A Stochastic Cartel Market Process

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Abstract

A dynamic version of discrete choice theory is presented in order to enable the explicit analysis of the interaction between the micro- and the macro-level of social systems. Suppliers in a cartel market face a social dilemma. They are modelled as boundedly rational decision makers with limited foresight.
A stimulus-response mechanism leads to an ergodic, time-discrete Markov-chain with a discrete state space. The resulting market dynamics exhibits a specific time-pattern of cartelisation and de-cartelisation. Most of the time, a cartel which significantly rations output is in existence. This happens though there is a permanent temptation to free ride.

Key-words

Cartel, social dilemma, bounded rationality, stochastic market process, Markov-chain, dynamic discrete choice, stimulus-response mechanism

1 Introduction

Cartels face a social dilemma: suppliers gain if they are able to collude but each single supplier gains even more if all others collude and he himself has a free ride. Cartel theory, as industrial economics in general, is dominated by the application of game theoretic techniques.[1] This approach asks whether there is a stable cartel or, if not, how to stabilise an unstable one.

In contrast, applied market theorist Erdmann (1997) argues that the permanent *instability* of cartels may be the *reason* for their long-term success. To deal with such phenomena, both social dilemma researchers and market theorists ask for new tools. Liebrand et al. (1992), for example, emphasise the need for methods that make an explicit analysis of the interaction between the micro- and the macro-level of social systems feasible.

To match this need, the present paper suggests an enhancement of discrete choice theory. A dynamic discrete choice model is set up which explicitly deals with the decisions of the suppliers on the micro-level of the market. As a result of their decisions, quantities and prices on the macro-level of the market are determined. In a feedback loop, these macro data are the basis for the decisions on the micro-level in the next period. Thus, a model of a market *process* is established.

Within this model, a cartel is considered which is permanently unstable. As will be shown, such an unstable cartel market may stochastically overcome the social dilemma of a cartel. The market dynamics exhibits a specific pattern which could be misperceived as being triggered exogenously or to be the result of structural breaks.

The paper is structured as follows: First, the building blocks of the model and the idea of dynamic choice theory are presented. Second, the basic assumptions of the model are explained. In the central section, the model is presented in detail. The model constitutes a Markov-chain. For two specific demand schemes, results in terms of stationary distributions and simulated realisations are discussed in the last section. A conclusion and outlook completes the paper.

2 The building blocks of the model

Three essential building blocks are fundamental to the model:

[1] For a first overview of cartel theory see Jacquemin and Slade (1989).

Stimulus-response mechanism The decision maker chooses her decision out of a set of alternatives according to a stimulus-response mechanism: the higher the expected gain from the alternative under consideration, the higher is the probability for this alternative to be chosen. The potential gain of each alternative is calculated by the comparison of the expected outcome of an alternative in relation to the status quo.

Strategic myopia As in evolutionary game theory, decision makers do not take the strategic consequences of their actions into consideration. They are strategically myopic.

Institutional inertia In cartel markets, the exit and entry of suppliers are rather rare events. On the institutional level most of the time nothing happens. This stylised fact of institutional inertia is straightforwardly modelled: to remain in the status quo is assumed to be the event with the highest probability.

3 Dynamic discrete choice

Standard discrete choice theory assignes to each alternative a specific set of attributes. This set is fixed and does not vary in time. With the contribution of De Palma and Lefèvre (1983), the attributes of an alternative may be state-dependent. According to the state of a social system, the attributes of the alternatives may vary.

The model put forward in the present paper is even more dynamic. Here, the set of alternatives itself depends on the state of the market which is straightforward for market processes. Being a member of a cartel, a supplier may stick with the cartel or leave it. As a member, he also has a say in determining the cartel rationing scheme. A non-member, on the other hand, may stay clear or join the cartel.

4 The basic assumptions of the model

A market of a homogeneous good is considered. For simplicity, demand is modelled according to a Cournot-price-demand-function. This function should be chosen in such a way that a *cartel problem* is depicted. A cartel problem is a *free rider problem*: establishing a cartel is profitable, if demand is *sufficiently inelastic*. A group of suppliers may raise their profits by rationing output as the decrease in output is overcompensated by an increase in market price. On the other hand, for a free rider problem to exist demand must not be *too* inelastic. It must not pay for a single supplier to ration output. Instead, he can raise his profit by leaving the cartel; the rise in his individual output is not outweighed by the decreased market price. Thus, a *social dilemma* is established: it pays for a group of suppliers to jointly ration output; once this is done, each cartel member has an incentive to free ride - the cartel is not stable.[2]

On the supply-side, N suppliers each have the same supply capacity, scaled to unity. Again for the sake of simplicity, there are just two states a supplier may be in: either he is independent and produces his unity-output; or he is in the one and only cartel which can be in existence and reduces his output to the agreed-upon cartel quota q. There are Q equidistant discrete quotas possible between zero and unity, excluding zero (which would imply non-production), including unity. So, any *state* i of the market can be described by a pair (n, q) where n is the number of suppliers organised in the cartel, and q is the rationing scheme applied by this cartel.

The *state space* may best be regarded as two-dimensional with the number n of suppliers in the cartel ranging from zero to N placed along the first axis, and the cartel quota q ranging from $1/Q$ to unity along the second axis. In total, there are $(N + 1) \cdot Q$ states.

As central piece of the model, the dynamics of the market is constituted by the transition probabilities $tp(i \rightarrow j)$ defined for each state pair (i, j). A transition probability $tp(i \rightarrow j)$ is the probability that, starting from state i in period t, in period $t + 1$ state j is approached. The transition probabilities from state i to all possible states j (including i itself) sum up to unity as in each period the market *must be* in a specific state, whichever that may be.

There are two crucial assumptions regarding the behaviour of the suppliers. First, it is assumed that the suppliers are *boundedly rational* in the following sense: they take the results of their actions into account but in a rather *myopic* and *non-strategic* way. At a specific time, each supplier evaluates all actions available to him assuming that the behaviour of all other suppliers remains constant. *The higher the profit* resulting from taking a certain action compared with status quo, *the higher is the probability* that this action will be taken. He calculates the market outcome, provided his action is performed, but he does not take into account the expected reactions of the other suppliers. This assumption of a *myopic stimulus-response mechanism* is complemented by another assumption: the action with the distinctively highest probability for all suppliers is to *remain* in *status quo*. On the one hand, this stochastically reflects the *institutional inertia* of real markets where neither permanent short run changes in the membership of cartels nor permanently fast changing rationing schemes can be observed. On the other hand, it makes the first assumption - at least partially -

[2]For the notion of *stability* of a cartel cf. Jacquemin and Slade (1989,427) and d'Aspremont et. al. (1983)

rational as, once a supplier has taken an action, he can reckon on the fact that, for a considerable time, no reaction will occur.

A comparison of an action a_j under evaluation with status quo a_i is performed in such a way that the difference in profit $\Delta U = U(a_j) - U(a_i)$ between the outcome of this action and the status quo is calculated. Then, a Fermi-like evaluation function F maps this difference in profit to the interval $(0, S)$.[3]

$$F: \quad \mathbb{R} \quad \rightarrow \quad (0, S), \qquad S \in (0, 1],$$

$$\Delta U \quad \mapsto \quad F(\Delta U) = S \cdot \frac{1}{1 + \exp(a\Delta U + b)}, \qquad a, b, \Delta U \in \mathbb{R}, \quad a < 0.$$

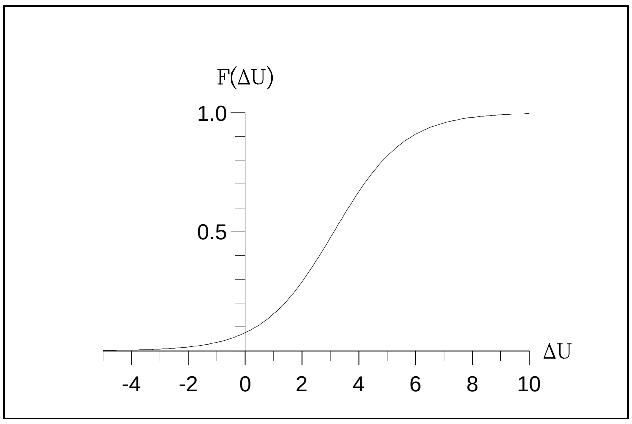

Figure 1: Fermi-like evaluation function, $S = 1, a = -0.8, b = +2.5$.

5 The model

Depending on the state of the market, a supplier may face three different decision situations:

Stay or leave As a member of the cartel, he may stay inside and just supply according to the cartel quota or he may step out and become an independent supplier.

Change quota? As a member of the cartel he has a say in deciding whether to lower or raise the cartel quota.

[3]For the class of Fermi-like evaluation functions cf. Brenner (1995,23).

Join or stay clear If he is not a member of the existing cartel or if there is no cartel at all, he may continue to be an independent supplier and supply his individually profit-maximising output of unity. Or together with other fellow suppliers, he joins or sets up the cartel.

The transition probability for cartel exit

Since each member of the cartel is confronted with the same temptation to free ride, it is straightforward to model the *individual* decision to leave the cartel *independently.* With n suppliers in the cartel, $\Delta U(n \to n-1)$ denotes the individual gain of leaving the cartel if exactly one supplier exits and the number of members is decreased from n to $n-1$. Based on ΔU, the individual transition probability $tp_{ind}(n \to n-1)$ is the probability that a *single* supplier exits the cartel:

$$tp_{ind}(n \to n-1) := \begin{cases} S_n \cdot \dfrac{1}{(1+\exp(a_n \Delta U(n \to n-1) + b_n))} & \text{for } \Delta U > 0, \\ 0 & \text{otherwise.} \end{cases}$$

The transition probability is set to zero for all non-positive values of ΔU. In deciding whether to leave the cartel, a supplier will make no error in the sense that he will not exit if there is no gain for him in doing so.

The values of the parameters S_n, a_n and b_n will be crucial for the shape of the transition probability function in dependence of the individual gain ΔU. In this paper, they will be identically chosen for the exit from as well as for the entry into the cartel.

Due to the fact that the individual exit decisions are made independently, the transition probability $tp(n \to n-k)$ for the *simultaneous* exit of k suppliers out of a cartel of n results as the $B(n; tp_{ind})$ binomial distribution.

The transition probability for quota changes

As a matter of computational convenience, a transition from a quota q shall only be possible to the neighbouring quotas $q^-(= q - 1/Q)$ and $q^+(= q + 1/Q)$ (with Q as defined above). Provided they exist, q^- is the next lower quota to q (tighter rationing), whereas q^+ is the next higher quota (less rationing). A transition to other quotas may take place as a result of sequential transitions and will thus afford more than just one period.

The decision situation is the same for all cartel members. To keep things simple, the group decision whether to change the cartel quota will not be modelled in detail. Instead, a single supplier will be regarded as representative of the group. The more a single cartel member could gain by shifting of the cartel quota, the higher the probability will be that such a shift will be agreed upon.

With the individual gain $\Delta U(q \rightarrow q^{\pm})$ of shifting the quota q to q^{+} or q^{-}, the transition probability $tp(q \rightarrow q^{\pm})$ is:

$$tp(q \rightarrow q^{\pm}) = S_q \cdot \frac{1}{(1 + \exp(a_q \Delta U(q \rightarrow q^{\pm}) + b_q))}$$

Again, the parameters S_q, a_q, and b_q are important determinants of the concrete shape of the evaluation function.

Different from the transition probabilities for the entry into and the exit out of the cartel, the evaluation function will not be set to zero for all non-positive values but will be supposed to be continuous on $\mathbb{R}$. There will be a small but positive probability for the quota to be shifted even if the cartel members lose from this change in output. This is a means of modelling *another* aspect of *bounded rationality*: decision makers may take wrong decisions. They occur with a rather low probability, but they cannot be ruled out completely. In this paper, the possibility of making errors will be restricted to the decision whether to change the cartel quota. This restriction is in no way essential for the logic or feasability of the model but it will make it much easier to comprehend its dynamics.

The transition probability for cartel entry

For the individual transition probability to enter the cartel, the same approach as for the individual calculation of cartel exit will be applied. In contrast to the decision situations already discussed, the question whether to join or create a cartel cannot be considered on an individual basis. As there is a strong temptation for the individual supplier to free ride, it does not pay for him to ration output individually. Only joint action of a group of suppliers will be sufficient for rationing output and thus raising the price in a way that allows each member of this group to profit from this action. The lower bound for the size of such a group will depend on the specific demand function considered, the concrete parameters of the model, *and* the actual state of the market.

Again, the way in which such a group is formed will not be modelled in detail. Instead, just two effects will explicitly be taken into consideration. First, it is supposed that the bigger the group under consideration the more difficult it is to coordinate suppliers to form such a group. This may seem rather ad hoc and should best be regarded as a reminder that explicit attention should be paid to the *coordination process* involved. Second, as a matter of *combinations*, how many different groups of a certain size *can* be formed depends on the size of the pool of suppliers from which a group of this size may be formed. The greater the number of possible groups, the higher the overall probability that such a group comes into existence should be.

With n suppliers in the cartel, the transition probability that m suppliers will join this cartel turns out to be:

$$tp(n \rightarrow n+m) := \begin{cases} \underbrace{S_n \cdot \frac{1}{(1+\exp(a_n \Delta U(n \rightarrow n+m) + b_n))}}_{tp_{ind}} \cdot \underbrace{\frac{1}{m}}_{gce} \cdot \underbrace{\binom{N-n}{m}}_{coe} & \text{for } \Delta U > 0, \\ 0 & \text{otherwise.} \end{cases}$$

The first term tp_{ind} models the individual calculation of a supplier, whereas the terms *gce* and *coe* reflect the group coordination effect and the combinatorial effect, respectively. The group coordination effect is simply modelled as hyperbola; to take combinatorial effects into account, the binomial coefficient is multiplied.

As with the transition probability for cartel exit, the probability for a group entry is set to zero for all non-positive values of ΔU.

When there is no cartel at all, there is no cartel quota q applied. In such a situation, the evaluation whether to set up a cartel is performed on the basis of an exogenously given quota q_e. This assumption is just convenient for computational purposes and helps keep the model comprehensible.

The probability to remain in status quo

With the transition probabilities defined, we are able to conclude to which states a transition in one time step may take place out of a state (n, q). One up to n suppliers may step out of the cartel, and m suppliers may join or set up a cartel. The upper bound for m is the number of suppliers outside the cartel, $N - n$. The lower bound, as mentioned above, depends on the demand function, the parameters of the model, and the state under consideration. If there is a cartel, the cartel quota may be changed, and a neighbouring quota be chosen. The transition probability to all other states is zero.

In *each* period the market must be in *some* state. This has two important implications: First, the sum of the probabilities $p_{i,t}$ to *be* in a *specific* state i in period t, summed up over *all* states, must be unity. Second, starting from a specific state in period t, in period $t+1$ the market must again be in some state, be it another one or the status quo. Therefore, the sum of the transition probabilities $tp(i \rightarrow j)$ over *all* states j

(including state i) must also be unity. As a consequence, the probability to remain in the status quo can be calculated as residual probability, i. e. the difference of unity and the sum of the transition probabilities to all other states j, excluding i:

$$tp(i \to i) = 1 - \sum_{j \neq i} tp(i \to j).$$

The magnitude of this sum is influenced by the parameters S_n and S_q. Their relation to each other determines the relative speed of the dynamics of the two processes under consideration: the entry and exit of suppliers, and the quota variation. The numeric values of these parameters have to be chosen so as to assure that the probabilities to remain in the status quo are all non-negative. For all non-pathological parameter sets these probabilities of remaining in the status quo will by far be higher than all other transition probabilities.

To complete our consideration, it should be mentioned that there is another way of remaining in the status quo: as the decision to leave the cartel, and the decision to join it are both independent, it may happen that m cartel members individually decide to drop out of the cartel and a group of m suppliers enters simultaneously. In general, such a coincidence is very unlikely and will not explicitly be considered in the following.

6 The Markov-chain

The transition probabilities constitute a homogeneous, time-discrete Markov-chain with finite and discrete state space. With the transition matrix $\mathbf{M}$, the dynamics of the system is given by equation[4]

$$\mathbf{p}_{t+1} = \mathbf{p}_t \cdot \mathbf{M}. \qquad (1)$$

There are two ways of perceiving such a stochastic process. The *stationary distribution* and specific *realisations* of the process are highly interrelated but they accentuate different *aspects* of a stochastic process.

From the theory of Markov-chains it is known that for ergodic Markov-chains the probability distribution converges towards an unique invariant distribution $\boldsymbol{\pi}$, whatever

[4]A good introduction and overview of stochastic processes in general and Markov-chains in particular is given by Çinlar (1975), Ross (1996), Taylor and Karlin (1994) and Jetschke (1989). In part of the literature equation (1) is called *master equation*.

the initial distribution $\mathbf{p}_1$ in period one has been. $\boldsymbol{\pi}$ is the fixed point of the iteration of equation (1) or, in other words, the solution of the set of equations

$$\begin{aligned} \mathbf{p} &= \mathbf{p} \cdot \mathbf{M} \\ \sum_{i=1}^{(N+1)\cdot Q} p_i &= 1. \end{aligned} \tag{2}$$

It can be shown that the Markov-chain constituted by the cartel model of this paper indeed has the ergodic property. So, by solving equation (2) for a given demand function and parameter set, the unique stationary limit distribution $\boldsymbol{\pi}$ can be derived.

By inspection of the stationary distribution, it it easy to conclude whether states with a cartel in operation have a significant probability. Based on this distribution, *averages* as such as the average market price can be calculated. What, however, cannot be deduced from the stationary distribution is the pattern of single realisations of the Markov-chain, i. e. what the sequence of market states looks like. Though the single realisations of a specific Markov-chain have much in common, each is individual and a prediction of the dynamics of a stochastic market process is only possible in a certain restricted sense.

7 Two exemplary models

In this section, the dynamics of two particular cartel models will be investigated. The number of suppliers N is set to 10 and there are 10 cartel quotas ranging from 0.1 to 1.0 in steps of 0.1. The Cournot-price-demand-function $P(x)$ maps x, the total quantity supplied to the market, to the market price P. To keep the model comprehensible, the total quantity supplied to the market is rescaled to the unit interval. The two models of this paper only differ in the price-demand-function applied, or, in other words, in ε, the elasticity of the market price due to a variation of output. The two functions are

$$\begin{aligned} P_1 : \quad & x \mapsto P_1(x) = x^{-2.2} & (\varepsilon = -2.2), \\ P_2 : \quad & x \mapsto P_2(x) = 0.37 \cdot \exp\left(x^{-0.8}\right) & (\varepsilon = -0.8\, x^{-0.8}), \end{aligned}$$

with

$$P_j : \quad (0,1] \quad \rightarrow \quad \mathbb{R}, \qquad (j = 1, 2).$$

The parameters are chosen such as to calibrate the base price without rationing to unity and to model specific elasticity profiles.

The exponent in the function P_1 is set to -2.2 which directly gives the elasticity of the market price ε due to a variation of output. For all cartel quotas, there is the same elasticity of -2.2: an decrease of the quantity supplied to the market is overcompensated by the thus caused increase in the market price. This may seem strange at first sight but is a way to model the social dilemma in a straightforward manner. There is both a strong incentive to form a cartel and a steady temptation to step out of it. The cartel is *permanently unstable* in the notion of standard cartel theory.

With function P_2 things are different. Here, elasticity varies with market supply. For high quantities, its values are below unity whereas for low quantities (i. e. a tight rationing) it is well above the elasticity of P_1.

A cartel which encompasses all suppliers is in the same decision situation as a monopolist. For such a situation, table 1 gives an overview of market prices and elasticities:

q	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
P_1	1.00	1.26	1.63	2.19	3.08	4.59	7.51	14.14	34.49	158.49
ε	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2	-2.2
P_2	1.00	1.09	1.22	1.39	1.66	2.10	2.95	5.05	13.79	202.33
ε	-0.80	-0.87	-0.96	-1.06	-1.20	-1.39	-1.67	-2.10	-2.90	-5.05

Table 1: Prices and elasticities for two different price-demand-functions, ten cartel quotas and a cartel encompassing all suppliers in the market.

In this paper, costs of production will not explicitly be taken into consideration. A close examination of the model shows that excluding production costs is just the special case of a parameter value of $\alpha = -1$ for the unit cost function[5]

$$\begin{aligned} k : (0,1] &\rightarrow \mathbb{R}, \\ q &\mapsto k(q) = q^{\alpha}, \quad \alpha \in \mathbb{R}. \end{aligned}$$

With no costs, profit maximisation coincides with turnover maximisation. For both price-demand-functions the optimal cartel quota would be 0.1 with a turnover of 15.84 ($158.49 \cdot 0.1$) or 20.23 ($202.33 \cdot 0.1$) compared to 1.0 ($1.0 \cdot 1.0$) for the uncartelised market.

[5]For further details cf. Haas (2000), Chapter 10.

7.1 Model 1

For the price-demand-function P_1, the monopoly or "big cartel" optimal profit would be 16 times greater than without supply-side coordination. A significant temptation, however, is the fact that a single supplier could more than double his profit by leaving the cartel while the remaining cartel members would have to be content with a quarter of their former profits.

This severe cartel problem notwithstanding, the *stationary distribution*, as displayed in figure 2, shows that most of the time a cartel is in existence. The states without a cartel or with just one cartel member ($n = 0$ and $n = 1, respectively$) have a very low probability. The distribution concentrates on a medium number of cartel members and moderate rationing. Tight rationing does occur with a significantly positive but rather low probability. The average market price which can be calculated using the stationary distribution is 1.77. So, although the suppliers are not able to stabilise their optimal market regime due to the social dilemma they face, they succeed in almost doubling the market price *on average*.

But how are single realisations looking like? Figures 3 to 10 display the result of a Monte Carlo simulation. It is a sequence of 377 states which the market sequentially occupied within a time horizon of 100.000 periods. Each figure displays information about four variables per state for a sequence of 50 states: the number of suppliers in the cartel (triangles), the cartel quota (crosses), the resulting market price (circles), and the time i. e. the number of periods the market has occupied this state (grey bars). A grey bar of a height of one unit (left vertical axis) indicates 100 periods. A grey bar reaching the upper edge of the figure stands for 1400 or more periods. The left vertical axis is scaled to directly show the number of cartel members and indirectly the cartel quota (unit divided by ten). The right vertical axis is the price axis. For convenient reading, the triangles, circles, and crosses are interconnected.

Starting with no suppliers in the cartel, a time series evolves with the cartel being in existence for all the 100.000 periods. Most generally speaking, there is a kind of saw-tooth oscillation of the number of cartel members: once a group of suppliers enters the cartel, free riding erodes this market device, until a new group enters again. Figures 3 to 5 and the left part of figure 6 show an oscillation regime where this cartelisation and de-cartelisation process happens with a modest frequency. In figure 6 a switching to an oscillation regime with a much higher frequency occurs. This fast regime only prevails for a short time and then another switch sets the market back to the modest regime as depicted in state 45 of figure 6. In the first states of figure 8 the fast regime sets in again followed by a transition back to the modest regime in the last states shown in this figure. An important insight is that the transitions between the regimes are not

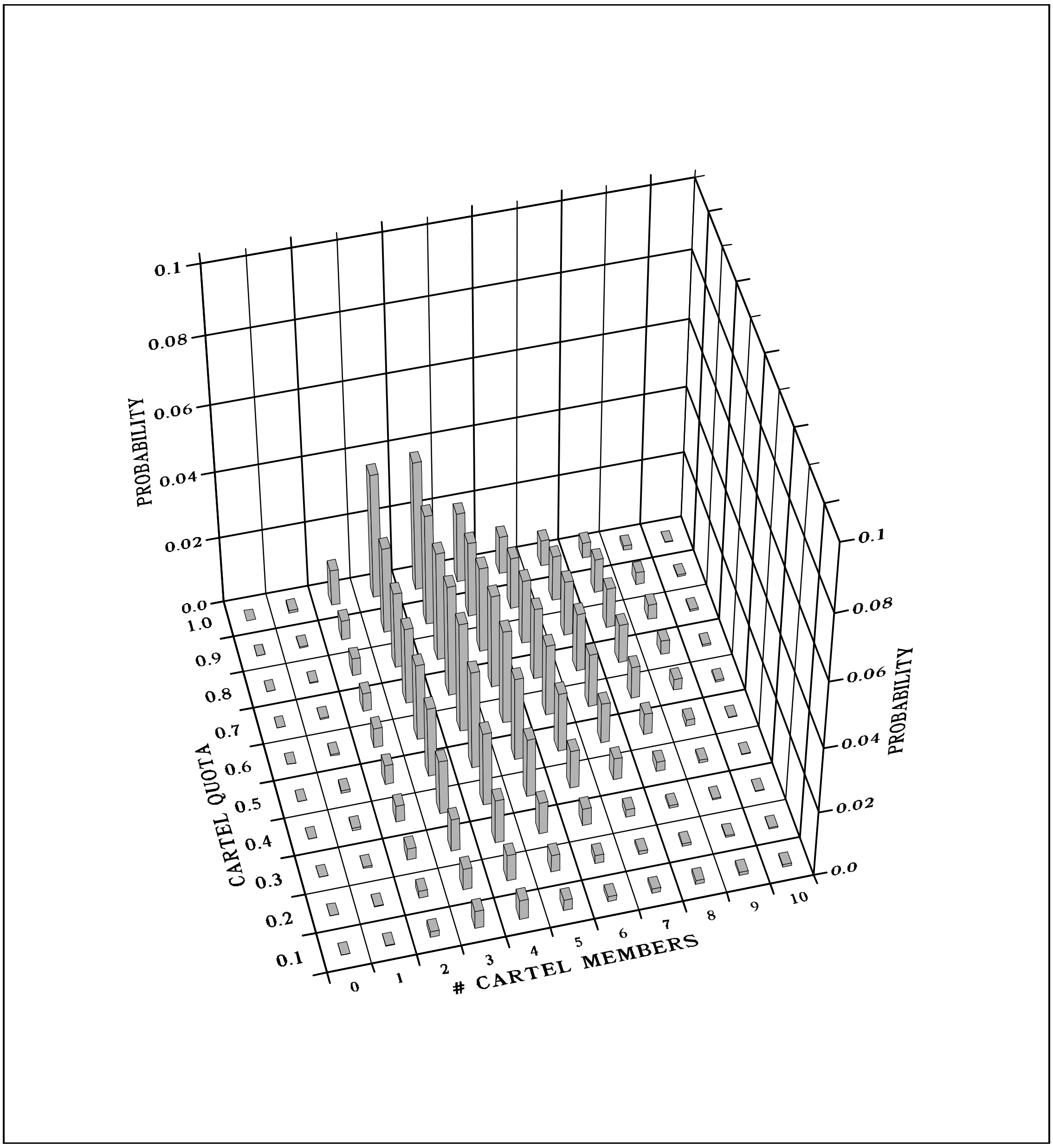

Figure 2: Stationary distribution for model 1.

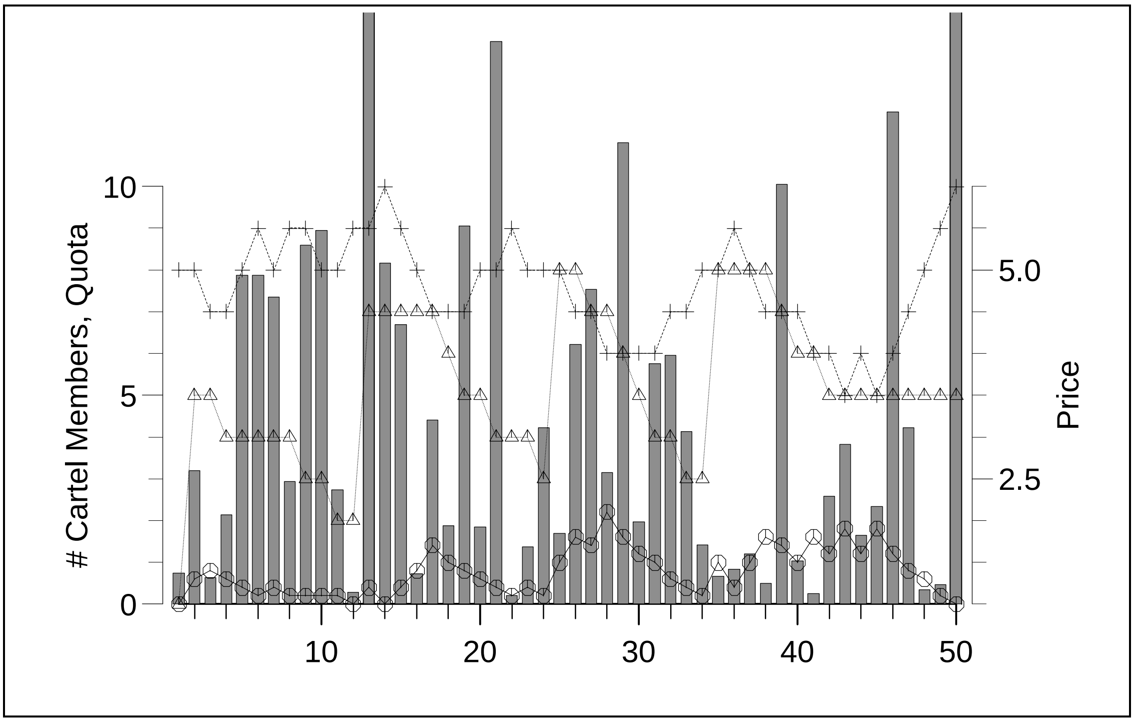

Figure 3: Modest oscillation regime.
Period 0 - 25.124 of a realisation of the cartel market.

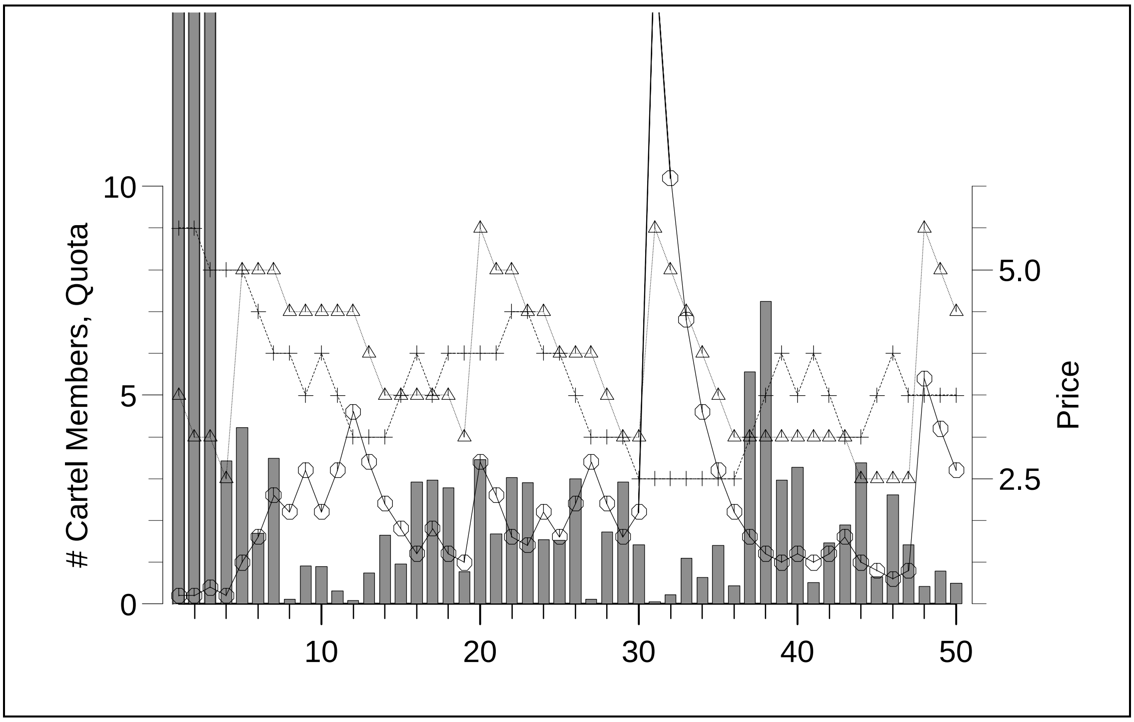

Figure 4: Modest oscillation regime.
Period 25.125 - 40.164 of a realisation of the cartel market.

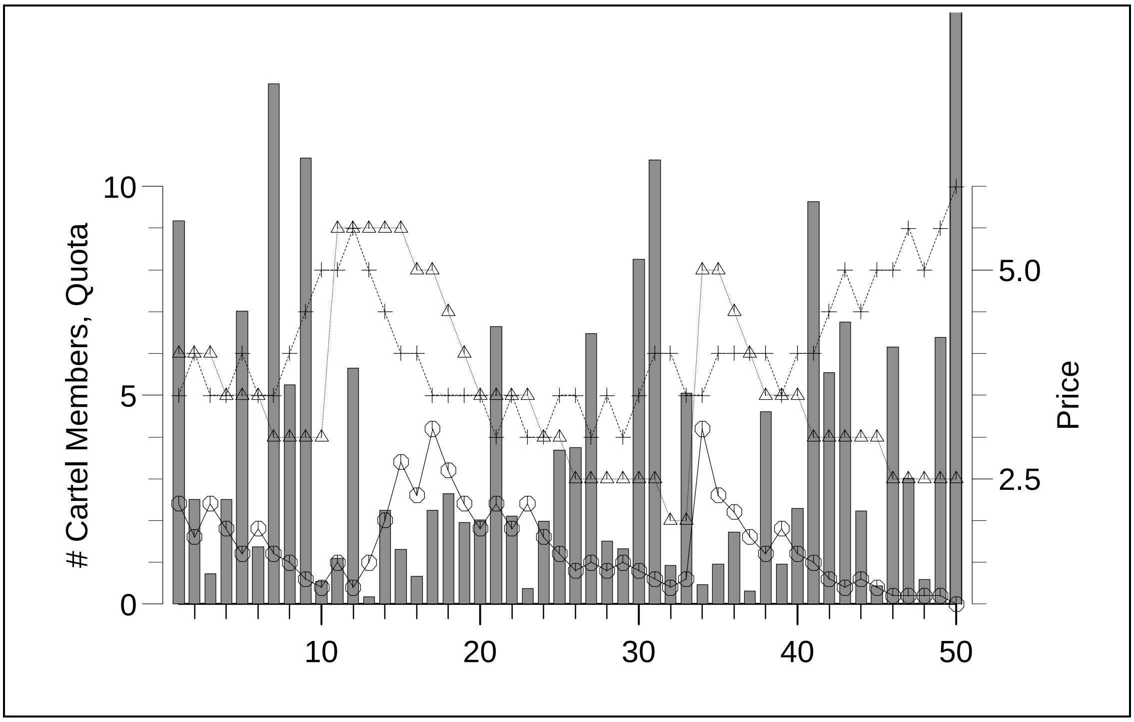

Figure 5: Modest oscillation regime.
Period 40.165 - 60.446 of a realisation of the cartel market.

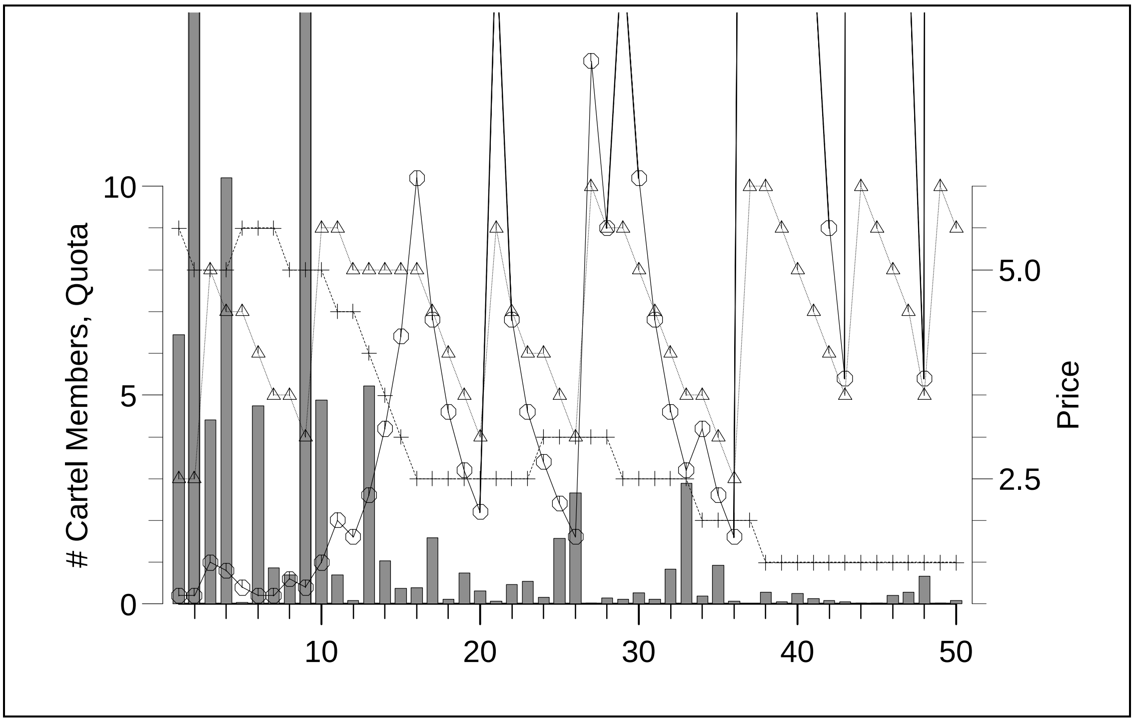

Figure 6: Transition to the fast regime.
Period 60.447 - 69.239 of a realisation of the cartel market.

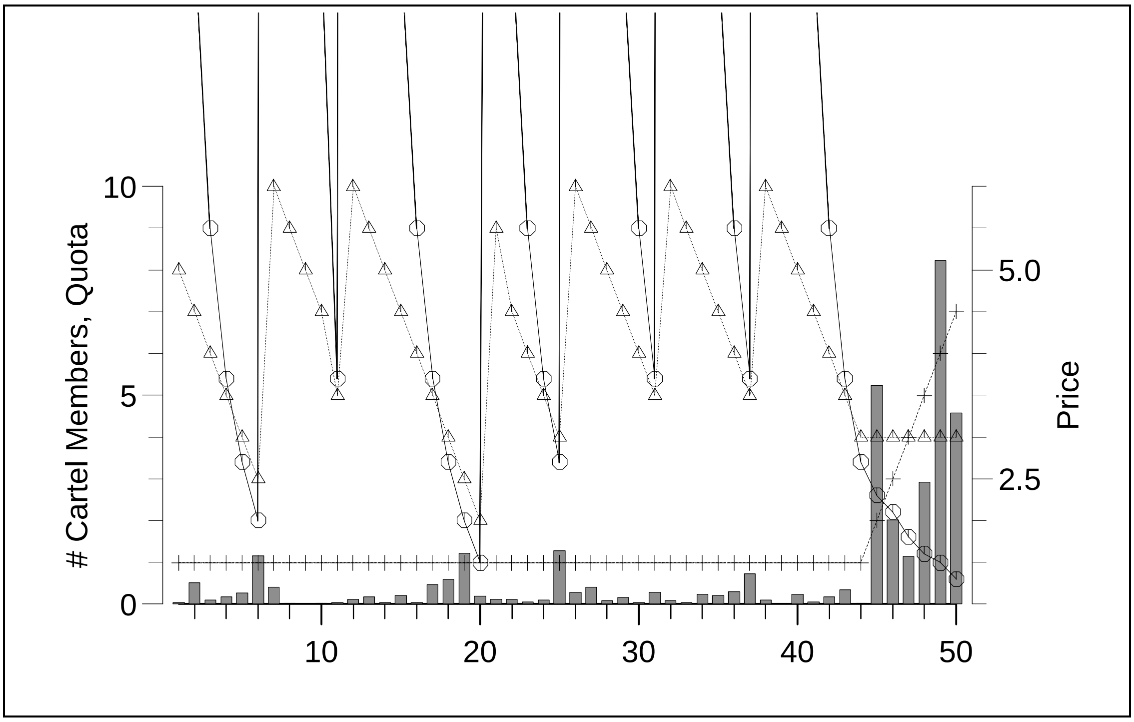

Figure 7: Fast regime and back transition.
Period 69.240 - 72.771 of a realisation of the cartel market.

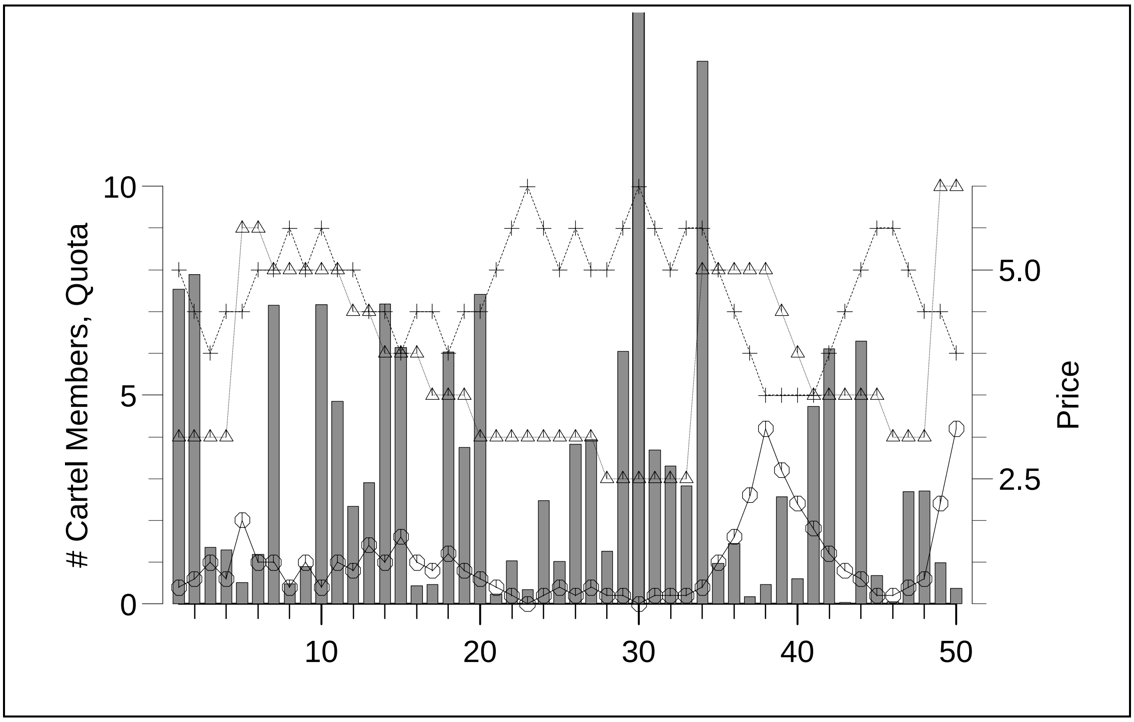

Figure 8: Modest oscillation regime.
Period 72.772 - 89.494 of a realisation of the cartel market.

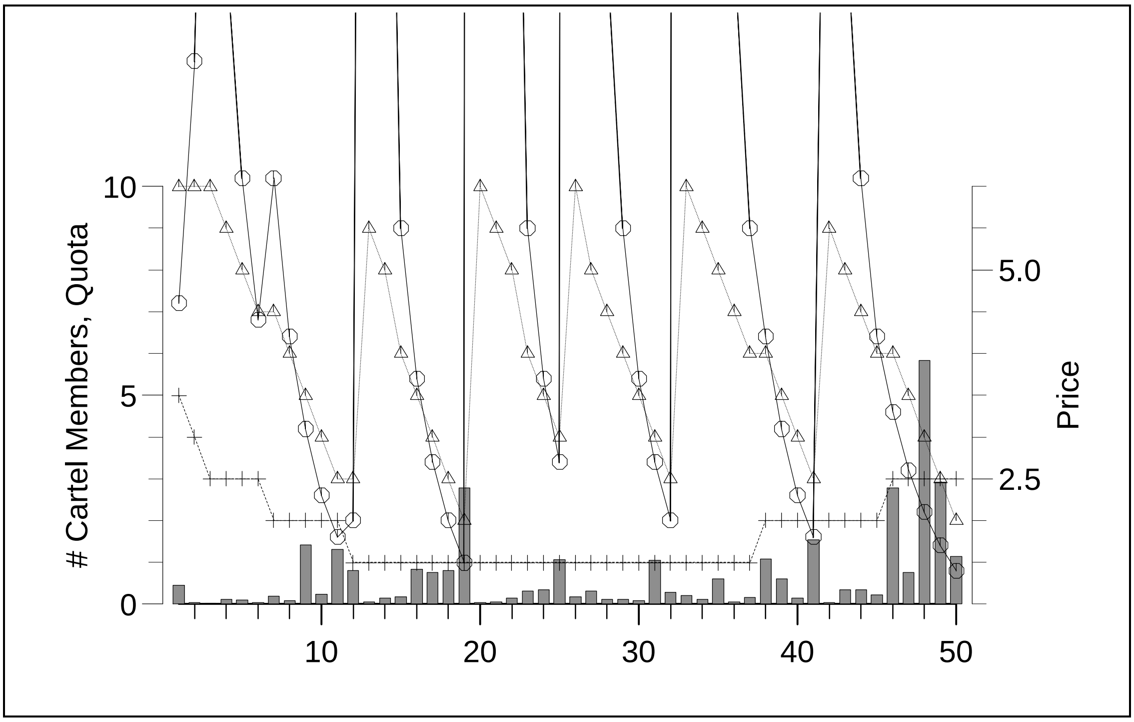

Figure 9: Second transition to the fast regime and back transition.
Period 89.495 - 92.829 of a realisation of the cartel market.

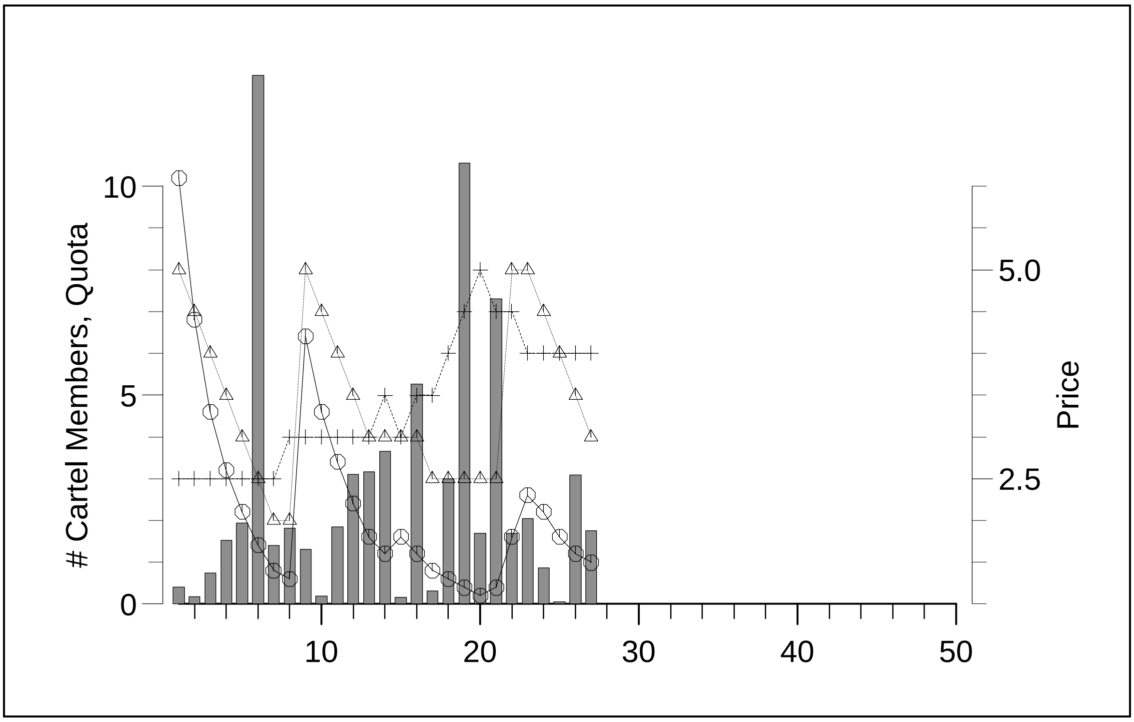

Figure 10: Modest oscillation regime.
Period 92.830 - 100.000 of a realisation of the cartel market.

triggered *exogenously* but are an *inherent endogenous* part of the system dynamics.

The computation and analysis of several time series of this process has shown that starting from the state without a cartel, the evolving realisations always have very individual properties. In general, however, they are characterised by a sequence of oscillation regimes similar to that of the realisation displayed.

7.2 Model 2

With the price-demand-function P_2, the stationary distribution resembles a one-point-distribution. Although the process has the ergodic property, the optimal point for the supply-side as a whole has a probability near to one as depicted in figure 11. This is due to the fact that with the demand function P_2 applied, in the cartel optimum there is *no* social dilemma: free riding does not pay. So, once the market is in this state of total cartelisation, the cartel is almost stable. Unfortunately, *outside* this state the same temptations as in model 1 prevail and so the question of how to get into the optimum state arises. Typically, it will take a lot of time to get there. Figure 12 displays the first 66.877 periods of a representative realisation of model 2. There is a kind of saw-tooth oscillation which persisted for 218.000 periods until the optimal state was eventually reached. This perfectly illustrates how far the stationary distribution may be misleading when it is taken as the only basis for predicting the dynamics of a stochastic market process.

8 Conclusion and outlook

Dynamic discrete choice In order to make the explicit analysis of the interaction between the micro- and the macro-level of social systems feasible, the present paper suggests an enhancement of discrete choice theory. Up to date, in discrete choice models the attributes of the alternatives may be state dependent, whereas the choice set itself is invariant. In the present approach, the set of alternatives depends on the state the market is in. This is a straightforward way to depict market processes.

Three building blocks A stimulus-response mechanism, strategic myopia and institutional inertia are the three essential building blocks of the cartel model.

Stochastic overcoming of a social dilemma Boundedly rational suppliers may succeed in stochastically overcoming the social dilemma. According to the present approach, although the cartel of this paper is *permanently unstable* in the notion

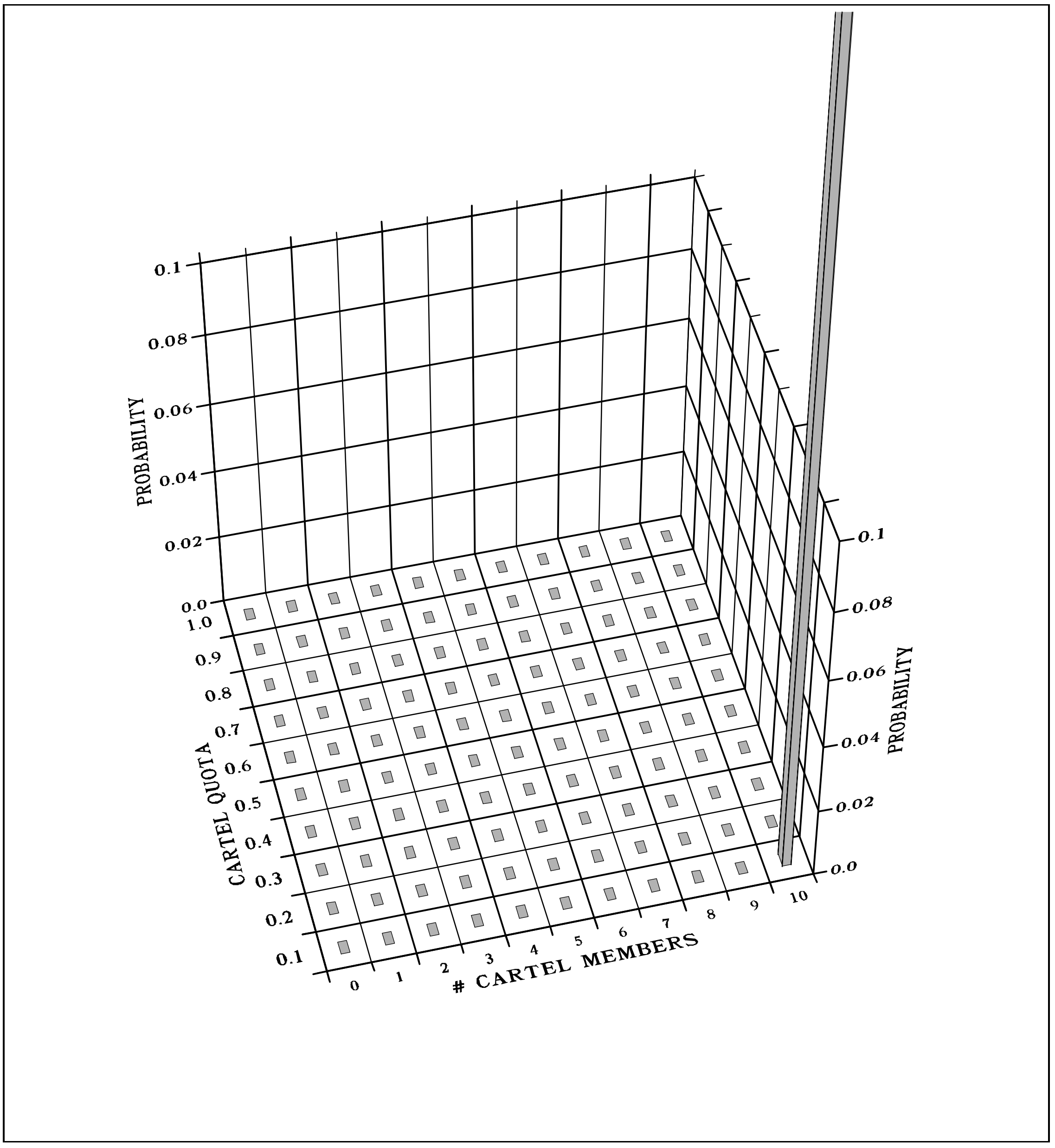

Figure 11: Stationary distribution of model 2.

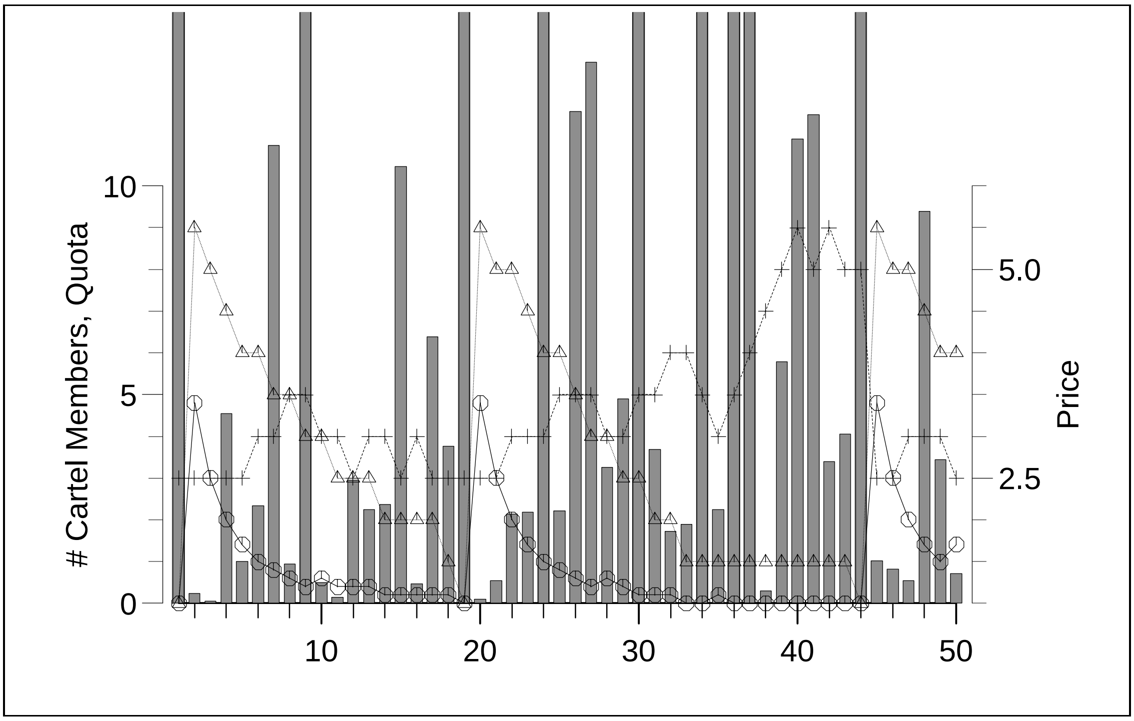

Figure 12: Period 0 - 66.876 of a realisation of model 2.

of standard cartel theory, it manages to survive for long time periods and is able to almost *double* the market price. This happens in spite of the fact that the social dilemma is neither "solved" nor "softened".

As a consequence, the notion and the conclusions of standard cartel theory should be applied with great care: they are misleading if it is appropriate to conceive the dynamics of a cartel market as a stochastic market process.

Endogenous dynamics The search for patterns in a set of realisations of our stochastic cartel market process revealed that generally a saw-tooth-like modest-frequency oscillation regime prevails. A group of suppliers joins the cartel, then one observes the sequential exit of single suppliers. At times, this regime is interrupted by transitions to a much faster regime which only lasts for a short while. This switching between regimes is a genuinely *endogenous* phenomenon and is not triggered from "outside". What in reality may be perceived as a structural change in the market or an effect of exogenous events could thus just be the outcome of the market dynamics itself.

Agent based Economics It is obvious that the model presented in this paper is still too "coarse-grained" to model the dynamics of a "real" cartel market. It is straightforward to replace the "dumb" suppliers of this paper by artificial agents which interact with other artificial agents on the demand side. Thus, complex expectation formation processes and the resulting market behaviour can be investigated. This opens up a promising agenda for future research.

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