

Explaining Exchange Rate Volatility with a Genetic Algorithm *

by

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Abstract

Motivated by empirical evidence, we construct a model where heterogeneous boundedly rational market participants rely on a mix of technical and fundamental trading rules. The rules are applied according to a weighting scheme. Traders evaluate and update their mix of rules by a genetic algorithm. Already for a low probability of fundamental shocks the interaction between the traders results in a complex behavior of the exchange rates. Simulations of the model produce a high volatility, unit roots of the exchange rate, a fuzzy relationship between news and exchange rate movements, cointegration between the exchange rate and its fundamental, fat tails of returns, a declining kurtosis under time aggregation, and evidence of mean reversion and of cluster in both volatility and trading volume.

Keywords

Exchange Rate Theory, Technical and Fundamental Trading Rules,
Genetic Algorithm

JEL Classification

F31, G14

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1 Introduction

Since the development of real time information systems and the decline in transaction costs following the liberalization of the capital markets in the mid 80's both daily foreign exchange turn-over and volatility of exchange rates have increased sharply. More and more the trading volume reflects very short-term transactions indicating a highly speculative component. By contrast, international trade transactions account for merely one percent of the total (Bank for International Settlement 1999). As reported by Taylor and Allen (1992) the market participants rely on rather simple trading strategies to determine their speculative investment positions.

The chartists-fundamentalists approach is a research direction which focuses on explaining such speculative transactions (Frankel and Froot 1986, de Grauwe et al. 1993, Kirman 1993). Of crucial importance in this class of models is the behavior of the so called chartists and fundamentalists because the interaction between the two groups has the potential of generating interesting non-linear dynamics. However, by analyzing only two groups a strong structure in the dynamics remains. To overcome this problem the models have to introduce some stochastics. More recently, some multi-agent models in the spirit of the chartists-fundamentalists models have emerged (see LeBaron 2000 for a survey). Since these models allow for many interacting heterogenous agents the structure in the data declines endogenously. Their significance is based upon the matching of basic stylized facts of the empirical data.

The aim of this paper is to develop a realistic, yet simple exchange rate model to get a deeper understanding of the driving forces underlying the foreign exchange dynamics. Rather than deriving the results from a well defined utility maximization problem, details from the market microstructure and psychological evidence are used to motivate the framework. We construct a model where heterogeneous boundedly rational market participants rely on a mix of technical and fundamental trading rules. The rules are applied according to a weighting scheme. Traders evaluate and update their mix of rules from time to time. To be able to understand the dynamics we concentrate only on a limited number of trading rules. The agents have the choice between three technical and three fundamental trading rules. The selection process is modeled by a genetic algorithm which has proven to be a useful tool for describing learning behavior in a variety of papers (for an overview see e.g. Dawid 1999). Thus, we derive the dynamics endogenously from learning processes on individual level rather than imposing random disturbances.

Our main results are: the interaction of the trading rules generates already in a simple setting a realistic behavior of exchange rates. For instance, the simulated time series resembles in their first moments a stochastic trend (unit roots). Further, simulations of the model produce a high volatility. Although the relationship between news and exchange rate movements is fuzzy in the short run, the exchange rate and its fundamental are found to be cointegrated over longer time periods. The returns of the generated exchange rates show a high kurtosis, indicating fat tails, which declines under time aggregation. Fat tails are also identified by the scaling behavior of the returns which roughly follow a power law. In addition, evidence of mean reversion and of cluster in both volatility and trading volume are found.

The paper is organized as follows. Section 2 presents a genetic algorithm exchange rate model. In section 3, some simulation results are discussed. The last section offers some conclusions and points at some extensions.

2 A Genetic Algorithm Exchange Rate Model

2.1 Description of the Market

In our fundamentalist-chartist-approach every trader has a collection of rules she might follow. At the beginning of each trading period she chooses a mix of these rules to determine her speculative positions. The selection of the rules depends on the past performance of the rules. Besides the agents imitate successful behavior and try to improve furtheron by experimentation and recombination. Every period the market is cleared, i.e. the technical and the fundamental positions add to zero.

Note that such a behavior of the agents is not irrational: Heiner (1983) argues that using simple behavioral rules results from the uncertainty when distinguishing preferred from less preferred options. For example, the complexity inherent in the foreign exchange market leads for every agent to a gap between his competence to make an optimizing decision and the actual difficulty involved with this decision. The wider the gap, the more likely the agents follow a rule governed strategy. Thus, agents cannot do much better than follow some adaptive scheme of behavior.

2.2 Technical Trading Rules

Technical analysis is a trading method that attempts to identify trends and reversals of trends by inferring future price movements from those of the recent past.¹ In the following we shortly introduce three technical trading rules which are all widely used. A deeper discussion of them is found in Murphy (1999). His book is often referred to as a manual of technical analysis. One of the rules is believed to identify trends, another one to spot trend reversals. The third rule is a combination of them. Note that technical analysis is a very common method to determine investment positions. As reported by Taylor and Allen (1992) most foreign exchange dealers place at least some weight on technical analysis.

Simple technical trading rules use only past movements of the exchange rate S as an indicator of market sentiment to identify trends. The most popular technical trading rule for trend analysis is the simple moving average rule. The demand in period t resulting from such a rule might be expressed as

$$d_t^{C,1} = \alpha^{C,1} \{0.8(\text{Log}S_{t-1} - \text{Log}S_{t-2}) + 0.2(\text{Log}S_{t-2} - \text{Log}S_{t-3})\}. \quad (1)$$

Equation (1) captures the typical behavior of chartists. In general, chartists buy (sell) foreign currency if the exchange rate rises (declines). Since the agents pay a stronger attention to the last trend the coefficient for the first extrapolating term is set higher than

¹ For a brief introduction into technical analysis see Neely (1997).

the second one (0.8 versus 0.2). The coefficient $\alpha^{C,1}$ calibrates the relationship of the amount of the demand of this rule to the other rules.

In the words of the chartists the moving average rule is a follower and not a leader since it never anticipates but only reacts to the dynamics. This disadvantage is dissolved by combining a short run and a long run moving average. For instance, the so called double crossover method produces a buy signal when the shorter average crosses above the longer. The demand from such a rule might be formalized as

$$d_t^{C,2} = \alpha^{C,2} \{ (\text{Log}S_{t-1} - \text{Log}S_{t-2}) - 0.5((\text{Log}S_{t-1} - \text{Log}S_{t-2}) + (\text{Log}S_{t-2} - \text{Log}S_{t-3})) \}, \quad (2)$$

where $\alpha^{C,2}$ is the reaction coefficient to adjust the demand. The first bracket contains the fast moving average and the second bracket the slow moving average. By this rule a trading signal is already generated when the actual trend of the exchange rate drops behind the long term trend.

However, when the market is not trending, that is the price fluctuates in a horizontal band, chartists often rely on oscillator techniques. By these rules overbought and oversold conditions of a market are indicated. For instance, a market is said to be overbought when it is near an upper extreme. Thus, the warning signal is given that the price trend is overextended and vulnerable. Momentum rules are the most popular application of oscillator analysis. Trading signals are derived by comparing the velocity (momentum) of price changes. Demand from such rules might be written as

$$d_t^{C,3} = \alpha^{C,3} \{ (\text{Log}S_{t-1} - \text{Log}S_{t-4}) - (\text{Log}S_{t-2} - \text{Log}S_{t-5}) \}. \quad (3)$$

This equation states that the chartists expect a future increase in the exchange rate when the observed change between period t-1 and t-4 relative to the change from period t-2 and t-5 starts increasing.

Note, that by (1), (2) and (3) chartists place a market order today in response to past price changes, i.e. price changes between period t and t-1 are disregarded. Such a lag structure is typical for technical trading rules, because only the past movements of the exchange rates are taken into account (Murphy 1999).

2.3 Fundamental Trading Rules

The fundamental trading rules deliver a buy (sell) signal, if the expected future exchange rate is above (below) the spot rate. Therefore, the crucial question arises how the agents form their expectations. We use empirical evidence as indicated, for instance, in Camerer (1995) or Slovic, Lichtenstein and Fischhoff (1988) to substantiate the fundamental expectation formation processes. In the following, we allow for three different kinds of (fundamental) expectation formation processes.

The first variant of the expectation formation process of the fundamentalists is modeled in a well known regressive manner, i.e. when the exchange rate deviates from its perceived equilibrium value S^F , the fundamentalists expect it to return. Therefore, we assume $E_t[S_{t+1}] = \beta S^F + (1-\beta)S_{t-1}$, where β stands for the expected adjustment speed of the exchange rate towards its fundamental. Since the expectation formation for the trading period t has to be made in advance the last available fundamental value is from t-1. The demand of fundamentalists might be written as follows

$$d_t^{F,1} = \alpha^{F,1} (E_t^{F,1}[S_{t+1}] - S_t) / S_t = \alpha^{F,1} (0.7S_{t-1}^F + 0.3S_{t-1} - S_t) / S_t \quad (4)$$

where the demand depends on the relative distance between the expected rate and the spot rate and the reaction coefficient $\alpha^{F,2}$. We assume β to be 0.7, that is, the agents expect an adjustment of 70 percent of the spot rate towards its fundamental.

The second expectation formation process is a variation of the first. The expectation formation is not only regressive but incorporates also an extrapolative component. We assume $E_t[S_{t+1}] = \beta S_{t-1}^F + (1-\beta)(S_{t-1} + (S_{t-1} - S_{t-2}))$ with $\beta=0.3$. Thus, the agent using this rule expects an adjustment of the exchange rate towards its fundamental of 30 percent, but corrects the speed of adjustment by the last recent exchange rate movement. For example, if the exchange rate converges towards its fundamental the speed of adjustment is expected to be higher. The demand follows as

$$d_t^{F,2} = \alpha^{F,2} (E_t^{F,2}[S_{t+1}] - S_t) / S_t = \alpha^{F,2} (0.3S_{t-1}^F + 0.7(S_{t-1} + (S_{t-1} - S_{t-2})) - S_t) / S_t. \quad (5)$$

Note that in the case of larger exchange rate movements the extrapolating term of the expectation formation may overcompensate the expected adjustment.

If the agents are uncertain about the fundamental exchange rate they allow themselves to be guided by past values of the exchange rate when forming new expectations. These function as “anchors“ in the individual judgement of the future exchange rate. This phenomenon is called anchoring heuristics. In such periods the expectation formation process with respect to the exchange rate is not only regressive but also anchored to the last few observations of the exchange rate. Assuming $E_t[S_{t+1}] = 0.2S_{t-1}^F + 0.4(S_{t-1} + S_{t-2})$, the demand modifies according to

$$d_t^{F,3} = \alpha^{F,3} (E_t^{F,3}[S_{t+1}] - S_t) / S_t = \alpha^{F,3} (0.2S_{t-1}^F + 0.4(S_{t-1} + S_{t-2}) - S_t) / S_t, \quad (6)$$

where the fundamentalists now use the exchange rate in t-1 and t-2 as an orientation for expectation formation.

2.4 News Arrival Process

Implicitly, we assume that the fundamentalists form their expectations of the fundamental exchange rate on the basis of a structural model that is correct on average. The perception of the fundamental value is due to the news arrival process. We assume that the fundamental value follows a geometric Brownian motion, hence its logarithm is given by the arithmetic Brownian motion

$$\text{Log}S_t = \text{Log}S_{t-1} + p\varepsilon_t, \quad (7)$$

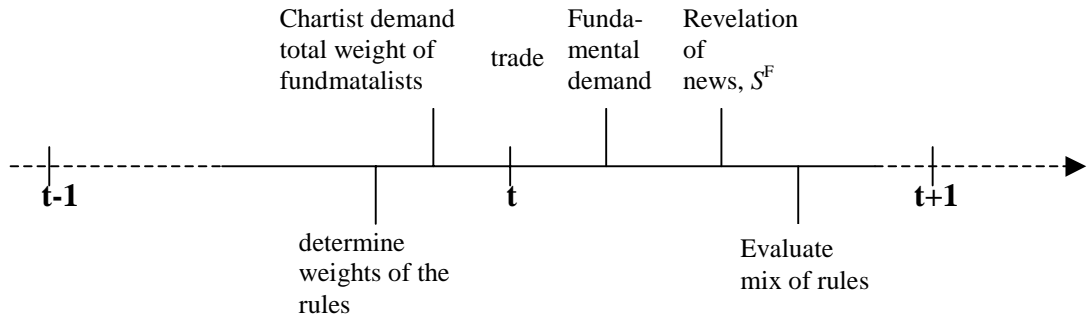
where the news ε_t are identically and independently distributed according to a Normal distribution with mean zero and (time invariant) variance σ_ε^2 . The random variable p is 1 with a probability of 20 percent and 0 otherwise. Thus, a shock hits the market on average every 5 periods.

2.5 Genetic Algorithm Application

The market is established by N=30 traders. Every trader has six rules she can use to determine her demand, three technical and three fundamental rules. In the first period she chooses by chance a mix of these rules and notices the success of her behavior.

Then she observes the market and looks for other rule mixes. She compares some of them and keeps the most promising one. By mistake it can occur that she chooses an inferior one. Furthermore, the trader sometimes tries to create new rule mixes by experimenting and recombining old ones.

Technically speaking: Before the beginning of a trading period the trader fixes the technical and fundamental fraction of her demand and gives a weight to every rule whereby the weights sum up to 1 for every trader. Through this the chartists' demand and the total weight of the fundamentalists is decided on. Via the market clearing the exchange rate S_t equalizes the demand of chartists and fundamentalists. After the revelation of the occurrence of news the success of the mix of the rules is evaluated.



This is formalized by using a GA: A trader i is encoded by a string $\rho_t^{i,j}$ of real numbers of length $l=8$. The first two elements encode the total weight of the fundamental and the technical positions respectively. The elements $\rho_t^{i,3}$ to $\rho_t^{i,8}$ define the relative weight of the three fundamental and the three technical rules. E.g. the weight of the momentum rule C3 is specified by

$$\gamma_t^{C,i,3} = \rho_t^{i,2} \cdot \rho_t^{i,8}, \quad (8)$$

with $\rho_t^{i,1} + \rho_t^{i,2} = 1$ and $\sum_{j=6}^8 \rho_t^{i,j} = 1$. Thus, the momentum rule contributes $\gamma_t^{C,i,3} \cdot d_t^{C,3}$ to the total demand of the trader i .

After the market-clearing exchange rate and the fundamental value of the exchange rate has been revealed, the traders assess their success relative to the other traders (fitness). The operators of the genetic algorithm are applied a follows. The fitness is defined by the profit of the last five periods and the valued actual stock.

$$fitness(t,i) = \sum_{k=1}^4 (-d_{t-k}^i \cdot S_{t-k}) + \sum_{k=0}^4 d_{t-k}^i \cdot S_t \cdot r_t \cdot c_t, \quad (9)$$

where $r_t = \max((1 - 0,5(\text{Log}10(S_t) - \text{Log}10(S_t^F))^2; 0.1)$ and

$c_t = \max(-6250(\text{Log}10(S_t) - \text{Log}10(S_{t-1}))^2 + 0.1; 0)$.

The stock is valued with a reduction r_t of the exchange rate s_t if the actual rate is far away from its fundamental value and with a surcharge c_t if there hasn't been much motion in the last periods.

The selection operator determines whether the actual string is maintained in then next generation. We have tested two selection methods; using roulette wheel selection a rule mix is selected according to its relative fitness. With tournament selection a trader picks two rule mixes by chance and takes the one with the higher fitness. The simulations are based on the latter. The mutation operator varies with probability p_{mut} an element of the string. The amount of change is uniform distributed over the interval $[-0.5;0.5]$. The crossover operator is realized as a single one-point-crossover. The application of the operators of the genetic algorithm can be interpreted as imitation of successful strategies (selection), experimentation (mutation) and communication (crossover).

2.6 Solution of the Model

In this model demand from international trade is neglected since trade transactions, in contrast to speculative transactions, are small in absolute magnitude. Using the market clearing condition

$$\sum_{i=1}^{30} \sum_{j=1}^3 \gamma^{C,i,j} d_t^{C,i,j} + \sum_{i=1}^{30} \sum_{j=1}^3 \gamma^{F,i,j} d_t^{F,i,j} = 0 , \quad (10)$$

the solution of the model is

$$S_t = \frac{\sum_{i=1}^{30} \gamma^{F,i,j} \sum_{j=1}^3 \alpha^{F,j} E_t^{F,j} [S_{t+1}]}{\sum_{i=1}^{30} \gamma^{F,i,j} \sum_{j=1}^3 \alpha^{F,j} - \sum_{i=1}^{30} \sum_{j=1}^3 \gamma^{C,i,j} d_t^{C,i,j}} . \quad (11)$$

Since (11) cannot be solved explicitly, we are going to simulate the dynamics in order to demonstrate that the underlying structure gives rise to complex exchange rate behavior as it is typically observed empirically.²

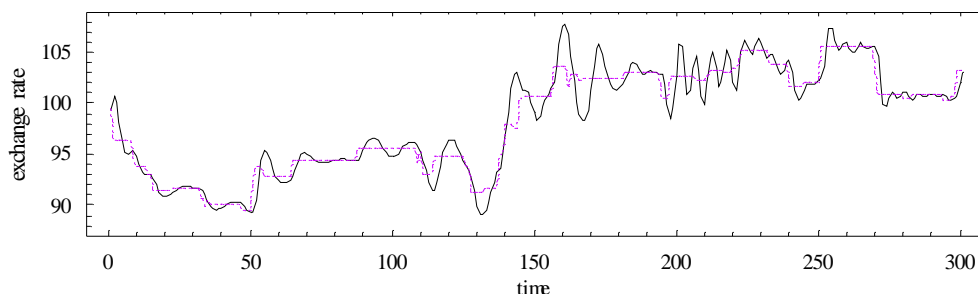
3 Simulation Results

Figure 1 contains the simulated dynamics for the exchange rate (solid line) and the stochastic development of its fundamental (dashed line) for 300 periods starting from period 100. Already a low probability of fundamental shocks suffices to generate more or less realistic exchange rate movements, where the exchange rate fluctuates around the perceived fundamental exchange rate. For some time the exchange rate stays close to its fundamental, however, this may change abruptly. Periods of lower volatility

² Note that if a low proportion of fundamentalists is confronted with a huge demand of chartists a very large price reaction is needed in order to match the demand. But this happens also from time to time in real financial markets. If the price reaction exceeds a certain treshold, the trading is then typically interrupted for a while to calm down the market. However, the parameter values for our simulations are choosen in such a way that the stability of the system maintaines.

alternate with periods of higher volatility. Further, the volatility of the exchange rate is far greater than the volatility of its fundamental.

Figure 1: Simulated Exchange Rate Behavior for 300 Periods



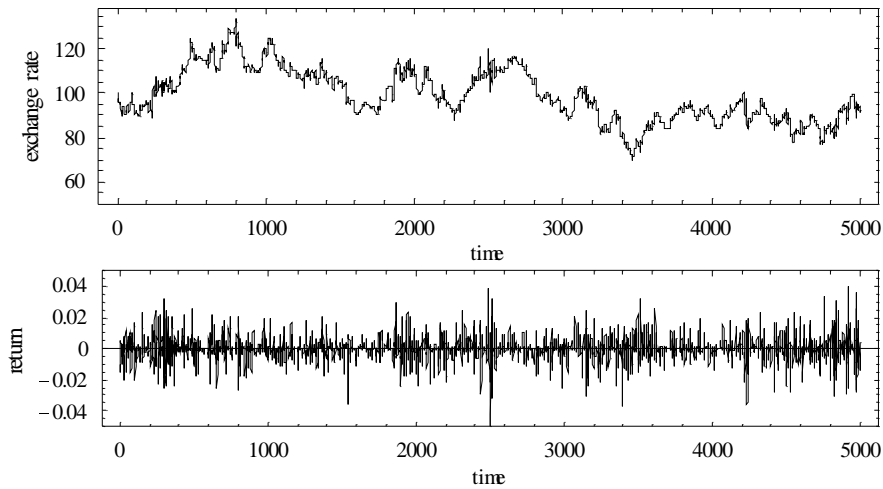
Notes: The solid line is the exchange rate, the dashed line represents the fundamental exchange rate, $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, $T=300$

Simplified, the dynamics might be explained as follows. Technical trading rules always produce some kind of buy or sell signal. On the basis of a feedback process a self-reinforcing run might emerge. But such a run cannot last because investment rules based on fundamentals work like a center of gravity. The more the exchange rate departs from the fundamental exchange rate, the stronger the influence of the fundamentalists, until eventually their increasing net position triggers a mean reversion. But this already indicates a new signal for the chartists and directly leads to the next momentum. The exchange rate is overshooting the fundamental exchange rate because chartism dominates the market in this phase. Heavy outliers can occur when the chartists have a clear trading signal and the influence of the fundamentalists is low.

Thus, we argue that the volatility of the foreign exchange market need not be caused solely by exogenous shocks; it might be explained at least partially by an endogenous nonlinear law of motion. The trading signals needed to keep the process going are generated by the agents themselves. This is exactly what Black (1986) has called noise trading. Black concludes that noise in the sense of a large number of small events is essential to the existence of liquid markets. The argument is that a person who wants to trade needs another person with opposite beliefs. To explain the high trading volume in the foreign exchange market it is not reasonable to assume that differences in beliefs are merely the result of different information. In our model noise (i.e. trading signals) is permanently produced by the agents themselves.

Figure 2 shows the exchange rate dynamics and the corresponding development of the returns (logarithm of price changes) for the first 5000 periods. A well-known stylized fact of the empirical literature is that exchange rate time series have a unit root (Goodhart et al. 1993). For various lag settings and values of the coefficients the null hypothesis that any shock to the exchange rate is permanent couldn't be rejected. Therefore, the time series resembles in its first moments a stochastic trend (Brownian motion).

Figure 2: Exchange Rate and Return Dynamics for 5000 Periods

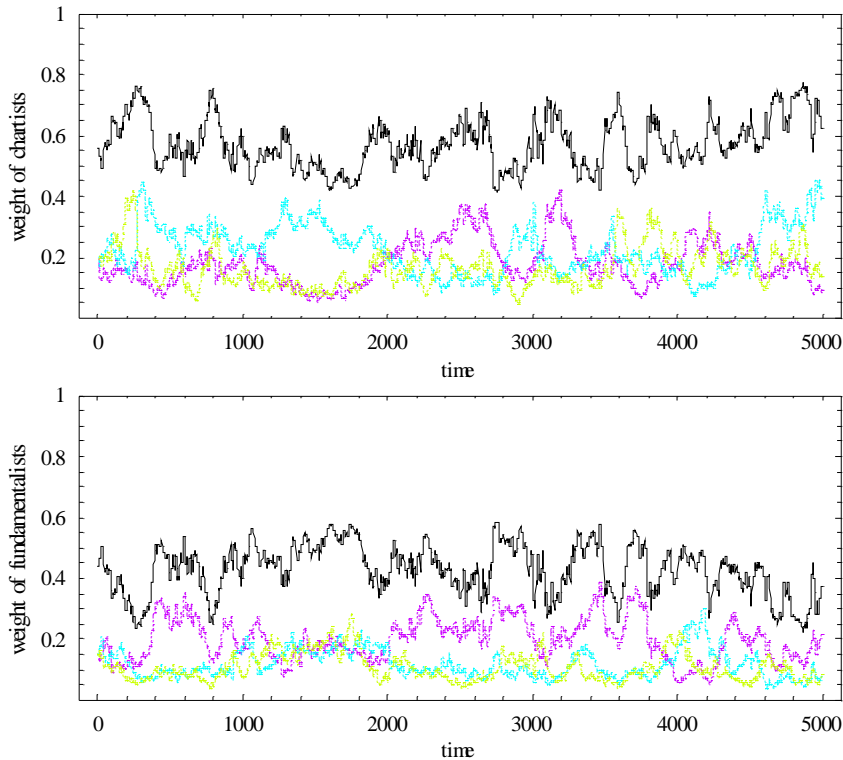


Notes: The top shows the exchange rate, the bottom the returns (logarithm of price changes), $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, $T=300$

Figure 3 displays in the top the aggregated weight of chartists and the development of the strength of the three technical trading rules for the first 5000 periods. The weight of chartists varies seemingly randomly between 40 to 80 percent. Overall, the strength of the technical trading rules is well mixed. However, every of the three technical trading rules dominates for some time the others. The bottom part of figure 3 presents the aggregated weight of fundamentalists and the development of the strength of the three fundamental trading rules. The influence of the fundamentalists is concentrated in the range from 20 to 60 percent. Although the fundamental trading rule with regressive expectations is the most used fundamental rule, for some periods also the other two fundamental trading rules become leading. Such a mix of trading strategies is pretty close to what is reported in survey studies (Taylor and Allen 1992).

The development of the power of the trading strategies in the time domain helps understanding the complicated exchange rate dynamics. Simplified, the exchange rate fluctuations, as can be seen in the figures 1 and 2, may be classified into different regimes. For instance, tranquil periods alternate with restless periods or longer swings in the exchange rate are followed by a faster zigzag course. These different regimes are the outcome of the development of the mix of the trading rules. Most importantly, the dynamics become less stable when the weight of chartists increases. Moreover, each of the trading rules has, due to its lag structure, a specific impact on the dynamics. Depending on how the technical and fundamental trading rules are matched, different regimes of the exchange rate fluctuations emerge. Of course, the change between the regimes is fluent. Since the agents choose in this model only from a set of six trading rules, the complexity of the exchange rate is naturally limited. However, an increase in the number of trading rules should intensify the complexity of the dynamics.

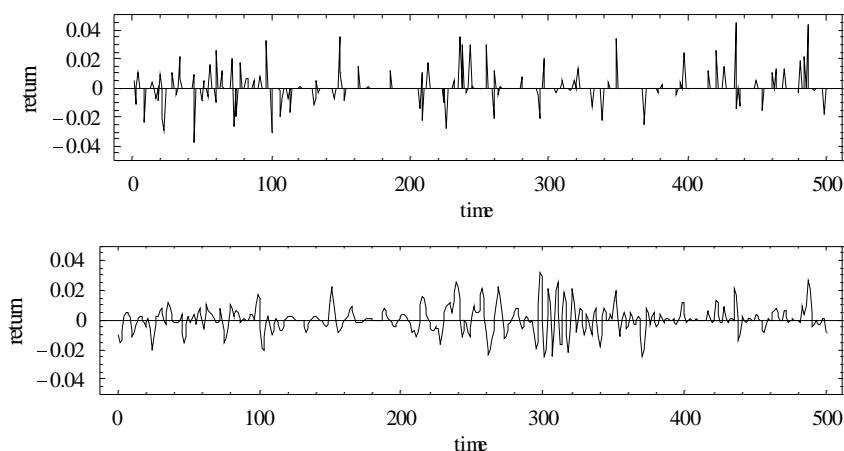
Figure 3: Weights of Chartists and Fundamentalists for 5000 Periods



Notes: The top displays the weight of chartists, the bottom the weights of fundamentalists; the red, the blue and the yellow course indicate the weight of the trading rules (C,1) and (F,1), (C,2) and (F,2), and (C,3) and (F,3), $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, $T=300$

Figure 4 shows in the top the news arrival process and in the bottom the resulting return dynamics of the exchange rate fluctuations for the first 1000 periods. According to Goodhart (1988) the empirical evidence indicates that the relationship between news and exchange rate movements is rather fuzzy. Both systematic underreaction and overreaction to news are reported. For minor news there is often no reaction at all. On the other side, large price movements unrelated to any news are also apparent. Simple visual inspection of figure 4 reveals the same findings in the simulated data. In general, one finds a correlation between the news arrival process and the returns. Especially when a larger shock hits the market the exchange rate behaves correctly. Nevertheless, the news are not incorporated immediately in the price. For minor shocks the reaction can even be wrong. Between the periods 300 and 350 the returns start to fluctuate heavily without any apparent reason. The explanation for these phenomena is rather simple: trading signals generated by technical trading rules can easily inforce or overcompensate the news effect.

Figure 4: The Relationship between News and Returns for 1000 Periods



Notes: The top of the figure shows the news arrival process, the bottom the return dynamics, $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, $T=1000$

This seems to indicate that the foreign exchange market is rather inefficient. A cointegration test allows to check this presumption. Variables are cointegrated if there is a linear combination that is stationary (Engle and Granger 1987). If the exchange rate tracks its fundamental, then the difference between the exchange rate and its fundamental should be stationary. In a broader sense, an equilibrium exists if the difference between the two variables doesn't get too large. We find strong support for cointegration. In empirical studies the PPP is often used as a proxy for the fundamental exchange rate and found to be cointegrated with the exchange rate (Dutton and Strauss 1997).

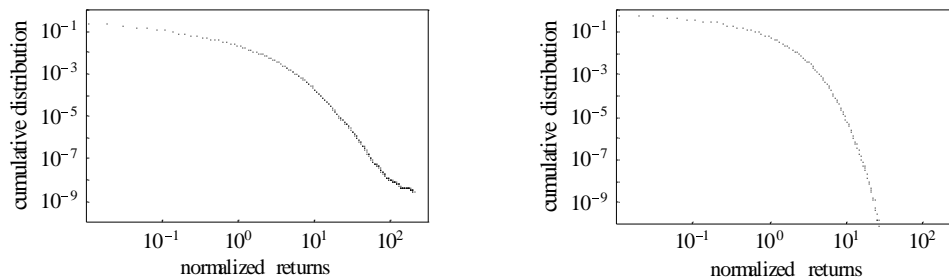
A lot of empirical work is done on describing the distribution of the returns. An important stylized fact says that the distribution of the returns reveals the property of fat tails (Guillaume et al. 1997). In contrast to a Normal distribution one finds a stronger concentration around the mean, more probability mass in the tails of the distribution and thinner shoulders. Estimations of the kurtosis are able to reveal the fat tail property. Furthermore, the empirically observed kurtosis declines under time aggregation. Table 1 displays estimates of the kurtosis under time aggregation for 65528 simulated exchange rates. In comparison, the kurtosis of a Normal distribution is given with 3. Since the random variables are normally distributed, the high kurtosis results from the model. Stronger outliers do not only occur as a consequence of normally distributed shocks. If, for instance, a medium demand by chartists is matched by a low weight of fundamentalists, the price reaction will also be strong.

Table 1: Estimated Kurtosis under Time Aggregation

time aggregation	1	2	5	10	50
kurtosis	13.2	12.1	7.11	4.51	3.51

Notes: $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, T = 65528

An alternative way to identify fat tails is to determine the tail index. Empirical studies indicate that the distribution of large price changes roughly follows a power law (Guillaume et al. 1997, Lux and Marchesi 1999, Farmer 1999). Figure 5 compares this scaling behavior for the simulated data with normal distributed returns (with identical variance). The tail index α , given as $F(|\text{return}|>x) \approx cx^{-\alpha}$, is estimated from the cumulative distribution of the positive and negative tails for normalized log-returns. The returns are normalized by dividing by the standard deviation. A regression on the largest 25 percent of the observations delivers a significant tail index of 3.48 which is in good agreement with results obtained from empirical data. The tail index of a Normal distribution, as can be seen at the slope in the right part of figure 5, is clearly higher and less significant (the r-squared drops from 0.977 to 0.877).

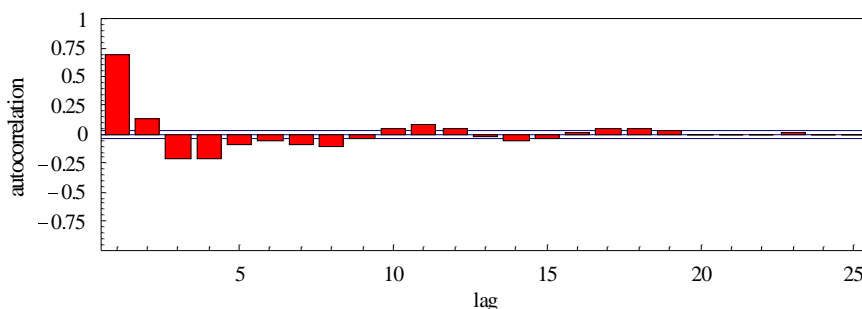
Figure 5: The Scaling Behavior of the Returns

Notes: The left shows the scaling behavior of the cumulative distribution of the positive and negative tails for normalized log-returns, $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, T=65528, the right shows the same as before but for Normal distributed returns with identical variance

Empirical results concerning serial autocorrelation of the returns of the exchange rates are not uniform. Cutler et. al. (1990) found that returns tend to be positively correlated at high frequencies and are weakly negatively correlated over longer horizons, thus exhibiting a mean reversion tendency. For other financial data, the mean reversion tendency is much stronger. Figure 6 displays the autocorrelation function for the first 5000 periods. The time series reveals significant mean reversion, since the autocorrelation lies for some lags clearly outside the ninety-five percent confidence intervals as given by $\pm 2/\sqrt{T}$, with T as the number of observations and the assumption of white noise of the returns. As already mentioned, the complexity of the exchange rate movements is bounded due to the limited number of the trading rules. If one allows for more rules, especially with a deeper lag structure, the autocorrelation declines. Further,

also the international trade has an impact on the dynamics. Especially in calm periods when the exchange rate is near its fundamental, randomly trade transaction break up the autocorrelation structure. In contrast to other financial markets, intervention operations of central banks have also an impact on the dynamics. However, in order to derive a simple exchange rate model we abstained from such extension.

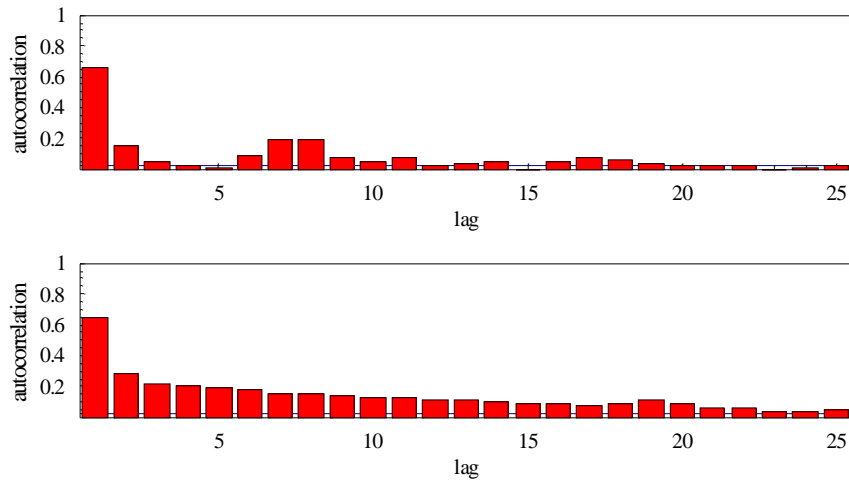
Figure 6: Autocorrelation Function of Returns



Notes: $\alpha^{c, 1}=0.75$, $\alpha^{c, 2}=1.5$, $\alpha^{c, 3}=0.75$, $\alpha^{f, 1}=0.75$, $\alpha^{f, 2}=1.5$, $\alpha^{f, 3}=0.75$, $T=5000$, ninety-five percent confidence intervals are plotted as $\pm 2 / \sqrt{T}$ (assumption of white noise)

Since the work of Mandelbrot (1963), the high short-term autocorrelation of the volatility and its clustering in periods of high and low volatility are well-known. Short-term autocorrelation is the result of market dynamics and not caused by clustered arrival of news. That is, after a new information has hit the market the agents need some time to deal with the shock. Typically, technical traders jump on the bandwagon and reinforce the market movement. Moreover, technical trading rules are also able to generate self-reinforcing trading signals of their own. The long-run clustering may be explained by different degrees of uncertainty triggered, for example, by an oil price shock or a political crisis. Figure 7 shows the autocorrelation function of the trading volume and the autocorrelation function of the absolute returns. Both time series exhibit a strong autocorrelation. The short-term autocorrelation is due to the trend following behavior of the chartists, the long term autocorrelation is a consequence of the variation in the matching of different trading rules which results in periods of lower and higher volatility (compare also figure2).

Figure 7: Autocorrelation Function of Trading Volume and Absolute Returns



Notes: $\alpha^{C,1}=0.75$, $\alpha^{C,2}=1.5$, $\alpha^{C,3}=0.75$, $\alpha^{F,1}=0.75$, $\alpha^{F,2}=1.5$, $\alpha^{F,3}=0.75$, $T=5000$ observations, ninety-five percent confidence intervals are plotted as $\pm 2/\sqrt{T}$ (assumption of white noise)

4 Conclusion

To sum up, this paper presents a model where heterogeneous boundedly rational market participants use a mix of technical and fundamental trading rules to determine their speculative investment positions. The trading rules and the selection of the strategies are not derived from a well-defined utility maximization problem but are based on empirical observations to describe speculators' behavior in a more realistic manner.

In conclusion, the model is able to explain various well-known stylized facts. On the one side, the computed time series looks in its first moments apparently random, as indicated for instance by the unit roots. On the other side, some (deterministic) pattern in the second moments of the time series like volatility cluster or mean reversion are also observable.

Since we focus only on six prominent trading rules, it remains, in contrast to financial market data, a higher structure in the simulated exchange rate time series. However, this can be solved by allowing for more rules or by introducing some random demand components. We abstained from such extension to concentrate on the major driving forces of the foreign exchange dynamics.

The perception of the fundamental exchange rate functions as an anchor for the development of the exchange rate over time. In this paper, the strong assumption that the agents on average are able to figure out the fundamental value is made. Thus, an important extension would be to allow for lasting mistakes in the perception of the fundamental exchange rate.

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