Extreme values and robust distribution analysis

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13th UK Stata Users Group meeting
Cass Business School (London), September 10-11, 2007

[outline]

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Context

"Distribution analysis"

Analysis of data modelled as realizations from some random variable Y

- characterize Y w.r.t. 'location', 'spread'/'skewness', 'modality'
- focus on other particular features, e.g.
  - measures of inequality, poverty, polarization (income data)
  - expected loss, value-at-risk (financial data)
- stochastic dominance comparisons (ordering RV w.r.t. risk or inequality)
- fit parametric models for the RV (e.g., Gamma distribution, Pareto, etc.)

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Analysis beyond 'central tendency'/'location' estimation (very) sensitive to extreme data

- data contamination (e.g., 'decimal point' encoding error')?
- 'valid' outliers?

Consequences are potential bias and high sampling uncertainty (even with large samples).

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Influence function examples – Inequality indices

from Cowell & Flachaire (2007)

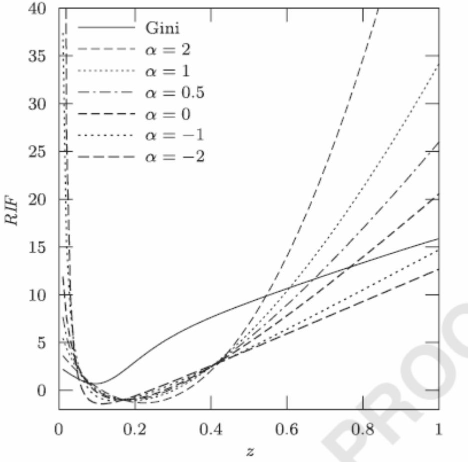
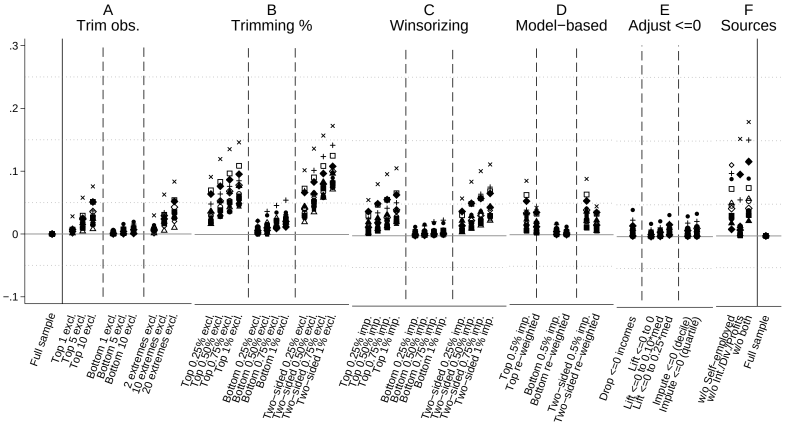


Fig. 1. *IF*s of generalised entropy I_E^α.

Impact of extreme incomes adjustments – Gini

from Van Kerm (2007)



Extreme incomes adjustments – GE(2)

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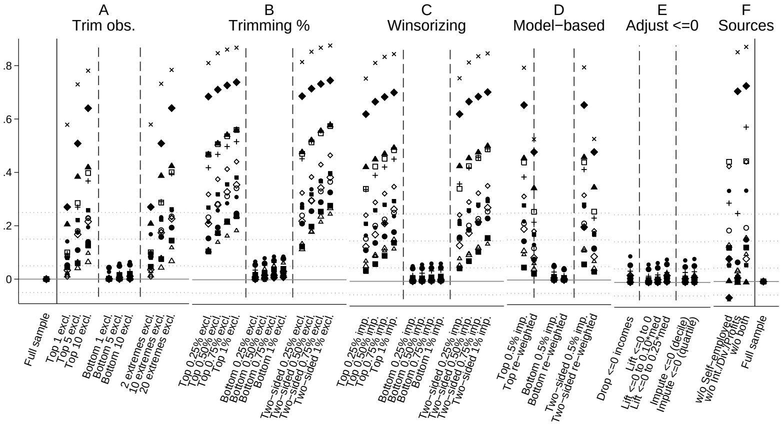

[outline]

Remedial actions

1. Identify and adjust extreme data: removal, re-coding
   - Relatively easy, but not efficient and dependence to ad-hoc trimming fractions
   - Impact can be substantial ... and difficult to justify
2. Rely on functional form assumptions:
   - model the full distribution parametrically (e.g. log-Normal, Gamma), so distribution fully characterized by just a few parameters
   - model only the tails of the distribution parametrically (e.g. Pareto)
   - But... classical ML estimators of distribution parameters are themselves non-robust to extreme values!

$\Longrightarrow$ Solution discussed in this talk: **Use "robust" estimators of model parameters** (instead of classical ML)

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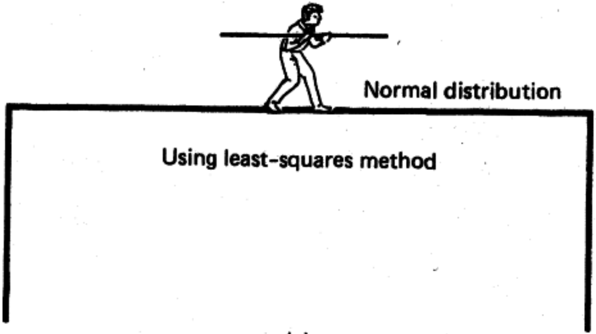
Robust estimation methods

(Hampel , 1986)



(a)

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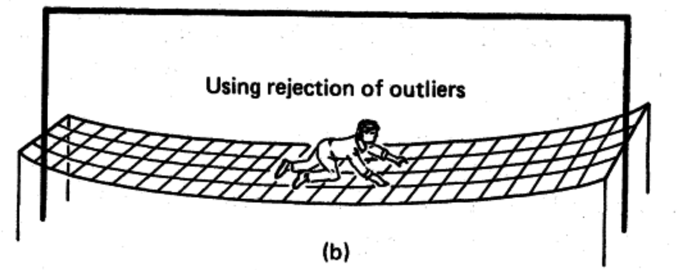

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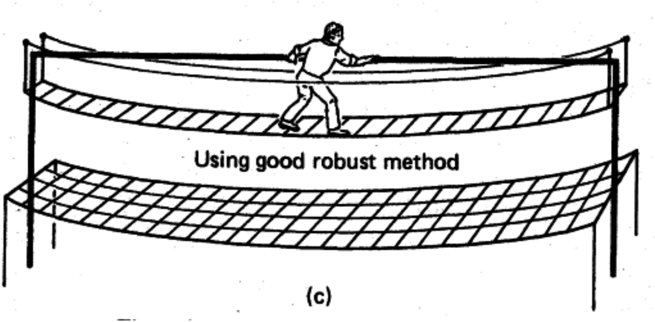

The estimation problem

Task

We want to fit a given parametric distribution f_θ to the available data: θ is a vector of parameters to be estimated.

ML estimation

Find θ^{ML} solution to $\sum_{i=1}^{N} s(x_i, \theta^{ML}) = 0$, where $s(x_i, \theta^{ML})$ is the score function: $s(x_i, \theta) = \partial \log(f_\theta(x_i))/\partial\theta$

Problem

The score function has unbounded influence function for almost all classic models of size distributions. Parameter estimates can therefore be driven to arbitrary values by data contamination...

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A robust alternative to classical ML

OBRE

- OBRE is also an M-estimator: θ solution to $\sum_{i=1}^{N} \psi(x_i, \theta) = 0$
- For ML: $\psi(x_i, \theta^{ML}) = s(x_i, \theta^{ML})$
- For OBRE:

$$\psi(x_i, \theta^{OB}) = (s(x_i, \theta^{OB}) - a(\theta^{OB}))W_c(x_i; \theta^{OB})$$

where

$$W_c(x_i; \theta^{OB}) = \min\left(1; \frac{c}{G(s(x_i, \theta^{OB}), a(\theta^{OB}), A(\theta^{OB}))}\right)$$

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Optimal B-Robust Estimators (OBRE) (ctd.)

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- $W_c(x;\theta^{OB})$ imposes a bound on influence function by downweighting extreme values (values deviating from model)
- c is a 'robustness' parameter to be determined ex ante (tune efficiency-robustness trade-off)
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- Iterative algorithm:
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- Solving equations for $a(\theta)$ and $A(\theta)$ also based on an iterative procedure

$\Longrightarrow$ Rather difficult problem, and very computer-intensive (esp. for numerical integration). So needs

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- Implementation is “relatively easy” with Mata (but familiarity with matrix algebra can help!)
- Uses a suite of existing commands by Stephen Jenkins to fit functional forms to unit record data by ML
  - just replace ML engine by home-brewed OBRE engine
  - i.e. call a Mata function, rather than `ml model`!

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    void gamma_obre(string scalar varname, string
    scalar sweight, string scalar touse, string
    scalar thenewvar, real scalar froma, real scalar
    fromb , real scalar c)
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  - the Mata function return a vector of parameter estimates along with a covariance matrix estimate
- To date I implemented Pareto Type I (1 param), log-Normal and Gamma (2 params) and Singh-Maddala (3 params)
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- Precision of numerical integration functions is important...
- ... and drives estimation speed
- Difficulty to set multiple tolerance and precision parameters – trade-off between speed and accuracy (still subject to changes...)
- As in ML estimation, using re-parameterization $\tilde{\theta} = \ln(\theta)$ can help convergence (in all models considered, $\theta > 0$)

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Output

```
Starting values (ML estimates): [ a = 4.430 ; b = 589.051 ]
Estimation with OBRE robustness constant set to c = 5

Iteration  1:  (....................) a = 5.116, b = 492.598
Iteration  2:  (....................) a = 5.366, b = 467.565
Iteration  3:  (....................) a = 5.466, b = 457.468
Iteration  4:  (..................) a = 5.516, b = 452.570
Iteration  5:  (................) a = 5.542, b = 450.096
Iteration  6:  (..............) a = 5.556, b = 448.812
Iteration  7:  (............) a = 5.563, b = 448.137
Iteration  8:  (..........)   a = 5.567, b = 447.780
Iteration  9:  (........)     a = 5.569, b = 447.591
Iteration 10:  (......)       a = 5.570, b = 447.490
Iteration 11:  (....)         a = 5.571, b = 447.435
Iteration 12:  (...)          a = 5.571, b = 447.407
Iteration 13:  (..)           a = 5.571, b = 447.391
Iteration 14:  (.)            a = 5.571, b = 447.383
```

y	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
a						
_cons	5.571198	.0580081	96.04	0.000	5.457504	5.684891
b						
_cons	447.3829	4.091696	109.34	0.000	439.3633	455.4025

Half CV^2	.089747
Gini coeff.	.23373
Theil	.087071

[outline]

Set-up

Monte Carlo simulation

1. Draw samples from known distributions
2. Add various kind of contamination – decimal point error – to a fraction of sample data
3. Estimate parameters from datasets using both ML and OBRE

- Pareto with sample size of 200
- log-Normal and Singh-Maddala with samples of size 1000

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Set-up (ctd.)

Types of contamination

1. 1% of obs. multiplied by 10
2. 1% of obs. divided by 10
3. 1% of obs. mulitplied by 10 and 1% of obs. divided by 10
4. 3% of obs. multiplied by 10
5. 3% of obs. divided by 10

Results

Pareto distribution

True parameter value: $\alpha = 3$

Model	root MSE		
	ML	c=5	c=2
No cont.	0.215	0.214	0.230
1% *10	0.261	0.252	0.231
3% *10	0.527	0.521	0.286

Results

log-Normal distribution

Model	Param.		root MSE		
			ML	c=5	c=3
No cont.	μ	8	0.017	0.017	0.017
	σ	.525	0.012	0.013	0.031
	Gini	0.290	0.006	0.007	0.017
	Theil	0.138	0.006	0.007	0.016
	$.5CV^2$	0.159	0.008	0.009	0.020
1% *10	μ	8	0.029	0.020	0.018
	σ	.525	0.050	0.020	0.021
	Gini	0.290	0.026	0.011	0.011
	Theil	0.138	0.027	0.011	0.011
	$.5CV^2$	0.159	0.037	0.014	0.014

Results

log-Normal distribution (ctd.)

Model	Param.	True	root MSE		
			ML	c=5	c=3
3% *10	μ	8	0.072	0.043	0.025
	σ	.525	0.131	0.070	0.016
	Gini	0.290	0.068	0.037	0.008
	Theil	0.138	0.078	0.040	0.009
	$.5CV^2$	0.159	0.111	0.054	0.011
3% /10	μ	8	0.070	0.047	0.025
	σ	.525	0.132	0.082	0.017
	Gini	0.290	0.068	0.043	0.009
	Theil	0.138	0.078	0.046	0.009
	$.5CV^2$	0.159	0.111	0.064	0.012

Results

Singh-Maddala distribution

Model	Param.	True	root MSE		
			ML	c=7	c=5
No cont.	α	2.8	0.128	0.145	0.301
	β	3500	297	283	590
	p	1.7	0.283	0.252	0.522
	Gini	0.289	0.008	0.009	0.016
	Theil	0.132	0.016	0.014	0.030
	$.5CV^2$	0.162	0.016	0.020	0.059
1% *10	α	2.8	0.297	0.243	0.370
	β	3500	720	572	751
	p	1.7	0.652	0.519	0.665
	Gini	0.289	0.032	0.021	0.027
	Theil	0.132	0.026	0.025	0.024
	$.5CV^2$	0.162	0.118	0.071	0.109

Results

Singh-Maddala distribution (ctd.)

Model	Param.	True	root MSE		
			ML	c=5	c=3
3% $\times$10	α	2.8	0.511	0.472	0.494
	β	3500	1145	1069	1004
	p	1.7	0.991	0.935	0.880
	Gini	0.289	0.088	0.073	0.055
	Theil	0.132	0.245	0.160	0.107
	$.5CV^2$	0.162	1.154	0.547	0.320
3% /10	α	2.8	0.578	0.521	0.253
	β	3500	1814	1306	788
	p	1.7	1.859	1.309	0.869
	Gini	0.289	0.022	0.021	0.021
	Theil	0.132	172.324	0.586	3.030
	$.5CV^2$	0.162	0.014	0.015	0.036

Main observations

- OBRE very useful with Pareto and, especially, log-Normal models
- OBRE useful too with Singh-Maddala, yet
  - choice of c matter – too much robustness not good with small contamination
  - too much contamination remains very harmful (look at impact on estimates of 'sensitive' inequality measures (Theil, $.5CV^2$)!) – even with OBRE
- Convergence problems with Gamma models – otherwise results similar to SM

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[outline]

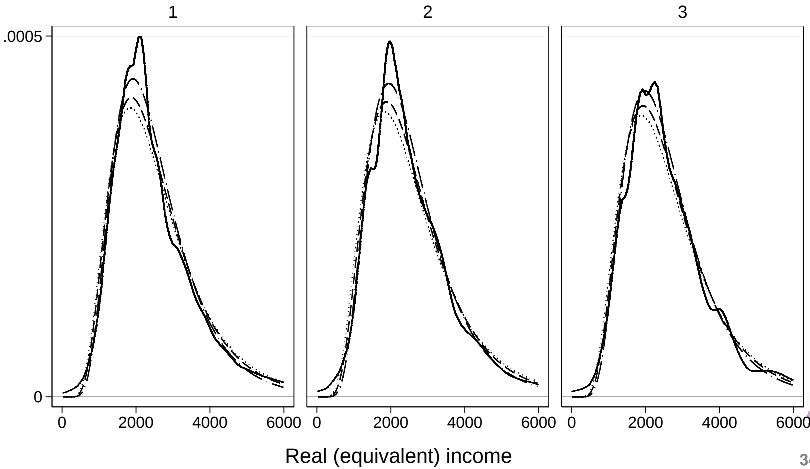
Data

PSELL-III

- Panel Survey “Liewen zu Letzebuerg”, waves 1(2003)-3(2005)
- Representative of residents in Luxembourg
- Real annual household income (in single adult equivalent)

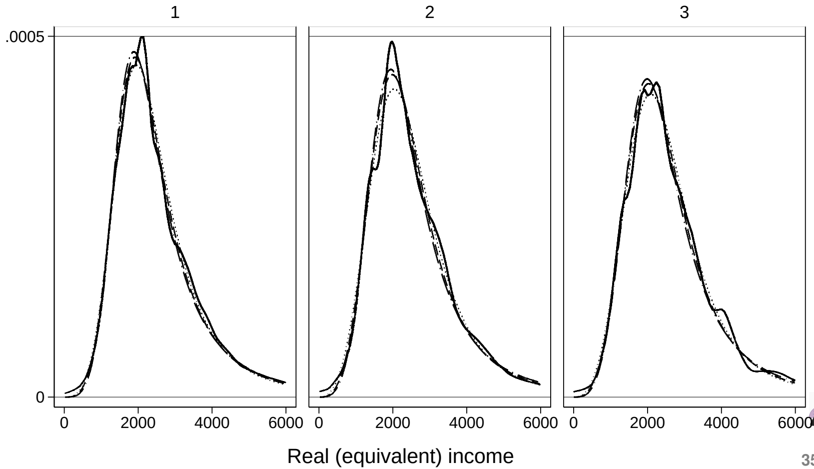
PDF estimates for log-Normal fit

OBRE improves fit, but not very good model



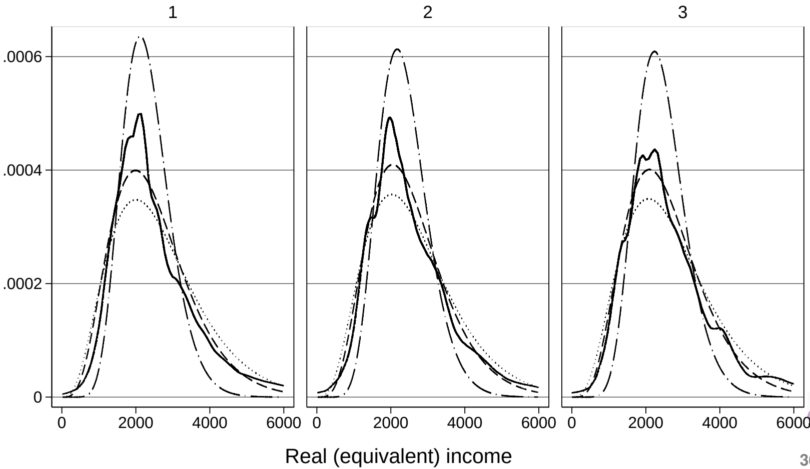
PDF estimates for Singh-Maddala fit

OBRE useful and much better fit

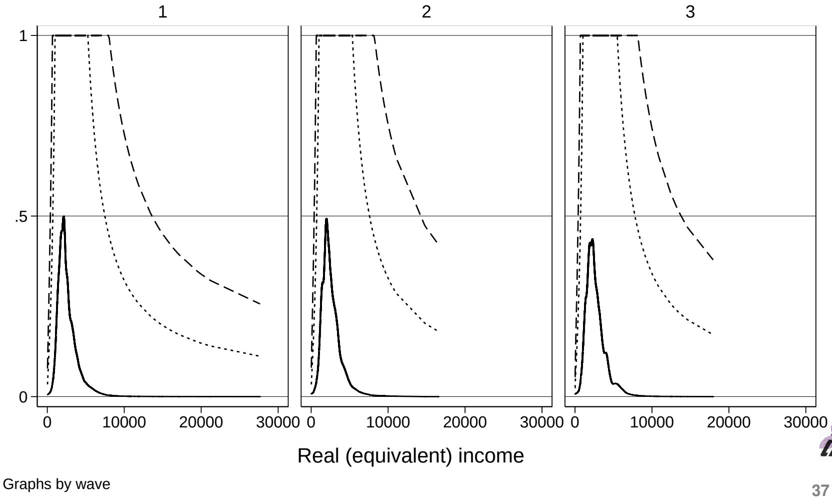


PDF estimates for Gamma fit

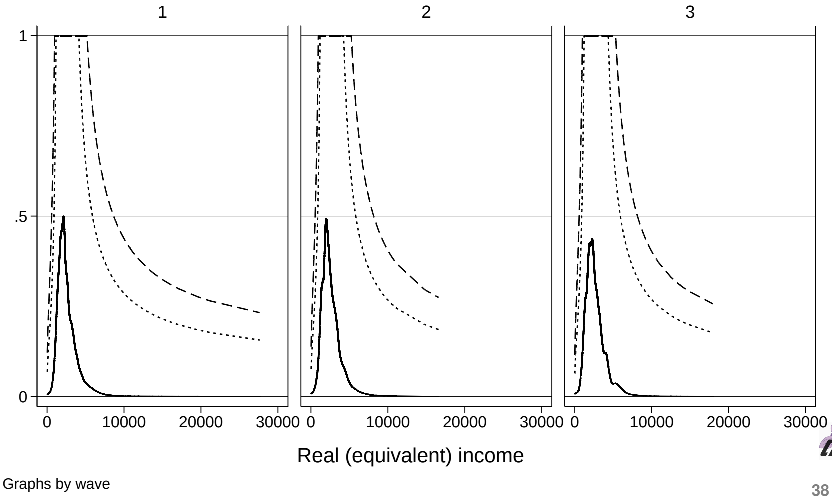
(does it call for any comment?)



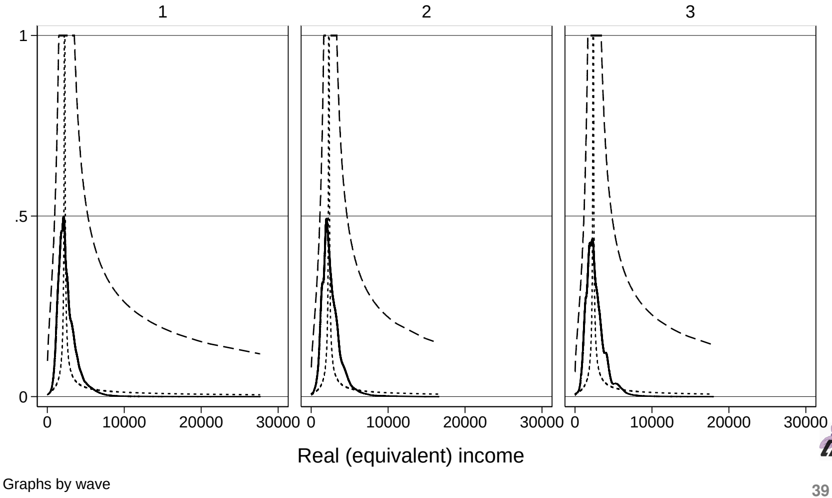
OBRE weights for log-Normal fit



OBRE weights for Singh-Maddala fit



OBRE weights for Gamma fit



[outline]

The principle

- More flexible approach is to focus on distribution tails
  - bulk of the data are taken at face value – use empirical CDF
  - parametric approach only for the tails – largest (and smallest?) observations are used to estimate a parametric model
  - empirical CDF combined with parametric CDFs for estimation of, say, inequality measures, stochastic dominance, etc.
- Under assumption that the CDF "decays as a power function" – i.e., has a heavy tail –, fitting a Pareto distribution to tail data is a valid choice: for $x \geq z$,

$$F(x) = 1 - \left(\frac{x}{z}\right)^{-\alpha}$$

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Pareto tail estimation

- OBRE estimator useful to avoid influence of contamination on Pareto parameter estimate α
- Main issue is the choice of z – value beyond which data are modelled parametrically

  $\Longrightarrow$ Pareto quantile plot and Hill's plot
  - Under Pareto model, linear relationship between $-\log(1 - F(x))$ and $\log(x)$ – so help detecting reasonable value of z
  - (yet difficulty associated with contamination at the very top)

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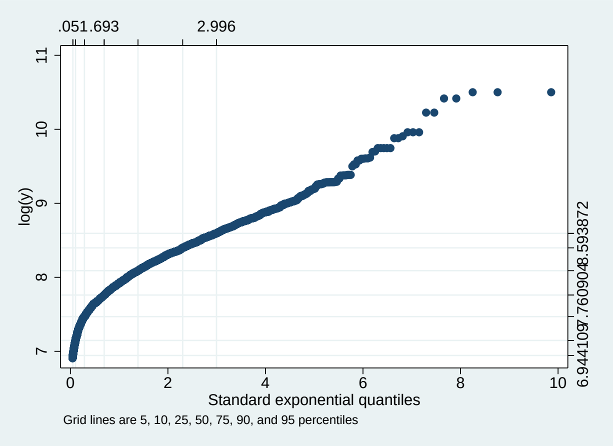
Pareto quantile plot



(Stata command `pareto_logqplot` available in package for Pareto tail modelling – coming soon on SSC!)

Concluding remarks

- Mata makes estimators such as OBRE feasible within Stata
- In theory, OBRE estimators have great relevance in distribution analysis... implementation in Stata may help putting this claim to broader practical assessment
- At present, it is (still) a prototype (but looks ok). Minor developments still needed for
  - fixing precision and tolerance thresholds
  - additional distributions (GB2?) – transplanting code to other distributions is easy, yet more convergence problems to be expected with higher number of parameters

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Cowell, F. A. & Flachaire, E. (2007), 'Income distribution and inequality measurement: The problem of extreme values', *Journal of Econometrics*, doi:10.1016/j.jeconom.2007.01.001 (forthcoming).

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