data and hence is available only every 10 years. The second comes from the Natality Statistics Analysis from National Center for Health Statistics, and is available at annual frequencies, but only since 1917. At the beginning of the period fertility is still in the midst of what is known to Demographers as the Demographic Transition, the marked fall in fertility that has occurred in all developed countries. This fall accelerates from 1920 to 1930 as can be seen in the Haines data. From the NSA data it appears that a good description would be:

1. High, and fairly constant fertility over the period from 1917 to 1924, with TFR at about 3.2 children per woman,
2. Falling fertility over the period from 1925 to 1932 (from TFR=3.2 to TFR=2.2),
3. Constant, but low, fertility over the period from 1933 to 1940, with the level at about TFR=2.2,
4. Rising fertility from 1941 to 1956, with TFR going from 2.2 up to 3.6,
5. High, stable fertility from 1957 to 1962 at about TFR=3.6,

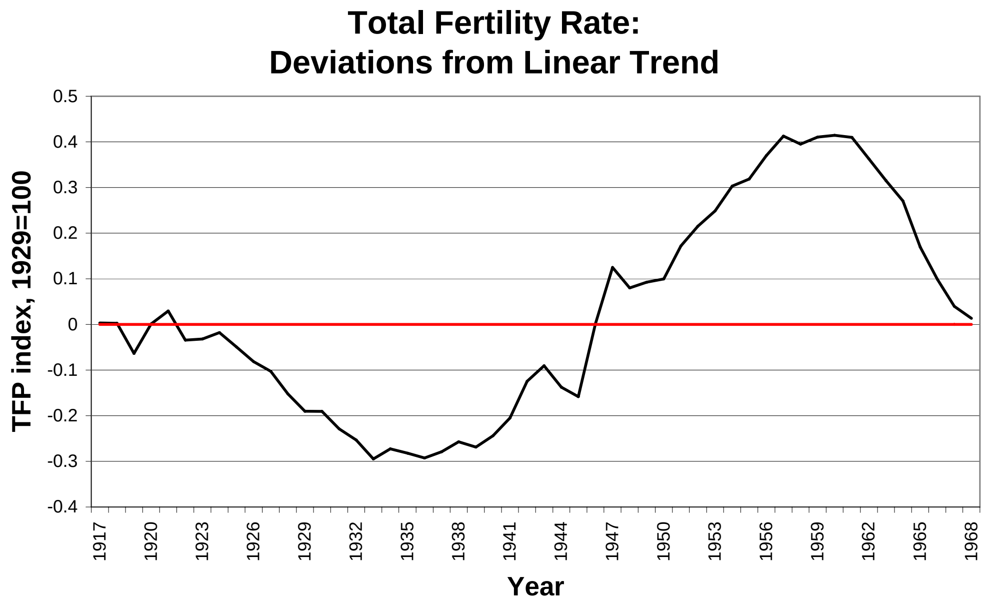


Figure 4:

6. Falling fertility over the remainder of the period from a TFR of 3.6 in 1963 to 2.5 in 1968.

Figure 4 shows a similar picture of fertility over this period. It shows the deviations from a fitted, linear, trend from 1900 to 1990 from Haines. The deviations at annual frequencies are calculated by subtracting the NSA data for the yearly observations from the fitted trend from Haines.[6] It shows a similar pattern to that described above.

Figure 5 shows the two series of deviations plotted on the same graph. Although it is not perfect, there is an impressive coincidence in timing that is no doubt the source of the original 'catching up' story in the Demography literature. The coefficient of correlation between the two annual series for the years 1917 to 1968 is 0.67, which suggests that the TFR is strongly procyclical. In sum, TFP was below trend from about 1910 to about 1940, while fertility was below trend from about 1925 to about 1940. They were both above trend over the 1945 to 1965 period with the TFP growth extending beyond this time.

This is not the whole story about the timing of the Baby Bust and Boom, obviously. One possibility is that the Baby Boom was simply a by-product of delayed

[6]Fitting a trend to the NSA data and calculating deviations from this trend gives virtually identical results.

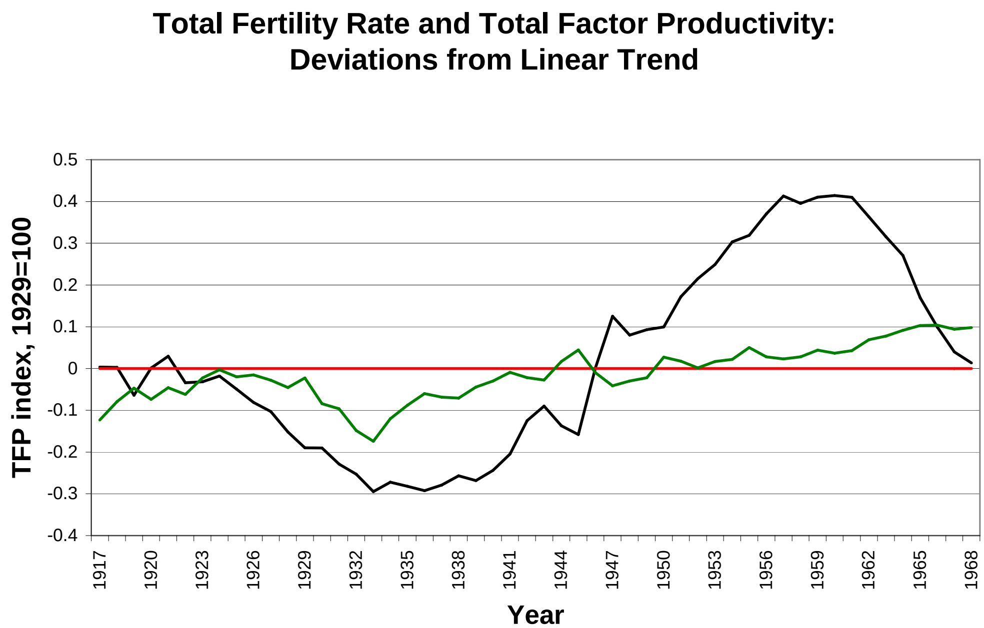


Figure 5:

fertility of those women whose fertility was low during the period of the Baby Bust. This is not true, however. In fact, women born roughly between 1905 and 1925 had lower completed fertility than did their mothers or daughters. This can be seen in Figure 6, which shows completed fertility by birth cohort over the period. This reaches a minimum with the 1908 birth cohort then increases back up to a peak with the 1938 birth cohort, falling thereafter. This pattern implies that if the baby boom is a 'catching up' of fertility from the baby bust, it is at the aggregate level, not at the level of the individual woman. See also Greenwood, et. al., (2005), and Doepke, et. al., (2005), for more on the make-up, across birth cohorts, of the fertility pattern during the baby boom.

2.2 Europe 1900-2000

The pattern of fertility in Europe[7] during the 20th century is, with the appropriate adjustments for the specific aspects of European economic history, altogether similar to that reported earlier for the U.S., with one important quantitative exception, that is the size of the post WWII Baby Boom. Consider the data about TFR for a

[7]Here and in what follows, the entity we call 'Europe' is composed of the following countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, UK.

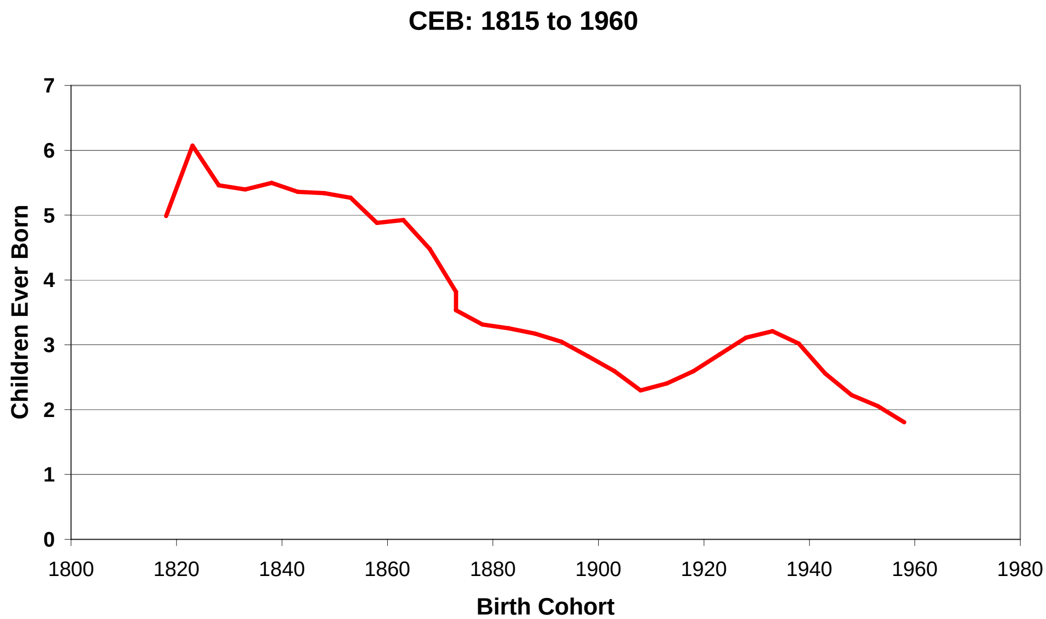


Figure 6:

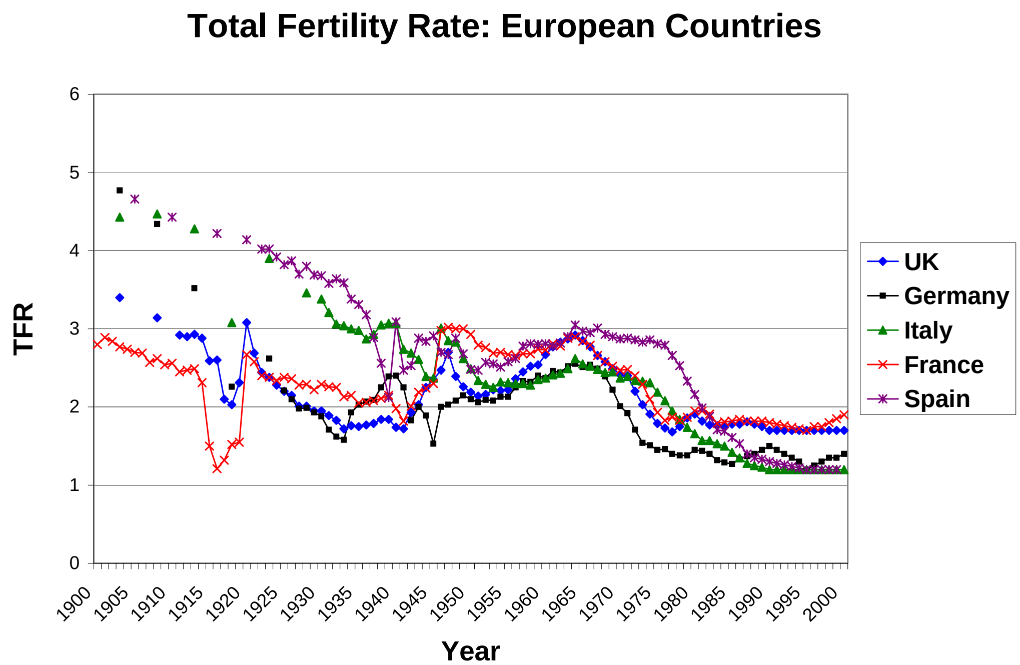
selected group comprising all the largest European countries, as reported in Figure 7. In France, Germany and the U.K. the First World War brings about a substantial drop in fertility, as its impact on the economic conditions of those countries is rather substantial; this seems to fit rather well with the general causal link we are exploring in this paper. Similarly, and with the significant exception of the UK, the 'depression era' was neither as dramatic nor as long lasting for Europe as it was in the U.S., which makes the much lower drop in fertility during that period also easy to understand from our point of view. So far so good, but things become less straightforward in the second part of the century.

The post WWII recovery was much stronger (in relative terms) in continental Europe than in the U.S., and especially so in France, Germany, and Italy; the 1950s and 1960s witnessed the European convergence phenomenon, with strong and above average growth in both income per capita and TFP. One would therefore expect an equally larger European Baby Boom, at least compared to the one in the U.S.. But this never materialized. With the possible exceptions of France and Spain, where the TFR peaked at around 3.0 in 1965, all the other European countries had much smaller 'Booms' than the U.S.. This is an important quantitative challenge for our modeling strategy. The smaller European Baby Booms were uniformly followed by a rapid decrease in fertility. In the last two decades, finally, TFR has reached much lower levels all over Europe than in the U.S.. The latter aspect seems coherent with the overall view we are pursuing, since the growth rates of both TFP and per capita income have been lower in Europe than in the U.S. for almost twenty years now. An additional and interesting challenge to the theory comes from the cross country variations in the timing of the second Bust, which are clearly visible in Figure 7 and in Figure ??? (need to add the figure comprising all other European countries). We observe that fertility rates dropped in France, Germany and the UK in the second half of the 1960s, in Italy in the middle 1970s, and in Spain in the early 1980s. The pattern of TFRs in the remaining 'Old Europe' countries are generally similar to the one described here (see Figure ???). This fits only partially with what we know about cross country variations in European economic growth and fluctuations. The so called 'productivity slowdown' that characterized the U.S. economy in the 1970s and the early part of the 1980s, was much less substantial, especially when measured via TFP growth rates, in Europe during those same years.[8] This makes the sudden and rapid reduction in European TFRs since the early 1970s hard to reconcile, at least quantitatively, with our causal mechanism. However, the European economic slowdown continued well into the 1980s and the 1990s, which makes the persistently lower European TFRs easier to understand, in light of our theory, relative to the behavior of the U.S. fertility rates.

2.3 Eastern Europe, 1980-2005

The recent evolution of fertility in Eastern European countries that were either part of the former USSR or members of the Warsaw Pact until the late 1980s, is also quite relevant as a test of the view considered here. All these countries underwent, albeit to different extents, a dramatic change in their political and socio-economic organization in the years between 1989 and the middle 1990s. These changes were certainly sudden and, to a large extent, unpredictable. They were also characterized by a dramatic and relatively long lasting drop in income *levels*, roughly during the years 1989-1995, and followed by several years of highly volatile (though overall positive) growth rates, roughly during 1995-2000. In the most recent years volatile growth has given way, at least for those countries that have joined or will soon be joining the E.U., to a period of relatively high and sustained growth in per capita income. As Table 1 shows, the drop in TFRs following the fall of the Communist regimes is quite dramatic and generalized across the whole spectrum of countries. It is hard to assess the extent to which the renewed economic growth since the late 1990s and, especially, after year 2000 has been reflected in a recovery of fertility rates. Interestingly, though, in those countries that have entered into the E.U. sphere and for which recent data are available (see e.g. Sobotka (2003)), there is widespread evidence of a sizeable rebounding in the TFRs since the late 1990s, which coincides exactly with the timing of economic recovery in those countries.

[8] Most European countries suffered a drop in their growth rates in the first half of the 1970s, which was, on the one hand, quite simultaneous (thereby making the different timing in the TFRs drop not so obvious to capture) and, on the other, less substantial than the one in the U.S.



Total Fertility Rate: European Countries
TFR
Year
UK
Germany
Italy
France
Spain
0
1
2
3
4
5
6
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000

Figure 7:

Table 1: TFR in Eastern Europe (1989 and 2000)

Country		Pop (000s)	TFR 1989	TFR 2000
Croatia		4,568	1.67	1.40
Czech Republic		10,278	1.87	1.14
Hungary		10,043	1.82	1.32
Poland		38,654	2.08	1.34
Slovak Republic		5,399	2.08	1.29
Slovenia		1,988	1.52	1.26
Former GDR		15,217	1.57	1.22
Bulgaria		8,190	1.90	1.30
Romania		22,456	2.21	1.31
Estonia		1,439	2.21	1.39
Latvia		2,424	2.05	1.24
Lithuania		3,699	1.98	1.27
Belarus		10,020	2.03	1.31
Russia		145,559	2.01	1.21
Ukraine		49,851	1.94	1.10

Source: Sobotka (2003)

[9]

2.4 European Demographic History, 1200-1800

We will tentatively follow a chronological order here, and briefly recall a few among the abundant historical episodes suggesting, across the centuries and across socio-economic organizations, the existence of a relatively stable relationship between oscillations in income growth rates and oscillations in fertility. There is a sense in which most of the historically recorded large swings in fertility and population levels, at least since the onset of the Demographic Transition, seem to fit the overall pattern outlined above: economic bonanzas bring high fertility while economic troubles

[9]The timing of the TFR's variations in the Eastern European countries is particularly interesting when compared to the rest of Europe. Following the Council of Europe (2001, 2002) division, we can divide the continent in the four areas of Western, Northern, Southern and Eastern Europe. Until 1989-1990, TFRs levels in Eastern Europe are very stable, at values slightly above 2.0, while they were falling since the middle 1970s in Southern Europe; at the end of the century the are at the same low levels, around 1.40, and both displaying signs of a small rebound. During the same period, TFRs in Western Europe are stable at 1.60, while in Northern Europe they are slowly cropping up from about 1.60 to 1.80.

engender low fertility.[10]

Without going back to the Neolithic transition, which has long been interpreted in the light of the basic Malthusian model by the classical demographic literature, or to the long centuries of economic and demographic depression that characterized Europe between the 5th and the 10th century A.D., it is enough to focus on the much better documented centuries of the late Middle Ages. In Europe (the only continent for which something akin to 'data' is available), these are the 'golden years' of the stationary Malthusian model, as we labelled it in the introduction. A classical portray of the Malthusian model at work, applied to the people of the Languedoc (Southern France) can be found in Le Roy Ladurie (1969); similar descriptions for Italy and Catalonia can be found, respectively, in Bellettini (1973), and Nadal (1982). It is important to note, though, that in order to map these earlier episodes into the analytical framework adopted here, mortality rates, their fluctuations and their, limited but relevant, endogeneity[11] also need to be taken into proper account. In any case, all available historical literature clearly document that, all along the period 1200-1800, periods of high economic growth were also periods of high fertility, while low fertility came almost invariably together with economic depression, be it caused by the plague, by wars, or simply by adverse economic shocks.

The century-long upswing of fertility in England and Wales between the middle of the 18th and the middle of the 19th century, overlaps with the initial phases of the Industrial Revolution. The increase in the growth rate of income per capita and of TFP, (the latter in particular during the second half of the named period) was sudden, at least by historical standards (Boldrin, Jones and Kahn (2005)). This correlation between unexpected surges in the underlying rate of economic growth and population growth characterizes, once declining mortality rates are taken into due account, also the 19th century evolution of continental Europe. Finally, however, the last period of the Demographic Transition where mortality is low and stable while income is growing at unprecedented rates and fertility is stable or declining remains a puzzle given the long established relation between fertility and the availability of economic resources.

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[10] To quote just one among the dozens of possible study, see the classical overview in Boserup (1983), stressing, as it is obvious, the crucial role played everywhere before the Industrial Revolution, by advances and retreats in agricultural productivity.

[11] See Livi-Bacci (1991) for a carefully argued skeptical view that the causation chain from economic depression to a sudden increase in mortality rates, via starvation/famine, had played a substantial role in the demographic equilibrium of Europe between the year 1000 and the end of the 19th century. Loschky (1976) is an earlier, but still relevant, evaluations of the pluses and minuses of the idea that economic conditions affect population growth mostly via the endogenous mortality channel.

3 A Simple Model of the Response of Fertility to TFP Shocks

In this section, we lay out a simple model of the response of fertility to period by period stochastic movements in TFP. To do this, we use a simple model of fertility based on that developed in Barro and Becker (1989) and Becker and Barro (1988). The simplification that we make is to assume that there is no physical (or human) capital in the model. Thus, the flow of income coming to a dynasty is solely due to wage income.

The dynasty head at time zero solves the maximization problem:

$$Max_{\{C_t,N_t\}} \qquad E_0\left(\sum_{t=0}^{\infty}\beta^t g(N_t(s^{t-1}))u\left[\frac{C_t(s^t)}{N_t(s^{t-1})}\right]\right)$$

subject to:

$$C_t(s^t)+\theta_t(s^t)N_{bt}(s^t)\leq N_t(s^{t-1})w_t(s^t),$$

$$N_{t+1}(s^t)\leq \pi N_t(s^{t-1})+N_{bt}(s^t),$$

$$N_0 \text{ fixed.}$$

Where $s^t=(s_0,s_1,....,s_t)$ is the history of shocks up to and including period t, w_t is the stochastic process for wages (assumed to be a function of the shocks), or incomes, θ_t is the cost of raising a child born in period t, C_t is aggregate consumption for the dynasty in the period (assumed split across all individuals of working age), N_{bt} is the number of new children born in the dynasty in period t and N_t is the number of dynasty members of working age alive in the period.

Note that each individual alive in period t is assumed to have a survival probability to period $t+1$ of π. This is unusual for a B&B fertility model, where it is usually assumed that individuals only have one period of active decision making. This corresponds to the assumption that $\pi=0$, one of the special cases we will discuss below. However, since some of the TFP movements that we want to discuss are at frequencies higher than a generation (in the fertility sense), it will be convenient to also consider cases in which $\pi>0$. Because of this choice of functional form (admittedly a gross simplification of actual dynastic survival processes), note that the model is equivalent to one in which the 'stock,' N, depreciates at rate $\delta=1-\pi$ over each period. This will allow us to use the analogy to a stochastic Ak model with less than full depreciation below.

We have assumed that the flow utility function is of the form $U(C,N)=g(N)u(\frac{C}{N})$, i.e., utility depends on both the size of the dynasty and per capita consumption.

Assuming that $g(N) = N^{\eta}$ and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we can rewrite this problem as:

$$\text{P1} \qquad Max_{\{C_t,N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \left[\frac{C_t}{N_t}\right]^{1-\sigma} /(1-\sigma) = \sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to:

$$C_t + \theta_t \left(N_{t+1} - \pi N_t\right) \leq N_t w_t.$$

There are two sets of parameter restrictions that satisfy the natural monotonicity and concavity restrictions for this functional form, both in terms of the aggregate, or dynasty variables, (C, N), and in terms of per capita values, $(N, c) = (N, \frac{C}{N})$. These are: i) $0 < \eta < 1$, $0 < \sigma < 1$ and $0 < \eta + \sigma - 1 < 1$, and ii) $\sigma > 1$ and $\eta + \sigma - 1 < 0$. We will explore both of these options below.

Notice that if $\eta + \sigma - 1 = 0$, then N does not enter the period utility function, and hence, N plays exactly the same role in this model as K does in a stochastic Ak model. There is one twist however. That is θ_t is also stochastic, at least in the case where childrearing is modeled as a time cost. In that case, $\theta_t = bw_t$ and hence, if w_t is stochastic, so are childrearing costs. This is an interesting twist, the periods when productivity is high are also those when capital is expensive. This might have important ramifications for the results. Also, in this case aggregate consumption, C, grows at the same rate as N, but per capita consumption is constant (without shocks). Other than that, the analogy is very close.

Let us introduce one additional piece of notation. Let N_{ft} be the fertile part of the working population, i.e. $N_{ft} = \lambda_t N_t$. Then children per fertile person - the fertility rate, n_{bt} can be expressed as $n_{bt} = \frac{N_{bt}}{N_{ft}}$. This is the model quantity that we will identify with the Total Fertility Rate in the Data.

3.1 Homogeneity Properties Planner's Problem

Recall that we want to study solutions to maximization problems of the form:

$$P(N_0, s_0) \qquad Max_{\{C_t,N_t\}} \qquad U_0(\{C_t, N_t\}) = E_0 \left[\sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$C_t + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi\right),$$

N_0 given.

Let $V(N_0, s_0)$ denote the maximized value of the objective at a solution (assuming one exists) and let $\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^{\infty}$, denote the solution itself.

Proposition 1 $V(\lambda N_0, s_0) = \lambda^\eta V(N_0, s_0)$ *and*
$\{(C_t^*(\lambda N_0, s_0), N_t^*(\lambda N_0, s_0)\}_{t=0}^\infty = \lambda\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^\infty$.

Proof. Step 1: $\{(C_t, N_t\}_{t=0}^\infty$ is feasible for $P(N_0, s_0)$ if and only if $\{(\lambda C_t, \lambda N_t\}_{t=0}^\infty$ is feasible for $P(\lambda N_0, s_0)$.

Step 2: $U_0(\lambda\{C_t, N_t\}) = \lambda^\eta U_0(\{C_t, N_t\})$.

Step 3: If the claim is false, then there is something 'better' for $P(\lambda N_0, s_0)$ than $\lambda\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^\infty$. Call this feasible plan $\{(C_t', N_t')\}_{t=0}^\infty$. ¿From Step 1, the plan $\frac{1}{\lambda}\{(C_t', N_t')\}_{t=0}^\infty$ is feasible for $P(N_0, s_0)$. ¿From step 2, $\frac{1}{\lambda}\{(C_t', N_t')\}_{t=0}^\infty$, gives higher utility than $\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^\infty$, a contradiction. ■

Thus, this is a standard homogeneous/homothetic type argument.

Because of this result, and because the problem is a stationary one if $(w(s), \theta(s))$ is assumed to be a first order Markov Process, we can characterize the solution through Bellman's Equation:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta E\left[V(N', s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \le (w(s) + \theta(s)\pi)\, N.$$

Because of the Proposition, this can be rewritten as:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta E\left[(N')^\eta\, V(1, s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \le (w(s) + \theta(s)\pi)\, N,$$

or,

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta\,(N')^\eta\, E\left[V(1, s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \le (w(s) + \theta(s)\pi)\, N.$$

Where the last step follows since N' is a function of s alone. (And not s'.)

Now, let $D(s) \equiv E\left[V(1, s')|s\right]$ to obtain:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta\,(N')^\eta\, D(s)$$

$$\text{s.t.} \qquad C + \theta(s)N' \le (w(s) + \theta(s)\pi)\, N.$$

From this, it follows that the FOC's for the problem are:

$$(FOC1) \qquad \frac{N^{\eta+\sigma-1}C^{-\sigma}}{1} = \frac{\beta\eta(N')^{\eta-1}D(s)}{\theta(s)},$$

$$(FOC2) \qquad C + \theta(s)N' = (w(s) + \theta(s)\pi)\, N.$$

These equations can be simplified to some extent. We have:

$$(FOC1) \qquad \left[\tfrac{C}{N}\right]^{-\sigma} = \tfrac{\beta\eta D(s)}{\theta(s)} \left[\tfrac{N'}{N}\right]^{\eta-1},$$

so that,

$$(FOC1) \qquad n = \tfrac{N'}{N} = \left[\tfrac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\tfrac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)}.$$

Substituting into $FOC2$ we get:

$$(FOC2) \qquad C + \theta(s)N' = (w(s) + \theta(s)\pi)\, N,$$

$$(FOC2) \qquad \tfrac{C}{N} + \theta(s)\tfrac{N'}{N} = (w(s) + \theta(s)\pi)\,,$$

$$(FOC2) \qquad \tfrac{C}{N} + \theta(s)\left[\tfrac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\tfrac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)} = (w(s) + \theta(s)\pi)\,.$$

It follows that $\frac{C}{N}$ and $\frac{N'}{N}$ are functions of the current shock only (although C and N' are not) and NOT the current level of the stock, N. Since this property plays a role in the ability of this type of model to exhibit a 'catching up' of fertility after a low shock, we state this as a formal proposition:

Proposition 2 *The solution to the Planner's Problem is to find $\frac{C}{N}$ and $\frac{N'}{N}$ which solve:*

$$\tfrac{C}{N} + \theta(s)\left[\tfrac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\tfrac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)} = (w(s) + \theta(s)\pi)$$

and,

$$\tfrac{N'}{N} = \left[\tfrac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\tfrac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)}.$$

Thus, the growth rate in population, $\frac{N'}{N}$ is a function of the current shock only, and not the size of the current stock, N.

Furthermore, if the fraction of fertile people in the population, λ_t, is independent of time, i.e. $\lambda_t = \lambda$, then the fertility rate also only depends on the current shock, s_t, and not the current stock, N_t

Proposition 3 *Let $N_f = \lambda N$. If λ is independent of time, then*

$$n_b = \tfrac{N_b}{N_f} = \tfrac{N' - \pi N}{N_f} = \tfrac{1}{\lambda}\left(\tfrac{N'}{N} - \pi\right).$$

That is, the fertility rate is independent on the size of the current stock, but, through $\frac{N'}{N}$, depends on this period's productivity shock.

This is an important qualitative propety of the model. That is, the central idea behind the notion that the Baby Boom is 'catching up' is that the size of the dynasty is 'too small' (relative to trend) at the end of WWII because of the Baby Bust. Thus, fertility is increased so as to bring the size of the 'stock' up to its desired level. However, due to this result, it follows that fertility in the model does NOT depend on the size of the dynasty, but only on the shock. Thus, this kind of model can never exhibit 'catch up' fertility of this type.

3.2 An Aside on Trend Growth in A

In this subsection we add to the previous analysis trend growth in productivity. This is assumed to be exogenous.

Thus, we will study solutions to maximization problems of the form:

$$P(\gamma,\beta;N_0,s_0) \qquad Max_{\{C_t,N_t\}} \qquad U_0(\{C_t,N_t\}) = E_0\left[\sum_{t=0}^{\infty}\beta^t N_t^{\eta+\sigma-1}\frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$C_t + \gamma^t\theta_t N_{t+1} \leq N_t\gamma^t\left(w_t + \theta_t\pi\right),$$

N_0 given.

We assume that $\gamma \geq 1$, is an exogenous constant.
Note that a sequence $\{C_t, N_t\}$ is feasible for this problem if and only if it satisfies:

$$\frac{C_t}{\gamma^t} + \theta_t N_{t+1} \leq N_t\left(w_t + \theta_t\pi\right)$$

Thus, defining $\hat{C}_t = \frac{C_t}{\gamma^t}$, this Problem can be written equivalently as:

$$P(\gamma,\beta;N_0,s_0) \qquad Max_{\{\hat{C}_t,N_t\}} \qquad U_0(\{\hat{C}_t,N_t\}) = E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\gamma^{1-\sigma}\right)^t N_t^{\eta+\sigma-1}\frac{\hat{C}_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$\hat{C}_t + \theta_t N_{t+1} \leq N_t\left(w_t + \theta_t\pi\right),$$

N_0 given.

Note that this problem is the same as:

$$P(1,\hat{\beta};N_0,s_0) \qquad Max_{\{\hat{C}_t,N_t\}} \qquad U_0(\{\hat{C}_t,N_t\}) = E_0\left[\sum_{t=0}^{\infty}\hat{\beta}^t N_t^{\eta+\sigma-1}\frac{\hat{C}_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$\hat{C}_t + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi\right),$$

N_0 given.

where $\hat{\beta} = \beta\gamma^{1-\sigma}$.

Note that this problem has no exogenous growth in it. Thus, we have:

Claim 4 *If $\{(\hat{C}_t^*, N_t^*)\}$ solves the problem $P(1, \hat{\beta}; N_0, s_0)$ for some $(1, \hat{\beta}; N_0, s_0)$, then $\{(\gamma^t \hat{C}_t^*, N_t^*)\}$ is the solution to the problem $P(\gamma, \hat{\beta}/\gamma^{1-\sigma}; N_0, s_0)$.*

Thus, to solve the problem with exogenously growing TFP, solve the one with $\gamma = 1$ and $\hat{\beta} = \beta\gamma^{1-\sigma}$, and then multiply the C sequence by γ^t. It follows that if $\frac{C}{N}$ is constant (or converges to a constant) in the solution to the problem with no growth, then, $\frac{C}{N}$ grows at rate γ in the one with exogenous growth.

4 The Stochastic AK Analogy: $\eta + \sigma = 1$, $\{s_t\}$ *i.i.d.*

In this section, we will specialize the model outlined above even further by assuming that $\eta + \sigma = 1$ and that the $\{s_t\}$ are i.i.d. There are several simplifications that occur under these assumptions. These are:

1. As noted above, in this case, the value function is homogeneous of degree $1 - \sigma$ (since $\eta = 1 - \sigma$) in N_0, $V(\lambda N_0, s_0) = \lambda^{1-\sigma} V(N_0, s_0)$.
2. Define $D(s) \equiv E\left[V(1, s')|s\right]$ for what comes below to simplify notation. Since the shocks are *i.i.d.*, it follows that $D(s) = E\left[V(1, s')|s\right] = E\left[V(1, s')\right]$.
3. $(FOC1)$ from Bellman's Equation (given above) simplifies to:

$$(FOC1) \qquad N' = \left[\frac{\beta\eta D(s)}{\theta(s)}\right]^{1/\sigma} C.$$

Furthermore, throughout this section, we will assume that:

$$w(s) = A(s) = As,$$

where the s are i.i.d. with $E(s) = 1$.

Finally, we will consider two extreme cases for the form of $\theta(s)$. In the first, we assume that only goods are needed to raise children, with $\theta(s) \equiv a$. In the second we assume that only time is used, $\theta(s) = bAs$.

Because of this result, we will abstract from trend growth through most of the remainder of the paper. In those cases where the solution to the model depends on the discount factor, we will use this result to 'calibrate' to the appropriate discount factor in the detrended model.

4.1 Goods cost only ($\theta_t = a$)

Here we assume that all cost of raising children can be summarized as a time invariant cost stated in terms of the consumption good. In this case an analytic solution to the Planner's Problem can be given. It is summarized in:

Proposition 5 *First suppose $\theta_t = a$ and assume that the shocks are i.i.d.. Then the problem has an analytical solution given by:*

$$\begin{cases} C = \varphi \left(As + a\pi\right) N \\ N' = \frac{(1-\varphi)}{a} \left(As + a\pi\right) N \end{cases}$$

where: $\varphi = \frac{1}{1+a^{\frac{\sigma-1}{\sigma}}\left(E(V(1,s'))\beta(1-\sigma)\right)^{\frac{1}{\sigma}}}.$

Proof:

From above the FOC's become:

$$(FOC1) \qquad N' = \left[\frac{\beta(1-\sigma)D}{a}\right]^{1/\sigma} C,$$

$$(FOC2) \qquad C + aN' = \left(As + a\pi\right) N.$$

Substituting $(FOC1)$ into $(FOC2)$, we obtain:

$$C + a\left[\frac{\beta(1-\sigma)D}{a}\right]^{1/\sigma} C = \left(As + a\pi\right) N, \text{ or,}$$

$$C\left[1 + a\left[\frac{\beta(1-\sigma)D}{a}\right]^{1/\sigma}\right] = \left(As + a\pi\right) N.$$

Thus,

$$C(N, s) = \varphi\left(As + a\pi\right) N, \text{ where,}$$

$$\varphi = \frac{1}{\left[1+a\left[\frac{\beta(1-\sigma)D}{a}\right]^{1/\sigma}\right]} = \frac{1}{\left[1+a^{\frac{\sigma-1}{\sigma}}(E(V(1,s'))\beta(1-\sigma))^{\frac{1}{\sigma}}\right]}.$$

It also follows that:

$C + aN' = (As + a\pi)\, N$, and so,

$N'(N,s) = \frac{(1-\varphi)(As+a\pi)}{a} N.$

Thus, using this, we can characterize the Value Function by:

(BE) $\qquad V(N,s) \equiv [C(N,s)]^{1-\sigma}/(1-\sigma) + \beta\,(N'(N,s))^{1-\sigma}\, D$

$$= [\varphi\,(As + a\pi)\, N]^{1-\sigma}/(1-\sigma) + \beta\left[\frac{(1-\varphi)(As+a\pi)N}{a}\right]^{1-\sigma} D = B\,[(As + a\pi)\, N]^{1-\sigma},$$

where,

$$B = \left[\frac{\varphi^{1-\sigma}}{1-\sigma} + \beta\left[\frac{(1-\varphi)}{a}\right]^{1-\sigma} D\right] = \left[\frac{\varphi^{1-\sigma}}{1-\sigma} + \beta\left[\frac{1-\varphi}{a}\right]^{1-\sigma} E\,[V(1,s')]\right].$$

Thus, integrating, we obtain:

$$E\,[V(1,s)] = BE\left[(As + a\pi)^{1-\sigma}\right].$$

This, along with the other equations above and the definitions of φ and B gives a complete solution to the problem.

Further, $C = \varphi\,(As + a\pi)\, N$ and $N' = \frac{(1-\varphi)}{a}\,(As + a\pi)\, N$ where φ is as described in the Claim. This completes the proof.

Note that φ and B depend on A, a and $E\left[(As + a\pi)^{1-\sigma}\right]$.

Using the characterization in the Proposition, it follows that fertility today (children per old person) is given by:

$$n_{bt} = \frac{N_{bt}}{\lambda_t N_t} = \frac{(1-\varphi)(As_t+a\pi)}{\lambda_t a} - \frac{\pi}{\lambda_t} = \frac{(1-\varphi)A}{\lambda_t a} s_t - \frac{\varphi\pi}{\lambda_t}, \text{ and,}$$

$$c_t = \frac{C_t}{N_t} = (\varphi\,(As_t + a\pi)) = \varphi A s_t + \varphi a\pi.$$

These depend on today's shock only. Thus, both the fertility rate and per capita consumption follow TFP movements procyclically in this case.

4.2 Time cost only ($\theta_t = bA(s_t)$)

In this section, we switch to the other extreme, and assume that all childrearing costs are in terms of time for the parents. That is, $\theta_t = bA(s_t) = bAs_t$. Under this assumption, the Planner's Problem has an analytic solution as summarized in the following Proposition.

Proposition 6 *Assume that $\theta_t = bA(s_t) = bAs_t$ and that the shocks are i.i.d.. Then the problem has an analytical solution given by:*

$$\begin{cases} C = \varphi(s)\,(1+b\pi)\,A(s)N \\ N' = \frac{(1-\varphi(s))}{b}\,(1+b\pi)\,N \end{cases}$$

where: $\varphi(s) = \frac{1}{\left[1+(bAs)^{1-1/\sigma}[\beta(1-\sigma)E[V(1,s')]]^{1/\sigma}\right]}$.

It follows that $\frac{N'}{N}$ is increasing in s if $\sigma > 1$ (a case which may not be of interest) and decreasing in s if $\sigma < 1$.

Proof: From above, the FOC's are:

$(FOC1)$ $\quad N' = \left[\frac{\beta(1-\sigma)D}{bAs}\right]^{1/\sigma} C,$

$(FOC2)$ $\quad C + bAsN' = (1+b\pi)\,AsN.$

Substituting $(FOC1)$ into $(FOC2)$, we obtain:

$$C + bAs\left[\frac{\beta(1-\sigma)D}{bAs}\right]^{1/\sigma} C = (1+b\pi)\,AsN, \text{ or,}$$

$$C\left[1 + bAs\left[\frac{\beta(1-\sigma)D}{bAs}\right]^{1/\sigma}\right] = (1+b\pi)\,AsN.$$

Thus,

$$C(N,s) = \varphi(s)\,(1+b\pi)\,AsN, \text{ where,}$$

$$\varphi(s) = \frac{1}{\left[1+bAs\left[\frac{\beta(1-\sigma)D}{bAs}\right]^{1/\sigma}\right]} = \frac{1}{\left[1+(bAs)^{1-1/\sigma}[\beta(1-\sigma)E[V(1,s')]]^{1/\sigma}\right]}.$$

It also follows that:

$$bAsN'(N,s) = (1-\varphi(s))\,(1+b\pi)\,AsN, \text{ or,}$$

$$N'(N,s) = \frac{(1-\varphi(s))(1+b\pi)AsN}{bAs} = \frac{(1-\varphi(s))(1+b\pi)N}{b}.$$

Notice that if $\sigma < 1$, then $1 - 1/\sigma < 0$ and so $s^{1-1/\sigma}$ is decreasing in s. It follows that in this case, $\varphi(s)$ is increasing in s. In this case then, it follows immediately that

$$\frac{N'(N,s)}{N} = \frac{1-\varphi(s)}{b}(1 + b\pi)$$

is decreasing in s.

In the opposite case, if $\sigma > 1$, then $1 - 1/\sigma > 0$ and so $s^{1-1/\sigma}$ is increasing in s. It follows that in this case, $\varphi(s)$ is decreasing in s and hence N'/N is increasing in s.

Thus, using this, we can characterize the Value Function by:

$$(BE) \qquad V(N,s) \equiv [C(N,s)]^{1-\sigma}/(1-\sigma) + \beta\,(N'(N,s))^{1-\sigma} D$$

$$= [\varphi(s)(1+b\pi)AsN]^{1-\sigma}/(1-\sigma) + \beta\left[\frac{(1-\varphi(s))(1+b\pi)AsN}{bAs}\right]^{1-\sigma} D = B(s)\,[sN]^{1-\sigma},$$

where

$$B(s) = \left[\frac{(\varphi(s)(1+b\pi)A)^{1-\sigma}}{1-\sigma} + \beta\left[\frac{(1-\varphi(s))(1+b\pi)A}{bAs}\right]^{1-\sigma} D\right]$$

$$= (1+b\pi)^{1-\sigma}\left[\frac{(\varphi(s)A)^{1-\sigma}}{1-\sigma} + \beta\left[\frac{(1-\varphi(s))}{bs}\right]^{1-\sigma} E\left[V(1,s')\right]\right].$$

Thus, integrating, we obtain:

$$E\left[V(1,s)\right] = E\left[B(s)1 \times s^{1-\sigma}\right] = E\left[B(s)s^{1-\sigma}\right].$$

4.3 Simulation

In this section we look at simple calibrated examples of the solution to the model given above. To do this, we must first choose parameter values for the model. A critical choice is the length of the period. Although it seems plausible that parents do consider the likely well-being of their children when making decisions about fertility, it is unlikely that these decisions are driven by quarter to quarter fluctuations of the kind studied in the Real Business Cycle literature. We consider two alternative, simple, specifications of the model. In the first, we assume that $\pi = 0$ so that it corresponds most closely to that of the original B&B model. Here, we assume that a period is approximately 10 years. At the other extreme, we use overall death rates to estimate π and use, $\pi = 0.8$. We lay out these two examples in the cases where (1) all costs of raising children are goods costs and (2) all costs are time costs.

We use the following additional facts to determine the parameters $\lambda, \sigma, \beta, \gamma$ and the standard deviation of productivity shocks, σ_s. Assuming that demographics are stationary and that a generation is 25 years with 85% of newborns surviving to age 25, we set the fraction of fertile people (women) in the population, $\lambda_t = \lambda = 0.5 * 0.85 = 0.425$. Using this we calibrate the intertemporal elsaticity of substitution parameter, σ, to match $E(n(s)) = (TFR * \lambda)^{\frac{10}{25}} = 1.087$ where $n(s) = \frac{N'(N,s)}{N}$, TFR is the average TFR over the period 1917 to 1968 and the exponent is the adjustment from annual data to $T = 10$ and 25 year generations. This is an important fact to match since we calculate percent deviations from it. We pick $\beta = (0.97)^{10}$ to match a 3% annual interest rate. Finally, from the TFP analysis given above, we find that the actual, estimated series for s_t decade by decade (beginning with 1910 to 1919) is $(0.935, 0.966, 0.905, 0.987, 1.026, 1.079)$, the growth rate is $\gamma = (1.017)^{10}$ and the standard deviation of shocks is $\sigma_s = 0.07$. We discuss alternative treatments of the data in Section 5 below.

4.3.1 Goods cost only ($\theta_t = a$)

Example 1: $\pi = 0$**,** $T = 10$ **years** In this case, it can be seen from the Proposition that:

$$n_{bt} = \frac{N_{t+1}}{\lambda N_t} = \frac{(1-\varphi)(As_t + a\pi)}{\lambda a} = \frac{(1-\varphi)A}{\lambda a} s_t, \text{ and,}$$

$$c_t = \frac{C_t}{N_t} = (\varphi\,(As_t + a\pi)) = \varphi A s_t.$$

It follows that $\overline{n_b} = E\{n_{bt}\} = \frac{(1-\varphi)A}{\lambda a}$ and thus,

$$\frac{n_{bt} - \overline{n_b}}{\overline{n_b}} = \frac{\frac{(1-\varphi)A}{\lambda a} s_t - \frac{(1-\varphi)A}{\lambda a}}{\frac{(1-\varphi)A}{\lambda a}} = s_t - 1.$$

Thus, in this case, the elasticity of fertility with respect to the current shock is 1, independent of the assumptions on the parameters.

The time series of both model and actual fertility are shown in Figure 8. As can be seen in the Figure, the model captures about 1/3 of the downswing in fertility in the 1930's and about 1/4 of the upswing in fertility during the Baby Boom.

Example 2: $\pi = 0.8$**,** $T = 10$ **years** In this example, we need to calibrate $A\_a \equiv \frac{A}{a}$. To do that we choose $c\_A \equiv \frac{c(1)}{A} = 0.75$ because, on average, a family spends about 25% of labor income on children. The choice of $\pi = 0.8$ corresponds to an expected active lifetime of about 50 years.

To calibrate the model, we solve for φ and A_a, using only the above targets.

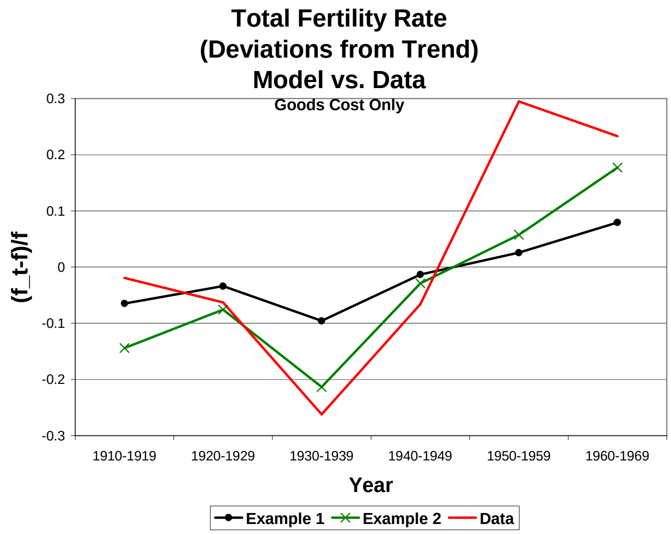

Figure 8:

From the Proposition we have

$$\begin{cases} n = (1-\varphi)(A\_a+\pi) \\ c\_A = \varphi\left(1+\frac{\pi}{A\_a}\right) \end{cases}$$

Since we know n, c_A and π, this is a system of equations in φ and A_a with the following solution.

$$\begin{cases} A\_a = \frac{n-\pi}{(1-c\_A)} \\ \varphi = \frac{c\_A}{(1-c\_A)}\frac{n-\pi}{(A\_a+\pi)} \end{cases}$$

To simulate the effect of changes in s_t on fertility, n_{bt}, only λ, π, A_a and φ are needed. In particular we do not need to know the values of σ, β or the distribution parameters of the shock process Also not needed are A and a separately.[12]

We find $A\_a = 1.14$ and $\varphi = 0.44$. Thus, we get $n_t = (1-\varphi)A\_a * s_t + (1-\varphi)\pi = 0.64 s_t + 0.44$. This implies that if $s_t = 0.9$, $\frac{n_{bt}-\overline{n_b}}{\overline{n_b}} = -0.223$. That is, a 10% deviation from trend in TFP results in an 22.3% reduction in fertility – the elasticity of fertility with respect to a shock to productivity is about 2.2.

4.3.2 Simulation: Time cost only ($\theta_t = bA(s_t)$)

In these examples, we set $b = 0.225$. This additional fact allows us to determine the intertemporal elasticity of substitution parameter, σ. We proceed as follows. First, we

[12]Solving for σ, given the facts above, gives $\sigma = 2.11$.

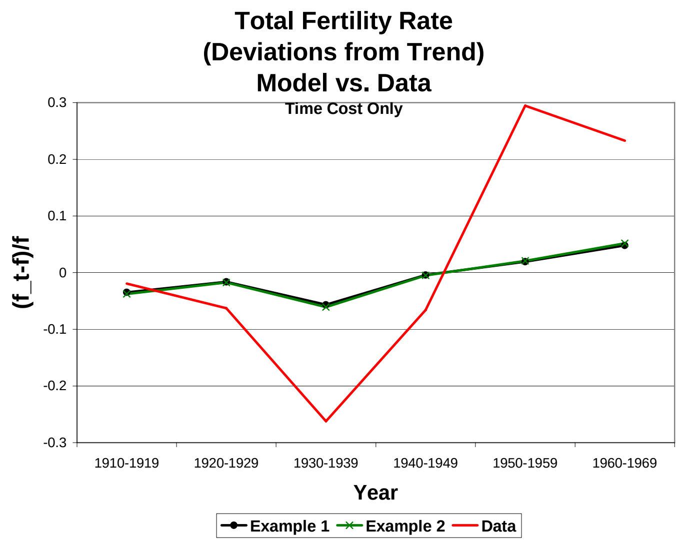

Figure 9:

use value function iteration to solve for policy functions. We then calculate $E(n(s))$ from the model and adjust σ, until $E(n(s)) = 1.087$. When $\pi = 0$ versus $\pi = 0.8$, the calibration gives a value of $\sigma = 6.7$ and $\sigma = 6.6$, respectively. We then feed in the observed sequence of shocks as before. Figure 9 plots the results for both examples. In this case, the effects of productivity shocks are rather small with an elasticity of about 0.6.

4.3.3 Summary

In sum, the response in fertility to productivity shocks is larger when all costs are in terms of goods and it is increasing in the survival rate. The intuition behind the lower elasticity in the time cost example is simple. If children cost time, during productivity slow downs, when the wage is low, the opportunity cost of having children is also low. This effect counteracts the usual intuition from the Ak analogy.

The intuition for the response being larger when survival is higher is as follows. In the case where survival is zero, the dynasty has to rebuild the entire population every period. Labor being the only input in production, to smooth consumption, it cannot invest too little in children. If the stock of people does not die out every period, it is much easier to adjust today's fertility rate to keep consumption high today and still have relatively high expected consumption tomorrow.

5 Extensions, etc.

There are several things that are of interest by way of extension of the simple models laid out here so far. First, there are those things that should be explored for the given state space of the model. These include:

1. Considering the case with $\eta + \sigma$ different from 1. This will not change some of the basic results – for example that the result that there is no 'catching up.' It might change some of the quantitative results however. It is unclear even the direction of these changes ex ante.

2. Including Autocorrelation in the shock in the model. Again, this will not change the negative 'catching up' result. How important it is depends on the length of a 'period' in the model. For example, the first order autocorrelation coefficient in the yearly data is about $\rho_1 = 0.86$, and assuming independence is fairly far off. On the other hand, the autocorrelation at 10 lags, $\rho_{10} = 0.28$, which is not statistically significantly different from 0.0. Thus, assuming independence of the shocks with a period of length 10 years is not such a bad assumption.

3. Alternative 'treatments' of the data. In the examples computed to this point, we have used a map between the model and data in which we have assumed that individual agents think that there is a common, exogenous, trend to TFP over the whole period of the data. The actual time series of the (log of) the Kendrick TFP data is shown in Figure 10, below. Thus, although it is not obviously insane to view it as with one, common trend over the period, it would also be reasonable to model the series as one with 'lower growth' near the beginning of the period, and 'higher growth' toward the end of the sample. Various alternatives would interesting to explore in more detail. For example, a version of the model where there are two, still exogenous, growth rates for TFP for the two periods, γ_1 and γ_2, with $\gamma_2 > \gamma_1$, with higher frequency fluctuations around these trends, would be an interesting generalization to explore. There are three (at least) serious conceptual issues that will arise. First, it is not clear, from visual inspection, when this 'change' should occur. Was the break in the early 1920's? Or did it occur later, during WWII for example. This affects the size of the 'shocks' (and even the sign in some cases). Second, did the individual decision-makers in the model 'know' that the break occurred? Or did they learn it, slowly, over time? Is it 'safe' or 'reasonable' to model this change as one that was viewed, ex ante, as a zero probability event? Is it 'safe' or 'reasonable' to model the corresponding opposite change (i.e., the event of growth returning to γ_1 after the change) as one that was viewed, ex post, as a zero probability event? These issues will make undertaking this venture challenging, but it is still doable.

Figure 10:

In addition to these, there are some that involve changing the state space.

1. Add physical capital to the model. This change may well alter the 'no catching up' result. It is not clear if analogs of the analytic results that we have provided in the N only case can be extended to this case. But, it is a realistic change.

2. Add a 'vintage structure' to the lifetimes of the dynasty. I.e., have realistic lifetimes, and overlapping members of the dynasty alive at each time. One advantage of this is that we can then try to address the question: What birth cohorts have low (high) fertility when a negative (positive) productivity shock occurs? How does this compare with which cohorts actuall had low (high) fertility during the 'bust' ('boom')?

A Simple version of this would be something like:

$$P(N_0, s_0) \qquad Max_{\{C_t,N_t\}} \qquad U_0(\{C_t, N_t\}) = E_0\left[\sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$C_t + \sum_{a=1}^{a_L-1} \theta_{at} N_{at} + \theta_{0t} N_{bt+1} \leq W_t,$

$W_t = \sum_{a=a_L}^{T} N_{at} p_a w_t,$

$N_t = \sum_{a=a_L}^{T} N_{at},$

$N_{a+1t+1} = \pi_a N_{at},$

$N_{1t} = \pi_1 N_{bt},$

$(N_{00}, N_{10}, ..., N_{T0})$ given.

6 Related Questions

- Do events similar to the coincidence in timing of the 'baby bust' and Depression in U.S. occur in other countries, and/or in different eras and are these documented? This 'story' does seem to be consisent with the facts that the Depression was smaller in most European countries and they experienced smaller baby busts and baby booms. (Check the volume of RED editted by Kehoe and Prescott on "Depressions of the 20th century," and see if we can find TFR data - if possible by age - to check to see whether a similar bust and subsequent catch up fertility happens in those countries during the depression times.)

- Fertility has been strikingly low for the last 20 years or so both in Russia and in many of the Eastern European countries. A similar hypothesis that has been mentioned as a possible explanation for why fertility has been so low in all of the former countries in the Soviet Union – i.e., the decline of output in the transition and the great economic uncertainty that accompanied the break up of the USSR. Can we document this?

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