Scaling and multi-scaling analysis in a market model with endogenous threshold dynamics

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Anomalous scaling laws appear in a wide class of phenomena where global dilatation invariance fails. The analysis of financial time series shows that the asymptotic behavior of the probability distribution of stock market returns is consistent with a power law decay at relatively short time scales while the shape of the Gaussian is recovered for montly returns. This change of behaviour also implies multiscaling in the moments of absolute returns. Anomalous scaling, or multiscaling, has also been detected in the autocorrelations of absolute returns for various market indices and currencies.

I propose a model of heterogeneous interacting traders which can explain some of the stylized facts of stock market returns. In the model, synchronization effects, which generate large fluctuations in returns, arise purely from communication and imitation among traders. The key element in the model is the introduction of a trade friction which, by responding to price movements, creates a feedback mechanism on future trading and generates volatility clustering. Scaling and multiscaling analysis performed on the simulated data is in good quantitative agreement with the empirical results.

JEL classification: G12, C63
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I. INTRODUCTION

The analysis of market indexes and exchange rates shows that the asymptotic behavior of the probability distribution of returns is consistent with a power law decay $P(r) \sim r^{-(1+\mu)}$ with an exponent $\mu \sim 3$ (Pagan (1996), Guillaume et al. (1997), Gopikrishnan et al. (1999)). Moreover, while stock market returns are uncorrelated on lags larger than a single day, the autocorrelation function of the volatility is positive and slowly decaying, indicating long memory effects. This phenomenon is known in the literature as volatility clustering (Ding et al. (1993), de Lima & Crato (1994), Ramsey (1997)).

Considerable attention has been devoted in detecting comovements of volatility with other economic variables in the attempt to interpret and capture the source of clustering effects in returns. In particular a lot of effort has been devoted to the analysis of correlations between volatility of returns and trading volume. Empirical evidence has been provided (Tauchen & Pitts (1983), Ronals et al. (1992), Pagan (1996)) of a positive cross correlation between these two quantities.

It is not settled yet whether the emergence of power law fluctuations and volatility clustering is due to external factors, like the arrival of new information, or to the inherent interaction among market players (for example herding behavior) and the trading process itself.

The aim of this paper is to understand which mechanisms in the process of trading can generate the statistical features observed in the financial data. My belief is that the interactions among traders and their heterogeneous nature by themselves, independently of other details of the microscopic environment, might be responsible for many of these features and for the large scale behavior of aggregate economic variables. In the next section I briefly describe the agent-based model (Iori (1999), Iori(2000)) used to generate price histories through numerical simulations and the simulation strategy. In section 3 anomalous scaling and multifractality are defined. In section 4 results of scaling and multi-scaling analysis are discussed. Section 5 concludes.

II. THE MODEL

The market consists of a market maker plus a number of noise traders. Traders buy from or sell to the market maker and respond to a signal which incorporates idiosyncratic preferences and the influence of the traders nearest to them. Only one kind of stock is traded, whose price is set by the market maker on the basis of the observed order flow.

Agents occupy the nodes of a $L \times L$ square lattice with periodic boundary conditions. At each time step t a given trader, i, chooses an action $S_i(t)$ which can take one of three values: $+1$ if (s)he buys one unit of the stock, -1 if (s)he sells one unit of the stock, or 0 if (s)he remains inactive. The trades undertaken by each player are bounded by his resources plus the constraint that (s)he can buy or sell only one indivisible unit at a time. Each agent i responds to a signal $Y_i(t)$:

$$Y_i(t) = \sum_{<i,j>} J_{ij} S_j(t) + A\nu_i(t) \tag{1}$$

where $< i, j >$ denotes that the sum is taken over the set of nearest neighbours of agent i. Noise ν_i represents a uniformly distributed shock to the agent's personal preference.

Under frictionless trading each agent would buy at the slightest positive signal and sell at the slightest negative one. I depart from this benchmark by assuming a trade friction which leads a fraction of the agents to being inactive in any time period. This friction can be interpreted, for example, as a transaction cost which is specific to each agent. Alternatively it could be interpreted as an imperfect capacity to access information. Formally I model this friction as an individual threshold which each agent's signal must exceed to induce him to trade. Each agent compares the signal he receives with his individual thresholds, $\xi_i^+(t), \xi_i^-(t)$, and undertakes the decision:

$$\begin{aligned} S_i(t) &= 1 \quad if \quad Y_i(t) \geq \xi_i^+(t) \\ S_i(t) &= 0 \quad if \quad \xi_i^-(t) < Y_i(t) < \xi_i^+(t) \\ S_i(t) &= -1 \quad if \quad Y_i(t) \leq -\xi_i^-(t) \end{aligned} \tag{2}$$

The $\xi_i(t)$ are chosen from a Gaussian distribution, with initial variance $\sigma_\xi(0)$ and mean $\mu_\xi(0)$, and are adjusted over time proportionally with movements in the stock price. I will consider the case $\mu_\xi = 0$ and $\xi_i^-(t) = -\xi_i^+(t)$. Agents' heterogeneity enters through the distribution of thresholds. The homogeneous traders scenario can be recovered in the limit when $\sigma_\xi = 0$.

Initially agents whose idiosyncratic signal exceeds their individual thresholds make a decision to buy or sell and subsequently influence their neighbors' according to eq. 2. A consultation round to make decisions is allowed before trading takes place. Traders decide sequentially and can revise past decisions on the basis of signals received from their neighbors. This process converges when no agent changes his decision. Once the decision making process is complete traders place their orders simultaneously.

The aggregate demand, $D(t)$, and supply, $Z(t)$, of stocks at time t are

$$D(t) = \sum_{i:S_i(t)>0} S_i(t) \qquad Z(t) = -\sum_{i:S_i(t)<0} S_i(t)$$

and the trading volume is $V(t) = Z(t) + D(t)$.

Traders buy from or sell to a market maker who, when the orders of the traders do not match, takes up the excess. In this way buy and sell orders are always filled. The market maker adjust prices according to the following pricing rule

$$P(t+1) = P(t)\left(\frac{D(t)}{Z(t)}\right)^{\alpha} \tag{3}$$

where the coefficient α, which measures the price speed adjustment, is assumed proportional to the overall trading volume

$$\alpha = aV(t)/L^2 \tag{4}$$

a is a constant and L^2 is the number of traders and represents the maximum number of stocks that can be traded at any time step. The asymmetric reaction of market makers to imbalance orders placed in periods of high versus low activity in the market is consistent with the empirically observed positive correlation between absolute price returns and trading volume.

Eventually a feedback mechanism is introduced such that price changes lead to an adjustment of next period's thresholds, $\xi_i(t+1)$:

$$\xi_i(t+1) = \xi_i(t)\frac{P(t)}{P(t-1)} \tag{5}$$

Note that there is a memory effect in the readjustment mechanism for thresholds, i.e. next period's threshold is proportional to last period's one and not to the initial one. My assumption that thresholds increases when prices increases is motivated primarily by the fact that if ξ_i arise from transaction costs, such as brokerage commissions, these are proportional to stock prices. The asymmetric response of threshold to the direction of price changes induce an asymmetric response of trading volume. See Iori (2000) for a discussion on the empirical justification of this assumption.

The outcomes of the model for different values of the parameters are simulated numerically. The analysis will be centered on the statistical properties of the distribution of stock returns and on the autocorrelation of absolute returns.

The dimension of the lattice is set at $L = 100$. Each agent is initially given the same amount of stocks $N_i(0) = 100$ and of cash $M_i(0) = 100P(0)$, where $P(0) = 1$. The market maker is given a number of stocks, N_m, which is a multiple m of the number of traders (L^2) and an infinite amount of money.

The initial value of the thresholds' variance $\sigma_\xi(0) = 1$ and $\mu_\xi(0) = 0$. The coefficient A in eq.(1) is fixed at $A = 0.2$ and the individual noise signals ν_i are uniformly distributed in the interval $(-1, 1)$.

In any trading round, $S_i(t)$ are set to zero at the beginning of the period. Then each agent observes his individual $\nu_i(t)$ and $\xi(t-1)$ following which an intra-period consultation with other agents takes place. The decision of each trader is updated sequentially following the rule in eqs.(1) and (2). Holding the value of $\nu_i(t)$ and ξ fixed, $Y_i(t)$ and $S_i(t)$ are iterated until they converge for each trader. At this point each trader places her order simultaneously, determining the values of $D(t), Z(t), V(t)$. Prices are adjusted at the end of the period by the market maker according to eq.(3) and eq.(4), feed back into thresholds according to eq.(6) and a new trading round begins.

III. SCALING AND MULTI-SCALING ANALYSIS

Scaling invariance, i.e. invariance under a global dilatation, plays a fundamental role in many natural phenomena and it is often related to the appearance of irregular forms which cannot be described by the usual geometry. A new class of geometrical objects, fractals, and a fractal dimension to characterize their shape have been introduced.

To characterize a fractal object a fractal dimension has been introduced by considering the number of hypercubes of edge l necessary to cover an object embedded in an D-dimensional space in the limit $l \to 0$

$$N(l) \sim l^{D_F} \tag{6}$$

The object is a fractal if D_F is larger than the topological dimension (while $D_F \leq D$).

A general way to characterize a multifractal object is to consider the scaling of an appropiate probability measure $\mu(x)$ over the object. One can define a coarse grained probability density

$$p_l(x) = \int_{\Lambda(x,l)} \mu(y) dy \tag{7}$$

which measure the 'mass' of a hypercube Λ, of size l and centered in the point x. In general $p_l(x)$ scales with an exponent α which depends on the particular point x:

$$p_l(x) \sim l^{\alpha} \tag{8}$$

If $\alpha = D_F$ the object is an homogeneous fractal. In $\alpha \neq D_F$ the object is said to be a multifractal and can be regarded as the superposition of different fractal sets

$$F(\alpha) = \{x \text{ such that } p_l(x) \sim l^{\alpha}\} \tag{9}$$

each one characterized by a different fractal dimension $f(\alpha)$, i.e. the number of hypercubes of size l necessary to cover a subset $F(\alpha)$ scale as:

$$n(\alpha) \sim l^{f(\alpha)} \tag{10}$$

The fluctuations of the exponents α are determined by a probability distribution which can be reconstructed from the knowledge of the mass moments scaling

$$< p_l(x)^q >= \sum_{k=1}^{N(l)} p_l(x(k))^{q+1} \sim l^{q d_{q+1}} \tag{11}$$

where d_q are the generalized dimensions. For an homogenous fractal $d_q = D_F$ for any q. The deviation from this linear scaling for the mass moments gives a measure of the degree of inhomogeneity.

It is possible to show (Paladin and Vulpiani (1987)) that the generalized dimensions d_q and the fractal dimensions $f(\alpha)$ of the sets $S(\alpha)$ are related via a Legendre transformation:

$$(q-1)d_q = \min_{\alpha}[\alpha q - f(\alpha)] \tag{12}$$

so that each moment q selects a particular exponent α.

Anomalous scaling laws appear in a wide class of phenomena where global dilatation invariance fails (see Paladin and Vulpiani (1987) and Bohr et al. (1998) for a review). Multifractality is not only associated to to the spatial fluctuations of an observable. Intermittent behaviour in dynamical system, i.e. strong time dependence in the degree of chaoticity, is accompanied by anomalous scaling with respect to time dilatations in the trajectory space. A measure of the degree of intermittency requires the introduction of an infinite set of exponents associated to the multifractal structure of an appropriately defined probability measure.

While multifractality is defined in terms of geometric properties of an appropriate probability measure, the notion of multiaffinity is introduced to characterize a a signal $\phi(x)$ such that

$$|\phi(x+r) - \ phi(x)| \sim r^{\zeta_q} \tag{13}$$

when the exponent ζ_q is a non linear function of q. If the non linear shape of the scaling exponents ζ_q is a consequence of intermittent behaviour multiaffinity also involves multifractality of an opportunely defined probability measure. Multifractality and fluctuation is time of the level of chaoticity are indeed two aspects of the same problem. Nonetheless the relation between spatial multi-fractality and temporal intermittency is not simple and the definition of a multifractal measure remains arbitrary unless theoretically motivated (like for example in turbolence).

IV. RESULTS

In order to compare the distribution of returns at different time scale one can define the normalized return

$$\tilde{r}_\tau(t) = \frac{1}{v_\tau} log \frac{P(t)}{P(t-\tau)} \tag{14}$$

where $v_\tau = < r_\tau^2(t) > - < r_\tau(t) >^2$ is averaged over the entire time serie of returns. In fig.(1) the cumulative probability distribution for the rescaled returns is plotted on a log-log scale at $\tau = 1$ and $\tau = 2^{12}$ (positive and negative returns were merged together by taking their absolute values). If the return distribution decays as a power law $P(r) \sim r{-}(1+\mu)$ than the cumulative distribution of finding a return larger than r goes like $P(\tilde{r} > r) \sim \frac{1}{r^\mu}$. While the distribution of returns is well approximated by a Gaussian at large τ I find a power-law behaviour for the cumulative distribution well approximated by an inverse cubic-law (shown in fig.(1) with a dashed line) for $\tau = 1$. This means that it is not possible to find an real number h such that the statistical properties of the rescaled variables $r_\tau(t)/\tau^h$ do not depend on τ.

A scaling behaviour of the probability distribution of returns would imply a similar scaling on the moments of $r_\tau(t)$, the so-called structure function, defined as

$$F_q(\tau) \equiv < |r_\tau(t)|^q > \sim \tau^{\xi(q)} \tag{15}$$

Independent random walks models always have a unique scaling exponent (the process is said self-affine) and $\xi(q) = hq$. For example in the Gaussian case $h = 1/2$. If ξ_q is a a non linear function of q, the process is called multi-affine. Multiscaling is an indication that r_t, even if uncorrelated, is a dependent stochastic process and it implies the presence of wild fluctuations. The larger the difference of ξ_q from a linear behaviour in q the wilder are the fluctuations. I analized the scaling of the moment of the distribution for the simulated data and detected multiscaling in analogy with real financial time series (Ghashghaie et al (1996), Baviera at al (1998)). In fig.(2) $\log(F_q(\tau))/q$ ia plotted versus $\log \tau$ for $q = 2, 4, 6$. For a random walk process the three curves would have the same slop. In fig.(3) ξ_q is plotted versus q. By using a step-wise linear regression, the slopes are estimated as being 0.47 for $q < 3$ and 0.14 for $q > 3$. The random walk approximation, with $\xi_q \sim q/2$, is compatible with the data only at $q < 3$. My results are in good agreement with the empirical analysis performed by Baviera et al (1998) on the DM/US exchange rate quotes. They also found that for $q < 3$ the slope is consistent with the random walk hypothesis, while for $q > 3$ the slope falls to a value of 0.256.

It is well known that absolute market returns are characterized by long range correlations. In fig.(4) the autocorrelation functions of absolute return $C_{|r|}$ and, for comparison, of simple returns C_r:

$$\begin{aligned} C_r(L) &= < r(t) r(t+L) > - < r(t) >< r(t+L) > \\ C_{|r|}(L) &= < |r(t)||r(t+L)| > - < |r(t)| >< |r(t+L)| > \end{aligned} \tag{16}$$

are plotted as a function of the time lag L. Fig.(4) shows that while returns are not correlated, the autocorrelation function of absolute returns is slowly decaying revealing the presence of long term memory. R/S analysis provides a precise test for inferring whether the decay of $C_{|r|}(L)$ is exponential (as in the GARCH model) or hyperbolic, i.e.

$$C_{|r|}(L) \sim L^{2H-2} \tag{17}$$

where H is the Hurst exponent. Lo (1991) pointed out that the simple R/S statistic may have difficulties in distinguishing between long-memory and short-term dependence. Given a time series X_t, Lo (1991) suggested to calculate the following modified R/S (MRS) statistic $Q(s, L)$:

$$Q(s, L) = \frac{1}{\sigma^*(s, L)} [\max_{1 \le u \le s} \sum_{i=1}^{u} (X_i - \bar{X}_s) - \min_{1 \le u \le s} \sum_{i=1}^{u} (X_i - \bar{X}_s)] \tag{18}$$

where

$$\sigma^*(s, L) = \frac{1}{s} \sum_{i=1}^{s} (X_i - \bar{X}_s)^2 + \frac{2}{s} \sum_{j=1}^{L} \omega_j(L) \sum_{i=j+1}^{s} (X_i - \bar{X}_s)(X_{i-j} - \bar{X}_s) \tag{19}$$

$$= \gamma_o(s) + 2\sum_{j=1}^{L}\omega_j(L)\gamma(s), \quad q < n$$

The weights $w_j(L)$ used are

$$\omega_j(L) = 1 - \frac{j}{L+1} \tag{20}$$

γ_j are the auto-covariance operators calculated up to a lag $L = \sqrt{n}$. $\bar{X}_s$ is the mean over a sample of size s. The case $L = 0$ gives the classical R/S statistic. The Hurst exponent, H, is calculated by a simple linear regression of $\log(Q(s,L))$ on $\log(s)$. If only short memory is present H should converge to $1/2$ while with long memory, H converges to a value larger than $1/2$. I divided the original volatility sample of size $n = 50000$ into n/s non overlapping intervals of size s and estimated $Q(s,L)$ for each of the intervals so defined and averaged it over all of them. Errors were estimated as the standard deviation of the $Q(s,L)$ over the n/s intervals. This procedure was repeated for different values of s in the range $2^4 < s < 2^{11}$. Results of the MRS statistics are plotted in fig.(5). I found a slope $H = 0.85$ over the whole range of considered values of s which indicates a hyperbolic decay in the autocorrelations, with an exponent $\beta = 2H - 2 = -0.3$. I tested this value of β against the plot of the absolute returns autocorrelations in fig.(4). The solid line is a power law curve with exponent $\beta = -0.3$. The agreement with the data is good throughout the considered values of L, up to $L = 500$. Empirical studies (Ding et al. (1993), Cont et al. (1997), Baviera et al. (1998), Pasquini & Serva (1999), Liu et al. (1999)) have estimated a value of $0.1 < \beta < 0.4$ for the absolute returns autocorrelation of many indices and currencies. My results are in good quantitative agreement with the empirical observations.

Multiscaling of volatility autocorrelations has been detected (Baviera et al. (1998)) through the analysis of the generalized correlations

$$C_\gamma(L) = <|r(t)|^\gamma |r(t+L)|^\gamma> - <|r(t)|^\gamma><|r(t+L)|^\gamma> \tag{21}$$

Chanching γ one can focus on correlations of returns of comparable size: small returns are more relevant at small γ while $C_\gamma(L)$ is dominated by large returns at large γ. If the absolute return serie $|r(t)|^\gamma$ shows long term memory

$$C_\gamma(L) \sim L^{-\beta_\gamma} \tag{22}$$

while if $|r(t)|^\gamma$ is an uncorrelated process than $\beta = 1$. Multi-scaling would be signaled by a non linear shape of β_γ.

The nature of the long term correlations can be better investigated through the analysis of cumulative returns instead of single returns. Following the notation in Baviera et al. (1998) we construct the variables $\phi_t(L,\gamma)$:

$$\phi_t(L,\gamma) = \frac{1}{L}\sum_{i=0}^{L-1}|r(t+i)|^\gamma \tag{23}$$

where the sum is taken over non overlapping intervals.

It can be shown (Baviera et al. (1998)) that the autocorrelations of powers of absolute returns have the same asymptotic behavior as the variance of cumulative absolute returns:

$$\delta(\phi_t(L,\gamma)) = <\phi_t(L,\gamma)^2> - <\phi_t(L,\gamma)>^2 \sim L^{-\beta(\gamma)} \tag{24}$$

We calculated $\delta(\phi_t(L,\gamma))$ when $\gamma = 1$ and found an exponent $\beta(1) \simeq 0.3$ in agreement with the MRS analysis.

The scaling exponent $\beta(\gamma)$ of the variance of the variables $phi_t(L,\gamma)$ is shown in fig.(6) for $0 < \gamma < 4$. In analogy with the NYSE index and the USD-DM exchange rate (see Baviera et al (1998), Pasquini & Serva (1999)) $\beta(\gamma)$ is not a constant function of γ revealing the presence of different anomalous scales. The convergence of $\beta(\gamma)$ to one reveals that large fluctuations are practically independent.

V. CONCLUSIONS

This paper has outlined a mechanism which can explain certain stylized facts of stock market returns. According to the model synchronization effects, which generate large fluctuations in returns, can arise purely from communication and imitation among traders, even in the absence of an aggregate exogenous shock. While aggregate news, i.e.

information on past price changes, plays a role in the model via the feedback effect on thresholds, this news is determined endogenously in the model.

In this paper an oversimplified communication structure has been assumed with agents arranged on a regular bidimensional lattice and interacting only with their nearest neighbors. A more realistic communication structure should be consider and the effects of different assumptions should be investigated. A preliminary analysis has been performed where agents are linked on a random graph to a given number of other agents chosen at random. I find in this case a decrease in the returns' volatility as the number of connected agents increases, revealing a stabilizing role for a larger communication network.

It has also been assumed, in a restrictive way, that only positive spillover effects are present, with agents always imitating each other in the model. I have relaxed this assumption introducing a number of contrarian traders which receive a signal to act in the direction opposite to the one predominant in their reference group. Preliminary results show that by increasing the number of contrarians the volatility decreases in the market.

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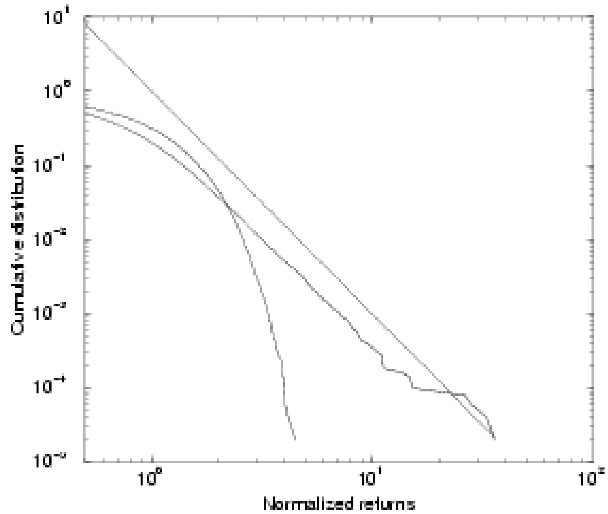

FIG. 1. Cumulative distribution of returns $\Pi(\bar{r}_\tau > r)$ at different time lag, $\tau = 1$ (solid), $\tau = 2^{11}$ (dotted). The dashed line, which is for reference, is a power law with an exponent $\mu = 3$. As can be seen there is a range of value of r for which the cumulative distribution is parallel to the reference line indicating consistency with the power law exponent. At large τ the distribution of returns converges to a Gaussian.

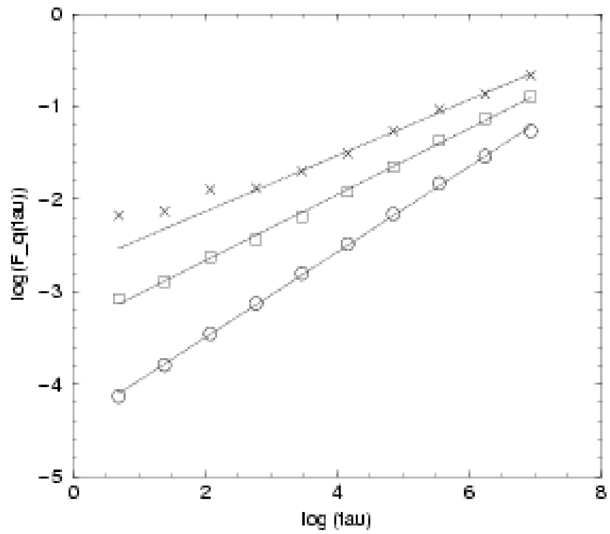

FIG. 2. Structure functions $\log(F_q(\tau))$ versus $\log(\tau)$. The three plots correspond to $q = 2$ (circles), $q = 4$ (squares) and $q = 6$ (crosses).

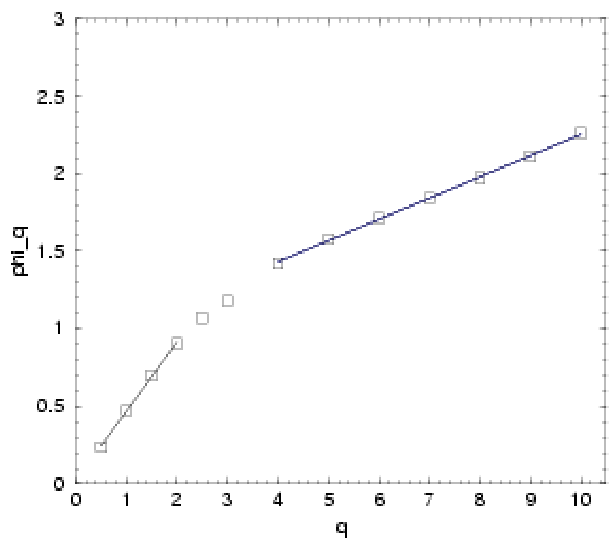

FIG. 3. Exponent ϕ_q versus q. The slope of the two regions are 0.47 at $q < 3$ and 0.14 at $q > 3$.

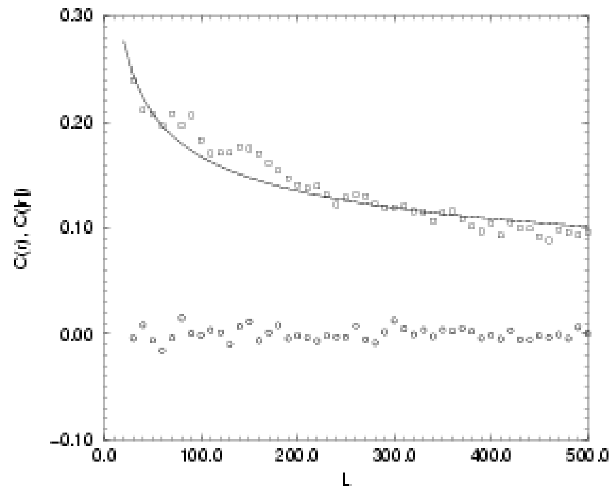

FIG. 4. Autocorrelations function of returns (circles) and absolute returns (squares) for $\alpha = aV(t)/N$ and $p = 1$. The solid line is a power law decay with the exponent $\beta = 0.3$ estimated from the MRS analysis of absolute returns.

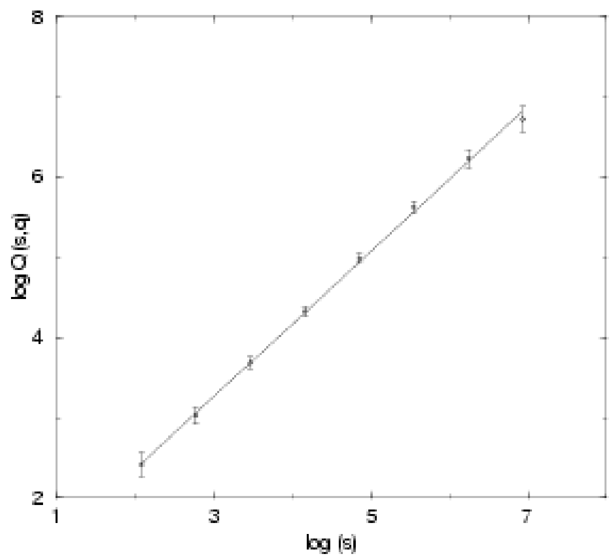

FIG. 5. Modified MRS statistics for absolute returns for $\alpha = aV(t)/N$ and $p = 1$. We plot the log of $Q(s, L)$, averaged over n/s non overlapping intervals, against the log of the sample size s. Errors are estimated as the standard deviation of $Q(s, L)$ over the n/s intervals. With a simple linear regression we find a slope $H = 0.85$.

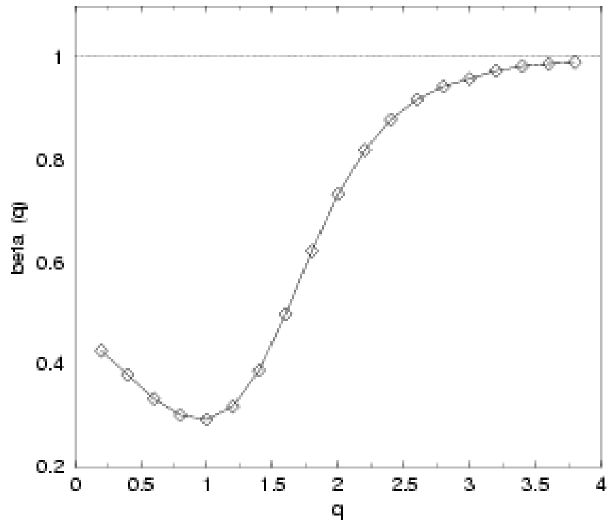

FIG. 6. Scaling exponent $\beta(\gamma)$ of the variance $var\bar{\phi}_t(L,\gamma)$ as a function of γ. Anomalous scaling $\beta(\gamma) < 1$ is shown

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