Another Look at the Identification of Dynamic Discrete Decision Processes: With an Application to Retirement Behavior

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Abstract

This paper presents a method to estimate the effects of a counterfactual policy intervention in the context of dynamic structural models where all the structural functions (i.e., preferences, technology, transition probabilities, and the distribution of unobservable variables) are nonparametrically specified. We show that agents' behavior, before and after the policy intervention, and the change in agents' utility are nonparametrically identified. Based on this result we propose a nonparametric procedure to estimate the behavioral and welfare effects of a general class of counterfactual policy interventions. We apply this method to evaluate hypothetical reforms in the rules of a public pension system using a model of retirement behavior and a sample of blue-collar workers in Sweden.

Keywords: Dynamic discrete decision processes; Nonparametric identification; Counterfactual policy interventions; Retirement behavior.

JEL: C14, C25, C61, D91, J26.

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1 Introduction

Discrete choice dynamic structural models have proven useful tools for the assessment of public policy initiatives (Wolpin, 1996). These econometric models have been applied to the evaluation of different economic policies, both factual and counterfactual, like welfare policies (Sanders and Miller, 1997, Keane and Moffit, 1998, and Keane and Wolpin, 2000), unemployment insurance (Ferrall, 1997), social security and retirement (Berkovec and Stern, 1991, and Rust and Phelan, 1997), patents regulation (Pakes, 1986, and Pakes and Simpsom, 1989), education policies (Eckstein and Zilcha, 1994, Eckstein and Wolpin, 1999, and Keane and Wolpin, 2001), contraceptive choice (Hotz and Miller, 1993), regulation on labor contracts (Aguirregabiria and Alonso-Borrego, 1999, and Rota, 2004), programs on child poverty (Todd and Wolpin, 2003), scrapping subsidies (Adda and Cooper, 2000), or regulation of nuclear plants (Rust and Rothwell, 1995).

A common feature of the econometric models in these applications is the parametric specification of structural functions like utility functions, technology, transition probabilities of state variables, and the probability distribution of unobservable variables.[1] These parametric models contrast with the emphasis on robustness and nonparametric specification that we find in other approaches to evaluate public policies. In particular, the literature on evaluation of treatment effects has emphasized the importance of a nonparametric specification of the distribution of unobservables to obtain robust results (see Heckman and Robb, 1985, Manski, 1990, and more recently Heckman and Smith, 1998, and Heckman and Vyltacil, 1999 and 2005). Though robustness is an important argument in favor of this reduced form approach, these methods have important limitations to evaluate counterfactual policies, to estimate welfare effects, to incorporate transitional dynamics, and to allow for general equilibrium effects.

In this paper we show that it is possible to use nonparametrically specified dynamic structural models to evaluate the effects of counterfactual policy interventions. The contribution of this paper is threefold. First, we show that agents' behavior, before and after the policy intervention, and the change in agents' utility are nonparametrically identified. Second, based on this identification result we propose a nonparametric method to estimate the behavioral and welfare effects of counterfactual policy interventions in this class of mod-

[1] Two exceptions are the semiparametric models in Taber (2000) and Heckman and Navarro (2004), where utilities are parametrically specified but the distribution of unobservable variables is nonparametric.

els. And third, we apply this method to evaluate hypothetical reforms in the rules of a public pension system using data of male blue-collar workers in Sweden. This application illustrates how the method can be used to obtain precise estimates of welfare effects, and the transitional dynamics of these effects, which do not rely on any parametric assumption on the primitives of the model.

The parametric specification of dynamic structural models is justified for the sake of parsimony, simplicity, and efficiency in the estimation. However, the economic content of dynamic structural models does not rest on the choice of a particular family of parametric functions for the primitives but on specification assumptions such as: the selection of the relevant decision and state variables; independence assumptions between unobservable variables and some observables; the stochastic structure of the transition probabilities of the state variables (e.g., which variables follow exogenous transitions, and which variables are endogenous and how); monotonicity and concavity assumptions of some primitive functions; specification of individual heterogeneity; or the equilibrium concept that is used.

As shown by Rust (1994) and Magnac and Thesmar (2002), the differences between the utilities of two choice alternatives cannot be identified in dynamic decision models even when the researcher "knows" the time discount factor, the probability distribution of the unobservables, and the transition probabilities of the state variables. This under-identification result contrasts with the identification of utility differences in static (i.e., not forward looking) decision models (see Matzkin, 1992). This paper takes a different look at the problem of nonparametric identification of dynamic decision models. Instead of looking at the nonparametric identification of the utility function we consider the identification of the behavioral and welfare effects of counterfactual policy changes. More specifically, we prove the identification of agents' choice probability functions and surplus functions associated with hypothetical policy interventions. We show that knowledge of the current utility function or of utility differences is not necessary to identify these counterfactual functions. These counterfactuals depend on the distribution of unobservables and on the difference between the present value of choosing always the same alternative and the value of deviating one period from that behavior. We show that these objects are identified under similar conditions as in static models. Therefore, though agents' preferences cannot be identified, we can identify the behavioral and welfare effects associated with changes in these preferences.

The rest of the paper is organized as follows. In section 2 we set up the model, the basic assumptions and the type of counterfactual policy experiments that we want to evaluate. Section 3 presents the identification results. In section 4 we describe the estimation procedure. The empirical application is presented in section 5. We summarize and conclude in section 6. Proofs of propositions are in the appendix.

2 Model

2.1 Framework and basic assumptions

Time is discrete an indexed by t. Consider an agent who has preferences defined over a sequence of states of the world from period $t = 0$ to $t = T$. A state of the world has two components: a vector of state variables s_t that is predetermined before period t; and a discrete decision $a_t \in A = \{0, 1, ..., J\}$ that the agent chooses at period t. The decision at period t affects the evolution of future values of the state variables. The agent's preferences over possible sequences of states of the world can be represented by a utility function $\sum_{j=0}^{T} \beta^j$ $U_t(a_{t+j}, s_{t+j})$, where $\beta \in [0, 1)$ is the discount factor and $U_t(a_t, s_t)$ is the current utility function at period t. The agent has uncertainty about future values of state variables. His beliefs about future states can be represented by a sequence of Markov transition probability functions $F_t(s_{t+1}|a, s)$. These beliefs are *rational* in the sense that they are the true transition probabilities of the state variables. Every period t the agent observes the vector of state variables s_t and chooses his action $a_t \in A$ to maximize the expected utility

$$E\left(\sum_{j=0}^{T} \beta^j \ U_t(a_{t+j}, s_{t+j}) \mid a_t, s_t\right). \tag{1}$$

Let $\alpha_t(s_t)$ and $V_t(s_t)$ be the optimal decision rule and the value function at period t, respectively. By Bellman principle of optimality the sequence of value functions can be obtained using the recursive expression:

$$V_t(s_t) = \max_{a \in A} \left\{U_t(a, s_t) + \beta \int V_{t+1}(s_{t+1}) \ dF_t(s_{t+1}|a, s_t)\right\} \tag{2}$$

For the rest of the paper we adopt a notation that omits the time subindex from functions and variables. We can include the time period t as a state variable of the model and therefore we can omit it as an index in the structural functions, in the optimal decision rule and in the value function. Note that a finite-horizon dynamic decision problem can be represented as an infinite-horizon problem if we just make the utility functions equal to zero for any state with $t > T$. We also omit the time subindex in the decision and state variables and use (a, s) to represent current values of these variables, and (a', s') for next period values.

From the point of view of the observing researcher there are three types of state variables. That is, $s = (x, \omega, \varepsilon)$ where: the vector x is observable to the researcher; the vector ε is unobservable; and the vector ω is unobservable but it can be inferred by the econometrician using data of a vector of outcome variables y and estimating the system of outcome equations:

$$y = h\left(a, x, \omega\right) \tag{3}$$

where y is a $q \times 1$ vector of variables and $h\left(., ., .\right)$ is a vector of q functions. For instance, in a model of firm behavior the researcher may observe a component of the profit function such as output, revenue or the wage bill. If y is firm's output then $h\left(a, x, \omega\right)$ would be a production function and ω a productivity shock. In a model of individual behavior where individuals maximize a utility that depends on consumption and leisure, the econometrician may observe individual earnings. In that case $h\left(a, x, \omega\right)$ would be an earnings function and ω is a shock in earnings.

Without loss of generality we can write the one-period utility as the sum of two components:

$$U(a, x, \omega, \varepsilon) = u(a, x, \omega) + \varepsilon(a, x, \omega), \tag{4}$$

where $u(a, x, \omega) \equiv E\left(U(a, x, \omega, \varepsilon) \mid a, x, \omega\right)$ and $\varepsilon(a, x, \omega) \equiv U(a, x, \omega, \varepsilon) - u(a, x, \omega)$. For the sake of notational simplicity we use $\varepsilon(a)$ instead of $\varepsilon(a, x, \omega)$, and the vector ε to represent $\{\varepsilon(a) : a \in A\}$. By definition, the random variables in ε have zero mean and are mean independent of x and ω.

We consider the following assumptions on the joint distribution of the state variables.

ASSUMPTION 1: The cumulative transition probability of the state variables factors as:

$$F(s'|a, s) = F_\omega(\omega'|\omega)\ F_\varepsilon(\varepsilon'|x')\ F_x(x'|a, x) \tag{5}$$

where $F_\omega(.|\omega)$, $F_\varepsilon(.|x)$ and $F_x(.|a, x)$ are distribution functions. That is: (a) ω follows an exogenous Markov process; (b) future ε's may depend on future values of x (e.g., heterocedasticity) but not on current values of a or x; and (c) future values of x depend on current values of x and current actions but not on current values of ω and ε. Furthermore: (d) $F_\varepsilon(.|x)$ is continuously differentiable and strictly increasing with support the Euclidean space; and (e) ω is a vector of continuous random variables and for any ω_0, ω_1 and w' with $\omega_0 < \omega_1$, we have that $F_\omega(\omega'|w_0) \geq F_\omega(\omega'|w_1)$.

Assumption 1 is based on Rust's conditional independence assumption (Rust, 1994), but it is more general than Rust's because it allows for the unobservable ω. Under this assumption the optimal decision rule $\alpha(x, \omega, \varepsilon)$ can be described as:

$$\alpha(x, \omega, \varepsilon) = \arg\max_{a \in A} \{ \ v(a, x, \omega) + \varepsilon(a) \ \} \tag{6}$$

where $v(a, x, \omega)$ is the present value of current and future utilities when current choice is a. That is,

$$v(a, x, \omega) \equiv u(a, x, \omega) + \beta \int \max_{j \in A} \{ \ v(j, x', \omega) + \varepsilon'(j) \ \} \ F_\omega(d\omega'|\omega) \ F_\varepsilon(d\varepsilon'|x') \ F_x(dx'|a, x) \tag{7}$$

The functions $v(0, x, \omega)$, $v(1, x, \omega)$, ..., $v(J, x, \omega)$ are called *conditional choice value functions*. The optimal decision rule represents individuals' behavior. Individuals' welfare is given by the value function $V(x, \omega, \varepsilon) = \max_{a \in A} \{ \ v(a, x, \omega) + \varepsilon(a) \ \}$.

For our econometric analysis it is convenient to define versions of these functions which are integrated over the unobservables in ε. The *optimal choice probability function* is defined as:

$$P(a|x, \omega) \equiv \int I\{\alpha(x, \omega, \varepsilon) = a\} \ F_\varepsilon(d\varepsilon|x) \tag{8}$$

The integrated valued function (Rust, 1994) $S(x, \omega)$ is defined as:

$$S(x, \omega) \equiv \int V(x, \omega, \varepsilon) \ F_\varepsilon(d\varepsilon|x) = \int \max_{a \in A} \{ \ v(a, x, \omega) + \varepsilon(a) \ \} \ dF_\varepsilon(\varepsilon|x) \tag{9}$$

To complete the model structure we should establish the relationship between the outcome variables in y and the utility function. Assumption 2 establishes that that the utility function $u(a, x, \omega)$ is additive in the outcome variables and in other component that does not depend on the outcome variables.

ASSUMPTION 2: The utility function $u(a, x, \omega)$ has the following form:

$$u(a, x, \omega) = \psi(x) \ y + c(a, x) \tag{10}$$

where $\psi(.)$ is a $1 \times q$ vector of positive valued functions, i.e., $\psi(x) = (\psi_1(x), ..., \psi_q(x))$ with $\psi_j(x) > 0$ for any j and any $x \in X$; y is the $q \times 1$ vector of outcome variables that we have defined in equation (3); and $c(., .)$ is a real valued function.

The set of structural functions that define the model is $\{\psi, \ h, \ c, \ \beta, \ F_\omega, \ F_\varepsilon, \ F_x\}$. This is the so called *model structure*. This paper concentrates in binary choice models, though our identification results and the estimation method can be generalized to the multinomial case. Consider the binary choice case where $a \in \{0, 1\}$. For notational simplicity we use $P(x, \omega)$ to denote $P(1|x, \omega)$.

2.2 An example: A model of retirement behavior

In this section we present a model of retirement behavior similar that follows Rust and Phelan (1997) and Karlstrom, Palme and Svensson (2004). Individuals have a utility function that is additively separable in consumption (C_t) and leisure (L_t). The marginal utilities of consumption and leisure may depend on individual characteristics such as age, marital status, family size, health status, etc. Some of these demographic variables are observable to the researcher (i.e., they are in the vector x_t) but some of them are unobservables (i.e., they are in ε_t). More specifically,

$$U_t = \psi_C(x_t)\ U_C(C_t) + \psi_L(x_t, \varepsilon_t)\ U_L(L_t) \tag{11}$$

where the functions $\psi_C(x_t)$ and $\psi_L(x_t, \varepsilon_t)$ capture individual heterogeneity in marginal utilities.

Every month t the individual decides whether to continue working ($a_t = 1$) or to retire from the labor force ($a_t = 0$). If the individual works, his hours of leisure are equal to $T(x_t) - H(x_t)$ and his monthly earnings (Y_t) are equal to labor earnings (W_t). If the individual decides to retire, then his hours of leisure are $T(x_t)$ and earnings are equal to retirement benefits (B_t). Thus, we can write $L_t = T(x_t) - a_t\ H(x_t)$ and monthly earnings as:

$$Y_t = a_t\ W_t + (1 - a_t)\ B_t \tag{12}$$

Labor earnings during the month are uncertain to the individual when he decides whether to retire. Suppose that $W_t = \exp\{w(x_t) + \omega_{t+1}\}$, such that: $w(.)$ is a function, and ω_t is a variable that follows a Markov process $\omega_{t+1} = \kappa(\omega_t) + e_{t+1}$ where $\kappa(.)$ is a function and e_{t+1} is the innovation of the process. The individual knows x_t and ω_t but he does not know the innovation e_{t+1} when he makes his decision.

Retirement benefits depend on retirement age (ra_t) and on *pension points* (pp_t): $B_t = B(ra_t, pp_t)$. The form form of the function $B(.,.)$ depends on the rules of the pension system. For instance, a very standard structure is:

$$B_t = \begin{cases} 0 & if \quad ra_t < ra_{\min} \\ pp_t\ \ (1 + \tau_1\,(ra_t - ra_*)) & if \quad ra_{\min} \leq ra_t < ra_* \\ pp_t\ \ (1 + \tau_2\,(ra_t - ra_*)) & if \quad ra_t \geq ra_* \end{cases} \tag{13}$$

where $ra_{\min}$, ra_*, τ_1 and τ_2 are policy parameters that characterize the function $b(.,.)$. More specifically: $ra_{\min}$ is the minimum retirement age; ra_* is the "normal" retirement age; τ_1

is a permanent actuarial reduction in benefits per month of early retirement; and τ_2 is a permanent actuarial increase in benefits per month of delayed retirement. In Sweden, the values of these parameters are $ra_{\min} = 60$ year, $ra_* = 65$ years, $\tau_1 = 0.5\%$ and $\tau_2 = 0.7\%$.

Pension points are a deterministic function of past earnings history. However, it turns out that for most systems the transition rule of pension points can be very closely approximated by a Markov process. For instance, that is the case for social security pensions in US (see Rust and Phelan, 1997, and Rust et al, 2000), for Germany (see Knaus, 2002), and for Sweden (see Karlstrom, Palme and Svensson, 2004). The variables ra_t and pp_t are part of the vector of observable state variables in x_t.

Since the individual has uncertainty about current labor earnings, the relevant current utility is the expected utility $E_t(U_t)$ where the information set at period t is $(a_t, x_t, \omega_t, \varepsilon_t)$. Suppose that consumption is proportional to earnings, with a proportionality constant that may depend on the state variables in x_t: i.e., $C_t = \lambda(x_t)\ Y_t$. And suppose that the function $U_C(.)$ is known. For instance, consider a constant relative risk aversion utility function $U_C(C_t) = C_t^{\gamma}$, where the parameter γ is known to the researcher. Then, we can write the utility function as:

$$E_t(U_t) \quad = \quad \psi_C(x_t)\ E_t\left(\lambda(x_t)^{\gamma}\ Y_t^{\gamma}\right) + \psi_L(x_t, \varepsilon_t)\ U_L\left(T(x_t) - a_t\ H(x_t)\right) \tag{14}$$

Define the functions $\psi(x_t) \equiv \psi_C(x_t)\ \lambda(x_t)^{\gamma}$, $\bar{\psi}_L(x_t) \equiv E(\psi_L(x_t, \varepsilon_t)|x_t)$, and $c(a_t, x_t) \equiv \bar{\psi}_L(x_t)\ U_L(T(x_t) - a_t H(x_t))$. And define also the variables $\varepsilon_t(a_t) \equiv \left(\psi_L(x_t, \varepsilon_t) - \bar{\psi}_L(x_t)\right)$ $U_L(T(x_t) - a_t H(x_t))$ and $y_t \equiv E_t\left(Y_t^{\gamma}\right)$. Then, we can write current utility as:

$$E_t U_t \quad = \quad \psi(x_t)\, y_t + c(a_t, x_t) + \varepsilon_t(a_t) \tag{15}$$

To show that this utility conforms to Assumptions 1 and 2, we still have to show that y_t is observable to the econometrician, and that we can write y_t as a function $h(a_t, x_t, \omega_t)$. Suppose that the econometrician observes labor earnings for those individuals who are working, and potential retirement benefits for every individual, working or not. Individuals do not have uncertainty about current benefits. Therefore, y_t is observable to the researcher if and only if $E_t(W_t^{\gamma})$ is observable for every individual, working or not. We have that:

$$E_t(W_t^{\gamma}) = \exp\left\{\gamma\ w(x_t) + \gamma\ \kappa(\omega_t) + \delta\right\} \tag{16}$$

with $\delta \equiv \ln\left(E\left(\exp\{\gamma e_{t+1}\}\right)\right)$. We know describe how $w(.)$, $\kappa(.)$ and δ can be nonparametrically identified. Given that $\ln W_t = w(x_t) + \kappa(\omega_t) + e_{t+1}$, and $\omega_t = \ln W_{t-1} - w(x_{t-1})$, we can write:

$$\ln W_t = w(x_t) + \kappa \left(\ln W_{t-1} - w(x_{t-1})\right) + e_{t+1} \tag{17}$$

The innovation e_{t+1} is unknown to the individual when he makes his decision. Therefore, e_{t+1} is independent of a_t, and it is also independent of x_t, x_{t-1} and W_{t-1}. The orthogonality conditions $E\left(e_{t+1}|a_t = 1, x_t, x_{t-1}, W_{t-1}\right) = 0$ provide moment conditions that allow us to estimate nonparametrically the functions $w(.)$ and $\kappa\,(.)$. Then, we can use the residuals of e_{t+1} to obtain an estimate of the parameter δ, i.e., $\hat{\delta} = \ln\left((1/n)\sum_{i=1}^{n} \exp\{\gamma \hat{e}_{i,t+1}\}\right)$.

2.3 Policy interventions

We want to evaluate the behavioral and welfare effects of an hypothetical policy intervention that modifies the current utility function. More specifically, we are interested in the evaluation of policies that modify the outcome functions such as the new (counterfactual) outcome functions become $h^*(a, x, \omega)$. That is,

$$h^*(a, x, \omega) = h(a, x, \omega) + \tau(a, x, \omega) \tag{18}$$

where h and h^* are the outcome functions before and after the policy intervention, respectively. The function τ represents the policy intervention and it is known to the researcher. Note that τ may depend on choice and state variables in a completely unrestricted way. We provide several examples to illustrate how general is this class of policy interventions.

EXAMPLE 1: Consider the retirement model in section 2.2. The outcome variable in this model is individual earnings. The type of policies that we can evaluate includes: policies that modify retirement benefits such as changes in the minimum and normal retirement age or changes in the discount for early retirement; policies that affect labor earnings such as a wage tax; or an hypothetical change in the relative risk aversion parameter.

EXAMPLE 2: Individual earnings are also an observable outcome in a model of educational choice. Some examples of policies that we can evaluate in this model are a change in returns to schooling, or a change in the costs of schooling.

EXAMPLE 3: Consider a dynamic model of firm input demand where the researcher observes firm output. In this model we can evaluate hypothetical changes in the production function parameters.

When we assume that the weighting function $\psi(x)$ is constant (i.e., $\psi(x) = 1$), then the class of counterfactual policies that we can evaluate using the method in this paper becomes more general. While this assumption seems quite strong for models of individual behavior, it can be more plausible for models of firm behavior where firms are assumed to maximize profits. Under this assumption, we provide identification results for any counterfactual policy intervention such that $u^*(a, x, \omega) = u(a, x, \omega) + \tau(a, x, \omega)$, where the function τ is known to the researcher, though the functions u and u^* are unknown.

Let P^* be the optimal choice probability function associated with the counterfactual structure. The difference between the functions P^* and P represents the behavioral effects of the policy from the point of view of the econometrician. Similarly, the difference between the functions $S^*(x, \omega)$ and $S(x, \omega)$ represents the welfare effects of the policy, where $S^*(x, \omega)$ is the integrated value function after the policy intervention. We are interested in the nonparametric estimation of the functions P^* and $S^* - S$.

3 Identification

Suppose that we have a random sample of individuals with information on the variables $\{a_t, a_{t+1}, x_t, x_{t+1}, y_t, y_{t+1}\}$ at some period t. As usual, we study identification with a very large (i.e., infinite) sample of individuals. Furthermore, we assume that the sample has variability over the whole support of the observable variables: $A \times X^2 \times Y^2$. This assumption of full-support variation is needed to identify the reduced form of the model. We assume that the outcome function $h(.,.,.)$ is identified without having to estimate the decision model. There are different conditions under which one can consistently estimate wage equations or production functions using instrumental variables or control function approaches which do not require the estimation of the complete structural model. We have provided an example of this in the retirement model in previous section 2.3.

ASSUMPTION 3: The outcome function $h(a, x, \omega)$ is a real valued function such that: (a) it is identified; (b) it is invertible with respect to ω, such that we can get $\omega = h^{-1}(y, x, \omega)$; (c) ω is a continuous random variable with support Ω; (d) the function $\tilde{h}(x, \omega) \equiv h(1, x, \omega) - h(0, x, \omega)$ is strictly increasing in ω; and (e) for any $x \in X$ there exits $\omega \in \Omega$ such that $\tilde{h}(x, \omega) + \tilde{c}(x) = 0$, where $\tilde{c}(x) \equiv c(1, x) - c(0, x)$.

It is clear that we can identify the transition probability function F_x on $A \times X^2$ from the transition probabilities $\Pr(x_{t+1}|a_t, x_t)$ in the data. Under Assumptions 3(a) and 3(b) the values of ω can be consistently estimated and we can treat ω as (indirectly) observable.

Therefore, F_ω is also identified on $\Omega \times \Omega$. It is also clear that we can identify the choice probability function $P(x, \omega)$ on $X \times \Omega$ from the probabilities $\Pr(a_t = 1|x_t, \omega_t)$ in the data.

However, without further restrictions, we cannot identify the structural functions $\{\psi$, c, $F_\varepsilon\}$. This is the case both in decision models where agents are forward looking (i.e., $\beta > 0$) and in models where agents are myopic (i.e., $\beta = 0$). In this paper we are not interested in the identification of $\{\psi, c, F_\varepsilon\}$ but in the functions P^* and $S^* - S$ associated with a counterfactual policy intervention that modifies the outcome function from h to h^*. We show that, under Assumptions 1 to 3, these functions are identified. For the sake of presentation, we start showing identification in a myopic version of the model.

3.1 Myopic model

Suppose that agents are not forward looking, i.e., $\beta = 0$. Then, the counterfactual choice probability function is:

$$\begin{aligned} P^*(x, \omega) &= \Pr\left(\psi(x)\ h^*(1, x, \omega) + c(1, x) + \varepsilon(1) \geq \psi(x)\ h^*(0, x, \omega) + c(0, x) + \varepsilon(0) \mid x, \omega \right) \\ &= F_{\tilde{\varepsilon}}\left(\tilde{h}^*(x, \omega) + \tilde{c}(x) \mid x \right) \end{aligned} \tag{19}$$

where $\tilde{h}^*(x, \omega) \equiv h^*(1, x, \omega) - h^*(0, x, \omega)$, $\tilde{c}(x) \equiv (c(1, x) - c(0, x))/\psi(x)$, and $F_{\tilde{\varepsilon}}(.|x)$ is the CDF of the random variable $\tilde{\varepsilon} \equiv (\varepsilon(0) - \varepsilon(1))/\psi(x)$ conditional to x. Equation (19) illustrates that the identification of $P^*(.)$ requires one to identify the functions $F_{\tilde{\varepsilon}}$ and $\tilde{c}$. The relationship between these functions and the *factual* reduced form probability function P is:

$$P(x, \omega) = F_{\tilde{\varepsilon}}\left(\tilde{h}(x, \omega) + \tilde{c}(x) \mid x \right) \tag{20}$$

Proposition 1 establishes the nonparametric identification of the functions $F_{\tilde{\varepsilon}}$ and $\tilde{c}$ and therefore of the counterfactual probability function P^*.

PROPOSITION 1: Let $\tilde{u}(X \times \Omega) \subseteq \mathbb{R}$ be the space of real values that the function $\tilde{h}(x, \omega) + \tilde{c}(x)$ can take. Under Assumptions 1 to 3, $\beta = 0$, and median$(\tilde{\varepsilon}|x) = 0$ we have that: (a) the function $\tilde{c}(.)$ is identified on X; (b) the function $F_{\tilde{\varepsilon}}(.|.)$ is identified over the set $\tilde{u}(X \times \Omega) \times X$; and (c) the counterfactual choice probability function P^* is identified over the set $(X \times \Omega)^* = \left\{ (x, \omega) \in X \times \Omega : \tilde{h}^*(x, \omega) + \tilde{c}(x) \in \tilde{u}(X \times \Omega) \right\}$.

The counterfactual probability function is identified on the set $(X \times \Omega)^*$ that is included in $X \times \Omega$. There are different cases in which $(X \times \Omega)^* = X \times \Omega$. *Case 1:* $\tilde{u}(X \times \Omega) = \mathbb{R}$. This is the case when the range of variation of the function $\tilde{h}(., .)$, or of the function $\tilde{c}(.)$, is the whole real line. Then, $\tilde{u}(X \times \Omega) = \mathbb{R}$ and this implies that $(X \times \Omega)^* = X \times \Omega$. *Case 2:* $\tilde{u}(X \times \Omega)$ is unbounded from above (below) and $\tilde{h}^* - \tilde{h}$ is positive (negative) valued. This is the case in applications where the outcome variable y has a lower bound at zero (e.g., output, earnings, revenue), and we consider a counterfactual policy that increases (decreases) the outcome variable for any possible value of (x, ω). For instance, a increase in the returns to schooling. *Case 3:* $\tilde{u}(X \times \Omega) \subset \mathbb{R}$ but $\tilde{h}^* - \tilde{h}$ is such that $\sup\{\tilde{h}^*(x,\omega) + \tilde{c}(x)\} \leq \sup\{\tilde{h}(x,\omega) + \tilde{c}(x)\}$ and $\inf\{\tilde{h}^*(x,\omega) + \tilde{c}(x)\} \geq \inf\{\tilde{h}(x,\omega) + \tilde{c}(x)\}$. That is, the policy that we want to evaluate is such that it reduces (increases) the utility differential in states where this differential is large (small).

Proposition 2 establishes the identification of the welfare effect function $\Delta(x,\omega) \equiv \left(S^*(x,\omega) - S(x,\omega)\right)/\psi(x)$.

PROPOSITION 2: Under the conditions in Proposition 1 the welfare effect function Δ is *nonparametrically identified. We can obtain this function as:*

$$\Delta(x,\omega) = \tau(0,x,\omega) + G\left(P^*(x,\omega), F_{\tilde{\varepsilon}}|x\right) - G\left(P(x,\omega), F_{\tilde{\varepsilon}}|x\right) \tag{21}$$

where $G\left(P, F_{\tilde{\varepsilon}}|x\right)$ is *McFadden's surplus function:*

$$G\left(P, F_{\tilde{\varepsilon}}|x\right) = P\ F_{\tilde{\varepsilon}}^{-1}(P|x) - \int_{-\infty}^{F_{\tilde{\varepsilon}}^{-1}(P|x)} \tilde{\varepsilon}\ dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x) \tag{22}$$

and $F_{\tilde{\varepsilon}}^{-1}(.|x)$ *is the inverse function of* $F_{\tilde{\varepsilon}}(.|x)$.

3.2 Dynamic model

We now consider the identification of counterfactual choice probabilities when agents are forward looking, i.e., when $\beta > 0$. The factual choice probability function is:

$$P(x,\omega) \quad = \quad F_{\tilde{\varepsilon}}\left(\tilde{v}(x,\omega) \mid x\right) \tag{23}$$

where $\tilde{v}(x,\omega) \equiv (v(1,x,\omega) - v(0,x,\omega))/\psi(x)$ is the *differential value function.* The counterfactual choice probability function is $P^*(x,\omega) = F_{\tilde{\varepsilon}}(\tilde{v}^*(x,\omega)|x)$, where $\tilde{v}^*$ is the differential value function after the policy change. We show in this section that the functions $F_{\tilde{\varepsilon}}$, $\tilde{v}^*$ and P^* are identified under the same conditions as in Proposition 1.

There is a main difference between the static and the dynamic models in the identification of behavioral effects. In the dynamic model we cannot identify current utility differences or any other function that depends only on preferences and not on agent's beliefs. That is, we cannot separate agents' preferences and agents' beliefs. Despite this under-identification of preferences, we can identify counterfactual choice probabilities.

For the sake of clarity, it is useful to describe our identification results in two steps. First, we show the identification of P^* when the function $F_{\tilde{\varepsilon}}$ is known. Second, we prove the joint identification of $F_{\tilde{\varepsilon}}$ and $\tilde{v}^*$.

3.2.1 Identification of behavioral effects when $F_{\tilde{\varepsilon}}$ is known

Suppose that the function $F_{\tilde{\varepsilon}}$ is known to the researcher. Then, it is clear that the function $\tilde{v}$ is identified from the factual choice probabilities: i.e., $\tilde{v}(x,\omega) = F_{\tilde{\varepsilon}}^{-1}\left(P(x,\omega)|x\right)$. However, in contrast to the static case, knowledge of the function $\tilde{v}$ is not enough to identify the counterfactual $\tilde{v}^*$. The reason is that $\tilde{v}^*$ is not just a function of $\tilde{v}$ and $h^* - h$ as in the static case. To obtain $\tilde{v}^*$ we need more information than just the factual value difference $\tilde{v}$. We show here that we can identify separately two components of $\tilde{v}$. Given this decomposition we can construct the counterfactual function $\tilde{v}^*$. Proposition 3 provides a characterization of the choice probability function that will be useful to identify and to estimate the counterfactuals.

PROPOSITION 3: The optimal choice probability function P is the unique fixed point of the mapping $\Psi(P)$, where

$$\Psi(P)(x,\omega) \equiv F_{\tilde{\varepsilon}}\left(\ \tilde{\varphi}(x,\omega) + \tilde{\delta}(x,\omega,P)\ \right) \tag{24}$$

and (1) $\tilde{\varphi}(x,\omega) = \varphi(1,x,\omega) - \varphi(0,x,\omega)$, where $\varphi(a,x,\omega)$ is the value of choosing alternative a today and then select alternative 0 forever in the future; and (2) $\tilde{\delta}(x,\omega,P) = \delta(1,x,\omega,P) - \delta(0,x,\omega,P)$, where $\delta(a,x,\omega,P)$ is the value of behaving optimally in the future minus the value of choosing always alternative 0, given that the current choice is a. These functions are recursively defined as follows:

$$\varphi(a,x,\omega) \quad = \quad \frac{u(a,x,\omega)}{\psi(x)} + \beta \int \varphi(0,x',\omega')\ dF_\omega(\omega'|\omega)\ dF_x(x'|a,x) \tag{25}$$

and

$$\delta(a,x,\omega,P) \quad = \quad \beta \textstyle\int \left(\ G\left(P(x',\omega'), F_{\tilde{\varepsilon}}|x'\right) + \delta(0,x',\omega',P)\ \right)\ dF_\omega(\omega'|\omega)\ dF_x(x'|a,x) \tag{26}$$

where $G\left(P, F_{\tilde{\varepsilon}}|x\right)$ has the same definition as in Proposition 2.

Proposition 3 establishes that we can decompose additively the value function $\tilde{v}$ in two functions : $\tilde{\varphi}$ and $\tilde{\delta}$. These two functions are not arbitrary. In particular, we show below that we can identify the function $\tilde{\varphi}$ and $\tilde{\delta}$ and that these functions, together with $F_{\tilde{\varepsilon}}$ are all what we need to construct the counterfactuals $\tilde{v}^*$ and P^*.

We now prove the identification of the function $\tilde{\varphi}$ when the distribution $F_{\tilde{\varepsilon}}$ is known. First, given $F_{\tilde{\varepsilon}}$ it is clear from that the surplus function $G(., F_{\tilde{\varepsilon}})$ is identified. Second, for any vector of probabilities P, equation (26) defines implicitly δ as the unique fixed point of a contraction mapping. Given G and β, this contraction mapping is known and therefore δ is also identified. And third, the optimal choice probability function is the only function that solves the functional equation: $P(x,\omega) = F_{\tilde{\varepsilon}}(\tilde{\varphi}(x,\omega) + \tilde{\delta}(x,\omega,P))$. Given that $F_{\tilde{\varepsilon}}$ is invertible and that $\tilde{\delta}(.,.,P) = \delta(1,.,.,P) - \delta(0,.,.,P)$ is known, we can identify $\tilde{\varphi}$ as:

$$\tilde{\varphi}(x,\omega) = F_{\tilde{\varepsilon}}^{-1}\left(P(x,\omega) \mid x\right) - \tilde{\delta}(x,\omega,P) \tag{27}$$

The functions δ, and $\tilde{\varphi}$ depend on agents' preferences and beliefs. Can we separately identify preferences and beliefs? No, without further restrictions. An assumption that identifies the utility function is the "normalization" $u(0,x,\omega) = 0$ for any (x,ω). Under this assumption we have that $\tilde{\varphi}(x) = u(1,x,\omega)$. This type of "normalization" is innocuous in static models because it does not affect the estimation of counterfactual probabilities, which only depend on utility differences and not on utility levels. However, this normalization is not innocuous in dynamic models. In dynamic models, the counterfactual choice probabilities depend on utility levels and not only on utility differences.

Proposition 4 shows that given the distribution function $F_{\tilde{\varepsilon}}$ we can identify the counterfactual choice probability function.

PROPOSITION 4: Suppose that the discount factor β, the distribution function $F_{\tilde{\varepsilon}}$, and the optimal choice probability function P are known. Then, the counterfactual choice probability function P^* is identified. In particular, P^* is the unique fixed point of the mapping $\Psi^*(P)$, where

$$\Psi^*(P)(x,\omega) \equiv F_{\tilde{\varepsilon}}\Big(\ \tilde{\varphi}(x,\omega) + T(1,x,\omega) - T(0,x,\omega) + \tilde{\delta}(x,\omega,P)\ \Big) \tag{28}$$

The functions $\tilde{\varphi}$ and $\tilde{\delta}$ are the same ones as in the "factual" mapping $\Psi(P)$ and they are identified from the factual choice probabilities. The function T only depends on the policy intervention and it can be obtained using the expression:

$$T(a,x,\omega) \quad = \quad h^*(a,x,\omega) - h(a,x,\omega) + \beta \int T(0,x',\omega')\ dF_\omega(\omega'|\omega)\ dF_x(x'|a,x) \tag{29}$$

3.2.2 Identification of behavioral effects when $F_{\tilde{\varepsilon}}$ is unknown

Under Assumptions 2 and 3 we can decompose the function φ in two components, $\varphi(a,x,\omega) = Y(a,x,\omega) + C(a,x)$ where the functions Y and C are implicitly defined by the recursive expressions:

$$\begin{aligned} Y(a,x,\omega) &= h(a,x,\omega) + \beta \int Y(0,x',\omega')\ dF_{\omega}(\omega'|\omega)\ dF_x(x'|a,x) \\ C(a,x) &= c(a,x) + \beta \int C(0,x')\ dF_x(x'|a,x) \end{aligned} \tag{30}$$

Therefore, the fixed point mapping $\Psi(P)$ can be written as:

$$\Psi(P)(x,\omega) \equiv F_{\tilde{\varepsilon}}\left(\ \tilde{Y}(x,\omega) + \tilde{C}(x) + \tilde{\delta}(x,\omega,P)\ |\ x\right) \tag{31}$$

where $\tilde{Y}(x,\omega) = Y(1,x,\omega) - Y(0,x,\omega)$ and $\tilde{C}(x) = C(1,x,\omega) - C(0,x,\omega)$. It is clear that, given the identification of h, the function $\tilde{Y}$ is identified. If the function $\tilde{\delta}$ were known the proof of identification of the probability distribution $F_{\tilde{\varepsilon}}$ would be very similar to the one in Proposition 1 for the static model. However, $\tilde{\delta}$ depends on $F_{\tilde{\varepsilon}}$ that is the object that we want to identify. Therefore, we have a "chicken-egg" problem we need to know $\tilde{\delta}$ to identify $F_{\tilde{\varepsilon}}$ but we need $F_{\tilde{\varepsilon}}$ to obtain $\tilde{\delta}$. If this problem has a unique fixed point, then the distribution of unobservable state variables is nonparametrically identified.

To show that $F_{\tilde{\varepsilon}}$ is identified we proceed in the following way. First, we show that the differential value function $\tilde{v}$ can be described as the unique fixed point of a mapping that depends only on the data and the discount factor β. This fixed point mapping $\Lambda(\tilde{v})$ is defined as:

$$\Lambda(\tilde{v})(x,\omega) = \tilde{Y}\left(x,\omega\right) - \tilde{Y}\left(x,\bar{\omega}(x)\right) + \tilde{\delta}\left(x,\omega,P,\tilde{v}\right) - \tilde{\delta}\left(x,\bar{\omega}(x),P,\tilde{v}\right) \tag{32}$$

The function $\bar{\omega}(x)$ is the value of ω that solves the equation $\tilde{v}(x,\omega) = 0$. Since the median of $\tilde{\varepsilon}$ is zero, we have that $F_{\tilde{\varepsilon}}(\tilde{v}(x,\bar{\omega}(x))|x) = 0.5$ and $P(x,\bar{\omega}(x)) = 0.5$. Therefore, we can obtain $\bar{\omega}(x)$ by solving the equation $P(x,\omega) = 0.5$ with respect to ω. Note that the function $\tilde{\delta}$ now has as arguments the functions P and $\tilde{v}$. This is because this function is based on a representation of the surplus function in terms of he functions P and $\tilde{v}$. More specifically,

$$G(x,\omega,P,\tilde{v}) \quad = \quad P(x,\omega)\ \tilde{v}(x,\omega) - \int_{-\infty}^{\omega} \tilde{v}(x,u)\frac{\partial P(x,u)}{\partial \omega}du \tag{33}$$

Proposition 5 shows that the mapping Λ is identified given $\tilde{Y}$ and P and that it is a contraction mapping. This Proposition also establishes that the functions $\tilde{\nu}$, $F_{\tilde{\varepsilon}}$ and $\tilde{\varphi}$ are nonparametrically identified.

PROPOSITION 5: Suppose that Assumptions 1 to 3 hold and that the discount factor β, the function $\tilde{Y}$, and the choice probability function P are known. Then, the mapping Λ is identified and it is a contraction mapping. It follows that the functions $\tilde{v}$ and $\tilde{\varphi}$ are identified on $X \times \Omega$, and that the probability distribution $F_{\tilde{\varepsilon}}$ is identified on $\tilde{\nu}(X \times \Omega) \times X$, where $\tilde{\nu}(X \times \Omega) \equiv \{\tilde{\nu}(x, \omega) : (x, \omega) \in X \times \Omega\}$.

Proposition 6 shows that the counterfactual probability function is nonparametrically identified and describes the procedure to compute this function.

PROPOSITION 6: Suppose that Assumptions 1 to 3 hold and that the discount factor β, the function $\tilde{Y}$, and the choice probability function P are known. Then, the counterfactual probability function P^* is identified. More specifically, P^* is the unique fixed point of the mapping Ψ^* defined in Proposition 4, where the functions $\tilde{\varphi}$ and $F_{\tilde{\varepsilon}}$ that appear in the definition of this mapping have been identified as we describe in Proposition 5.

Proposition 7 establishes the identification of the welfare effect function $\Delta \equiv (S^* - S)/\psi$.

PROPOSITION 7: Under the conditions in Proposition 6 the welfare effect function $\Delta \equiv (S^* - S)/\psi$ is *identified. We can get this function as:*

$$\Delta(x, \omega) = T(0, x, \omega) + \delta(0, x, \omega, P^*) - \delta(0, x, \omega, P) + G\left(P^*(x, \omega), F_{\tilde{\varepsilon}}|x\right) - G\left(P(x, \omega), F_{\tilde{\varepsilon}}|x\right) \tag{34}$$

4 Estimation method

This section presents a nonparametric procedure for the estimation of counterfactual choice probabilities that is based on the previous identification results. Suppose that we have a random sample of the agents' decisions, state variables and outcome variable at two consecutive periods. We use the subindex i to represent an observation for agent i. The sample size is n. We describe here a nonparametric procedure for the estimation of the functions P^* and Δ.

Step 1: Estimation of the outcome function h and the transition probabilities F_ω and F_x. Once we have estimated h, we can obtain the value of ω as residuals: $\omega_i = h^{-1}(y_i, a_i, x_i)$. We use these residuals at the two consecutive periods to estimate the transition probability function F_ω using a kernel method. Similarly, we estimate the transition function F_x.

Step 2: Estimation of the choice probability function P. We use a estimator that guarantees the smoothness and monotonicity of the estimator with respect to ω. In particular, we use the isotonic-smooth (IS) kernel estimator proposed by Mukerjee (1988) and Mammen (1991) and extended by Mukerjee and Stern (1994) to models with multiple explanatory variables. The estimator can be defined in two steps. Suppose that the observations have been sorted with respect to the variable ω, such that $\omega_1 \leq \omega_2 \leq ... \leq \omega_n$. The first step is an *isotonic regression* for $\{a_i\}$ on $\{\omega_i\}$*:*

$$\hat{P}_I(x_i, \omega_i) = \max_{s \leq i} \min_{t \geq s} \frac{\sum_{j=s}^{t} a_j}{t - s + 1} \tag{35}$$

The second step introduces smoothing by using a Nadaraya-Watson kernel estimator where the dependent variable is the isotonic regression $\{\hat{P}_I(x_i, \omega_i)\}$ and the explanatory variable is ω.

$$\hat{P}_{IS}(x, \omega) = \frac{\sum_{i=1}^{n} K\left(\frac{\omega - \omega_i}{b_n}\right) \hat{P}_I(x_i, \omega_i)}{\sum_{i=1}^{n} K\left(\frac{\omega - \omega_i}{b_n}\right)} \tag{36}$$

where b_n is the bandwidth. This estimator was first proposed by Mukerjee (1988). This estimator is always a smooth function and, when the kernel function is symmetric with maximum at zero, it is necessarily a non-decreasing function. It is consistent, asymptotically normal and first order asymptotically equivalent to the Nadaraya-Watson estimator. Therefore, the monotonicity restriction does not improve the first order asymptotics of the estimator. Mammen (1991) derived a second order approximation to the variance and bias of this estimator and showed that imposing the monotonicity restriction does reduces the finite sample variance and bias. Different Monte Carlo experiments have also found very significant gains in the finite sample performance (see Mammen, 1991, Dette et al., 2003, Gjbels, 2004, Aguirregabiria and Vicentini, 2005). These experiments show also that the isotonic-smooth estimator typical has better finite sample properties than the smooth-isotonic estimator in which we reverse the order of the kernel and isotonic regressions. Based on the experiments in Aguirregabiria and Vicentini (2005), we use cross validation for choice of bandwidth, where the cross-validation function is defined as if the dependent variable were the isotonic regression $\{\hat{P}_I(x_i, \omega_i)\}$.

Step 3: Estimation of the function $\bar{\omega}$. The function $\bar{\omega}$, from X into Ω, is defined as the value of ω that solves the equation $P(x, \omega) = 0.5$. Given our estimate $\hat{P}_{IS}$ of P, we use Newton's method to find $\bar{\omega}(x)$. Strict monotonicity of our estimator $\hat{P}_{IS}(x,.)$, implies that $\bar{\omega}(x)$ is unique and Newton's method always converges to this unique value.

Step 5: Estimation of the mapping Λ *and the function* $\tilde{v}$. The mapping Λ is defined in equation (32). Given our estimates in Steps 1 to 3, we can construct a consistent estimate of this mapping. This estimate is also a contraction mapping and its unique fixed point is a consistent estimator of the value function $\tilde{v}$.

Step 6: Estimation of the distribution function $F_{\tilde{\varepsilon}}$. Our estimator of the function $\tilde{v}$ is continuous and strictly increasing in ω. Therefore, there is an inverse function $\tilde{v}^{-1}(x,v)$ such that, for any $(x,v) \in X \times \tilde{v}(X \times \Omega)$, we have that $\tilde{v}(x, \tilde{v}^{-1}(x,v)) = v$. The model implies that $P(x,\omega) = F_{\tilde{\varepsilon}}(\tilde{v}(x,\omega))$. Therefore, it is clear that for any $(x,v) \in X \times \tilde{v}(X \times \Omega)$ we can obtain $F_{\tilde{\varepsilon}}(v|x)$ as $P(x, \tilde{v}^{-1}(x,v))$. Thus, our estimator of $F_{\tilde{\varepsilon}}$ is:

$$\hat{F}_{\tilde{\varepsilon}}\,(v \mid x) = \hat{P}_{IS}(x, \tilde{v}^{-1}(x,v)) \tag{37}$$

where $\tilde{v}^{-1}(x,v)$ is our estimator of the inverse function of $\tilde{v}$. For every value of x, the value $\tilde{v}^{-1}(x,v)$ can be easily obtained using Newton's method. Again, the strict monotonicity of $\hat{v}(x,.)$ guarantees that Newton's method always converges to $\tilde{v}^{-1}(x,v)$.

Step 7: Estimation of the functions $\tilde{\delta}$, $\tilde{C}$ and $\tilde{\varphi}$. Given $\tilde{v}$ and $\hat{F}_{\tilde{\varepsilon}}$, we have just to follow the definition of $\tilde{\delta}$ to obtain an estimator of this function. We know that $\tilde{\varphi}(x,\omega) = \tilde{Y}(x,\omega) + \tilde{C}(x)$. By definition of $\bar{\omega}(x)$, we have that $\tilde{C}(x) = -\tilde{Y}(x, \bar{\omega}(x)) - \tilde{\delta}(x, \bar{\omega}(x))$. Therefore, our estimator of the function $\tilde{C}$ is just the application of this formula using our estimates of $\tilde{Y}$, $\bar{\omega}$ and $\tilde{\delta}$. Then, the estimator of $\tilde{\varphi}$ is $\tilde{Y}$+ $\hat{C}$, where $\hat{C}$ is the estimator of $\tilde{C}$.

Step 8: Estimation of the mapping Ψ^* *and the functions* P^* *and* Δ. Given our estimate of $\tilde{\varphi}$, $\tilde{\delta}$ and $F_{\tilde{\varepsilon}}$ we can construct a consistent estimator of the mapping Ψ^* that we defined in Proposition 3. This mapping is a contraction and its unique fixed point is a consistent estimator of P^*. Finally, we apply the formula that defines the function Δ is Proposition 7 to obtain a consistent estimator of the welfare effect function.

The main computational cost in this procedure comes from the computation of the fixed points of the contraction mappings $\hat{\Lambda}$ and $\hat{\Psi}^*$. This cost is equivalent to solving the dynamic programming problem twice. It is of the same order of magnitude as estimating a parametric version of the model using the two-step method in Hotz and Miller (1993), or the nested pseudo likelihood algorithm in Aguirregabiria and Mira (2002). The Monte Carlo experiments in the next section provide an idea of the simplicity of this method. For a model with two state variables, $10,000$ cells in the state space, and $1,000$ observations, the CPU time of the whole method was less than six seconds using a program written in GAUSS language and an Intel Pentium processor of 2.2MHz. Though the computational cost increases exponentially with the number of cells in the state space, it is clear that we can use this method for any dynamic programming model that we can solve once in a reasonable amount time.

We do not derive in this paper the asymptotic distribution of our estimator of P^*. However, this estimator is consistent and asymptotically normal under standard regularity conditions. The Nadaraya-Watson estimator of P is consistent, and the estimators in steps 3 to 8 are continuous and differentiable functions of the estimator $\hat{P}_{IS}$. Therefore, all these estimators are consistent. The derivation of the rate of convergence of $(\hat{P}^* - P^*)$ is a more complicated problem that we do not consider in this paper. In any case, the computation of a consistent estimator of the asymptotic variance using a delta method is a complicated task. Furthermore, it is likely that this asymptotic variance is not a good approximation to the finite sample variance. In this context, a bootstrap method could be a most convenient and precise method to estimate the variance of this estimator.

5 An application

This section presents an application of this methodology to evaluate the effects of hypothetical reforms in the social security pension system in Sweden. The main purpose of this application is to illustrate the implementation of the method and to show that it can provide meaningful results in relevant contexts. There are several reasons that have motivated the choice of this particular application. The dataset has been used before by Karlstrom, Palme and Svensson (2004, KPS hereinafter) to estimate a parametric dynamic structural model of retirement. We consider a nonparametric version of their model. This previous study provides a useful benchmark to compare our results. Furthermore, the dataset is publicly available and it can be download from the web page of the *Journal of Applied Econometrics*. Therefore, interested readers can replicate our results.

The model is the one described in section 2.2. The data come from Longitudinal Individual Panel (LINDA).It is a subsample from LINDA with information on blue-collar workers in Sweden who were born between 1927 and 1931. The observation period is 1983 to 1997. The sample in KPS includes cohorts from 1927 to 1940. However, we have preferred to include only those cohorts for which most individuals are already retired by 1997. Our working sample has 1,063 individuals and 12,081 observations. Table 1 presents summary statistics for the variables that we use in this paper. Table 2 contains the empirical distribution of retirement age, including the percentage of censored observations.

TO BE COMPLETED

6 Summary and Conclusions

This paper presents a nonparametric approach to evaluate the behavioral and welfare effects of counterfactual policies using dynamic discrete structural models. The computational cost of this method is equivalent to solving the dynamic programming problem twice (i.e., before and after the policy change), and it applies both to finite horizon and infinite horizon decision processes. We have applied this method to evaluate the effects on individuals' retirement behavior of an hypothetical reform that changes the minimum retirement age to receive a social security pension. Our method can be also applied to test a parametric model. More specifically, this testing procedure tests whether the parametric assumptions in the model, as a whole, introduce a significant bias in our estimates of the effects of a policy.

APPENDIX

PROOF OF PROPOSITION 1.

[**1**] The factual choice probability function is:

$$P(x,\omega) \;=\; F_{\tilde{\varepsilon}}\left(\tilde{h}(x,\omega)+\tilde{c}(x) \mid x\right)$$

The function $P(.,.)$ is identified on $X\times\Omega$. Furthermore, by Assumptions 1(d) and 3(d), this probability function is strictly monotonic in ω. Define the function $\omega^*(x,u)$ from $X\times\tilde{u}(X\times\Omega)$ into Ω such that $\omega^*(x,u)$ is the value of ω that solves the equation $\tilde{h}(x,\omega)+\tilde{c}(x)=u$. Assumption 3(d) implies that $\omega^*(.,.)$ exits is a well-defined function on $X\times\tilde{u}(X\times\Omega)$. Assumption 3(e) implies that, for any $x\in X$, the pair $(x,0)$ belongs to $X\times\tilde{u}(X\times\Omega)$. Since $median(\tilde{\varepsilon}|x)=0$, we have that $P(x,\omega^*(x,0))=0.5$. Identification and invertibility of $P(.,.)$ implies that $\omega^*(x,0)$ is identified for any $x\in X$. Given $\omega^*(x,0)$ we can identify $\tilde{c}(x)$ as:

$$\tilde{c}(x)=-\tilde{h}(x,\omega^*(x,0))$$

Therefore, the function $\tilde{c}(.)$ is identified on X.

[**2**] Now, we prove the identification of $F_{\tilde{\varepsilon}}$. Given that we know $\tilde{c}(.)$, we can obtain $\omega^*(x,u)$ as the value of ω that solves the equation:

$$\tilde{h}(x,\omega)=u+\tilde{h}(x,\omega^*(x,0))$$

Then, by construction we have that for $(x,u)\in X\times\tilde{u}(X\times\Omega)$,

$$F_{\tilde{\varepsilon}}\left(u \mid x\right)=P(x,\omega^*(x,u))$$

Thus, $F_{\tilde{\varepsilon}}$ is identified on $X\times\tilde{u}(X\times\Omega)$.

[**3**] The counterfactual choice probability function is $P^*(x,\omega)=F_{\tilde{\varepsilon}}\left(\tilde{h}^*(x,\omega)+\tilde{c}(x)\mid x\right)$. Given that h^* is known and $F_{\tilde{\varepsilon}}$ and $\tilde{c}$ are identified, it is clear that $P^*(x,\omega)$ is identified at any pair (x,ω) such that $\tilde{h}^*(x,\omega)+\tilde{c}(x)\in\tilde{u}(X\times\Omega)$.

PROOF OF PROPOSITION 2. Using the definition of the integrated value function, we have that the welfare effect function is:

$$\begin{array}{rcl}\Delta(x,\omega) & = & \dfrac{1}{\psi(x)}\displaystyle\int \max_{a\in A}\{\ u^{*}(a,x,\omega)+\varepsilon(a)\ \}\, dF_{\varepsilon}(\varepsilon|x) - \dfrac{1}{\psi(x)}\displaystyle\int \max_{a\in A}\{\ u(a,x,\omega)+\varepsilon(a)\ \}\, dF_{\varepsilon}(\varepsilon|x) \\ & = & \tau(0,x,\omega) \\ & + & \displaystyle\int \max\left\{\ \tilde{h}^{*}(x,\omega)+\tilde{c}(x)-\tilde{\varepsilon}\ ;\ 0\right\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x) - \displaystyle\int \max\left\{\ \tilde{h}(x,\omega)+\tilde{c}(x)-\tilde{\varepsilon}\ ;\ 0\right\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x)\end{array}$$

Now,

$$\begin{array}{rcl}\displaystyle\int \max\left\{\ \tilde{h}(x,\omega)+\tilde{c}(x)-\tilde{\varepsilon}\ ;\ 0\right\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x) & = & P(x,\omega)\ E\left(\tilde{h}(x,\omega)+\tilde{c}(x)-\tilde{\varepsilon}\ |\ x,\, \tilde{\varepsilon}\leq \tilde{h}(x,\omega)+\tilde{c}(x)\right) \\ & = & P(x,\omega)\ \left(\tilde{h}(x,\omega)+\tilde{c}(x)\right) - \displaystyle\int_{-\infty}^{\tilde{h}(x,\omega)+\tilde{c}(x)} \tilde{\varepsilon}\ dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x)\end{array}$$

And given that the optimal choice probability function $P(x,\omega)$ is $F_{\tilde{\varepsilon}}(\tilde{h}(x,\omega)+\tilde{c}(x)|x)$ and that $F_{\tilde{\varepsilon}}(.|x)$ is invertible, we have that:

$$\begin{array}{rcl}\displaystyle\int \max\left\{\ \tilde{h}(x,\omega)+\tilde{c}(x)-\tilde{\varepsilon}\ ;\ 0\right\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x) & = & P(x,\omega)\ F_{\tilde{\varepsilon}}^{-1}(P(x,\omega)|x) - \displaystyle\int_{-\infty}^{F_{\tilde{\varepsilon}}^{-1}(P(x,\omega)|x)} \tilde{\varepsilon}\ dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}|x) \\ & = & G\left(P(x,\omega), F_{\tilde{\varepsilon}}|x\right)\end{array}$$

Thus,

$$\Delta(x,\omega) = \tau(0,x,\omega) + G\left(P^{*}(x,\omega), F_{\tilde{\varepsilon}}|x\right) - G\left(P(x,\omega), F_{\tilde{\varepsilon}}|x\right)$$

PROOF OF PROPOSITION 3. For notational simplicity we use z to denote the pair (x, ω) and the function $F(z'|a,z)$ to represent $F_\omega(\omega'|\omega)F_x(x'|a,x)$.

[**1**] Given the definition of the surplus function $G(P, F_{\tilde{\varepsilon}}|z)$, we have that:

$$\frac{1}{\psi(x)} \int \max\left\{v(0,z)+\varepsilon(0)\ ;\ v(1,z)+\varepsilon(1)\right\} dF_\varepsilon(\varepsilon|z) \quad = \frac{v(0,z)}{\psi(x)} + G(P(z), F_{\tilde{\varepsilon}}|z)$$

Solving this expression in equation (7) that defines the conditional choice value function $v(a,z)$, we have that:

$$\frac{v(a,z)}{\psi(x)} \quad = \quad \frac{u(a,z)}{\psi(x)} + \beta \int \frac{v(0,z')}{\psi(x')}\ dF(z'|a,z) + \beta \int G(P(z'), F_{\tilde{\varepsilon}}|z')\ dF(z'|a,z)$$

We can apply the same decomposition to the value $v(0,z')/\psi(x')$ that appears in this expression. If we do this, we get:

$$\begin{aligned}\frac{v(a,z)}{\psi(x)} \quad &= \quad \frac{u(a,z)}{\psi(x)} + \beta \int \frac{u(0,z')}{\psi(x')}\ dF(z'|a,z) + \beta^2 \int \frac{v(0,z'')}{\psi(x'')}\ dF(z''|0,z')\ dF(z'|a,z) \\ &+ \quad \beta \int G(P(z'), F_{\tilde{\varepsilon}}|z')\ dF(z'|a,z) + \beta^2 \int G(P(z''), F_{\tilde{\varepsilon}}|z'')\ dF(z''|0,z')\ dF(z'|a,z)\end{aligned}$$

If we continue applying the decomposition to $v(0,z'')/\psi(x'')$, $v(0,z''')/\psi(x''')$, and so on, we get:

$$\begin{aligned}\frac{v(a,z)}{\psi(x)} \quad &= \quad \frac{u(a,z)}{\psi(x)} + \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int \frac{u(0,z^j)}{\psi(x^j)} \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right)\ dF(z'|a,z)\right] \\ &+ \quad \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int G(P(z^j), F_{\tilde{\varepsilon}}|z^j) \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right)\ dF(z'|a,z)\right]\end{aligned}$$

The first two terms in the right hand side provide the present value of choosing alternative a today and then alternative 0 forever in the future given that the current state is z. The third term in the right hand side is the difference between the value of behaving optimally in the future and the value of choosing always alternative 0. Then, given the definitions of $\varphi(a,z)$ and $\delta(a,z)$ in the enunciate of this Proposition, it is clear that $v(a,z)/\psi(x) = \varphi(a,z) + \delta(a,z,P)$ where:

$$\varphi(a,z) = \frac{u(a,z)}{\psi(x)} + \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int \frac{u(0,z^j)}{\psi(x^j)} \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right)\ dF(z'|a,z)\right]$$

and

$$\delta(a,z,P) = \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int G(P(z^j), F_{\tilde{\varepsilon}}|z^j) \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right)\ dF(z'|a,z)\right]$$

Given these expressions, it is straightforward to show that we can obtain $\varphi(a,z)$ and $\delta(a,z,P)$ recursively as:

$$\varphi(a,z) \;\; = \;\; \frac{u(a,z)}{\psi(x)} + \beta \int \varphi(0,z') \; dF(z'|a,z)$$

and

$$\delta(a,z,P) \;\; = \;\; \beta \textstyle\int \left(\; G(P(z'),F_{\tilde{\varepsilon}}|z') + \delta(0,z',P) \; \right) \; dF(z'|a,z)$$

[**2**] Thus, $v(a,z)/\psi(x) = \varphi(a,z) + \delta(a,z,P)$. This implies that the expression $P(z) = F_{\tilde{\varepsilon}}(\tilde{v}(z)|z)$ can be rewritten as:

$$P(z) = F_{\tilde{\varepsilon}}(\tilde{\varphi}(z) + \tilde{\delta}(z,P))$$

Therefore, the optimal choice probability function P is a fixed point of the mapping $\Psi(P)$ where $\Psi(P)(z) \equiv F_{\tilde{\varepsilon}}(\tilde{\varphi}(z) + \tilde{\delta}(z,P))$. This is a particular case of the fixed point probability mapping in Aguirregabiria and Mira (2002). Proposition 1(i) in Aguirregabiria and Mira (2002) shows that this mapping has a unique fixed point.

PROOF OF PROPOSITION 4. By Proposition 3, the counterfactual probability function P^* is the unique fixed point of the mapping $\Psi^*(P)$, where

$$\Psi^*(P)(x,\omega) \equiv F_{\tilde{\varepsilon}}\left(\; \tilde{\varphi}^*(x,\omega) + \tilde{\delta}^*(x,\omega,P) \; \right)$$

where $\tilde{\varphi}^*$ and $\tilde{\delta}^*$ are the functions associated with the counterfactual utility function $u^*(a,x,\omega)$.

[1] **Identification of** $\tilde{\delta}^*$. By the definition of δ in Proposition 3, we can see that this function depends on the probability distribution $F_{\tilde{\varepsilon}}$ and on the discount factor β. Since these two functions are invariant in our policy experiment, we have that $\tilde{\delta}^*(x,\omega,P) = \tilde{\delta}(x,\omega,P)$ and this function is identified.

[2] **Identification of** $\tilde{\varphi}^*$: Taking into account the definition of φ in the proof of Proposition 3 we have that:

$$\varphi^*(a,z) \;\; = \;\; \frac{u^*(a,z)}{\psi(x)} + \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int \frac{u^*(0,z^j)}{\psi(x^j)} \;\; \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right) \; dF(z'|a,z)\right]$$

Given that $u^*(a,z)/\psi(x) = u(a,z)/\psi(x) + \tau(a,z)$ we have that:

$$\begin{aligned}
\varphi^*(a,z) \;\; &= \;\; \frac{u(a,z)}{\psi(x)} + \sum\nolimits_{j=1}^{\infty} \beta^j \left[\int \frac{u(0,z^j)}{\psi(x^j)} \;\; \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right) \; dF(z'|a,z)\right] \\
&+ \;\; \tau(a,z) + \sum\nolimits_{j=1}^{\infty} \beta^j \;\; \left[\int \tau(0,z^j) \;\; \left(\prod\nolimits_{i=2}^{j} dF(z^j|0,z^{j-1})\right) \; dF(z'|a,z)\right] \\
&= \;\; \varphi(a,z) + T(a,z)
\end{aligned}$$

where $T(a,z)$ is the term associated with the present value of the function τ. Therefore,

$$\tilde{\varphi}^*(x,\omega) = \tilde{\varphi}(x,\omega) + T(1,x,\omega) - T(0,x,\omega)$$

The function T can be obtained as the fixed point of the contraction mapping:

$$T(a,z) = \tau(a,z) + \beta \int T(0,z') \; dF(z'|a,z)$$

Since the function τ is known, it is clear that T is identified.

[3] **Identification of** P^***.** P^* is the unique fixed point of the mapping $\Psi^*(P)$, where:

$$\Psi^*(P)(x,\omega) \equiv F_{\tilde{\varepsilon}}\left(\; \tilde{\varphi}(x,\omega) + T(1,x,\omega) - T(0,x,\omega) + \tilde{\delta}(x,\omega,P) \mid x\right)$$

We have shown that the functions $\tilde{\varphi}$, T and $\tilde{\delta}$ are identified. Thus, given $F_{\tilde{\varepsilon}}$, the counterfactual probability function is identified.

PROOF OF PROPOSITION 5. The proof proceeds in three steps. First, we show that the surplus function can be written in terms of the arguments $\tilde{v}(x,\omega)$ and P, instead of the arguments $P(x,\omega)$ and $F_{\tilde{\varepsilon}}$. Second, we show that the function $\tilde{v}$ is the unique fixed point of a contraction mapping that depends only on the identified functions $\tilde{Y}$ and P. Finally, we show that given $\tilde{v}$, $\tilde{Y}$ and P we can identify $F_{\tilde{\varepsilon}}$ and $\tilde{\varphi}$.

[1] An alternative representation of the surplus function. The surplus function for this binary choice model is $G(x,\omega) = \displaystyle\int \max\left\{\tilde{v}(x,\omega) - \tilde{\varepsilon}\ ;\ 0\right\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon})$. This function depends on $\tilde{v}(x,\omega)$ and $F_{\tilde{\varepsilon}}$. Given the one-to-one relationship between $\tilde{v}$ and P, we have shown that $G(x,\omega)$ can be written in terms of $P(x,\omega)$ and $F_{\tilde{\varepsilon}}$. Now, we show that the function can be written in terms of $\tilde{v}(x,\omega)$ and the functions $\tilde{v}$ and P.

$$G(x,\omega) = P(x,\omega)\ \tilde{v}(x,\omega) - \int_{-\infty}^{\omega} \tilde{v}(x,u)\frac{\partial P(x,u)}{\partial \omega} du$$

To emphasize the dependence with respect to $\tilde{v}$ and P we write this function as $G(x,\omega,P,\tilde{v})$.

Associated with this surplus function we can redefine the function $\tilde{\delta}$ as $\delta(1,x,\omega) - \delta(1,x,\omega)$ where:

$$\delta(x,\omega) \quad = \quad \beta \textstyle\int \left(\ G(x',\omega') + \delta(0,x',\omega')\ \right)\ dF_{\omega}(\omega'|\omega)\ dF_{x}(x'|a,x)$$

We also write $\tilde{\delta}(x,\omega,P,\tilde{v})$ to emphasize that this function depends on $\tilde{v}$ and P.

[2] Fixed point mapping for $\tilde{v}$. The function $\tilde{v}$ solves the functional equation $\tilde{v}(x,\omega) = \tilde{\varphi}(x,\omega) + \tilde{\delta}(x,\omega,P,\tilde{v})$. However, we cannot use this representation to identify $\tilde{v}$ because this fixed point mapping depends on $\tilde{\varphi}$ that is unknown. Now, we define $\tilde{v}$ as the fixed point of a different mapping that is identified. Define the function $\bar{\omega}(x)$ is the value of ω that solves the equation $\tilde{v}(x,\omega) = 0$. Since the median of $\tilde{\varepsilon}$ is zero, $F_{\tilde{\varepsilon}}(\tilde{v}(x,\bar{\omega}(x))|x) = 0.5$ and $P(x,\bar{\omega}(x)) = 0.5$. Therefore, we can obtain $\bar{\omega}(x)$ by solving the equation $P(x,\omega) = 0.5$ with respect to ω. Therefore, $\bar{\omega}(x)$ is identified on $X \times \tilde{v}(X \times \Omega)$. Remember that $\tilde{v}(x,\omega) = \tilde{Y}(x,\omega) + \tilde{C}(x) + \tilde{\delta}(x,\omega)$. Therefore, by definition of $\bar{\omega}(x)$, we have that $\tilde{C}(x) = -\tilde{Y}(x,\bar{\omega}(x)) - \tilde{\delta}(x,\bar{\omega}(x))$. Taking this into account we can write:

$$\tilde{v}(x,\omega) = \tilde{Y}(x,\omega) - \tilde{Y}(x,\bar{\omega}(x)) + \tilde{\delta}(x,\omega,P,\tilde{v}) - \tilde{\delta}(x,\bar{\omega}(x),P,\tilde{v})$$

The right hand side of this equation is the mapping Λ evaluated at function $\tilde{v}$ and point (x,ω), i.e., $\Lambda(\tilde{v})(x,\omega)$.

Now, we show that Λ is a contraction mapping and therefore it has a unique fixed point. To prove this we use Blackwell's sufficient conditions for a contraction (see Theorem 3.3 in Stockey and Lucas, 1989). These sufficient conditions are monotonicity and discounting.

(a) Monotonicity: We should prove that for any two functions $\tilde{v}^0$ and $\tilde{v}^1$ such that $\tilde{v}^1(x,\omega) - \tilde{v}^0(x,\omega) \geq 0$ for any $(x,\omega) \in X \times \Omega$, then $\Lambda(\tilde{v}^1)(x,\omega) - \Lambda(\tilde{v}^0)(x,\omega) \geq 0$ for any $(x,\omega) \in X \times \Omega$. Using the definition of the mapping Λ above, a sufficient condition for the second inequality is that $G(x,\omega,P,\tilde{v}^1) - G(x,\omega,P,\tilde{v}^0) \geq 0$ for any $(x,\omega) \in X \times \Omega$. Note that:

$$G(x,\omega,P,\tilde{v}^1) - G(x,\omega,P,\tilde{v}^0) = P(x,\omega)\ \left(\tilde{v}^1(x,\omega) - \tilde{v}^0(x,\omega)\right)\ - \int_{-\infty}^{\omega} \left(\tilde{v}^1(x,u) - \tilde{v}^0(x,u)\right) \frac{\partial P(x,u)}{\partial \omega} du$$

Solving by parts the integral, it is straightforward to show that:

$$G(x,\omega,P,\tilde{v}^1) - G(x,\omega,P,\tilde{v}^0) = \int_{-\infty}^{\omega} P(x,u) du \geq 0$$

(b) Discounting: We should prove that the exists some constant $\lambda \in [0,1)$ such that for any function $\tilde{v}$, any constant c, and any $(x,\omega) \in X \times \Omega$ we have that $\Lambda(\tilde{v}+c)(x,\omega) \leq \Lambda(\tilde{v})(x,\omega) + \lambda c$. We start obtaining $\tilde{G}(x,\omega,P,\tilde{v}+c)$.

$$\begin{aligned} G(x,\omega,P,\tilde{v}+c) &= P(x,\omega)\ \left(\tilde{v}(x,\omega)+c\right)\ - \int_{-\infty}^{\omega} \left(\tilde{v}(x,u)+c\right) \tfrac{\partial P(x,u)}{\partial \omega} du \\ &= G(x,\omega,P,\tilde{v}) + c\ P(x,\omega) - c\ \int_{-\infty}^{\omega} \tfrac{\partial P(x,u)}{\partial \omega} du = G(x,\omega,P,\tilde{v}) \end{aligned}$$

Therefore, there is discounting in the surplus function. Furthermore, given the definition of Λ, it is clear that $\Lambda(\tilde{v}+c)(x,\omega) = \Lambda(\tilde{v})(x,\omega)$, i.e., there is discounting in the mapping Λ.

[3] **Identification of $\tilde{v}$.** The mapping Λ is identified and it is a contraction. Therefore, its unique fixed point $\tilde{v}$ is identified on $\tilde{v}(X \times \Omega)$.

[4] **Identification of $F_{\tilde{\varepsilon}}$.** The function $\tilde{v}$ is continuous and strictly increasing in ω. Therefore, there is an inverse function $\tilde{v}^{-1}(x,v)$ such that, for any $(x,v) \in X \times \tilde{v}(X \times \Omega)$, we have that $\tilde{v}(x,\tilde{v}^{-1}(x,v)) = v$. The model implies that $P(x,\omega) = F_{\tilde{\varepsilon}}(\tilde{v}(x,\omega))$. Therefore, it is clear that for any $(x,v) \in X \times \tilde{v}(X \times \Omega)$ we can obtain $F_{\tilde{\varepsilon}}(v|x)$ as $P(x,\tilde{v}^{-1}(x,v))$. Thus, $F_{\tilde{\varepsilon}}$ is identified.

[5] **Identification of $\tilde{\varphi}$.** We know that $\tilde{\varphi}(x,\omega) = \tilde{Y}(x,\omega) + \tilde{C}(x)$ and we have shown above that $\tilde{C}(x) = -\tilde{Y}(x,\bar{\omega}(x)) - \tilde{\delta}(x,\bar{\omega}(x))$. Given $F_{\tilde{\varepsilon}}$ and $\tilde{v}$, the function $\tilde{\delta}$ is identified and therefore $\tilde{C}$ and $\tilde{\varphi}$ are identified as well.

PROOF OF PROPOSITION 6. By Proposition 3, P^* is the unique fixed point of the mapping Ψ^*. This mapping depends on the known functions $T(1,.) - T(0,.)$, on the discount factor β, and on the functions $\tilde{\varphi}$ and $F_{\tilde{\varepsilon}}$. By Proposition 5, the functions $\tilde{\varphi}$ and $F_{\tilde{\varepsilon}}$ are identified given β, $\tilde{Y}$, and P. Therefore, the mapping Ψ^* and its unique fixed point P^* are identified.

PROOF OF PROPOSITION 7. By definition, we have that:

$$\begin{aligned}
\Delta(x,\omega) &= \frac{1}{\psi(x)} \int \max_{a \in A} \{\ v^*(a,x,\omega) + \varepsilon(a)\ \}\, dF_\varepsilon(\varepsilon|x) - \frac{1}{\psi(x)} \int \max_{a \in A} \{\ v(a,x,\omega) + \varepsilon(a)\ \}\, dF_\varepsilon(\varepsilon|x) \\
&= v^*(0,x,\omega) - v(0,x,\omega) + G\left(P^*(x,\omega), F_{\tilde{\varepsilon}}|x\right) - G\left(P(x,\omega), F_{\tilde{\varepsilon}}|x\right)
\end{aligned}$$

By Proposition 3, $v^*(0,x,\omega) - v(0,x,\omega) = T(0,x,\omega) + \delta(0,x,\omega,P^*) - \delta(0,x,\omega,P)$. Thus,

$$\Delta(x,\omega) = T(0,x,\omega) + \delta(0,x,\omega,P^*) - \delta(0,x,\omega,P) + G\left(P^*(x,\omega), F_{\tilde{\varepsilon}}|x\right) - G\left(P(x,\omega), F_{\tilde{\varepsilon}}|x\right)$$

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Table 1
Summary Statistics
1,063 individuals. Cohorts 1927-1931

Variable	Mean	Std. Dev.	Min	Max	# Obs.
Female	0.339	0.473	0	1	1,063
Married	0.748	0.434	0	1	1,063
Retired in 1997	0.934	0.248	0	1	1,063
Retirement Age	64.16	1.82	53	69	1,063
Wage at Retirement Age (in thousands of Swedish Kronas)	161.56	48.48	37.00	334.00	993
Pension Points at Retirement Age	3.77	1.28	0.10	6.50	993

Table 2
Empirical Distribution of Retirement Age
1,063 individuals. Cohorts 1927-1931

Retirement Age	Individuals (%)	Number of Individuals Not Retitred in1997
53	2 (0.2)	0
54	1 (0.1)	0
55	2 (0.2)	0
56	1 (0.1)	0
57	6 (0.6)	0
58	4 (0.4)	0
59	7 (0.7)	0
60	40 (4.0)	0
61	32 (3.2)	0
62	36 (3.6)	0
63	54 (5.4)	0
64	191 (19.2)	0
65	564 (56.8)	0
66	29 (2.9)	22
67	13 (1.3)	14
68	9 (0.9)	12
69	2 (0.2)	13
70	0 (0.0)	9