Technology Adoption Under Uncertainty in General Equilibrium*

Julien Hugonnier† Erwan Morellec‡ Aude Pommeret§

February 2006
PRELIMINARY AND INCOMPLETE
PLEASE TO NOT QUOTE WITHOUT THE AUTHORS' PERMISSION

Abstract

Investment is often irreversible, especially at the aggregate level. This paper proposes and solves a general equilibrium model of technology adotpion when investment in the new technlogy is irreversible. In contrast to prior research, we consider a setup where the returns on technology adoption are uncertain. We find that even without learning by doing it may be optimal for the representative agent to wait before acquiring the technlogy. We relate the timing of investment to the risk aversion of the representative agent and demonstrate that the value of waiting to invest quickly disappears with the introduction of risk aversion in an equilibrium framework.

Keywords: Time varying MRS; Growth Options; General Equilibrium.

JEL Classification Numbers: G10; G11; G12; G31.

---

*We thank ... We remain responsible for any errors or deficiencies. Julien Hugonnier and Erwan Morellec acknowledge financial support from NCCR FinRisk.

†University of Lausanne and FAME. Postal address: Institute of Banking and Finance, Ecole des HEC, University of Lausanne, Route de Chavannes 33, 1007 Lausanne, Switzerland. E-mail address: julien.hugonnier@unil.ch.

‡University of Lausanne, FAME, and CEPR. Postal address: Institute of Banking and Finance, Ecole des HEC, University of Lausanne, Route de Chavannes 33, 1007 Lausanne, Switzerland. E-mail address: erwan.morellec@unil.ch.

§University of Lausanne. Postal address: Departent of Economics, Ecole des HEC, University of Lausanne, Route de Chavannes 33, 1007 Lausanne, Switzerland. E-mail address: aude.pommeret@unil.ch.

1 Introduction

Technology adoption has become an important research topics in economics in the recent years. The considerable weight of adoption cost of technology (twenty times those for innovation according to Jovanovic, (1997)) has directed interest of growth theorists in general, towards the study of problems of diffusion and adoption of new technologies. The recent boom in information technologies and the associated employment and growth opportunities has stimulated a lot of research in this field. The objective was to treat the two following questions: given the state of the economy, is it optimal to switch to a new technology? And if the decision is to adopt, what is the optimal timing of adoption? Indeed, a too quick adoption would be a disaster on the one hand because the return to adoption is uncertain, and on the other hand because of the lack of complementarity between the new technology and the actual structure of the economy (for instance the composition of skills in the economy, see Galor and Tsiddon (1997)). The issue we want to adress in this paper concerns the possibilty to delay technology adoption. In particular, we focus on the conditions under which the representative agent switches with some delay to the new technology and on the determinants of the size of this delay?

In a deterministic environment with no learning-by-doing, only the absence of adoption or an immediate adoption can be optimal. Parente (1994) and (2000) develop models in which the firms accumulate expertise in their technology which allows them to operate the technology more efficienty. Technological upgrading implies the depreciation of the pre-existing specific human capital and the need to learn the new technology. Thus there is learning-by-doing and firms can choose the timing of adoption. Nevertheless, there is no possibility to stick to a given technology, upgrading always occurs which makes Parente's approach unadapted to handle technological sclerosis cases. Jovanovic and Nyarko (1996) considers another process for learning, which becomes Bayesian in nature since output loss occurs as long as agents do not estimate correctly some productivity parameters. In their setting, it is possible to stick to one particular technology but the structure seems too complicated to be considered in gemeral equilibrium. Boucekkine *et alii* (2004) just allow for one switch (see also Saglam (2002) for a finite number of switches) and assume that technological progress is embodied in capital goods. They study how learning affects adoption delays.

However, the decision to adopt a new technology is highly sensitive to uncertainty in the environment. When considering the opportunity to adopt a new technology at the macroeconmic level, we consider long term benefits, whose size are not known at all with certainty. It is therefore legitimate to consider the technology adoption in a stochastic framework. Industrial economics, first has considered technology adoption in a static setting when there is uncertainty about whether it will succeed or fail. The focus is mainly on the effect on market competition. Moreover, Jensen (1992) stresses welfare effects of adoption and points out that the incentives of firms to adopt a new process need not coincide with social welfare. Second, finance and macroeconomics may also help determining the formulation of optimal technology adoption under uncertainty.

Indeed, technology adoption shares the irreversible characteristics of any investment while it is now largely acknowledged that uncertainty joined with irreversibility has a dramatic impact on the investment decision. A large literature shows that a value to wait for new information before making the investment is embedded in the investment opportunity. In such a framework, optimal adoption has extensively been studied in partial equilibrium in which technological adoption is treated as the decision to exercise a real option. Abel and Eberly (2002) and (2004) study the optimal adoption of the stochastic latest technology. By constrast, in Roche (2003), it may be optimal for an upgrading firm to keep some distance with the frontier technology. Grenadier and Weiss (1997) also focuses on investment opportunities in stochastic technological innovations. In a sequential investment framework, adopting an innovation provides the firm with an option value to learn. Pavlova (2001) introduces the leaning-by-doing of Parente (1994) into the firm's choice of under uncertainty. The author shows that uncertainty does not matter for adoption if the only cost of technology comes from learning. Finally, Alvarez and Stenbacka (2001) studies optimal timing of when to adopt an incumbent technology, incorporating as an embedded option a technologically uncertain prospect of opportunities for updating the technology in response to the emergence of future superior versions of the technology. It develops a new mathematical approach for finding the optimal exercise thresholds both of the ordinary real option associated with the updating decision and of the compound real option associated with the incumbent technology.

All this literature rests on the fact that uncertainty joined with a sunk adoption cost generates a significant value to wait for new information. As a result, investment projects should only be undertaken when the value of the new technology exceeds the adoption cost by a potentially large premium. It generates an optimal delay for technology adoption, and allows for an optimal timing. This delay depends on technological characteristics of the firm and on characteristics of the environment (for instance the size of uncertainty).

Since policies are formulated in partial equilibrium settings in this literature, they ignore the interaction between optimal consumption and optimal technology adoption. Theoretical analyses which mix general equilibrium with real options are a lot more recent and very limited in number. Hugonnier *et al.* (2006) studies the optimal investment timing in a general equilibrium with a representative agent similar to that of Cox *et alii* (1985). Investment is irreversible and requires the use of a fixed amount of existing capital and it generates an expansion in the scale of the capital stock by a constant factor. They show that feedback effects of lumpy investments on optimal consumption decisions can severely erode the option value of waiting even for moderate levels of risk aversion. Note moreover, that Wang (2001) studies how technologies change endogenously in an equilibrium and the effect of technological changes on the equilibrium. Nevertheless, it does not explicitely derive the value-maximizing policies.

In this paper, we develop a stochastic general equilibrium model to study the optimal adoption of a new technology. The principal characteristics are the following.

- The representative agent optimally chooses her consumption path and the timing of technology adoption in order to maximize her expected lifetime utility.

- Technological progress is embodied, meaning only investments undertaken after the adoption will benefit from the new technology. Indeed, there is evidence that technological progress is largely investment specific. For instance, we observe that the probability that a peak of investment occurs is increasing with time (see among others Caballero, Engle and Haltiwanger, (1995), Cooper, Haltiwanger and Power (1999)). Moreover, as time passes, the relative price of capital goods is declining and the ratio equipment-GDP is raising. Therefore, investment decisions and technological progress seem to be interrelated. It is then relevant to consider the adoption of an *embodied* technology (see for instance Cooley *et alii* (1997), Boucekkine, Germain and Licandro (1997), Boucekkine and Pommeret (2004)).

- We allow for learning-by-doing, that is, there is an efficiency loss at the time of the switch. Once the technology has been adopted, the representative agent learns as time passes and will eventually asymptotically eliminate the expertise gap.

We solve the model backward and we show that having the opportunity to switch to a new technology generates an option value which is highly sensitive to the value of the risk aversion. In particular, for high values of the risk aversion, the optimal timing of adoption become close to that which would prevail in the absence of uncertainty. We also study the determinants of the threshold which triggers adoption. We show that there exists a hump-shaped relationship between risk aversion and this threshold. moreover, the larger the uncertainty the the later the adoption. Finally, the higher the learning speed, the sooner the technology adoption.

2 The model

We consider an infinite horizon production economy in continuous time. Uncertainty is represented by a probability space $(\Omega, \mathbb{F}, \mathcal{F}, P)$ on which is defined a standard Brownian motion B. The filtration $\mathbb{F}$ is the usual augmentation of the filtration generated by the Brownian motion and we let $\mathcal{F} := \cup_{t\geq 0}\mathcal{F}_t$ so that the true state of nature is solely determined by the path of the Brownian motion. All processes are adapted to the filtration $\mathbb{F}$ and all statements involving random quantities hold either almost surely or almost everywhere depending on the context.

Production in the consumption sector is generated from capital by the linear production technology:

$$dY_t = A_t K_t dt + \sigma_t K_t dB_t. \tag{1}$$

Equation (1) shows that the accumulated flow of of output over a time interval of length dt consists of two components. First, the flow of output reflects a deterministic component $A_t K_t$ that represents the mean rate of output per unit of time. Second, it depends on stochastic component that can be interpreted as a productivity shock. Throughout the analysis, this shock is governed by a temporally independent, Normally distributed, stochastic process with zero mean and variance $\sigma^2 dt$. The overall size of the stochastic disturbance in output varies with the existing capital stock and, hence, with the size of current output. The factor A_t depends on technology changes and will be described later.

We consider that the consumption good can be either consumed or used as an input in the production of the capital goods. The production function in the capital goods sector is then [see Greenwood, Hercowitz and Krusell (1997)]:

$$dK_t = q\left(A_t K_t - c\right)dt + \sigma_t K_t dB_t \tag{2}$$

where c denotes the (endogenous) consumption rate. In this equation, q represents the productivity in the capital goods sector. Technological progress is investment specific and, hence, an increase in q will rise the productivity of new capital without affecting the producivity of the whole stock of capital. Units of capital of different vintages can then be aggregated using the appropriate weights on past investment.

At any time τ, a new production technology can be adopted. Investment is irreversible and requires the use of capital I. The new production production technology has higher productivity $q_N > q_I$ and can have an impact on both the level of uncertainty $\sigma_N \neq \sigma_I$ and the efficiency of the consumption good sector A_t. In our setup, a reduction in the efficiency parameter A_t may reflect a lower expertise in the use of the new capital good. Indeed, following Parente (1994) and Greenwood and Jovanovic (2001), we allow for learning effects, that is, the deterministic part of the productivity varies with time:

$$A_t = \begin{cases} A_I & \text{for } 0 \leq t < \tau \\ A_N - A^* e^{-\theta(t-\tau)} & \text{for } \tau < t \end{cases} \tag{3}$$

In this specification, $A_N - A^*$ gives the efficiency loss at the time of the switch (it could be an increasing function of the technology differential $\frac{q_N}{q_I}$. Once the technology has been adopted, the representative agent learns as time passes and will eventually asymptotically eliminate the expertise gap. The learning speed is measured by θ.

The agent is endowed with an initial capital stock $K_0 > 0$ and has *exclusive access* to the risky production technology and the ensuing option to adopt the new technology. His preferences over consumption plans are represented by the lifetime expected utility functional

$$E\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right] = E\left[\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt\right], \tag{4}$$

where $1 \neq R > 0$ is the agent's constant relative risk aversion and $\rho \geq 0$ is his subjective rate of time preference. While most of our results hold for general utility functions satisfying the Inada conditions, we focus on this simple specification of the model because it allows us to obtain explicit results in some cases of interest.

The optimal switching time should maximize the intertemporal utility of the representative agent subject to the non negativity constraint and the law of motion of capital:

$$dK_t = q_i \left(A_t K_t - C\right) dt + \sigma_i K_t dB_t, \tag{5}$$

where A_t is defined in equation (3) and where $i = N$ (resp. I) for $\tau < t$ (resp. $0 \leq t < \tau$). The central planner's problem consists in maximizing the lifetime utility subject to the non negativity constraint and the above dynamic budget constraint. In order to facilitate the presentation of our result, let Θ denote the set of admissible plans, that is the set of consumption and investment plans (c, τ) such that

$$E\left[\int_0^\infty e^{-\rho s} |U(c_s)| ds\right] < \infty$$

and the corresponding solution to (5) is non negative throughout the infinite horizon. With this notation, the value function of the central planner is

$$V(K_0) := \sup_{(c,\tau)\in\Theta} E\left[\int_0^\infty e^{-\rho t} U(c_t) dt\right].$$

This program can be solved in two stages. We first solve for the optimal consumption behavior of the representative agent assuming that the new technology has been adopted. This will provide us with a boundary condition at the time of the technology adoption. Using this condition we then determine the optimal time for adopting the new technology.

3 Equilibrium after technology adoption

In this section, we present the analysis assuming that the new technology has already been adopted. Therefore, it provides the boundary condition for computing the equilibrium of our general model with technology adoption. Note that it also provides a benchmark for the economy with the technology adoption option. The economy is then rather close to Cox, Ingersoll and Ross (1985) economy. Note however, that first, there exist learning effects in the model. Due this learning effect, central planner program has to take into account the a law of motion of the deterministic part of the productivity:

$$dA = \theta(A_N - A_t)dt \tag{6}$$

Second, we assume from now on that there is no discount factor affecting the intertemporal utility. Introducing a discount factor woud prevent from analytically solve the model without changing the nature of the results (as could be shown using simulations[1]).

The central planner's program consists in maximizing the lifetime utility (4) subject to the non negativity constraint, the law of motion of the determinist part of productivity (6) and the law of motion of capital:

$$dK = q_F\left(A_t K - C\right)dt + \sigma_F K dz_t \tag{7}$$

Since we assume that the new technology has already been adopted, the set of admissible plans Θ collapses to the set of consumptions plans such that

$$E\left[\int_0^\infty |U(c_s)|\, ds\right] < \infty$$

and the corresponding solution to (7) is non negative throughout the infinite horizon. With this notation, the value function of the central planner is

$$W(K_0, A_N - A^*) := \sup_{c\in\Theta} E\left[\int_0^\infty U(c_t)dt\right] \tag{8}$$

The value function is finite if and only if the marginal propensity to consume is strictly positive. Using Itô's lemma, the Bellman equation the value function $W(K_t, A_t)$ has to satisfy is quite standard ; note however that it takes into account the law of motion of the deterministic part of productivity:

$$0 = \max_{C_t}\left\{\frac{C_t^{1-\gamma}}{1-\gamma} + q_N W_K(A_t K_t - C_t) + \frac{1}{2}\sigma_F^2 K_t^2 W_{KK} + \theta(A_N - A_t)W_A\right\}$$

[1] See also Hugonnier *et alii* (2005).

Under the above assumptions for preferences and technology, a unique equilibrium exists in this economy with no technological change if and only if the marginal propensity to consume is strictly positive, with

$$C_t = (W_K q_N)^{-1/\gamma} = g(A_t) q_N^{-1/\gamma} K_t \tag{9}$$

This provides some parametric restrictions once the function $g(A_t)$ is identified. This is usually achieved by solving the Bellman equation but in this problem with a law of motion for the deterministic part of productivity, it also requires the use of the following condition:

$$\begin{aligned}
\lim_{A_t \to A_N} g(A_t) &\to \ell \\
\Leftrightarrow \quad \frac{1-\gamma}{\gamma}\left(\frac{\gamma\sigma_F^2}{2} - q_N A_N\right) > 0 &\Rightarrow \begin{cases} \frac{\gamma\sigma_F^2}{2} > q_N A_N & \text{if } \gamma < 1 \\ \frac{\gamma\sigma_F^2}{2} < q_N A_N & \text{if } \gamma > 1 \end{cases}
\end{aligned}$$

As a result, we obtain

$$g(A_t) = \left\{ \int_0^\infty q_N^{\frac{\gamma-1}{\gamma}} \exp^{-\frac{1-\gamma}{\gamma}\left[\left(\frac{\gamma\sigma_F^2}{2} - q_N A_N\right)\alpha + \frac{q_N}{\theta}(A_N - A_t)(1-e^{-\theta\alpha})\right]} d\alpha \right\}^{-1}$$

The lifetime utility of the agent can also be computed explicitely. Specifically, plugging the optimal consumption policy above in equation (8) and computing the conditional expectation we obtain:

$$W(K_t, A_N - A^*) = \frac{K_t^{1-\gamma}}{1-\gamma}\left[g(A_N - A^*)\right]^{-\gamma} \tag{10}$$

4 Equilibrium in the absence of new technology

In this section, we deduce from the previous section the optimal path of consumption in the the absence of an opportunity to switch to a new technology. Indeed, it corresponds to the case in which both embodied and disembodied technologies are at their initial levels and of course there exists no learning-by doing. It provides a boundary condition for computing the equilibrium of our general model with technology adoption since the agent has to be always better off when having the opportunity to switch.

Taking the program of the previous section and replacing A_N by A_I,and q_N by q_I in the absence of learning-by-doing ($\theta \to \infty$) provides the benchmark of an economy with no technological change. The representative consumer's optimal consumption policy collapses to the very standard one:

$$\begin{aligned} C_t &= \Lambda K_t \\ \Lambda &= \frac{1-\gamma}{\gamma} q_I^{-1} \left(\frac{\gamma \sigma_I^2}{2} - q_I A_I \right) \end{aligned} \tag{11}$$

Under the above assumptions for preferences and technology, a unique equilibrium exists in this economy with no technological change if and only if the marginal propensity to consume is strictly positive, which is achieved if:

$$\frac{1-\gamma}{\gamma} \left(\frac{\gamma \sigma_I^2}{2} - q_I A_I \right) > 0 \Rightarrow \begin{cases} \frac{\gamma \sigma_I^2}{2} > q_I A_I & \text{if} \quad \gamma < 1 \\ \frac{\gamma \sigma_I^2}{2} < q_I A_I & \text{if} \quad \gamma > 1 \end{cases}$$

Finally the lifetime utility of the agent in the economy with no switching opportunity is the following

$$W_0(K_t) = \left[\frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma \sigma_I^2}{2} - q_I A_I \right) \right]^{-\gamma} \frac{K_t^{1-\gamma}}{1-\gamma} \tag{12}$$

Recall that it cannot be greater than the lifetime utility of the agent in an economy with a switching opportunity. We shall consider such a condition in the next section.

5 Equilibrium before the optimal switching time

The central planner's problem, that is the choice of an optimal consumption plan and of an optimal technology adoption timing, is given by the maximization of the intertemporal utility function (4) subject to the law of capital accumulation (5).

5.1 Formulation

Once the new technology has been adopted, the central planner optimally follows the consumption plan described by equation (9). Therefore, the value function at the time of the switch has to satisfy the following value matching and smooth pasting conditions:

$$V(K_\tau, A_I) = W(K_\tau - \beta, A_F - A^*) \tag{13}$$

$$V_K(K_\tau, A_I) = W_K(K_\tau - \beta, A_F - A^*) \tag{14}$$

where K_τ is the level of the capital stock for which it is optimal to switch which implicitely determines the optimal switching time τ. The two other conditions the value function has to satisfy are the following:

$$W_0(K_t) \leq V(K_t) \qquad \forall t \tag{15}$$

$$W_0(K_t) \leq W(K_t, A_F - A^*)\ \forall t \Leftrightarrow \begin{cases} g(A_N - A^*) \leq \frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma\sigma_I^2}{2} - q_I A_I\right) & \text{if} \quad \gamma < 1 \\ g(A_N - A^*) \geq \frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma\sigma_I^2}{2} - q_I A_I\right) & \text{if} \quad \gamma > 1 \end{cases} \tag{16}$$

The first condition (equation (15)) has to be satisfied since it is always possible for the representative agent to indefinitely postpone the adoption of the new technology. The second condition (equation (16)) ensures that there exists an optimal switching date, that is, in the absence of cost in terms of capital to switch to the new technology, the agent would choose to immediately switch for any current level of capital accumulation.

An application of the dynamic programming principle allows focusing on the optimal technology adoption time and on the optimal consumption plan prior to the adoption. The central planner's program becomes then:

$$\begin{aligned} V(K_0 \qquad) &= \sup_{(c,\tau)\in\Theta} E\left[W(K_\tau - \beta, A_F - A^*) 1_{\{\tau<\infty\}} + \int_0^\tau \frac{C(t)^{1-\gamma}}{1-\gamma} dt\right] \qquad (17) \\ &\text{subject to} \\ &dK = q_I \left(A_I K - C\right) dt + \sigma_I K dz_t \quad \text{for } 0 \leq t < \tau \end{aligned}$$

Solving this program, we first determine the expression of the optimal consumption path and of the marginal value of capital before the switch which has to satisfy conditions (15), (16) and (14). Using the boundary condition (13)) it is then possible to derive the optimal timing for the switch.

5.2 Marginal value of capital

We consider the economy at times when it is not already optimal to adopt the new technology. This period is usually refered to as the continuation period. We solve the program before the switch in order to obtain the optimal consumption path and the value of capital which is needed to derive the optimal timing to switch to the new technology. The program before the switch is simply:

$$\begin{aligned} V(K_t)|_{t<\tau} &= \sup_{c\in\Theta} E\left[\int_0^\infty U(c_s)ds\right] \\ &\quad \text{subject to} \\ &\quad dK = q_I\left(A_I K - C\right)dt + \sigma_I K dz_t \quad \text{for } 0 \le t < \tau \end{aligned}$$

Using Itô's lemma, the Bellman equation the value function before technology adoption $V(K_t)|_{t<\tau}$has to satisfy is:

$$0 = \max_{C_t}\left\{\frac{C_t^{1-\gamma}}{1-\gamma} + q_I V_K(A_I K_t - C_t) + \frac{1}{2}\sigma_I^2 K_t^2 V_{KK}\right\}$$

The existence of an option to switch to the new technology prevents from directly solving for the value of the program; instead we solve for the marginal value V_K using the smooth pasting condition (14):

$$V_K = \left[(W_{0K}(K_t))^{1/\gamma} + \underbrace{\left[(W_K(K_\tau - \beta))^{1/\gamma} - (W_{0K}(K_\tau))^{1/\gamma}\right]\left(\frac{K_\tau}{K_t}\right)^{2q_I A_I/\left(\gamma\sigma_I^2\right)}}_{D=\text{part due to the option to switch}}\right]^\gamma \tag{18}$$

Moreover, the optimal consumption path is given by:

$$C_t = (V_K q_I)^{-1/\gamma}$$

Note first that the marginal propensity to consume is strictly postive which means that there exists a unique equilibrium in the economy. Second, the marginal value of capital differs significantly from the one which can be derived in the absence of technological change from equation (13) ; this is due existence of an option to switch which generates an option value taken into account in the marginal value of capital. In the absence of such an option, $D = 0$ and the marginal value of capital reduces to W_{0K}.

- if $\gamma < 1$, condition (16) ensures that $D \geq 0$. There is no prblem of existence of V_K in this case. The marginal value of capital in the economy with the opportunity to adopt a new technology is greater or equal to the marginal value of capital in an economy without this opportunity. It means that consumption at each time is smaller or equal in an economy with an opportunity to switch to a new technology compared to the consumption which prevails in an economy in which the opportunity does not exists. Indeed, if the intertemporal elasticity of substitution of the representative agent is greater than one, meaning she likes to substitute in time, she is ready to reduce her current consumption in order to foster technology adoption.

- on the contrary, if $\gamma > 1$, the sign of D is a priori ambiguous.
  - if $D < 0$ the part due to the option in the expression of the marginal value of capital is negative. Therefore, consumption at each time would be greater or equal in an economy with an opportunity to adopt a new technology compared to the consumption which prevails in an economy in without such an opportunity. In this case, the agent would not like to substitute in time and the option to adopt would be an incentive to increase her consumption today to smooth the path which is expected to grow more once the technology is adopted. This would of course delay adoption. But such a consumption can never happen since for $K_t < K_{\inf}$, ie. for small values of K_t which have to be considered, we obtain that $V_K^{1/\gamma}$becomes negative and the program is no longer defined. Note that the expression of $K_{\inf}$ is the following:

$$K_{\inf} = \left[K_\tau^{2q_I A_I / \left(\gamma \sigma_I^2\right)} \left(\frac{1}{K_\tau} - \frac{\left(\frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma \sigma_I^2}{2} - q_I A_I \right) \right)}{\left[g(A_N - A^*) \right] (K_\tau - \beta)} \right) \right]^{\frac{1}{2q_I A_I / \left(\gamma \sigma_I^2\right) - 1}}$$

  - if $D > 0$, condition (15) no longer holds when integrating V_K (since $\gamma > 1$ implies that $2q_I A_I / \sigma_I^2 > 1$) ; therefore, $D > 0$ cannot be considered.
  - if $D = 0$, both $(V_K)^{1/\gamma}$ is positive which ensures that the program is defined and condition (15) is satisfied. It is therefore the only solution we can consider if $\gamma > 1$. Note that it implies that consumption remains unaffected by the existence of the option to switch to the new technology. It does not mean that the option to switch has no value but that its value is independent from the level of wealth.

6 Optimal adoption of the new technology

The stock of capital K_τ is such that, at the time of the switch, the value with the initial technology ($V(K_\tau, A_I)$) is equal to the value with the new technology once the cost β is paid ($W(K_\tau - \beta, A_F - A^*)$). Therefore, K_τ has to satisfy:

$$V(K_\tau, A_I) = W(K_\tau - \beta, A_F - A^*)$$

We need to consider two cases, depending on value of relative risk aversion with respect to unity.

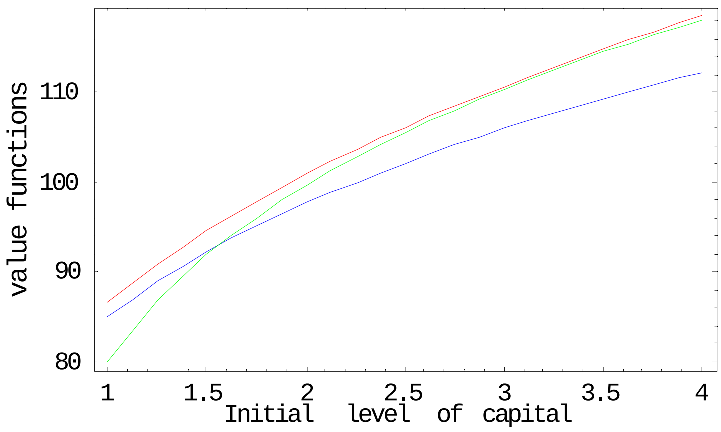
- If $\gamma < 1$, K_τ has to satisfy:

$$\int_0^{K_\tau} V_K(K, A_I)dK = W(K^* - \beta, A_F - A^*) \qquad \text{since } V(0, A_I) = 0 \text{ for } \gamma < 1 \tag{19}$$

Recall that V_K is itself a function of K_τ (see equation (18)). Moreover, $W(K^* - \beta, A_F - A^*)$ contains an integral which comes from the existence of learning-by-doing. Equation (19) has to be solved numerically.

Simulations are driven using the following values for the parameters : $A_N = 0.5$; $q_N = 0.2$; $\gamma = 0.8$; $\sigma_I = \sigma_F = 0.8$; $\theta = 0.6$; $A^* = 0.1$; $A_I = 0.5$; $q_I = 0.1$; $\beta = 0.5$.

Figure 1 : Technology adoption and uncertainty before and after adoption



In this example, the switch to the new technology happens when the stock of capital reaches $K = 3.22$.

- If $\gamma > 1$, we have shown previously that $D = 0$. It has a direct implication on the optimal level of capital which triggers technology adoption. Using (12) and (10) we obtain:

$$D = 0 \Leftrightarrow K_\tau = \frac{\beta}{1 - \left[\frac{\left(\frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma\sigma_I^2}{2} - q_I A_I\right)\right)}{[g(A_N - A^*)]}\right]}$$

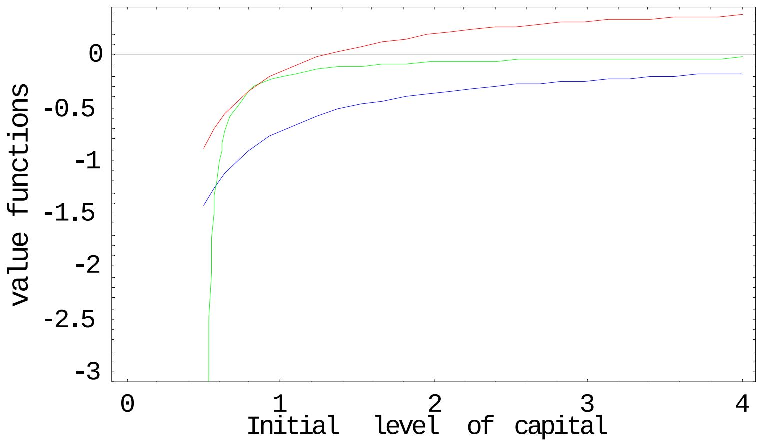
and the boundary condition allows deriving the value of the program before the switch:

$$V(K_t) = \left[\frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma\sigma_I^2}{2} - q_I A_I\right)\right]^{-\gamma} \frac{1}{1-\gamma}\left(K_t^{1-\gamma} - K_\tau^{1-\gamma}\right) + \frac{(K_\tau - \beta)^{1-\gamma}}{1-\gamma}\left[g(A_N - A^*)\right]^{-\gamma}$$

Note that in this case, both K_τ and $V(K_t)$ are defined if $\gamma > 1$.

Simulations are driven using the following values for the parameters : $A_N = 3$; $q_N = 5$; $\gamma = 2$; $\sigma_I = \sigma_F = 0.8$; $\theta = 0.6$; $A^* = 0.1$; $A_I = 3$; $q_I = 1$; $\beta = 0.5$.

Figure 2 : Technology adoption and risk-aversion

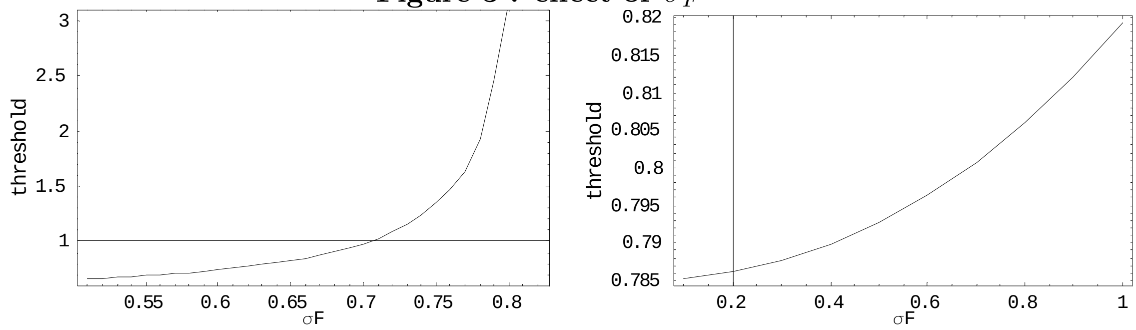


In this example, the switch to the new technology happens when the stock of capital reaches $K = 0.81$.

6.1 The sensitivity of the optimal threshold with respect to parameters

- effect of uncertainty after the switch

Figure 3 : effect of σ_F

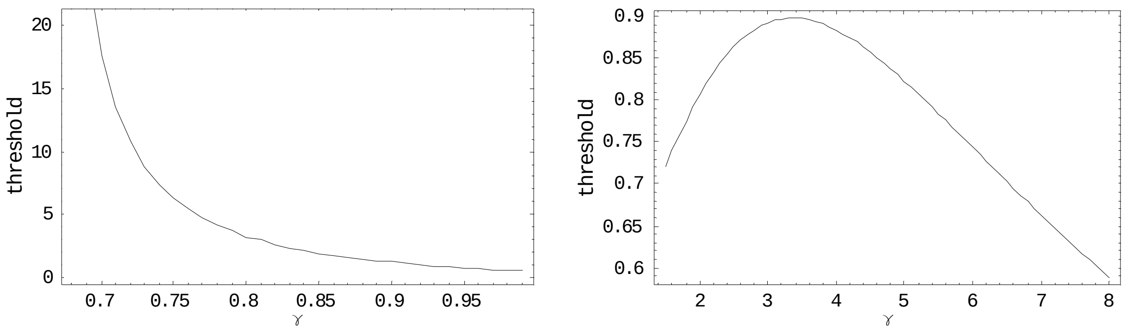


$\gamma < 1$ $\gamma > 1$

Unambiguously, an increase of uncertainty after the switch increases the level of caiptal which triggers the switch : adoption is delaied.

- effect of the relative risk aversion

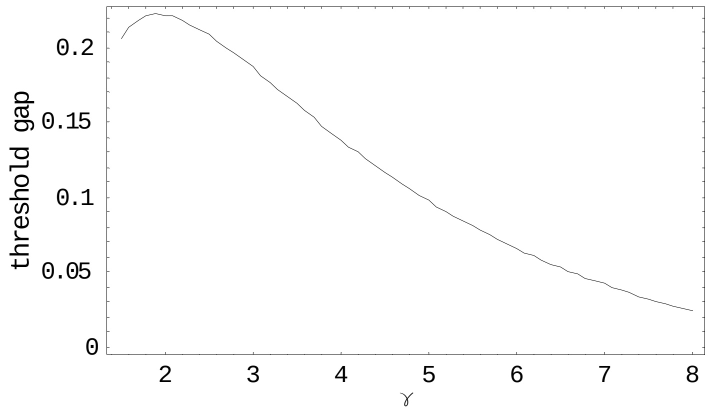
Figure 4 : effect of γ



$\gamma < 1$ $\gamma > 1$

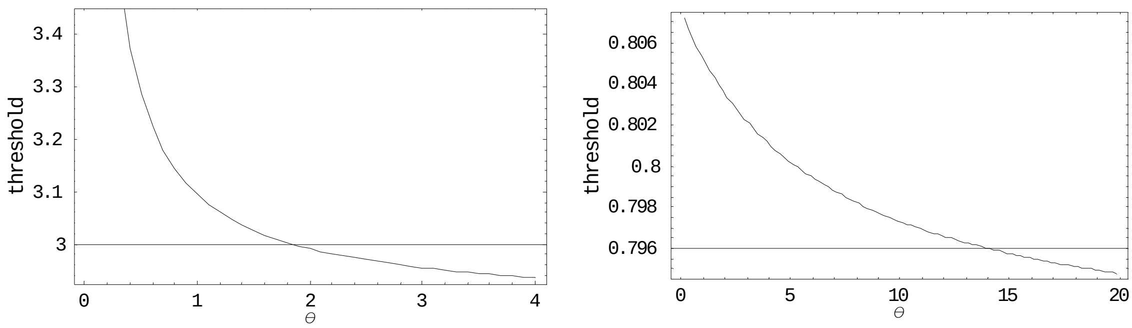
An increase in the relative risk aversion parameter reduces the delay before adoption for $\gamma < 1$. In the case $\gamma > 1$, it only happens for big enough values of γ. Note moreover, that as γ increases, the gap between the threshold computed by taking into account of the option value to switch and the threshold for which the value with no opportunity to switch equals the value after the switch strongly decreases:

Figure 5 : γ and the threshold gap



- Effect of learning

Figure 6 : effect of θ



$\gamma < 1$ $\qquad\qquad$ $\gamma > 1$

A suggested by intuition, an increase in the learning speed speeds up adoption.

7 Optimal consumption path

Figure7: consumption and capital dynamics with (C and K)

and without ($C0$ and $K0$) opportunity to switch

$\gamma < 1$

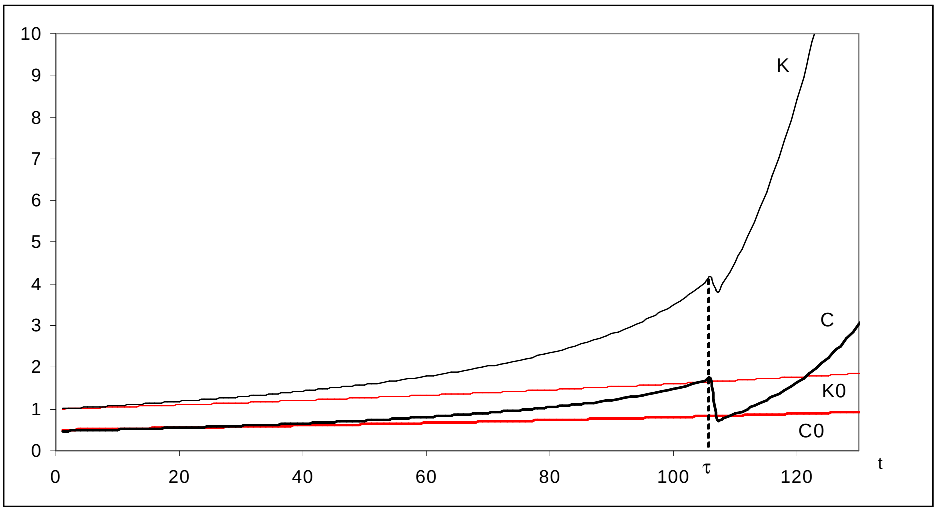
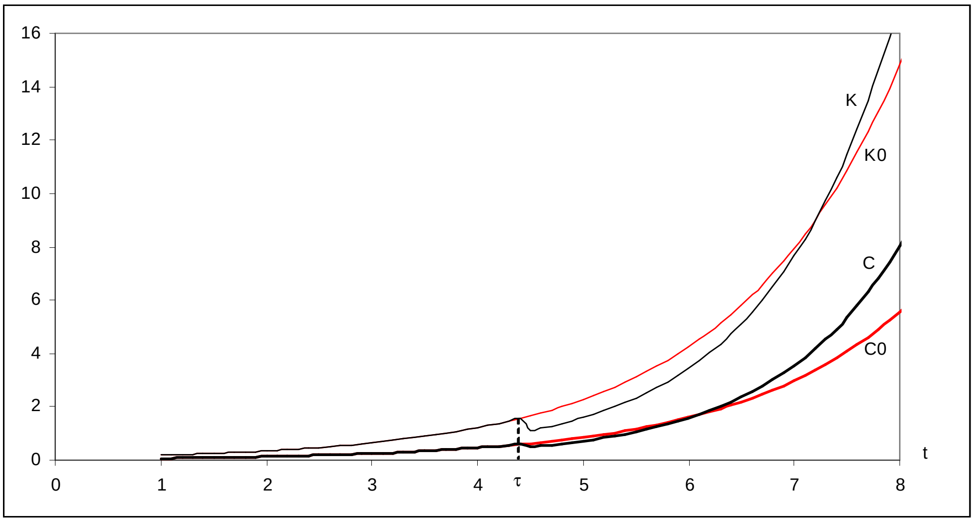


Figure 8 :consumption and capital dynamics with (C and K) and without ($C0$ and $K0$) opportunity to switch

$\boldsymbol{\gamma > 1}$



8 Appendix

8.1 Solving the program after the switch

We consider the following controlled process :

$$dK_t = K_t(q_t - c_t)dt + \sigma K_t dz_t$$

where $q_t = q_N A_t$ and $c_t = q_N C_t$.We need to find the process such that $\left[K_t H_t + \int_0^t H_s d_s ds\right]$ is a local martingale, with $H(.)$ being the stochastic Lagrangian multiplier.

$$E\left[K_T H_T + \int_0^T H_s c_s ds\right] \leq K_0$$

where K_0 is the initial level of the stock of capital. Since $K_T H_T \geq 0$, we have

$$E\left[\int_0^T H_s c_s ds\right] \leq K_0$$

and through monotonous convergence

$$E\left[\int_0^\infty H_s c_s ds\right] \leq K_0$$

Here:

$$dH_t = -H_t d\left(\frac{q_t + \nu_t}{\sigma}\right) dz_t + H_t \nu_t dt$$

with $H_0 = 1$ and $\nu_t : \Omega \times [0, T] \to \Re$. Then

$$\begin{aligned} d\left(H_t K_t + \int_0^t H_s c_s ds\right) &= H_t c_t dt + H_t \left(q_t K_t - c_t\right) dt + \sigma H_t K_t dz_t \\ &\quad - K_t H_t \left(\frac{q_t + \nu_t}{\sigma}\right) dz_t + K_t H_t \nu_t dt - K_t H_t (q_t + \nu_t) dt \\ &= K_t H_t \left(\sigma - \frac{q_t + \nu_t}{\sigma}\right) dz_t \end{aligned}$$

$$\begin{aligned} E\left[\int_0^\infty U(C_s)ds\right] &\leq E\left[\int_0^\infty U(C_s)ds + \lambda\left(K_0 - \int_0^\infty H_\tau^{(\nu)} d\tau\right) ds\right] \\ &\leq E\left[\int_0^\infty U^*(\lambda H_s^{(\nu)})ds + \lambda K_0\right] \quad \text{where} \quad U^*(y) = \sup_{x>0}\{u(x) - xy\} \end{aligned}$$

which has to be true whatever ν but it is an equality if the left handside is maximized and the righthandside is minimized.

$$\begin{aligned}
& \min_{\nu_t} E\left[\int_0^\infty U^*(\lambda H_t^{(\nu)})dt\right] \\
\Leftrightarrow \quad & \min_{\nu_t} \Gamma E \int_0^\infty \exp\left[\Gamma \int_0^t \nu_s ds - \frac{1}{2}\Gamma \int_0^t \left(\frac{q_s+\nu_s}{\sigma}\right)^2 ds + \Gamma \int_0^t \left(\frac{q_s+\nu_s}{\sigma}\right) dz\right] dt \\
\Leftrightarrow \quad & \min_{\nu_t} \Gamma E \int_0^\infty M_t \exp\left[\Gamma \int_0^t \left(\nu_s - \frac{1}{2}\left(\frac{q_s+\nu_s}{\sigma}\right)^2 + \frac{1}{2}\Gamma\left(\frac{q_s+\nu_s}{\sigma}\right)^2\right) ds\right] dt \\
\Leftrightarrow \quad & \min_{\nu_t} \left(1-\frac{1}{\gamma}\right) \int_0^\infty \exp\left[\left(1-\frac{1}{\gamma}\right) \int_0^t \left(\nu_s - \frac{1}{2\gamma}\left(\frac{q_s+\nu_s}{\sigma}\right)^2\right) ds\right] dt
\end{aligned}$$

for $\Gamma = 1 - 1/\gamma$ and where M_t is a martingale

Note that ν is deterministic since we minimize since only deterministic variables enter the equation. The first order condition provides:

$$1 - \frac{1}{\gamma\sigma}\left[\frac{q_t+\nu_t}{\sigma}\right] = 0 \Leftrightarrow \widehat{\nu}_t = \gamma\sigma^2 - q_t$$

Therefore

$$d\widehat{H}_t = \widehat{H}\left(\gamma\sigma^2 - q_t\right) dt - \widehat{H}_t\left(\gamma\sigma\right) dz_t$$

So it is indeed a martingale.

Since $H_t K_t + \int_0^t H_s c_s ds$ is a martingale, the corresponding process for K is

$$\begin{aligned}
\widehat{K}_t &= E\left[\int_t^\infty \frac{\widehat{H}_s}{\widehat{H}_t}\left(\lambda \widehat{H}_s\right)^{-1/\gamma} ds\right] \\
&= \frac{\lambda^{-1/\gamma}}{\widehat{H}_t} E\left[\int_t^\infty \widehat{H}_s^{1-1/\gamma} ds\right] \\
&= \frac{\lambda^{-1/\gamma}}{\widehat{H}_t} E\left[\int_t^\infty \widehat{H}_t^{1-1/\gamma} \exp\left[\Gamma \int_t^s \left(\gamma\sigma^2 - q_u\right) - \frac{1}{2}\left(\gamma\sigma\right)^2 + \frac{1}{2}\Gamma\left(\gamma\sigma\right)^2 du\right] \frac{M_s}{M_t} ds\right] \\
&= \lambda^{-1/\gamma} \widehat{H}_t^{-1/\gamma} E\left[\int_t^\infty \frac{M_s}{M_t} \exp\left[\Gamma \int_t^s \left(\frac{\gamma\sigma^2}{2} - q_u\right) du\right] ds\right] \\
&= \lambda^{-1/\gamma} \widehat{H}_t^{-1/\gamma} \underbrace{\int_t^\infty \exp\left[-\Gamma \int_t^s \left(q_u - \frac{\gamma\sigma^2}{2}\right) du\right] ds}_{G(q_t)}
\end{aligned}$$

From which we deduce

$$\frac{\widehat{K}_t}{G(q_t)} = \lambda^{-1/\gamma}\widehat{H}_t^{-1/\gamma} = (U')^{-1}(\lambda\widehat{H}_t) = \widehat{c}_t = q_N\widehat{C}_t$$

And

$$V_K(K_t, A_t) = K_t^{-\gamma}g(A_t)^{-\gamma} = K_t^{-\gamma}\left[G(q_t)\right]^{\gamma} q_N^{\gamma-1}$$

Therefore

$$g(A_t)^{-\gamma} = \left[G(A_t)\right]^{\gamma} q_N^{\gamma-1} = \left\{\int_t^{\infty} q_N^{\frac{\gamma-1}{\gamma}} \exp\left[-\Gamma\int_t^s\left(q_u - \frac{\gamma\sigma^2}{2}\right)du\right]ds\right\}^{\gamma}$$

Moreover,

$$\begin{aligned}
q_u &= q_N A_t = q_N\left(A_N - A^* e^{-\theta u}\right)\\
&= q_N\left(A_N - A^* e^{-\theta t}\right) + q_N\left(A^* e^{-\theta t} - A^* e^{-\theta u}\right)\\
&= q_t + q_F A^* e^{-\theta t}\left(1 - e^{-\theta(u-t)}\right) = q_t + (q_N A_N - q_t)\left(1 - e^{-\theta(u-t)}\right)\\
&= q_N A_N\left(1 - e^{-\theta(u-t)}\right) + q_t e^{-\theta(u-t)}
\end{aligned}$$

Noting $\alpha = s - t$ and with $q_t = q_N A_t$

$$\begin{aligned}
& g(A_t)^{-\gamma}\\
=\ & \left[G(A_t)\right]^{\gamma} q_N^{\gamma-1}\\
=\ & \left\{\int_0^{\infty} q_N^{\frac{\gamma-1}{\gamma}} \exp\left[\Gamma\int_0^{\alpha}\left(\frac{\gamma\sigma^2}{2} - q_N A_N\left(1 - e^{-\theta\tau}\right) - q_N A_t e^{-\theta\tau}\right)d\tau\right]d\alpha\right\}^{\gamma}\\
=\ & \left\{\int_0^{\infty} q_N^{\frac{\gamma-1}{\gamma}} \exp\left[\Gamma\left(\alpha\left(\frac{\gamma\sigma^2}{2} - q_N A_N\right) + \frac{q_N}{\theta}\left(A_N - A_t\right)\left(1 - e^{-\theta\alpha}\right)\right)\right]d\alpha\right\}^{\gamma}
\end{aligned}$$

8.2 Solving the program before the switch

The Hamiltonian-Jacobi-Bellman equation corresponding to the problem before the switch is:

$$\sup_{C_t}\left\{\frac{C_t^{1-\gamma}}{1-\gamma} + q_I V_K(A_I K_t - C_t) + \frac{1}{2}\sigma_I^2 K_t^2 V_{KK}\right\} = 0$$

The first order condition gives the optimal consumption path:

$$C_t = (V_K q_I)^{-1/\gamma}$$

Replacing consumption by its optimal expression gives :

$$q_I V_K A_I K_t + \frac{1}{2}\sigma_I^2 K_t^2 V_{KK} + \frac{\gamma}{1-\gamma}\left(V_K q_I\right)^{1-1/\gamma} = 0$$

Noting $h(K_t) = V_K(K_t)$ and multiplying by $[h(K_t)]^{(1-\gamma)/\gamma}$

$$q_I h^{1/\gamma} A_I K_t + \frac{1}{2}\sigma_I^2 K_t^2 h_K h^{(1-\gamma)/\gamma} + \frac{\gamma}{1-\gamma} q_I^{1-1/\gamma} = 0$$

Noting $f = [h(K_t)]^{1/\gamma}$ it becomes

$$q_I f A_I K_t + \frac{1}{2}\gamma\sigma_I^2 K_t^2 f' + \frac{\gamma}{1-\gamma} q_I^{1-1/\gamma} = 0$$

We postulate $f = \frac{C_1}{K_t} + C_2 K_t^{C_3}$ (which implies $V_K = \left[\frac{C_1}{K_t} + C_2 K_t^{C_3}\right]^{\gamma}$ as in the text) The first order condition gives the optimal consumption path:

$$C_t = \left(\frac{C_1}{K_t} + C_2 K^{C_3}\right)^{-1} q_I^{-1/\gamma}$$

Replacing in the Bellman equation, and solving for C_1 and C_3, leads to:

$$h(K_t) = V_K(K_t) = \left[\frac{\gamma q_I^{1-1/\gamma}}{(1-\gamma)\left(\gamma\sigma_I^2/2 - q_I A_I\right)}\frac{1}{K_t} + C_2 K_t^{-2q_I A_I/\left(\gamma\sigma_I^2\right)}\right]^{\gamma}$$

To determine C_2 we have to consider the level of capital K^* which triggers the switch to the new economy. Indeed the value function has to satisfy:

$$V(K_t, K^*) = W(K^*) - \int_{K(t)}^{K^*} h(K^*, y)dy$$

where $W(K^*)$ is the value function in the new economy at the time of the switch. This can be rewritten:

$$\begin{aligned} V(K_t, K^*) - W(K) &= W(K^*) - W(K) - \int_{K(t)}^{K^*} h(K^*, y)dy \\ &= \int_{K(t)}^{K^*} W'(y) - h(K^*, y)dy \geq 0 \text{ whatever } K(t) \end{aligned}$$

And at the time of the switch when a part β of the capital is used to reach the new economy, we have:

$$V_K(K_t) = W_K(K_t - \beta)$$

where C_2 is a constant that can be determined using the smooth pasting condition:

$$C_2 = \left[\frac{1}{\left[g(A_F - A^*)\right](K^* - \beta)} - \frac{1}{\left[\frac{1-\gamma}{\gamma} q_I^{\frac{1-\gamma}{\gamma}} \left(\frac{\gamma\sigma_I^2}{2} - q_I A_I\right)\right] K^*} \right] K^{*2q_I A_I/(\gamma\sigma_I^2)}$$

9 References

Abel, A. and J. Eberly, 1996, "Optimal Investment with Costly Reversibility," *Review of Economic Studies* 63, 581-593.

Abel, A. and J. Eberly, 2002, "Q for the Long Run", Unpublished Manuscript, University of Pennsylvania.

Abel, A. and J. Eberly, 2004, "Investment, Valuation and Growth Options", Unpublished Manuscript, University of Pennsylvania.

Alvarez L. and R. Stenbacka, 2001, "Adoption of Uncertain Multistage Technology Projects: a Real Options Approach", *Carnergie-Rochester Conference Series on Public Policy* 48, 101-135.

Arrow, K., and A. Fisher, 1974, "Environmental Preservation, Uncertainty, and Irreversibility," *Quarterly Journal of Economics* 88, 312-319.

Bentolila, S., and G. Bertola, 1990, "Firing Costs and Labor Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 57, 381-402.

Bertola, G., and R. Caballero, 1994, "Irreversibility and Aggregate Investment," *Review of Economic Studies* 61, 223-246.

Boucekkine, R., M. Germain, and O. Licandro, 1997, "Replacement Echoes in the Vintage Capital Growth Model", *Journal of Economic Theory*, 74 (2): 333-348.

Boucekkine, R. and A. Pommeret,2004, "Energy Saving Technical Progress and Optimal Capital Stock: The Role of Embodiment", *Economic Modelling*, 21: 429-444.

Boucekkine, R., C. Saglam and T. Vallée, 2004, "Technology adoption under embodiment: A two-stage optimal control approach," *Macroeconomic Dynamics* 8, 250-271.

Caballero, R., E. Engel, and J. Haltiwanger,1995, "Plant-Level Adjustment and Aggregate Investment Dynamics", *Brookings Papers on Economic Activity*, 2 : 1-54.

Caballero, R., and R. Pindyck, 1996, "Uncertainty, Investment, and Industry Evolution," *International Economic Review* 37, 641-662.

Cooley, T., J. Greenwood and M. Yorukoglu, 1997, “The Replacement Problem”, Journal of Monetary Economics, 40: 457-499.

Cooper, R., J. Haltiwanger, and L. Power,1999, “Machine Replacement and the Business Cycle: Lumps and Bumps”, *American Economic Review*, 89: 921-946.

Cox, J., J. Ingersoll and S. Ross, 1985, ”An Intertemporal General Equilibrium Model of Asset Pricing”, *Econometrica*, 53, 363-384.

Décamps, J.P., T., Mariotti, 2004, “Investment Timing with Learning Externalities,” *Journal of Economic Theory* 118, 80-102.

Décamps, J.P., T., Mariotti, and S. Villeneuve, 2005, “Investment Timing under Incomplete Information,” *Mathematics of Operations Research* Forthcoming.

Dixit, A., 1989, “Entry and Exit Decisions under Uncertainty”, *Journal of Political Economy* 97, 620-638.

Galor,O. and D. Tsiddon, 1997, “Technological Progress, Mobility and Economic Growth”, *American Economic Review*, 87, 363-382.

Dixit, A. and R. Pindyck, 1994, *Investment Under Uncertainty*, Princeton, NJ: Princeton University Press.

Greenwood, J., Z. Hercowitz and P. Krusell , 1997, “Long-run Implications of Investment-Specific Technological Change,” *American Economic Review* 87, 342-362.

Greenwood, J. and B. Jovanovic, 2001, “Accounting for Growth,” In E.Dean, M. Harper and C.Hulten (eds), *New Directions in Productivity Analysis*, NBER Studies in Income and Wealth Volume 63, Chicago: Chicago University Press.

Grenadier, S., and A. Weiss, 1997, “Investment in Technological Innovations: An Option Pricing Approach,” *Journal of Financial Economics* 44, 397-416.

Guo, X., J. Miao, and E. Morellec, 2005, “Irreversible Investment with Regime Shifts,” *Journal of Economic Theory* 122, 37-59.

Hugonnier J., E. Morellec, and S.Sunderasan, 2005, “Irreversible Investment in General Equilibrium”, Working Paper, University of Lausanne.

Jensen R., 1992, “Innovation Adoption and Welfare under Uncertainty”, *The Journal of Industrial Economics*, 40(2), 173-180.

Jovanovic,B.,1997, “Learning and Growth” in D.Kreps and K. Wallis (eds.), *Advances in economics*, vol2, 318-339, London: Cambridge University Press.

jovanovic B. and Y Nyarko, 1996, “Learning-by-Doing and the Choice of Technology”, *Econometrica*, 64, 1299-1310.

Leahy, J., 1993, “Investment in Competitive Equilibrium: The Optimality of Myopic Behavior”, *Quarterly Journal of Economics* 108, 1005-1033.

Ljungqvist, L., and T. Sargent, 2000, *Recursive Macroeconomic Theory*, MIT Press, Cambridge MA.

McDonald, R., and D. Siegel, 1986, “The Value of Waiting to Invest,” *Quarterly Journal of Economics* 101, 707-728.

Parente, S., 1994, “Technology Adoption, learning by doing, and Economic Growth,” *Journal of Economic Theory* 63, 346-369.

Parente, S., 2000, “Learning-by-Using and the Switch to Better Machines”*, Review of Economic Dynamics* 3, 675-703.

Pavlova A., 2001, “Adjustment Costs, Learning-by-Doing, and Technology Adoption under Uncertainty”, mimeo Sloan Institute of Technology, MIT.

Pindyck, R., 1988, “Irreversible Investment, Capacity Choice, and the Value of the Firm”, *American Economic Review* 78, 969-985.

Pindyck, R., 1991, “Irreversibility, Uncertainty and Investment,” *Journal of Economic Literature* 29, 1110-1148.

Roche, H., 2003, “Optimal Scrapping and Technology Adoption under Uncertainty”, Unpublished Manuscript, Instituto Tecnologico Autonomo de Mexico.

Saglam, C, 2002, “Optimal Sequence of Technology Adoptions with Finite Horizon via Multi-Stage Optimal Control”, Mimeo, IRES-Université Catholique de Louvain.

Wang, T., 2001, “Equilibrium with New Investment Opportunities,” *Journal of Economic Dynamics and Control* 25, 1751-1773.