

Welfare Costs, Long Run Consumption Risk, and a Production Economy. *

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PRELIMINARY AND INCOMPLETE

Abstract

The main goal of this paper is to measure the welfare costs of business cycles in a production economy in which the representative agent has low risk aversion and - at the same time - the equity premium and the co-movements of aggregate quantities and market returns are comparable to what observed in historical data. In order to do so, I consider a production economy in which the representative agent has Epstein-Zin-Weil(1989) preferences, productivity has a Long Run Risk component and there are capital adjustment costs. In this way, I try to bridge the gap between the current Long Run Risk asset pricing literature, in which quantities are taken as exogenous, and the standard macroeconomic business cycle models. Preliminary results from a benchmark exchange economy suggest that when there is a Long Run Consumption Risk and the representative agent prefers early resolution of uncertainty, the implied total welfare costs of the consumption uncertainty range from 12% to 20%. (JEL classification: E20, E32, G12, D81)

KEYWORDS: Production Economy, Long-Run Risk, Asset Pricing, Recursive Utility

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1 Introduction

Since Lucas(1987) measuring the welfare costs of business cycle fluctuations has been an on going challenge carrying on important policy implications. Low business cycle costs would suggest that it's not efficient for society to devote more resources to *further* stabilize consumption since the benefits would be modest even with the same policies. On the other hand, producing evidence that the postwar business cycles were costly doesn't directly imply that a further stabilization is desirable or possible. Still, it encourages researchers and policymakers to look for new channels through which economic policies could effectively reduce the residual cyclical fluctuations and produce relevant welfare benefits. Using a business cycle model with time additive log preferences and serially uncorrelated consumption fluctuations, Lucas(1987) calculated that individuals would be willing to sacrifice at most .1% of their lifetime consumption for policies devoted to remove the residual amount of business cycle risk. Since then, various researchers have revisited Lucas's calculation looking for new evidence of larger welfare costs, focusing on two aspects of his stylized model: the absence of persistent aggregate consumption fluctuations and the presence of counterfactual implications for asset market facts.

Reis(2005) points out that assuming no serial correlation in the consumption fluctuations substantially reduces the *amount of busyness cycle risk* and its associated cost. He then shows empirically also that consumption is actually very persistent and it's not possible to reject the existence of a unit root (as in Hall(1980)).

Mehra-Prescott(1985) and the subsequent literature about the equity premium puzzle have largely documented that CRRA preferences - workhorse in benchmark business cycle models - cannot produce Sharpe-Ratios as high as the ones observed in the data unless the relative risk aversion coefficient is calibrated to incredibly high values in turn implying approximately even one hundred times bigger welfare costs for business cycles.

In a model with standard time additive CRRA preferences it's impossible to reconcile business cycle and asset market facts.¹ The implied measure of the welfare costs is crucially affected by the preferences of the researchers. Those ones that - like Lucas(1987) - choose to give more importance to the aggregate quantities properties, calibrate the risk aversion to low values and report lower welfare costs. Those ones that care for the equity premium, instead, calibrate the risk aversion coefficient to larger values and find higher costs.

¹Rouwenhorst(1995) shows that a production economy with standard time-additive CRRA preferences cannot solve the puzzle since when the risk aversion is high the consumption process becomes too smooth.

Tallarini(2000) offers a partial resolution to this problem by studying the costs of uncertainty in a production economy where productivity follows an exogenous stochastic trend and the representative agent has Epstein-Zin-Weil(1989) preferences with elasticity of intertemporal substitution equal to one. The relative risk aversion and the subjective discount factor are calibrated in order to match the level of the market Sharpe-ratios and the real interest rate observed in the data. He is able to match simultaneously several key features of both the aggregate quantities and the market returns. He finds very high costs ranging from 13% to 10283%, but these results are mostly due to the fact that he doesn't solve the equity premium puzzle.²

The main challenge of this paper is to measure the welfare costs of business cycles in a production economy in which the representative agent has low risk aversion and - at the same time - the equity premium and the co-movement of the aggregate quantities are comparable to what is observed in historical data. For this reason, I follow a recent finance literature that has proposed a new possible resolution for the equity premium puzzle. I consider an economy in which the representative agent cares for the timing of resolution of uncertainty according to Epstein-Zin-Weil(1989) preferences and consumption is simultaneously affected by two different sources of uncertainty. In particular, I assume that aggregate consumption has a unit root and that its drift is subject to small but very persistent deviations from its unconditional mean. This source of uncertainty takes the name of Long Run Risk since it produces low frequency fluctuations whose volatility is almost negligible over a short horizon but is bigger over long horizons. Although it's difficult to identify such a small Long Run Risk component from consumption data (Hansen-Heaton-Li(2005)), several recent papers show that this might be a reasonable assumption in light of the sensible improvements that such models afford in explaining key features of asset data.³ For example, Bansal-Yaron(2004) show that in an exchange economy with Long Run Risk and Epstein-Zin-Weil(1989) preferences it is possible to reconcile consumption and asset prices properties with low risk aversion and an elasticity of intertemporal substitution slightly bigger than one.⁴ Their results might suggest, at first, that the implied costs of uncertainty should be low due to the fact they manage to keep risk aversion to a low level. However, studying a benchmark exchange economy I show that trading off risk

²In Tallarini(2000) the risk aversion coefficient ranges from 45 to 180).

³See Bansal-Gallant-Tauchen(2004), Hansen-Heaton-Li(2005), Bansal-Dittmar-Lundblad(2005), Parker-Juillard(2005), Kiku(2005), Bansal-Dittmar-Kiku(2005), Croce-Lettau-Ludvigson(2005).

⁴Bansal-Yaron(2004) calibrate their risk aversion in the range [7.5 10]. Colacito-Croce(2005) manage to calibrate their relative risk aversion coefficient to an even lower level, 4.25.

aversion with Long Run Risk in order to match the historical equity premium produces even higher welfare costs that range from 11% to ???%. A calibration in which risk aversion is low, in fact, requires a large amount of Long Run Risk in order to match the assets market data. Focusing also on the role of the intertemporal elasticity of substitution, I show that agents that are more willing to substitute current consumption with future consumption experience higher welfare losses similarly to what found in Obstfeld(1994).

Although I have still to complete the analysis of the welfare costs in the production economy, these preliminary results suggest that it's difficult to produce a low costs of consumption fluctuations once one commits to take asset prices data seriously. In the production economy, I introduce Long Run Risk in the productivity growth rate and capital adjustment cost as in Jerman(1997) in order to match the volatility of the market price-dividend ratio and the market excess returns. As Barlevy(2003) points out, the welfare loss of business cycle fluctuations can be quite high in this case. In fact, in order to match the observed elasticity of the investment-to-capital ratio with respect to the marginal Tobin's q a high curvature in the adjustment cost function is required. This implies a reduction of the average growth rate of consumption by between .3 and .5 percentage points. For this reason, in order to isolate the different channels through which uncertainty produces welfare costs, it's my intention to carefully explore the role that both the long run risk component and the stochastic trend have on the volatility and the average growth rate of consumption and investment. Of course, I will give particular attention to the effects of precautionary savings motives on the steady state of the economy and I will commit to produce asset returns as high and as volatile as those observed in the data.

A interesting and crucial complication of this analysis is that when the representative agent has Epstein-Zin-Weil(1989) preferences his continuation values enter the first order conditions. So, in order to simulate the model, one has to solve for the value function first. By setting the intertemporal elasticity of substitution to one, Tallarini(2000) is able to solve his model by using the methods in Hansen and Sargent(1995). However, Kiley(2001) concludes that "...it is the intertemporal elasticity of substitution that is critical for quantity fluctuations after a shock...".⁵ Kiley(2001) and Bansal-Yaron(2004) findings suggest that exploring the role of the intertemporal elasticity of substitution has to be an important step of this analysis and for this reason I adopt a more flexible approximation

⁵Kiley(2001) uses a very stylized business cycle model with production and Epstein-Zin-Weil(1989) preferences for which there exists a closed form solution.

method than Tallarini(2000)'s one.⁶ This approach, even if computationally intensive, provides me a laboratory in which to examine simultaneously the implications of Long Run Risk and preference parameters on basic business cycle facts. This will help me to bridge the gap between the current Long Run Risk asset pricing literature, in which quantities are taken as exogenous, and the standard macroeconomic business cycle models.

Reconciling the asset markets fact with the aggregate quantities behavior has proved a challenge for modern stochastic dynamic general equilibrium models. Jerman(1998), Lettau-Uhlig(2000), Boldrin-Christiano-Fisher(2001) have proposed models based on preferences with habit formation. In particular, Jerman(1998) is able to produce low risk free rate, high equity premium, high volatility for the excess returns and relative volatilities for consumption, investment and output in the order of what observed in the data by introducing also capital adjustment costs. However, as pointed out in Boldrin-Christiano-Fisher(2001) and Lettau-Uhlig(2000), his model produces a countercyclically response of labor to a persistent shock to productivity. Boldrin-Christiano-Fisher(2001) propose a two sector economy that doesn't generate this counterfactual behavior but, on the other side, predicts a negative serially correlation for consumption and a too much volatile risk free rate. By contrast, in Tallarini(2000), the interest rate and the excess returns are too smooth. Even if the Sharpe-ratios are successfully close to those ones observed in the asset market, the highest annualized equity premium produced is about .44%. A production economy with long run risk and Epstein-Zin-Weil(1989) preferences is instead potentially able to match both the level and the volatility of the risk free rate and the market returns once adjustment costs are introduced in order to allow fluctuations of the price of capital.

On the other hand, looking at the business cycles welfare costs literature, this paper extends the analysis of Obstfeld(1994) studying also long run consumption fluctuations both in an exchange economy and a production economy and relates also to Alvarez-Jerman(2003). Adopting an approach that doesn't require the specification of preferences and that instead uses just asset prices, they show that low frequencies consumption fluctuations can be much more costly than fluctuations corresponding to business cycle frequencies. Their findings look consistent with the preliminary results found in the exchange economy.

The rest of this paper is organized as follows. Section 2 describes the relevance of the preferences parameters for the welfare costs and the asset market implications in an exchange economy. In Section 3 I present the growth model, I show the effect of the long

⁶See Carroll(2005) and Barillas-Fernández-Villaverde(2005).

run risk on the consumption, investment and labor. I compute the welfare costs with production and I study the role of relative risk aversion and the elasticity of intertemporal substitution. Section 4 concludes.

2 The exchange economy

2.1 Economy setup

I study an economy in which time is discrete and there is a representative agent who has Epstein-Zin-Weil(1989) recursive preferences taking the following form:

$$U_t = \left[(1 - \delta) C_t^{1 - \frac{1}{\Psi}} + \delta \left(E_t \left[U_{t+1}^{1 - \gamma} \right] \right)^{\frac{1}{\theta}} \right]^{1 - \frac{1}{\Psi}} \quad (1)$$

where γ is the coefficient of risk aversion, Ψ is the intertemporal elasticity of substitution and $\theta \equiv \frac{1 - \gamma}{1 - 1/\Psi}$.

The intertemporal marginal rate of substitution of this agent is:

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\Psi}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1 - \gamma} \right]^{\frac{1}{1 - \gamma}}} \right)^{\frac{1}{\Psi} - \gamma} \quad (2)$$

In order to study the properties of this economy I need to specify the consumption process. In an endowment economy, thanks to the feasibility condition, consumption will be equal to the exogenous endowment process. I assume that there is no storage technology and that the aggregate endowment has no durable component and I model the consumption growth rate⁷ as in Bansal-Yaron(2004):⁸

$$\Delta c_{t+1} = \mu + x_t + \sigma \epsilon_t^c \quad (3)$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_t^x \quad (4)$$

$$\begin{bmatrix} \epsilon^c \\ \epsilon^x \end{bmatrix} \sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \quad (5)$$

⁷From now on, I adopt the convention of denoting log-variables in small letters. For example, $c_{t+1} = \log C_{t+1}$, $m_{t+1} = \log M_{t+1}, \dots$

⁸For a more general specification see Hansen-Heaton-Li(2005)

The parameter ρ is calibrated to be close to one while the ratio $\frac{\sigma_x}{\sigma}$ is calibrated in order to be small so that consumption growth is not highly serially correlated. On the base of this calibration strategy, the long run component x_t is a small but persistent deviation of the consumption drift from its unconditional mean μ .

Since I want to take seriously the market returns data, I calibrate the preferences in order to match the mean and the volatility of the risk free rate and the excess returns in the post-war US data. While the model specified in (1)-(5) gives precise implications for the risk free rate and the asset paying consumption, it doesn't give any idea of how an asset that pays dividends should be priced. In order to take into account these elements, similarly to Bansal-Yaron(2004), I assume that the growth rate of dividends evolves in the following way:⁹

$$\Delta d_{t+1} = \mu_d + \phi x_t + \phi_c \sigma \epsilon_{t+1}^c + \sigma_d \epsilon_{t+1}^d \quad (6)$$

$$\begin{bmatrix} \epsilon^c \\ \epsilon^x \\ \epsilon^d \end{bmatrix} \sim iidN \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \quad (7)$$

The parameters (σ_d, ϕ, ϕ_c) allow me to calibrate the overall volatility of dividends and its correlation with consumption. The parameters (ϕ, ϕ_c) determine the relative importance of the idiosyncratic shock ϵ_{t+1}^c and the long run component.

I assume that the securities markets are complete in order to have a simple asset pricing model. Let V_t^d denote the ex-dividend price-dividend ratio of a claim to an asset that pays a dividend stream growing as in (6)-(7), and let V_t^c denote the ex-dividend price-consumption ratio of a share of a claim to the aggregate consumption stream. From the first order condition for optimal consumption choice and the definition of returns:

$$\begin{aligned} 1 &= E_t \left[M_{t+1} R_{t+1}^d \right], R_{t+1}^d \equiv \frac{V_{t+1}^d + 1}{V_t^d} e^{\Delta d_{t+1}} \\ 1 &= E_t \left[M_{t+1} R_{t+1}^c \right], R_{t+1}^c \equiv \frac{V_{t+1}^c + 1}{V_t^c} e^{\Delta c_{t+1}} \end{aligned}$$

Thanks to the homogeneity of the preferences the following holds:

$$V_t^c = \frac{1}{1 - \delta} \left(\frac{U_t}{C_t} \right)^{1 - \frac{1}{\psi}} \quad (8)$$

⁹Dividends are a sub-component of the total endowment, the residual part corresponds to labor income.

It's also possible to rewrite (2) as:

$$M_{t+1} = \delta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\Psi}} (R_{t+1}^c)^{\theta-1} \quad (9)$$

Finally, I need to specify the information set of the agent. For simplicity, I focus on the benchmark case in which the agent has full information:

$$E_t[\cdot] \equiv E[\cdot \mid \{x_k, \epsilon_k^x, \epsilon_k^c, \epsilon_k^d\}_{k=-\infty}^t]$$

The agent observes not only the dividends and consumption growth but also their specific components.¹⁰

2.2 The costs of uncertainty

2.2.1 Definitions

I define the cost of uncertainty as the percentage increase of consumption $\Lambda > 0$ that one has to give to the agent in every period and along every history in order to make him indifferent between the consumption process $\{C^i\}$ and a less risky consumption process $\{C^j\}$:

$$U(\{(1 + \Lambda)C^i\}) = U(\{C^j\})$$

Let u^j denote the log of the utility-consumption ratio for the generic consumption process $\{C^j\}$, the following holds:

$$\lambda \equiv \log(1 + \Lambda) = u^j - u^i \quad (10)$$

In order to have a measure of the cost, all I have to do is to compute the value of the utility-consumption ratio in log units for the two different consumption process and calculate their difference.

In the economy described above, given any calibration for $\{\mu, \sigma, \sigma_x\}$ there are three dif-

¹⁰A different information structure is analyzed in Croce-Lettau-Ludvigson(2005)

ferent consumption processes I look at:¹¹

$$\begin{aligned} C^{tr} & : \Delta c_{t+1} = \mu \\ C^{id} & : \Delta c_{t+1} = \mu + \sigma \epsilon_{t+1}^c \\ C^{lr} & : \Delta c_{t+1} = \mu + \sigma \epsilon_{t+1}^c + x_t \end{aligned}$$

The total cost of uncertainty can be computed by comparing the utility that the agent would have in an economy with consumption $\{C^{tr}\}$ with that one associated to the process $\{C^{lr}\}$:

$$\lambda^{Tot} = u^{tr} - u^{lr} \quad (11)$$

This cost can be decomposed in two parts. The cost of the idiosyncratic risk is:

$$\lambda^{id} = u^{tr} - u^{id} \quad (12)$$

and, simply by difference, the cost of the long run risk is $\lambda^{Tot} - \lambda^{id}$, equivalent to:

$$\lambda^{lr} = u^{id} - u^{lr} \quad (13)$$

2.2.2 The special case $\Psi = 1$

The case in which $\Psi = 1$ is interesting for two different reasons: the first one is that this is what Lucas(1987) has assumed¹² and the second one is that it's possible to get an *exact* solution for the value function.

According with (1)-(5) the following results hold:

$$u_t^1 \equiv \log \left(\frac{U_t^{(\Psi=1)}}{C_t} \right) = \mu_u + U_u x_t \quad (14)$$

$$U_u = \frac{\delta}{1 - \rho\delta} \quad (15)$$

$$\mu_u = \frac{\delta}{1 - \delta} [\mu + .5(1 - \gamma)U_u^2\sigma_x^2 + .5(1 - \gamma)\sigma^2] \quad (16)$$

¹¹Here, "tr" stands for trend, "id" for iid uncertainty, "lr" for uncertainty with long run risk.

¹²When $\Psi = 1$ the Epstein-Zin-Weil(1989) aggregator collapses into a log function.

Let μ^L denote the unconditional mean of the consumption growth rate $E[C_{t+1}/C_t]$; in order to preserve this mean I impose:

$$\mu = \mu^L - \frac{1}{2}(\sigma^2 + \frac{\sigma_x^2}{1 - \rho^2}) \quad (17)$$

Using (11)-(17) I get:

$$\lambda^{id} = .5\gamma\sigma^2 \frac{\delta}{1 - \delta} \quad (18)$$

$$\lambda^{lr} = .5 \left[1 + (\gamma - 1) \frac{\delta^2}{(1 - \rho\delta)^2} \right] \sigma_x^2 \frac{\delta}{1 - \delta} \quad (19)$$

Tallarini(2000) has discussed (18). I want to study (19). From the formulas above we see that the magnitude of the two costs depend crucially on the relative risk aversion γ and the subjective discount factor coefficient δ . It's possible to obtain the following simple expression for the ratio of the two costs:

$$\frac{\lambda^{lr}}{\lambda^{id}} \approx \frac{\frac{\sigma_x^2}{\sigma^2} \delta^2}{(1 - \rho\delta)^2}$$

Here we see two different forces at work: the persistence and the relative magnitude of the long run component. The smaller this component is with respect to the idiosyncratic shock (low $\frac{\sigma_x^2}{\sigma^2}$), the smaller its relative cost is. On the other side, the closer ρ is to unity, the higher the ratio of the costs is.

The choice of parameters for the long run component will be very important for the measure of the welfare costs. I decide to calibrate $\rho = .979$ as in Bansal-Yaron(2004) in order to match the serial correlation of the price-dividend ratio¹³ and will remain unaltered through my analysis. Using the quarterly real consumption of non-durable and services from B.E.A., from 1948:02 to 2005:02, I find that the annualized volatility of the consumption growth rate is about 1.2% while its first auto correlation is about 20%. These two moments are enough to calibrate $\{\sigma, \sigma_x\}$. As in Bansal-Yaron(2004), I assume that the agent decision horizon is monthly and I calibrate my model at a monthly frequency, but I decide to target the quarterly statistics. In order to keep the analysis simple, I prefer to take into account the time aggregation effect by MonteCarlo simulations. So, given a calibration for $\{\sigma, \sigma_x\}$, I simulate the consumption growth rate over a sample

¹³Equation (25) shows why the serial correlation of the price-dividend ratio depends on the persistence of x_t .

of 840 months, I recover the consumption series in levels by normalizing $C(0) = 1$, I aggregate the consumption process over quarters and I compute the quarterly growth rate over a sample of length 280. At this point I compute the sample mean and the sample standard deviation of the quarterly growth rate and I repeat this procedure 500 times. Finally I look at the average of both the two sample statistics. In Table 1 I report my benchmark calibration, the moments I want to match, their values in the data and the results from the simulations of the model. In order to keep the autocorrelation of the quarterly consumption growth low, the volatility of the long run component has to be small (in fact in this calibration $\sigma_x = 2\%\sigma$).

For a given $\{\rho, \sigma, \sigma_x\}$, I am now able to study the role of the two preference parameters $\{\gamma, \delta\}$. In figure 1, I plot the welfare cost of the i.i.d. consumption shock and the total cost of uncertainty with respect to the aversion coefficient and the subjective discount factor. Under the benchmark calibration $\lambda^{lr}/\lambda^{id}$ is about 70% and this explains why $\lambda^{Tot} \approx 2\lambda^{id}$. This is anticipating that to trading off risk aversion with Long Run Risk will not necessarily reduce the total cost of uncertainty. All of the cost functions are increasing in the relative risk aversion parameter and the subjective discount factor, especially when the latter approaches the value of 1. Finally, it's worth noticing that in the range of the parameters plotted in Figure 1, the total cost of uncertainty is always above 5% and this is a quite high number if compared to those obtained by Lucas(1987). In the next section I study the impact that the elasticity of intertemporal substitution can have on the cost functions.

2.2.3 $\Psi \neq 1$

The role of the elasticity of intertemporal substitution is interesting for at least two reasons: first, from an empirical point of view, whether the elasticity is bigger or smaller than one is still a debate; second, standard business cycle models with time-additive C.E.S. preferences usually adopt $\Psi = 1/\gamma < 1$, while Bansal-Yaron(2004), Colacito-Croce(2004), Croce-Lettau-Ludvigson(2005) need $\Psi > 1$ to match key features of the assets market data. When the elasticity of intertemporal substitution is different from one it's impossible to find an exact solution for the utility-consumption ratios, however it's possible to approximate them. Hansen-Heaton-Lee(2005) approximate the utility function around $\Psi = 1$ by a quadratic expansion. Bansal-Yaron(2004), instead, log-linearize the model. In figure 2 I show that the results are similar for both approximation methods and, moreover, the elasticity of intertemporal substitution has a strong positive effect on the welfare

costs. This is a genuine feature of Epstein-Zin-Weil(1989) stressed also in Obstfeld(1994). Focusing on the case in which consumption is a martingale, Obstfeld(1994) is able to get a closed form solution for the costs of the consumption fluctuations and shows that the costs can be expressed as a function of the effective growth and the effective discount rate of the representative agent. While the coefficient of risk aversion enters the effective growth rate with negative sign and adjusts consumption growth according with its volatility, a higher intertemporal elasticity of substitution increases the effective discount factor contributing to rise the welfare costs. A particular interesting case is that one of indifference to the timing of uncertainty that is obtained when $\Psi = 1/\gamma$. Under this condition the Epstein-Zin preferences are equivalent to standard time separable CES preferences with risk aversion coefficient γ . The results just obtained suggest that the CES utility function should produce lower costs since higher risk aversion will be associated with a lower elasticity of intertemporal substitution. Figure 3 shows what happens to the welfare costs when preferences are time separable. The risk aversion coefficient ranges from .5 to 60 and the results are based on the log-linearization of the model.¹⁴

First of all, it's important to notice that for high values of the relative risk aversion the welfare costs are lower than before by two orders of magnitude. The costs are higher than those ones reported in Lucas(1987), but that's due to the fact that in my experiment log consumption is integrated of order one, while in the Lucas computations consumption is trend stationary.

With this preferences the agent doesn't discount anymore future payoff according with the continuation value of this utility and for this reason the costs of uncertainty are lower. It's interesting to notice that the costs of the long run component and that of the idiosyncratic shock are no longer proportional to each other. Moreover, the costs of the Long Run Risk are negative for low values of the relative risk aversion (higher values of the elasticity of intertemporal substitution).

[THE BEHAVIOR OF THE COST OF THE LONG RUN COMPONENT HAS TO BE EXPLAINED BETTER. I HAVE TO ADD HERE THE COMPUTATIONS OF THE WELFARE COSTS FOR THE BASIC CES CASE SIMILAR TO OBSTFELD(1994).]

¹⁴The Hansen-Heaton-Li(2005) quadratic approximation is accurate only for values of the IES very close to 1.

2.3 Asset market implications

2.3.1 The special case $\Psi = 1$

Under the assumption that $\Psi = 1$, it's possible to find exact solutions for the stochastic discount factor, the risk free rate and the price-consumption ratio:

$$m_{t+1}^1 = \mu_m - x_t - \gamma\sigma\epsilon_{t+1}^c + (1 - \gamma)U_u\sigma_x\epsilon_{t+1}^x \quad (20)$$

$$\mu_m \equiv \log(\delta) - \mu - \frac{(1 - \gamma)^2}{2}(\sigma^2 + U_u^2\sigma_x^2) \quad (21)$$

$$r_t^f = -\log(\delta) + \mu - .5\gamma^2\sigma^2 + x_t \quad (22)$$

$$V_t^c = \frac{1}{1 - \delta} \quad (23)$$

$$r_{t+1}^c = -\log(\delta) + \Delta c_{t+1} \quad (24)$$

In order to have predictions for the market price-dividend ratio and the market's excess returns I log-linearize as Bansal-Yaron(2004):¹⁵

$$\begin{aligned} v_t^d &\equiv \log(V_t^d) \approx a_0 + a_1x_t \\ r_{t+1}^d &= \bar{r}^d + x_t + \phi_c\sigma\epsilon_{t+1}^c + \sigma_d\epsilon_{t+1}^d + \kappa_m a_1\sigma_x\epsilon_{t+1}^x \end{aligned}$$

The model implies the following variance and mean for the excess returns:

$$V_t(r_{t+1}^{ex}) = (\phi_c\sigma)^2 + \sigma_d^2 + (\kappa_m a_1\sigma_x)^2 \quad (25)$$

$$E[r_{t+1}^{ex}] = \gamma\phi_c\sigma^2 + (\gamma - 1)U_u\kappa_m a_1\sigma_x^2 - \frac{1}{2}V(r_{t+1}^{ex}) \quad (26)$$

The economic intuition behind this asset pricing model has already been largely explained by Bansal-Yaron(2004), so I will underline just two particular features that affect the welfare costs. Equation (22) shows that the unconditional mean of the risk free rate decreases if the relative risk aversion coefficient increases. So, when it's possible to match the equity premium with a low risk aversion coefficient, a higher subjective discount factor is needed in order to keep the risk free rate level on line with the data. This generates an important tension on the costs of uncertainty since I have already shown that a higher subjective discount factor increases the welfare loss, while a lower risk aversion reduces it.

¹⁵ $\kappa_m \equiv \exp(a_0)/(1 + \exp(a_0))$, $a_1 = (\phi - 1)/(1 - \kappa_m\rho)$ and a_0 is found by numerical integration methods.

The second term in equation(26), instead, shows that it's possible to have low risk aversion and a high equity premium if the volatility of the long run shock σ_x is high enough. Once again, there are two forces simultaneously at work pushing the welfare costs in different directions.

I am now ready to calibrate the risk aversion coefficient and the individual discount factor by simulating the model and making sure it produces statistics as close as possible to those ones observed in the data. In Table 2 and Table 3 I show two different calibrations for two opposite scenarios. In the first scenario, that I denote as LLR, the Long Run Risk is affecting both the consumption and the dividend growth and it's the only component responsible for their contemporaneous correlation (I impose $\phi_c = 0$). In the the second scenario, denoted as IID, both the consumption and the dividend growth rate are perfectly *i.i.d.* over a monthly frequency and their correlation is totally due to the short run risk component $\phi_c \epsilon_{t+1}^c$.

In the economy with with long run risk the total welfare costs are about 2.5 times those ones computed in the IID economy despite of the fact that the risk aversion is calibrated at a lower level. Perhaps, the most surprising thing is that in the IID case the costs of the idiosyncratic shock are lower by about 1% than in the LRR case. That's due to the fact that in economy with Long Run Risk the discount factor of the agent has to be calibrated at an higher level in order to match the risk free rate mean. It's also worth to notice that under the assumption of i.i.d. growth, the price dividend is not fluctuating over time and the volatility of the returns is for this reason three times smaller than in the data. In the economy with LRR, instead, the model is much closer to the data. An other important remark regards the fact that the cost of the long run component reported in Table 2 can be considered like a lower bound with respect to what assumed in the current finance literature about the Long Run Risk. For example, Bansal-Yaron(2004) calibrate $\sigma_x = 4.4\% \sigma$ (in Table 2 $\sigma_x = 2\% \sigma$) and, at the same time, keep the serial correlation of consumption close to what suggested by annual data.¹⁶ In the next section I will discuss the costs implied by their calibration in which the elasticity of intertemporal substitution is assumed bigger than one.

2.3.2 $\Psi \neq 1$

Bansal-Yaron(2005) show that it's possible to get results that match matter the asset prices data by calibrating the intertemporal elasticity of substitution above one. A log-

¹⁶They consider also pre-war data.

linear approximation of the price-dividend implies that:

$$v_t^d \approx a_0 + \frac{1}{1 - \rho\kappa_m} \left(\phi - \frac{1}{\Psi} \right) x_t$$

By No-Arbitrage, the price-dividend is equal to conditional expected value of all future discounted dividends. The long run component enters the expected value of both future dividends (with loading ϕ) and discount factor (with loading $-\frac{1}{\Psi}$). The coefficient $\frac{1}{1 - \rho\kappa_m}$ captures the fact that dividends are discounted over an infinite horizon. A high elasticity of intertemporal substitution let the the price-dividend ratio be steeper with respect to the long run component. In this way the dividend claim is riskier and, holding fix all the other parameters, the equity premium is higher.

On the other side, the unconditional mean of the risk free rate satisfies the following condition:

$$E[r_t^f] = -\log(\delta) + \frac{1}{\Psi}\mu + \frac{1 - \theta}{\theta} E[r_{t+1}^c - r_t^f] - \frac{1}{2\theta} \text{var}_t(m_{t+1}) \quad (27)$$

The second term in (27) shows how a higher intertemporal elasticity of substitution helps to maintain low the risk free rate mean despite of consumption growth.¹⁷

[TO BE COMPLETED.]

3 The production economy

3.1 Capital accumulation

In this section I present the model I use to study the business cycle and evaluate the welfare costs. In order to keep the analysis as simple as possible, I first focus only on the consumption/saving problem of the representative agent and I assume a constant labor supply. In section 3.2 I introduce fluctuations in labor and I study its aggregate co-movements with consumption and investment.

¹⁷As long as $\Psi > 1$ and $\gamma > 1$ the third term in (27) is negative.

3.1.1 The model

The representative has preferences defined only on aggregate consumption:

$$U_t = \left[(1 - \delta)C_t^{1 - \frac{1}{\Psi}} + \delta \left(E_t [U_{t+1}^{1 - \gamma}]^{\frac{1}{1 - \gamma}} \right)^{1 - \frac{1}{\Psi}} \right]^{\frac{1}{1 - \frac{1}{\Psi}}}$$

$$0 \leq C_t$$

The consumption good is produced according to a constant returns to scale neoclassical production function:

$$Y_t = K_t^\alpha [A_t(n_t)]^{1 - \alpha}$$

where K_t is the fixed stock of capital carried into date t , n_t is the labor input at t and A_t is an aggregate productivity shock. The productivity growth rate evolves as described below:

$$\begin{aligned} \Delta a_{t+1} &= \mu + x_t + \sigma \epsilon_{a,t+1} \\ x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\ \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \end{aligned}$$

The resource constraint of this economy is:

$$C_t + I_t \leq Y_t$$

The capital stock evolves according to:

$$K_{t+1} = (1 - \delta_k)K_t + G\left(\frac{I_t}{K_t}\right)K_t$$

where

$$G\left(\frac{I_t}{K_t}\right) = \left[\frac{a_1}{1 - \frac{1}{\tau}} \left(\frac{I_t}{K_t}\right)^{1 - 1/\tau} + a_2 \right]$$

The rate of depreciation of capital is denoted by δ_k and the function $G(\cdot)$ transforms investment in new capital as in Jerman(1998). When $\tau \rightarrow +\infty$ there are no adjustment

costs. When $\tau \rightarrow 0$ the adjustment costs are infinite. When $\tau \rightarrow 0$ and $\delta_k = 0$ the production economy collapses to an exchange economy since the capital stock doesn't move over time and output fluctuations are due only to exogenous productivity shocks.

The agent is endowed with \bar{n} units of time that he can devote to leisure (denoted by l_t) or labor, i.e.

$$n_t + l_t \leq \bar{n};$$

Since his utility is not affected by leisure, the representative agent will always find optimal to offer $n_t = \bar{n}$ units of labor.

3.1.2 Equilibrium

In this economy the allocation that solves the planner's problem can be decentralized by means of competitive markets (Sargent(2004)). It's then possible to find the competitive equilibrium allocation by solving the planner's problem.

Let's define the following stationary variables:

$$\{c_t, i_t, y_t, k_t, u_t\} \equiv \left\{ \frac{C_t}{A_{t-1}}, \frac{I_t}{A_{t-1}}, \frac{Y_t}{A_{t-1}}, \frac{K_t}{A_{t-1}}, \frac{U_t}{A_{t-1}} \right\}$$

Let $s_t \equiv [\Delta a_t, x_t, k_t]$ denote the vector of the states of the economy. Let $u(s)$ be the value of the planner's problem evaluated at the optimum for given state s . The planner's problem can be rewritten in the following recursive way:

$$u(s) = \max_{c, k'} \left[(1 - \delta)c^{1 - \frac{1}{\Psi}} + \delta e^{(1 - \frac{1}{\Psi})\Delta a} \left(E_s [u(s')^{1 - \gamma}]^{\frac{1 - \frac{1}{\Psi}}{1 - \gamma}} \right) \right]^{\frac{1}{1 - \frac{1}{\Psi}}}$$

s.t.

$$\begin{aligned} c &\geq 0, \quad k' \geq 0 \\ c + i &= y \equiv e^{(1 - \alpha)\Delta a} k^\alpha \bar{n}^{1 - \alpha} \\ k' e^{\Delta a} &= (1 - \delta)k + G\left(\frac{i}{k}\right)k \\ x' &= \rho x + \sigma_x \epsilon'_x \\ \Delta a' &= \mu + x + \sigma_a \epsilon'_a \end{aligned}$$

Although this problem is similar to that one solved in Tallarini(2001), there are three basic differences. The state space has one extra dimension (the long run component x). There are adjustment costs in capital. The intertemporal elasticity of substitution is not constrained to be one.¹⁸

Prices and returns are derived from the solution to the planning problem as follows.

The marginal value of standardized capital is equal to the marginal rate of transformation between capital and consumption:

$$q_t = \frac{1}{G'(\frac{i_t}{k_t})}$$

The returns per unit of normalized capital are¹⁹

$$R_{t+1} \equiv \frac{q_{t+1} + d_{t+1}}{q_t}$$

where the dividend are defined as:

$$d_{t+1} \equiv \alpha \frac{y_{t+1}}{k_{t+1}} - \delta_k q_{t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} G'(\frac{i_{t+1}}{k_{t+1}})$$

The first order conditions of the planner imply the following no-arbitrage equation:

$$\mathbf{E}_t \left[\delta \left(e^{\Delta a_t} \frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\Psi}} \left(\frac{u_{t+1}}{\mathbf{E}_t[u_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\Psi}-\gamma} R_{t+1} \right] = 1$$

The stochastic discount factor takes exactly the same form of that one derived in (2). The risk free rate is:

$$R_t^f = \mathbf{E}_t \left[\delta \left(e^{\Delta a_t} \frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\Psi}} \left(\frac{u_{t+1}}{\mathbf{E}_t[u_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\Psi}-\gamma} \right]^{-1}$$

[To be completed]

¹⁸I solve this problem by numerical methods. See Miranda-Fackler(2002), Judd(2004), Barillas-Fernández-Villaverde(2005).

¹⁹The total market returns will be $R_{t+1}^m = R_{t+1} \frac{k_{t+1}}{k_t} e^{\Delta a_t}$

3.1.3 The welfare costs

[To be completed]

3.2 Capital accumulation and endogenous labor supply

3.2.1 The model

[To be completed]

3.2.2 The welfare costs

[To be completed]

4 Conclusions

[To be completed]

APPENDIX: Computational methods

In the paper I use computational methods to solve for the asset prices schedules in the exchange economy and the planner's problem in the production economy. The following sections explains which methods I use.

A.1 Exchange Economy

[To be completed]

A.2 Production Economy

[To be completed]

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TABLE 1
CALIBRATION WITH LONG RUN CONSUMPTION RISK

| | | Moment | Data | Model |
|------------|--------|-----------------------------|-------|-------|
| μ^L | .00164 | $E[C_{t+1}/C_t]$ | 2.00% | 1.99% |
| σ | .00405 | $\text{std}[C_{t+1}/C_t]$ | 1.20% | 1.20% |
| σ_x | .0081% | $\text{ACF}_1[C_{t+1}/C_t]$ | .20 | .23 |
| ρ | .97900 | $\text{ACF}_1[p_t - d_t]$ | .97 | .97 |

All the statistics are based on quarterly data and are annualized. The entries for the model are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency.

TABLE 2
MODEL CALIBRATION: LRR CASE ($\Psi = 1$)

| Parameter | Value | Moment | Data | Model |
|----------------|---------|-------------------------------|----------------|-----------------|
| μ^L | .00164 | $E[C_{t+1}/C_t]$ | 2.00% | 1.99% |
| σ | .00405 | $\text{std}[C_{t+1}/C_t]$ | 1.20% | 1.20% |
| σ_x | .0081% | $\text{ACF}_1[C_{t+1}/C_t]$ | .20 | .23 |
| ρ | .97900 | $\text{ACF}_1[p_t - d_t]$ | .97 | .97 |
| μ_d^L | .0014 | $E[D_{t+1}/D_t]$ | 1.84% | 1.84% |
| ϕ | 14 | $\rho_{\Delta c, \Delta d}$ | .13 | .13 |
| ϕ_c | 0 | - | - | - |
| σ_d | 1.0125% | $\text{std}[D_{t+1}/D_t]$ | 4.23 | 4.23 |
| δ | .99748 | $E[r_t^f]$ | 1.1% | 1.06% |
| γ | 20 | $E[r_{t+1} - r_t^f]$ | 6.19% | 5.95% |
| - | - | $\text{std}[r_{t+1} - r_t^f]$ | 16.4% | 14.81% |
| - | - | $\text{std}[r_t^f]$ | 1.35% | .4% |
| | | λ^{id} | λ^{lr} | λ^{Tot} |
| Welfare costs: | | 6.49% | 4.46% | 10.95% |

All the statistics are based on quarterly data and are annualized. The entries for the model are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency.

TABLE 3
MODEL CALIBRATION: IID CASE ($\Psi = 1$)

| Parameter | Value | Moment | Data | Model |
|----------------|---------|-------------------------------|----------------|-----------------|
| μ^L | .00164 | $E[C_{t+1}/C_t]$ | 2.00% | 1.99% |
| σ | .00405 | $\text{std}[C_{t+1}/C_t]$ | 1.20% | 1.20% |
| σ_x | 0 | $\text{ACF}_1[C_{t+1}/C_t]$ | .20 | .21 |
| ρ | 0 | $\text{ACF}_1[p_t - d_t]$ | .97 | 0 |
| μ_d^L | .0014 | $E[D_{t+1}/D_t]$ | 1.84% | 1.87% |
| ϕ | 0 | - | - | - |
| ϕ_c | .4385 | $\rho_{\Delta c, \Delta d}$ | .13 | .13 |
| σ_d | 1.3992% | $\text{std}[D_{t+1}/D_t]$ | 4.23 | 4.09 |
| δ | .9953 | $E[r_t^f]$ | 1.1% | 1.09% |
| γ | 25.2 | $E[r_{t+1} - r_t^f]$ | 6.19% | 6.13% |
| - | - | $\text{std}[r_{t+1} - r_t^f]$ | 16.4% | 4.9% |
| - | - | $\text{std}[r_t^f]$ | 1.35% | 0 |
| | | λ^{id} | λ^{lr} | λ^{Tot} |
| Welfare costs: | | 4.57% | 0% | 4.57% |

All the statistics are based on quarterly data and are annualized. The entries for the model are based on 500 simulations each with 840 monthly observations that are time-aggregated to a quarterly frequency.

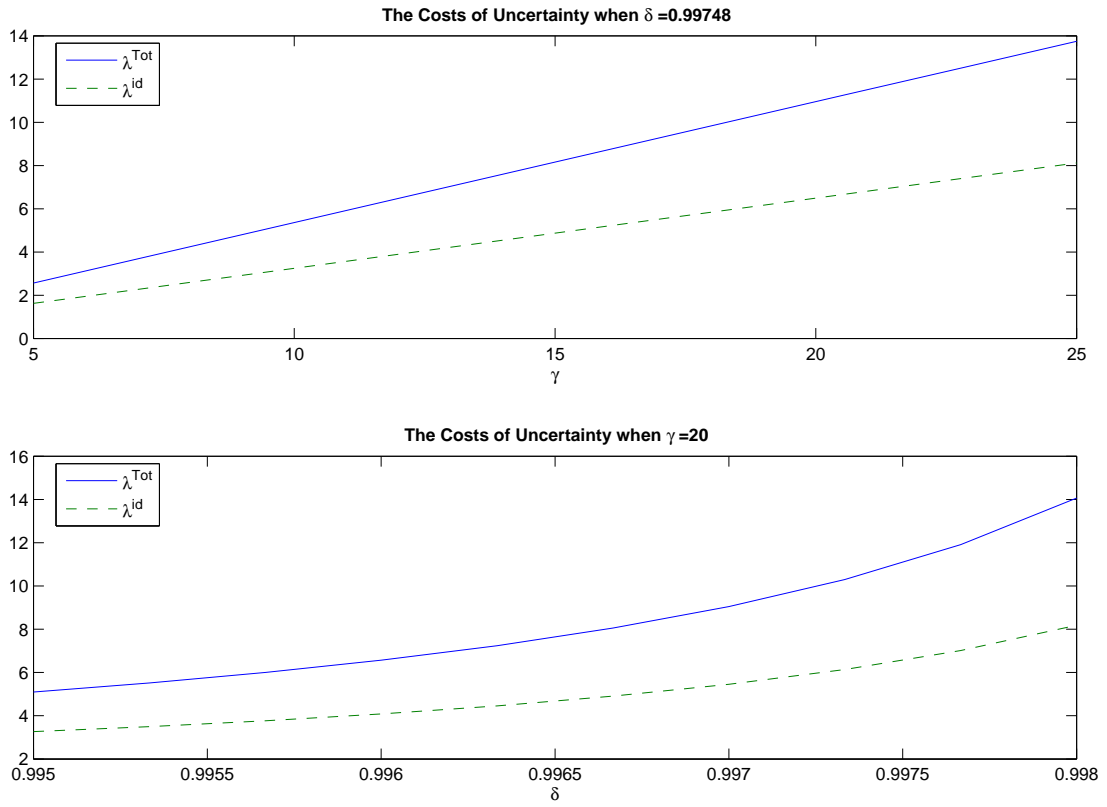


FIG. 1
THE WELFARE COSTS AS FUNCTION OF (γ, δ)

The parameters $(\mu^L, \rho, \sigma, \sigma_x)$ are calibrated to the values reported in Table 1.

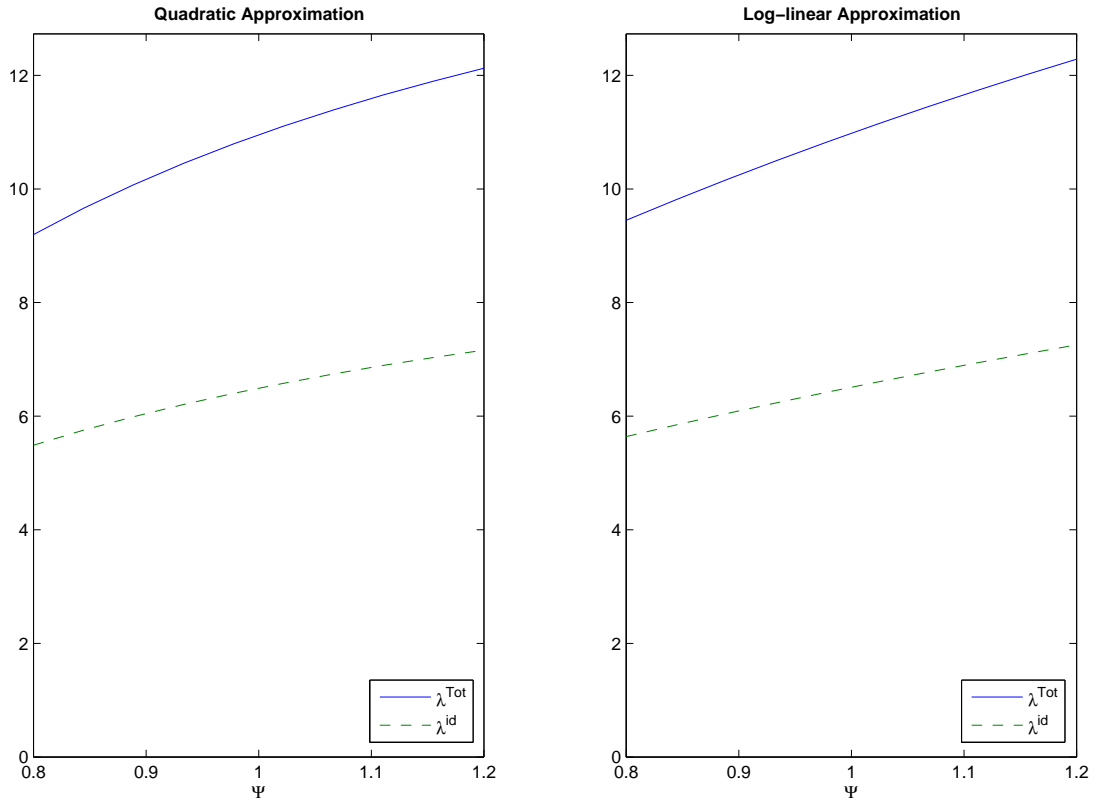


FIG. 2
THE WELFARE COSTS AS FUNCTION OF THE EIS

The parameters $(\mu^L, \rho, \sigma, \sigma_x)$ are calibrated to the values reported in Table 1. In this figure, $\gamma = 20$ and $\delta = .99748$.

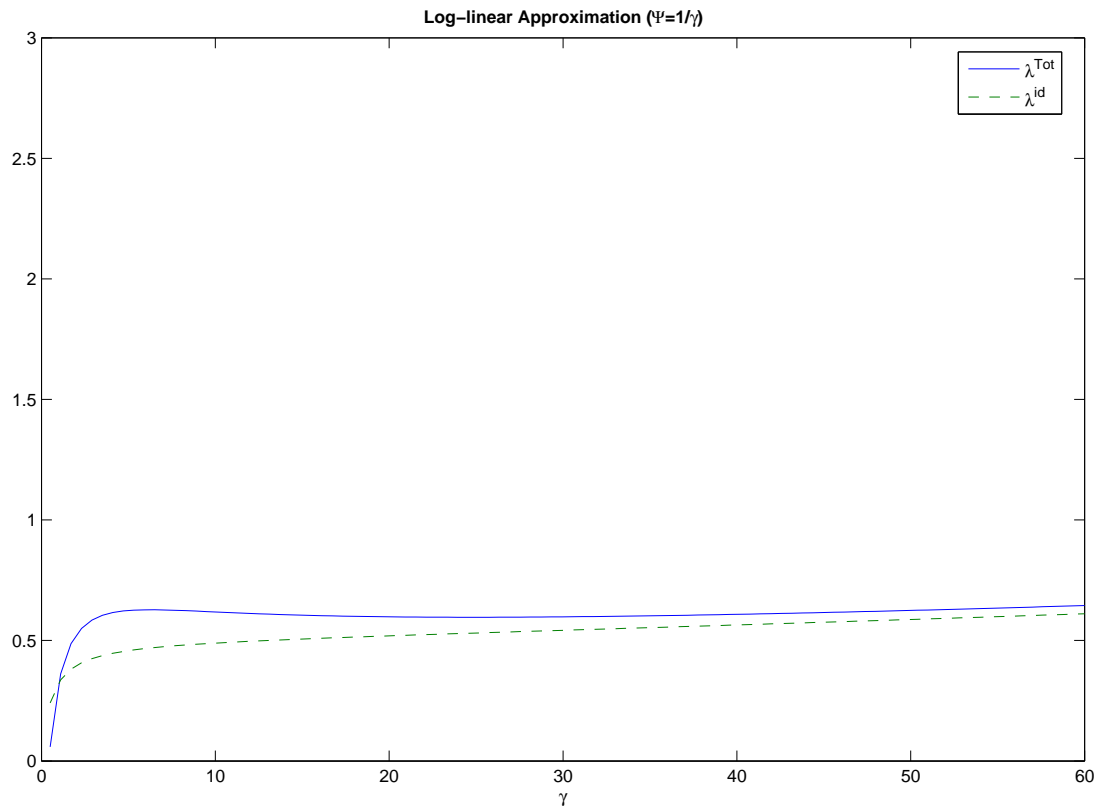


FIG. 3
THE WELFARE COSTS IN THE CES CASE

The parameters $(\mu^L, \rho, \sigma, \sigma_x)$ are calibrated to the values reported in Table 1. In this figure, $\delta = .99748$.