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Extensions to `gllamm`

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Extensions to `gllamm`:

- More response processes
  - Ordinal responses
  - **I. Nominal responses and rankings**
- Structural equations for the latent variables
  - **II. Regressions of latent variables on observed variables**
  - Regressions of latent variables on other latent variables
- Parameter constraints
- `gllapred` for posterior means and probabilities
- A manual

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Generalised Linear Latent and Mixed Models (GLLAMMs)

- Conditional expectation of response

$$g(E[y|\mathbf{x},\mathbf{u}]) = \eta$$

where g is a link function and η is the linear predictor.

- Linear predictor:

$$\eta = \boldsymbol{\beta}'\mathbf{x} + \sum_{l=2}^{L}\sum_{m=1}^{M_l} u_m^{(l)}\boldsymbol{\lambda}_m^{(l)\prime}\mathbf{z}_m^{(l)} \quad \text{for identification, } \lambda_{\mathrm{m1}}^{(\mathrm{l})} = 1$$

- Conditional distribution of response is from exponential family
- Latent variables can be factors or random coefficients:
  - Random coefficient: one explanatory variable multiplies the latent variable
  - Factor: The items are treated as level 1 units and a linear combination of dummy variables for the items multiplies the latent variable

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Response Processes

- The response variables may be of mixed type - requiring mixed links and families:

Links
identity
reciprocal
logarithm
logit
probit
scaled probit
compl. log-log

Families
Gaussian
gamma
Poisson
binomial

Polytomous responses
ordinal logit
ordinal probit
ordinal compl. log-log
multinomial logit

- Heteroscedasticity: The dispersion parameter for the Gauss and gamma families can differ between responses or depend on covariates
- Offsets
- Many response processes: multivariate survival, discrete survival data, rankings, ceiling/floor effects

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I. Nominal responses and rankings

- Nominal or unordered categorical responses:
  - Party voted for
  - Treatment selected for a patient
  - Brand of ketchup bought

  One of A alternatives is 'selected': *first choice* data.
- Multinomial logit model (polytomous logistic regression):
  - linear predictor for alternative a is V^a, e.g., $V^a = \beta_0^a + \beta_1^a \text{Age}$
  - The probability that f is the 'chosen' alternative is

$$\Pr(f) = \frac{\exp(V^f)}{\sum_{a=1}^{A} \exp(V^a)}$$

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Latent Response Derivation of Multinomial Logit Model

- Associated with each alternative is an unobserved 'utility' U^a (latent response). The alternative with the highest utility is selected. Depending on the situation, utility means attractivess or usefulness (voting/purchasing), cost-efficitveness (treatments), etc. of the alternative.

$$U^a = V^a + \epsilon^a$$

- f is chosen if

$$U^f > U^g \text{ for all } g \neq f$$

  or

$$U^f - U^g = V^f - V^g + (\epsilon^f - \epsilon^g) > 0$$

- If the error term ϵ^a has an extreme value distribution of type I (Gumbel), then the differences $(\epsilon^f - \epsilon^g)$ have a logistic distribution and it follows that (McFadden, 1974)

$$\Pr(f) = \frac{\exp(V^f)}{\sum_{a=1}^{A} \exp(V^a)}$$

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British Election Study

- voters who voted Conservative, Labour, Liberal in 1987 and 1992 elections.
- Variables:
  - `male`, `age`, `manual` (father a manual worker)
  - `rldist`: distance between voter and party on left-right dimension constructed from respondent's and paty's position on 4 scales, e.g. more effort to redistribute wealth→less effort
  - `price`: judgement how much prices have risen
  - Expanded or "exploded" data

	serialno	year	party	chosen	rldist
10.	11	92	con	1	.5031
11.	11	92	lab	0	30.12
12.	11	92	lib	0	16.18
13.	13	87	con	0	22.86
14.	13	87	lab	1	.3622
15.	13	87	lib	0	1.894
16.	13	92	con	0	20.62
17.	13	92	lab	1	.0567
18.	13	92	lib	0	1.507
19.	15	87	con	0	25.32

  - `zrldist`, `zprice` are standardised versions

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Multinomial logit in gllamm

- Data in expanded form: alternative sets (analogous to risk sets)
- Dummy variables `lab` and `lib` for Labour and Liberal and interactions with all subject-specific explanatory variables

```
gen lab_age = lab*age
gen lib_age = lib*age
```

- Multinomial logit model with a random effect of `zrldist`:

$$V_{ij}^{a} = \boldsymbol{\beta}^{a\prime}\mathbf{x}_{ij} + (\alpha + u_i)d_{ij}^{a}$$

where i indexes the voter, j indexes the election and d_{ij}^{a} is the distance between voter and party on the left-right political dimension.

```
eq beta1: zrldist
gllamm party zrldist lab87 lib87 lab92 lib92 lab_mal lib_mal   /*
  */ lab_age lib_age lab_man lib_man lab_zpri lib_zpri, nocons /*
  */ i(serialno) expand(occ chosen o) f(binom) l(mlogit) eq(beta1)
```

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party	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
zrldist	-1.060069	.0494498	-21.44	0.000	-1.156989	-.9631496
lab87	.5425648	.2526952	2.15	0.032	.0472912	1.037838
lib87	.157179	.2357233	0.67	0.505	-.3048302	.6191881
lab92	.6552893	.2613689	2.51	0.012	.1430156	1.167563
lib92	-.1244899	.2459711	-0.51	0.613	-.6065843	.3576046
lab_mal	-.8032313	.137831	-5.83	0.000	-1.073375	-.5330876
lib_mal	-.6890129	.1298547	-5.31	0.000	-.9435235	-.4345024
lab_age	-.3636854	.0465448	-7.81	0.000	-.4549116	-.2724592
lib_age	-.2127401	.0430409	-4.94	0.000	-.2970987	-.1283815
lab_man	.8029466	.1472146	5.45	0.000	.5144113	1.091482
lib_man	-.0675791	.1309883	-0.52	0.606	-.3243114	.1891532
lab_zpri	.573825	.0766719	7.48	0.000	.4235507	.7240992
lib_zpri	.4458234	.0706312	6.31	0.000	.3073889	.584258

```
Variances and covariances of random effects
------------------------------------------------------------------------------
***level 2 (serialno)

    var(1): .27935136 (.07560918)
------------------------------------------------------------------------------
```

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Multinomial Logit Model For Rankings

- Rankings are orderings of alternatives (parties, treatments, brands) according to preference or some other characteristic.
- Associated with each alternative a is an unobserved utility U^a

$$U^a = V^a + \epsilon^a$$

  where ϵ^a has an extreme value distribution (Gumbel)
- Let r^s be the alternative with rank s. Then the ranking $R = (r^1, r^2, \cdots r^A)$ is obtained if

$$U^{r^1} > U^{r^2} > \cdots > U^{r^A}$$

- The probability of a ranking R is (Luce, 1959)

$$\Pr(R) = \frac{\exp(V^{r^1})}{\sum_{s=1}^{A} \exp(V^{r^s})} \times \frac{\exp(V^{r^2})}{\sum_{s=2}^{A} \exp(V^{r^s})} \times \cdots \times \frac{\exp(V^{r^A})}{\sum_{s=A-1}^{A} \exp(V^{r^s})}$$

- At each ‘stage’, a first choice is made among the remaining alternatives
- A subject’s contribution to the likelihood is identical to the contribution of a stratum to the partial likelihood in Cox’s regression

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Rankings for British Election Study

- First choice: party voted for
- Rankings: the parties were rated on a five point scale

  strongly against → strongly in favour
- The parties not voted for are ranked into second and third place using the rating scales. (In 6.5% of votes, the party voted for did not have the highest score)
- original data

	serialno	year	occ	rank	party
7.	11	87	3	1	con
8.	11	87	3	2	lib
9.	11	87	3	2	lab
10.	11	92	4	1	con
11.	11	92	4	2	lib
12.	11	92	4	3	lab

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Data preparation for rankings

- "Exploding the data to alternative sets" using `stsplit`

```
egen maxr = max(rank), by(occ)
gen chosen=1
gen id=_n
stset rank, fail(chosen) id(id)
stsplit , at(failures) strata(occ) riskset(occstage)
replace chosen=0 if chosen==.
drop if rank==maxr
```

	serialno	year	occstage	party	chosen
11.	11	87	7	con	1
12.	11	87	7	lab	0
13.	11	87	7	lib	0
14.	11	92	9	con	1
15.	11	92	9	lab	0
16.	11	92	9	lib	0
17.	11	92	10	lib	1
18.	11	92	10	lab	0

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Analysing rankings

- There are a number of possible random structures for election within voter within consituency.
- Example: correlated random coefficients for lab and lib at voter level

$$V_{ij}^a = \boldsymbol{\beta}^{a\prime}\mathbf{x}_{ij} + \alpha d_{ij}^a + u_{1i} z_{1ij}^a + u2ix_{2ij}^a$$

where z_{1ij} and x_{2ij} are dummy variables for Labour and Liberal, respectively.

– random coefficients of Labour and Liberal induce longitudial correlations across elections for Labour and Liberal, respectively.

– correlation between random coefficients of Labour and Liberal induces both cross-sectional and longitudinal correlations between the utilities for Labour and Liberal.

```
eq lab: lab
eq lib: lib
gllamm party zrldist lab87 lib87 lab92 lib92 lab_mal lib_mal   /*
  */ lab_age lib_age lab_man lib_man lab_zpri lib_zpri, nocons /*
  */ i(serialno) expand(occstage chosen o) f(binom) l(mlogit) /*
  */ nrf(2) nip(10) eqs(lib lab)
```

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```
log likelihood = -2647.2917

>>> fixed effects omitted

Variances and covariances of random effects
-----------------------------------------------------------


***level 2 (serialno)

    var(1): 18.918555 (2.2960479)
    cov(1,2): 9.0408098 (1.1798509) cor(1,2): .82567035

    var(2): 6.3374322 (.88875039)
-----------------------------------------------------------
```

Assuming uncorrelated random coefficients (using `nocor` option, gives a log-likelihood of -2800.2076

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II. Regressions of latent variables on observed variables

Structural equations for the latent variables

Regress the latent variables on other latent and explanatory variables

$$\mathbf{u} = \mathbf{B}\mathbf{u} + \mathbf{\Gamma}\mathbf{w} + \boldsymbol{\zeta}$$

- $\mathbf{u} = (u_1^{(2)}, u_2^{(2)}, \cdots, u_{M_2}^{(2)}, \cdots u_1^{(l)}, \cdots, u_{M_l}^{(l)}, \cdots, u_{M_L i}^{(L)})'$ (M elements)
  - factors
  - random coefficients
- $\mathbf{B}$ is an upper diagonal $M \times M$ matrix of regression coefficients
- $\mathbf{\Gamma}$ is an $M \times p$ matrix of regression coefficients
- $\mathbf{w}$ are p explanatory variables
- $\boldsymbol{\zeta}$ is an M dimensional vector of errors/disturbances (same level as corresponding elements in $\mathbf{u}$).

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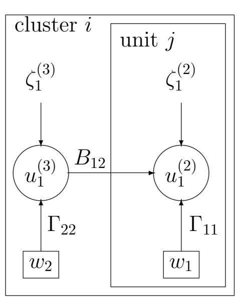
Theoretical example

$$\begin{bmatrix} u_1^{(2)} \\ u_1^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & B_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1^{(2)} \\ u_1^{(3)} \end{bmatrix} + \begin{bmatrix} \Gamma_{11} & 0 \\ 0 & \Gamma_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} \zeta_1^{(2)} \\ \zeta_1^{(3)} \end{bmatrix}$$



- School level factor regressed on (and measured by) school level variables
- (a) School level factor affects pupil level factor (e.g. ability)
- (b) School level factor affects pupil level random coefficient (e.g. rate of increase in performance)

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Example: Logistic regression with covariate measurement error

- Data and notation
  - Effect of fibre intake (continuous, measured twice on a subset of subjects) on coronary heart disease (CHD present/absent) (Morris, Marr and Clayton, 1977)
  - Responses are dietary fibre intake ($j = 1, 2$) and coronary heart disease ($j = 3$)
  - u_i is ith subject's true dietary intake (- population mean)
- Measurement model for fibre intake: y_{i1}, y_{i2} conditionally independently normally distributed with

$$E[y_{ij}|\mathbf{u}] = \beta_j + u_i\lambda_j, \quad j = 1, 2 \ (\beta_1 = \beta_2, \ \lambda_1 = \lambda_2 = 1)$$

- Disease model: y_{i3} conditionally binomial with

$$\text{logit}(E[y_{i3}|\mathbf{u}]) = \beta_3 + u_i\lambda_3 \quad \lambda_3 \text{ is } \log(\text{OR})$$

- GLLAMM
  - z_{1ij} is 1 for the element(s) corresponding to fibre and 0 otherwise.
  - z_{3ij} is 1 for the element corresponding to CHD, 0 otherwise.

$$\eta_{ij} = \boldsymbol{\beta}'\mathbf{z}_{ij} + u_i\boldsymbol{\lambda}'\mathbf{z}_{ij} \qquad \mathbf{z}_{ij} = (z_{1ij}, z_{3ij})'$$

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Diet example in `gllamm`, see STB53, sg129

	id	resp	diet	chd	var
425.	217	3.06	1	0	1
426.	217	0	0	1	2
427.	218	3.14	1	0	1
428.	218	0	0	1	2
429.	219	2.75	1	0	1
430.	219	2.7	1	0	1
431.	219	0	0	1	2

`diet` is z_1 and `chd` is z_3

```
eq id: diet chd
gllamm resp diet chd, nocons i(id) eqs(id) link(ident logit) /*
*/ fam(gauss binom) lv(var) fv(var) nip(30)
```

Including other covariates

- **Direct effect of x on y_3**

$$\begin{aligned}\text{Measurement model}: E[y_{ij}|\mathbf{u}] &= \beta_j + u_i, \quad j = 1,2 \\ \text{Disease model}: \text{logit}(E[y_{i3}|\mathbf{u}]) &= \beta_3 + \beta_4 x + u_i\lambda_3\end{aligned}$$

- **Indirect effect of x on y_3**

$$u_i = \gamma x + \zeta_i,$$

where ζ_i is a residual error term

$$\begin{aligned}\text{Measurement model}: E[y_{ij}|\mathbf{u}] &= \beta_j + \gamma x + \zeta_i \\ \text{Disease model}: \text{logit}(E[y_{i3}|\mathbf{u}]) &= \beta_3 + \gamma\lambda_3 x + \zeta_i\lambda_3\end{aligned}$$

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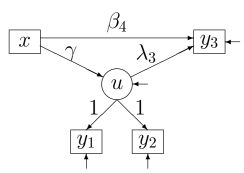
$\Rightarrow$ would requires nonlinear constraint for the coefficients if we couldn't regress $\mathbf{u}$ on explanatory variables

- **Direct and indirect effect of x on y_3**

$$\begin{aligned}\text{Measurement model}: E[y_{ij}|\mathbf{u}] &= \beta_j + \gamma x + \zeta_i \\ \text{Disease model}: \text{logit}(E[y_{i3}|\mathbf{u}]) &= \beta_3 + (\beta_4 + \gamma\lambda_3)x + \zeta_i\lambda_3\end{aligned}$$

(would not require a constraint for the coefficients)



Including effect of occupation (bus staff vs bank staff)

	id	resp	diet	chd	var
425.	217	3.06	1	0	1
426.	217	0	0	1	2
427.	218	3.14	1	0	1
428.	218	0	0	1	2
429.	219	2.75	1	0	1
430.	219	2.7	1	0	1
431.	219	0	0	1	2

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- `gllamm` syntax without `occ`

```
eq id: diet chd
gllamm resp diet chd, nocons i(id) eqs(id) link(ident logit) /*
*/ fam(gauss binom) lv(var) fv(var) nip(30)
```

- `gllamm` syntax with direct and indirect effects of `occ` (dummy for bus staff)

```
eq f1: occ
gen occc=occ*chd
gllamm resp diet chd occc, nocons i(id) eqs(id) link(ident logit) /*
*/ fam(gauss binom) lv(var) fv(var) nip(30) geqs(f1)
```

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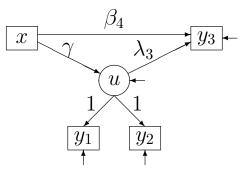
Results

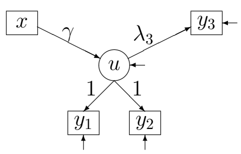
- direct and indirect effect of x on y_3



Log-likelihood=-186.90

Parameters	Estimates	SE
λ_3	-1.95	0.73
γ	-0.12	0.03
β_4	-0.19	0.34
σ^2	0.02	0.003
var(u)	0.07	0.007

- Indirect effect of x on y_3



Log-likelihood=-187.05

Parameters	Estimates	SE
λ_3	-1.86	0.70
γ	-0.12	0.03
σ^2	0.02	0.003
var(u)	0.07	0.007