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Title: "Is it possible to study jointly chaotic and ARCH behaviour? Application of a noisy Mackey-Glass equation with heteroskedastic errors to the Paris Stock Exchange returns series "

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1. Introduction

The discovery of deterministic chaos has changed our view of erratic behaviour. The enormous interest in deterministic systems derives from two opposite characteristics of these dynamical systems. First, they are unpredictable on the long term and second, short term predictions may be possible. This means that time series, which looks random at first glance, may in fact be predictable on short time scales. Therefore, as most specialists in the field would probably say, purely deterministic chaos seems to be hard to find in financial data. The detection of chaotic structures in stock markets is usually complicated by the large noise component inherent in the underlying dynamical system.

Most recent empirical works, applying sophisticated statistical procedures such as correlation dimension method, have shown that stock returns are highly complex. The estimated correlation dimension is high and there is little evidence of low dimensional deterministic chaos. This result is well justified: noise and uncertainty play an important role in financial markets. Evidence against low dimensional chaos includes not only high estimated (and unstable) correlation dimension, but also very little evidence of out-of-sample predictability. For example, Scheinkman and LeBaron (1989) present some evidence for a strange attractor with correlation dimension equal to 6. They are very careful in stating their claim however: *"…..the data are not incompatible with a theory where some of the variation in weekly returns could come from non-linearities as opposed to randomness and are not compatible with a theory that predicts that returns are generated by i.i.d. random variables"* (Scheinkman and LeBaron (1989, p. 332)). This evidence is thus consistent with a noisy chaotic model, but may also be consistent with a non-linear stochastic time series model.

From a practical viewpoint, distinguishing between noisy chaos (i.e. a chaotic system disturbed by dynamical noise) and randomness is a very difficult task. Noisy chaotic series can have zero autocorrelations at all lags and therefore, from a linear statistical viewpoint, *noisy chaos may be indistinguishable from white noise.*

Early works of Brock and Hommes (1998), Lux (1995, 1998), Malliaris and Stein (1999), Gaunersdorfer (2000), and Chiarella et al. (2000) showed that *structural non-linear financial markets models may lead to market instability and chaos*. In these non-linear models, asset price complex fluctuations are triggered by an interaction between a stabilising force driving prices back towards their fundamental value when the market is dominated by fundamentalists, and a destabilising force driving prices away from their fundamental value when the market is dominated by speculative noise traders. The distributions of returns derived from chaotic trajectories of the models [e.g. Lux (1998), Iori (1999)] share important characteristics of empirical data: volatility clustering, they exhibit high peaks around the mean as well as fat tails (leptokurtosis) and become less leptokurtotic under time aggregation. The introduction of endogenously determined transitions probabilities (e.g. Lux, 1998), or the modelling using noisy chaotic systems (e.g. Malliaris and Stein, 1999), give a new dimension of noisy chaos applications in finance.

Taking the complex behaviour in stock markets into account, we think that a more robust approach than the traditional stochastic one, is to model the observed data by *a non-linear dynamical system disturbed by dynamical noise*. In fact, we construct a model having negligible or even zero correlations in the conditional mean, which corresponds to the deterministic component, but a rich structure in the conditional variance (volatility), which characterises the stochastic component. In time series analysis the time dependence of conditional variance is referred to as heteroskedasticity. The model is a noisy Mackey-Glass equation with errors that follow a GARCH(p,q) process (henceforth MG-GARCH(p,q)).

The MG-GARCH(p,q) model permits us to capture volatility clustering phenomenon, according to which, stock prices fluctuations are characterised by episodes of low volatility, with small prices changes, irregularly interchanged by episodes of high volatility, with large price changes. Nevertheless, the particularity of this model is that volatility clustering is interpreted as an endogenous phenomenon. If stock markets are dominated only by fundamentalists, prices would be determined using the deterministic part of the MG-GARCH(p,q) model. But in reality, there are different types of traders, having different trading strategies and expectations about future prices and dividends of a risky asset. The interactions between them cause the volatility clustering.

The main objective of this article is the identification of the underlying process of the Paris Stock Exchange returns series (CAC40). For this reason, we apply different test concerning, the research of long memory components [fractional integration test, Geweke and Porter-Hudak (1983)], and chaotic structures [Grassberger and Procaccia correlation dimension (1983), and Gençay and Dechert Lyapunov exponents (1992) methods]. Then, we estimate and forecast the CAC40 returns series by the MG-GARCH(p,q) model. Finally, we compare this model with recent forecasting methods (Principal components regression, and Radial basic functions). Since a part of this methodology has already used in Kyrtsou and Terraza (2000a), we prefer briefly presenting the empirical procedure in figure 1 and reporting only the results in the following section. We must note that this paper completes the findings in Kyrtsou and Terraza (2000a), with the application of the MG-GARCH(p,q) model permitting to study jointly chaotic and ARCH behaviour.

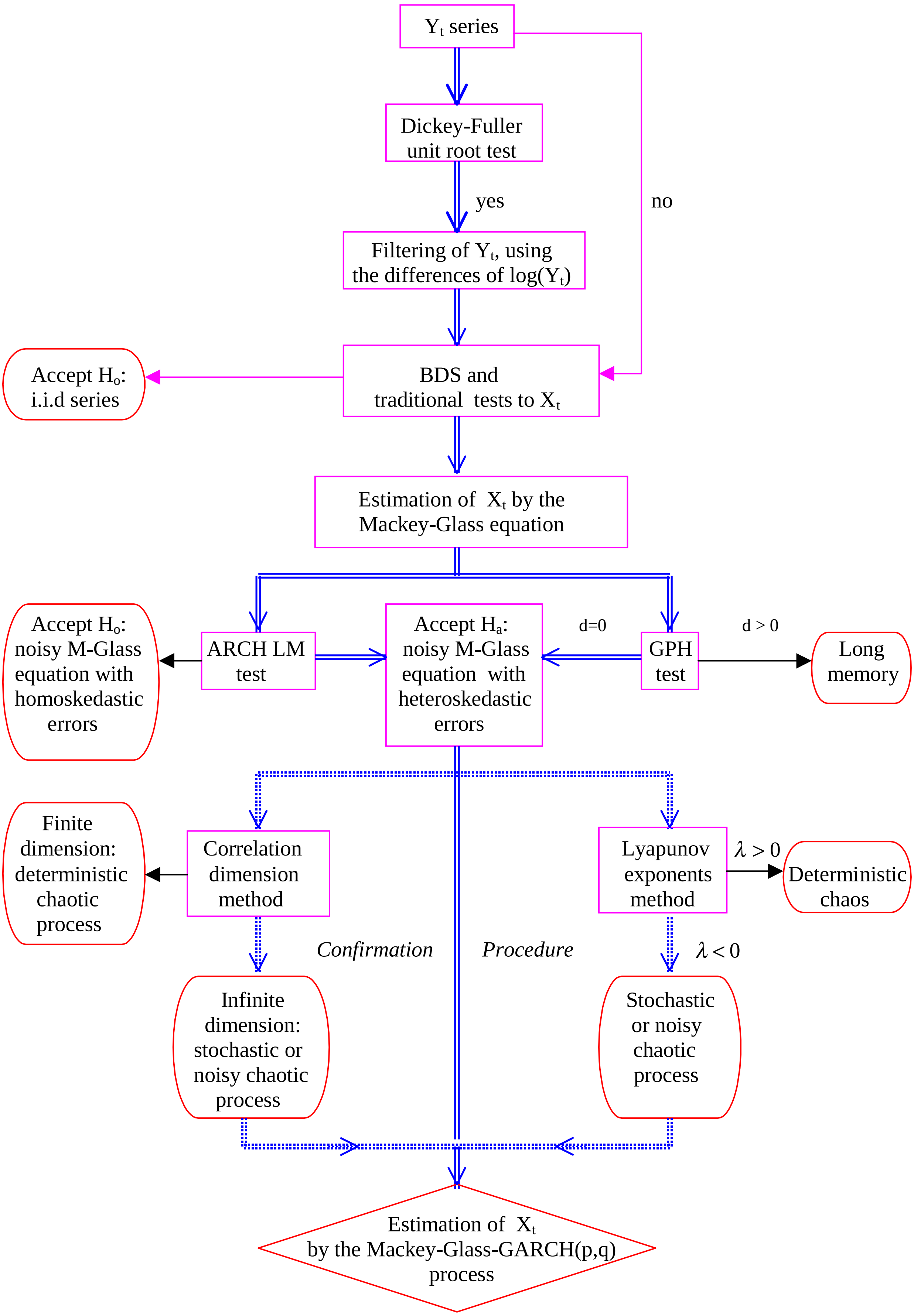

Figure 1: Empirical procedure. Double lines show the path, we have followed.

2. Empirical results

2.1 Data and tests results

The data considered in this study were daily index returns series of the French Stock Exchange (CAC40, during the period 09/07/1987-05/28/1999, 3060 observations). After applying the augmented Dickey-Fuller unit root test, we find that a unit root exists in the CAC40 series (DF value = 0.7294 > critical value at 5%). So, the data were first log-differenced (DLCAC40, figure 2 in appendix) (DF value = -53.48 < critical value at 5%).

Afterwards, we apply the BDS test to the CAC40 returns series, in order to discern the absence or the existence of linear or non-linear dependence. The results reported in Table 1 reveal that the i.i.d. null is rejected ($|W|>1.96$).

Table 1: BDS test results for the DLCAC40 series

Epsilon/sigma	**0.5**	**1**	**1.5**	**2**
m = 2	4.2522	5.6170	7.6004	9.0979
m = 3	5.5562	7.3283	9.8005	12.056
m = 4	6.9679	8.9349	11.376	13.817
m = 5	8.7084	10.235	12.588	14.822

Calculating descriptive statistics, the leptokurtosis in the data is revealed by high kurtosis coefficient and Jarque-Bera (kurtosis = 9.21 and J.B = 5017,6). The ARCH LM test result confirms the presence of heteroscedasticity (TR^2 = 47,145 > $\chi^2(1)$). Table 2 presents the spectral regression estimates of the fractional differencing parameter d for the CAC40 returns series over the in-sample period. A choice must be made with respect to the number of low-frequency periodogram ordinates used in the spectral regression. Improper inclusion of medium or high-frequency periodogram ordinates will contaminate the estimate of d. At the same time too small a regression sample will lead to imprecise estimates. For this precise reason, we put $v = T^{0.5}$. To raise estimation efficiency, the known theoretical variance of the regression error $\pi^2/6$ is imposed in the construction of the t-statistic for d. As table 2 reports, there is strong evidence that the CAC40 returns series exhibits short memory. Nevertheless, we cannot conclude that an ARCH process generates the CAC40 series. The fractional integration coefficients for both series are identical (for more details see Kyrtsou and Terraza, 2000a).

Table 2: Fractional integration coefficient for the DLCAC40 series

Methods	**d**	**Standard error**	**Test for d = 0**	**p-value**
GPH	0.18469491	0.29168536	0.63319911	0.52660363

Tables 3, presents the correlation dimension estimates for the DLCAC40 series. Clearly, these results reveal that as embedding dimension m is increased from 2 to 10, the correlation dimension estimates do not converge to a stable value. This is a well-known characteristic of stochastic processes. The interest in this stage, is the behaviour of correlation dimension estimator, in the case of a noisy chaotic and a pure stochastic process. In both cases correlation dimension is very high and thus it is very difficult to distinguish between the two processes (Kyrtsou and Terraza, 2000a).

Table 3: Correlation dimension estimates for the DLCAC40 series

m	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C.D	1.918	2.928	3.794	4.562	5.091	5.477	5.878	6.244	6.522

Afterwards, in order to get a complete description of the CAC40 series dynamic behaviour, we also compute the Lyapunov exponents of the log-differenced price series. The reliability of the Lyapunov exponents on the basis of which one can distinguish between low-dimensional chaos and stochastic processes, is well-known. Therefore, the algorithm developed by Wolf et al. (1985), which is used to estimate the rates of exponential growth of small perturbations to initial conditions, is very sensitive to the noise level. Thus, taking the high noise level in financial series into account, we are not sure that Wolf et al. Lyapunov exponent estimator will be robust. So that, we avoid biased estimators because of noise, we apply the algorithm proposed by Gençay and Dechert (1992) based on feedforward neural networks. The obtained results for the DLCAC40 series are reported in table 4. In the first columns is given the number of hidden units. Then, we have the largest Lyapunov exponents $\lambda(1)$ (for the first input) and $\lambda(2)$ (for the second). The two last columns give the mean squared error (MSE) and the Schwarz information criterion (SIC). The best Lyapunov exponent is which that minimises the previous criteria. Looking at the table 4, we see that the minimum MSE value is obtained, when we use 16 hidden units. In this case, $\lambda(1)$=*0.2954235^{e}-03* and $\lambda(2)$= *-0.970291*. Respectively, we obtain the minimum SIC when we use 4 hidden units. The corresponding Lyapunov exponents are $\lambda(1)$= *0.5352910^{e}-03* and $\lambda(2)$= *-1.047435*. For both cases, $\lambda(1)$ is positive and $\lambda(2)$ negative. Consequently, we consider that there is not clear evidence for pure stochastic process. Similar results are also obtained by Gençay and Liu (1996), for a noisy logistic map, a noisy Hénon map, and a noisy Mackey-Glass delay equation. With the addition of noise, the largest Lyapunov exponent becomes greatly negative. The fact that $\lambda(1)$ is slightly positive could be due to the existence of an *underlying high-dimensional chaotic system*, which generates random similar behaviour, because of *hidden dimensionality*.

Table 4: Lyapunov exponents estimates for the DLCAC40 series

Hiddens	**Lambda(1)**	**Lambda(2)**	**MSE**	**SIC**
1	-0.1839252^{e}-02	-0.9607019	0.381505^{e}-07	-17.0686
2	*0.1341960^{e}-02*	-1.051362	0.293360^{e}-07	-17.3208
3	*0.4474912^{e}-02*	-1.107096	0.360923^{e}-07	-17.1031
4	*0.5352910^{e}-03*	-1.047435	0.285112^{e}-07	*-17.3284*
5	-0.1775347^{e}-03	-1.048775	0.282644^{e}-07	-17.3266
6	-0.1860099^{e}-03	-1.048154	0.282796^{e}-07	-17.3155
7	*0.5006879^{e}-03*	-1.023327	0.287912^{e}-07	-17.2871
8	*0.1201093^{e}-03*	-1.057145	0.280426^{e}-07	-17.3030
9	-0.1789163^{e}-03	-1.061870	0.282179^{e}-07	-17.2862
10	-0.1937435^{e}-03	-0.9916856	0.271461^{e}-07	-17.3145
11	*0.1551437^{e}-03*	-0.9593365	0.270472^{e}-07	-17.3076
12	*0.2185285^{e}-03*	-1.060679	0.279938^{e}-07	-17.2627
13	-0.9673228^{e}-04	-1.048142	0.282721^{e}-07	-17.2423
14	*0.1971973^{e}-03*	-1.059729	0.280766^{e}-07	-17.2388
15	*0.1827083^{e}-03*	-0.9544411	0.272428^{e}-07	-17.2584
16	*0.2954235^{e}-03*	-0.970291	*0.268500^{e}-07*	-17.2625
17	*0.2153128^{e}-03*	-1.056791	0.279382^{e}-07	-17.2122
18	*0.2276746^{e}-03*	-1.059496	0.280041^{e}-07	-17.1994
19	*0.3131472^{e}-03*	-0.9656978	0.269641^{e}-07	-17.2267
20	-0.1554122^{e}-03	-1.043172	0.278274^{e}-07	-17.1847

The application of the correlation dimension and the Lyapunov exponents methods cannot permit us to bring a clear conclusion in favour of a noisy chaotic or a pure stochastic process. *A possible explanation is that financial series may include both chaotic and heteroskedastic structures*. To test this hypothesis, we fit a noisy Mackey-Glass model with errors that follow a GARCH(1,1) process, to the CAC40 returns series.

2.2 Mackey-Glass-GARCH(1,1) model

As the majority of stock indices, the Paris Stock Exchange returns series does not present significant autocorrelations. It is indistinguishable from white noise (figure 3 and table 7, in appendix). Therefore, the results of the BDS test, presented in the previous section, have shown that the CAC40 returns series is not an i.i.d. process. This dependence in the mean can not be detected by an ARMA process. On the contrary, when we estimate a noisy Mackey-Glass[1] model the coefficients are widely significant[2].

Our model is a discretized variant of the deterministic chaotic Mackey-Glass delay equation[3] plus a noise.

$$X_t = X_{t-1} + a\frac{X_{t-\tau}}{1+X_{t-\tau}^2} - bX_{t-1} + \varepsilon_t = a\frac{X_{t-\tau}}{1+X_{t-\tau}^2} - (b-1)X_{t-1} + \varepsilon_t = a\frac{X_{t-\tau}}{1+X_{t-\tau}^2} - \delta X_{t-1} + \varepsilon_t$$

[1] We also estimated a logistic equation but the coefficients were not significant.
[2] These results are available upon authors.
[3] Mackey and Glass (1977) equation is an infinite dimensional system, but its attracting set dimension varies as the delay parameter τ is changed. We have found that for τ =1 the dimension is about 7 or greater.

We use $\tau = 1$.

After applying the ARCH LM test to the residuals series of the noisy Mackey-Glass equation, the existence of heteroscedasticity is well confirmed ($TR^2=39.44 > \chi^2(1)$).

Consequently, the stochastic part in the previous model is a heteroskedastic noise $\varepsilon_t | I_t \sim N(0, h_t)$, where h_t is the conditional variance.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

Then, we estimate the Mackey-Glass-GARCH(1,1) process (henceforth MG-GARCH(1,1)). The results are presented in the following table:

Table 4: Estimates for MG-GARCH(1,1) model for the DLCAC40 series

Coefficient	Value	t-Statistics
α	187,46	145.10**
δ	187,38	145.20**
α_0	0.0000792	1.8874*
α_1	0.14999	4.2839**
β_1	0.5999	7.607**

* : coefficient statistically significant at 10%

**: coefficients statistically significant at 5%

Looking at the table 4 we can see that the coefficients of the MG-GARCH(1,1) model are significant. The model detects an important part of non-linearities in the returns series. For the standardised residuals[4] the kurtosis has been reduced from 9.2102 to 5.646, and the Jarque-Bera value from 5,017.61 to 946.03 (figures 6 and 7, in appendix). In order to test the robustness of this conclusion, we also apply the BDS test to the standardised residuals series. The results presented below show, that dependencies have been eliminated. The autocorrelations study of the standardised residuals, confirms this result (figures 4, 5 and table 8, in appendix). The $|W|$ statistics is clearly inferior to the critical value 1.96. Therefore, the remaining high kurtosis and Jarque-Bera, are not due to heteroskedastic structure in the data, because applying the ARCH LM test to the squared residuals, we find that $TR^2= 1.205 < \chi^2(1)$).

Table 5: BDS test results for the residuals of the MG-GARCH(1,1) model

Epsilon/sigma	0.5	1	1.5	2
m = 2	-1.6928	-1.7504	-1.2508	-0.71201
m = 3	-1.4647	-1.3558	-0.54833	-0.34387
m = 4	-0.90981	-0.64331	-0.3037	1.2809
m = 5	-0.15938	-0.17411	1.0398	1.9026

Finally, we compare the forecasts of the CAC40 returns series, obtained by using the MG-GARCH(1,1), the GARCH(1,1), the naive prediction, the random walk, the Principal Components Regression (PCR)[5] and the Radial Basic Functions (RBF)[6]. In order to evaluate the performance of the six forecasting methods we use the normalised mean square error (NMSE) index, as defined by Farmer and Sidorowich (1987):

[4] The residuals of the MG-GARCH(1,1) process.

[5] See Kugiumtzis et al. (1998).

[6] See Casdagli (1989).

$$\text{NMSE} = \frac{\sum_{t \in P} (x_t - \hat{x}_t)^2}{\sum_{t \in P} (x_t - \bar{x}_P)^2}$$

where $\hat{x}_t$ is the predicted value of x_t, P represents the samples for out-of-sample prediction, and $\bar{x}_P$ is the mean value of P. We prefer using NMSE because the division by the estimated variance eliminates the dependence on the range of the data. If NMSE=0, the predictions are perfect; NMSE$\geq$1 indicates that the performance is no better than the mean value predictor. The NMSE values obtained by the six methods are summarised in table 6. As can be seen, the MG-GARCH(1,1) gives the best forecasts.

Table 6: Normalised Mean Squared Error for the forecasts of the DLCAC40 series

Steps ahead	**RBF**	**Mackey-G GARCH**	**Mean (PCR)**	**GARCH**	**Naive**	**Random Walk**
2	1.324172124	0.920453939	1.322494599	1.072239166	1.32168934	1.324172124
3	0.71659296	0.951630225	0.823201636	0.984493173	0.943101453	0.71659296
4	1.335082917	0.946319914	0.956974088	0.978389408	0.93634015	1.335082917
5	1.050936002	0.944227294	0.819394604	0.968726315	0.745238289	1.050936002
10	1.134384017	0.982015221	0.879301895	0.991544386	1.872776554	1.134384017
15	1.271814332	0.994110633	1.066608116	0.999884373	1.739011616	1.271814332
20	1.267923024	0.988908906	1.025008975	0.994233467	1.741206249	1.267923024
Mean of Columns	1.157272	0.9610951**	0.9848027	0.9985014	1.3284806	1.1572657

** = best forecasting method

3. Conclusion

This paper investigated whether the behaviour of the CAC40 returns series is governed by noisy chaotic dynamics. With the use of the BDS and the fractional integration tests as well as the correlation dimension and the Lyapunov exponents methods, this paper's primary findings are as follows:

- The CAC40 returns series is not an i.i.d. process and exhibits short memory.
- Our results on the estimation of correlation dimension and Lyapunov exponents provide evidence that the CAC40 returns series is likely generated from a high-dimensional chaotic (noisy chaos) or a pure stochastic process.
- Thus, a MG-GARCH(1,1) model is fitted to the returns series, and it appears to capture an important part of non-linearities.
- Finally, obtained prediction results show that, according to the normalised mean square error criterion, MG-GARCH(1,1) and chaotic models outperform the GARCH, naive and random walk models. Paris stock market has become increasingly complex and therefore less amenable to forecasting over long time.

The previous findings confirm the idea that chaotic and *ARCH phenomena can be studied jointly in stock markets*. Thereby, *heteroskedasticity is interpreted endogenously*. Heterogeneity of expectations about future prices and dividends is the main source of fluctuations in returns.

APPENDIX

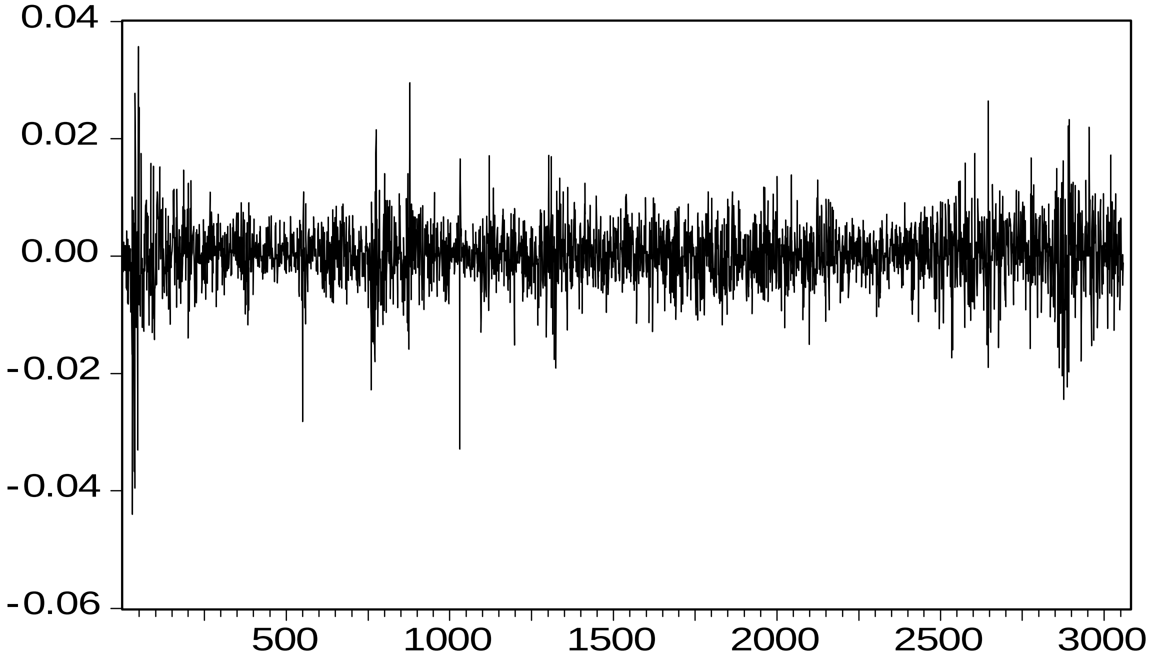

Figure 2: Paris Stock Exchange returns series

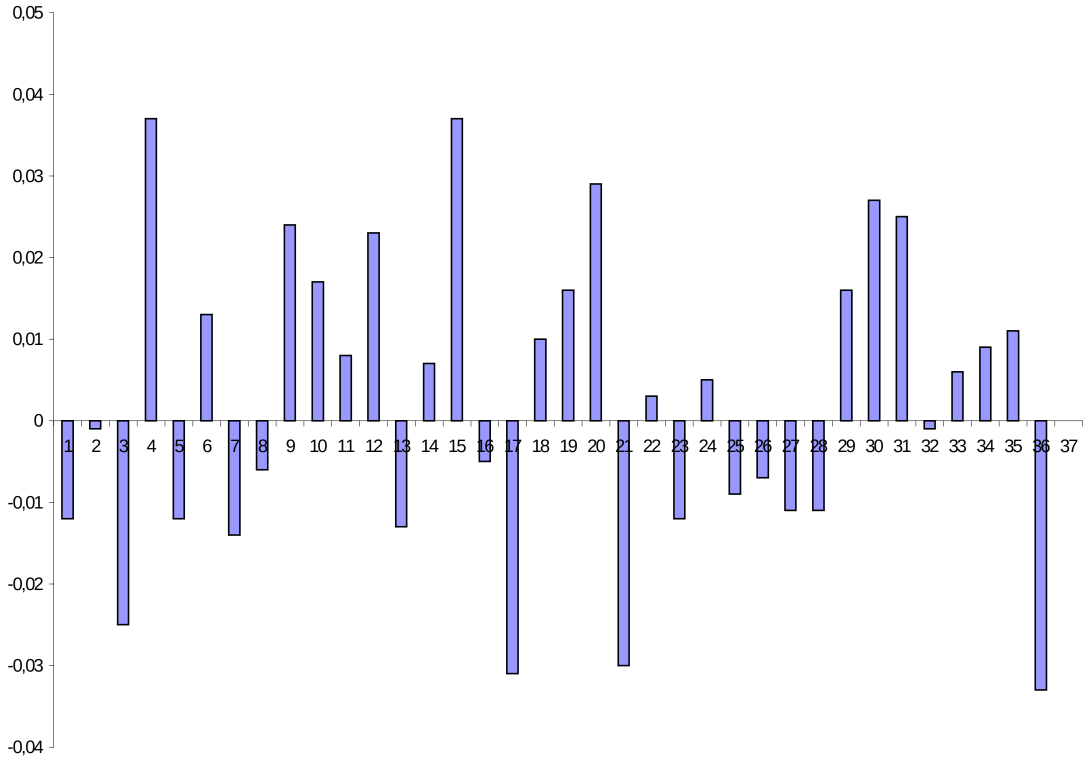

Figure 3: Autocorrelations of the CAC40 returns series

Table 7 : Autocorrelations, and Q-Statistics for the DLCAC40 series

Lags	AC	PAC	Q-Stat	Prob
1	0.033	0.033	3.4126	0.065
2	0.006	0.005	3.516	0.172
3	-0.036	-0.037	7.5278	0.057
4	0.009	0.012	7.7848	0.100
5	-0.004	-0.004	7.8242	0.166
6	0.006	0.004	7.9213	0.244
7	-0.004	-0.004	7.9778	0.335
8	-0.014	-0.014	8.6008	0.377
9	0.045	0.047	14.824	0.096
10	0.030	0.027	17.571	0.063
11	0.009	0.005	17.798	0.086
12	0.026	0.029	19.930	0.068
13	-0.031	-0.032	22.853	0.043
14	-0.001	0.002	22.854	0.063
15	0.021	0.023	24.249	0.061
16	0.007	0.002	24.400	0.081
17	-0.028	-0.027	26.875	0.06
18	0.006	0.008	27.003	0.079
19	0.021	0.019	28.394	0.076
20	0.024	0.020	30.234	0.066
21	-0.031	-0.036	33.252	0.043
22	-0.005	-0.001	33.329	0.057
23	-0.003	0.002	33.354	0.075
24	0.013	0.008	33.874	0.087
25	-0.006	-0.007	33.985	0.108
26	-0.013	-0.013	34.538	0.122
27	-0.004	-0.002	34.592	0.150
28	-0.003	-0.004	34.622	0.181
29	0.022	0.019	36.110	0.170
30	0.042	0.039	41.440	0.080
31	0.020	0.018	42.670	0.079
32	-0.010	-0.008	42.995	0.093
33	0.006	0.012	43.119	0.112
34	0.016	0.012	43.876	0.100
35	-0.002	-0.004	43.889	0.144
36	-0.031	-0.029	46.912	0.105

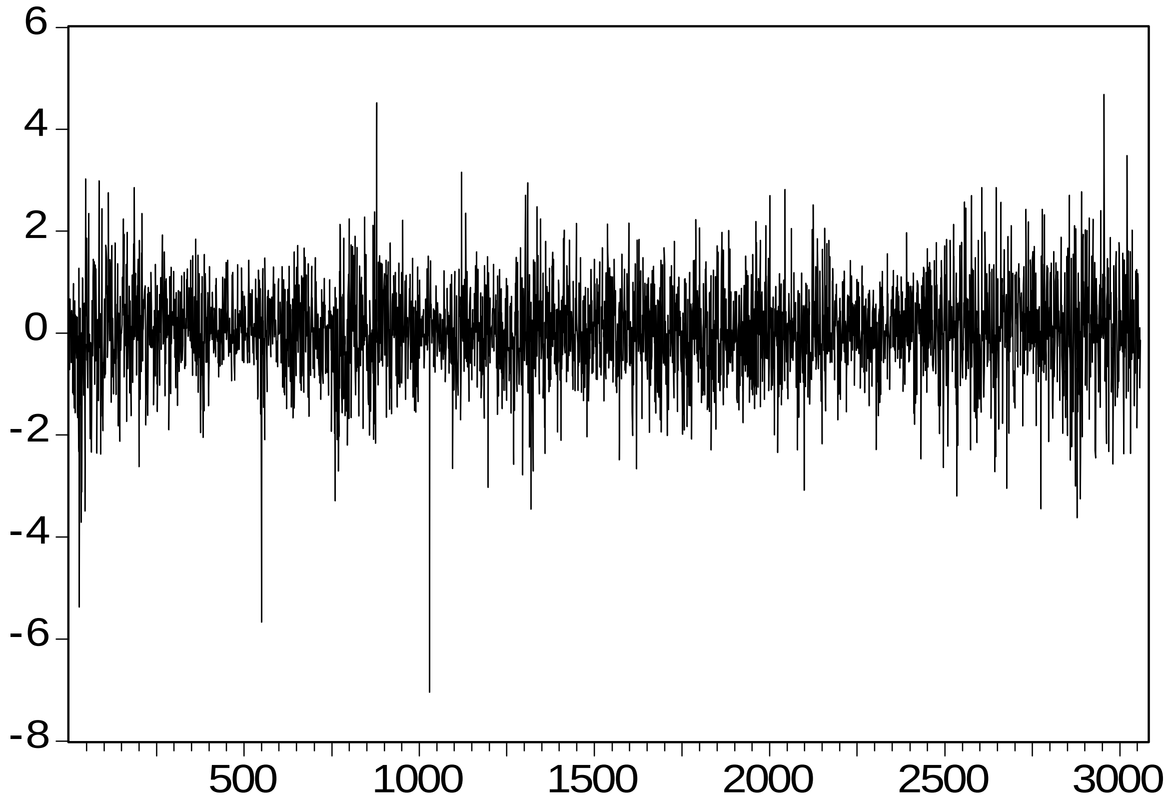

Figure 4: Residuals series of the MG-GARCH(1,1) process

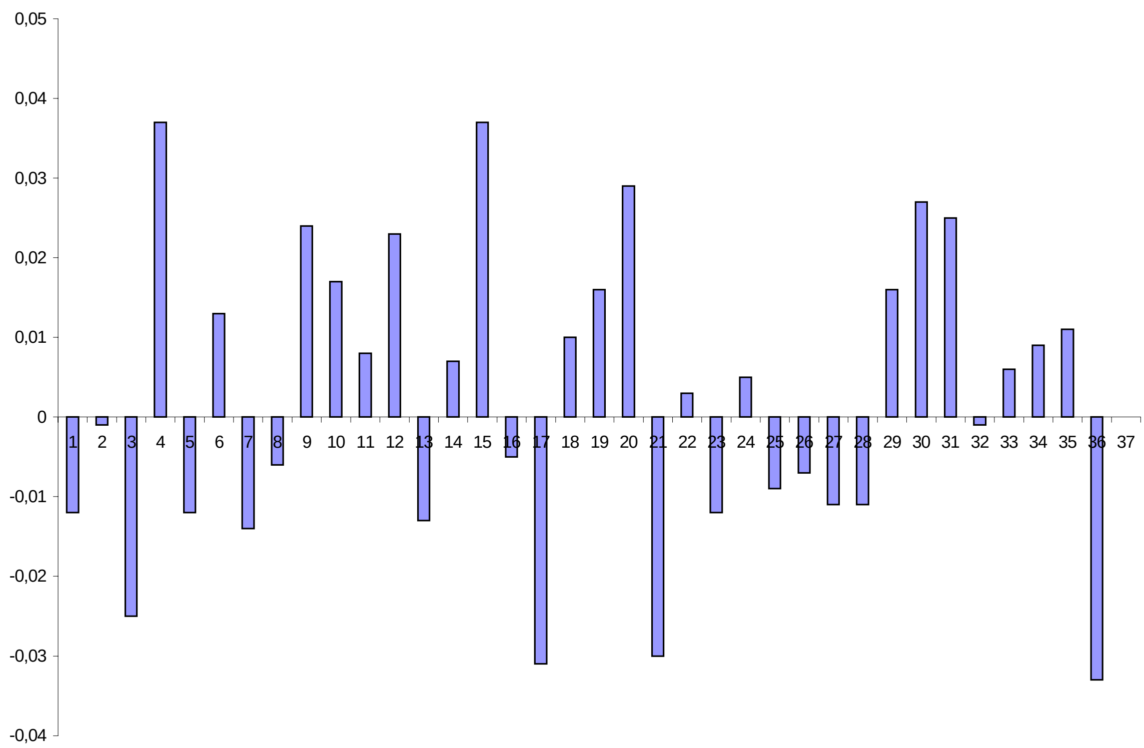

Figure 5: Autocorrelations for the residuals series of the MG-GARCH(1,1) process

Table 8 : Autocorrelations, and Q-Statistics for the residuals series of the MG-GARCH(1,1) process

Lags	AC	PAC	Q-Stat	Prob
1	-0.012	-0.012	0.4613	0.497
2	-0.001	-0.001	0.4666	0.792
3	-0.025	-0.025	2.4259	0.489
4	0.037	0.036	6.5477	0.162
5	-0.012	-0.011	6.9803	0.222
6	0.013	0.012	7.5124	0.276
7	-0.014	-0.012	8.0822	0.325
8	-0.006	-0.008	8.1959	0.415
9	0.024	0.025	9.9588	0.354
10	0.017	0.016	10.822	0.372
11	0.008	0.009	11.026	0.441
12	0.023	0.024	12.604	0.398
13	-0.013	-0.014	13.160	0.436
14	0.007	0.007	13.306	0.503
15	0.037	0.038	17.557	0.287
16	-0.005	-0.006	17.636	0.346
17	-0.031	-0.028	20.515	0.249
18	0.010	0.009	20.804	0.289
19	0.016	0.014	21.602	0.305
20	0.029	0.028	24.117	0.237
21	-0.030	-0.029	26.838	0.176
22	0.003	0.003	26.863	0.216
23	-0.012	-0.011	27.345	0.242
24	0.005	-0.002	27.430	0.285
25	-0.009	-0.007	27.685	0.323
26	-0.007	-0.009	27.851	0.366
27	-0.011	-0.010	28.238	0.399
28	-0.011	-0.012	28.608	0.433
29	0.016	0.014	29.359	0.446
30	0.027	0.025	31.615	0.386
31	0.025	0.027	33.605	0.342
32	-0.001	0.003	33.610	0.389
33	0.006	0.008	33.735	0.432
34	0.009	0.007	34.004	0.468
35	0.011	0.010	34.389	0.497
36	-0.033	-0.029	37.774	0.388

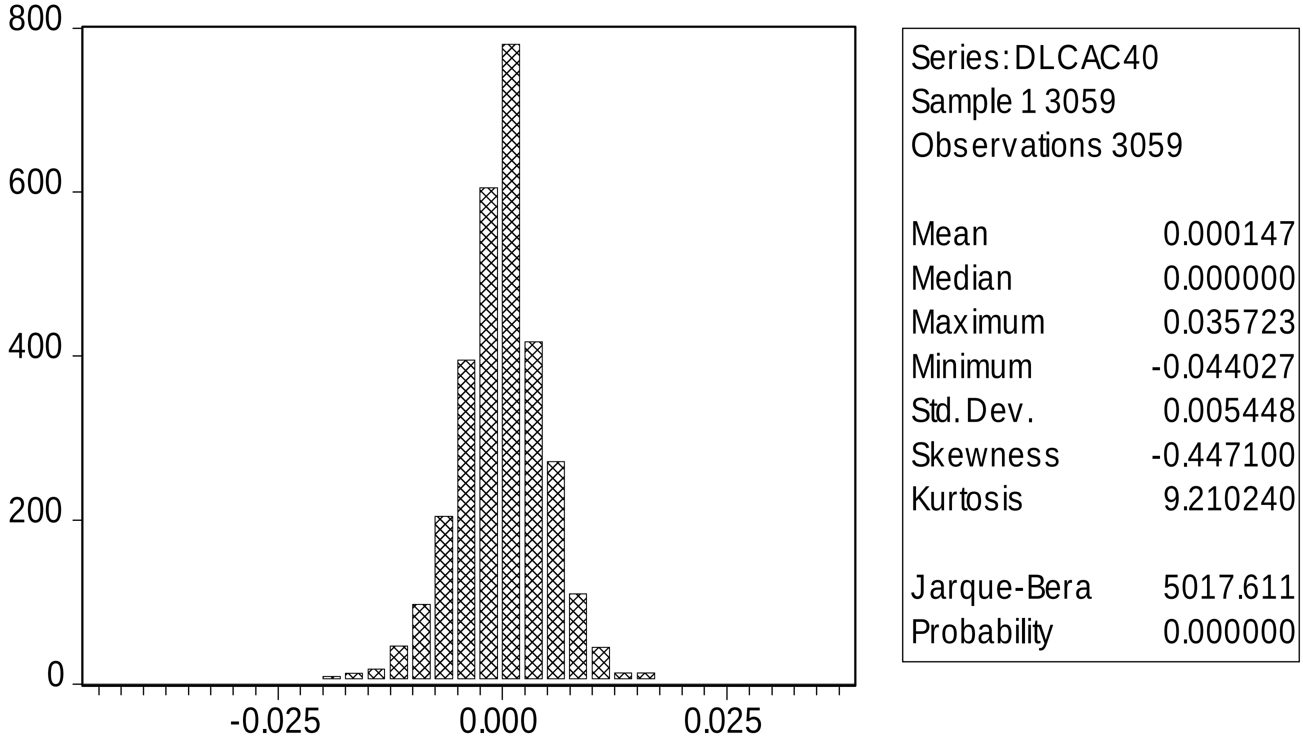

Figure 6: Sample statistics for the DLCAC40 series

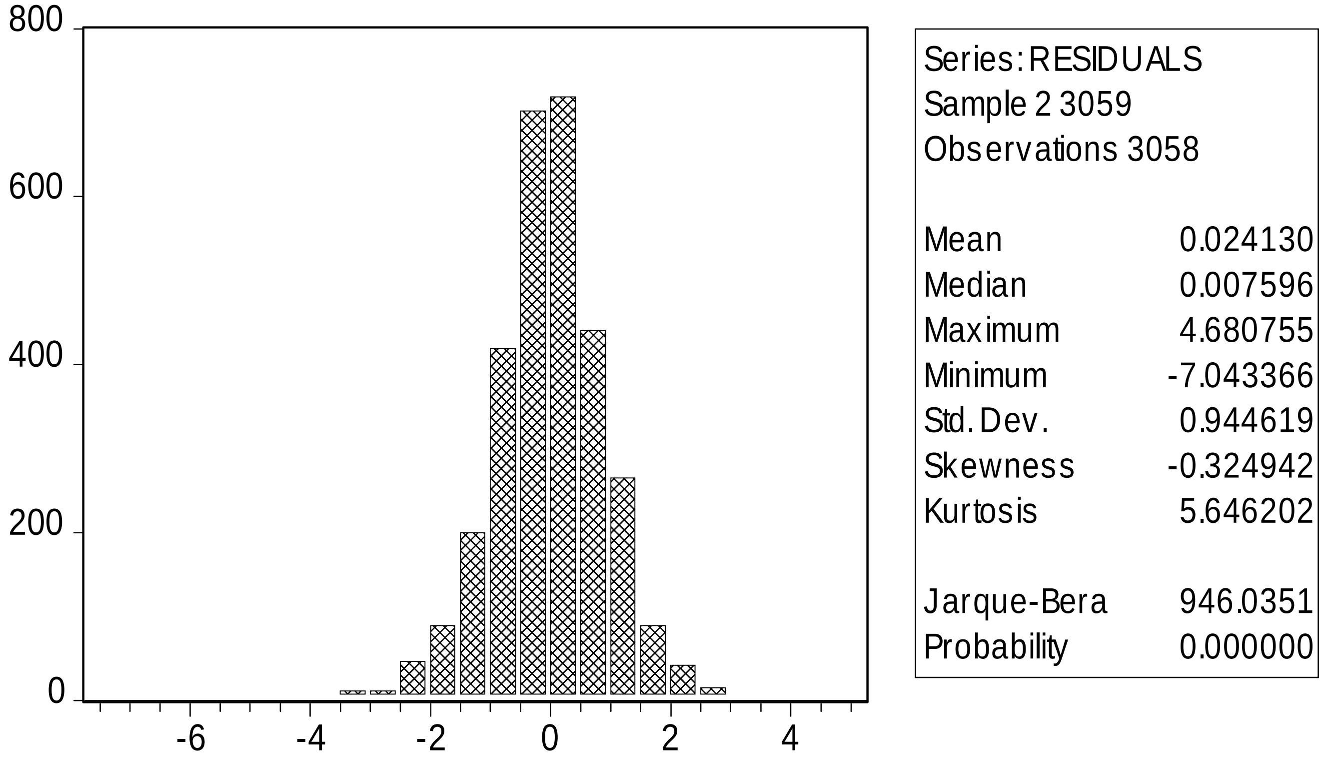

Figure 7: Sample statistics for the residuals series of the MG-GARCH(1,1) process

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