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Ridit splines with applications to propensity weighting

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What are ridits?

- The distribution of a random variable X can be specified by its **Bross ridit function**[2] $R_X(\cdot)$, defined by the formula

$$R_X(x) \;=\; \Pr(X < x) \;+\; \frac{1}{2}\Pr(X = x).$$

- *So*, ridits are like ranks, but expressed on a scale from 0 (below the bottom–ranking value) to 1 (above the top–ranking value).
- The word was chosen to be like logit and probit, as the prefix stands for "with respect to an identified distribution".
- The **Brockett–Levene ridit function**[1] $R^*_X(\cdot)$ is defined (on a scale from -1 to 1) as a *difference* between probabilities,

$$R^*_X(x) \;=\; \Pr(X < x) \;-\; \Pr(X > x) \,,$$

and should *always* be used to calculate the Bross ridit function

$$R_X(x) \;=\; \frac{1}{2}\left[R^*_X(x) + 1\right] \,,$$

avoiding the precision problems of adding tiny half–probabilities to huge probabilities.

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- These ridits may be on a reverse scale (using the `reverse` option) and/or on a percentage scale (using the `percent` option), as with the `ridit` module of Nick Cox's `egenmore`.
- *However*, `wridit` also allows weights, so the ridits can be with respect to the distribution of the variable in a **target population**.
- In particular, zero weights are allowed, so the user can define ridits for the zero–weighted observations with respect to the distribution of the variable in the nonzero–weighted observations.
- *For instance*, in the `auto` data, we can define ridits of `length` with respect to the length distribution in US cars by zero–weighting non–US cars, or *vice versa*.
- On the default Bross scale, these ridits may be 0 in the former case for non–US cars, or 1 in the latter case for US cars.

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- If the user has installed the SSC packages `bspline`[3] and `polyspline`[4] as well as `wridit`, then the user can compute an unrestricted **reference–spline basis** in the ridit of an X–variable.
- This spline basis will have the advantage that the corresponding parameters of a fitted model will be values of the ridit spline at a list of values on the ridit scale, ranging from 0 to 1 (such as 0, 0.25, 0.50, 0.75 and 1).
- These fitted parameters will be mean values of the outcome variable, corresponding to X–values equal to percentiles of X (such as the minimum, median, maximum, and 25th and 75th percentiles).
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Example: Mileage and car length in the `auto` data

- We will demonstrate our methods in the `auto` data, with 1 observation for each of 74 car models.
- We will regress fuel efficiency in US/Imperial miles per gallon with respect to a ridit spline in car length in US/Imperial inches.
- We will use `wridit` to define the ridits of car length, and `polyspline`[4] to define an unrestricted cubic reference–spline basis in the ridits.
- We will then use `rcentile`[4] to estimate the percentiles corresponding to the reference ridits.
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Computing ridits using `wridit`

After loading the `auto` data, we use `wridit` to generate a new variable `lengthridit`, containing ridits (on a percentage scale) for the variable `length`:

```
. wridit length, percent generate(lengthridit);

. lab var lengthridit "Ridit (%) of Length (in.)";

. desc lengthridit, fu;

              storage   display    value
variable name   type    format     label      variable label
------------------------------------------------------------------------------------------------------
lengthridit     double  %10.0g                Ridit (%) of Length (in.)

. summ lengthridit;

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 lengthridit |         74          50    29.04986   .6756757   99.32432
```

Note that the Bross ridits (on a percentage scale) are *strictly* bounded between 0 and 100 percent, and have a mean of *exactly* 50 percent.

Computing a cubic ridit spine basis in `length`

We use the SSC package `polyspline`[4] to generate a basis of 5 cubic reference splines `rs_1` to `rs_5` in the ridit variable, corresponding to percentages of 0, 25, 50, 75 and 100, respectively:

```
. polyspline lengthridit, power(3) refpts(0(25)100) gene(rs_) labprefix(Percent@);
5 reference splines generated of degree: 3

. desc rs_*, fu;

              storage   display    value
variable name   type    format     label      variable label
------------------------------------------------------------------------------------
rs_1            float   %8.4f                 Percent@0
rs_2            float   %8.4f                 Percent@25
rs_3            float   %8.4f                 Percent@50
rs_4            float   %8.4f                 Percent@75
rs_5            float   %8.4f                 Percent@100
```

Note that we have labelled them using the `labprefix()` option of `polyspline`.

Percentiles corresponding to the 5 reference percentage ridits

To estimate the inverse ridits (also known as percentiles) corresponding to our 5 reference percentage ridits, we use the SSC package `rcentile`[4] to compute percentile car lengths in inches:

```
. rcentile length, centile(0(25)100) transf(asin);
Percentile(s) for variable: length
Mean sign transformation: Daniels' arcsine
Valid observations: 74
95% confidence interval(s) for percentile(s)
  Percent    Centile    Minimum    Maximum
        0        142  -9.0e+307        142
       25        170        164        174
       50      192.5        179        198
       75        204        200        212
      100        233        233   9.0e+307
```

Percentiles 0 and 100 are estimated as the minimum and maximum lengths, respectively, with lower and upper confidence limits (respectively) equal to minus and plus infinity (respectively). *However*, we are not really interested in confidence limits here, because. . .

Mean mileages corresponding to the 5 reference percentage ridits

... `length` is the *X*–variable, and we are really interested in the *conditional* means of the *Y*–variable `mpg`, corresponding to our 5 *sample* percentile lengths. We estimate these using `regress`:

```
. regress mpg rs_*, noconst vce(robust);

Linear regression                               Number of obs     =         74
                                                F(5, 69)          =     757.73
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9778
                                                Root MSE          =     3.4072

------------------------------------------------------------------------------
             |               Robust
         mpg |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rs_1 |    29.2563    2.17573    13.45   0.000     24.91584    33.59677
        rs_2 |   25.66597   .9778877    26.25   0.000     23.71514     27.6168
        rs_3 |   19.43958   .6659589    29.19   0.000     18.11103    20.76813
        rs_4 |   18.01778   .5218036    34.53   0.000     16.97681    19.05875
        rs_5 |   12.68334   1.043106    12.16   0.000      10.6024    14.76427
------------------------------------------------------------------------------
```

These estimates and confidence limits are expressed in miles per gallon, and in an alien–looking format. *However* . . .

Percentile lengths and mean mileages corresponding to the 5 reference percentage ridits

. . . if we collect the percentiles in an output dataset (or resultsset) using `xsvmat`, and collect the estimated mean mileages in a second resultsset using `parmest`, and reconstruct the `Percent` variable in the second resultsset using `factext`, and merge the 2 resultssets by `Percent` to form a single resultsset in memory, then we can list the percents, percentile lengths, and conditional mean mileages as follows:

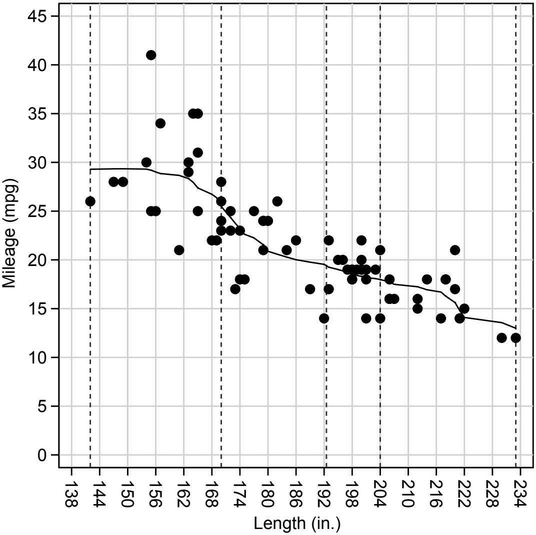
```
. list Percent Centile parm estimate min* max*, abbr(32);

     +------------------------------------------------------+
     | Percent   Centile   parm   estimate   min95   max95 |
     |------------------------------------------------------|
  1. |       0       142   rs_1      29.26   24.92   33.60 |
  2. |      25       170   rs_2      25.67   23.72   27.62 |
  3. |      50     192.5   rs_3      19.44   18.11   20.77 |
  4. |      75       204   rs_4      18.02   16.98   19.06 |
  5. |     100       233   rs_5      12.68   10.60   14.76 |
     +------------------------------------------------------+
```

This format is easier to understand. *However* . . .

Plot of fitted and observed car mileages against car length

- ... we can be even more informative if we `append` the resultsset to the original dataset and create some graphics.
- Here, we have scatter–plotted the observed mileages, and line–plotted the fitted mileages, against car length.
- The horizontal–axis reference lines show the positions of car length percentiles 0, 25, 50, 75 and 100.



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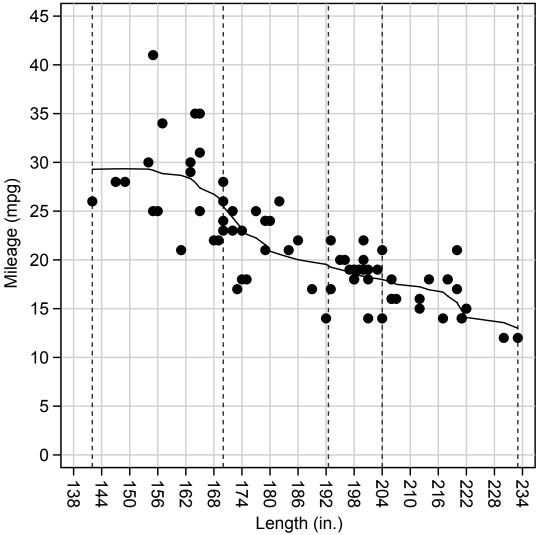

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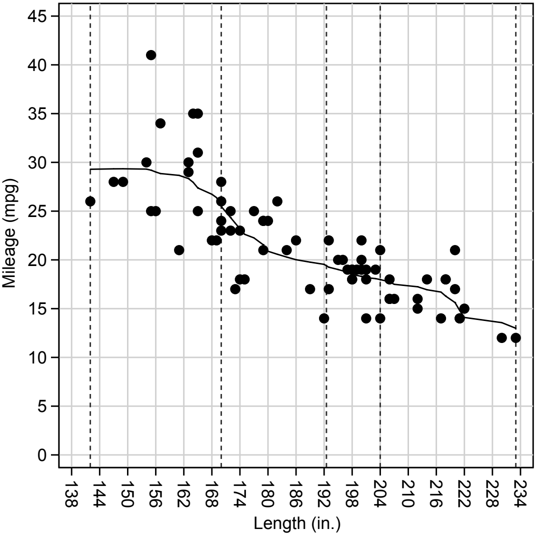

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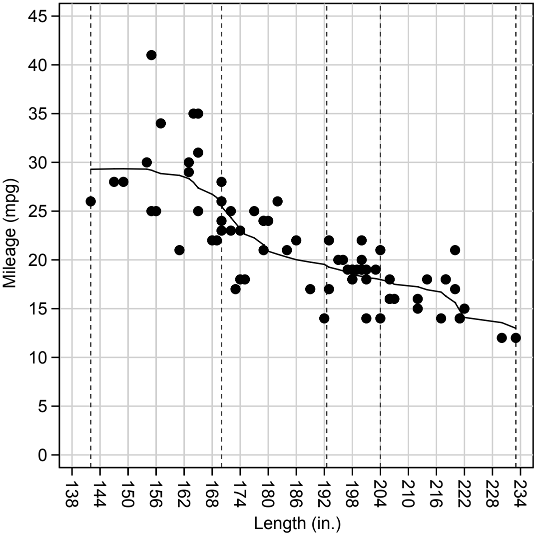

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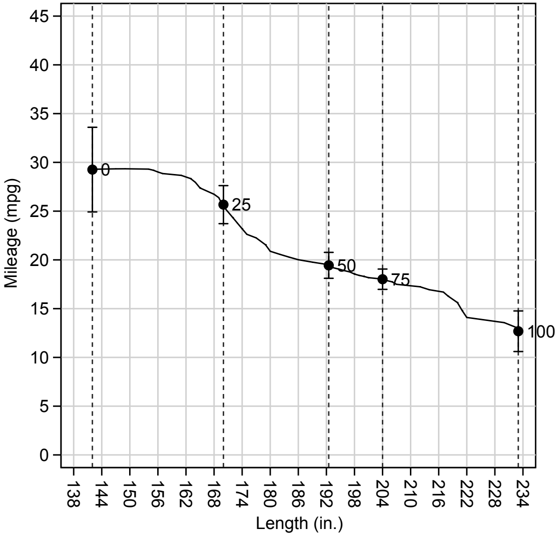

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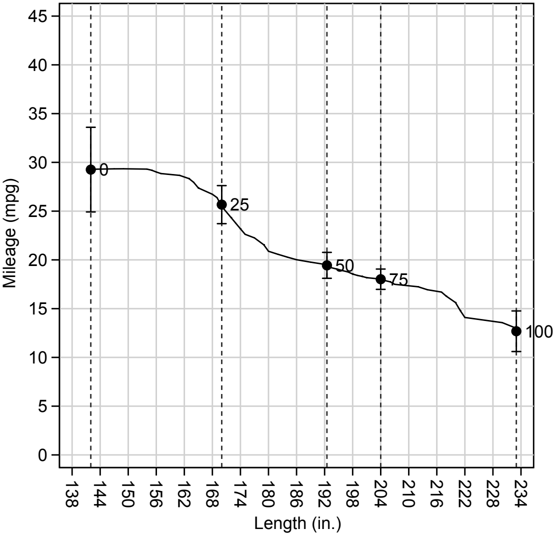

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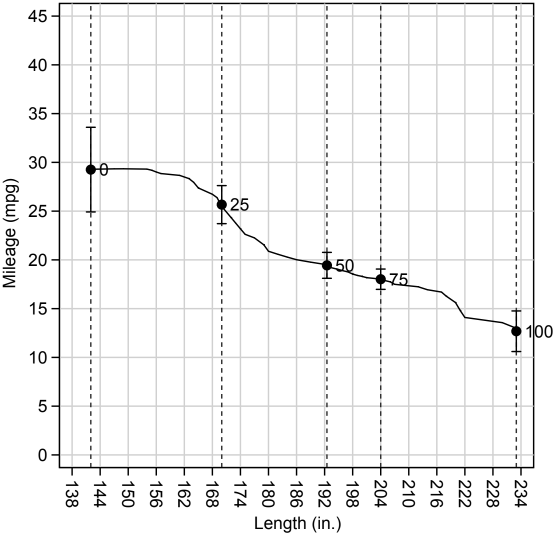

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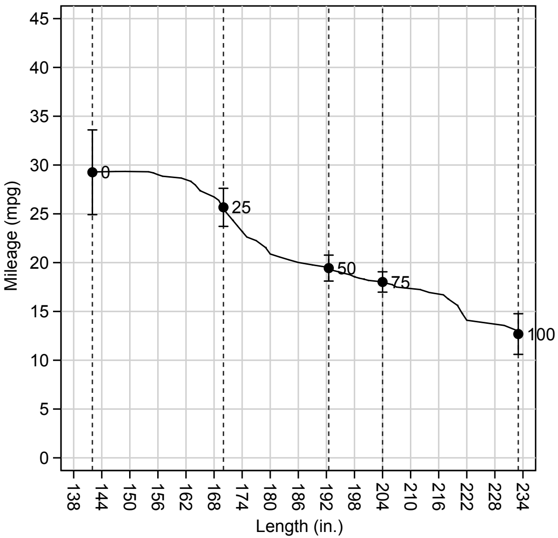

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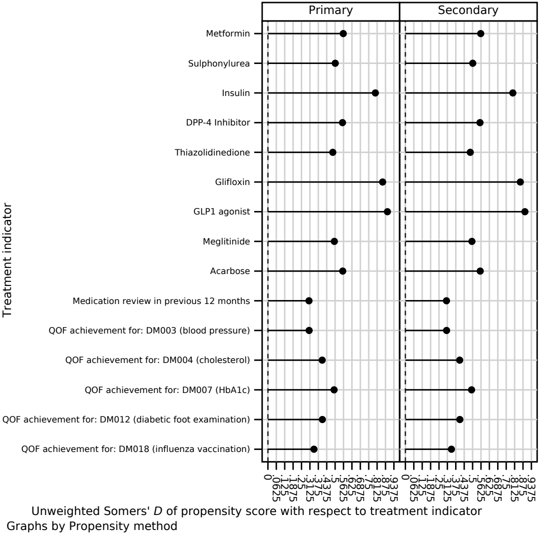
- Values for the same treatment are practically identical between the two propensity methods.



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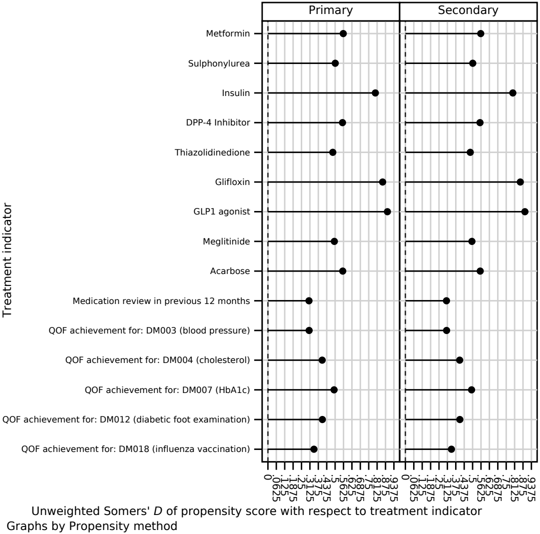

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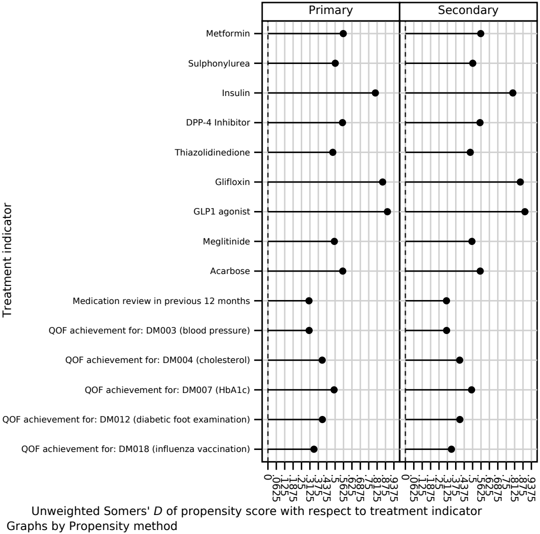

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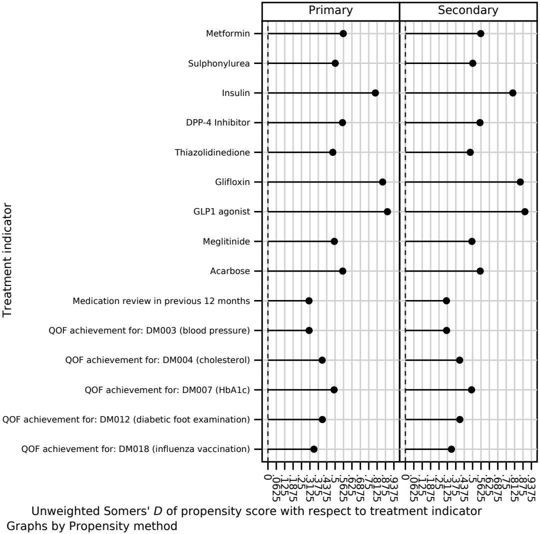

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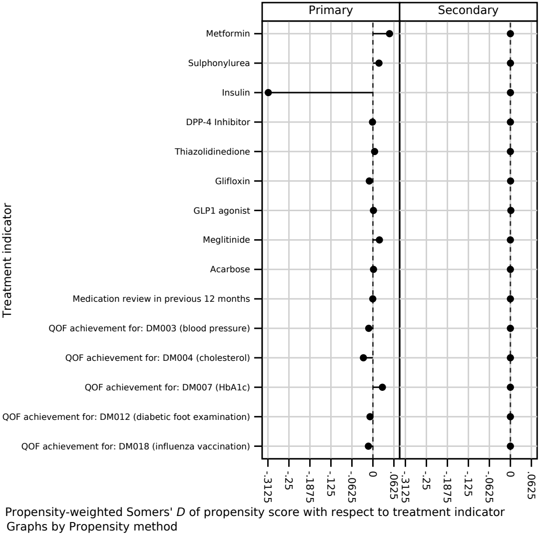

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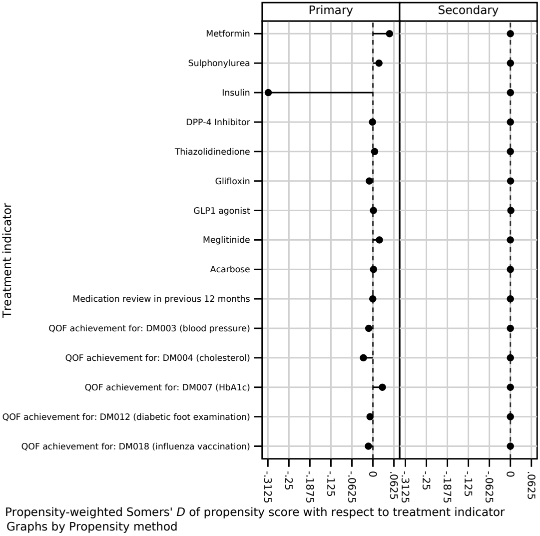

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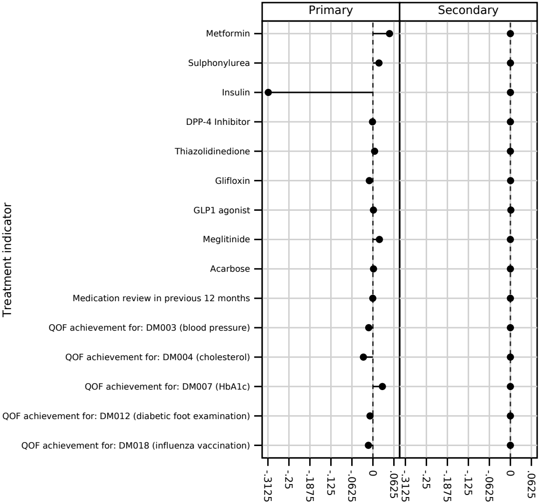

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Graphs by Propensity method

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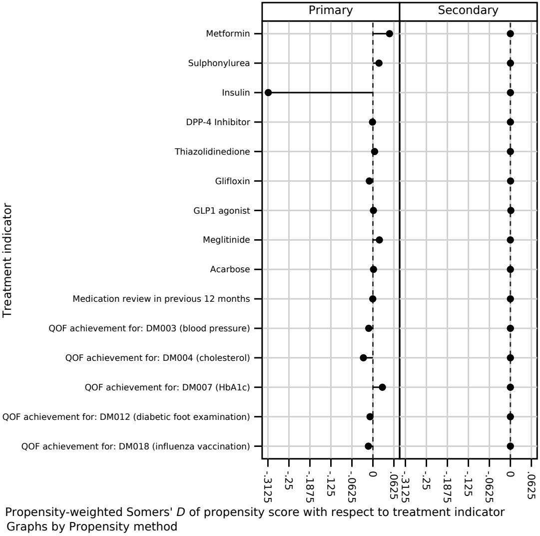

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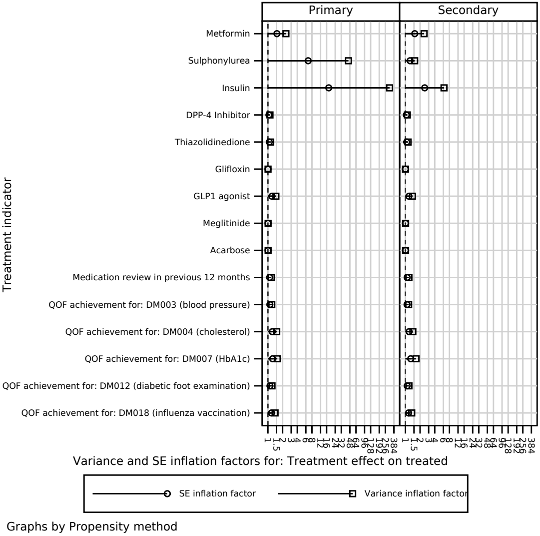

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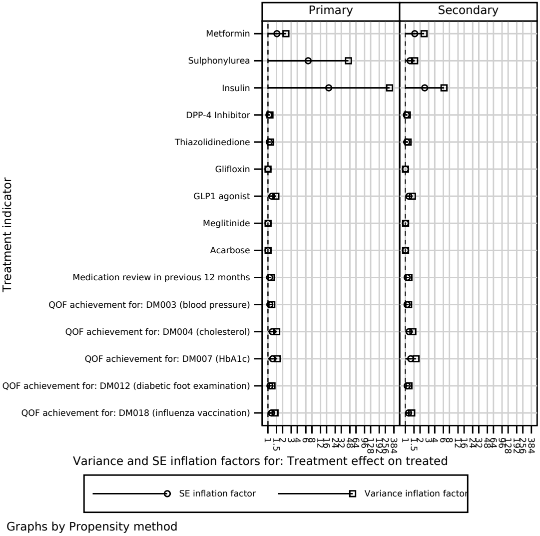

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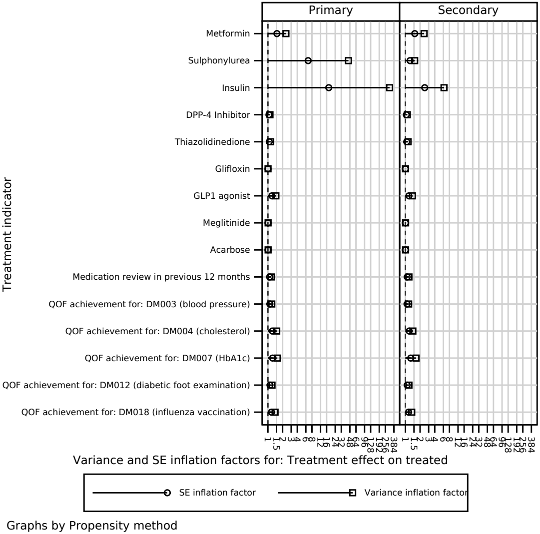

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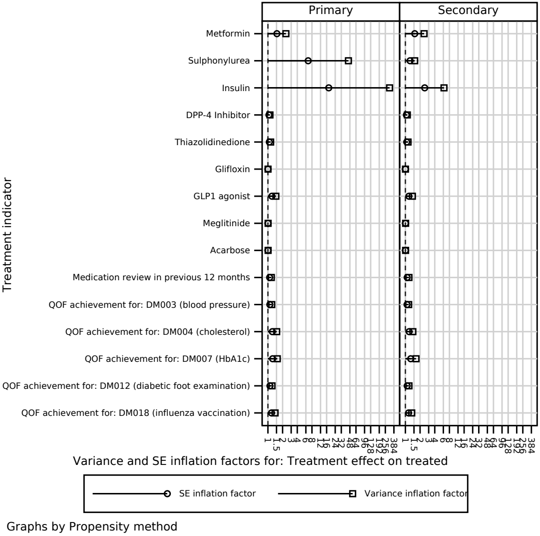

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This presentation, and the do–file producing the `auto` data examples, can be downloaded from the conference website at *http://ideas.repec.org/s/boc/usug17.html*

The packages used in this presentation can be downloaded from SSC, using the `ssc` command.