Learning and Adaptive Artificial Agents:

Analysis of an evolutionary economic model

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Abstract

We study a simple overlapping generations economy as an adaptive learning system. The learning is via a so-called genetic algorithm process. We first investigate performances of Holland's standard GA (SGA), Arifovic's augmented GA (AGA), and Birchenhall's selective transfer GA (STGA), Bullard and Duffy (BDGA) as a model of population learning. In addition, we also investigate these learning algorithms variant. Second, compared to population learning, we also implement the GAs as a model of individual learning. An "*ecological*" approach showing "*inter-generation*" aspect of the GA to learning problems is therefore modelled. Finally, We visit a further approach called "*open learning*" model, about endogenising learning in which agents learn how to learn. The results we obtain confirm previous statement that the stability of the Pareto superior equilibrium of the model, i.e. the low inflation equilibrium, is robust independent of precise learning variant. Furthermore, we show that individual agents with heterogeneous learning schemes eventually coordinate on the equilibrium. We offer the interpretation of convergence to the equilibrium.

I. Introduction

The last years have been seen an extraordinary flourishing of works studying learning and adaptive behaviour in diverse fields. Following the fashion of computer innovation, there has been a growing interest in application to economic models of learning procedure developed in evolutionary computation tools such as genetic algorithms. Accordingly then, the use of computer simulation based on the related genetic algorithms (GAs) has largely taken by many researchers, for example, Axelord (1987), Marimon, McGrattan and Sargent (1990), Arifovic (1994, 1995a, 1995b), Arifovic and Eaton (1995), Dawid (1994, 1996a, 1996b), Birchenhall (1994, 1995), Birchenhall et al (1996), Bullard and Duffy (1999), Riechmann (1998, 1999), and Vriend (2000).

Such works may illustrate an uneasy acceptance of the assumption of perfect foresight or rational expectation. Under the assumption, the analysis of the single representative agent in economic modelling may produce an inconsistency with interpretations of results of general equilibrium analysis. The perfect foresight hypothesis means not only that the market as a whole is able to establish an equilibrium for period *t* commodity, but also that simultaneously all agents in the market are able to predict all prices (or interest rates) that will obtain on the market in the future. Hence, agents must have precise models on aggregates of a kind in mind, which permits them to do the required computation. However, while the perfect foresight and rational expectation assumptions have became a standard feature of general equilibrium economic theory, the equilibria that are optimal and determinate will fail in an overlapping generation economy.

In contrast to the study of perfect foresight or rational expectation, the evolutionary economic model takes a viewpoint that heterogeneous agents will learn adaptively from the population experience or individual experience possibly learn to predict correctly. The line of research is to theorise as to how such a learning process might work and whether systems with expectations so defined would actually converge to rational expectation equilibrium. Sacco (1994) argued that an "ecological approach" to modelling of learning problems suggested that the notion of rational expectations is not a useful benchmark for the characterization of rational behaviour. However, many researchers have argued that perfect foresight and rational expectation seem to be reasonable first approximations and can be justified as the eventual outcome of learning process which is usually unspecified.

We study a simple overlapping generation economy as an adaptive learning system. There are two populations co-existing in each period of time. A significant departure to representative agent in economic modelling is a relaxation of hypothesis of perfect foresight or rational expectations. As a result, individual agents in the economy have heterogeneous beliefs concerning realisation of possible outcomes. With the existence of heterogeneity in the economy, the actual outcome may or may not be identical to any particular individual agent's expectation. When the actual outcome feeds back to individual agents' beliefs, individual agents learn to adaptively adjust their own beliefs. The learning is via a so-called genetic algorithm process.

The framework proposed here is identical to the one considered in Bullard and Duffy (1999)'s work. Two prime questions raised are, firstly the explanation of appearance of convergence to the Pareto superior equilibrium, and secondly the robustness of convergence to the equilibrium. In addition, we will look at a so-called "*spiteful behaviour*" in which one player might hurt himself in order to hurt the other player more. The spiteful behaviour may influence the reproduction process in a genetic algorithm learning through its effect on the relative fitness of strategies belief (Vriend, 2000).

We first investigate performances of Holland's standard GA (SGA), Arifovic's augmented GA (AGA), and Birchenhall's selective transfer GA (STGA) as a model of population learning. We also revisit the version of Bullard and Duffy (1999) GA. In addition, we also modify these learning algorithms. The results are compared to the results of their originals. Second, compared to population learning, we also implement the GAs as a model of individual learning. An "*ecological*" approach showing "*inter-generation*" aspect of the GA to learning problems is therefore modelled. Our work suggests that the stability of the Pareto superior equilibrium of the model is robust i.e. independent of the precise algorithm used.

The first part of the study focus on GA-like learning algorithms. Following the context, we visit a further approach called "*open learning*" model, about endogenising learning in which agents learn how to learn. The results we obtain re-confirm previous statement that the stability of the Pareto superior equilibrium of the model is robust. Furthermore, agents with heterogeneous learning schemes eventually learn the rational expectation. However, the approach is tentative, carrying no guarantee of satisfaction at current stage.

II. The overlapping generation economy

We will begin by studying a special case of the overlapping generation economy in which there is a single perishable commodity and a fixed supply of fiat money in each period introduced by a government. There are two co-existing populations in the economy. Each agent in the population only lives for two periods. Time is discrete with integer $t \in (-\infty, \infty)$. There is no growth of population in which the population in each generation is fixed. Therefore, the whole population of agents at any date is $2 \times N$ where N is the number of agents in each generation. To keep thing simple, we will assume that all agents born in generation t are endowed with an amount w_1 of the consumption good in the first period of life, and an amount w_2 of the consumption good in the second period of life, where $w_1 > w_2 > 0$. In the first period of life, agents may choose to simply consume their endowments, or they may choose to save a fraction of their first period endowment in order to increase consumption in the second period of life. Since the commodity is non-storable, agents in this economy can save only by trading a portion of their consumption good for fiat money. This is the only possibility to transfer wealth from young to old. Fiat money is used for the purpose of transfer. Therefore, individual agent born at time *t* solves the following problem:[1]

$$\max_{c_t^i(t), c_t^i(t+1)} U(c_t^i, c_{t+1}^i) = \ln c_t^i(t) + \ln c_t^i(t+1), \qquad (1)$$

subject to an budget constraint:

$$c_t^i(t) + c_t^i(t+1)\beta^i(t) \leq w_1 + w_2\beta^i(t), \qquad (2)$$

where $\beta^i(t)$ denotes agent *i*'s time *t* forecast of the gross inflation factor between dates *t* and *t+1*.

A difference of the overlapping generation economy different from a representative agent economy is that at any point in time there are agents of different ages. When they are in the first period of life, they have to decide how many they are going to consume and savings, according to the endowment w_1 and the forecast of gross inflation factor, $\beta^i(t)$. When they are old, they only can consume an endowment w_2, plus the savings that was made when they were young. In addition, a heterogeneity is captured by the fact that individual agent has a different belief regarding the appropriate value of the unknown parameter $\beta^i(t)$. The heterogeneity relaxes the assumption of perfect foresight or rational expectation.

Hence, agents in the economy have heterogeneous beliefs concerning realization of possible outcomes, which is an inflation factor. Accordingly then, individual agent form expectation with his own belief and forecasts future prices using the simple specification:

$$F^i[P(t+1)] = b^i(t)P(t), \qquad (3)$$

where $b^i(t)$ denotes the parameter that agent *i* of generation *t* uses to forecast next period's price. At the first glance, all agents use the same specification for their forecasts. However, forecast models are actually made differently across agents because individual agents form different expectations. In this study, individual agents' beliefs are encoded and represented by binary strings.[2] It is thought of that the formations of agents' expectations are through a building-block structure.[3] As it will

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[1] The model used here is identical to the version of Bullard and Duffy model (1998). See section XII for more details or see Bullard and Duffy (1998).

[2] To see how we encode an individual agent's belief, please see appendix A.

[3] In terms of Goldberg (1989), highly fit, short-defining-length schemata, building blocks, are propagated generation to generation by giving exponentially increasing samples to the observed best; all this goes in parallel. In our context, the building blocks or schemata just mean agent's forecast

become clear as the genetic algorithm learning proceeds. However, at the moment, we may think the building block of that the beliefs are constantly organized and reorganized themselves to adapt into the changing environment through the contacts of mutual accommodation and mutual rivalry.[4] These local interactions enable the agents to exploit information and, simultaneously, to explore new information. Once the building blocks had been processed, adjusted and refined and thoroughly debugged through experience and competition, the agents generally can adapt and build better expectations and forecasts. In short, each agent updated not a particular variable but an expectation formation they are employing to forecast the inflation factor.

After individual agents form their own expectations, individual agents can make decisions on amounts of consumption and savings, according to their budget constraints. In this model, the possibility of borrowing by agents is ruled out. Thus when forecasts of the inflation factor are equal to or exceed an upper bound, a highest inflation factor that agents would need to forecast in order to achieve a feasible equilibrium in the model, agents simply consume their endowments and save nothing.[5]

In the model, individual agents' realised lifetime utilities depend on two components: the consumption of first period and the consumption of second period which, in turn, in part depends on the realised inflation in the time.[6] Therefore, the more accurate the agent's forecast, the higher is the agent's realised lifetime utility. It will be the agent's interest to approximate the realised value of the unknown parameter *b* as closely as possible.[7] The realised inflation depends on all agents' beliefs. Agent is also aware that actual outcome may or may not identical to his own expectation. When the actual outcome feeds back to individual agents' beliefs, individual agents could gradually learn to update their own beliefs. Therefore, in the economy, each agent is learning how to make a good forecast. When agent is learning to make a forecast, he also has to consider as well as be affected by other agents' learning behaviour in the economy. This means that when beliefs are updated as a result of local interactions between agents, changes are made to all agents. There is the problem of co-ordination between agents. In addition, agent's belief is time varying, agent may or may not change belief over time because a good forecast today does not mean that the forecast will be good tomorrow. Therefore agents' beliefs are environment dependable. Certainly, in the evolutionary modelling, agent is adaptive in which he is learning how to form expectation and then correctly make forecasts in the time and co-ordinate with

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models.

[4] In fact, we have to keep in mind that the beliefs not agents are the evolutionary entities. As it will become clear below, it could be thought as that individual agents are choosing beliefs in each period of time.

[5] Following Bullard and Duffy (1998), the highest inflation factor, λ, equals to w_1 / w_2 .

[6] See section XII.

[7] See section XII.

other agents. The particular adaptive learning process we will use in the study is a so-called genetic algorithm's learning scheme. We discuss the genetic algorithm in the next section.

III. Genetic Algorithm learning

The Genetic Algorithm (GA) is a computational model of evolution, currently the most prominent and widely used model of evolution in artificial-life systems. The GA uses Darwin's basic principles of natural selection and mutation, and a cross breeding to create solutions for problems, in general. Excellent introductions to GAs are available elsewhere including Holland's original (Holland, 1992), Goldberg's class tutorial (Goldberg, 1989) and Michalewicz's contemporary development. Birchenhall's summary (1995) is a good brief of overview of the GA. Here we consider the GA as an economically and socially meaningful model of adaptive learning. We address interpretation of the GA.

III.1 Genetic algorithm as a model of adaptive learning

Technically speaking, the GA is a search algorithm and complementary tool for optimising problems.[8] The GA functioned as a highly parallel mathematical algorithm that transformed a population of individual mathematical entities, each with an associated fitness value, into a new population. The GA operates after the Darwinian principles of natural selection and "*survival of fittest*", and after naturally occurring genetic operations.[9] However, because GA attempts to mimic the way species become adapted to their respective environmental niches, the research based on the related GAs has its implicitly metaphor. Although the role of metaphor in science is ambiguous and ubiquitous, its use not only is important but also provides a solution to reconcile demarcation and gap inherently caused between sciences. Peirce (1958, p.46) wrote:

> *"The higher places in science in the coming years are for those who succeed in adapting the methods of one science to the investigation of another. That is what the greatest progress of passing generation has consisted in. Darwin adapted biology to the methods of Malthus and the economics; Maxwell adapted to the theory of gases the methods of the doctrines of chances, and to electricity the methods of hydrodynamics. Wundt adapts to psychology the method of physiology; Galton adapts to the same study the method of the theory of errors; Morgan adapted to history a method from biology; Cournot adapted to political economy the calculus of variations."*

Importantly, metaphor can help generate response leading to novelty and creativity

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[8] In nature, evolution does not necessary lead to optimum. However, it could be a target.

[9] Spence not Darwin invented the term of "survival of fittest". Spence also popularised the term of "*evolution*". In fact, there is a difference between fitness and survival. See Hamilton (1991) for more details. Following the basis, Metcalf's (1998) definition of the fitness of economic institution can be adopted. The fitness is defined as the "propensity" to accumulate i.e. better-adapted entity leaves increased numbers of offspring.

(Davidson, 1980). Laudan (1977) argued that the source of creativity in science is through the amalgamation of different underlying references, so that already existing but previously separate ideas may fertilize mutually and therefore produce a sum greater than their constitute parts and cumulative process. Certainly, we do not have enough knowledge in either human behaviour or natural phenomenon. When much certain knowledge about real world is lacking, a sort of integrated idea is critical. The field of Artificial Intelligence (AI) provided a sort of glue in integrating the ideas from underlying disciplines, such as biology, engineering, psychology, computer science, economics, *etc*., by comparing them in terms of their power for solving various types of problems. Some modelling techniques have emerged over the last few decades, for example, the symbolic approach (rules, case-based reasoning and fuzzy logic), the connectionist approach (neural nets), the inductive approach (machine learning) and the evolutionary approach (genetic algorithms and genetic programming).

The evolutionary process of the GA has been adequately used to model the adaptive behaviour of a population of bounded rational agents interacting within an economic system. The role of metaphor in the interpretation is that the learning process of human incorporates imitation, communication and innovation effects analogy to reproduction, combination, and mutation in biological evolution. However, there is an argument of the interaction between the natural processes of evolution and learning (Belew, 1990). Although some biologists discredited the suggestion that behaviours acquired through individual experience can be transmitted to future generations, learning alters the shape of the search space in which evolution operates and thereby provides good evolutionary paths towards sets of co-adapted alleles (Hinton and Nowlan, 1987). Particularly, in social and economic sphere a variety of institutional and cultural device permit the codification and transmission of acquired experience through time (Hodgson, 1993). Therefore, not only will agents seek to alter their behaviour in order to improve their chances of success, they actively seek to affect selection environments in their favour. Given beliefs are distributed across the economic population, evolution of agents' beliefs in the study can be viewed as a "*process of distributed learning*" (Birchenhall, 1995).[10]

The evolutionary thinking and evolutionary tools, like the GA, undoubtedly affected many scientific theories, including economic theories. In this respect, such a use of metaphor helps understand our behaviour and the world, on the one hand. On the other hand, we have to carefully accommodate them in the hope of that we can manipulate underlying forces.

III.2 Learning with genetic operators

As its name suggests, the GA draws inspiration from the process of natural

[10] Birchenhall (1995) suggested that, given technical knowledge is distributed across the economic population, technological change can be viewed as a process of distributed learning.

selection found in nature. However, the algorithm is not necessarily limited to the study of biological phenomena. While the GA is an evolutionary algorithm, it can be applied to a wide range of phenomena where learning takes place over time. Nowadays, the important role of imitation or rote learning in economic behaviour can be widely accepted. On closer look there is a difference between rote learning and imitation learning. The former requires a trust in the stability of the environment and a phase of "What is good today will be good tomorrow". Maynard Smith's replicator model (1982) presented this kind of learning (Selten, 1991 and Mailath, 1992). The latter is based on a discussion of Charles Darwin's notion of natural selection, which Spencer encapsulated in the phase "*survival of the fittest*". Given a factor of randomness underlying the process of natural selection, imitation is noise, i.e. biased imitation. When imitation is regarded as a process of learning analogy to the evolutionary process in nature, a mapping, from the more successful ideas or beliefs being replicated faster to the higher fitness being propended to accumulate, is built up. An evolutionary model is basically the formalization of such an idea.

As the same observation that was made in relation to the selection operator of the GA where proportionate selection operates on a population is found, the selection operator has been seen as the modelling of an imitation effect within a population. Recent literatures about learning, especially in the evolutionary game theory, have addressed the importance of imitation effect. The selection operator has been a sound interpretation within the mainstream of economic learning theory (Dawid, 1996b).[11]

There comes to a connection the theory of genetic algorithm leaning to evolutionary game theory. The basic argument is in the discussion of property of stability of genetic algorithms.[12] Riechmann (1998) argues that a concept of evolutionary stability will be: "*A population is evolutionarily stable if it is resistant against changes in its composition*."[13] Standard versions of Goldberg GA (1989) will displays an Ljapunov stability of genetic algorithm learning in which in the long run social behaviour will remain within a certain corridor of social behavioural patterns (Riechmann, 1999). However, a modified version of GA with election operator might not show such a property. As it will become clear as we discuss simulation results.

Selection alone cannot make exchanging of concepts in the process of

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[11] There are three important differences between GA and the replicator dynamics. See Dawid (1996b) for more details.

[12] Metcafle (1998) argued that in the sense of socio-economic evolution, the dynamic analysis from the need to identify the uniform states should be discarded. In particular, it does not strong depend on the related notion of equilibrium. When Sargent made a move toward learning agents with the help of artificial intelligence and focused on convergence to equilibrium with his resistance against relinquishing the neoclassical notion of an equilibrium, it is not strictly compatible with the evolutionary principles (Sent, 1998).

[13] While the concept of ESS is based on symmetric games only, games in GA represent a one against the rest where a large collection of possibly heterogeneous agents subject to nonpairwise field effects or the term "playing the field" used by Friedman (1991).

learning. What is appealing intuitively in the GA is that crossover and mutation combine to search potentially pregnant new concepts. Dawid (1996b) gave an excellent interpretation on these. The whole process in the genetic algorithm makes up the building-block structure in which all agents' beliefs are updated. Having those in mind, we actually interpret the evolution of beliefs not agents in the adaptive learning system. In that respect, we could think that a belief, like a meme in terms of Dawkins (1989), is a replicator that need host in whose brain it is imprinted.[14] Being memes, they must be something that can carry information, for example a belief, a norm or a theory, which can be transmitted to others, and copied. When agents have an idea about realised belief in the future, individual agents have not, graven in his brain, an exactly identical copy or correct belief, but heterogeneous beliefs. However, there is an essence of belief, which is present in the head of every individual who is trying to figure out what the realised belief will be.[15] The belief like meme can be divided into components, such that some believe component X but not component Y and then separate beliefs (memes) are caused. For example, in a binary string, a good component could be 1 in the first position and the third position. Therefore, when the binary string has the length 4, there are four combinations for such a kind of component. With respect to the concept of Holland's schemata (1992), we can imagine that there is a population of *N* individuals with the binary genetic length of *L*. As a result, there are $M = \left|2^L\right|$ possible strings (beliefs) and $B = \left|3^L\right|$ possible schema (belief components). Each string is a member of $B = \left|3^L\right|$ defining schema. Each observation on the fitness of a string provides information on $M = \left|2^L\right|$. Sargent (1993, p76) also pointed out that the concept of schemata is "*equivalence classes of strings*". Members of the equivalence classes are instances of the corresponding schemata.

III.3 Learning level

Holland's GA is a model of population learning in that it simultaneously involved a parallel search within a set of population composed of many entities in a

[14] Dawkins (1989) introduced a term, meme (plural memes), a replicator or a unit of imitation in culture and social level. The forces of selection result in memes being propagated by copying and transmission processes analogous to biological processes, which move memes in the meme pool, between people, and conserve them in time. The memes transmission is subject to continuous mutation and blending. The differential "survival value" of memes that results from such selection and transmission processes leads to changes in memes frequencies in the cultural pool in time (Plotkin, 1997). The survival value, according to Dawkins, results from its great psychological appeal. The existence of survival is, if only in the form of a meme with high survival value, or infective power, in the environment provided by human culture. It does not mean value for a gene in a gene pool but value for a meme in a meme pool.

[15] The essence corresponds to a specific principle of inclusion to be a population.

solution landscape. The notion of population or social learning is that there is interaction between entities to produce the effect of differential rates of growth and survival (Darden and Cain, 1989). In the cognitive science, it is clearly understood that the mind obtains much of its power by working in parallel i.e. various parts of the brain simultaneously respond to information and it is the combined results of these parallel processes that govern the final response. A population is a collection to have members assigned to the population on a basis of specific principles of inclusion. What matters here is the entities within the population compete in a common environment, subject to a same selective pressure (Metcalf, 1998). Clearly in the adaptive learning system, agents' beliefs share some attributes in common but they are also different enough for selection to be possible: they are not exactly identical entities. To survive, beliefs, like replicators, need hosts in whose brains they are imprinted and accordingly then the hosts are identified within the population. It is the replicator's best interest to maximize the fitness of its hosts when the relevant set of selection pressures is specified: all agents want to make forecast as close the true value as possible and therefore in return get as high the realized utilities as possible.

When there is learning in an interactive setting, there are two underlying processes, a change in the perception of the underlying environments and a change in these environments themselves. It could generally be the case that the dynamics of learning and the dynamics of the underlying forces as such will interact with each other (Vriend, 2000). This implies that learning is on individual level rather than population level. The basic concept is that the individual learning is on the basis of reflective self-consciousness but population learning bases the experiences of the population. In the study, such a concept of individual learning can be modelled under the ecological approach in which individual agent has in mind a population of competing beliefs, and agent's experience of forecasting inflation acts as a selection mechanism for these beliefs, by assigning an fitness to those beliefs that enhance the forecast performance of the agent.

IV. Learning algorithm variant

Nowadays, there are many variations of the genetic algorithms. However, most of these variations still keep the original principles of Holland's GA. Three main genetic operators, selection, recombination and mutation, constitute such a framework of the genetic algorithm learning as the standard genetic algorithm (SGA), augmented genetic algorithm (AGA), Bullard and Duffy GA (BDGA), and selective transfer genetic algorithm (STGA). Hence, we will discuss the variations of genetic operators used in the study, in turn.

IV.1 Selection operator

First, in a common form of selection the new population $P^{'}$ from the old population P is built up element by element, in the manner of a biased *roulette wheel*.

Each new bit-string is selected at random from among the elements of the old population *P*, where the probability selecting a bit-string is proportional to its fitness. This selection operator is the same as the process underlying the replicator dynamics.

Second is the probability selection. This form of selection the new population $P^{'}$ is in the manner of a random number generator. Each new bit-string is selected at random from among the members of the old population, where the probability selecting a bit-string is by the random number generator built in a programming language. It can be seen that this selection is quite randomly.

Third is the Top 50 selection. Each time the top 50% of the old population *P* are selected. The rest of the new population $P^{'}$ will be produced by a method called *randomize()* which randomly produces a binary string of length *N*.

Fourth is the section selection. The first step is to select the top 50% of the old population. The second step is to select members from the third one-fourth part of the old population and it will produce 17.5% of the new population. The third step is to select members from fourth one-fourth part of the old population and will produce 7.5% of the new population. Then, the rest 25% of the new population will be produced by the method called *randomize()*. It is the hope to harmonize members of the new population.

Fifth is the tournament selection. Each time two bit-strings of the old population are selected at random and their fitness values are compared. Then the new bit-string is selected from the one with the highest fitness.

The SGA, AGA, and STGA use roulette wheel selection. Bullard and Duffy (1999)'s GA used tournament selection. These selection operator variants are used to replace selection operators of SGA, AGA, BDGA, and STGA when we modify these learning algorithms.

IV.2 Crossover operator

Now, we can make a step to crossover. The simplest way how to do this is to choose randomly one crossover point and everything before this point copy from a first parent and then everything after a crossover point copy from the second parent. Crossover probability says how often will be crossover performed. If there is no crossover, offspring is exact copy of parents. If there is a crossover, offspring is made from parts of parents' chromosome. If crossover probability is 100%, then all offspring is made by crossover. If it is 0%, whole new generation is made from exact copies of chromosomes from old population. The current experiments allowed 16 different crossover rates, varying from 0.25 to 1.00 increments of 0.05 (Grefenstette, 1986).

There are other ways to make a crossover; for example, we can choose more crossover points. Birchenhall (1996)'s STGA used two-point crossover, plus a selective transfer factor.[16] This factor is an internal selection mechanism i.e. operates within the population, which filter the potential string parings before mating occurs. As a result, the progeny is offered up for testing by an external selection environment. The will shifts the population learning from a first-order systems analysis to a second-order systems analysis that contains both internal and external selection mechanisms (Windrum, 1998). In particular, the selective transfer is based on “one-way transfer” of strings, not an exchange. Windrum (1998) suggested that such filtering mechanisms in the one hand, take into account the time, resources and capabilities required to develop an concept and, on the other hand, select between alternative ideas, throwing out impracticable or nonsense novel solutions.

Crossover can be rather complicated and very depends on encoding of chromosome. Specific crossover made for a specific problem can improve performance of the genetic algorithm. However, it is essential requirement of any proposed algorithm that it behaves sensibly in situations we understand.

IV.3 Mutation operator

After the crossover is performed, the mutation takes place. This is to prevent falling all solutions in the population into a local optimum of solved problem. Mutation changes randomly the new offspring. For binary encoding we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Mutation probability says how often will be parts of chromosome mutated. If there is no mutation, offspring is taken after crossover (or copy) without any change. If mutation is performed, part of chromosome is changed. If mutation probability is 100%, whole chromosome is changed, if it is 0%, nothing is changed. Mutation is made to prevent falling GA into local extreme, but it should not occur very often, because then GA will in fact change to random search. The current experiments allowed eight values for the mutation rate, increasing exponentially from 0.0 to 1.0 (Grefenstette, 1986).

IV.4 Election operator

The election operator discards the products of crossover and/or mutation if their potential fitness is less than the original or parent strings. With the “*one-to-one*” election, a child string replaces a parent string only if the potential fitness of that child is greater than the fitness of that parent. Another version is the “*best two*” election. Once crossover and/or mutation are completed, the election operator then chooses the best two strings out of the four strings (two newborns and two originals). The election operator is important in identifying the convergence to state or equilibrium. Without election, there are no constraints on the GA process. Arifovic (1994) indicated that GA

[16] The concept of the selective transfer is based on Bandura (1986)’s a social learning theory that includes both internal and external aspects of human behaviour and learning. More details about STGA see Birchenhall (1996).

is never convergence to any equilibrium without election operator.[17] Also, Birchenhall (1995) suggested that the presence of election operator is important if population convergence is to be a feature of models. The augment GA and STGA use the "*one-to-one*" election operator. The "*best two*" election operator is applied in Bullard and Duffy's GA.

X. The Simulation results[18]

Our study focuses on an exhaustive simulation and investigates the performances of these learning algorithms. The entire data set of simulations in this study is available from the author upon request. When our study may fit into a research program of agent-based modelling, the investigation of simulation results is in many ways. First, it can aid intuition and explanation of rationality, for example, rationality and bounded rationality issue in the current study. Second, it shows a stylised "emergent properties" of the system, for example, the Pareto superior state in the current study. Third, we did not prove any theorems here and use simple and explicit natural rules to investigate the emergent properties resulting from interactions between individuals.

In population learning, for each original learning algorithm, we performed each experiment both with and without scaling operator. In addition, when we modified selection operator of these learning algorithms, we only performed each experiment with scaling operator. Therefore, each learning algorithm has 36 experiment designs. In each experiment design, 100 simulations are performed. In total, we have 14,400 simulations. In individual learning, two learning algorithms are performed, STGA and BDGA. Each algorithm has 48 experiment designs. The scaling operator is applied to all simulations. In total, we have 9,600 simulations.[19]

Table X.1 provides a summary of experiment designs. There are three main catalogs of experiment designs. Each represents a particular interest under current studying. The fist one is a base experiment, "*S*" for short. The purpose of experiments is to investigate the effect of change in population size and length of string. The second one is to investigate whether it will be more difficult for a particular algorithm to converge when the two stationary equilibria closer together. In the current model, the increase in government finance moves the two stationary equilibria closer together. We

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[17]Rudolph (1994) used a mean of homogeneous finite Markov chain analysis to prove that a canonical genetic algorithm (CGA) will never converge to an optimum regardless of the initialisation, crossover, operator and objective function. However, the CGA's variants always maintain the best solution in the population.

[18] For structure of population learning, individual learning and open learning, see appendix B C, and D, respectively.

[19] In fact, in population learning, there are 72 experiment designs for each learning algorithms. In total, we have 28,800 simulations. Some of them were not reported here. These simulations were executed on a Laptop computer with Pentium I-150 MHz processor. In total, it took approximately three months to finish the simulations of population learning and one month to finish the simulations of individual learning.

denoted the *"IG"* for such experiments. The final one is to investigate whether increase probability of a particular belief adopted can fool agent's learning to the rational expectation equilibrium. Following the study of Bullard and Duffy (1999), this is by an increase in the maximum inflation forecast from λ to *MAX*. By doing this, there is Ψ of all possible forecasts having a zero savings decision.

$$\Psi = \frac{(2^L - 1) - (\lambda/_{MAX})(2^L - 1)}{2^L - 1}$$

where, $\lambda = \frac{endowment1}{endowment2}$ and *L* is the length of string. We denoted the *"IMF"* for such experiments.

In population learning, the rule to name an experiment design is following. The first position is the name of particular experiment design as described above. The second position is the size of population. The third position is the length of string. In individual learning, the first position is the name of particular experiment design as described above. The second position is the number of agents. The third position is the number of strings for each individual agent. The fourth position is the length of string. These naming rules are applied to open learning as well.

Table X.2 and Table X.3 provide the summary of results for population and individual learning, respectively. The summary is based on 14,400 simulations and 9,00 simulations for population and individual learning, respectively. In order to investigate performances between learning algorithms, some statistics are also calculated. The *"Mean"* is the average iteration of convergence and the *"STDEV"* is the standard deviation of iteration of convergence. In addition, there comes an issue of consistency between how fast is the convergence, and how big is the variance of iteration of convergence. We used a statistic called relative dispersion to measure the consistency. The value is obtained by the equation below.

$$V = \frac{STDEV}{Mean}$$ [20]

The name of any statistic with *"_L"* means that the statistic measures convergence to the low rational expectation belief (LREB) and with *"_C"* measures convergence to any state (CS), including the LREB. Moreover, in order to measure a probability of convergence to the LREB, a successful rate of convergence (SOL) to the LREB is calculated by the frequency of convergence to LREB divided by the total number of simulations. A successful rate of convergence to any state (SOC) is also calculated by a similar procedure. [21] In addition, to measure how accuracy one learning

[20] The smallest value of V is zero when STDEV equals to zero. In this case, we have always a same iteration of convergence in each simulation.

[21] The GA is an evolution-based approach, an approach so called "evolutionary computation"; it is a probabilistic algorithm in which there is a factor of randomness to affect the movement of an

algorithm converges to LREB, a ratio of SOL/SOC is calculated. The value measures how precisely one learning algorithm converges to the LREB. When the value equal to one, it means that once one learning algorithm converges, it converges to the LREB.

X.1 Main finding

The main result shows that, in most of the experiments, the low inflation rational expectation equilibrium (LRE) of the model emerged. In some experiments, other convergence results are emerged. There are some experiments that convergence fail to obtain within our simulation criterion.[22]

From these experiments, the low inflation equilibrium is sustained and the high inflation equilibrium is refuted. This is in contrast to the property of the model under rational expectation assumption, that is, the high inflation rational expectation equilibrium (HRE) is the stable attractor. Our result supports the result of Bullard and Duffy (1999) and is accord with many studies in which the same kind of learning scheme is applied. Arifovic (1995), for example, obtained the low-inflation stationary equilibrium for overlapping generation economies in which agents learn through genetic algorithm. The result is also consistent with the result of other adaptive expectations scheme, for example, Lucas's (1986) past average of prices and Marcet and Sargent (1989)'s least square learning.

X 2.1 Performance of population learning

Consider the probability of convergence. Compared to BDGA, STGA and AGA, the SGA always has the lowest probability of convergence (see SOL column in Table X.2). Even though sometimes it converged, the iteration of convergence is longer than the others (see the Mean_L and Mean_C columns in Table X.2). One explanation is the effect of election operator. In SGA, there is no election operator, which is not true in BDGA, STGA, and AGA. As a result, the election operator has responsibility for the probability of convergence.[23]

With respect to the speed of convergence, the BDGA has the best performance. The mean iteration of convergence to any state and convergence to the low inflation equilibrium are 36.44 and 33.95 both with scaling factor, respectively, compared to 68.13 and 66.81 in STGA, 60.37 and 53.44 in AGA and 519.96 and 559.02 in SGA.[24]

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evolutionary system. With the same parameterisation for a GA, there is no guarantee for our simulations to have the same iteration of convergence to LREB and have always convergence to LREB.

[22] Two main criterions are applied. First, if the system does not converge within 1000 generations, it fails. Second, once convergence, all agents in the economy, including the old and the young population, have the same belief.

[23] One criterion we put is that any simulation has to end within 1000 iterations. Therefore, we did not know whether the probability increases if allowing longer iterations. However, a property can be referred from our current results is that with the same criterion, SGA has always the lowest probability of convergence and longer iteration of convergence than AGA, BDGA, and STGA.

[24] The similar results can also be found in simulations without scaling factor.

From the results, we see the effects of different election operators on convergence speed. The BDGA has the strictest election rule (best-two election) in which the two newborns with the highest fitness are chosen out of the four strings (two newborns and their parent). However, in STGA and AGA, the newborn is chosen if its fitness is more than its original (one-to-one election). The difference between the two election operators is that in the case of the best-two election operator, the two newborns always have the highest fitness; however, in the case of the one-to-one election, the two newborns are not always have the highest fitness. As a result, the election operator has the responsibility for the speed of convergence and different election operators cause different speeds. The explanation is that the "*best-two*" election in BDGA is more likely to destroy variety resulting from crossover and mutation operators and therefore shorten the time of convergence, than the "*one-to-one*" election in STGA and AGA. However, election operator carries out no guarantee of global optimum, i.e. effectiveness of search.

With the election operator so programmed, the fate that the GA has no room for dynamic stability under the property of economic equilibrium might be refuted.[25] As long as the mutation operator is preceded before the election operator, there is always chance for making a GA converge and stay forever, if convergence to an optimum. In other words, if the mutation operator is applied in a normal way, the only chance for making a GA converge is to modify the GA.[26] In addition, the beneficiary exploration in GA will be confined to the election operator. It takes a risk to reduce the "*robust*" of GA and to induce a possibility of inefficient search.[27]

This effect of election operator also can be shown on the procedure of selection. Take a comparison between STGA and BDGA. While the election operator is applied to the procedure of tournament selection in BDGA, it does not apply to the natural selection i.e. roulette wheel in STGA.[28] With the tournament selection, chromosomes being selected and put into reproduction pool are those chromosomes in top 50 of parents. It is not true in roulette wheel selection. Consider chromosomes' fitness taking values between 0 and 1. The values of chromosomes' fitness are randomly located between 0 and 1. The tournament selection has high probability to

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[25] Dawid (1996b) argued that the GA has room for dynamic stability. When there is an enough proportion of population being mutated, the system may shift to another state in which the incumbent belief is no longer dominant. As a result, the landscape of search space is also changed. However, when mutation is always terribly small, the opportunity of that case is almost impossible. In addition, the property of dynamic stability of genetic algorithms is critical. As discussed before, in the long run, genetic algorithms have dynamic stability with the enough small probability of mutation; however, in the very long run, so stability could not be sustained forever.

[26] Even we introduce a time-varying mutation probabilities alone, it would not help at all times. This may confirm the insight that the selection operator is the key problem of the GA. See below discussion.

[27] In fact, according to our investigation from these simulation results, once the adaptive system converged to LREB, with election operator so programmed in BDGA, AGA, and STGA, there is no chance to get out of LREB. Also see discussion below.

[28] The roulette wheel is also applied to AGA.

choose any chromosome with fitness greater than 0.5. After tournament reproduction, the values of chromosomes' fitness are more like to locate between 0.5 and 1 in the mating pool. If the procedure is repeated infinitely, the tournament selection will result in all chromosomes having the value of fitness equal to 1. When environment is static, tournament selection may do a good job in searching good solution. When environment is time varying such a selection procedure may have cost in searching good solution. The selection procedure is a main force to destroy the variety. As the force is much more intensive in tournament selection where the searching put too much attention in exploitation, than in other selections, the benefit of variety in GA will be lost, where searching keeps in exploration.

Another interesting phenomenon we can find is that having the same election operator in AGA and STGA, the speed of convergence in STGA is longer than in AGA (see Mean_L and Mean_C columns in Table X.2).[29] This may be due the procedure of two-point crossover in selective transfer operator. Clearly, the crossover cannot combine certain combinations of features encoded on chromosomes. It is not possible for the one-point crossover to get a string to be matched by a schema with two or more high performance schemata (Michalewicz, 1996). Consider there are two high performance schemata S_1 and S_2, and two strings s_1 and s_2 matched by S_1 and S_2, respectively.

$$S_1 = (00{*}{*}{*}{*}10) \quad s_1 = (00010110)$$
$$S_2 = ({*}{*}{*}11{*}{*}{*}) \quad s_2 = (01011010)$$

With the two-point crossover in selective transfer, we may get an offspring, $s_1^{'}$ matched by a schema, $S_1^{'}$ having combinations of features encoded on its parent.[30]

$$S_1^{'} = (00{*}11{*}10) \quad s_1^{'} = (00011010)$$ [31]

However, there is a disadvantage of destroying building blocks i.e. structure of scheme, for two-point crossover. In the case of one-point crossover, we select a structure to be exchanged among *l - 1* (where *l* is the length of string) structures at random. With a two-point crossover, there are C_2^l different ways of picking the two cross points and C_2^l structures caused. As a result, each structure is less likely to be picked during a particular cross and therefore more mixing and less (original) structure to be preserved i.e. fewer (original) schemata can be preserved. However, the cost may

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[29] The procedure of crossover in AGA is the same as in the SGA. Here we focus on a difference between standard crossover and selective transfer.

[30] There are schemata that two-point crossover cannot combine as well. In addition, the ability to have the combination of features does not mean that the resulting feature will be better. This is quite reasonable inference from a viewpoint of evolution. In particular, in an adaptive system in the current study, the fitness of a particular feature, single or combinations, is environment dependence. However, what is good for the combination is that important messages can be carried out forward to the next generation. Having the chance of combinations of features encoded on chromosomes, it may improve the efficiency of search.

[31] Here, under selective transfer, we assume that there is an insider transfer with two cutting point, position third and 6^{th}. See Birchenhall (1996) for more details.

be compensated by inherent properties of one-way transfer and second-order systems analysis in selective transfer that contains both external and internal selection mechanism where mutation and selective transfer are made subject to an internal evaluation of their merit prior to inclusion in an agent's belief formation. Only those transfers or mutation that are likely to improve the agent's forecast are undertaken (Windrum, 1998).

In other words, the processes introduce a factor of variety in agents' beliefs and simultaneously reject many new beliefs long before they are given a chance to show their worth in the real world.[32] Hence, the factor of variety in selective transfer has responsibility for the low speed of convergence in STGA. However, one would expect the cost to be compensated by sensible behaviour of STGA in the current program. This can be investigated by the values of SOL and SOL/SOC. Without scaling, the STGA have the highest probability of successfully convergence to the low inflation equilibrium, which is 93%, compared to 88% in BDGA, 62% in AGA, and 5% in SGA. The property holds with respect to their modified versions. The modified STGA with roulette wheel replaced by tournament has the highest accuracy of convergence to LREB, which is 97%. In addition, in the issue of consistency concerning the stability of iteration of convergence, the values of V_L and V_C in STGA are lower than in BDGA. STGA has a more stable iteration of convergence than BDGA.

As the selection operator is a main force to destroy the variety, one would expect the selection operator in a genetic algorithm to be responsible for the convergence. When the modified SGA replaces selection operator with the tournament selection, the mean iteration of convergence is dramatically reduced by 80%.[33] On average, the mean iteration of convergence is reduced by 75%. This is an average over experiment designs in S_60_8, IG_60_8, and IMF_60_8 for AGA and STGA. The results suggested that selection operator significantly affects the mean iteration of convergence.

From these experiments, only three of five selection mechanisms are effective. They are tournament selection, probability selection, and roulette wheel selection. The Top50 selection and Section selection fail to convergence in all experiments. One explanation is that when the variety factor is emphasized, the system is in a very unstable state and becomes very noisy. This is a dilemma between exploitation of and exploration of information in GAs. A different story is investigated for the BDGA. The mean iteration and probability of convergence in original BDGA with scaling are 33.94

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[32] Remember that in the system, agents' beliefs are dynamic i.e. time varying. A new belief rejected today does not mean that the belief will be bad in later periods. In that case, convergence to the particular belief will be prolonged.

[33] In a particular experiment design of S_30_4, the mean iteration of convergence is reduced down to 96%. The mean iteration of convergence to the LRE in original SGA is 633 and that in the modified SGA with a replacement of tournament selection operator is 24.47. The result is not reported here.

and 91%, respectively. However, when selection operator is replaced with the roulette wheel selection in the modified BDGA, the mean iteration and probability of convergence to the LRE are 167.88 and 94%, respectively.[34] Obviously, there is a trade-off between the speed and the probability of convergence. This trade-off can also be found in a comparison between BDGA and STGA. The mean iteration of convergence in STGA is approximately as twice as in BDGA (65 to 36 both without scaling). However, the value of SOL in STGA is higher than in BDGA (93% to 88% both without scaling). Moreover, there is also a trade-off between the speed of convergence and accuracy of convergence to LREB. The accuracy of convergence (SOL/SOC) in STGA without scaling is 96% that is higher than 90% in BDGA. Particularly, when the selection operator of BDGA is replaced by roulette wheel, the accuracy of convergence in the modified BDGA is 94.3% that is very similar to the accuracy of convergence in STGA, which is 94.5%. Among these algorithms, the STGA always has the highest accuracy of convergence to LREB (see column of SOL/SOC in Table X.2).

Both the selection and election forces control the speed of convergence where searching emphasizes in exploitation. What the searching needs is to balance exploitation and exploration. From these experiment results, the modified STGA with roulette wheel replaced by tournament may do a good job in balancing exploitation and exploration. The mean iteration of convergence to LREB is 34 and the probability of and accuracy of convergence to LREB are 91% and 92%, respectively, in the BDGA. However, the mean iteration of convergence to LREB is 25 and the probability of and accuracy of convergence to LREB are 96% and 97%, respectively, in the modified STGA with selection operator replaced by tournament. The speed of convergence and probability and accuracy of convergence improve. This is due to the combined result of the two forces. First, the property of internal selective transfer makes STGA more explorative and second, tournament selection and one-to-one election operators exploit the exploration effectively.

There are other findings and we summarize as followings.[35]

(1) The longer the length of bit-string, the longer the timing of convergence.
(2) Generally speaking, the population size does not affect the speed and the property of convergence too much. From these experiment designs, the population size of 60 is enough, regardless of the length of bit-string. In some experiments, the increase of population size improves the speed and probability of convergence.

[34] For modified BDGA, the similar result can be found with the replacement of probability selection. However, the value of SOL is reduced from 91% to 87%.

[35] These findings are based on results of individual experiment designs. We do not report these results here. However, these results can be requested from authors.

(3) There is no difference in algorithms between with and without the inclusion of scaling factor.[36]

(4) From experiments of IG_30_4 to IG_60_8, we can find that there is no difficult for genetic algorithms to sustain the low inflation equilibrium when the two equilibria are very close to each other.[37] However, the increase in government finance leads to an increase in the iteration of convergence.

(5) From experiments of IMF_30_4 to IMF_60_8, the increase in the maximum forecast does not affect the results of genetic algorithms and its properties. Agents did not be fooled by initial environment. By learning from population experience, agents eventually can learn the rational expectation and coordinate with others in a varying environment. Enlarging the domain of forecasts leads to an increase in the iteration of convergence. Interesting enough, the IMF experiment does not lead to an increase in the mean number of iteration of convergence in AGA but a reduction. In AGA, the values of Mean_L both for experiment IMF_30_8 and IMF_60_4 are smaller than experiments S_30_8 and IMF_60_4.

X 2.2 Individual learning

The model of individual learning can be modelled under the ecological approach in which the learning mechanism is based on the genetic algorithm. Now rather than each agent only with a belief in population level, it is assumed that each agent has in mind a set of different beliefs that compete to be used by the agent as a basis for his forecast. These beliefs are again modelled as the binary string with attached to each belief a fitness measure of its strength or success, i.e. the expected lifetime utility generated by that belief if it was activated.[38] These expected lifetime utilities are evaluated using the most recent actual inflation rate $\beta(t-1)$. The beliefs that had been more successful recently are more likely to be chosen. Hence each period an individual only chooses one of beliefs to make the inflation forecast and put it onto the contest. In return, his actual lifetime utility is evaluated when he is old. Then the genetic algorithm learning is used and to modify the set of beliefs in exactly the same way as it was applied to the set of beliefs presented in the population level above. What a difference here is that individual agents have different sets of beliefs in their minds and therefore the genetic algorithm is applied to these individual sets of beliefs. Instead of learning by looking how well the other agents with different beliefs were doing, an

[36] In the study, the linear scaling is applied. However, there are other scaling methods. We do not intend to claim that scaling is useless.

[37] In these experiments, the government finance is 0.45. The two equilibrium points move from between 1.333 and 3 to between 1.6 and 2.5.

[38] The learning mechanism has an analogy to the learning mechanism for *N*-armed bandit problems (Arthur, 1993). There is a set of arms available to the agent. Agent is figuring out (learning) the “right”

individual agent now evaluates how well he had been doing in the past when he used the set of beliefs himself. In this respect, the adaptive learning system can be described as ecology of sets of competing beliefs or forecasts.[39] When individual agent's information about inflation factor is represented by his set of beliefs, i.e. a population of beliefs, the information is within individual only and differs from individual to individual. In social-economic system, a family can be considered as a unit carrying out such information. In this context, every population of beliefs represents every family's information bundle about forecasting the inflation factor. With an overlapping generations structure, it shows an the inter-generation aspect of the GA where information is transmitted from generation to generation.

X 2.3 Performance of individual learning

In order to illustrate the individual learning and economise on executing time, we only performed the individual learning for STGA and BDGA.[40] Table X.3 shows the summary of the individual learning results. The simulation results based on individual learning model are similar to those based on population learning. The Pareto superior equilibrium is again sustained and the Pareto inferior equilibrium is refuted. An important phenomenon is investigated. Compared to population learning, the emergent property in individual learning is quite homogeneous. All of simulations show that once the adaptive system has converged, the Pareto superior equilibrium emerges. In other words, convergence with the individual learning GA is very neat.

The increase in government finance, experiments IG_60_30_4 to IG_120_60_8, not only leads to an very significant increase in mean number of iterations of convergence as well as standard deviation but also indicates that coordination is made much more difficult when equilibria are closer together. In particular, for STGA, some experiment designs fail to converge or the probability of convergence is very low. Such a phenomenon is more seriously when increasing the maximum forecast above λ i.e. experiment IMF, where more agents will initially choose to save zero. Now, not only STGA but also BDGA, many experiments fail to converge and the probability of convergence is also very low. The result suggests that agents learning through individual experience are more difficult to coordinate with the others than learning through population experience. Without reference to population experience, individual agent may monotonously choose a strategy belief that is good in the past according to his own experience.

There are other findings. We summarized these results below.

(1) The increase in number of agents increases the mean iteration of convergence. It indicated a common sense that there is a difficulty in

arm by trials and errors.

[39] The setting is similar to Arifovic's multiple-population GA used to study a cobweb model.

[40] The executing time for individual learning is approximately five times than population learning.

coordination between agents when the number of agents in the economy is large.

(2) The increase in number of strings, i.e. the actual set of beliefs available to individual agent, reduces the iteration of convergence. However, this has to be harmonized with an increase in number of agents.

(3) The increase in length of string increases iteration of convergence. This fits into an intuition. Increase in the length of string enlarges the whole search space. Therefore, when agents have too many potential information in mind, agents take longer to learn the rational expectation and coordinate with others when the realised belief is time varying.

The probability of convergence for STGA and BDGA in individual learning is smaller than in population learning. The intuition is following. The way we model the individual learning is under the ecological approach in which there are multiple populations in the system. Global coordination is achieved between populations. Therefore, technically, when increasing the number of agents (number of populations) or space of beliefs (number of strings), the landscape for the GA to search is also enlarged at the same time. One would expect an increase in the search time and difficulty of coordination. However, from these simulations, the cost is compensated by the accuracy of convergence to Pareto superior equilibrium. In all of experiment designs, the value of SOL/SOC is always equal to 1, independent of any experiment.

When the individual learning is modelled under the ecological approach in which all agents have the same learning scheme, naturally it is reasonable to think of individual agents has its own learning scheme different from the other agents.[41] In other words, we want to investigate a situation where there a collection of agents with multiple learning schemes instead of only one learning scheme used in population and individual learning. The learning environment is described below.

XI Open learning

As there is no standard learning mode, a common characteristic of most learning models are often ad hoc and very specific in which they might not be derived from an explicit behavioural model or are tailored for a specific context. When we have been applied the genetic algorithm and its variants to belief learning, a speculative simulation is to provide an environment in which there is no any specific genetic algorithm process to be established in advance. In the case of individual learning, individual agent learns according to his own past experience only without communicating with the others. In the absence of reference to population experience, there are difficulties for agents to coordinate with each other and eventually learn the rational expectation. Therefore, here we open a tunnel for these artificial agents to

[41] Remember here a genetic population not a string represents an individual agent.

communicate with each other with respect to their learning experiences. We called it "*open learning*".

Similar to the individual learning, each individual agent also has a set of beliefs in mind and each period only one of these beliefs is activated. Then, from every now each agent must decide what is the whole procedure of his learning, i.e., what is a conjunction of selection, crossover, and mutation. In other words, agent has to choose one selection approach, one crossover approach, and one mutation approaches to construct his own genetic algorithm learning.[42] A binary string represents individual agent's learning scheme. Therefore, there is a population of binary strings representing a collection of individual agents' learning schemes. Once a genetic algorithm learning was constructed, it is applied to update the set of beliefs an individual agent has in mind in exactly the same way as a genetic algorithm was applied to the individual learning. Hence, there are different schemes of genetic algorithm learning operating on different sets of beliefs. Then, each individual agent chooses a binary string representing his belief resulting from his own learning process. The belief that has been more successful recently is more like to be chosen. The procedure is the same as that in individual learning. What difference in the open learning is that individual agents' learning schemes are updated through population experience. Each bit string representing agent's learning scheme is assigned with an average fitness measure of its strength or success. The average fitness is a mean value over the set of beliefs an individual agent has in mind, with attached to each belief a lifetime utility value if it is activated.[43] Each period agent look around how to construct a learning process from the population experience, and choose a belief from the set of beliefs to make the forecast. Hence, agent not only learns how to forecast individually but also learns how to learn from population experience.

XI. 1 Performance of open learning

When the open learning is intended to illustrate a possibility of learning how to learn and investigate the emergent property of the system, the result should be regarded as suggestive. We performed three experiments that are S_60_30_4, IG_60_30_4, and IMF_60_30_4.[44] In the current study, four main types of genetic algorithm learning

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[42] In our program, each individual has four selection approaches that are Section Selection, Probability Selection, Roulette Wheel Selection, and Tournament, and three crossover approaches, which are Standard Crossover, Elitism Crossover, and Elitism Best Crossover, and two mutation approaches, which are Standard Mutation and Elitism Mutation. In total, there are 24 combinations of learning schemes.

[43] As individual learning schemes operate on individual sets of beliefs, it is not adequately to evaluate success of a learning scheme by a fitness generated by a belief, chosen and put onto the contest, from the set of beliefs. It should be an average fitness of the set of beliefs used to evaluate the success of learning scheme.

[44] To perform these experiments, they are more time consumable than population and individual learning. For example, we used five days to perform the experiment S_60_30_4 based on a laptop computer with Intel P-I 150MHz processor. In addition, as in population and individual learning, the open learning uses the same convergence criteria.

schemes investigated, that are Birchenhall's selective transfer GA (stga), Bullard and Duffy's GA (bdga), Arifovic's augumented GA (aga), and standard GA (sga). For each type, three modified versions are also investigated, where the selection operation of its own original version is replaced. Therefore, in total, we catalog sixteen types of genetic algorithm learning. Prefix *"S", "PRO", "RW" and "T"* to a name of learning scheme correspond to that learning scheme using Section selection, Probability selection, Roulette Wheel selection, and Tournament selection. In addition, we used STGA to represent an assemblage of stga, s_stga, pro_stga, and t_stga, BDGA to represent an assemblage of bdga, s_bdga, pro_bdga, and rw_bdga, AGA to represent an assemblage of aga, s_aga, pro_aga, and t_aga, and SGA to represent an assemblage of sga, s_sga, pro_sga, and t_sga. The results are summarized in Table XI.1, XI.2 and XI.3.

First, with respect to the accuracy of convergence, the performance of open learning is the same as the performance of individual learning, where once convergence, the system converges to the LREB (See Table XI.3, SOL/SOC column). In addition, in all simulations, the probability of convergence to low inflation equilibrium is always one that is higher than in individual learning. Second, agents do not have greater difficulty coordinating on the LREB when the LREB is closer to the HREB (experiment IG_60_30_4). In addition, increase the probability that more agents initially choose to save zero, i.e. the changes in the set of possible forecast rules, does not affect our convergence results (experiment IMF_60_30_4).

Third, the mean iteration of convergence to the LREB in the three experiments is quite similar to each other. This suggests that agents with more learning rules will have no greater difficulty coordinating on the LREB independent of any experiment design. In fact, the result of open learning suggests that even though agents have different learning schemes, they can still coordinate their belief on the low rational expectation equilibrium. We call the situation "*coordination of multiple types*". This phenomenon is quite common in nature. Image a tank of aquarium. In the tank, there are several different species to compete with each other. From time to time, some new species may appear, like laver and aquatic. Of course, some species may die out and some still alive. The most important thing in the aquarium tank system is that these species can co-exist with each other and maintain an ecological balance in the tank where each species has its own survival rule. Therefore, in the open learning, individual agents will not stick on a particular learning rule forever instead they change rules from time to time in order to compete with the others. Once coordinated, individual agents have a same strategy belief, but individual agents chose the same strategy belief referring to their own learning schemes. These learning schemes are different from each other.

Forth, from the result of Table XI.1, eight learning schemes are significantly investigated and the mean frequency in each period of each type is also shown. Table XI.2 shows the result of convergence to the LREB. The most two common types are s_stga and s_bdga in the case of all simulations (Table XI.1) and s_bdga and bdga in the case of convergence to the LREB (table XI.2). When the four assemblage types are considered, the most two common types are STGA and BDGA in any case. The results correspond to our previous results of population and individual learning where the majority of simulations converged when STGA or BDGA is applied. In addition, we found that majority of agents choose the Section selection in organizing their own learning schemes. This may suggest that the best thing for individual agents is to keep a variety of strategy beliefs in a more complicated environment. Furthermore, our results also suggested that learning without any filter i.e. the election operation is meaningless and unfavorable. None of frequency of SGA type learning is greater than 1.

XII. Discussion

In applying the computational algorithm to the adaptive learning system, interpretation is both more and less limited. As the methodological role of computer simulations in studying economic models is not well developed, some researchers give little weight to and question the reliability of such work. The major advantage is that we can study models that do not involve the restrictive assumptions that would be required to produce analytical results.

In the model, the dynamic environment comes from the interactions between agents of the economy. Therefore, the landscape that the GA is searching is state dependence i.e. changing from time to time. In a sense, none of strategy beliefs can guarantee to bring agent a highest lifetime utility at all times.[45] However, a question remains as to why agents coordinate on the strategy low rational expectation belief (LREB) and therefore the economy converges to the low rational expectation equilibrium i.e. Pareto superior equilibrium. We discuss this issue below.

In this specification of the model, the life cycle choice of individual agent $i \in [1, N]$ solves the maximisation problem:

$$\max_{c_t^i(t),\, c_t^i(t+1)} U(c_t^i, c_{t+1}^i) = \ln c_t^i(t) + \ln c_t^i(t+1), \qquad (1)$$

such that

$$c_t^i(t) + c_t^i(t+1)\beta^i(t) \le w_1 + w_2\beta^i(t), \qquad (2)$$

where $w_1 > w_2 > 0$. $c_t^i(t+j)$ denotes consumption in period $t + j$ by the agent i born at time t and $\beta^i(t)$ denotes agent *i's* time t forecast of the gross inflation factor between dates t and $t + 1$ according to a simple forecast rule, equation (3) below:

$$F^i[P(t+1)] = \beta^i(t)P(t), \qquad (3)$$

P(t) denotes the time *t* price of the consumption good in terms of fiat money, and

[45] Agent with a low or high belief has forecast model that forecast a low or high inflation factor. Here, we use a strategy low or high belief to represent such a concept.

$F^i[P(t+1)]$ is agent *i's* time *t* forecast of the price of the consumption good at time *t+1*.

Combining the first order conditions with the budget constraint (2), we can find the first period consumption decision for all *N* agents in any generation is given by:

$$c_t^i(t) = \frac{w_2}{2}[\lambda + \beta^i(t)], \qquad (4)$$

where $\lambda = w_1 / w_2$. Therefore, individual agent *i's* saving decision at time *t* is the same and is given by:

$$s_t^i(t) = w_1 - c_t^i(t) = \frac{w_2}{2}[\lambda - \beta^i(t)]. \qquad (5)$$

The expected consumption decision in second period of lifetime for all *N* agents in any generation is given by:

$$c_t^i(t+1) = \frac{w_2}{2\beta^i(t)}[\lambda + \beta^i(t)], \qquad (6)$$

The government prints fiat money at each date *t* in the amount *M(t)* per capita. The government uses this money to purchase a fixed, per capita amount *g* of the consumption good in every period according to equation (6) below:

$$g = \frac{M(t) - M(t-1)}{P(t)}. \qquad (6)$$

Now money supply is no longer constant. As a result we have aggregate money supply in period *t:*

$$M(t) = M(t-1) + gP(t) \qquad (7)$$

It is assumed that these government purchases do not yield agents any additional utility. Since agents can save only by holding fiat money, the money market clearing condition is that aggregate savings equals the aggregate stock of real money balance at every date *t:*

$$S(t) = \sum_{i=1}^{N} s_t^i(t) = N\frac{M(t)}{P(t)} \qquad (8)$$

From equation (1), we have:

$$U(c_t^i, c_{t+1}^i) = \ln c_t^i(t) + \ln_{t+1}^i(t+1) = \ln c_t^i(t) \times c_{t+1}^i(t+1) \qquad (1')$$

Then, substituting equation (5) into (6), we have realised second consumption;

$$c_{t+1}^i(t+1) = \frac{w_2}{2\beta^*}(\lambda + \beta^*) = w_2 + \frac{s_t^i(t)}{\beta^*} \qquad (6')$$

Substituting equation (6') and (4) into (1'), we have:

$$U(c_t^i, c_{t+1}^i) = \ln\frac{w_2^2}{4\beta^*}[(\lambda+\beta^*)^2 - (\beta^i(t) - \beta^*)^2] \qquad (9)$$

β^* is the realised inflation in period *t+1* and $\beta^i(t)$ is agent *i's* forecast value made in period *t*. Therefore, when the smaller the value of $(\beta_t^i(t)-\beta^*)$ i.e. the smaller the forecast error, the larger the agent's utility, other thing being equal. It will be agent's best interest to make his forest as precise (close to β^*) as possible. Selection force will favour those beliefs that produce inflation forecast close to the realised inflation β^*.

In addition, suppose that there are two strategy β' and β''. From equation (9), we have:

$$U(\beta'')\geq U(\beta'), \quad iff \quad \beta^* \geq \frac{\beta'+\beta''}{2} \quad as \quad \begin{matrix}\beta^*>\beta' \\ \beta''>\beta'\end{matrix}, \qquad (10)$$

$$U(\beta'')\geq U(\beta'), \quad iff \quad \beta^* \leq \frac{\beta'+\beta''}{2} \quad as \quad \begin{matrix}\beta^*<\beta' \\ \beta''<\beta'\end{matrix}, \qquad (11)$$

A strategy belief (A) closer to β^* than the other strategy belief (B), higher or lower the strategy beliefs (B), has potential gains subject to (10) and (11).

Take the first derivative of equation (9), we have:

$$\frac{\partial U^*}{\partial \beta^i(t)} = -\frac{w_2^2}{2\beta^*}(\beta^i(t)-\beta^*) \begin{matrix} <0 & if & \beta^i(t)>\beta^*, & decreasing \ in \ \beta^i(t) \\ =0 & if & \beta^i(t)=\beta^*, & constant \\ >0 & if & \beta^i(t)<\beta^*, & increasing \ in \ \beta^i(t) \end{matrix} \qquad (12)$$

and

$$\frac{\partial U^*}{\partial \beta^*} = \frac{w_2}{4}\left(1-\frac{\lambda^2}{\beta^{*2}}\right) \begin{matrix}\geq \\ \equiv 0 \\ \leq\end{matrix} \quad as \quad \lambda \begin{matrix}\leq \\ \equiv \\ \geq\end{matrix} \beta^* \qquad (13)$$

Therefore, agent with strategy low belief (LB) producing low inflation forecast has higher lifetime utility. This also suggested the disadvantage of strategy high belief (HB). In terms of GA, the fitness assigned to strategy LB that forecast a low inflation factor is higher than the fitness assigned to strategy HB that forecast a high inflation factor. The strategy LB has selective advantage.

In the question, our computer program provides two dynamic frame windows.[46] One shows the evolution of relationship between lifetime utility and belief (U-B window). The other shows the evolution of agents' beliefs over time (B-T window). From investigating the U-B window, strategies belief a little below a median degree have higher lifetime utilities than others.[47] From the B-T window, we also

[46] Unfortunately, due to personal technical problem, we cannot report the dynamic frame window on a sheet. The windows are shown only during the executing time. Further refinement will be done in the future. When the maximum feasible belief is 4, the median belief will be 2. The Java program is available from the author upon request.

[47] In fact, we found that the lifetime utility produced by strategy low rational expectation belief always higher than the lifetime utility produced by strategy high rational expectation belief. In addition, the lifetime utility produced by strategy average belief (an mean of population beliefs) is also higher than the lifetime utility produced by strategy high rational expectation belief. The lifetime utility produced

found that strategies being selected out in the first place are those strategies HB not strategies LB. Therefore, convergence to the high rational expectation equilibrium seems to be very unlikely.[48] This finishes the first explanation. However, it is naturally to ask why agents do not simply choose the strategy lowest belief (LTB) that forecast a zero inflation factor.[49] This leads to the second explanation.

Again from equations (7) and (8), we have

$$M(t)-M(t-1)=gP(t) \qquad (7')$$

$$\bar{S}(t)=\sum_{i=1}^{N} s_t^i(t)/N=\frac{M(t)}{P(t)} \qquad (8')$$

Substitute (8') into (7'), we have:

$$g=\bar{S}(t)-\frac{\bar{S}(t-)}{\beta^*(t)}, \qquad (14)$$

Therefore, we have:

$$\beta^*(t)=\frac{\bar{S}(t-1)}{\bar{S}(t)-g}, \qquad (15)$$

From equation (8), we have:

$$\bar{S}(t)=\sum_{i=1}^{N} s_t^i(t)/N=\frac{1}{N}\sum\frac{w_2}{2}(\lambda-\beta^i(t))=\frac{w_2}{2}(\lambda-\bar{\beta}(t)) \qquad (16)$$

Combine equations (15) and (16), we have:

$$\beta^*(t)=\frac{\frac{w_2}{2}(\lambda-\bar{\beta}(t-1))}{\frac{w_2}{2}(\lambda-\bar{\beta}(t))-g}=\frac{\lambda-\bar{\beta}(t-1)}{\lambda-\bar{\beta}(t)-\frac{2g}{w_2}}, \qquad (17)$$

When selection force favours those beliefs that produce inflation forecast close to the realised inflation $\beta*$, it is also truth for $\bar{\beta}$. Therefore, we have properties below:

$$\beta^*(t)\begin{matrix}\bar{\geq}\\ \equiv \\ \leq\end{matrix}\bar{\beta}(t) \quad \begin{matrix} if & \beta^*>\bar{\beta} & increase & \beta^i\text{'s} & \rightarrow & \bar{\beta}(t)\uparrow \\ if & \beta^*<\bar{\beta} & decrease & \beta^i\text{'s} & \rightarrow & \bar{\beta}(t)\downarrow \end{matrix} \qquad (18)$$

Combining equations (17) and (18), we have:

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by strategy low rational expectation belief is very close to that produced by strategy average belief. An exception is that when initially all members have high beliefs, the relation between belief and utility is positive i.e. the higher the belief, the higher the utility. However, such a case is very rarely.

[48] Therefore, β^* is away from the basin of attractor.

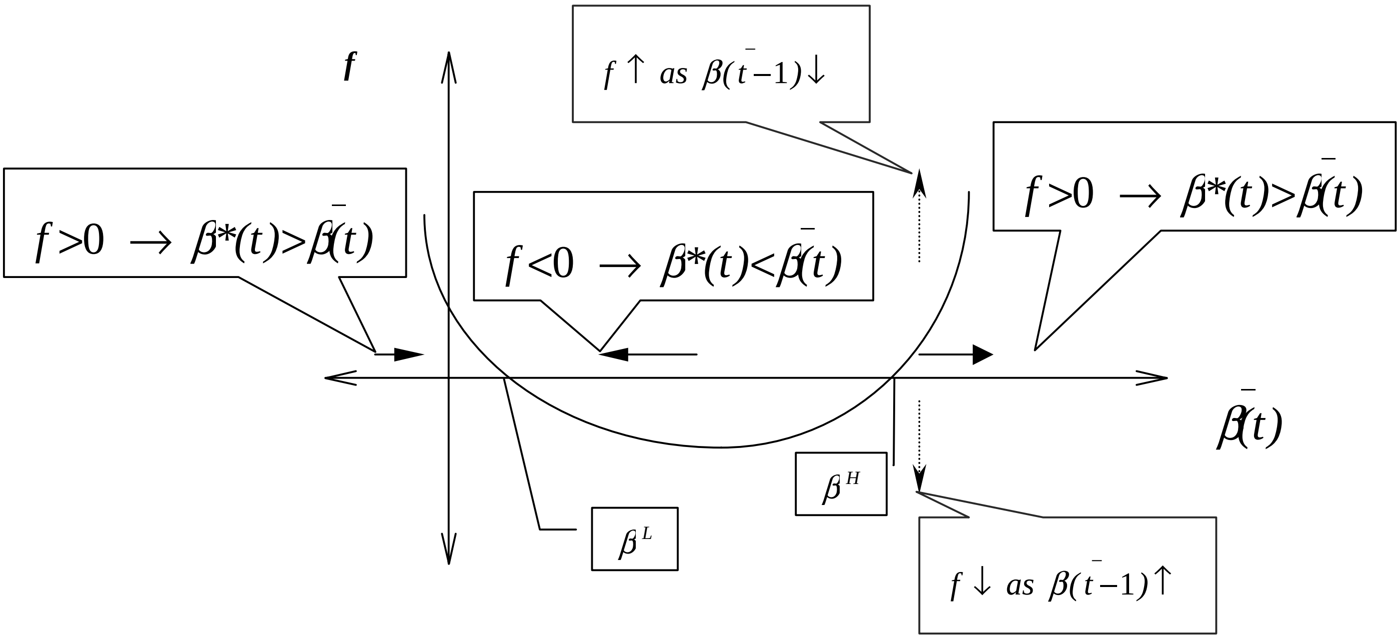
[49] In the current study, the minimum inflation factor is zero.

$$\frac{\lambda-\bar{\beta}(t-1)}{\lambda-\bar{\beta}(t)-\frac{2g}{w_2}} \begin{matrix}\geq\\ \equiv\\ \leq\end{matrix} \bar{\beta}(t) \qquad (17')$$

Solve equation (17'), we have

$$f=\bar{\beta}_t^{\,2}-(\lambda-\frac{2g}{w_2})\bar{\beta}(t)+(\lambda-\bar{\beta}_{t-1}) \begin{matrix}\geq\\ \equiv\\ \leq\end{matrix} 0 \qquad (19)$$

Figure XII.1



It may be difficult to draw a static graph of *f* due to the nature of time varying, i.e. both $\bar{\beta}$ and β^* is changing from period to period. However, it is possible to have a qualitative analysis of *f* and a possible graph of *f* can be shown in Figure XII.1. First, the nature of *f* also corresponds to (18). When $\bar{\beta}$ is less than β^*, there is "*pull-up*" force, to pull $\bar{\beta}$ up to β^***.** On the other hand, when $\bar{\beta}$ is higher than β^*, there is "*pull-down*" force to pull $\bar{\beta}$ down to β^***.** In addition, the movement (up or down) of *f* is also affected by $\bar{\beta}(t-1)$. When *f=0* and $\beta^*(t)=\bar{\beta}(t)=\bar{\beta}(t-1)$, we have two convergence states i.e. the high inflation equilibrium, β^H **,** and low inflation equilibrium, β^L. From Figure XII.1, we see that β^L is a stable state. Therefore, we have an inverse dynamics different from the dynamics under the assumption of rational expectation. The Pareto superior equilibrium, i.e. the low inflation steady state, is a stable attractor in the learning dynamics.

In fact, we also found these properties from our simulation results. In particular we have examined three convergence cases below. The three cases are low rational expectation belief (LREB), low belief (LB), and high belief (HB). In each case, we suppose that there is one member of young generation retreats from his previous belief to a much higher belief (MHB), much lower belief (MLB), bid for current realised belief (RB), and bid for the low rational expectation belief (LREB).

From the Table XII.1, we see that in the case of convergence to HB higher than LREB, it would be wise for any player to retreat to a belief lower than the HB but higher than LREB at least. However, in the case of convergence to LB, there is potential gain to retreat to a belief close to LREB where $j \succ i$. By doing this, the mutant hurts himself, but he hurts the other player, i in our case, even more.[50] The potential gain results from the so-called spiteful behaviour. In particular, selfish and spiteful behaviour can be expected in an evolutionary model and it can have a selective advantage (Hamilton, 1970).

Table XII.1

Welfare comparison based on lifetime utility after strategy retreated

	Case 1 LREB*1 (=1.333)		Case 2 LB*2 (=1.235)				Case 3 HB*3 (=2.509)			
Retreat Strategy	MHB	MLB	MHB	MLB	RB	LREB	MHB	MLB	RB	LREB
Incumbent i	worse off	better off	worse off	worse off	worse off	worse off	worse off	better off	better off	better off
Mutant j	worse off	worse off	worse off	worse off	worse off	worse off	worse off	better off	better off	better off
Welfare Comparison	$i \succ j$	$i \succ j$	$i \succ j$	$i \succ j$	$j \succ i$	$j \succ i$	$i \succ j$	$j \succ i$	$j \succ i$	$j \succ i$

MHB: much higher belief. MLB: much lower belief. LREB: low rational expectation belief.
Incumbent: player stick to his previous belief. Mutant: player switch his belief to another.
$i \succ j$: player *i's welfare improves more than* player *j*'s. $j \succ i$: player *j's welfare improves more than* player *i*'s.
*1: In the case we investigated, MLB<LREB<MHB. In addition, when convergence to LREB, LREB equals to RB and every individual agent has lifetime utility value of 1.674.
*2: In the case we investigated, MLB<LB<RB<LREB<MHB and every individual agent has a lifetime utility value of 1.68.
*3: In the case we investigated, LREB<RB<MLB<HB<MHB and every individual agent has a lifetime utility value of 1.525.

To make thing more easily to understand, we concluded the three cases below.

Case 1:

$$MLB \rightarrow LREB \leftarrow MHB$$

Case 2:

$$MLB \rightarrow LB \rightarrow RB \rightarrow LREB \leftarrow MHB$$

[50] In this case, when the mutant switches to the LREB strategy, his lifetime utility is 1.679 and his loss of welfare is –0.0006731. However, the incumbent's lifetime utility is 1.678 and his loss of welfare is –0.0008031.

Case 3:

$$LREB \leftarrow RB \leftarrow MLB \leftarrow HB \leftarrow MHB$$

Therefore, both spite effect and advantage of strategy LB are investigated. The heterogeneity in the current adaptive learning system, initially agents may not behave in the manner of strategy low belief and the economy is not in a matured state. After agents come into this economy and communicate with each other, they gradually recognize dominance of strategy LB facing the nature of time varying. Therefore, $\bar{\beta}$ and β^* will be located in the neighborhood of β^L. Again, from investigating the B-T windows, we also see $\bar{\beta}$ wanders about β^* and close to β^L after the first iterations. Eventually, individual agents coordinate on the low rational expectation equilibrium, where $\bar{\beta} = \beta^* = \beta^L$. In other words, the genetic algorithm learning evolved the strategy LREB as successful as the best strategy belief in the economy.

In addition, a nature that lifetime utility of one player increases as the action chosen by the other decreases (any strategy belief below HB in case 3), is captured. In particular, the action chosen by the other player creates an incentive, due to a selective force, for the remaining players to choose the same action. This is a property of positive feedback or spillovers. (Cooper, 1999).[51] It is produced by effects of interactions between agents in the economy, due to contributions from these agents involved. Individual agents may not internalize the spillovers. Therefore, equilibria may be dominated by some other feasible outcomes. As we can see from the examples above, the high inflation equilibrium belief is dominated by some other relative strategies low belief, including the LREB.

For full validity, a further remark is necessary, even if it is a bit of beyond the scope of the study. There comes a special flair of genetic algorithm about stability. Riechmann (1999) argued that the genetic algorithm ends up in a kind of Ljapunov stability, where a constant subset of the set of all genetic populations is reached.[52] In the long run there is a constant distribution of all genetic populations (states). Therefore, every state can be reached from every other state with a positive measure. However, in the current study, while genetic algorithms having the election operator in the last step, there is a force to stop agents to experiment furthermore. The election operator will reject any experiment that is inferior to the current best strategy but probably superior in later periods. Hence, it is impossibility to leave one uniform population (state) that is

[51] Cooper (1999) indicated the implication of spillovers or positive feedback, often termed strategic complementarity. It is central to a characterisation of coordination games.

[52] The difference between genetic populations is in their different compositions. Remember that when the length of string is *L*, the number of uniform population i.e. population consisting of only one type of

an evolutionarily stable population (state). In other words, once it reaches the state, it will stay forever. This is quite different from the property of stability in genetic algorithms without the election operation in the last step. Therefore, genetic algorithms with the election operator in the last step show an asymptotic stability in which decisions of the artificial agents cease to change.

XIII. Conclusion

When we apply genetic algorithms to the economic learning problems, our conclusions are in many ways. The first one concerns the interpretation of emergence of convergence to the low inflation equilibrium. In the first place, it is agents' best interest to make an inflation forecast as close to the realised inflation as possible, i.e. the concept of "*survival of fitness*". There is selective pressure on individual agents' utilities resulting from the outcomes of agents' strategies belief having forecast models that forecast inflation factors. As far as the selective pressure concerns, there are two underlying processes. On the one hand, due to the presence of dominance of strategy low belief and spillover effect, there is selective pressure in favour of strategies low belief. Strategies high belief will be selected out and therefore the economy might not have chance to end up in the high inflation equilibrium. The only equilibrium in the model left is the low inflation equilibrium.[53] On the other hand, there is an adverse force against the dominance of the strategy low belief, due to the presence of spiteful behaviour. Hence, these two processes drive agents coordinating on the low rational expectation belief (LREB) and therefore the economy converges to the low inflation equilibrium, i.e. Pareto superior equilibrium. This result is robust independent of precise algorithms used.

The second one concerns the performance of genetic algorithms learning. The performance heavily depends on features of genetic operators. Comparison of our simulations suggests that there is a trade off between speed of convergence and accuracy and probability of convergence resulting from combined processes of selection, election and crossover variants to balance the exploration and exploitation. We show that selective transfer genetic algorithm (STGA) with tournament selection operator has a very reasonable performance.

The third one concerns the learning variant. In the study, our artificial adaptive agents eventually can learn rational expectations and coordinate on the low inflation equilibrium under population learning, individual learning, and open learning. It will be more difficult for these agents to learn the rational expectations and coordinate on the low inflation equilibrium under individual learning. However, in

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individual is 2^L. In total, we have 2^{NL} genetic populations, where N is population size.

[53] Remember that in the model, there are two steady states. One is the high inflation equilibrium in which all agents have a low rational expectations belief; the other one is the low inflation equilibrium in which all agents have a high rational expectation belief.

open learning in which there is a tunnel allowing these individual agents to learn their own learning schemes with reference to population experience, the result is as neat and successful as best one. On the one hand, this suggested that people with different learning schemes more efficiently learn the rational expectations and coordinate on the low inflation equilibrium. On the other hand, this suggested that people tend to learn how to learn from social experience and then learn from the basis of reflective self-consciousness.

Finally, we also showed that in the study, agents tend to use the BDGA and STGA learning when there are many learning schemes available. This suggests that the two learning algorithms behave sensibly. However, we have to keep in mind that it might be a problem to interpret the result when these artificial inductive procedures may be arbitrary. It is still an empirical issue concerning what learning schemes people tend to use.

Table X.1 Parameterisation of Simulation

Population Learning		Individual Learning	
	S_30_4		S_60_30_4
			S_60_30_8
1. Standard		1. Standard	S_60_60_4
Government Finance = 0.333	S_30_8	Government Finance = 0.333	S_60_60_8
Maximum Inflation Belief = 4		Maximum Inflation Belief = 4	S_120_30_4
	S_60_4		S_120_30_8
	S_60_8		S_120_60_4
			S_120_60_8
	IG_30_4		IG_60_30_4
			IG_60_30_8
2 Increase Government finance		2 Increase Government finance	IG_60_60_4
Government Finance = 0.45		Government Finance = 0.45	IG_60_60_8
Maximum Inflation Belief = 4		Maximum Inflation Belief = 4	IG_120_30_4
	IG_30_4		IG_120_30_8
	IG_60_4		IG_120_60_4
	IG_60_8		IG_120_60_8
	IMF_30_4		IMF_60_30_4
			IMF_60_30_8
3 Increase maximum inflation forecast		3 Increase maximum inflation forecast	IMF_60_60_4
Government Finance = 0.333		Government Finance = 0.333	IMF_60_60_8
Maximum Inflation Belief = 5	IMF_30_8	Maximum Inflation Belief = 5	IMF_120_30_4
	IMF_60_4		IMF_120_30_8
	IMF_60_8		IMF_120_60_4
			IMF_120_60_8

1. S: Standard parameterization; IG: Increase government finance; IMF: Increase maximum inflation belief
2. In population learning, the rule to name an experiment design is following.
    (1) The name of experiment design as described above.
    (2) Population size
    (3) Length of string
3. In individual learning, the rule to name an experiment design is following:
    (1) The name of experiment design as described above.
    (2) Number of agents
    (3) Size of strings for each agents
    (4) Length of string
4. The rate of crossover is 100% and the rate of mutation is 3.3%. However, SGA cannot converge for all experiments using these parameterizations. After several trials, the rates of crossover and mutation used are 90% and 0.33% in SGA.

Table X.2 Summary of Population Learning

	Mean_L	STDEV_L	Mean_C	STDEV_C	SOC	SOL	SOL/SOC	V_C	V_L
SGA									
no_scaling	617.462	238.0635	566.927	239.6544	0.1033	0.05417	0.524366	0.42273	0.38555
scaling	559.017	246.1257	519.962	241.0484	0.06583	0.04833	0.734176	0.46359	0.44028
M_SGA									
TOP50	fail	fail	fail	fail	fail	fail	fail	fail	fail
SECTION	fail	fail	fail	fail	fail	fail	fail	fail	fail
PRO	fail	fail	fail	fail	fail	fail	fail	fail	fail
TOURNAMENT(S)	109.562	142.889	107.084	134.7031	0.2975	0.25833	0.868336	1.25792	1.30419
AGA									
no_scaling	51.9569	25.91185	56.9332	44.60146	0.69833	0.61917	0.886636	0.7834	0.49872
scaling	53.44	29.0991	60.3688	56.64016	0.68917	0.6175	0.896009	0.93824	0.54452
M_AGA									
TOP50	fail	fail	fail	fail	fail	fail	fail	fail	fail
SECTION	fail	fail	fail	fail	fail	fail	fail	fail	fail
PRO	fail	fail	fail	fail	fail	fail	fail	fail	fail
TOURNAMENT(S)	21.5337	4.368802	25.6165	20.75886	0.6867	0.5933	0.863987	0.81037	0.20288
STGA									
no_scaling	65.8657	77.14256	66.5993	77.11294	0.9733	0.93083	0.956368	1.15786	1.17121
scaling	66.8069	73.84029	68.1252	77.77912	0.97167	0.91917	0.945969	1.14171	1.10528
M_STGA									
TOP50	fail	fail	fail	fail	fail	fail	fail	fail	fail
SECTION	fail	fail	fail	fail	fail	fail	fail	fail	fail
PRO(S)	115.45	83.5063	114.167	82.14307	0.98	0.93	0.94898	0.7195	0.72331
TOURNAMENT(S)	24.9135	18.10529	24.9799	18.26721	0.9967	0.963	0.966188	0.73128	0.72673
BDGA									
no_scaling	36.6789	79.35905	38.6457	81.55487	0.97833	0.8825	0.902047	2.11032	2.16361
scaling	33.9494	65.87079	36.4437	71.65454	0.98417	0.90583	0.920406	1.96617	1.94026
M_BDGA									
TOP50	fail	fail	fail	fail	fail	fail	fail	fail	fail
SECTION	fail	fail	fail	fail	fail	fail	fail	fail	fail
PRO(S)	208.244	135.3196	201.924	132.2973	0.96	0.87	0.90625	0.65518	0.64981
Roulette(S)	167.879	112.6841	166.47	111.3596	0.9933	0.9367	0.943018	0.66895	0.67122

Table X.3 Summary of Individual Learning

	Mean_L	STDEV_I	Mean_C	STDEV_(	SOC	SOL	SOL/SOC	V_C	V_L
STGA									
no_scalin;	142.083	179.741	142.083	179.741	0.67583	0.67583	1	1.26505	1.26505
scaling	149.033	181.767	149.033	181.767	0.64	0.64	1	1.21964	1.21964
BDGA									
no_scalin;	120.164	180.528	120.164	180.528	0.70833	0.70833	1	1.50234	1.50234
scaling	114.391	174.015	114.391	174.015	0.69667	0.69667	1	1.52123	1.52123

Table XI.1 Open learning by all simulation

Frequency	stga	bdga	aga	sga	s_stga	s_bdga	s_aga	s_sga	pro_stga	pro_bdg	pro_aga	pro_sga	t_stga	rw_bdga	t_aga	t_sga	others
S_60_30_4	4.2422	4.22545	0.95389	0.45842	4.39899	4.31828	0.99127	0.49172	4.05527	4.0016	0.90251	0.46083	4.2744	4.27122	0.97066	0.49242	20.4909
IGF_60_30_4	4.24023	4.36096	0.93035	0.46599	4.2614	4.31822	0.97311	0.46902	4.13031	4.14735	0.9491	0.46599	4.22654	4.18528	0.96357	0.47748	20.4351
IMF_60_30_4	4.18024	4.19762	0.93615	0.4683	4.39266	4.35672	0.97074	0.48444	4.05301	4.04717	0.90905	0.44708	4.26602	4.18219	0.94607	0.4639	20.6986
Mean Frequency	4.22089	4.26134	0.94013	0.46424	4.35102	4.33107	0.97837	0.48173	4.07953	4.06537	0.92022	0.45797	4.25565	4.2129	0.9601	0.47793	20.5415

Frequency	STGA	BDGA	AGA	SGA
S_60_30_4	4.24272	4.20414	0.95458	0.47585
IGF_60_30_4	4.21462	4.25295	0.95403	0.46962
IMF_60_30_4	4.22298	4.19593	0.9405	0.46593
Mean Frequency	4.22677	4.21767	0.94971	0.47047

Table XI.2 Open learning by convergence

Frequency	stga	bdga	aga	sga	s_stga	s_bdga	s_aga	s_sga	pro_stga	pro_bdg	pro_aga	pro_sga	t_stga	rw_bdga	t_aga	t_sga	others
S_60_30_4	3.48841	5.9702	0.70902	0.32947	5.91846	6.42964	1.28353	0.62086	1.46854	1.52194	0.35017	0.32947	5.78932	3.72641	1.30505	0.56954	20.3502
IGF_60_30_4	3.05888	6.45671	0.74255	0.3311	5.61256	6.11037	1.35211	0.56754	1.54468	1.62688	0.38929	0.3311	5.97276	3.32117	1.38629	0.74302	20.1372
IMF_60_30_4	3.64235	5.80095	0.78271	0.33228	6.11142	6.20143	1.26963	0.6483	1.52577	1.49841	0.33466	0.15623	5.82435	3.64909	1.33029	0.66178	20.2304
Mean Frequency	3.39655	6.07595	0.74476	0.33095	5.88081	6.24714	1.30176	0.61223	1.513	1.54908	0.35804	0.27227	5.86214	3.56556	1.34054	0.65811	20.2392

Frequency	STGA	BDGA	AGA	SGA
S_60_30_4	4.16618	4.41205	0.91194	0.46233
IGF_60_30_4	4.15897	4.37878	0.96756	0.49319
IMF_60_30_4	4.27597	4.28747	0.92932	0.44964
Mean Frequency	4.20038	4.35943	0.93627	0.46839

Table XI.3 Open learning by first time convergence in each run

	Frequency	SOL/SOC	Mean	STDEV
S_60_30_4	100	1	103.58	90.5
IGF_60_30_4	100	1	111.78	92.5
IMF_60_30_4	100	1	120.3	76.54265
Average	100	1	111.8867	86.51422

Appendix A The Overlapping Generation Model Under Learning

Consequence of individual agents' beliefs

Instead of maintaining the perfect foresight knowledge of future prices, all *N* agents who are in the first period of lifespan at time *t* forecast future prices using the simple linear model:

$$F^i[P(t+1)] = b^i(t)P(t), \qquad (1)$$

$b^i(t)$ denotes the parameter that agent $i = 1,2,...,N$ of generation *t* uses to forecast next period's price. While all N agents use the same specification (1) for their forecast model, each agent may have a different belief regarding the appropriate value of the unknown parameter *b*. We restrict agent's belief regarding the parameter *b* to fall in the interval:

$$0 \le b^i(t) \le \lambda, \qquad \forall \;\; i, t \; .$$

The lower bound ensures that price forecasts are always nonnegative. The upper bound of λ represents the highest inflation factor that agents would need to forecast in order to achieve a feasible equilibrium.

Agents' forecast take place in a sequence of periods, indexed by $t = 1,2,...\infty$ for each agent. A matrix represents these forecasts below:

$$\begin{pmatrix} b^1(0) & b^1(1) & . & . & b^1(\infty) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ b^N(0) & b^N(1) & . & . & b^N(\infty) \end{pmatrix}$$

Encoding Agent's Belief

We use binary string to represent agent's belief. Let the bit string for agent *i* at time t be given by the vector, *i.e.*, a chromosome:

$$\langle a_{i1}(t), a_{i2}(t),..., a_{il}(t) \rangle$$

where $a_{ij}(t) \in \{0,1\}$.

The mapping from a binary string $\langle a_{i1}, a_{i2},..., a_{il} \rangle$ into a real number, the parameter estimate $b^i(t)$ in our case is straightforward and completed in two steps:

(1) converting the binary string $\langle a_{i1}, a_{i2},..., a_{il} \rangle$ from the base 2 to base 10:

$$d_i(t)=\sum_{j=1}^{l} a_{ij}(t)\bullet 2^{l-j}$$

(2) finding a corresponding real number $b^i(t)$

$$b^i(t)=\frac{d_i(t)}{d_{\max}}\bullet\lambda$$

where $d_{\max}=\sum_{s=1}^{l} 2^{l-s}$, the maximum possible decoded value,

and $\lambda = w_1 / w_2$, the maximum gross inflation factor that the agent would need to forecast in order to achieve a feasible equilibrium. Therefore, a string, (1010) represents a real number 2.667, since

$$d_i = (1010)_2 = 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 = 10$$

$$d_{\max} = (1111) = 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0 = 15$$

and $b^i = \frac{10}{15}\bullet 4 \approx 2.667$, $\lambda = 4$.

Updating beliefs

Aggregate savings of the economy is given by:

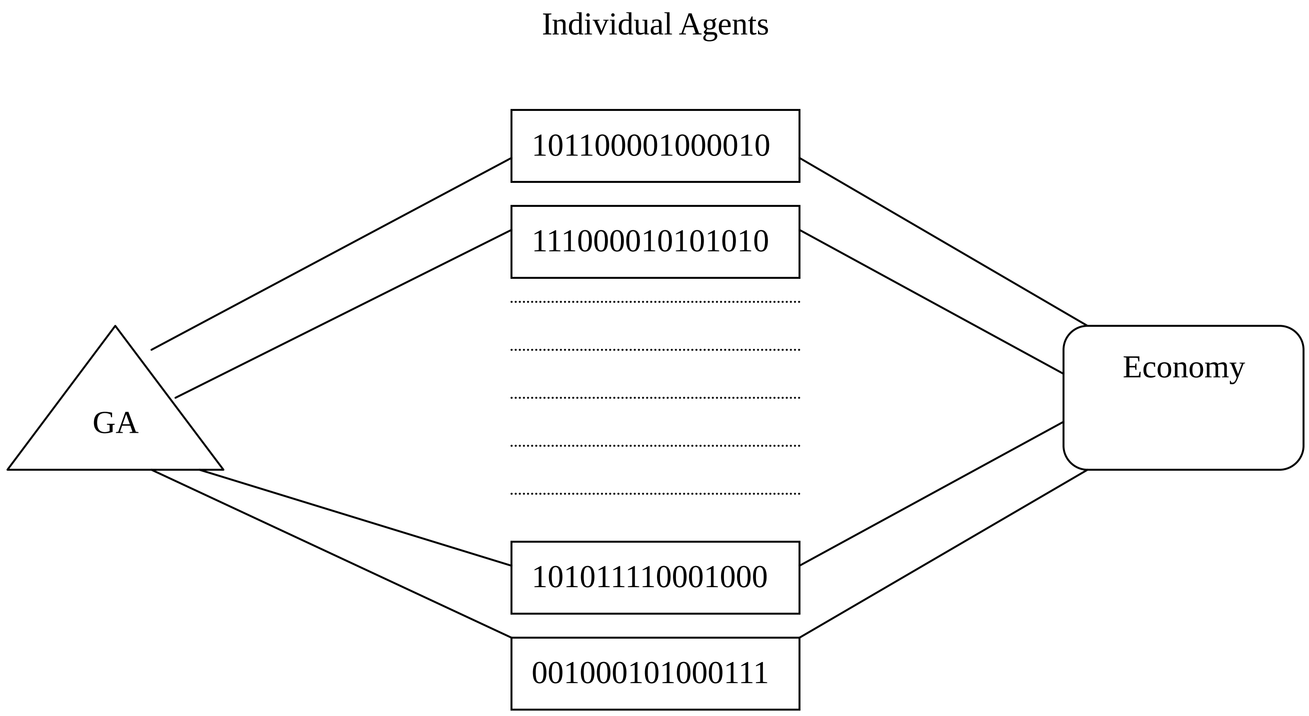
$$S(t)=\sum_{i=1}^{N} s_t^i(t) \qquad (2)$$

Let this equation equals to equation (8) in appendix A and use equation (7) in appendix A to substitute out for real money balances. Therefore, the realized inflation factor $\beta(t-1)$ is given by:

$$\beta(t-1)=\frac{P(t)}{P(t-1)}=\frac{S(t-1)}{S(t)-Ng} \qquad (3)$$

Once $\beta(t-1)$ is known, agents' forecasts made by generation *t-1* can be evaluated. Also, the realized lifetime utilities of agents, born at period *t-1*, is evaluated. We use the lifetime utility as fitness of entity in the genetic algorithm.

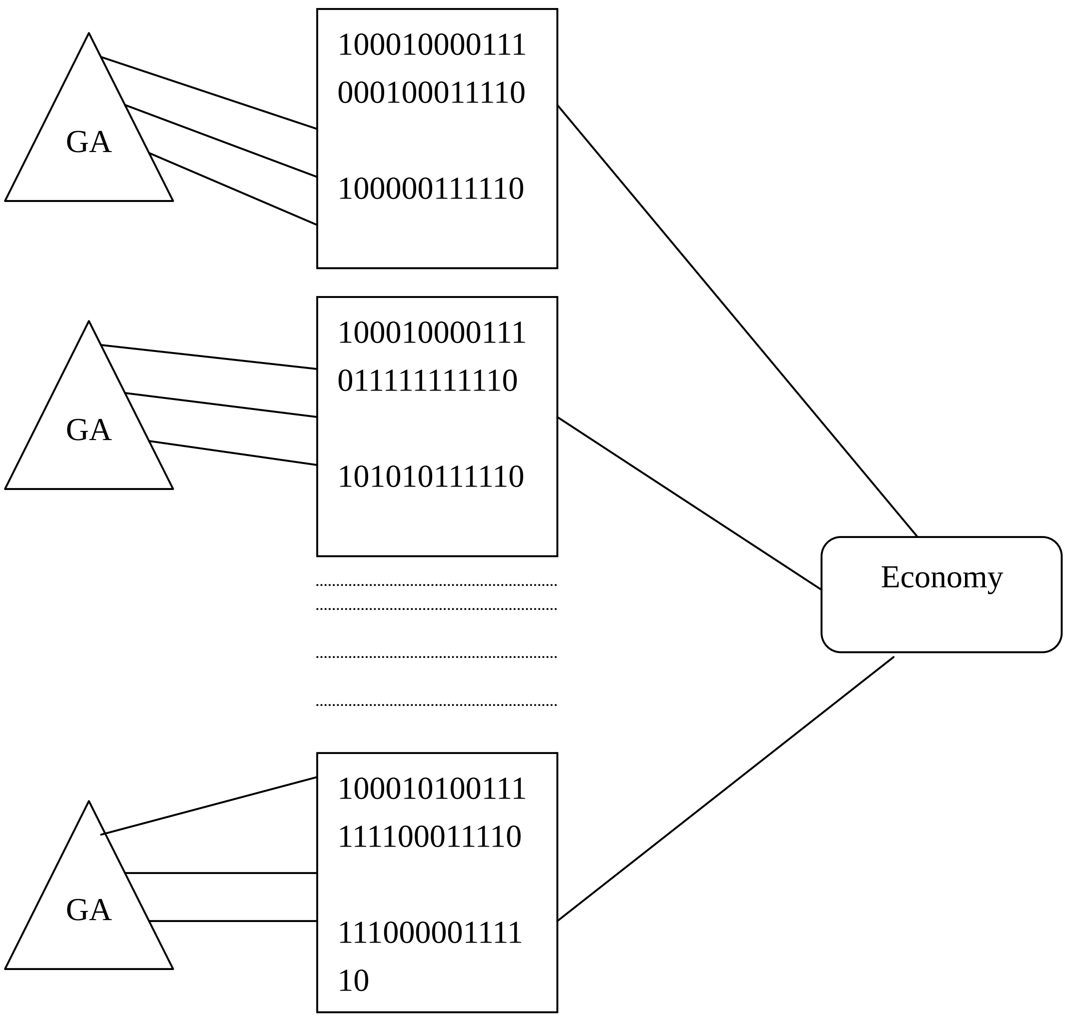
Appendix B. Population Learning



Population Learning GA

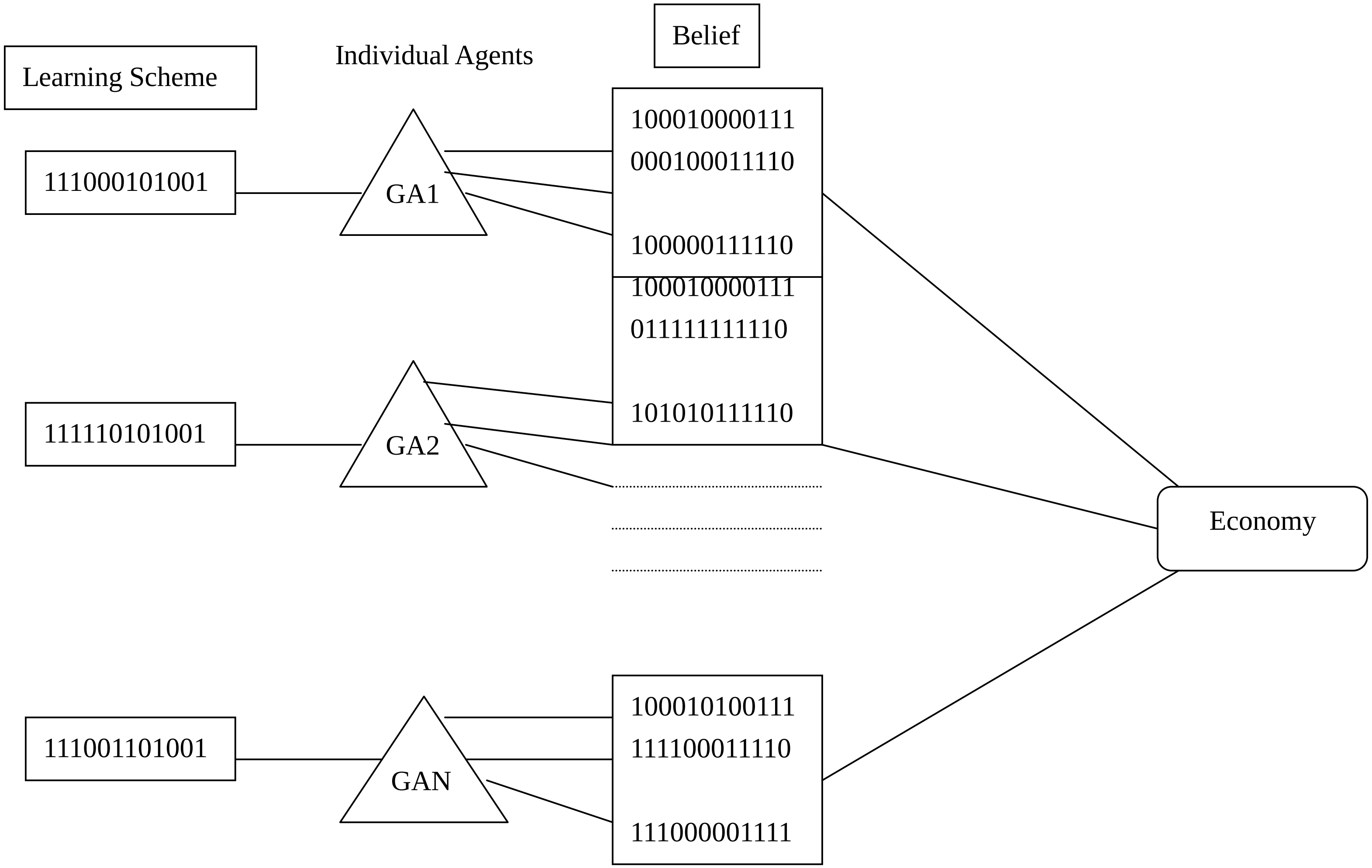
Appendix C Individual Learning

Individual Agents



Individual Learning GA

Appendix D Open Learning



Open Learning GA

References

Arifovic, J., (1994) “Genetic algorithm learning and the cobweb model,” *Journal of Economic Dynamics and Control* 18, 3-28.

Arifovic, J., (1995) “Genetic algorithms and inflationary economies,” *Journal of Monetary Economics* 36, 219-243.

Arifovic, J., (1996) “The Behavior of the exchange rate in the genetic algorithm and experimental economics,” *Journal of Political Economy* vol. 104. no 3, 510-541.

Arifovic, J. and C., Eaton (1995) “Coordination via genetic learning,” *Computational Economics* 8, 181-203.

Arthur, W. B. (1995) “Self-Reinforcing Mechanisms in Economics” in *Economics, Cognition, and Society Series*, Lichback, Mark Irving eds, Ann Arbor: University of Michigan Press, 1995.

Axelrod, R., (1990) *The Evolution of Cooperation*, Penguin.

Bandura, A., (1986) *Social Foundation of Thought and Action: A Social Congitive Theory*, Prentice-Hall: Englewood Cliffs, New York.

Belew, R. K. (1990) “Evolution, learning, and culture: computational metaphors for adaptive algorithms,” *Complex Systems* 4, p11-49.

Binmore, K. (1992) *Fun and Game*s, D. C. Heath & Co., Lexington.

Birchenhall, C. R. (1994) “Evolutionary Games and Genetic Algorithms,” *School of Economics Discussion Paper*, University of Manchester.

Birchenhall, C. R. (1995) “Technical change and genetic algorithms,” special issue in Genetic Algorithms on *Computational Economics* 8, 223-253.

Birchenhall, C. R., N. Kastrinos and S. Metcalfe (1996) “Genetic algorithms in evolutionary modeling,” *Journal of Evolutionary Economics* 7, 375-393.

Blume, L. E. and D. Easley (1982) “Learning to be rational,” *Journal of Economic Theory* 25, 340-351.

Bullard, J. and John Duffy (1999) “Using genetic algorithms to model the evolution of heterogeneous beliefs,” *Computational Economics* vol 13(1), p41-60.

Cooper, R. W. (1999) *Coordination Games: Complementarities and Macroeconomics*, Cambridge: Cambridge University Press.

Darden, L. and Cain J., (1989) Selection type theories, *Philosophy of Scienc*e 56, 106-129.

Davidson, D. (1980) *Essays on Action and Events*, Oxford: Oxford University Press.

Dawid, H., (1996b) “Genetic algorithms as a model of adaptive learning in economic systems,” *Central European Journal for Operations Research and Economics* 4(1), 7-23.

Dawid, H. (1994) “A markov chain analysis of genetic algorithms with a state depenent fitness function,” *Complex System* 8, 497-417.

Dawid, H., (1996a) “Learning of cycles and sunspot equilibria by genetic algorithms,” *Journal of Evolutionary Economics* 6, 361-373.

Dawkins, R. (1989) *The Selfish Gene*, Oxford University Press.

Goldberg, David E. (1989) *Genetic Algorithms inSearch, Optimization and Machine Learning*, Addison-Wesley.

Grefenstette, John J., (1986) "Optimization of control parameters for genetic algorithms," *IEEE Transactions on Systems, Man and Cybernetics* vol. SMC-16, no 1, January/February.

Hamilton, W. D. (1970) " Selfish and spiteful behaviour in an evolutionary model," *Nature* vol. 228 December, 1218-1220.

Hamilton, D. (1991) *Evolutionary Economics*, University of New Mexico Press.

Harsanyi, J. and R. Selten (1988) *A General Theory of Equilibrium Selection in Games*, Cambridge: Cambridge University Press.

Hintyon, G. E. and Steven J. Nowlan (1987) "How learning can guide evolution," *Complex Systems* 1, 495-502.

Hofbauer, J. and Sigmund K. (1988) *Evolutionary Games and Population Dynamics*, Cambridge University Press, Cambridge, UK.

Hodgson, G. M. (1993) *Economics and Evolution*, Polity Press.

Holland, J. H. (1992) *Adaptation in Natural and Artificial Systems*, A Bradford Book, MIT Press.

Laudan, L. (1977) *Progress and its Problems: Towards a Theory of Scientific Growth*, London: Routledge and Kegan Paul.

Lucas, R. E. Jr (1986) "Adaptive behaviour and economic theory," *Journal of Business* vol. 59, no. 4

Mailath, G. J. (1992) "Introduction: Symposium on evolutionary game theory," *Journal of Economic Theory* 57, 259-277.

Marcet, A. and Thomas J. Sargent (1989) "Least-squares learning and the dynamics of hyperinflation" in the book oof W.A. Barnett, J. Geweke, and K. Shell, eds., *Economics Complexity: Chaos, Sunspot, Bubbles and Nonlinearity*, Cambridge University Press: Cambridge, MA.

Marimon R., E. McGrattan and Thmos J. Sargent (1990) "Money as a medium of exchange in an economy with artificial intelligent agents," *Journal of Economic Dynamics and Control* 14, 329-373.

Maynard Smith J (1982) *Evolution and the Theory of Games*, Oxford: Oxford University Press.

Metcalfe, J. S. (1998) "Evolutionary concepts in relation to evolutionary economics," *CRIC Working paper* No 4, January, University of Manchester.

Michalewicz, Z. (1996) *Genetci Algorithms + Data Structures = Evolution Programs*, Springer.

Peirce, C. S. (1958) Collected Papers of Charles Sanders Peirce, vol. 7: *Science and*

Philosophy, ed. A. W. Burks, Cambridge, MA:Harvard University Press.
Plotkin, H. (1997) *Evolution in Mind*, Penguin Books.
Riechmann, T. (1998) "Genetic algorithms and economic evolution," *Discussion paper* No 219, University Hannover.
Riechmann, T. (1999) "Learning and behavioral stability,“ *Journal of Evolutionary Economics* 9, 225-242.
Rudolph, G., (1994) "Convergence analysis of canonical genetic algorithms," *IEEE Transactions on Neural Networks* vol. 5, no 1, January/February.
Sacco, P. L. (1994) "Can people learn rational expectations?" *Journal of Evolutionary Economics* 4, 35-43.
Sargent, T. J., (1993) *Bounded Rationality in Macroeconomics*, Clarendon Press, Oxford.Selten, R. (1991) "Evolution, learning and economic behavior," Games and Economic Behavior 3, 3-24.
Selten, R. (1991) "Evolution, learning and economic behaviour," *Games and Economic Behaviour* 3, 3-24.
Sent, Esther-Mirjam (1998) *The Evolving Rationality of Rational Expectations*, Cambridge University Press.
Vriend, N., (2000) "An illustration of the essential difference between individual and social learning, and its consequences for computational analyses," *Journal of economic dynamic and control* vol 24(1), p1-19.
Windrum, P., (1998) *The Population Dynamics of Innovation: Modelling Scientific and Industrial Knowledge Systems*, PhD Thesis, University of Manchester.