Path diagrams as a notational formalism (Noodling around with pictures)

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Introduction

Sometimes path diagrams are used in lieu of mathematical notation to define and explain models.

How far can they be pushed?

Maths and Paths — Covariates

Mathematical notation for a covariate

Maths and Paths — Covariates

Mathematical notation for a covariate

$$x$$

Maths and Paths — Covariates

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$$x$$

Path notation for a covariate

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$$x$$

Path notation for a covariate

$$\boxed{\mathrm{x}}$$

Maths and Paths — Dependent variables

Mathematical notation for a dependent variable

Mathematical notation for a dependent variable

$$y$$

Maths and Paths — Dependent variables

Mathematical notation for a dependent variable

$$y$$

Path notation for a dependent variable

Maths and Paths — Dependent variables

Mathematical notation for a dependent variable

$$y$$

Path notation for a dependent variable

$$\boxed{\mathrm{y}}$$

Maths and Paths — Parameters/Coefficients

Mathematical notation for a parameter

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Mathematical notation for a parameter

$$\beta$$

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Mathematical notation for a parameter

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Mathematical notation for a parameter

$$\beta$$

Path notation for a parameter

$$\xrightarrow{\quad\quad\beta\quad\quad}$$

Maths and Paths — Parameters/Coefficients

Mathematical notation for a parameter

$$\beta$$

Path notation for a parameter

$$\xrightarrow{\qquad\quad\beta\qquad\quad}$$

Paths are truly parameters in a linear form, and must connect observed or latent variables. So better is

Maths and Paths — Parameters/Coefficients

Mathematical notation for a parameter

$$\beta$$

Path notation for a parameter

$$\xrightarrow{\quad\beta\quad}$$

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$$y = \beta x \ + \ \ldots$$

$$\boxed{x} \xrightarrow{\quad\beta\quad} \boxed{y}$$

Maths and Paths — Latent variables

Mathematical notation for a latent variable

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Mathematical notation for a latent variable

$$\mu_i \quad \textit{unobserved}$$
$$\mu_i \sim \textit{Normal}(0, \sigma_\mu)$$

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Mathematical notation for a latent variable

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Path notation for a latent variable

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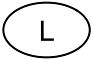
Path notation for a latent variable



A digression on learning to love latent variables

They can be:

- an error term
- a random intercept
- part of a random slope
- a frailty
- a creator/modeler of endogeneity
- a creator/modeler of sample selection
- a creator/modeler of endogenous treatment effects
- part of a switching model
- or any other latent/unobserved thingy

Linear regression

Mathematical notation for a linear regression

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x1_i + \beta_2 x2_i + \epsilon_i \\
\epsilon_j &\sim \mathit{Normal}(0, \sigma_\epsilon)
\end{aligned}
$$

Linear regression

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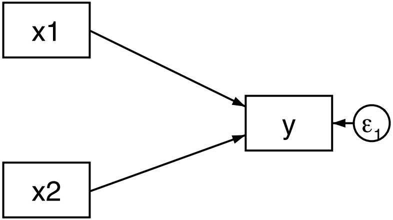
Path notation for a linear regression



Linear regression

Mathematical notation for a linear regression

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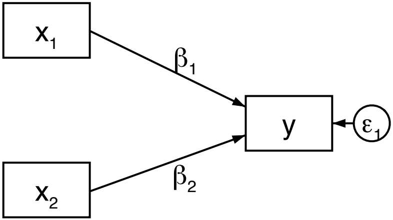
Path notation for a linear regression — with coefficients



Linear multilevel model

Mathematical notation for a multilevel model

$$
\begin{aligned}
y_{ji} &= \beta_0 + \beta_1 x1_{ji} + (\beta_2 + \mu 1_j) x2_{ji} + \mu 0_j + \epsilon_{ji} \\
\mu 0_j &\sim Normal(0, \sigma_{\mu 0}) \\
\mu 1_j &\sim Normal(0, \sigma_{\mu 1})
\end{aligned}
$$

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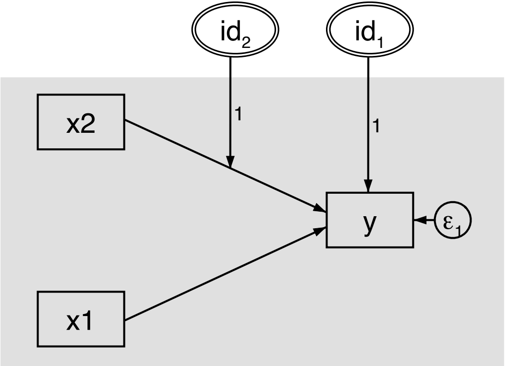
Path notation for a multilevel model

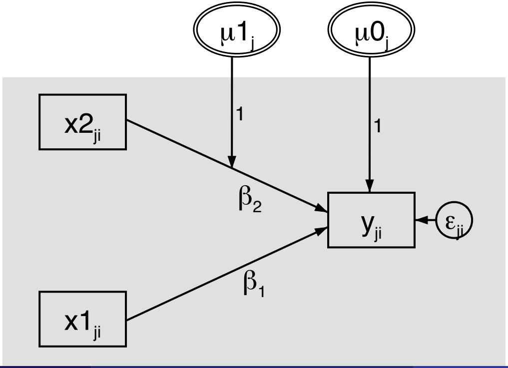


Linear multilevel model

Mathematical notation for a multilevel model

$$\begin{aligned} y_{ji} &= \beta_0 + \beta_1 x1_{ji} + (\beta_2 + \mu1_j)x2_{ji} + \mu0_j + \epsilon_{ji} \\ \mu0_j &\sim Normal(0, \sigma_{\mu0}) \\ \mu1_j &\sim Normal(0, \sigma_{\mu1}) \end{aligned}$$

Path notation for a multilevel model — with maths



A digression on the SEM Builder

Every diagram in this talk was drawn with it.

They each took 2 or 3 minutes to draw.

They can all be estimated by **`sem`** or **`gsem`**.

Let's draw one ...

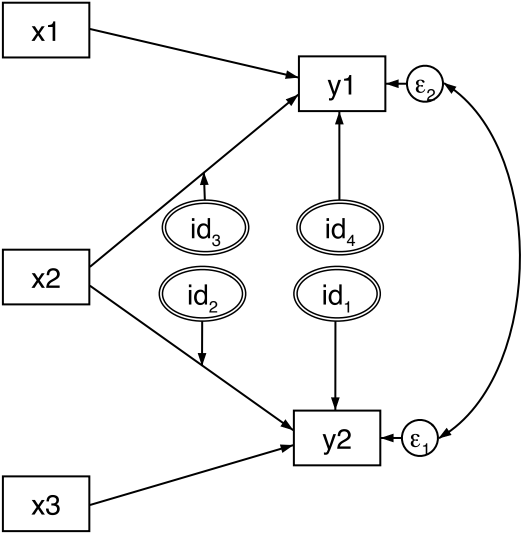
If you are reading this online, you just had to be there.

Linear multivariate (seemingly unrelated) multilevel model

Mathematical notation

$$
\begin{aligned}
y1_{ji} &= \beta_{0a} + \beta_1 x1_{ji} + (\beta_2 + \mu 1_j)x2_{ji} + \mu 0_j + \epsilon 1_{ji} \\
y2_{ji} &= \beta_{0b} + \beta_3 x3_{ji} + (\beta_4 + \mu 2_j)x2_{ji} + \mu 3_j + \epsilon 2_{ji} \\
\mu 0_j &\sim \textit{Normal}(0, \sigma_{\mu 0}) \\
\mu 1_j &\sim \textit{Normal}(0, \sigma_{\mu 1}) \\
\mu 2_j &\sim \textit{Normal}(0, \sigma_{\mu 2}) \\
\mu 3_j &\sim \textit{Normal}(0, \sigma_{\mu 3}) \\
j &\quad \textit{indexes levels}
\end{aligned}
$$

Linear multivariate (seemingly unrelated) multilevel model



Variations on the linear multivariate multilevel model

What if the random effects are shared?

- Easy enough in the mathematical notation, but easier to see in the path diagram. Let's edit the path diagram ...
- *I am sorry once again to those following at home.*

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What if the random coefficients on $x2$ are the same for both y's?

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- Must explicitly constrain paths/coefficients from latent variables.

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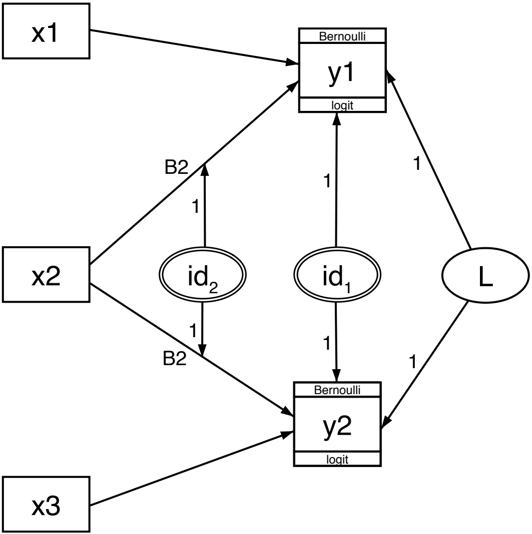
- Must explicitly constrain paths/coefficients from latent variables.

What if the outcomes were 0/1 and we wanted to model them as logistic?

- We would add some probabilistic statements to our mathematical notation. And likely some verbal explanation.
- We can also designate this on the path diagram ...

Final Variation



Time series — ARMAX model

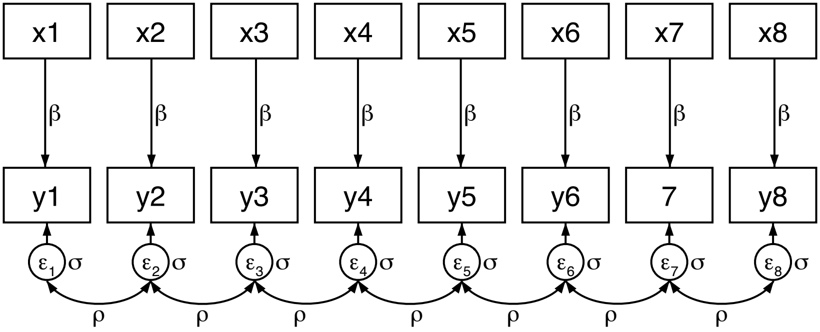
Mathematical notation

$$
\begin{aligned}
y_t &= \beta x_t + \epsilon_t \\
\epsilon_t &= \rho \epsilon_{t-1} + \mu_t \\
\mu_t &\sim Normal(0, \sigma_\mu)
\end{aligned}
$$

Mathematical notation

$$y_t = \beta x_t + \epsilon_t$$
$$\epsilon_t = \rho\epsilon_{t-1} + \mu_t$$
$$\mu_t \sim Normal(0, \sigma_\mu)$$

Path notation



Regression with sample selection — Heckman style

Mathematical notation

$$y_i = \beta_0 + \beta_1 x1_i + \beta_2 x2_i + \epsilon_i$$
$$s_i = \lambda_0 + \lambda_2 x2_i + \lambda_3 x3_i + \xi_i$$
$$cov(\epsilon, \xi) \neq 0$$
$$y_i \textit{ observed when } s_i > 0$$

Regression with sample selection — Heckman style

Mathematical notation

$$y_i = \beta_0 + \beta_1 x1_i + \beta_2 x2_i + \epsilon_i$$
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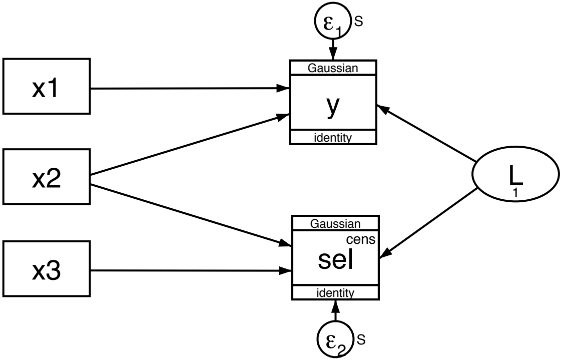
Path notation



Logistic model with endogenous covariate

Mathematical notation

$$Pr(y_i) = logit(\beta_0 + \beta_1 x1_i + \beta_2 z_i + UC_i)$$
$$z_i = \gamma_0 + \gamma_1 x2_i + UC_i + \nu_i$$
$$cov(\epsilon, \xi) = 0$$

Logistic model with endogenous covariate

Mathematical notation

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$$z_i = \gamma_0 + \gamma_1 x2_i + UC_i + \nu_i$$
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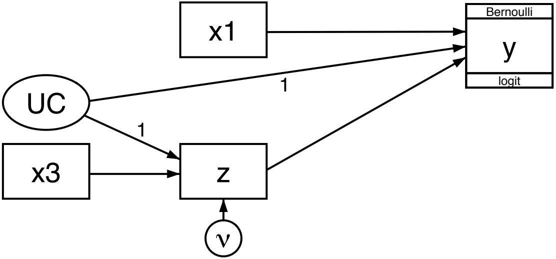
Path notation



Selection with an endogenous covariate in a multilevel framework

Regression component

$$y_{ji} = \beta_0 + (\beta_1 + \mu 1_j) x1_{ji} + \beta_2 z_{ji} + UC_{ji} + \mu 0_j + \epsilon_{ji}$$

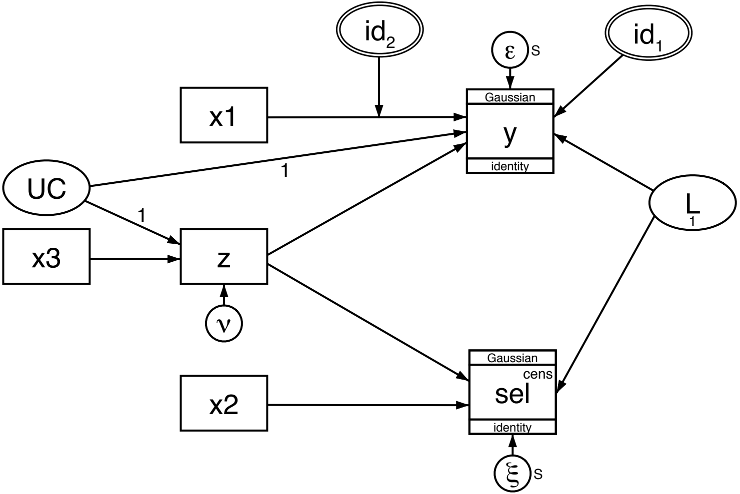
Selection component

$$s_{ji} = \lambda_0 + \lambda_1 z_i + \lambda_2 x2_i + \xi_i$$
$$cov(\epsilon, \xi) \neq 0$$
$$y_{ji} \; observed \; when \; s_{ji} > 0$$

Endogenous component

$$z_j i = \gamma_0 + \gamma_1 x3_j i + UC_j i + \nu_i$$
$$cov(\epsilon, \nu) = 0$$
$$cov(\xi, \nu) = 0$$

Selection with an endogenous covariate in a multilevel framework — path diagram



Closing

Compared to math, path diagrams are:

- easier to understand
- more attuned to human perception
- less precise
- less flexible
- more amenable to noodling

Don't choose.
Use what works — words, maths, paths.