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Spatial Restrictions and Coalition Formation: A Computational Approach

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1 Introduction

The existence of cooperation between diverse agents is a well recognized phenomenon. Cities made up of diverse districts, countries agreeing upon environmental reforms and competing companies working on industry standards are all examples of coalitions. The motive for the formation of most of the coalitions centers around gains in efficiency. Individual agents often gain from working with other agents. Cities, for example, are made up of districts that often have divergent interests yet agreeing on common policy allows for a broader tax base, more resources and, potentially a more efficient government. Pro sports leagues consist of teams that compete violently for entertainment dollars, and then share their profits, in order to perpetuate their own ability to make future profits. Many competing businesses have joined together to agree upon industry standards in order to promote greater profits for all members of the industry, even though, in an environment without industry standards, some businesses would do better. The existence and benefits of these coalitions are clear, but the process of how these coalitions form, why other coalitions do not form, and why some coalitions last while others break up quickly is not at all clear.

Consider the well known game of "Split the Pie". In this game, three agents must split a payoff through a simple majority rule. The traditional result is that two of the three agents will form a coalition and vote that each member of the coalition is to receive half of the payoff. This result makes no claim as to which two will be able to form this coalition, thus the expected payoff to each agent is their Shapley value[1]: 1/3 of the pie. The problem with this result is obvious: without knowing which two agents will form a coalition, this result cannot be applied to any real situations. The result as suffers from instability. If any two agents make an agreement, the third agent has an incentive to make one of them an offer of more than one half, to join with them. Thus no stable solution exists. The dynamics of this game are vital to applying the theory to actual economic problems, yet the process by which agents might contact each other and negotiate an agreement is uniformly ignored. Agents in this game may be constrained in numerous ways including the order in which they are contacted, their ability to communicate with one another as well as their negotiating skill. All of

[1]The Shapley value, in this case, represents the expected payoff to each agent before play begins.

these constraints effect, not only the actual outcome of the game, (ie: which equilibrium is chosen) but the expected value of the game to each player. The Shapley value may no longer be the correct measure of value to the players involved.

The original "Split the Pie" game assumes that all of the players are free to communicate with each other and that there is no particular order in which agents make contact. If, however, it was known that two of the agents would meet first, the Shapley value for those two agents would each be 1/2 and 0 for the third player, a drastically different outcome. The fact that negotiations do follow a process, and not full, instant communication and settlement, suggest that the unstructured or traditional Shapley value may not, in fact be the best measure of the value of the game to a particular player. The other factor to recognize is that of spatial restrictions. Consider the game in which the agents are restricted by a location in space. This space may represent a physical location, political preferences or any other type of restriction that limits communication between agents. If, for example, in the three agent "Split the Pie" game, agents one and three are restricted to communicate only with agent two and two may communicate with both one a three (see figure 1), the results change drastically.

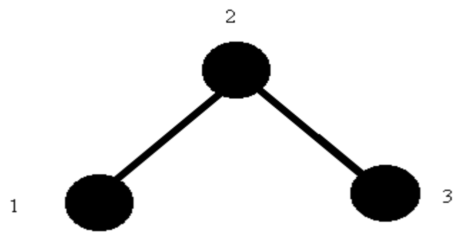


Figure 1

Now the coalition between agents one and two are not possible. Instead of splitting the pie evenly, now agent two can expect to claim most of the pie, giving only a minimal amount to one of the remaining agents. This example shows the importance of understanding the spatial structure of any model, because the end results can be greatly affected by it.

This paper will focus on the formation of coalitions around a public good. This differs from the "Split the Pie" game in that the agents have an incentive to form a grand coalition, not just a sub coalition, because there is an efficiency gain to having a larger coalition. A tax on wealth will be used to finance the public good, thus each member of the coalition must agree upon the tax rate. A traditional approach finds the most efficient tax rate for the grand coalition and Game Theoretic models hope to explain why the efficient solution will or will not prevail. The computational model proposed in this paper will attempt to address the difficult issues surrounding the process by which coalitions actually form. Agents will face spatial restrictions, but will be allowed to choose between coalitions based upon which offers the most beneficial tax rate. Five different structures of four agents will be used in order to examine the effects that spatial structure has on coalition formation and the negotiation process.

There are a many possible applications of this type of model. The model may add insight to the issue of growth, and in some cases shrinking of cities in the United States and countries throughout the world. International agreements, particularly centering around environmental issues is also a natural application. Section 2 discusses previous attempts at explaining coalition formation and the need for a computational approach. Section 3 describes the particulars of the model to be used and defines the experiment that will be performed in order to analyze these restrictions. Section 5 discusses the results of the experiment and section 6 concludes by discussing the potential and uses of this type of approach as well as some of the applications of the model.

2 The Computational Approach

The issues surrounding coalition formation are well known and much studied. The majority of the literature, however, suffers from three problems. The first is the lack of attention to the dynamics of the situation. The process by which agents come together and negotiate is vital to knowing what outcomes to expect as well as a potential means of selecting between multiple equilibria. Second, there are rarely any restrictions placed upon the ability of agents to communicate with one another. We have already seen the drastic affect this may have on the expected outcome of a game. Third, the potential of several small coalitions forming is ruled out by the structure of the proposed games Chander and Tulkens [2]. Thus situations in which two or more coalitions between multiple agents have formed cannot be examined by these tools. There have been many writers in this area that have often used either normal goods, and derived solutions from axioms or public goods and examined efficient and stable solutions, often regarding international environmental agreements.

The cooperative approach to coalition formation recognized the problems and turned to graphs to represent coalition. Each node of a graph represents a player, and an edge between nodes represents a link between players (either bidirectional or unidirectional depending upon the approach). One of the early uses of graphs comes from Aumann and Myerson [1] who defined cooperation structures. These structures were graphs that showed how the order in which links between players formed, greatly affected the potential Shapley value for each agent. The Myseron value that they propose utilizes a sub-game perfect solution to the formation game, that reduces to the Shapley value in the general case. Their approach to the dynamics of the problem is to state that there is some predefined "rule of order" that is known by all players. This result is only applicable however under certain conditions. Two important conditions are that the rule of order is exists and is known by all players and that there are not too many players. "Too many players" is a vague term that refers to the idea that the game can be too big for a sub-game perfect solution to be applied. The game tree may grow too large for any player to be able to inspect the entire tree never mind solve it. There are results on these issues in complexity theory which will not be discussed here.

Other attempts (Qin [7]) have furthered the use of cooperation structures, but fall prey to the same flaws as much of the previous work: only one large coalition is considered. We need only look into history at World War II to see that restricting anyone not in the coalition to be singletons is not a valid assumption. The Axis and Allies were subcoalitions of countries. Most of the previous work would not have been able to recognize a coalition between the United Kingdom and the United States, if it had previously recognized the coalition between Italy and Germany.

The problems of previous literature that have been outline above, can best be attacked with computational tools. Attempting to analyze a three player game with spatial restrictions is not too computationally difficult, as we have seen. The results of the three player model cannot be directly extended to games with four or more players and as the number of players increase, the possible coalitions grow rapidly. Thus a computational approach seems to be the most efficient means to analyze any particular game. This approach adds a great deal more than just efficiency however. The model discussed in this paper is an agent based model, which means that the results are not determined by a any particular solution, but rather by the interaction of autonomous, proactive agents.

Each agent will have the ability to make its own decisions regarding the coalition it would like to join, and how the tax rate should be set. This approach allows us to address the three main concerns that were raised previously with regard to the research in coalition formation. First, by allowing the agents to make their own decisions within a model of the bargaining and dynamic process, we can begin to understand the actual process by which coalitions are formed. Second, the computational complexity of the problem of allowing spatial restrictions is more easily dealt with due to the great advantage computers have over us in doing calculations. And finally, because agent's are free to choose any feasible coalition, agents are free to form subcoalitions.

The issue of dynamics in the problem of coalition formation is not unrecognized. The problem is not whether dynamics are important, but how do we model them. In order to understand the rational by which the dynamics were modeled here, an example will prove useful. Let us consider four young men (we will call them John, Paul, George and Ringo, listed in order of talent) who are interested in forming a rock band. The benefit of forming a band is the same for all members of the band, but not enjoyed by anyone not in the band. Thus the benefit of the band is an excludable public good. The cost of forming a band, however, is shared unequally by all members of the band, based upon their endowed musical talent. When the band gets together to practice, all members must be present, but the beauty of the music is more due to the more talented members than the less talented members. This is equivalent to an equal percentage tax on talent where the tax represents a percentage of a week that is spent rehearsing with the band.

Now, consider the situation in which our potential band members all have a different level of endowed musical ability which we can denote by a_i. The level of music produced, M, is a function of the percentage, t, of a given week the band spends practicing and the total talent level of the entire band, B. Explicitly:

$$M = t \sum_{i=1}^{n} a_i \quad i \in B \tag{1}$$

And the payoff to any member i of the band B can be represented as:

$$u_i = \alpha_i (1 - t) a_i + \beta_i \ln t \sum_{i=1}^{n} a_i \quad i \in B \tag{2}$$

Where α_i and β_i are exogeneous parameters for each potential band member.

Now, there are several possibilities regarding the relationships between John, Paul, George and Ringo. They may all know each other, one of them may know all of them but none of the rest are acquainted or each one may know two of the others. Whatever the relationships are, the formation of the band must follow some sort of process. The concept that all four simultaneously decided that they would like to be part of a rock band, and simultaneously contact each other to discuss how much they should practice, is not at all realistic. A more realistic process would be a scenario in which one of them, Paul for example, conceives of a band and calls John first, then he calls George and George calls Ringo. It is through this chain of communication that offers to join the band at a given practice level are made. Changing the order of communication, (ie: having Ringo call Paul who talks to George who then discusses it with John) may have a very different outcome than the original order. John may have very similar talent level to Paul, thus when Paul calls he listens and they might be able to make an agreement, if however the least talented member of the group, Ringo, calls John then they may not be able to agree upon anything because they are so different, thus a band might not form at all.

3 The Model

The model proposes one process by which coalitions may form. The process proposed begins with the idea that a coalition of any size, must begin with two agents getting together and agreeing to work together. Once two agents are together, a third agent must be contacted, and it is left up to one of the agents currently in the coalition to contact the third agent. When the third agent is contacted, those two agents must agree, then the agent in both of the two agent coalitions may be able to join the two coalitions to create one three member coalition. To better envision how this works, let us return to our band.

We begin by considering the situation in which all potential members know each other. If John calls Paul and the two of the agree to form a band at a particular practice level, then a band has been formed, whether anyone else joins or not. Now John calls George and they agree upon a practice level. Thus two separate bands have been created, with John as their common member. John now has the power to join the two bands to create one three member band with a single practice level. Due to the heterogeneity of the agents, the practice level that the two separate bands agreed upon could be very different, thus the combination of the bands will require some negotiation between John and his band mates to make an agreement. The agreement that these agents may come to could be very different if George was the originating member of both bands, because he has a lower talent level than both John and Paul, thus the practice level between the two separate bands will be higher and George will only join the two bands if the practice level continues to be beneficial to him.

This process can be captured by constructing a tree that includes all of the possible paths a group of agents may take in order to form a feasible coalition. A coalition is feasible if each agent is connected to at least one other agent in the coalition. The simplest way to construct this tree is to begin with the grand coalition of all N potential agents. This grand coalition is the root node of the tree. From this coalition all feasible N-1 coalition nodes, are linked. Then from each N-1 node each feasible N-2 coalition node is linked. This process continues until each path has reach a singleton node, representing a lone agent.

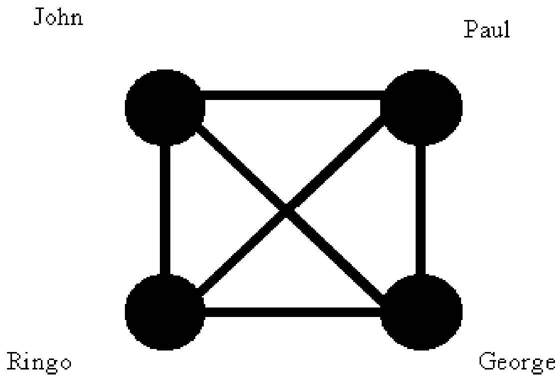


Figure 2

In the case of our band, with the fully connected graph in Figure 2 where each edge represents a bidirectional link between the nodes that each represent an agent, this begins with the root node of four agents, from which each feasible three member coalition is represented by a node that is linked back to the grand coalition. From each three member coalition comes all of the feasible two member coalitions and from them the singletons as in Figure 3.

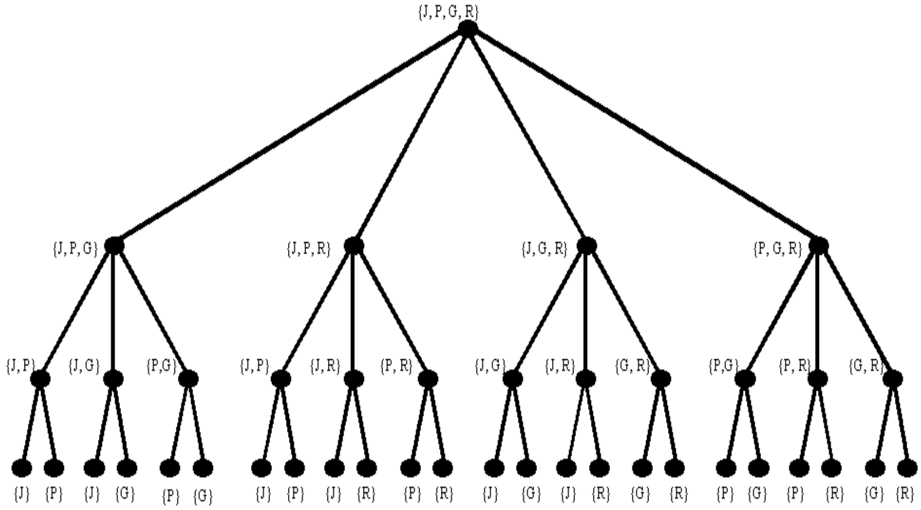


Figure 3

The tree that is pictured in Figure 3 represents the case in which all agents know each other. Some of these paths to the grand coalition are, however, not feasible if that is not the case. If Ringo knows John, Paul and George, but they do not know each other at all, as represented in Figure 4a, then a different tree, Figure 4b, is produced. It is in this fashion that this model allows the investigation of both how process and spatial restrictions affect the outcome of the bargaining process.

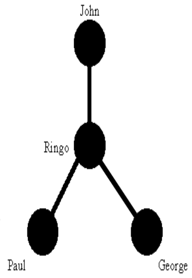


Figure 4a

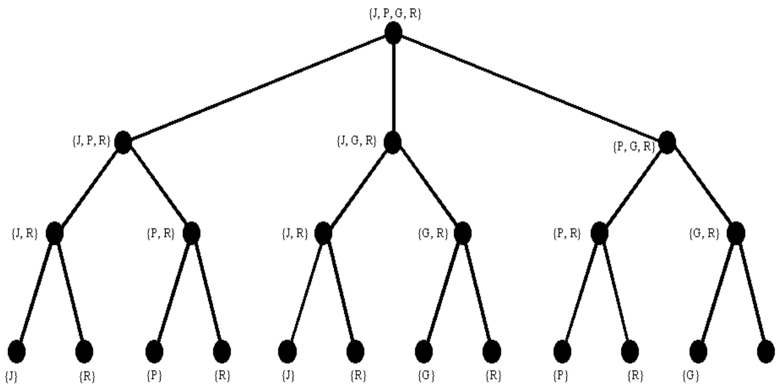


Figure 4b

Any model of the process must of course define how agents make decisions as well as how agreements are made. Agents in this model are placed on a particular path. Once on that path, they may compare all of the nodes that are directly linked to whichever node they are currently located at, and move to whichever they most prefer, based upon the tax rate (or practice rate in our example) that exists at that node.

The tax rates at each node are proposed by an Auctioneer. The Auctioneer at each node would like to attract as many complete coalitions as possible. Thus if the Auctioneer resides at a three member node, and only one agent is there, the Auctioneer will adjust the tax rate in favor of the missing agents. The Auctioneer may adjust the tax rate by adding or subtracting an exogeneously determined increment, based upon the desires of the minority agents. So, at a three member node, if only one agent is there, the other two agents vote on whether they prefer a higher or lower rate. If the two agents agree, then the rate is adjusted accordingly, if they do not agree, then the rate remains unchanged. Agents know whether they would prefer a higher or lower tax rate by evaluating the derivative of their payoff function.

In order to fully explore a given tree, multiple instances of each agent will be released, at randomly chosen starting points. One instance of each agent is referred to as a generation. A time step refers to a round in which every agent currently in the tree makes a decision on whether to move or not, then each Auctioneer adjusts the tax rate. One generation will be released into the tree at each time step, until all generations have been released. A given number of steps will follow at which point the agents will be stopped from further movement.

Once the model is stopped, the location of the instances reveals which coalitions are most likely to form. If all instances of each agent are located at the grand coalition, for example, it is fair to point to the grand coalition as the probable end result of the negotiation process, and whatever tax rate exists at that node can be interpreted as the most likely tax rate to exist in that grand coalition. The model does not restrict instances to forming complete coalitions, at any step . Thus, when the movement of agents is ended, it is possible for an instance to be at a node of a three member coalition, but without complementary instances of other agents, thus they are not really in a coalition. To use our example, this is the situation in which Ringo calls John to form a band and John is not interested, even if Ringo gives him the choice to set the practice rate. That particular coalition could then be ruled out as one that would not form, just as a node that attracts no instances could be discarded.

4 The Experiment

In order to properly use the model, a detailed experimental process must be defined. The goal of the experiment is to explore how imposing a process and spatial restrictions affect the bargaining process, within the context of the public goods problem that has previously been posed. In order to do this, various parameters must be specified, as well as the process by which the experiment will be conducted. First, however, the choice of utility function should be discussed. The quasilinear utility function used in this experiment was chosen for its simplicity. The problem is well defined, and there is one global maximum point. This set up allows us to examine purely the effects of structure on the problem, without having to concern ourselves with a complicated topology, in which there are several local maximum points. The payoff function of the agents could scarcely be more basic, thus we can be sure that any variation in the outcome that is different from other approaches, may be attributable to the process and the spatial restrictions imposed.

The experiment will use five of possible arrangements of four unlabeled agents, in which a coalition of all four agents is feasible. We will begin the experiment with the fully connected graph and follow with each of the other possibilities. All five graphs and their associated trees are shown in Appendix

A.

Agent	Talent Level
John	2000
Paul	1500
George	1000
Ringo	500

Table 1

Each agent is endowed with a certain talent level in our problem. These are specified, rather arbitrarily for the experiment in Table 1. In order to fully explore the effects of the restrictions, within each tree, the agents will be assigned to nodes in four different ways as detailed in Table 2.

Assignment	Player 1	Player 2	Player 3	Player 4
1	George	Paul	Ringo	John
2	George	Ringo	Paul	John
3	George	John	Paul	Ringo
4	Ringo	George	Paul	John

Table 2

Each assignment of each tree will then be run 5 times, with 1000 instances of each agent and a total of 3000 steps. The initial tax rate in all cases will be 0.0000001 and the increment used by the auctioneer will be 0.0000001. Finally, the parameters α_i and β_i will all be set to 1.

5 The Results

The full results are shown in Appendix B. The first set of results, the Graph that should be compared to all others, is graph 1, the fully connected situation. As expected, the different assignments do not affect the outcomes that can be seen in this situation, and there is very little variation in the outcomes. The grand coalition is always the overwhelmingly most likely coalition to occur, with very few singletons. There is some variation in the tax rate that exists however. This suggests, that while the end result of a grand coalition is inevitable, the tax rate in that coalition depends upon how the agents come together. Thus there are multiple equilibria, even in the fully connected case. If a certain path is known (much like Aumann and Myerson's "rule of order") then that particular path can be analyzed with this model to select the appropriate tax rate. So we can feel confident that in the fully connected case, the full four member band will form. It should be noted that there is a unique efficient tax rate for each coalition (see Table 3) and this rate, for the grand coalition does not occur in our experiment.

Coalition	Efficient Rate
{J, P, G, R}	0.0008
{J, P, G}	0.000667
{J, P, R}	0.00075
{P, R, G}	0.001
{J, P}	0.000571
{J, G}	0.000667
{J, R}	0.0008
{P,G}	0.0008
{P, R}	0.001
{G, R}	0.00133
{J}	0.0005
{P}	0.000667
{G}	0.001
{R}	0.002

Table 3 - Efficient tax rates found by maximizing the additive welfare function for all members of a coalition.

Now we turn to Graph 2 which has one fully connected agent and three agents connected only to the central agent. Three of the four assignments in this case yield very similar results to the fully connected case. In fact these results are even more stable, as the grand coalition of all four band members always selects the same tax rate. This changes however, in assignment 2, in which Ringo, the least talented agent is the central agent. In these cases, the grand coalition may not be stable. Examining the output from this assignment, runs 1 and 5 show a cycling number of grand coalitions. In each one, a number of instances of agent 2 leave the coalition, every other period, only to rejoin it the next period. This is akin Ringo, attempting to conspire with two other agents (in this case John and George) to form their own band without Paul. When Ringo does leave, his coconspirators do not follow, as the prevailing rate at the new band is not as beneficial to them. There are also a number of instances of both John and Paul that choose to remain on their own, as opposed to joining any band with Ringo in it.

Next we look at Graph 3. Graph 3 reliably produces the grand coalition, and three of the four assignments have unique tax rates. Assignment 1 however has several different rates. Thus assignment 1 has no unique solution, but a Grand Coalition is an equilibrium in all assignments.

Graph 4, which has two central agents and two agents connected only through one other agent also provides some insight. In this case, two of the four assignments converge to the grand coalition just as the fully connected case. Assignment 4, which places George and Paul (our mid level agents) in the central positions, also converge uniformly to the grand coalition, however, the tax rate is variable. Thus there are multiple equilibira in this situation, indicating that, in order to fully understand the outcome, the process must be taken into account. Then, turning our attention to assignment 1, in which Paul and Ringo are placed in the connected positions. Here, the grand coalition is likely to form, but not nearly as likely as in the other assignments, and the tax rate that the grand coalition converges to is much lower than the rate that exists at any of the other grand coalitions, for this graph. We can see that John is creating the assignment in this distribution, as he chooses to be alone fairly frequently. This is because the agent who can connect him to the rest of the group is Ringo, the least talented of the bunch. John is reluctant to join Ringo in any band, because of the high tax rate that Ringo will demand. Ringo is in a position of power here, because, if John rejects his offer, he can still join with Paul, while if John refuses Ringo's offer, he must be a singleton.

Graph 5, like Graph 4, yields mostly results consistent with the fully connected graph. This is not the case for assignment 2, however, which leaves Ringo in the most connected position. Once again, the grand coalition is not entirely stable, as Ringo continuously attempts to draw Paul (postition 4) and George (position 1) out of the quartet and into a power trio. John and Paul also prefer to be solo on a few occasions. The prevailing tax rate in the grand coalition is again different from run to run, suggesting again that it is important to know the path the agents are most likely to take to the grand coalition, in order to know which equilibrium to expect. The multiple equilibria are present in assignments 1, 3 and 4 as well, even though the complete and stable grand coalition is formed in each run.

These results reinforce the thesis of this paper: process and restrictions matter. Placing the agents in different locations does change the outcome that can be expected in a negotiation. Each situation may call for a different dynamic, but the dynamic can not be ignored. The results are however, from one specific experiment, with many built in assumptions. The reliability of these results for more general cases can only be determined through rigorous sensitivity analysis. The analysis performed on this model varied wealth levels, starting tax rates and the auctioneer's rule. All of these variations were run repeatedly on the fully connected graph (Graph 1) and on Graph 2. The results of these runs matched the results of the experiment in a qualitative nature. The instability and multiple equilibria of Graph 2 persist and the stable but multiple equilibria of Graph 1 persist as well. This sensitivity analysis adds to the veracity of the thesis that process and restrictions have real affects on the outcome.

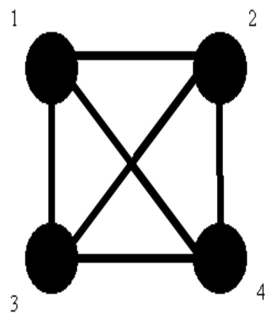
6 Conclusion

The model of this paper is the starting point for two separate veins of research. The first is further theoretical exploration. These results were achieved with a very simple decision process. Further research should, however, begin to consider a more complex approach by letting agents interact with each other, as opposed to an auctioneer. Whether this would be through a learning process, genetic algorithms or other approach is of course dependent upon the particular interests and problem. Learning may not be appropriate in all situations. Also in the theoretical vein, the complexity issues that create the need for this type of analysis should be explored further.

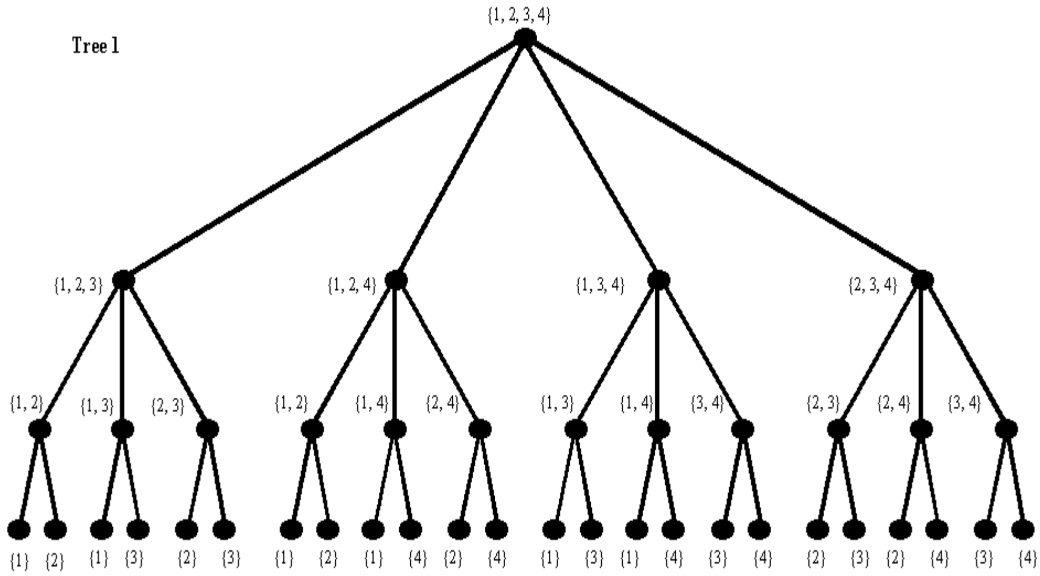
The application of these models to particular problems of interest is also a very furtive area for researchers. This model was built with the intention of applying it to geographically restricted agents, forming, and leaving coalitions whether they be called cities, countries or states, around public goods. It could also be applied rather directly to international agreements about peace, trade or environmental standards. There are of course, several other applications for models that properly asses the coalition formation process including industrial organization settings and and other private goods scenarios.

A Experiment Graphs and Trees

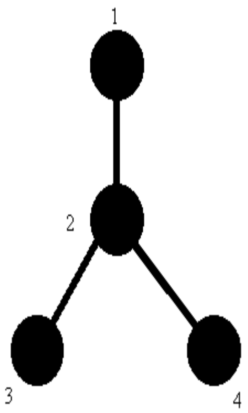
Graph 1



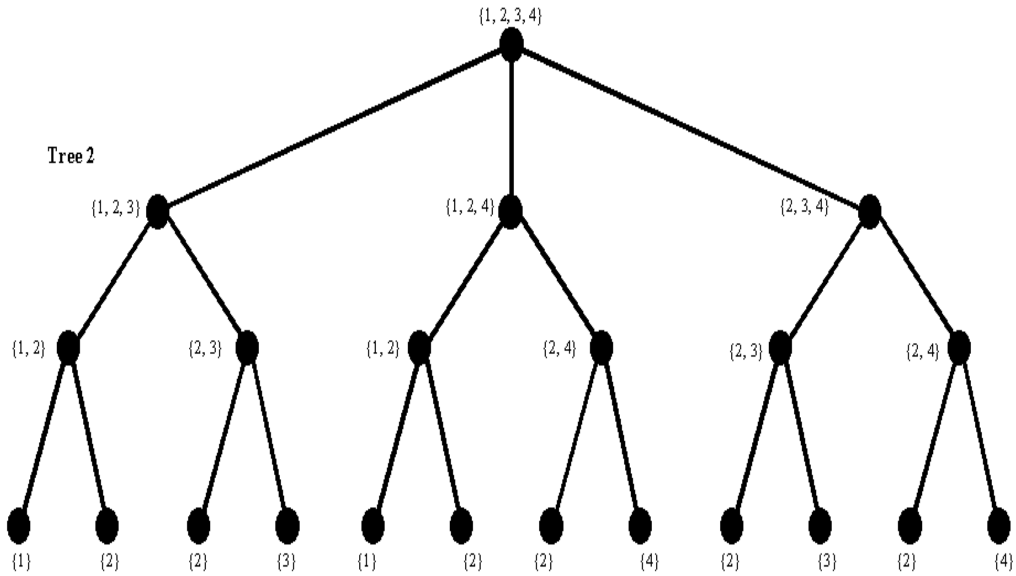
Tree 1



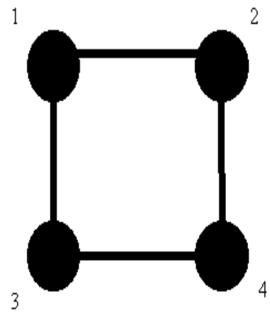
Graph 2



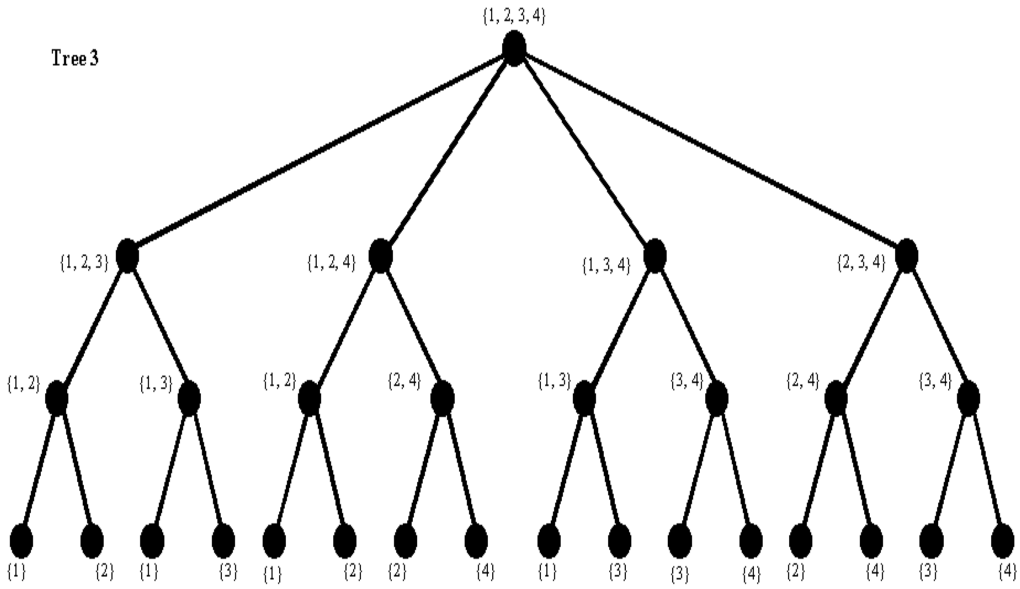
Tree 2



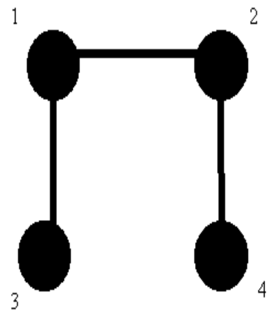
Graph 3



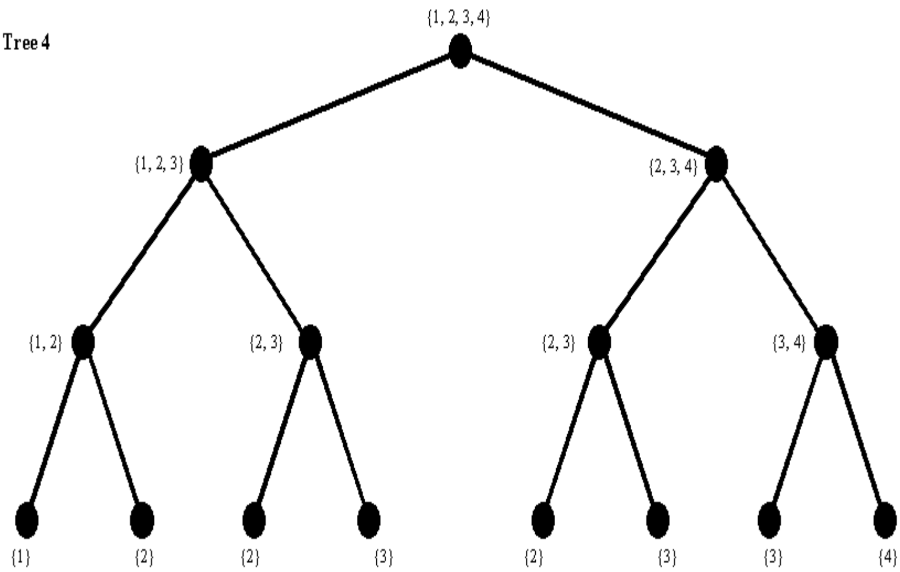
Tree 3



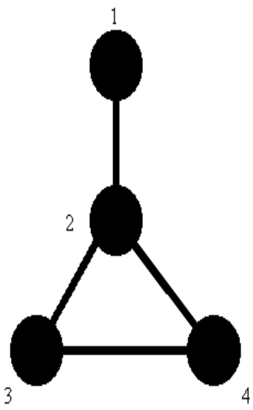
Graph 4



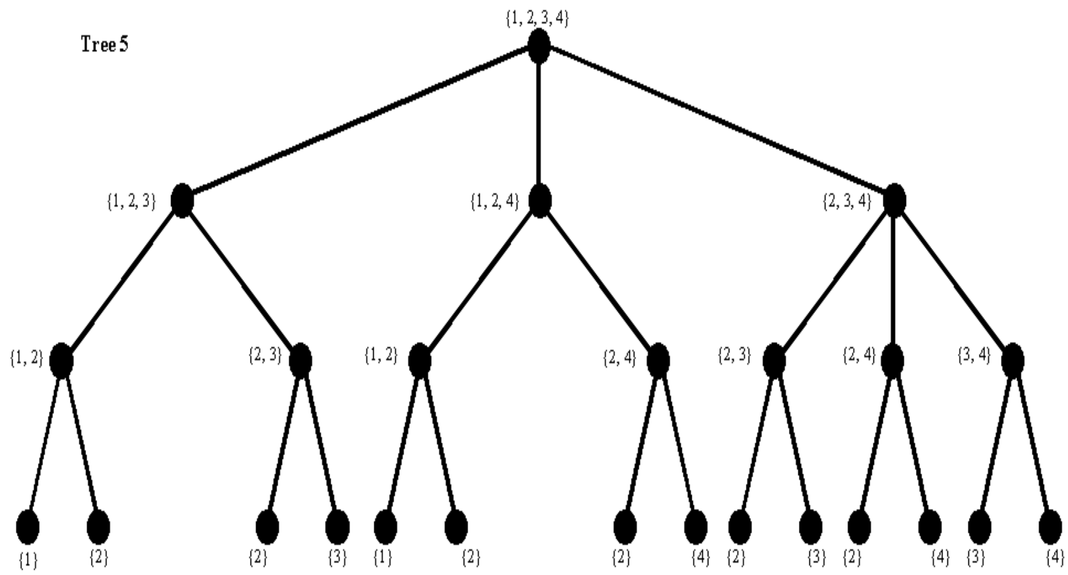
Tree 4



Graph 5



Tree 5



B Experiment Data

The tables of data each represent the output from five runs of a particular distribution within a particular tree/graph. The number of complete coalitions of each possible coalition is listed with the prevailing tax rate in parentheses below. Square brackets list extraneous instances and there type. Cells that are split represent a cycling between the values in the cells.

Coalition / Run	1		2	3	4	5
{1,2,3,4}	998 (0.000566)	996 (0.000658)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0		0	0	0	0
{1,2,4}	0		0	0	0	0
{1,3,4}	0	[4−3] (0.001)	0	0	0	0
{2,3,4}	0		0	0	0	0
{1,2}	0		0	0	0	0
{1,3}	0		0	0	0	0
{1,4}	0		0	0	0	0
{2,3}	0		0	0	0	0
{2,4}	0		0	0	0	0
{3,4}	0		0	0	0	0
{1}	0		0	0	0	0
{2}	0		0	0	0	0
{3}	0		0	0	0	0
{4}	2 (0.0005)		0	0	0	0

Tree 1

Distribution 1

Coalition / Run	1		2	3	4	5
{1,2,3,4}	998 (0.000658)	999 (0.000567)	1000 (0.000667)	1000 (0.000667)	998 (0.000667)	1000 (0.000667)
{1,2,3}	0		0	0	0	0
{1,2,4}	[2 2] (0.001)	0	0	0	0	0
{1,3,4}	0		0	0	0	0
{2,3,4}	0		0	0	0	0
{1,2}	0		0	0	0	0
{1,3}	0		0	0	0	0
{1,4}	0		0	0	0	0
{2,3}	0		0	0	0	0
{2,4}	0		0	0	0	0
{3,4}	0		0	0	0	0
{1}	0		0	0	0	0
{2}	0		0	0	0	0
{3}	0		0	0	2 (0.000667)	0
{4}	1 (0.0005)		0	0	0	0

Tree 1

Coalition / Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000667)	995 (0.000667)	998 (0.000662)/(0.000562)	996 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	2 (0.001)	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	2 (0.0005)	0
{3}	0	0	5 (0.000667)	0	4 (0.000667)
{4}	0	0	0	0	0

Tree 1

Distribution 3

Coalition / Run	1	2		3	4	5
{1,2,3,4}	999 (0.000667)	999 (0.000566)	998 (0.000658)	999 (0.000667)	999 (0.000567)	999 (0.000667)
{1,2,3}	0	0		0	0	0
{1,2,4}	0	0	[2 1] (0.001)	0	0	0
{1,3,4}	0	0		0	0	0
{2,3,4}	0	0		0	0	0
{1,2}	0	0		0	0	0
{1,3}	0	0		0	0	0
{1,4}	0	0		0	0	0
{2,3}	0	0		0	0	0
{2,4}	0	0		0	0	0
{3,4}	0	0		0	0	0
{1}	0	0		0	0	0
{2}	0	0		0	0	0
{3}	1 (0.000667)	0		1 (0.000667)	1 (0.000667)	1 (0.000667)
{4}	0	1 (0.0005)		0	1 (0.0005)	0

Tree 1

Distribution 4

Coalition	Run 1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree2

Distribution 1

<table>
<tr><td>Coalition</td><td>Run</td><td colspan="2">1</td><td colspan="2">2</td><td>3</td><td>4</td><td colspan="2">5</td></tr>
<tr><td colspan="2">{1,2,3,4}</td><td>925
(0.000608)</td><td>850
(0.000602)</td><td colspan="2">948
(0.0005)</td><td>989
(0.000667)</td><td>922
(0.000667)</td><td>875
(0.000555)</td><td>819
(0.000644)</td></tr>
<tr><td colspan="2">{1,2,3}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{1,2,4}</td><td>0</td><td>[150- 2]
(0.001)</td><td>[51 – 2]
(0.000851)</td><td>[52 – 2]
(0.000851)</td><td>0</td><td>0</td><td>[69 – 2]
(0.001)</td><td>[181 – 2]
(0.001)</td></tr>
<tr><td colspan="2">{1,3,4}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{2,3,4}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{1,2}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{1,3}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{1,4}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{2,3}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{2,4}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{3,4}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{1}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{2}</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td><td>0</td><td colspan="2">0</td></tr>
<tr><td colspan="2">{3}</td><td colspan="2">55
(0.000667)</td><td colspan="2">1
(0.000667)</td><td>11
(0.000667)</td><td>78
(0.000667)</td><td colspan="2">79
(0.000667)</td></tr>
<tr><td colspan="2">{4}</td><td colspan="2">75
(0.0005)</td><td colspan="2">52
(0.0005)</td><td>4
(0.0005)</td><td>53
(0.0005)</td><td colspan="2">125
(0.0005)</td></tr>
</table>

Tree2

Distribution 2

Coalition Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 2

Distribution 3

Coalition	Run	1	2	3	4	5
{1,2,3,4}		1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}		0	0	0	0	0
{1,2,4}		0	0	0	0	0
{1,3,4}		0	0	0	0	0
{2,3,4}		0	0	0	0	0
{1,2}		0	0	0	0	0
{1,3}		0	0	0	0	0
{1,4}		0	0	0	0	0
{2,3}		0	0	0	0	0
{2,4}		0	0	0	0	0
{3,4}		0	0	0	0	0
{1}		0	0	0	0	0
{2}		0	0	0	0	0
{3}		0	0	0	0	0
{4}		0	0	0	0	0

Tree 2

Distribution 4

Coalition Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000691)	1000 (0.000756)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 3

Distribution 1

Coalition Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	996 (0.000658)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	[4−2] (0.001)	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	2 (0.0005)	0	0	0

Tree 3

Distribution 2

Coalition / Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 3

Distribution 3

Coalition / Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	999 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	1 (0.000667)	0	0	0
{4}	0	0	0	0	0

Tree 3

Distribution 4

Coalition / Run	1	2	3	4	5
{1,2,3,4}	791 (0.0005)	827 (0.0005)	742 (0.0005)	744 (0.0005)	872 (0.0005)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	1 (0.000667)	0
{3}	0	0	0	0	0
{4}	209 (0.0005)	173 (0.0005)	258 (0.0005)	256 (0.0005)	128 (0.0005)

Tree 4

Distribution 1

Coalition / Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 4

Distribution 2

Coalition	Run	1	2	3	4	5
{1,2,3,4}		1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}		0	0	0	0	0
{1,2,4}		0	0	0	0	0
{1,3,4}		0	0	0	0	0
{2,3,4}		0	0	0	0	0
{1,2}		0	0	0	0	0
{1,3}		0	0	0	0	0
{1,4}		0	0	0	0	0
{2,3}		0	0	0	0	0
{2,4}		0	0	0	0	0
{3,4}		0	0	0	0	0
{1}		0	0	0	0	0
{2}		0	0	0	0	0
{3}		0	0	0	0	0
{4}		0	0	0	0	0

Tree 4

Distribution 3

Coalition Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000708)	1000 (0.000667)	1000 (0.000667)	1000 (0.0007)	1000 (0.0007)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 4

Distribution 4

Coalition / Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.00057)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 5

Distribution 1

Coalition / Run	1		2		3		4	5	
{1,2,3,4}	994 (0.000583)	994 (0.000641)	992 (0.000635)	996 (0.000589)	984 (0.000615)	992 (0.000608)	998 (0.000667)	990 (0.000658)	995 (0.000566)
{1,2,3}	0		0		0		0	0	
{1,2,4}	[12 – 2] (0.001)	0	[8 – 2] (0.001)	0	[16 – 2] (0.001)	0	0	10 (0.001)	0
{1,3,4}	0		0		0		0	0	
{2,3,4}	0		0		0		0	0	
{1,2}	0		0		0		0	0	
{1,3}	0		0		0		0	0	
{1,4}	0		0		0		0	0	
{2,3}	0		0		0		0	0	
{2,4}	0		0		0		0	0	
{3,4}	0		0		0		0	0	
{1}	0		0		0		0	0	
{2}	0		0		0		0	0	
{3}	0		3 (0.000667)		1 (0.000667)		2 (0.000667)	0	
{4}	6 (0.0005)		4 (0.0005)		8 (0.0005)		1 (0.0005)	5 (0.0005)	

Tree 5

Distribution 2

Coalition Run	1	2	3	4	5
{1,2,3,4}	1000 (0.000667)	1000 (0.00057)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}	0	0	0	0	0
{1,2,4}	0	0	0	0	0
{1,3,4}	0	0	0	0	0
{2,3,4}	0	0	0	0	0
{1,2}	0	0	0	0	0
{1,3}	0	0	0	0	0
{1,4}	0	0	0	0	0
{2,3}	0	0	0	0	0
{2,4}	0	0	0	0	0
{3,4}	0	0	0	0	0
{1}	0	0	0	0	0
{2}	0	0	0	0	0
{3}	0	0	0	0	0
{4}	0	0	0	0	0

Tree 5

Distribution 3

Coalition	Run	1	2	3	4	5
{1,2,3,4}		1000 (0.000667)	1000 (0.00057)	1000 (0.000667)	1000 (0.000667)	1000 (0.000667)
{1,2,3}		0	0	0	0	0
{1,2,4}		0	0	0	0	0
{1,3,4}		0	0	0	0	0
{2,3,4}		0	0	0	0	0
{1,2}		0	0	0	0	0
{1,3}		0	0	0	0	0
{1,4}		0	0	0	0	0
{2,3}		0	0	0	0	0
{2,4}		0	0	0	0	0
{3,4}		0	0	0	0	0
{1}		0	0	0	0	0
{2}		0	0	0	0	0
{3}		0	0	0	0	0
{4}		0	0	0	0	0

Tree 5

Distribution 4

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