Panel Unit Root Tests with Structural Breaks

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Motivation

- Structural breaks are shocks which are exogenous to the model but have a lasting effect.
- They mislead unit root tests to accept the null of unit root when in fact it is stationary.
- Ignorance of structural breaks can distort power of tests and lead to deceptive conclusions.
- In real world, structural breaks can be caused by many factors, including changes in policy regime or important worldwide events, e.g., the Great Depression, World War II, oil price shock, Covid19.

Contribution

- **xtbunitroot** implements the econometric method suggested by Karavias and Tzavalis (2014).
- It performs panel unit root tests that allow for breaks in the intercepts of the individual series or in both intercepts and linear trends.
- **xtbunitroot** allows for one or two breaks at either known or unknown dates.
- It is the first Stata command which allows for panel unit root tests with structural breaks and can be viewed as a complement to the official **xtunitroot** command.

Underlying models

- M1 model tests against a structural break in the intercepts of the series:

$$y_{i,t} = \varphi y_{i,t-1} + (1-\varphi)[\alpha_{1,i} I(t \leq b) + \alpha_{2,i} I(t > b)] + u_{i,t}$$

- $\alpha_{1,i}$ and $\alpha_{2,i}$ are the fixed effects before and after the break.
- The break happens on date b and the notation $I(\cdot)$ denotes indicator function.

Underlying models

- M2 model tests against a structural break in both intercepts and linear trends:

$$
\begin{aligned}
y_{i,t} = \varphi y_{i,t-1} + \varphi[\beta_{1,i} I(t \leq b) + \beta_{2,i} I(t > b)] \\
+(1-\varphi)[\alpha_{1,i} I(t \leq b) + \alpha_{2,i} I(t > b)] \\
+(1-\varphi)[\beta_{1,i} t I(t \leq b) + \beta_{2,i} t I(t > b)] + u_{i,t}
\end{aligned}
$$

- $\beta_{1,i}$ and $\beta_{2,i}$ are coefficients of linear trends before and after the break.
- The break is allowed to be in $I_1 = \{1, 2, ..., T-1\}$ for M1 and in $I_2 = \{2, ..., T-2\}$ for M2.
- Both models can be extended to the case of two structural breaks.

Hypothesis

- The null hypothesis H_0 : All panel time series are unit root processes without breaks($\varphi = 1$).
- The alternative hypothesis H_1 : Some or all of the panel time series are stationary processes with breaks($\varphi < 1$).
- Structural breaks can only occur under the alternative hypothesis (Zivot and Andrews, 1992).
- The alternative hypothesis is homogeneous across different individuals but it is also evidenced that the test has power against heterogeneous alternatives (Karavias and Tzavalis, 2016).

Test Statistics

Known breaks

- For a given break date b, autoregressive parameter φ can be estimated with the following pooled least squares estimator:

$$\widehat{\varphi} = (\sum_{i=1}^{N} y_{i,-1}' Q_m^b y_{i,-1})^{-1} (\sum_{i=1}^{N} y_{i,-1}' Q_m^b y_i), \;\; m = \{M1, M2\}$$

- The orthogonal projection matrix Q_m^b is defined as $Q_m^b = I_T - X_m^b (X_m^{b\prime} X_m^b)^{-1} X_m^{b\prime}$, where $X_{M1}^b = (e_1, e_2)$ and $X_{M2}^b = (e_1, e_2, \tau_1, \tau_2)$.
- Karavias and Tzavalis (2014) show that the estimator $\widehat{\varphi}$ is inconsistent and must be modified.

Test Statistics

Known breaks

- The inconsistency of $\widehat{\varphi}$ is given by:

$$B^b = \underset{N\to\infty}{plim}(\widehat{\varphi}-1) = \frac{tr[\Lambda' Q_m^b]}{tr(\Lambda' Q_m^b \Lambda)}, for\ m = \{M1, M2\}$$

- $T \times T$ matrix Λ is defined as: $[\Lambda]_{r,c} = 1$ if $r > c$ and 0 otherwise, where $r, c \in \{1, ..., T\}$.
- The consistent test statistics is:

$$Z(b) = \sqrt{N}[C^b(k_u, \sigma_u^2)]^{-\frac{1}{2}}(\widehat{\varphi} - 1 - B^b) \xrightarrow{L} N(0,1)$$

- Assume errors are not serially correlated, the estimated variance is:

$$C^b(k_u, \sigma_u^2) = \{k\sum_{j=1}^{T}[A^b]_{j,j}^2 + 2\sigma_u^4 tr(A^{b^2})\}\{\sigma_u^2 tr(\Lambda' Q_m^b \Lambda)\}^{-2}$$

Test Statistics

Known breaks

- The consistent estimators of parameters k_u and σ_u^2 are given by Harris and Tzavalis (2004).
- For heteroskedastic errors, $C^b(k_u, \sigma_u^2)$ is replaced with

$$\bar{C}^b(k_u, \sigma_u^2) = \frac{1}{N}\sum_{i=1}^{N}\{[k_i \sum_{j=1}^{T}[A^b]_{j,j}^2 + 2\sigma_{u,i}^4 tr(A^{b^2})][\sigma_{u,i}^2 tr(\Lambda' Q_m^b \Lambda)]^{-2}\}$$

- For independently, normally distributed errors:

$$C^b = 2tr[(A^b)^2]/tr(\Lambda' Q_m^b \Lambda)^2$$

Test Statistics

Unknown breaks

- If the break date is unknown, one break point b_{min} is chosen which minimize test statistic over all possible break dates (Zivot and Andrews, 1992)

$$\min \mathcal{Z} = \min_{b \in I_m} Z(b) \ \textit{for} \ \ m = \{M_1, M_2\}$$

- Following Karavias and Tzavalis (2019), a bootstrap algorithm is implemented to derive the critical values and p-values of $\min \mathcal{Z}$.

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Basic Syntax

`xtbunitroot` *varname* [*if*] [*in*] [, trend known(*integer integer*) unknown(*numlist integer*) normal csd het nobootstrap]

- Dataset must be xtset before using the command.
- If no option is specified, the default will be M_1 model with a single break in the intercept, at an unknown date with 100 bootstrap replications.

Main Options

- `known`(*break1 break2*)
  - the number and places of breaks. This option must be used when the dates of the breaks are known.
- `unknown`(*numbreaks numboot*)
  - the number of unknown breaks and the number of bootstrap replications.
- trend
  - the common breaks affect both intercepts and trends.
- normal
  - the errors are normally distributed.
- csd
  - demeaning procedure for cross-sectionally dependent errors.
- het
  - errors are cross-sectionally heteroskedastic.

Empirical examples

- To examine the stationarity of banking variable: returns on assets.
- The quarterly data is collected from the Federal Deposit Insurance Corporation (FDIC), composed of a random sample of 500 banks, from 2018q3 to 2020q4.
- This period includes the COVID19 pandemic which may have caused breaks in the intercepts and trends of the series (started in 2020q1).

Results

- First assume that the date of the break is known to be 2020q1.
- Errors are assumed to be normally distributed but cross-sectionally dependent.
- The output below shows that the null hypothesis of non-stationarity is rejected at 1% significance level.

Output

```
. use xtbunitroot_example.dta, clear

. xtset fed_rssd time
       panel variable:  fed_rssd (strongly balanced)
        time variable:  time, 1 to 10
                delta:  1 unit

. xtbunitroot roa, known(7) normal csd
Karavias and Tzavalis (2014) panel unit root test for roa
------------------------------------------------------------------------------
H0: All panel time series are unit root processes
H1: Some or all of the panel time series are stationary processes
------------------------------------------------------------------------------
Number of panels:                 500      Number of periods:        10
Number of breaks:                 1

Cross-section dependence:         Yes      Linear time trend:        No
Cross-section heteroskedasticity: No       Normal errors:            Yes
Result: the null is rejected               Known break date(s):      7
------------------------------------------------------------------------------
                   Statistic   5% Asymtotic critical-value      p-value
------------------------------------------------------------------------------
 Z-statistic       -15.3360              -1.6450                0.0000
------------------------------------------------------------------------------
```

Output

- Secondly the break date is assumed to be unknown and determined from the data.

```
. xtbunitroot roa, unknown(1) normal csd
Karavias and Tzavalis (2014) panel unit root test for roa
------------------------------------------------------------------------------
H0: All panel time series are unit root processes
H1: Some or all of the panel time series are stationary processes
------------------------------------------------------------------------------
Number of panels:                  500       Number of periods:        10
Number of breaks:                  1         Bootstrap replications:   100
Cross-section dependence:          Yes       Linear time trend:        No
Cross-section heteroskedasticity:  No        Normal errors:            Yes
Result: the null is rejected                 Estimated break date(s):  6
------------------------------------------------------------------------------
                     Statistic    5% Bootstrap critical-value      p-value
------------------------------------------------------------------------------
 minZ-statistic      -25.1273             9.3046                   0.0000
------------------------------------------------------------------------------
```

Output

- Similarly the null hypothesis is rejected at 1% significance level.
- When the break date is unknown, the command reports the estimated break date: observation 6 in this case, which corresponds to 2019q4.
- Result: Returns on assets are found to be stationary with structural breaks in the intercepts.

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Conclusion

- This presentation has introduced a new Stata command **xtbunitroot** suggested by Karavias and Tzavalis (2014).
- The test is more powerful when structural break exists in intercepts and trends.
- It allows for one or two breaks under the alternative which can be either known or unknown.
- The command can be downloaded in stata using: **ssc install xtbunitroot**

References I

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Thanks!