Polarization under Incomplete Markets and Endogenous Labor Productivity

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Abstract

We explore the accumulation of assets in the presence of limited insurance against idiosyncratic shocks, borrowing constraints and endogenous labor productivity due to the so-called “nutrition curve”. We show that in such an environment, any stationary equilibrium is characterized by a polarized distribution of wealth. That is, there are only extremely rich and extremely poor agents.

Keywords: Idiosyncratic shocks, incomplete markets, labor supply.
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1 Introduction

In this paper we study the long run distribution of wealth corresponding to an economy where agents are subject to idiosyncratic shocks of unemployment, and where all markets are competitive but insurance markets are incomplete. We show that the stationary distribution of wealth is polarized. That is, in the long run there are only extremely rich and extremely poor agents, with no middle class in between, hence, there is no mobility between the two extremes. In addition, in our model the equilibrium return to assets is equal to the rate of time preference, and there are many distributions of wealth compatible with equilibrium. We think these results are interesting for at least, the three reasons we explain below.

First, in related incomplete markets models, such as Aiyagari (1994) and Huggett (1993, 1997), the stationary distribution of wealth puts positive mass in all of the support of the distribution, and thus, there is a lot of mobility. Furthermore, in these models, the equilibrium prices are such that the return to assets is smaller than the rate of time preference, and they uniquely determine the long run distribution of wealth. The main departures from the standard incomplete markets setup that explain our results are that labor supply of agents in the good (employment) state is endogenous, and that their labor productivity depends on their level of consumption. The assumption of endogenous labor has been widely used in many studies about real business cycles (see for instance, Kydland and Prescott (1982) and the vast literature following that paper), and its effects under incomplete markets economies have been previously studied, among others, by Rios-Rull (1994), Diaz-Gimenez (1998), Obiols-Homs (2003), and Marcet, Obiols-Homs and Weil (2004) (MOW from now on). The interest in economics in the linkage between nutrition intakes and productivity of labor goes back, at least, to Leibenstein (1957) and Mazumdar (1959).[1] In addition, there is a large body of empirical results supporting this link.[2] Therefore, we do not perceive these two assumptions as implying a large deviation from existing theory or empirical evidence.

Second, in several papers studying the implications of incomplete insurance

[1]Also, Mirlees (1975), Stiglitz (1976, 1982), and Dasgupta and Ray (1986), among many others, formalized and extended the efficiency wages hypothesis, and pointed to the so-called "nutrition curve" as an explanation for involuntary unemployment.

[2]See Strauss (1986), and the related studies in Pitt and Rosenzweig (1985), Behrman and Deolalikar (1987, 1990), Pitt, Rosenzweig and Hassan (1990), and Subramanian and Deaton (1996).

with respect to real business cycles, taxation, goodness of fit of the model, political economics, etc., the authors choose to calibrate their model economy so that at the steady state, the predicted distribution of wealth reproduces a predetermined set of observations (such as measures of inequality and skeeweness) of actual distributions (see for instance Krusell and Smith (1998) and Pijoan-Mas (2003), among others). This strategy can be seriously misleading because the predictions of the models are extremely non robust to small changes, such as the introduction of the "nutrition curve".

Third, our results suggest that perfect competition together with the development of credit markets, will not reduce inequality nor favor growth and development, as is sometimes asserted in policy recommendations. Rather, what is needed to reduce inefficiencies is what standard competitive theory a la Arrow-Debreu dictates: complete insurance markets. Some government intervention should be designed if polarization is to be avoided (Atkeson and Lucas).

The intuition for our results can be described as follows. As shown in MOW, when leisure is a normal good a sufficiently rich agent chooses not to work because of a wealth effect on her labor supply. The assumption that the efficiency units of labor supplied by agents increases with their consumption (although it is bounded above) implies that at low consumption levels, labor productivity is also low, and therefore, agents prefer not to work, and in stead, enjoy their leisure. This means that in our model, both, rich agents that can afford a high consumption level, and poor agents that barely consume, do not face uncertainty because they choose not to work irrespectively of their occupational status. Then, if the return of assets was larger (or smaller) than the rate of time preference, asset holdings of *all* agents would be, in the long run, sufficiently large (or sufficiently small) so that non of them would work. At those asset levels the assets market cannot clear. Thus, the only possible equilibria are such that the return of assets is equal to the rate of time preference. With those prices, however, the assets of agents that receive a sufficiently long sequence of good (or bad) idiosyncratic shocks converge to the levels where they choose not to work. This is why the distribution of wealth is polarized in the long run. Furthermore, it is possible to construct a continuum of equilibrium distributions simply by putting more/less mass of agents beyond the critical levels of assets where they choose not to work. In other words, in our model the Monotone Mixing Condition sufficient for uniqueness in Theorem 2 in Hopenhayn and Prescott (1992), is not satisfied.

One may argue that our story for polarization through the effects of the "nutrition curve" is relevant in developing/rural economies, but that it has little to do in developed economies with a welfare state. After all, informal evidence from developing countries suggests that in those economies income and wealth are rather unequally distributed. However, it is worth emphasizing that a polarized distribution in the long run is the result of non convexities in budgets sets (together with leisure being a normal good).

The paper continues as follows: section 2 introduces the assumptions that are sustained throughout the analysis; section 3 characterizes optimal decision rules; section 4 states the main results of the paper about the stationary distribution of wealth. The appendix contains all proofs.

2 The model

2.1 Assumptions

Time is discrete and goes on forever. The economy consists of a continuum of ex-ante identical consumers, indexed by i, uniformly distributed in the unit interval. Consumers have instantaneous preferences $U(c,l)$ defined over consumption and leisure, as formalized in assumptions A1-A3.

A1: $U(c,l) = u(c) + v(l) : R_+ \times [0,1] \to R$, is continuous and differentiable.

A2: $u(c)$ is increasing and concave. There are $0 \leq \underline{c} < \bar{c}$, such that $\lim_{c \to 0} u'(c) = 1/\underline{c}$, and such that $\lim_{c \to \bar{c}} u'(c) = 0$.

A3: $v(l)$ is strictly increasing and strictly concave, with $\lim_{l \to 0} v'(l) = +\infty$.

Notice that in A2 we do not necessarily require an Inada condition on zero consumption (this case is obtained with $\underline{c} = 0$). Also, the case of $\bar{c} < \infty$ allows for satiation in consumption. These possibilities will play a central role in several results of the paper. To the extent that $\underline{c}$ can be made arbitrarily small, and that $\bar{c}$ can be made arbitrarily large, assumption A2 does not seem too restrictive. A3 is a standard assumption in models with endogenous labor supply decisions. We introduce leisure in the preferences mainly for technical reasons: as shown in MOW, with endogenous leisure/labor supply there may be a stationary distribution of wealth even if the return to savings equals the rate of time preference.

An agent's endowment has two parts. The first part consists of one unit of time, and of θ units of the consumption good (non random, and the same for all agents). We think of θ as fruits that land produces espontaneously. The second part is an idiosyncratic endowment of labor productivity (like employment shocks, or health shocks). We therefore model this idiosyncratic state as a random variable s_t that takes the values 0 and 1. Formally,

A4: $s_t^i \in S \equiv \{1, 0\}$ with $\sum_{s'} \pi_{s'|s} = 1$ and $\pi_{s'|s} > 0$ for all $s, s' \in S$, and is independent across i.

The assumptions introduced up to now are similar to those in other models with idiosyncratic shocks, such as Aiyagari (1994), Huggett (1993, 1997), and Krusell and Smith (1998), with the only exception of assuming endogenous labor as it is introduced in MOW. We now depart from the standard setup by assuming that the productivity of the time spent at work depends on the level of consumption of the agent. That is, if a consumer spends $(1 - l_t)$ units of her time at work, then her labor supply in efficiency units is given by $P(c_t)(1 - l_t)$ in the "good" or productive state, and zero otherwise. The function $P(c)$ satisfies:

A5: i) $P(c) : R_+ \to [0, 1]$, is continuous, differentiable, strictly increasing and concave, with $P'(c) < 1$ for all $c \geq 0$; ii) $P(0) = 0$ and $\lim_{c\to\infty} P(c) = 1$.

The empirical evidence supporting A5 can be found, for instance, in Strauss (1986).[3] The concavity assumption in part i) simplifies the derivation of the results, which, as we discuss later, would also follow under the assumption of convex-concave $P(c)$ similar to that in Dasgupta and Ray (1986). The normalization of $P(0) = 0$ seems appropriate if we think of time periods as weeks or longer spells. That productivity is constant in the limit is assumed in an effort to keep the model close to the usual case for large consumption.

We assume that output in period t is given by a linear function of aggregate labor in efficiency units, such that one unit of labor is worth one unit of output. To complete the model, we assume that consumers can save in a riskless bond (for simplicity, we assume that this is the only available asset in the economy). Our interpretation is that there is a central authority that gives credit balances at a cost q and that accepts saving accounts offering a return $(1 - q)/q$. That is, to obtain a credit balance of one unit of consumption goods in the next period, the consumers must pay q units of goods in the current period. Finally, bond holdings are subject to a borrowing limit such that consumers face the constraint $b_t^i \geq \underline{B}$.

We introduce for technical reasons the following two assumptions.

A6: i) There are $c^- > 0$ and $c^+ < \infty$ such that, for all $c \in [c^-, c^+]$, the identity $u'(c)P(c) - v'(l)(1 - P'(c)(1 - l)) \equiv 0$, implicitly defines a function $l_f : [c^-, c^+] \to [0, 1]$; ii) furthermore, $u'(c)/(1 - P'(c)(1 - l_f(c))$ is

[3] Strauss (1986) contains many other interesting references.

monotonically decreasing in c.

A7: Given any $\overline{\gamma}$, define c^0 and c^1 (as a function of $\overline{\gamma}$) by the following equations

$$\frac{u'(c^1)}{1 - P'(c^1)(1 - l^f(c^1))} = u'(c^0) = \overline{\gamma}$$

We require that if $u'(c^-) > \overline{\gamma} > u'(c^+)$ then

$$c^1 - P(c^1)(1 - l^f(c^1)) \neq c^0. \tag{1}$$

The identity in A6 i) is simply the result of combining the first order conditions for optimality with respect to consumption and leisure for an agent's problem when leisure is interior (see section 2.2). Since the introduction of the function P in A5 makes the budget set to be concave, A6 i) ensures that for levels of $c \in [c^-, c^+]$ there is only one possible level for leisure satisfying the optimality conditions. The existence of c^- and c^+ will be established in Lemma 1 below. The expression in A6 ii) corresponds to the lagrange multiplier associated to the budget constraint of an agent's problem (see Equation (3) in section 2.2). It is straightforward (but tedious) to show that a sufficient condition for A6 ii) is $cP'(c)/P(c) > -u''(c)/u'(c)c$. That is, the Coefficient of Relative Risk Aversion cannot be too large relative to the growth rate of the efficiency of labor hours.

2.2 An agent's problem

The problem of an agent that discounts expected future utility with a factor $\beta \in (0, 1)$ (which is the same for all agents), and that faces a constant price q for bonds, can be written formally as follows:

$$\begin{aligned} \max_{\{c_t, l_t, b_t\}} \quad & E \sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(l_t)\} \\ \text{s. to} \quad & c_t + qb_t \leq \theta + s_t P(c_t)(1 - l_t) + b_{t-1}, \\ & c_t, l_t \geq 0,\ l_t \leq 1,\ b_t \geq \underline{B}, \end{aligned} \tag{2}$$

and the usual transversality condition.

Remark 1: $q > 0$, otherwise the consumer would be willing to issue an infinite amount of debt.

It is instructive to study first the set of constraints in the previous problem. To this end, assume for a moment that the optimal stochastic process for

$\{b_t\}$ solving the previous problem has already been found, and define net wealth as $\tilde{\theta} = \theta + b_{t-1} - q_t b_t$. With this definition the budget constraint of an employed agent can be written as $c \leq \tilde{\theta} + sP(c)(1-l)$. Then, we can study the relation between c and l for different values of $\tilde{\theta}$. If we write $l(c, \tilde{\theta}) = 1 - (c - \tilde{\theta})/P(c)$, then $\lim_{c \to \tilde{\theta}} l_c(c, \theta) < 0$ and $\lim_{c \to \tilde{\theta}} l_{cc}(c, \theta) > 0$. That is, the set of constraints is not convex. Since we have an Inada condition at zero leisure, and marginal utility of consumption is bounded for zero consumption, indifference curves are rather flat for low levels of leisure and not too steep for low levels of consumption. This means that there may be corner solutions for leisure when $\tilde{\theta}$ is sufficiently small. That is, leisure is not a normal good for low levels of net wealth: a poor agent will choose not to work because although her opportunity cost of leisure is small, the return to her work is even smaller. Likewise, an agent with a sufficiently large $\tilde{\theta}$ will choose not to work because the opportunity cost of her leisure increases with her consumption (or wealth) (see Figure 1 for a diagrammatic exposition).

We will now establish more formally these facts. The first order conditions necessary for optimality that a solution to the problem in (3) must satisfy can be written as follows (the conditions correspond, respectively, to c_t, l_t, and b_t):

$$u'(c_t) \leq \gamma_t \left(1 - s_t P'(c_t)(1 - l_t)\right), \text{ and } c_t \geq 0, \tag{3}$$

$$v'(l_t) \leq \gamma_t s_t P(c_t) + \eta_t, \text{ and } l_t \geq 0, \tag{4}$$

$$q\gamma_t \geq \beta E_t[\gamma_{t+1}], \text{ and } b_t \geq \underline{B}, \tag{5}$$

and with respect to the multipliers associated to the budget and time constraints in period t (denoted, respectively, γ_t and η_t):

$$c_t + qb_t \leq \theta + b_{t-1} + s_t P(c_t)(1 - l_t), \text{ with } \gamma_t \geq 0, \tag{6}$$

$$l_t \leq 1, \text{ with } \eta_t \geq 0. \tag{7}$$

Complementary slackness implies that, for each of the above pairs of inequalities, at least one of them holds as an equation. Equation (3) shows that marginal utility of consumption is modified by the factor $(1 - s_t\, P'(c_t)(1 - l_t))$, which is the marginal income that it costs to consume a marginal unit of consumption; Equation (4) is the usual condition that, for an interior solution for leisure, the marginal rate of substitution between consumption and leisure must be equal to the wage $P(c_t)$. Notice that A6 rules out the possibility that for a given level of $c_t > 0$, there are several values for l_t in

$[0, 1]$ such that Equations (3) and (4) are simultaneously satisfied. That is, A6 guarantees that Equations (3) and (4) are necessary and sufficient for an optimum. Finally, Equation (5) is the familiar intertemporal condition for models with savings. Lemma 1 introduces a useful result.

Lemma 1: i) If $s_t = 0$ then $l_t = 1$; ii) There is a $c^- > 0$ such that $l_t = 1$ whenever $s_t = 1$ and $c_t \leq c^-$; iii) There is a finite $c^+ > c^-$ such that $l_t = 1$ whenever $s_t = 1$ and $c_t \geq c^+$.

The previous lemma simply says that an agent in the unemployment state chooses not to work, and that there are two critical levels of consumption such that, beyond them, an agent in the employment state chooses not to work. Since we see very poor people lying around in the streets (in countries without a welfare state), our model seems to match this informally collected bit of reality, while the usual model would predict that very poor agents would always work very hard.

The following Proposition 1 describes several properties of the solution to the utility maximization problem in Equation (3).

Proposition 1: Assume A1-A6, and R2. The expected present value of utility associated to the solution of the problem in Equation (3) can be represented by a continuous, bounded, and non decreasing funtion $V(b_{-1}, s_0)$, and there continuous policies such that $c_t = c(b_{t-1}, s_t)$, $l_t = l(b_{t-1}, s_t)$, $b_t = b(b_{t-1}, s_t)$, and such that $V(b_{t-1}, s_t)$ is attained.

3 Results

We now examine several properties of the stochastic processes that solve the problem in (3). We will assume that $\underline{B} < B = \theta/(q-1)$. The amount B corresponds to the maximum debt that a consumer is able to repay with probability 1 in the following period when $b_{t-1} = B$. This means that the central authority would never extend a credit balance beyond B, and therefore, $b_t > \underline{B}$ and $q_t \gamma_t = \beta E[\gamma_{t+1}]$ in all periods.

3.1 Characterization of policy functions

Remark 2: c_t and b_t are continuous and increasing in b_{t-1}, for both $s_t = 0$ and $s_t = 1$.

Notice that L1 and R2 imply that there are b^- and b^+, such that $l_t = 1$ whenever $b_{t-1} \leq b^-$, and also, whenever $b_{t-1} \geq b^+$.

Proposition 1: Assume $\beta/q = 1$. Then for all $j = 0, 1, ...$, and for both, $s_{t+j} = 0$ and $s_{t+j} = 1$: i) If $c_t \geq c^+$, then $c_{t+j} = c_t$ and $b_{t+j} = b_{t-1}$; ii) If $c_t \leq c^-$, then $c_{t+j} = c_t$ and $b_{t+j} = b_{t-1}$.

Proposition 2: i) Assume $\beta/q < 1$. If $b_{t-1} \leq b^-$, then $b_t \leq b_{t-1}$ (with strict inequality if $b_{t-1} > B$) for both, $s_t = 0$ and $s_t = 1$; ii) Assume $\beta/q > 1$. If $b_{t-1} \geq b^+$, then $b_t > b_{t-1}$ for both, $s_t = 0$ and $s_t = 1$.

Proposition 3: Assume $\beta/q \leq 1$. There is a $b_{min} > B$ such that if $b_{t-1} \in (B, b_{min})$, then $b_t = B$.

3.2 Limiting behavior for fixed q

Proposition 4:

i) If $b^- \leq b_{-1} \leq b^+$ and $\beta/q = 1$ then consumption converges a.s.. The only possible limit points of consumption are c^- and c^+. Formally:

$$Prb\left(\lim_{t\to\infty} c_t = c^-\right) + Prb\left(\lim_{t\to\infty} c_t \geq c^+\right) = 1. \tag{8}$$

ii) If $1 < \beta/q$ and $b^- \leq b_{-1} \leq b^+$, then consumption converges a.s. to $\bar{c}$.

iii) If $1 > \beta/q$ and $b^- \leq b_{-1} \leq b^+$, then consumption converges a.s. to 0.

3.3 Properties of stationary distributions

In a competitive equilibrium it must be satisfied that the stochastic process for q is such that the bonds market clears:

$$\int_0^1 b_t^i di = 0,\ t = 0, 1, ... \tag{9}$$

Proposition 5:

i) Only if $b/q = 1$ there can be a stationary distribution of wealth.

ii) Any stationary distribution of wealth is polarized and without mobility. More precisely, any distribution of wealth that implies a measure of consumption with $\mu_c((c^-, c^+)) > 0$ cannot be a stationary distribution.

ii) There exist many stationary distributions.

4 Appendix

Proof of Lemma 1

i) Equation (4) reads $v'(l_t) \leq \eta_t$ when $s_t = 0$. Since $v'(l_t) > 0$ for $l_t < \infty$, then $\eta_t > 0$ and thus, $l_t = 1$ from the complementary slackness in (7). ii) Equation (3) implies that γ_t is bounded for all $c_t > 0$. Since $P(c_t) \to 0$ as $c_t \to 0$, then the term $\gamma_t P(c_t)$ in Equation (4) converges to 0 as $c_t \to 0$. Therefore, there is a c^- sufficiently small such that $v'(1) = \gamma_t P(c^-)$, thus for all $c_t < c^-$, $\eta_t > 0$ and then, $l_t = 1$. iii) It follows from Equation (3) that γ_t can be made arbitrarily small by making c_t arbitrarily large. Since $P(c_t)$ is bounded, then there is a c^+ sufficiently large such that $v'(1) = \gamma_t P(c^+)$. As before, for all $c_t > c^+$, $\eta_t > 0$ and then, $l_t = 1$.

Proof of Proposition 1

Let $\bar{B}$ be an arbitrarily large (but finite) upper bound for bond holdings and let $X = [\underline{B}, \bar{B}]$ denote the space of bonds, with $\mathcal{X}$ Borel subsets. Let also $\mathcal{S}$ denote the σ-algebra associated to S, and let $(E, \mathcal{E}) = (X \times S, \mathcal{X} \times \mathcal{S})$ be the product space. We also need the following objects:

Let $\Phi : E \to R_+ \times [0,1] \times X$ be given by

$$\begin{aligned}\Phi(b,s) \quad = \quad & \{(c,l,b') : c_t + qb_t \leq \theta + s_t P(c_t)(1-l_t) + b_{t-1}, \\ & c_t, l_t \geq 0,\, l_t \leq 1,\, b_t \geq \underline{B}\}. \qquad (10)\end{aligned}$$

Let $\Gamma : E \to X$ be given by

$$\Gamma(b,s) = \{b' \in X : (c,l,b') \in \Phi(b,s) \text{ for some } (c,l) \in R_+ \times [0,1]\}, \qquad (11)$$

and let

$$F(b,s,b') = max_{c,l \in R_+ \times [0,1]}(u(c) + v(l)) \text{ s. t. } (c,l,b') \in \Phi(b,s).$$

Under A1-A6 and the restrictions in $\Phi(b,s)$ the hypothesis for theorems 9.2 and 9.6 in Stokey and Lucas (1998) hold, thus in the space of bounded continuous functions $C(E)$ with the sup-norm there exists a unique solution to the functional equation

$$V(b,s) = sup_{b' \in \Gamma(b,s)}\{F(b,s,b') + \beta E_t[V(b',s')]\}, \qquad (12)$$

and it coincides with the expected present value associated to the problem in sequence form in Equation (3).

Proof of Proposition 1

i) The allocation $(c_t, l_t, b_t) = (\omega + b_{t-1}(1-q), 1, b_{t-1})$ is feasible and satisfies all the first order conditions. We will show that this is the only allocation doing so. First, $c_t \geq c^+$ implies that $b_{t-1} \geq b^+$, and c_t increasing in b_{t-1} implies that the agent faces no idiosyncratic uncertainty (since $l_t = 1$ in both, the good and bad state). Since $l_t = 1$, it follows from Equations (3) and (5) that $u'(c_t) = u'(c_{t+1})$. Since marginal utility of consumption is decreasing in c_t, and c_t is increasing in b_{t-1}, the previous equation cannot be satisfied if $b_t > b_{t-1}$. The same argument rules out the choice of b_t slightly below b_{t-1}. Finally if the agent chooses a large c_t, so that b_t is so small that in the following period she would choose to work in case $s_{t+1} = 1$, then it follows from Equations (3) and (5), that $u'(c_t) = E[\gamma_{t+1}]$. This is not possible either. To see this, notice that since $b_t < b_{t-1}$, then in the good state in $t+1$ we would have $u'(c_{t+1}) > u'(c_t)$, and from Equation (3) we have $\gamma_{t+1} > u'(c_{t+1})$. A similar reasoning shows that in the bad state in $t+1$ we would also have $\gamma_{t+1} = u'(c_{t+1}) > u'(c_t)$. It would follow that $u'(c_t) = E[\gamma_{t+1}] > u'(c_t)$, a contradiction. The proof of ii) uses a similar argument.

Proof of Proposition 2

i) Since $b_{t-1} \leq b^-$, then $l_t = 1$ and consumption is the same for both, $s_t = 0$ and $s_t = 1$. Suppose, towards a contradiction, that $b_t \geq b_{t-1}$. It follows from Equation (3) and (5) that $u'(c_t) < E[\gamma_{t+1}] \leq \gamma_t = u'(c_t)$, where the first inequality uses that $\beta/q < 1$, the second uses R2, A6, and the fact that $b_t \geq b_{t-1}$, and the equality follows because $l_t = 1$. Contradiction. The proof of ii) uses a similar argument.

We will use the following definitions: $\Omega^- = \{\omega \in \Omega : c_t(\omega) = c^- \text{ for some } t\}$, and $\Omega^+ = \Omega \setminus \Omega^-$.

Proof of Proposition 4

i) From the FOC for capital γ_t is a non negative martingale, and by A2 and A6, it is bounded, so that it converges a.s. to a random variable $\overline{\gamma}$. We now prove that consumption converges a.s.. Notice that either $\omega \in \Omega^-$, or that $\omega \in \Omega^+$. If $\omega \in \Omega^-$, then $c_{t+j} = c^-$ for $j = 0, 1, 2...$, by P1. Assume now that $\omega \in \Omega^+$. Then $b_{-1} \in (b^-, b^+)$ and R2 implies that $c_t(\omega) > 0$ for all t. Therefore,

$$\frac{u'(c_t(\omega))}{1 - s_t(\omega)P'(c_t(\omega))(1 - l_t(\omega))} = \gamma_t(\omega), \forall t,$$

and c_t converges a.s. to the set $\{c^1(\overline{\gamma}(\omega)), c^0(\overline{\gamma}(\omega))\}$, where each element of this set is defined analogously as in A7. To prove (8) note that if $u'(c^-) > \overline{\gamma}(\omega) > u'(c^+)$, by assumption A7 deficit uncertainty is positive so that, by the usual argument, the budget constraint would explode and the consumption would be unfeasible or suboptimal, so that this is impossible. The only possibilities are that $u'(c^-) = \overline{\gamma}(\omega)$, or that $\overline{\gamma}(\omega) = u'(c^+)$, and in these cases consumption must converge to c^- or to c^+. Notice that in both cases, labor supply is zero.

ii) Again, γ_t is a bounded martingale, so it converges a.s. to $\overline{\gamma}$. Since convergence a.s. implies convergence in probability, then $\gamma_t \to \overline{\gamma}$ in probability. The Euler equation implies

$$\gamma_0 = (\beta/q)^t \ E_0(\gamma_t)$$

for all t and, since $(\beta/q)^t \to \infty$ then γ_t converges in mean to 0. Since, by the Chebysheff inequality, convergence in absolute mean implies convergence in probability, we have $\gamma_t \to 0$ in probability. Therefore, $\gamma_t \to 0$ a.s., so that $u'(c) \to 0$ which can only occur if consumption goes to saturation.

iii) Now $(\beta/q)^t \to 0$ so $E_0(\gamma_t) \to \infty$. This implies that $Prb(\nu_t = 0$ at some $t) > 0$, otherwise, $E_0(\gamma_t) = E_0\left(\frac{u'(c_t)}{1-s_t P'(c_t)(1-l_t)}\right) < u'(0) < \infty$. The fact that $P(\nu_t = 0$ at some $t) > 0$ implies that $b(\cdot, 0)$ is flat at $b = -r\Omega$, otherwise it is impossible that the positivity constraint on consumption is binding. This implies that we can go to zero consumption in finitely many steps so we go there with probability one (this argument must be completed ...).

Proof of Proposition 4

i) Obvious from the form of the policy function.

ii) If a distribution of wealth has positive probability between 0 and c^+, from previous proposition, wealth goes to the limits, so some mass would escape to 0 or c^+ (or c^s) and this could not be a stationary distribution of wealth.

iii) since at c^+ consumption moves for other interest rates (it moves up if $\beta/q > 1$ and down if $\beta/q < 1$ only c^+ is a possible limit.

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