Semivariogram estimation and panel data structure in spatial models: an empirical analysis

Théophile Azomahou*

Abstract

Some insights are provided on constructing contiguity matrices for spatial models when only the X-Y latitude-longitude coordinates of the centroids from spatial units are available. The spatial weights matrix is computed from the so-called "range" of the two- dimensional semivariogram estimation, that is, the plot of semivariances against the sampling interval. The resulting contiguity matrix is then included in an error-components model for a regressive spatial autoregressive process. The model is estimated in two stages using the sequential estimator suggested by Chamberlain (1982, 1984). To overcome the computational difficulties that beset spatial processes, the data, in the first stage, are treated as T cross-sections whose parameters are estimated by a pseudo maximum likelihood procedure. In the second stage, nonlinear restrictions that combine both weak simultaneity and correlation effects are imposed in the application of the minimum distance method. The presence of a weighting matrix precludes direct and/or linear restrictions on parameters of interest. This framework is applied to estimating spatial patterns in residential water demand.

Keywords: Minimum distance estimator, panel data, semivariogram estimation, spatial dependence, water demand.

JEL Classification: C13, C23, D12

BETA, Université Louis Pasteur,Strasbourg, Economics Department; 61, avenue de la Forêt Noire; F-67085 Strasbourg Cedex; France; phone: (+33) 3.90.41.40.71; fax: (+33) 3.90.41.40.50; e-mail:azomahou@cournot.u-strasbg.fr

1 Introduction

It is often relevant to consider the spatial distribution of phenomena such as diffusion patterns in counties or states of a country, or as a map of points of occurrences. This yields the spatial analysis of the so-called lattice data, i.e., observations for a fixed and given set of locations in space. Related applied econometric areas are various. LeSage and Dowd (1997) used this methodology to examine the spatial contiguity influences on state price level formation. A similar framework has been used by Case (1991) to describe spatial patterns in household demand for rice in some Indonesian districts. Recent examples of empirical works that explicitly incorporate spatial dependence concern, among others, the analysis of innovation decisions, Hautsch and Klotz (1999), the forecasting of cigarette demand using panel data, Baltagi and Li (1999), real wages variation to local and aggregate unemployment rates over time, Ziliak *et al.* (1999) and the estimation of a hedonic model for residential sales transactions, Bell and Bockstael (2000).

In a regression framework, spatial autocorrelation (more generally spatial dependence) is the situation where the dependent variable or/and the error term of a regression function, at each location, is correlated with observations on the dependent variable or/and values of the error term at other locations. As pointed out by Anselin (1988), ignoring this structure when it is actually existing results in mis-specification and bias in estimation. While most studies focus on cross-sectional specifications, spatial models for panel data have not received much attention.

As outlined by Case (1991), fixed effect specifications can be used to control for spatial components in panel data. But in some cases, when there is no intra-regional variation in variables of interest, a spatial modeling approach may be more appropriate. This is the case for example when the variation in the variable depends upon distance between points. Then, there is a perfect correlation between the variables of interest and the fixed effects. The same paper discusses the gains in information and efficiency which are achieved by spatial random effects modeling, and shows that when specific effects are uncorrelated with right hand side variables, there are clear benefits to spatial specification. More generally, it can be argued that the equicorrelated structure of individual dependence that is typically specified for the error-components in panel data models does not allow for distance decay effects. Furthermore, the equicorrelation is associated with the time, and not the individual, dimension of the data set. Such a structure is not adequate for estimating spatial patterns in panel data. This study provides empirical advances on this topic.

In this study I consider estimating panel data autoregressive models in the framework of Chamberlain's (1982, 1984) sequential estimator. We specify a mixed regressive spatial autoregressive model. This specification defines a class of random fields, i.e., models derived from processes indexed by space, time and cross-sectional dimensions. I work with a row-standardized spatial weighting matrix, i.e., the spatial weight matrix is normalized so that the rows sum to unity. This standardization produces a spatial lagged variable that represents a vector of average values from neighboring observations. The specification is assumed to be the true data generating process which relates observations with reference to points in space and time. Then the model id estimated in two stages.

To overcome the computational difficulties that beset spatial processes and assuming the errors to be normally distributed, the data, in the first stage, are treated as T cross-sections whose parameters are estimated by a pseudo-maximum likelihood procedures. Under suitable regularity conditions, this stage provides both unrestricted consistent parameters estimates, including the spatial coefficient, and scores which are used to compute the consistent asymptotic covariance needed for the second stage. Then, nonlinear restrictions that combine both weak simultaneity and correlation effects are imposed in the application of the minimum distance method. Three cases of restrictions are considered: the fixed slopes, the all identical parameters and the time-varying parameters. The minimum distance estimates are computed for each case and is consistent and asymptotically efficient. We used this specification for empirical application.

The empirical analysis consists in examining the spatial variations of residential water demand for the French department of "Moselle", including electricity price effects. At this stage, it is important to explain why electricity price can be used as additional regressor in the model specification and why the data at hand are appropriate to spatial context.

Indeed, as indicated by Hansen (1996), when estimating the determinant factors of residential water demand, we may expect to observe the indirect effects of energy variables, according to water consumption between different water-using tasks. Indeed, water is consumed by households jointly with different tasks which involves use of water and in most cases sizable amounts of energy and other goods (appliances, etc.). Table 5 in Appendix 6.2 reports the daily distribution (on average) of French residential water consumption between household tasks. About 40% of this distribution is concerned with water heating (mainly by electricity). We combine this consideration with spatial aspects for two reasons.

The first reason is related to the empirical concern. Several studies have pointed out a regionalized water consumption behavior of households living in municipalities concerned by our study, which is linked to the availability of water resources. See, e.g., INSEE (1998) for more details on this purpose.[1]). Furthermore as will be seen later, these municipalities have been organized in two spatial sectors for the need of water network management. In this context, the specification udes may be viewed as a model of endogenously changing tastes, which allows to check for social interdependence by testing the extent to which households look to a reference group when making water consumption decisions. It may also be thought of as indicating the magnitude and the direction of interactions between consumers with respect to the availability of water resources. The second reason is attached to the theoretical framework. As outlined earlier and as will be seen in data description, there is no intra-regional variation in water prices and variations in this variable depend on distance between minicipalities. As a result, a spatial approach seems more appropriate and should be preferred above pure fixed effect modeling in this specific application. Furthermore in our panel, the number of waves is less than the number of cross-sections. Then an framework like one suggested by Whittle (1954) can not be applied.[2] All these reasons motivate the use of the spatial approach adopted here.

Section 2 presents the model. The proposed specification combines elements of spatial modeling and panel data framework using the minimum distance approach. Section 3 is dedicated to data. We describe the sampling and basic descriptive statistics. Correlograms are computed to check for spatial patterns. We also use nonparametric density estimation to identify spatial sectorial tendance in the distribution of water average price. Estimation results involving both parameters estimate and spatial elasticities are presented in Section 4. Concluding remarks are given in Section 5.

[1] Tableaux de l'Economie Lorraine 1997/1998 (Tables of Lorraine Economics

[2] Whittle (1954) suggests that if panel data are available and if the time dimension is sufficiently large, $T > N$, one can consider, e.g., a seemingly unrelated regression specification, or an error components model to permit for cross-sectional correlation, and estimate the cross-sectional correlations through the time dimension of the panel.

2 Spatial model for panel data

The materials used here originate in Chamberlain (1982, 1984). . See also Crépon and Mairesse (1996) for a suitable presentation of the Chamberlain approach for linear models, and **?** for nonlinear models. The Chamberlain approach use restrictions implied by model assumptions on moments to obtain asymptotically efficient estimates without imposing conditional homoskedasticity or independance over time on the disturbance of the model. This estimator is sequential in that in a first stage, the moments of the variables are computed up to the second order, forming a set of auxiliary parameters. In a second stage, the parameters of interest are estimated on the basis of their relations with the auxiliary parameters, using minimum distance or asymptotic least squares estimator.

The basic feature of the Chamberlain method is that the sample covariance, say, the set of moments of the response variable and regressors can be summarized by a specific combination of moments by the so-called Π matrix, which is formed of the coefficients of the predictors of the response variable given the whole set of regressors (leads and lags) at each period. This means that if there are T waves in the panel and K explanatory variables, the $\Pi = [\pi_{t,j}]$ matrix of dimension $T \times KT$ is obtained by stacking one above the other the row vectors of demension $1 \times KT$ of the coefficients for the T separate year regressions. The restricted estimates from the minimum distance applied to the unrestricted Π matrix are more efficient under general conditions than the usual "within estimator" computed on the deviation of the variable from their individual means. In what follows, we state the basic relations between Π and parameters of primary and secondary interest implied by covariance restrictions in the framework of autoregressive spatial models. Then, we discuss the form of the estimating equations and of resulting restrictions on the Π matrix.

Consider a spatial autoregressive model for panel data containing a spatial lag of the response variable as additional regressor. Then the model has the following structure

$$y_{it} = \sum_{j \neq i} \rho \omega_{ij} y_{jt} + \sum_{k=1}^{K-1} x_{it}^{(k)} \beta_k + \mu_i + \eta_{it}, \quad |\rho| < 1 \tag{1}$$

$$i = 1, \cdots, N;\ j = 1, \cdots, N;\ t = 1, \cdots, T.$$

where y_{it} is the i-th observation on the dependent continuous variable at period t, $x_{it}^{(k)}$ is the i-th observation for the k-th explanatory variable, y_{it} is the j-th contiguous to i. ρ is scalar spatial coefficient and the β's are $k-1$ parameters of the remaining explanatory variables. ρ and β are parameters of primary interest. ω_{ij} is an element of the spatial weighting matrix, the computation of which based in semivariogram estimation is given in Appendix. The correlated individual effect is given by μ_i and η_{it} denotes a idiosyncratic i.i.d error term.

Let $\tilde{x} = [x_{it}, \omega_{ij} y_{jt}]$ be a block component of regressors in (**??**). In a linear regression framework, μ_i can be a general non-linear function of the $\tilde{x} = [x_{it}$'s that is decomposed into its minimum mean squared error predictor and orthogonal error

$$\mu_i = \lambda' \tilde{x}_i + v_i \tag{2}$$

with $\tilde{x}_i = (x'_{i1}, x'_{i2}, \cdots, x'_{iT}, y'_{j1}, y'_{j2}, \cdots, y'_{jT})'$. Restrictions on λ will allow the exclusion of the regressors for wich one may assume non-correlation with the heterogeneity term μ_i. Here, we will assume that the regression function $E(\mu_i|\tilde{x}_i)$ is actually linear and that $E(v_i \tilde{x}_i) \neq 0$, that is v_i indenpendent of the $\tilde{x}_i$'s included in (2). If we define $\underline{y}_i = (\underline{y}'_1, \cdots, \underline{y}'_T)'$, $\underline{x}_i = (\underline{x}_1^{(1)\prime}, \cdots, \underline{x}_T^{(1)\prime}, \cdots, \underline{x}_T^{(K-1)\prime})'$ and $\underline{u}_i = (\iota_T \mu + \underline{\eta}'_1, \cdots, \iota_T \mu + \underline{\eta}'_T)'$, which are respectively block matrices $(1 \times T)$ and $(1 \times KT)$ with each block being of $(N \times 1)$ dimension. The relation (1) can be rewritten as

$$\underline{y}_i = \underline{\rho} W \underline{y}_i + \Gamma(\underline{\beta}) \underline{x}_i + \underline{u}_i \tag{3}$$

where $\Gamma(\underline{\beta}) = (\beta_1, \cdots, \beta_{k-1}) \otimes I_T$. Factorising relation (3) from the left we can also write

$$[I - \underline{\rho} W]^{-1} \underline{y}_i = \Gamma(\underline{\beta}) \underline{x}_i + \underline{u}_i \tag{4}$$

So far, we have assumed that some component of $\underline{x}_i$ to be correlated with μ_i but not with η_{it}. Let us denote $\Phi = E(\underline{u}_i \underline{x}'_i)$, the covariance matrix between $\underline{u}_i$ and $\underline{x}_i$. Projecting equation (4) on $\underline{x}_i$ introduces the Π matrix

$$[I - \underline{\rho} W]^{-1} \Pi = \Gamma(\underline{\beta}) E(\underline{x}_i \underline{x}'_i) E(\underline{x}_i \underline{x}'_i)^{-1} + \Phi E(\underline{x}_i \underline{x}'_i)^{-1} \tag{5}$$

with $\Pi = E(\underline{y}_i \underline{x}'_i) E(\underline{x}_i \underline{x}'_i)^{-1}$. Then non-linear restrictions are obtained in term of Π and parameters of primary and secondary interest, $\underline{\rho}$, $\underline{\beta}$, and $\underline{\lambda}$:

$$\Pi = \text{vec} \left\{ [I - \underline{\rho} W]^{-1} \Gamma(\underline{\beta}) + [I - \underline{\rho} W]^{-1} \Phi E(\underline{x}_i \underline{x}'_i)^{-1} \right\} \equiv f(\theta) \tag{6}$$

with $\Pi = (\Pi'_1, \cdots, \Pi'_T)'$, $\theta = (\underline{\rho}', \underline{\beta}', \underline{\lambda}')'$.

From (6) note that the presence of a spatial weighting matrix precludes direct and/or linear restrictions on parameters of interest, inducing necessarily nonlinear restrictions that combine both weak simultaneity and correlation effects.[3]

In a first stage, assuming the errors to be normally distributed, Π_t can be estimated by maximazing the marginal log-likelihood function based on the distribution of y_{it} conditional on non-correlated regressors. In a second stage non-linear restrictions (6) can be imposed by applying minimum distance estimator

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \left[\hat{\Pi} - f(\theta)\right]' S_n \left[\hat{\Pi} - f(\theta)\right] \tag{7}$$

where $S_n \overset{\text{a.s.}}{\to} S_0$ a positive definite symmetric matrix. In the case of spatial models, a consistent estimate of S_n is not trivial since the computation of S_n make use of the spatial weights matrix in the likelihood function from the first stage.

In order to derive the asymptotic distribution of $\sqrt{N}(\hat{\Pi} - \hat{\Pi}_0)$, let Ω_0 denote a consistent approximation of S_0 such that

$$\Omega_0 = \frac{1}{N}\left[J_0^{-1} I_0 J_0^{-1}\right] \tag{8}$$

where $J_0 = \text{diag}\{J_1, \cdots, J_T\}$ is a block diagonal matrix with elements

$$J = E\left(-\frac{\partial^2 \psi(y, X; W, \theta_0)}{\partial\theta\partial\theta'}\right) \tag{9}$$

and elements of I_0 are

$$I = E\left(\frac{\partial\psi}{\partial\theta}(y, X; W, \theta_0)\frac{\partial\psi}{\partial\theta}(y, X; W, \theta_0)'\right) \tag{10}$$

A consistent estimator $\hat{\Omega}$ of Ω is obtained as follows. Let $\psi_i(y, X; W, \rho, \beta, \sigma^2)$ denote the log-likelihood for one observation at each multi-period. That is

$$\begin{aligned}\psi_i(y, X; W, \rho, \beta, \sigma^2) = &-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\sigma^2 + \frac{1}{N}\ln|A| \\ &- \frac{1}{2\sigma^2}\left[\sum_{j\in J}\left(\mathbf{1}_{[i=j]} - \rho\omega_{ij}\right)y_j - \sum_k X_{ik}\beta_k\right]^2\end{aligned} \tag{11}$$

[3]The spatial autoregressive model on consideration combines naturally two known cases of estimating equations: correlated effects and weak simultaneity due to the presence of the spatially lagged dependent variable. In correlated effects case, it is assumed that the past, present and future values of the explanatory variables are not correlated with the time varying disturbance η_{it}, but that they can be correlated with the specific effects μ_i. Weak simultaneity corresponds to the case of pre-determination or weak exogeneity of the explanatory variables (or some of them) that allow for both "feedback effects" of the response variable which can be interpreted as spatial diffusion patterns.

where $j = 1, \cdots, J$ is the set of communities contiguous to a community i and $\mathbf{1}_{[i=j]}$ denotes an indicator function. Taking partial derivatives of (11) with respect to the parameters yields:

$$\frac{\partial \psi_i(\cdot)}{\partial \beta_k} = \frac{1}{\sigma^2}\left[\sum_{j\in J}\left(\mathbf{1}_{[i=j]} - \rho\omega_{ij}\right) y_j - \sum_h X_{ih}\beta_h\right] X_{ik} \tag{12}$$

$$\frac{\partial \psi_i(\cdot)}{\partial \rho} = \frac{1}{\sigma^2}\left[\sum_{j\in J}\left(\mathbf{1}_{[i=j]} - \rho\omega_{ij}\right) y_j - \sum_h X_{ih}\beta_h\right] \sum_{j\in J}\omega_{ij}y_j + N^{-1}\xi \tag{13}$$

$$\frac{\partial \psi_i(\cdot)}{\partial \sigma^2} = -\frac{1}{2\sigma^2} + \frac{1}{2\sigma^4}\left[\sum_{j\in J}\left(\mathbf{1}_{[i=j]} - \rho\omega_{ij}\right) y_j - \sum_k X_{ik}\beta_k\right]^2 \tag{14}$$

with

$$\xi = \frac{\partial}{\partial \rho}\ln(I - \rho W) = -\mathrm{tr}\left([I - \rho W]^{-1} W\right)$$

Let $\hat{\mathrm{v}} = \left[\frac{\partial \psi_i(\cdot)}{\partial \beta_k} \mid \frac{\partial \psi_i(\cdot)}{\partial \rho} \mid \frac{\partial \psi_i(\cdot)}{\partial \sigma^2}\right]_{\theta=\hat{\theta}}$ be a block element of $\hat{I}$ obtained by stacking the vector of derivatives evaluated at parameters estimates. The empirical variances matrix $\hat{I}$ of individual scores is given by the cross product of $\hat{\mathrm{v}}_{t,s}$ for $t \neq s$. The estimate $\hat{\Omega}$ of Ω is computed as $\hat{J}^{-1}\hat{I}\hat{J}^{-1}$ by replacing theoretical expectations by sample means.

3 Data

The data considered here rely on a lattice sample coming from the French network of drinking water distribution. The sample is constituted by 115 neighboring municipalities located in the north-east of France, in the "department of Moselle ". Households living in these communities are supplied with drinking water by a private operator. The data are collected with a biannual frequency, from 1988.1 to 1993.2. We have then a balanced panel of 1380 spatial observations. Thus, we are concerned with the residential water consumption per municipality expressed in m3 per household. A detailed data analysis can be found in Azomahou (1999). We refer the reader to this study for a complete data description.

The explained variable is the aggregate water consumption per community expressed in cubic meter per house. Urban communities are larger than rural ones. So as to consider homogeneous observations and in order to reduce the community size effect, each consumption value has been divided by the total number of households per community in 1990, the year of the last inventory available. It also constitutes the last period when the population general census was conducted by the offices of the National Institute of Statistics and Economic Studies "(INSEE)".

National statistics indicate an average water consumption tendency around 120 m3 per house and per year. These figures vary from one house to another. Old houses are light on water consumption whereas high standing dwellings with gardens can consume around 180 m3. When we compare these indicators with those computed from the sample, we notice that the average consumptions recorded are of the same magnitude. Minimum values can be considered as the consumption of rural communities. These tendencies are also indicative of the standard of living of the population considered. As a whole, there is no outliers in consumption values. Note however some high values for 1989.2, 1991.1 and 1991.2 where we observe 74.55, 75.56 and 75.04 m3 respectively. This may result from extra consumptions in addition to purely domestic ones. It may be the case for households having small businesses or farms as described above. These statistics support, on average, the relative statility of our data.

Disposable income statistics are characterised by very low values. Consider for example the year 1990 where the minimum values are the lowest, that is, 31,820 FF per household paying tax. One obtains a monthly disposable income figure of 2,651.66 FF. Supposing that this household paying tax is made up of a single, the latter earns around the "minimum insertion income" in France. This shows the difficulty usually encountered in recording income data. Other reasons explain these low values. Indeed, various studies conducted by the "National Institut of Statistics" show that, in the department of "Moselle", taxable incomes under-estimate by 30% on average the actual household incomes.[4] This under-estimation is extremely high for the self-employed (43%), and even more for self-employed farmers (57%). Moreover, even if we know that the consequences of the economic crisis on the evolution of global wages has been compensated by a strong increase of social benefits and a slight increase in taxes, the "Moselle" departement is below the national indicators.

The organization and the management of water distribution in France pertain to public service liability. The price results from a negociation between local authorities and the water distributor which may be the local collectivity itself or a private company. Average price values clearly indicate relevant patterns. The average price increases continuously on the twelve biannual periods. This increase shows three

[4] Tableaux de l'Economie Lorraine 1997/1998 (Tables of Lorraine Economics), (INSEE (1998)).

figures. From 1988.1 to 1989.2 the average price is below 7 FF; from 1990.1 to 1991.2 it is below 8 FF and from 1992.1, the tendency is higher than the previous ones. This last tendency indicates an important modification in the water price structure. As a whole, the price variable suggests a clustering pattern. It also presents an increasing dispersion within clusters with stable minimum values (around 3,5 FF).

According to the water supplier, the communities are organized in two sectors, but there is some doubt about the exact number of sectors. We denote each sector by a dummy variable (dummy 1 for sector 1 and zero for sector 2). Out of 115 communities, 65.2% belong to sector 1. The sectors correspond to two distinct areas of water management. This spatial arrangement is mainly due to the network management issues (water transportation and various treatments to make water drinkable) and is closely linked to the various elements of water prices.[5] The marginal price of water is the same within a given sector but varies between sectors. Thus, we know that there is no intra-regional variation in the marginal price. But the average price varies from one community to another when the fixed charges of water are included. Moreover, the laws on water of november 1992, by the so-called "M-49 directive", have strongly modified the working orders of water agencies.[6] This modification has been translated into a high increase in water prices. The aim is to let customers pay for the effective price of water, and no more for the water service.

Finally, note that the meteorological values presented here are not dummy variables as in many studies, but the thrue values recorded by the Regional Center of Meteorological Studies.

4 Estimation results

We use the model specified above to carry out empirical estimation on data described in the previous section.[7] Tables 1 and 2 present the unrestricted maximum likelihood estimates for the mixed regressive spatial autoregressive model for the twelve time

[5]To make ideas clear, we computed the correlation coefficient between the average price variable and the sector dummy for each time period: (-0.33, -0.32, -0.38, -0.38, -0.36, -0.39, -0.35, -0.34, -0.35, -0.38, -0.35, -0.41). There is evidence of correlation.

[6]Set up on November 10th, 1992 (its implementation date) the "M-49 directive" imposes to water services (supply and cleaning up) the rule of budget balance. They are forced not to make their general budget support the water spendings (building up and maintenance of network, equipments, cleaning up...).

[7]GAUSS procedures to implement these calculations are available from the author on request.

periods. A Lagrange multiplier test shows rejection of the alternative spatial error specification for most cases except for 1988.2, 1991.1, 1992.1 and 1993.1.

For these cases, spatial dependence remains in the residuals and our specification is clearly rejected. Thus, a mixed autoregressive spatial moving average model, i.e., a model with a spatial lag dependent variable as well as a spatial moving average process in the error will be more appropriate. In the other cross-sections, the spatial dependence has been adequately dealt with. A spatial Breusch-Pagan test for spatial heteroskedasticity clearly indicates that heteroskedasticity patterns remain in the specification.

Characteristics variables: proportion of persons below 19 years, proportion of workers, proportion of unemployed, community equipments, density of population appear to be stable over time. Some of them (proportion of persons below 19 years, proportion of workers and proportion of unemployed) are highly significant in the unrestricted cross- sectional estimates. Note that the average price of water becomes significant only from 1990.1 on. The intercept varies widely but is not significant.

Table 3 reports the results from the asymptotic least squares for the two sets of restrictions. From the general specification described in the modelling section, we obtain various estimates by imposing two restrictions. The first restriction is that of fixed slopes expressed as:

$$g(\hat{b}(\theta), a) = \begin{pmatrix} \hat{\theta}_x^1 - \theta_x \\ \hat{\theta}_x^2 - \theta_x \\ \vdots \\ \hat{\theta}_x^T - \theta_x \end{pmatrix} = O \tag{15}$$

with $\hat{\theta}_t = (\hat{\theta}_t^0 \; \hat{\theta}_t^x)'$, $t = 1, \cdots T$, where θ_t^0 and $\hat{\theta}_t^x$ denote respectively the parameters vector of varying intercept and the parameters vector of fixed slopes for the period t, and $a = \theta_x$. The second restriction is that of all identical parameters:

$$g(\hat{b}(\theta), a) = \begin{pmatrix} \hat{\theta}_1 - \theta \\ \hat{\theta}_2 - \theta \\ \vdots \\ \hat{\theta}_T - \theta \end{pmatrix} = O \tag{16}$$

with $\hat{b} = (\hat{\theta}_1, \cdots, \hat{\theta}_T)'$ and $a = \theta$. For each case, the ALS estimates may be computed by generalized least squares procedures.

Table 1: Unrestricted ML regressive spatial autoregressive estimates (continued)

	Cross-section estimates (and standard errors)					
Variable	1988.1	1988.2	1989.1	1989.2	1990.1	1990.2
Intercept	128.33	-157.20	-190.32	58.00	-653.15	-160.67
	(157.10)	(171.10)	(267.64)	(244.24)	(310.40)	(256.57)
Disposable Income	0.524	0.036	-0.564	-0.887	0.238	0.063
	(0.665)	(0.590)	(0.717)	(0.736)	(0.605)	(0.605)
Water price	-1.346	-0.860	-1.508	-1.702	-2.093	-1.391
	(1.275)	(1.195)	(1.082)	(1.209)	(1.116)	(1.137)
Electricity price	0.194	0.189	0.483	0.125	0.599	0.349
	(0.130)	(0.108)	(0.264)	(0.235)	(0.284)	(0.240)
Rainfall	-0.946	0.269	-1.220	-1.384	0.862	-0.552
	(0.380)	(0.406)	(0.373)	(0.426)	(0.376)	(0.318)
Mean temperature	-1.138	16.087	3.419	8.371	25.220	2.924
	(6.469)	(8.481)	(6.521)	(6.257)	(9.068)	(6.989)
Persons $<$ 19 years	-1.396	-1.694	-1.028	-1.239	-0.576	-0.882
	(0.575)	(0.530)	(0.596)	(0.607)	(0.600)	(0.582)
Workers	-2.662	-1.478	-2.194	-0.726	-2.724	-1.716
	(0.651)	(0.585)	(0.700)	(0.695)	(0.647)	(0.659)
Unemployed	-2.977	-2.624	-3.321	-2.740	-3.477	-3.124
	(0.603)	(0.561)	(0.603)	(0.614)	(0.621)	(0.616)
Equipments	-0.409	-0.678	-0.281	-0.211	-0.084	-0.078
	(0.352)	(0.330)	(0.359)	(0.366)	(0.359)	(0.359)
Density of population	0.166	0.123	0.225	0.004	0.316	0.081
	(0.400)	(0.374)	(0.405)	(0.415)	(0.412)	(0.410)
Spatial lagged variable	0.273	0.119	0.281	0.323	-0.004	0.310
	(0.282)	(0.325)	(0.288)	(0.292)	(0.335)	(0.292)
Diagnostics tests, (p-value)						
LM spatial error	0.704	3.911	0.219	0.826	0.269	0.374
	(0.401)	(0.047)	(0.639)	(0.363)	(0.603)	(0.540)
Spatial B-P heteroskedas.	13.753	7.778	19.826	13.538	16.634	25.224
	(0.131)	(0.556)	(0.019)	(0.139)	(0.054)	(0.002)
Number of cross-section obs.			115			

Table 2: Unrestricted ML regressive spatial autoregressive estimates (end)

	Cross-section estimates (and standard errors)					
Variable	1991.1	1991.2	1992.1	1992.2	1993.1	1993.2
Intercept	-248.72	-423.66	-72.88	-21.60	-50.34	-320.07
	(348.13)	(330.08)	(346.51)	(317.15)	(322.76)	(350.23)
Disposable Income	0.067	0.100	0.127	-0.225	0.147	-0.269
	(0.612)	(0.518)	(0.457)	(0.443)	(0.439)	(0.483)
Water price	-1.212	-3.595	-2.177	-0.729	-1.949	-3.092
	(1.167)	(1.025)	(0.898)	(0.915)	(0.689)	(0.757)
Electricity price	0.443	0.500	0.166	0.215	0.147	0.482
	(0.346)	(0.297)	(0.334)	(0.276)	(0.306)	(0.281)
Rainfall	-1.384	-0.127	0.011	-0.911	0.002	-0.356
	(0.606)	(0.306)	(0.384)	(0.264)	(0.280)	(0.319)
Mean temperature	-9.589	10.831	10.025	3.240	13.723	7.660
	(8.177)	(8.561)	(7.646)	(12.058)	(9.354)	(8.216)
Persons $<$ 19 years	-0.829	-0.458	-0.697	-0.463	-1.138	-0.445
	(0.651)	(0.605)	(0.566)	(0.537)	(0.550)	(0.567)
Workers	-2.533	-2.088	-2.209	-1.613	-1.796	-2.517
	(0.719)	(0.685)	(0.689)	(0.638)	(0.646)	(0.652)
Unemployed	-3.088	-2.928	-3.067	-2.878	-2.479	-3.132
	(0.690)	(0.646)	(0.623)	(0.586)	(0.606)	(0.644)
Equipments	-0.022	0.135	-0.166	0.015	-0.555	0.204
	(0.402)	(0.379)	(0.364)	(0.335)	(0.345)	(0.365)
Density of population	-0.096	0.453	0.114	0.187	-0.004	0.523
	(0.457)	(0.435)	(0.416)	(0.392)	(0.406)	(0.421)
Spatial lagged variable	0.386	0.134	0.484	0.260	0.233	0.287
	(0.273)	(0.299)	(0.243)	(0.285)	(0.299)	(0.284)
Diagnostics tests, (p-value)						
LM spatial error	6.386	0.074	7.863	1.194	9.798	1.093
	(0.011)	(0.785)	(0.005)	(0.274)	(0.001)	(0.295)
Spatial B-P heteroskedas.	13.458	20.644	21.721	15.658	19.717	42.132
	(0.142)	(0.014)	(0.009)	(0.074)	(0.019)	(0.000)
Number of cross-section obs.			115			

Table 3: Asymptotic least squares estimates

Variable	Restriction 1 (fixed slopes) coef.	std.err	t-stat.	Restriction 2 (all fixed parameters) coef.	std.err	t-stat.
Intercept	—	—	—	4.496	40.886	0.109
Disposable Income	0.092	0.145	0.637	0.205	0.170	1.201
Water price	-1.998	0.253	-7.878	-2.385	0.264	-9.017
Electricity price	0.240	0.055	4.324	0.201	0.038	5.226
Rainfall	-0.367	0.082	-4.474	-0.079	0.041	-1.901
Temperature	5.780	1.992	2.901	0.594	0.425	1.399
Persons $<$ 19 years	-0.991	0.155	-6.390	-0.959	0.183	-5.241
Workers	-2.104	0.175	-11.965	-2.131	0.201	-10.584
Unemployed	-2.931	0.167	-17.544	-2.800	0.196	-14.217
Equipments	-0.223	0.097	-2.292	-0.271	0.115	-2.347
Density of population	0.149	0.111	1.341	0.175	0.130	1.338
Spatial lagged variable	0.271	0.078	3.437	0.289	0.091	3.163
R^2		0.692			0.603	
$\bar{R}^2$		0.656			0.565	
$\chi^2(5\%)$		94.165			133.972	
d.o.f		143			121	
Number of obs ($N \times T$)			1380			

The minimum distance tests indicate no rejection for our restrictions (fixed slopes and all fixed parameters). Nevertheless, imposing additional restrictions may lead to rejection. For the first restriction, the estimated coefficients appeared to be significant except for the disposable income and the density of population variables. The other coefficients have the expected sign, except perhaps for the coefficient of the electricity price variable which is positive, which a priori appears to be surprising.

Indeed, although complementarity between the two goods (water and electricity) may be expected, the positive sign for the parameter of electricity average price variable indicates that, for the sample concerned, water and electriciy display substituability patterns. This means that an increase in electricity average price may result in more water consumption by residential consumers. This a priori surprising result is in contradiction with the study of Hansen (1996) where the energy cross-price parameter is found to be negative. Our cross-effects estimates suggest that changes in electricity average price may induce modifications in the distribution of residential water consumption for different uses. That is to say, the share of residential water consumed in connection with electricity may decrease with electricity price, whereas the remainder (the share of residential water consumed without energy) does not. We have noticed in section 1 that about 40% of daily residential water consumption in France is concerned with heating. 60% would not be, which partly explains our result. This may also indicate that consumers take into account the electricity block pricing structure where water consumption occurs effectively. For the second restriction, meteorological variables (rainfall and mean temperature) are no longer significant but are of the expected sign.

The spatial coefficient is also highly significant, which confirms the modelling framework. Here, the spatial behavior may be viewed in two ways. First, we can argue that households are actually influencing their neighbours. The water consumption behavior of other households affects the consumption of a given household through social proximity. In this sense, the estimated spatial coefficients represent a direct measure of externality. The significant spatial pattern may also be interpreted as the reaction of households with respect to the availability of water resources.

5 Conclusion

Statistical inferences in models for spatial processes are highly susceptible to misspecifications in the contiguity matrix. In this paper, we take advantage of geostatistical tools using the semivariogram to describe the degree of spatial dependence in an economic attribute, and to suggest a way for computing a spatial weight matrix for spatial econometric models. As shown, our approach is feasible and provides a meaningful alternative when little information on the spatial structure of the attribute being studied is available to the investigator.

So, what can we learn through this study? First, the estimated spatial lagged parameter is strongly significant, which means that households living in the same geographic area have approximately similar water consumption behaviors. Finally, some comments concerning price variables are in order. We find evidence that consumers respond jointly to water and electricity average price, not to water average price only. The use of electricity average price as additional regressor improved the model specification. A crucial assumption behind the maximum likelihood method used is that the disturbances are normally distributed. As pointed out by LeSage (1996), violation of this can arise from outliers or spatial enclave effects where a small cluster of observations display aberrant behavior. This aspect remains to be examined.

6 Appendix

6.1 Computation of the contiguity matrix

Given the importance of the choice of the proper weights for the interpretation of spatial models, some inherent characteristics of weights matrices must be satisfied. These necessary regularity conditions such that asymptotics may be invoked to obtain the properties of estimators and test statistics. For example one may ensure mixing conditions such that:

$$\sum_{i=1}^{n} |\omega_{ij}| \leq \kappa_\omega, \qquad \text{for all } i = 1, \cdots, n; n \geq 1$$

$$\sum_{j=1}^{n} |\omega_{ij}| \leq \kappa_\omega, \qquad \text{for all } j = 1, \cdots, n; n \geq 1$$

where κ is a finite constant. The above relations mean that the row and column sums of the weighting matrix are uniformly bounded in absolute value. Specifically, this assumption means that weights must be nonnegative and restricts the extent of correlations relating to the elements of u such that:

$$n^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} |\text{corr}(u_i, u_j)| \leq \kappa < \infty \qquad \text{for all } n \geq 1$$

where $\text{corr}(u_i, u_j)$ denotes the correlation between u_i and u_j.

From now, suppose that the spatial units are not all contiguous and that the investigator does not have any prior information to guide him for the choice of the contiguity matrix, other than the X-Y latitude-longitude coordinates of the centroids from each spatial units. What can he do ? First, choosing an appropriate metric, he may compute a distance matrix and use it directly as weights. But as drawback, this choice usually leads to explosive values for spatial coefficients; that is ρ and λ will no longer be less than one.

Another way consists in specifying W as a function of distance (inverse distance, negative exponentials of distance etc.), see Cliff and Ord (1981) for details. However an important problem results from the incorporation of unkown parameters to be estimated in the weights. These parameters are usually determined a priori from a step which is not included in the whole spatial analysis. Such approach turns out to be a circular reasoning. Indeed, the spatial structure which the investigator may wish to discover from the sample is assumed to be known before data analysis.

Finally the investigator may compute a binary contiguity matrix based on critical distance bands. However, recall that any prior notion of which meaningful distance ranges is available to the researcher.[8] So he may try several distance bands until finding a "good one". But this is not a scientific way to proceed. Moreover, the true spatial process underlying the system is no longer well stated. This is a typical issue one may face in empirical studies. Our proposed approach to handle this problem is to bring in relations between geostatistics and spatial econometrics. We suggest to use the two-dimensional semivariogram estimation for computing the weights matrix from the so-called "range". That is the plot of semivariances against the sampling interval.[9] The resulting contiguity matrix can then included in an error-components model for a regressive spatial autoregressive process.

Semivariogram allows to assert the spatial variability of an attribute. It results from the plot for semivariances against sampling interval or distance lag. Because of its ease of computation and lack of issue when the variance is non-stationary, the semivariance is a robust tool having the properties of optimum interpolation and structure recognition. In this section, we show how the semivariogram can contribute to constructing spatial weights in a meaningful way. Then, we give an application.

Consider a series of observations for an economic attribute $Z(s_i)$ measured at locations $\{s_i : i = 1, \cdots, N\}$. The semivariance denoted $\gamma(h)$ of the increment $Z(s_i) - Z(s_j)$ is defined as one-half the variance of this increment and is computed as:

$$2\gamma(h) = \text{var}\left[Z(s_i) - Z(s_j)\right] \tag{17}$$

For points separated by distance h, relation (17) can be rewritten:

$$2\gamma(h) = \text{E}[Z(s+h) - Z(s)]^2 \tag{18}$$

[8]Such prior notions may be for instance, economic organization of spatial agglomerations, travel time, social network theory, etc.

[9]To the best of my knowledge, this approach is new in its application for computing spatial weights matrices.

The semivariance is estimated as:

$$2\hat{\gamma}(h) = \frac{1}{|N(h)|} \sum_{N(h)} [Z(s_i) - Z(s_j)]^2 \tag{19}$$

where the sum is over $N(h) \equiv \{(i,j) : s_i - s_j = h\}$ and $|N(h)|$ is the number of distinct elements of $N(h)$. For asymptotic considerations, the variance and the covariance of $\{2\hat{\gamma}(h) : h = 1, \cdots, H\}$ for H fixed are shown to be $O(1/n)$, Cressie (1985). Then a mathematical model may be fitted to the realisation of sample values at points $\{s_i : i = 1, \cdots, N\}$ using either least squares or maximum likelihood. Commonly used theoretical semivariogram models have been classified by Journel and Huijbregts (1978). More technical details on semivariograms can be found in Cressie (1991).

Theoretically, the (two-dimentional) semivariogram should pass through the origin, because differences between points and themselves are zero. Often, the semivariogram cut the $\gamma(h)$ axis at a positive value of it. This is the so-called "nugget variance effect", which means that at the shortest sampling interval (lag = 1), there is a random residual variation which is not spatially correlated. Then, the graph will rise gradually to a point called the "range". This level is termed the "sill". At this point, the semivariogram levels out. Then, the range represents the distance in which points are spatially dependent. The presence of a sill and a constant semivariance at lags greater than the range mean that observations spaced by distances greater than the range may be considered as spatially independent. Our idea is to use the range as a meaningful indicator upon which the structure of the W matrix can be based.

Semivariances estimation based on these data is carried out in two stages. (i) Sample two-dimensional semivariograms were computed. (ii) Transition theoretical curves were fitted (by weighted least squares) to these points using the "spherical" specification:

$$\gamma(h;\theta) = \begin{cases} 0 & h = 0 \\ c_0 + c\left[(3/2)(\|h\|/a) - (1/2)(\|h\|/a)^3\right] & 0 < \|h\| \leq a \\ c_0 + c & \|h\| \geq a \end{cases} \tag{20}$$

with $\theta = (c_0, c, a)'$; $c_0 \geq 0$ is the nugget variance, $c \geq 0$ is the sill and $a \geq 0$ denotes the range.

We propose a "corrected form" of the range as computation criterion for the spatial weights matrix. Indeed, at this stage, edge effects (due for instance to sampling) may result in spurious spatial dependance. To avoid this, we compute confidence limits for the range. Hence, a weights matrix can be constructed as follows.

Let W denote a spatial weights matrix with elements ω_{ij}, D the distance matrix with elements d_{ij} and $\mathcal{A}$ the range obtained from the semivariance estimation. A binary contiguity relation can be defined as:

$$\omega_{ij} = \begin{cases} 1 & \text{if } d_{ij} \in [\underline{\mathcal{A}}; \bar{\mathcal{A}}] \\ 0 & \text{otherwise and if } i = j \end{cases} \tag{21}$$

where $\underline{\mathcal{A}}$ and $\bar{\mathcal{A}}$ denote respectively the lower and upper asymptotic 95% confidence limits for the range. Sample characteristics and semivariogram estimation results for 1988.1 and for 1993.1 are reported in Table 4. Semivariogram plots are given in Figure 1.

Sample characteristics and semivariogram estimation results for 1988.1 and for 1993.1 are reported in Table 4. Standard errors estimates are in brackets. Semivariogram plots are given in Figure 1. Dots correspond to estimated semivariance points computed for each lag. The superimposed dashed lines show the weighted least squares fit of the spherical model. The spatial variability of residential water consumption is then asserted. Spatial autocorrelation occurred up to 37.53 km both for 1988.1 and 1993.1, which suggests the use of approximately the same W matrix for the two cross-section.[10]

6.2 Distribution of French daily residential water consumption between household tasks

6.3 Definition of variables and related data sources

The data were provided by "La Compagnie Générale des Eaux (Direction Régionale Est" (General Company of Water(s)), "la Direction Générale des Impôts de la Moselle" (Regional Tax Center), "le Centre Départemental de la Météorologie de la Moselle" (Regional Center of Meteorological Studies) and "l'Institut National de la Statistique et des Études Économiques" (National Institute of Statistics and Economic Studies). The variables used in this study are the following.

[10] We have computed the characteristics of the distance (using Euclidean metric) matrix based on X-Y latitude- longitude coordinates of the centroids from each municipality. We find that the range fails within the median and the third quartile distance, respectively 29.273 km and 41.641 km.

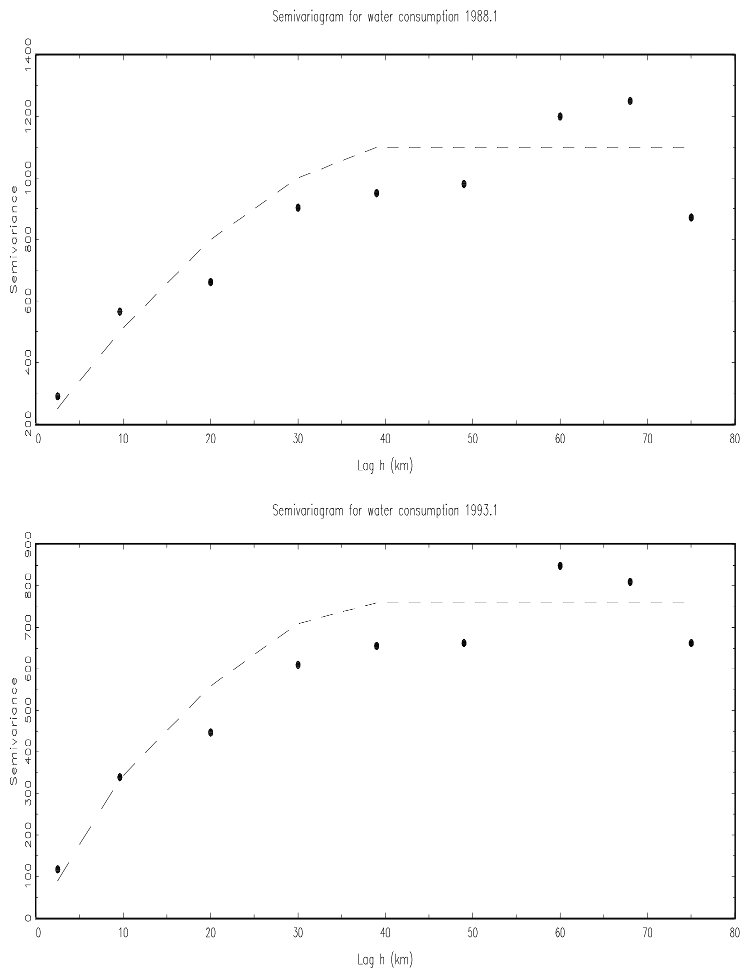

Figure 1: Robust estimated semivariograms for residential water consumption. Dots correspond to estimated semivariance points computed for each lag. The superimposed dashed lines show the weighted least squares fit of the spherical model. Spatial autocorrelation occurred up to 37.53 km both for 1988.1 and for 1993.1.

Table 4: Sample characteristics and the semivariogram model estimation for residential water consumption per municipality expressed in m3 per household. Standard errors estimates are in brackets.

Sample characteristics and estimated parameters	Water consumption 1988.1	Water consumption 1993.1
Sample characteristics:		
mean	69.68	72.24
std. dev.	27.75	26.51
min.	1.11	0.81
max.	153.15	157.33
obs.	115	115
Estimated parameters:		
c_0 (nugget variance)	14.40 (7.71)	10.82 (5.63)
c (sill)	1008 (366.54)	757.1 (270.39)
a (range)	37.53 (10.17)	37.53 (9.21)
Lags	9	9

Dependent variable: aggregate residential water consumption per community expressed in m3 per house.

Explanatory variables

- Water average price in "FF" per m3 (computed to include fixed charges),
- Electricity average price in "FF" per kwh,
- Disposable income per household paying taxes,
- Rainfall in m,
- Mean temperature in degree Celsius,
- Proportion of persons below 19 years,
- Proportion of workers,
- Proportion of unemployed,
- Index of equipments,
- Density of population,
- Spatial lagged dependent variable.

Table 5: Water-using tasks (*Source:* "General Company of Waters")

Water consuming tasks	Proportion
Drink	1%
Cooking (heated)	6%
Dish washing (heated)	10%
Clothes washing (heated)	12%
Toilets	39%
Personal hygiene (heated)	20%
Outdoor use (including sprinkling)	6%
Other uses	6%

References

ANSELIN, L. (1988): *Spatial Econometrics Methods and Models.* Dordrecht: Kluwer.

AZOMAHOU, T. (1999): "Estimation of Spatial Panel Data Models Using a Minimum Distance Estimator: Application," *Working Paper BETA, no 9912, Louis Pasteur University, Strasbourg, Economics Department*, p. 26 pages.

BALTAGI, B. H., AND D. LI (1999): "Prediction in the Panel Data Model with Spatial Correlation," *Texas A&M University, Department of Economics*, p. 17 pages.

BELL, K. P., AND N. E. BOCKSTAEL (2000): "Applying the Generalized-Moments Estimation Approach to Spatial Problems Involving Microlevel Data," *The Review of Economics and Statistics*, (82–1), 72–82.

CASE, A. (1991): "Spatial Patterns in Household Demand," *Econometrica*, (59), 953 –965.

CHAMBERLAIN, G. (1984): "Panel Data," *Handbook of Econometrics*, (2), 1248–1313.

CLIFF, A. D., AND J. K. ORD (1981): *Spatial Processes Models & Applications.* London Pion.

CRÉPON, B., AND J. MAIRESSE (1996): "The Chamberlain Approach," *The Econometrics of Panel Data, A Handbook of the Theory with Applications*, pp. 323–395.

CRESSIE, N. (1985): "Fitting Variogram Models by Weigthed Least-squares," *Mathematical Geology*, (17), 563–586.

CRESSIE, N. (1991): *Statistics for Spatial Data.* Willey-Interscience.

HANSEN, L. G. (1996): "Water and Energy Price Impacts on Residential Water Demand in Copenhagen," *Land Economics*, (72–1), 66–79.

HAUTSCH, N., AND S. KLOTZ (1999): "Estimating the Neighborhood Influence on Decision Makers: Theory and an Application on the Analysis of Innovation Decisions," *Center of Finance and Econometrics, University of Konstanz*, p. 26 pages.

INSEE (1998): "Tableaux de l'Economie Lorraine 96/97," *INSEE, Lorraine.*

JOURNEL, A. G., AND C. J. HUIJBREGTS (1978): *Mining Geostatistics.* Academic Press.

LESAGE, J. P. (1996): "Spatial Modelling of Housing Values in Toledo," *Working Paper.*

LESAGE, J. P., AND M. R. DOWD (1997): "Analysis of Spatial Contiguity Influences on State Price Level Formation," *International Journal of Forcasting*, (2), 245–253.

WHITTLE, P. (1954): "On Stationary Processes in the Plane," *Biometrica*, (41), 434–449.

ZILIAK, J. P., B. A. WILSON, AND J. A. STONES (1999): "Spatial Dynamics and Heterogeneity in The Cyclicality of Real Wages," *The Review of Economics and Statistics*, (81–2), 227–236.