Toward an Integration of Social Learning and Individual Learning in Agent-Based Computational Stock Markets: The Approach Based on Population Genetic Programming*

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Abstract

Artificial stock market is a growing field in the past few years. The essence of this issue is the interaction between many heterogeneous agents. In order to model this *complex adaptive system*, the techniques of evolutionary computation have been employed. Chen and Yeh (2000) proposed a new architecture to construct the artificial stock market. This framework is composed of a single-population genetic programming (SGP) based adaptive agent with a SA (Simulated Annealing) learning process and a *business school*.

However, one of the drawbacks of SGP-based framework is that the traders can't work out new ideas by themselves. The only way is to consult researchers in the business school. In order to make the traders more intelligent, we employ multi-population GP (MGP) based framework with the mechanism of *school*. This extension is not only reasonable, but also has the economic implications. How do the more intelligent agents influence the economy? Are the econometric properties of the simulation results based on MGP more like the phenomena found in the real stock market? In this paper, the comparison between SGP and MGP is studied from two sides. One is related to the micro-structure, traders' behavior and believe. The other is macro-properties, the properties of time series. The line of research is helpful in understanding the foundation of economics and finance, and constructing more realistic economic models.

Keywords: Evolutionary Computation, Genetic Programming, Agent-Based Modeling, Artificial Stock Market, Simulated Annealing

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1 Background and Motivation

Artificial stock market has been a hot topic in the fields of *agent based computational economics* and *finance*. Genetic algorithms, artificial neural net and genetic programming have been used to model this framework. The main difference between these approaches is twofold.

- representation
- social learning vs. individual learning

Different representation constitutes different strategy space. Similarly, different style of learning explains different kind of human behavior. Both of them may induce different phenomena. Therefore, in order to obtain meaningful and reasonable results, employing the appropriate representation and learning behavior are the most important steps in studying agent-based computational economics and finance. In Lucas (1986),

> In general terms, we view or model an individual as a collection of decision rules (rules that dictate the action to be taken in given situations) and a set of preferences used to evaluate the outcomes arising from particular situation-action combinations. These decision rules are continuously under review and revision; new decision rules are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. (pp. 217)

From the viewpoint of representation, if a decision rule can *hopefully* be written and implemented as a computer program, and since every program in terms of its input-output structure can be understood as a function. Then, based on the language of LISP program, every function can be represented as a LISP S-Expression, and hence a parse tree. This representation of decision rule is exactly what genetic programming does. Consequently, the Lucasian adaptive economic agent can be modeled as

- evolving a population of decision rules
- evolving a population of functions
- evolving a population of programs
- evolving a population of LISP S-Expression
- evolving a population of parse trees

Moreover, from the perspective of genetic programming, these decision rules can be reviewed and revised under the genetic operators (including reproduction, crossover and mutation). The performance of new decision rules are validated based on the fitness function. Selection is then conducted under the *survival-of-the-fittest principle* which approximates the concept of *rules producing desirable outcomes supplant those that do not.* Therefore, we can restate the Lucasian economic agent in terms of the language of genetic programming:

> In general terms, we view or model an individual as a population of *LISP programs* (*which is generated by a set of function and terminals*) and *a set of fitness functions* used to evaluate the performance arising from particular LISP programs. These programs are continuously under review and revision based on the genetic operators, new programs are tried and tested against *experience*, and selection is conducted according to the *survival-of-the-fittest principle*

However, when we extend this idea to model a society of economic agents, a population of genetic programming is then employed. Each agent in this society is formed by a genetic programming. The action is determined by his own decision rules (strategies) and fitness function(s). The social and economic activities are the aggregate phenomena generated from these agents' interaction and coordination. This is the concept of multi-population genetic programming (MGP) which is distinguished from single-population genetic programming (SGP). In Vriend (2000), the implications of SGP/SGAs and MGP/MGAs are distinguished from *social* and *individual* learning. In other words, in social learning, agents learn from other agents' experience, whereas in individual learning, agents learn from their experience and thinking. Therefore, what Lucas (1986) mentioned focused on *individual learning*. Moreover, due to the criticisms given by Harrald (1998),[1] it further demonstrated the importance of multi-population GP/GAs.[2] However, there also exists problems in the MGP/MGAs modeling. The important phenomena found in economic activities, such as *following the herd* and *rumors dissemination*, or the style of social learning, are totally missing in the architecture of MGP/MGAs. In the past few years, MGAs is widely employed in the economic modeling. Although it passes the Harrald's criticisms, whereas the concept of Lucasian economic agent is not well captured in the representation of GAs. Also, the traditional representation of GAs is the fixed-length bit string, the important economic activity, such as *innovation* and *creation*, can't be modeled by GAs. Therefore, genetic programming seems to be the best tool to model the Lucasian economic agent.

The problem mentioned above is twofold. Firstly, SGP ignores individual learning. Moreover, the traditional approach of SGP fails to pass the Harrald's criticism. Secondly, the architecture of MGP merely focuses on individual learning. Therefore, we need a new architecture to integrate both of the key feature of *social learning* and *individual learning*. In Chen and Yeh (2000), a new architecture was proposed to solve Harrald's criticism. The mechanism of "school" is introduced into SGP framework. However, it is still a type of social learning. The concept of Lucasian adaptive economic agent is not modeled. In this paper, we extend the previous research to a new architecture which is a multi-population GP framework with the mechanism of school. This framework passes Harrald's criticism, it also integrates both of social learning and individual learning. This extension is very reasonable, while the economic implications of this extension are more important.

In Lucas (1986), he didn't mention how many decision rules are necessary (or enough) to model an individual. To some extent, this is related to the *memory* or *intelligence*. In other words, the more intelligent the traders are, the more decision rules (ideas) they have. From this perspective, the traders in our previous research (Chen and Yeh, 2000) are very naive. Each trader has only one idea in his mind. They are unable to work out new ideas by themselves. The only source of knowledge is the business school. On the other hand, the traders in this paper are more intelligent. Each trader has k (for example, 20) ideas. They have the ability of reasoning and imagination to create useful strategies. Therefore, we are interested in how the level of traders' intelligence influences the economy. Is the economy more stable (or unstable) if the traders are more intelligent? Is the interaction between these more intelligent agents (or inherent complexity) more complicated (or simpler)? Consider a market composed of many traders. Each trader is modeled by a computer. The traders in our previous framework (Chen and Yeh, 2000) behave like basic computers with the ability of validation only. They have no way to reason. That means, the traders have only the basic *intelligence*. On the other hand, the traders in this paper behave like supercomputers (very intelligent traders). They could

[1] He mentioned that the traditional distinction between the *phenotype* and *genotype* in biology and doubted whether the adaptation can be directly operated on the genotype via the phenotype in social processes. In other words, it is not easy to justify why we can learn or know other agents' strategies (phenotype) by means of their actions (phenotype).

[2] Arifovic (1995a, 1996), Miller (1996), Vila (1997), Arifovic, Bullard and Duffy (1997), Bullard and Duffy (1998a, 1998b, 1999), Staudinger (1998) are examples of SGA, while Andrews and Prager (1994), Chen and Yeh (1996, 1997a, 1997b, 1998), and Chen, Duffy and Yeh (1996) are examples of SGP. Examples of MGA can be found in Palmer et al. (1994), Tayler (1995), Arthur et al. (1997), Price (1997), Heymann, Pearzzo and Schuschny (1998).

be the econometricians at school, or the technicians in the stock market. They have the powerful abilities in computing (in computation complexity and speed) and reasoning. The interesting questions is, how are these supercomputers (more intelligent traders) different from those basic computers (less intelligent traders)? Of course, they may quickly discover the useful knowledge. Does it imply more efficient market just what the textbooks told us? If so, what is the reason behind it? Is it because they quickly realize the fundamental of market? Or the interaction between these intelligent traders makes the market more complicated such that they can't predict others' behavior and market dynamics well. In principle, the knowledge base of these intelligent traders is larger, so they have higher probabilities to get useful information. In other words, their adaptability is much higher than those who are less intelligent. Therefore, the survivability is also higher. However, they may cause the market more complicated beyond control. Their survivability is then reduced. Which one is the most possible outcome needed to be studied.

As to the macro-phenomena, we also care about whether the econometric properties of time series are affected by the level of traders' intelligence. Does it still have the stylized facts found in our previous simulations? More importantly, are the emergent properties more richer? If the answer is *yes*, what is the driving force behind it? If *no*, how does it happen?

The psychological activities of each trader are also important. When does the trader intend to look for new strategies? When does he decide to do it by himself? However, researchers and traders may have different focuses. Researchers care about the accuracy of prediction (prediction accuracy oriented), while traders care about how to make money (profit oriented). Both of these criteria don't necessarily lead to the same goal. It may cause traders to make wrong decisions when they consult with researchers. This further impacts the market dynamics. In the real world, there are many different types of traders. Sometimes, they could be "econometricians", "technicians" or none of them. The diversity of traders' behavior and believe is the key phenomenon in economics and financial market, it is what the conventional approaches lack. In the more general framework proposed in this paper, we can study the dynamics of interaction between these different types of traders and their impact on the market dynamics. Of course, in a more general point of view, according to Lucas (1986), each trader has *a set of preferences* to evaluate outcomes arising from particular situation-action combinations. Therefore, the performance of traders' action may be influenced by several criteria. They could be profit, prediction accuracy, or others. Basically, these criteria could also evolve over time. However, it is not easy to model this idea. Also, the architecture used in this paper is more complicated than that used in Chen and Yeh (2000). Therefore, we have to be very prudent when we employ more complicated model.

The questions proposed above are not easy to answer. Of course, we can't have the answers by means of deductive processes. The architecture used in this paper provides us the chance to understand these issues. We can trace the simulations step by step, record the information about the dynamics of traders' behavior, including the prediction about the stock price in the next period, the amount of stock holding, the strategies they use, how sophisticated the strategies are...., and so on.

In the past few years, the new field of *artificial stock market* has been emphasized by several researchers. The reason of this field growing fast is that it opens a broader view, so we can study basic problems in the financial stock market. For example, why are the herd behavior, volatility clustering (*autoregressive conditional heteroskedasticity, ARCH*), *excess kurtosis* (*fat-tail distribution*), *bubbles* and *crashes*, *chaos*, and *unit roots* usually found in the financial markets? And, how do they happen?[3]

The stock market is known as a *complex adaptive system*, the traditional techniques which are top-down perspective can't serve this purpose. Furthermore, the technique tends toward the *agent-based modeling* which

[3]See, for example, Lux (1995, 1997, 1998), Lux and Marchesi (1998), and LeBaron, Arthur and Palmer (1999).

is a *bottom-up* approach. Such idea is more appropriate and reasonable to model social or economic activities. Genetic programming serves this task better than other evolutionary techniques.[4] It is not only a new approach, but also a methodological innovation to economics.

In sum, the advantages of this new framework is as follows:

- It captures the concept of Lucasian adaptive economic agent.
- It allows us to discuss the influence of the level of traders' intelligence.
- It allows more rooms to discuss the psychological activities which determine when the traders visit "school" to improve their strategies and when they would like to modify their thought by themselves.

In this primary study, we focus on the characteristics of multi-population GP with the mechanism of school. Besides replicating the stylized facts, the issues related to the comparison between SGP-based and MGP-based simulations are discussed. In Section 2, the analytical model of our artificial market is described. The experimental design is provided in Section 3. In Section 4, we analyze the simulation results and the concluding remarks is given in Section 5.

2 The Framework of Artificial Stock Market

The basic framework of the artificial stock market considered in this paper is the standard asset pricing model employed in Grossman and Stiglitz (1980). The dynamics of market is determined by an interaction of many heterogeneous agents. Each of them, based on his forecast of the future, maximize his expected utility.

2.1 Description about Traders

For simplicity, we assume that all traders share the same *constant absolute risk aversion* (CARA) utility function,

$$U(W_{i,t}) = -exp(-\lambda W_{i,t}) \tag{1}$$

where $W_{i,t}$ is the wealth of trader i at time period t, and λ is the degree of relative risk aversion. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest. One is the riskless interest-bearing asset called *money*, and the other is the risky asset known as the *stock*. In other words, at each period, each trader has two ways to keep his wealth, i.e.,

$$W_{i,t} = M_{i,t} + P_t h_{i,t} \tag{2}$$

where $M_{i,t}$ and $h_{i,t}$ denotes the money and shares of the stock held by trader i at time t. Given this portfolio ($M_{i,t}$,$h_{i,t}$), a trader's total wealth $W_{i,t+1}$ is thus

$$W_{i,t+1} = (1 + r)M_{i,t} + h_{i,t}(P_{t+1} + D_{t+1}) \tag{3}$$

where P_t is the price of the stock at time period t and D_t is per-share *cash dividends* paid by the companies issuing the stocks. D_t can follow *a stochastic process* not known to traders. Given this wealth dynamics, the goal of each trader is to myopically maximize the one-period expected utility function,

$$E_{i,t}(U(W_{i,t+1})) = E(-exp(-\lambda W_{i,t+1}) \mid I_{i,t}) \tag{4}$$

[4]The advantages of genetic programming has been stressed in LeBaron (2000).

subject to

$$W_{i,t+1} = (1+r)M_{i,t} + h_{i,t}(P_{t+1} + D_{t+1}), \tag{5}$$

where $E_{i,t}(.)$ is trader i's conditional expectations of W_{t+1} given his information up to t (the information set $I_{i,t}$), and r is the riskless interest rate.

It is well known that under $CARA$ utility and Gaussian distribution for forecasts, trader i's desire demand, $h^*_{i,t+1}$ for holding shares of risky asset is linear in the expected *excess return*:

$$h^*_{i,t} = \frac{E_{i,t}(P_{t+1} + D_{t+1}) - (1+r)P_t}{\lambda \sigma^2_{i,t}}, \tag{6}$$

where $\sigma^2_{i,t}$ is the conditional variance of $(P_{t+1} + D_{t+1})$ given $I_{i,t}$.

The key point in the agent-based artificial stock market is the formation of $E_{i,t}(.)$. In this paper, the expectation is modeled by genetic programming. The details is described in next section.

2.2 The Mechanism of Price Determination

Given $h^*_{i,t}$, the market mechanism is described as follows. Let $b_{i,t}$ be the number of shares trader i would like to submit a bid to buy at period t, and let $o_{i,t}$ be the number trader i would like to offer to sell at period t. It is clear that

$$b_{i,t} = \begin{cases} h^*_{i,t} - h_{i,t-1}, & h^*_{i,t} \geq h_{i,t-1}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

and

$$o_{i,t} = \begin{cases} h_{i,t-1} - h^*_{i,t}, & h^*_{i,t} < h_{i,t-1}, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Furthermore, let

$$B_t = \sum_{i=1}^{N} b_{i,t} \tag{9}$$

and

$$O_t = \sum_{i=1}^{N} o_{i,t} \tag{10}$$

be the totals of the bids and offers for the stock at time t, where N is the number of traders. Following Palmer et al. (1994), we use the following simple rationing scheme:[5]

$$h_{i,t} = \begin{cases} h_{i,t-1} + b_{i,t} - o_{i,t}, & \text{if } B_t = O_t, \\ h_{i,t-1} + \frac{O_t}{B_t} b_{i,t} - o_{i,t}, & \text{if } B_t > O_t, \\ h_{i,t-1} + b_{i,t} - \frac{B_t}{O_t} o_{i,t}, & \text{if } B_t < O_t. \end{cases} \tag{11}$$

All these cases can be subsumed into

$$h_{i,t} = h_{i,t-1} + \frac{V_t}{B_t} b_{i,t} - \frac{V_t}{O_t} o_{i,t} \tag{12}$$

where $V_t \equiv \min(B_t, O_t)$ is the volume of trade in the stock.

Based on Palmer et al.'s *rationing scheme*, we can have a very simple price adjustment scheme, based solely on the *excess demand* $B_t - O_t$:

$$P_{t+1} = P_t(1 + \beta(B_t - O_t)) \tag{13}$$

[5] This simple rationing scheme is chosen mainly to ease the burden of intensive computation. An realistic alternative is to introduce the double auction price mechanism.

where β is a function of the difference between B_t and O_t. β can be interpreted as speed of adjustment of prices. One of the β functions we consider is:

$$\beta(B_t - O_t) = \begin{cases} tanh(\beta_1(B_t - O_t)) & \text{if } B_t \geq O_t, \\ tanh(\beta_2(B_t - O_t)) & \text{if } B_t < O_t \end{cases} \tag{14}$$

where *tanh* is the *hyperbolic tangent function*:

$$tanh(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{15}$$

The price adjustment process introduced above implicitly assumes that the total number of shares of the stock circulated in the market is fixed, i.e.,

$$H_t = \sum_i h_{i,t} = H. \tag{16}$$

In addition, we assume that dividends and interests are all paid by cash, so

$$M_{t+1} = \sum_i M_{i,t+1} = M_t(1+r) + H_t D_{t+1}. \tag{17}$$

2.3 Formation of Adaptive Traders

As to the formation of traders' expectations, $E_{i,t}(P_{t+1} + D_{t+1})$, we assume the following functional form for $E_{i,t}(.)$.

$$E_{i,t}(P_{t+1} + D_{t+1}) = \begin{cases} (P_t + D_t)(1 + \theta_1 f_{i,t} * 10^{-4}), & \text{if } -10000.0 \leq f_{i,t} \leq 10000.0, \\ (P_t + D_t)(1 + \theta_1), & \text{if } f_{i,t} > 10000.0, \\ (P_t + D_t)(1 - \theta_1), & \text{if } f_{i,t} < -10000.0 \end{cases} \tag{18}$$

The population of $f_{i,t}$ (i=1,...,N) is formed by genetic programming. That means, the value of $f_{i,t}$ is decode from its GP tree $gp_{i,t}$. According to the martingale hypothesis, the trader holds martingale belief if $E_{i,t}(P_{t+1} + D_{t+1}) = P_t + D_t$. Therefore, the *cardinality* of set $\{i \mid E_{i,t}(P_{t+1} + D_{t+1}) - (P_t + D_t) = 0\}$, denoted by $N_{1,t}$, gives us the information *how well the efficient market hypothesis is accepted among traders*.

As to the subjective risk equation, we modified the equation originally used by Arthur et al. (1997).

$$\sigma^2_{i,t} = (1 - \theta_2)\sigma^2_{t-1|n_1} + \theta_2[(P_t + D_t - E_{i,t-1}(P_t + D_t))^2]. \tag{19}$$

where

$$\sigma^2_{t|n_1} = \frac{\sum_{j=0}^{n_1-1}[P_{t-j} - \overline{P}_{t|n_1}]^2}{n_1 - 1} \tag{20}$$

and

$$\overline{P}_{t|n_1} = \frac{\sum_{j=0}^{n_1-1} P_{t-j}}{n_1} \tag{21}$$

In other words, $\sigma^2_{t-1|n_1}$ is simply the *historical volatility* based on the past n_1 observations.

2.4 Single-Population Based Business School

Before introducing the different architecture of multi-population GP, we have to review the mechanism of "business school" once again. The business school serves as a faculty of researchers. Traders can consult with them when they face the peer pressure or losing lot of money. However, the researchers and traders may have different focus. Traders care about the models or strategies which are helpful for making money. While the researchers put attention on the accuracy of forecasting, for example, *mean absolute percentage error* (MAPE). Therefore, the business school considered here can be viewed as a collection of forecasting models. Then, the single-population can be applied to model its evolution.

Each researcher (forecasting model) is represented by a tree (GP parse tree). The school will be evaluated with a prespecified schedule, say once for every m_1 trading days. The procedure proceeds as follows.

At the evaluation date t, the business school will generate a group of new forecasting models in order to fit (or survive in) the new situation. Each forecasting model $gp_{i,t-1}$ at period $t-1$ will be examined by a *new model* which is generated from the same business school at period $t-1$ by one of the following three genetic operators, reproduction, crossover and mutation, each with probability p_r, p_c and p_m (Table 1). The tournament selection is applied in the procedures of three genetic operators as follows:

- Reproduction:
  Two forecasting models (GP trees) are randomly selected from $GP_{i,t-1}$. The one with lower MAPE over the last m_2 days' forecasts is chosen as the *new model*.

- Mutation:
  Two forecasting models are randomly selected from $GP_{i,t-1}$. The one with lower MAPE over the last m_2 days' forecasts is chosen as the candidate with the probability of p_M (In Table 1, the probability is 0.3) being mutated. No matter the candidate is mutated or not, the one (the new one if it is mutated) is chosen as the *new model*.

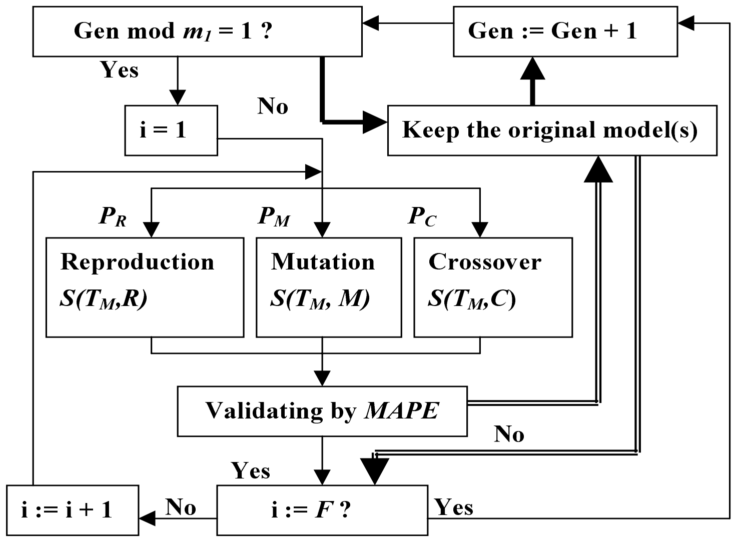
- Crossover:
  Two pairs of forecasting models are randomly chosen, say ($gp_{j_1,t-1}$,$gp_{j_2,t-1}$) and ($gp_{k_1,t-1}$,$gp_{k_2,t-1}$). The one with lower MAPE in each pairs is chosen as *parent*. One of the two children which is born by the crossover of their parents is randomly chosen as the *new model*.

Therefore, each forecasting model at period $t-1$ is compared with the new model generated by one of the three genetic operators based on the criterion of MAPE. The lower one is selected as the new forecasting model in next period (generation). The following is a pseudo program of the procedure of **Business School** (Also see Flowchart 1). Table 1 is an example of the specification of the control parameters to evolve the business school.

Procedure [Business School]

0. **begin**
   1. Calculate $MAPE(gp_{i,t})$
   2. A = **Random**(R,C,M) with (p_r,p_c,p_m)
   3. If A = C, go to step (11).
   4. (gp_1, gp_2) = (**Random**(GP_{t-1}),**Random**(GP_{t-1}))
   5. Calculate $MAPE(gp_1)$ and $MAPE(gp_2)$.
   6. gp_{new} = **Tournament Selection** ($MAPE(gp_1)$,$MAPE(gp_2)$)
   7. If A = R, go to step (17).
   8. $gp_{new} \leftarrow$ **Mutation** (gp_{new})
   9. Calculate $MAPE(gp_{new})$
   10. Go to step (17)
   11. Randomly select two pairs of trees from GP_{t-1}
   12. Calculate MAPE of these two pairs of GP trees

Flowchart 1 : Evolution of the Business School



***T_M* : Tournament selection according to MAPE**

***$S(T_M,i)$* : Selection procedure according to tournament selection with the criterion MAPE based on i (genetic operator)**

***i* : R, M, C represent reproduction, mutation and crossover respectively**

***m_1* : Evaluation cycle**

***F* : Number of faculty member**

gp_1 = **Tournament Selection** (Pair 1)
gp_2 = **Tournament Selection** (Pair 2)
$(gp_1, gp_2) \leftarrow$ **Crossover** (gp_1, gp_2)
gp_{new} = **Random**(gp_1, gp_2)
$gp_{i,t}$ = **Tournament Selection** $(MAPE(gp_{i,t-1}), MAPE(gp_{new}))$
end

2.5 The Interaction between Traders' Behavior and Business School

The main distinction between SGP and MGP is on the formation of traders' behavior. In the architecture of MGP, we allow the traders to think about how to react the environment by themselves. Therefore, at the evaluation date t, each trader has to make a decision. Should he change his mind (the strategy used in the previous period)? If the answer is yes, where should he consult? the business school or himself.

The way we use to model this psychological activities can be summarized as the following procedure. First, whether each trader change his mind or not depends on his net change of wealth over the last n_2 days compared with other traders. Let $R_{i,t}$ be his rank and $\Delta W_{i,t}^{n_2}$ be this net change of wealth of trader i at time period t, i.e.,

$$\Delta W_{i,t}^{n_2} \equiv W_{i,t} - W_{i,t-n_2}, \tag{22}$$

Table 1: Parameters of the Stock Market (I)

The Stock Market	
Shares of the stock (H)	100
Initial money supply (M_1)	100
Interest rate (r)	0.1
Stochastic process (D_t)	Uniform distribution, U(5.01,14.99)
Price adjustment function	*tanh*
Price adjustment (β_1)	0.2×10^{-4}
Price adjustment (β_2)	0.2×10^{-4}
Parameters of Genetic Programming	
Function set	$\{+, -, \times, \%, Sin, Cos, Exp, Rlog, Abs, Sqrt\}$
Terminal set	$\{P_t, P_{t-1}, \cdots, P_{t-10}, P_t + D_t, \cdots, P_{t-10} + D_{t-10}\}$
Selection scheme	Tournament selection
Tournament size	2
Probability of creating a tree by reproduction	0.10
Probability of creating a tree by immigration	0.20
Probability of creating a tree by crossover	0.35
Probability of creating a tree by mutation	0.35
Probability of mutation	0.3
Probability of leaf selection under crossover	0.5
Mutation scheme	Tree mutation
Replacement scheme	(1+1) Strategy
Maximum depth of tree	17
Maximum number in the domain of Exp	1700
Number of generations	10000

Then, the probability for the trader i changing his mind at period t is determined by

$$p_{i,t} = \frac{R_{i,t}}{N}. \tag{23}$$

Equation (23) means that the trader with higher rank faces higher peer pressure. Hence, he has higher motivation to change mind.

Second, in addition to the peer pressure, each trader also cares about his own satisfaction. That means, traders intend to improve their growth rate of income. Let the growth of income over the last n_2 days be

$$\delta_{i,t}^{n_2} = \frac{\Delta W_{i,t}^{n_2} - \Delta W_{i,t-n_2}^{n_2}}{\mid \Delta W_{i,t-n_2}^{n_2} \mid}, \tag{24}$$

and let $q_{i,t}$ be the probability that trader i will look for new strategies at the end of the tth period, assume that it is determined by

$$q_{i,t} = \frac{1}{1 + exp^{\delta_{i,t}^{n_2}}}. \tag{25}$$

Therefore, the traders make great (less) progress have lower (higher) probability to change mind.

$$\lim_{\delta_{i,t}^{n_2} \to \infty} q_{i,t} = 0, \tag{26}$$

Table 2: Parameters of the Stock Market (II)

Business School	
Number of faculty members (F)	500
Criterion of fitness (Faculty members)	MAPE
Evaluation cycle (m_1)	20
Sample Size (MAPE) (m_2)	10
Search intensity in Business School (I_s^*)	5
Traders	
Number of traders (N)	100
Number of ideas for each trader	20
Degree of RRA (λ)	0.5
Criterion of fitness (Traders)	Increments in wealth (Income)
Sample size of $\sigma^2_{t\|n_1}$ (n_1)	10
Evaluation cycle(n_2)	1
Sample size (n_3)	10
Initial probability of consulting business school ($p_{i,t}^{sm}$)	0.5
Search intensity by Trader itself (I_h^*)	5
θ_1	0.5
θ_2	0.0133

and

$$\lim_{\delta_{i,t}^{n_2} \to -\infty} q_{i,t} = 1. \tag{27}$$

Based on the description above, we know the probability ($r_{i,t}$) that trader i decides to change mind.

$$r_{i,t} = p_{i,t} + (1 - p_{i,t})q_{i,t} = \frac{R_{i,t}}{N} + \frac{N - R_{i,t}}{N} \frac{1}{1 + exp^{\delta_{i,t}^k}} \tag{28}$$

However, we have not yet mentioned how a trader come up with a new idea. In order to model this process, we introduce a probability measure to describe this psychological activity. Let $p_{i,t}^{sm}$ be the probability that trader i would like to look for new ideas from business school. On the other hand, the probability that trader i decide to work out new ideas by himself is $1 - p_{i,t}^{sm}$. This probability is determined by

$$p_{i,t}^{sm} = \begin{cases} p_{i,t-1}^{sm} - (r_{i,t} - r_{i,t-1})p_{i,t-1}^{sm}, & \text{if } \ r_{i,t} - r_{i,t-1} \geq 0, \ \text{Case1}, \\ p_{i,t-1}^{sm} - (r_{i,t} - r_{i,t-1})(1 - p_{i,t-1}^{sm}), & \text{if } \ r_{i,t} - r_{i,t-1} < 0, \ \text{Case1}, \\ p_{i,t-1}^{sm} + (r_{i,t} - r_{i,t-1})(1 - p_{i,t-1}^{sm}), & \text{if } \ r_{i,t} - r_{i,t-1} \geq 0, \ \text{Case2}, \\ p_{i,t-1}^{sm} + (r_{i,t} - r_{i,t-1})p_{i,t-1}^{sm}, & \text{if } \ r_{i,t} - r_{i,t-1} < 0, \ \text{Case2}, \\ p_{i,t-1}^{sm}, & \text{Case3}. \end{cases} \tag{29}$$

where Case1 means that trader i looked for new idea from business school at period $t-1$, Case2 means that trader i made new idea by himself at period $t-1$, and Case3 means that trader i didn't change mind at period $t-1$.

The idea of Equation (29) is very straightforward. If a trader has high motivation to change mind, he will think about whether the result is due to the wrong decision, for example, consulted with researchers in the business school, made in the previous period or not. Therefore, he is prone to reduce the confidence about business school.

Once a trader decides to go to business school, he has to consult with one researcher at school randomly (or pick up one forecasting model at school randomly). Then, he compares the new idea with his old one used in the previous period based on the criterion of MAPE by means of calculating the stock price and dividends over the last n_3 trading days. If the new idea outperforms his old idea, he will adopt the new one. Otherwise, he will look for new one at school once again until either he succeeds or he fails for I_s^* times. Of course, it is very possible that the trader decides to work out a new idea by himself. The new idea is also generated by three genetic operators which is happened in his mind. He has to compare the new idea with the old one based on the net change of wealth over the last n_2 trading days.[6] If the new one outperforms the old one, he will adopt it. Otherwise, he will think about it once again until he succeeds or he fails for I_h^* times. These procedures are summarized as a pseudo program listed below (Also see Flowchart 2).

Procedure 1 [Come up with a new idea]

0. **begin**
    1. If go the business school, go to Procedure 2
    2. Go to Procedure 3
3. **end**

Procedure 2 [Visiting Business School]

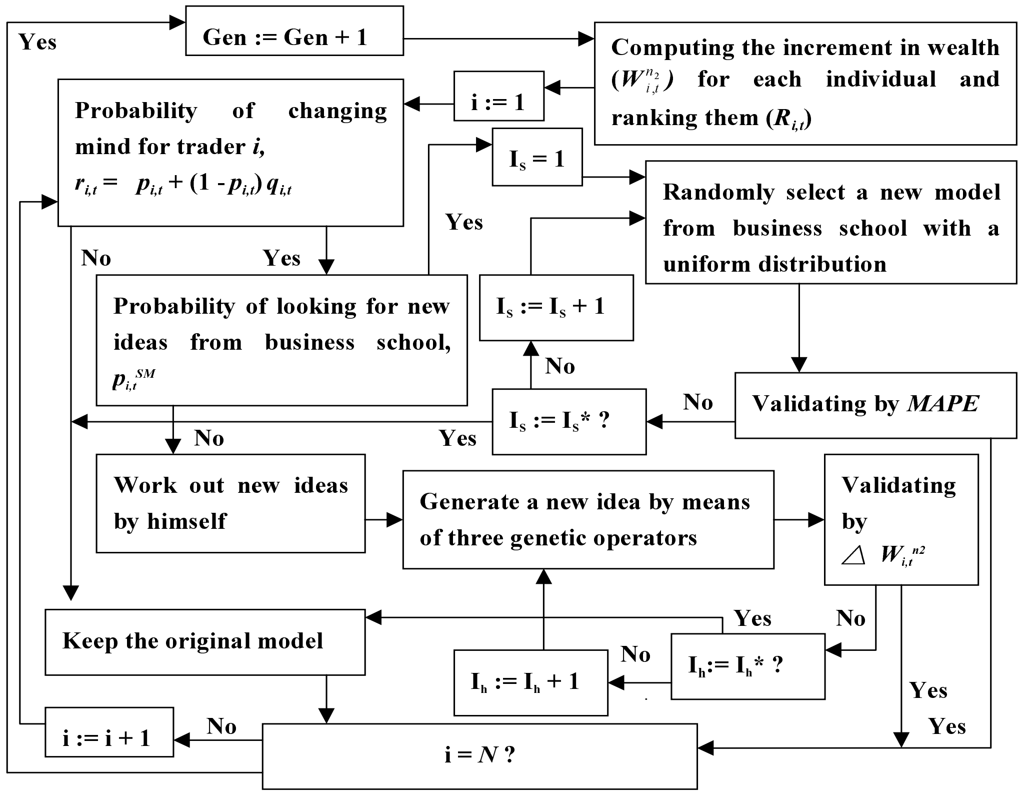
0. **begin**
    1. Calculate $MAPE(f_{i,t})$
    2. $I_s \leftarrow 1$
    3. Randomly select a $gp_{j,t}$ ($\sim U[1, 500]$)
    4. Calculate $MAPE(gp_{j,t})$
    5. If $MAPE(gp_{j,t}) < MAPE(f_{i,t})$, go to Step (10)
    6. $I_s \leftarrow I_s + 1$
    7. If $I_s \leq I_s^*$, go to step (3)
    8. $f_{i,t+1} = f_{i,t}$
    9. Go to Step (11)
    10. $f_{i,t+1} = gp_{j,t}$
11. **end**

Procedure 3 [Work out new ideas by himself]

0. **begin**
    1. Calculate $\Delta W_{i,t}^{n_2}$ $(W_{i,t} - W_{i,t-n_2})$
    2. $I_h \leftarrow 1$
    3. Generate a new $gp_{i,t,j}$ based on one of three genetic operators
    4. Calculate $\Delta W_{i,t,j}^{n_2}$
    5. If $\Delta W_{i,t}^{n_2} < \Delta W_{i,t,j}^{n_2}$, go to Step (10)
    6. $I_h \leftarrow I_h + 1$
    7. If $I_h \leq I_h^*$, go to step (3)
    8. $f_{i,t+1} = f_{i,t}$
    9. Go to Step (11)

[6] Of course, the new idea doesn't really be used. We can assume that the trader used the new idea since n_2 trading days before, then calculate its performance over these n_2 trading days.

Flowchart 2 : Traders' Search Process



N : **Number of traders**

10. $f_{i,t+1} = gp_{i,t,j}$

11. **end**

3 Experimental Designs

In order to facilitate us to understand the influence of "intelligent" agents, we consider three different scenarios, Market A, B and C. The difference between these markets are shown in Table 3. In Market A, the SGP based market is simulated. The results are compared with that in the Market B where traders work out new ideas by themselves rather that consult with researchers. The difference between these two simulations provides the effects of prediction accuracy oriented and profit oriented agents. Market C is a more realistic one. The agents can adapt themselves to modify the confidence about the school (or themselves). This design coincides with part of human's psychological activity.

Based on these different designs, simulations are conducted according to the parameters shown in Table 1 and 2. Each trader in our simulations has twenty ideas in his mind. These ideas also evolve from generation to generation. Therefore, the traders are very adaptive. In Table 4, the important variables related to the traders and market are summarized. These are helpful for us to go one step further to analyze our simulation results. For example, the number of martingale believers ($N_{1,t}$) tell us how many traders hold martingale belief at period t. The time series $\{N_{1,t}\}$ also provides us the information how the market dynamics interact with the

Table 3: The Market Structure

Market	Architecture	Probability of consulting business school
A	SGP with business school	1.0
B	MGP with business school	0.0
C	MGP with business school	Adaptive adjustment, Equation (29)

Table 4: Time Series Generated from the Artificial Stock Market:

Aggregate Variables	
Stock price	P_t
Trading volumes	V_t
Totals of the bids	B_t
Totals of the offers	O_t
# of martingale believers	$N_{1,t}$
# of traders registered to business school	$N_{2,t}$
# of traders with successful search in business school	$N_{3,t}$
# of traders registered to himself	$N_{4,t}$
# of traders with successful thinking	$N_{5,t}$
Individual Trader	
Forecasts	$f_{i,t}$
Subjective risks	$\sigma_{i,t}$
Bid to buy	$b_{i,t}$
Offer to sell	$o_{i,t}$
Wealth	$W_{i,t}$
Income	$\Delta W^1_{i,t}$
Rank of profit-earning performance	$R_{i,t}$
Complexity (depth of $f_{i,t}$)	$k_{i,t}$
Complexity (# of nodes of $f_{i,t}$)	$\kappa_{i,t}$

traders' believe. We are also interested in how well the traders "live" in the market. Do they change mind usually? Do they benefit from business school or their mind? These subjects can be refereed to ($N_{2,t}$,$N_{4,t}$) and ($N_{3,t}$,$N_{5,t}$). As to the $k_{i,t}$ and $\kappa_{i,t}$, they are complexity measures of traders' strategies in terms of GP-tree. How the complexity of traders' strategies coevolve with market dynamics is also an important issue. From the viewpoint of econometrics, the typical phenomena found in the financial markets are analyzed in the paper. For example, is $\{P_t\}$ normal or stationary? is return series ($\{R_t\}$) independently and identically distributed? Or, is $\{R_t\}$ nonlinearly dependent? Does $\{R_t\}$ have the property of GARCH...., and so on.

4 Simulation Results

Based on the experimental design given in Table 1 and 2, a single run with 10000 generations (periods) is conducted for Market A, B and C respectively. The time series data is further divided into ten subperiods. The basic statistics, econometric properties and the important variables shown in Table 4 for each subperiod are then calculated. In this paper, the simulation results are analyzed from two parts. One is the macro-properties, the econometric properties of time series. The other is micro-structure, traders' behavior and believes. This approach provides us the information about the relation between the traders' believes and market dynamics. Therefore, we can understand how these *intelligent* traders influence the market.

4.1 Macro-properties

As to the properties of time series, whether this artificial stock market can replicate the stylized facts found in the financial markets or not is the first question for the researchers working on this field will face.

1. Are stock prices and stock return normally distributed?
2. Does the price series have a unit root?
3. Are stock return independently and identically distributed?

The time series of the stock price in the last 9000 periods for each market is drawn in Figure 1, 2 and 3 respectively.[7] In these figures, the range of price fluctuation in Market B is higher than that in Market A and C. However, the homogeneous rational expectations equilibrium price under full information is

$$P_f = \frac{1}{r}(\overline{d} - \lambda\sigma^2 h) \tag{30}$$

where r is interest rate, $\overline{d}$ is the average of dividends, σ^2 is the variance of dividends series and h is the average of shares of the stock for each trader. Therefore, the fundamental price (P_f) in these markets is 58.375. It implies that the market composed of profit oriented traders tends to overestimate the *intrinsic value* of the stock.

Stock return is derived by

$$r_t = \ln(P_t) - \ln(P_{t-1}) \tag{31}$$

Figure 4, 5, and 6 are the time series plots of the stock return, and the basic statistics of the time series of stock price and return are given in Table 5 and 6. According to the Jarqu-Bera normality test, neither the stock price nor return follows normal distribution. Moreover, the leptokurtosis of the stock return also confirms the fat tail phenomenon usually found in the financial data. In Table 7, the result of Dickey-Fuller test shows that there exists a unit root in the price series for each market except the first subperiod of Market A and the second subperiod of Market C based on 99% significance level.

As to the third question, it is related to the classical version of *efficient market hypothesis*. Technically speaking, the market is efficient if there exists no linear and nonlinear structure in the return series. Here, we employ the procedure proposed by Chen, Lux and Marchesi (1999). First, the Rissanen's predictive stochastic complexity (**PSC**) is used to filter the linear signal. Once the linear signal is filtered, if there is any structure left in the residual, it must be nonlinear. Therefore, the most frequently used nonlinear test, **BDS test**, is then applied to the residual series. However, there are two parameters needed to be chosen. One is the distance parameter (ϵ standard deviations), and the other is the *embedding dimension* (DIM). Here, the result of BDS test is performed under $\epsilon = 1$ and DIM=2,3,4,5. In Table 8, we found that there exists linear structure in the three markets, while the R^2 is very low. Moreover, most periods fail to reject the nonexistence of nonlinear signal. However, it is well known in econometrics that nonlinearity could be found in the second moment. The

[7] In the beginning of our simulation, trader are unaware of the characteristics of the market. Therefore, price is highly overestimated. However, as time goes on, traders gradually modify their expectations. Price then goes down below $90. In order to focus on the major range of price fluctuation, we only report the last 9000 periods.

Figure 1 : Time Series Plot of the Stock Price (Market A)

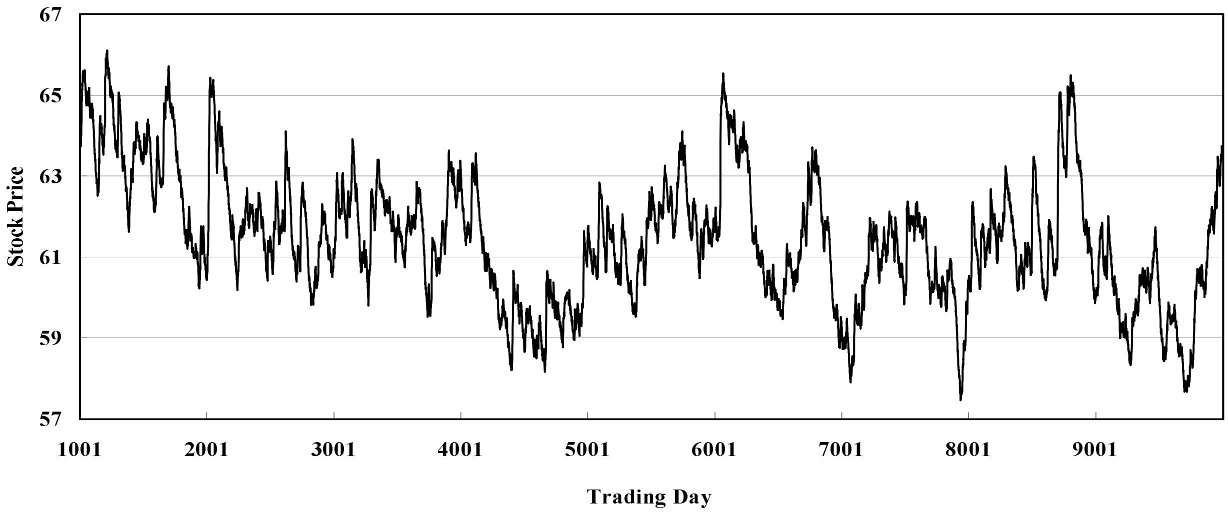


Figure 2 : Time Series Plot of the Stock Price (Market B)

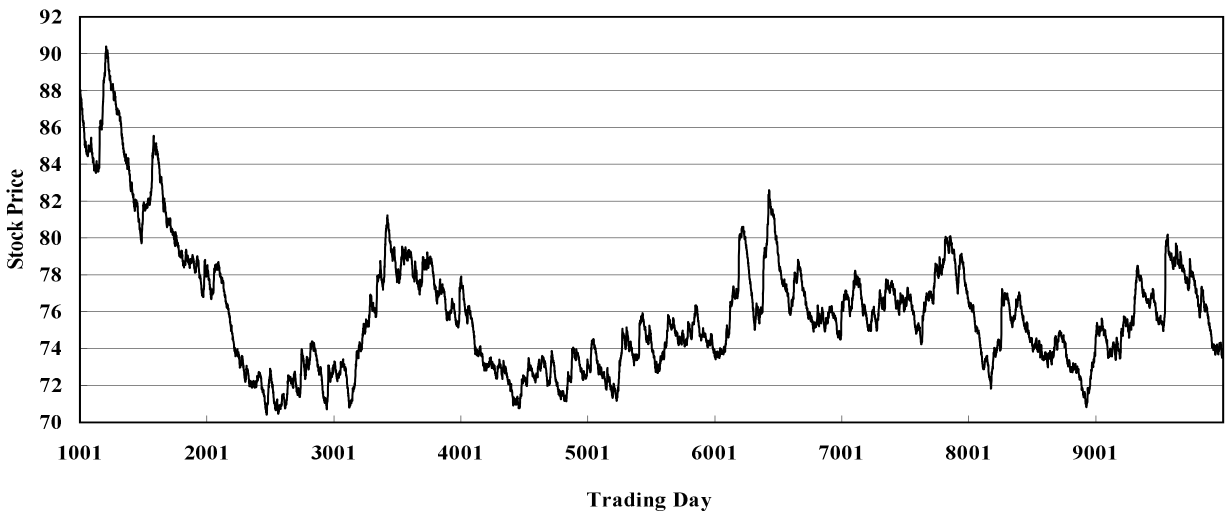


Figure 3 : Time Series Plot of the Stock Price (Market C)

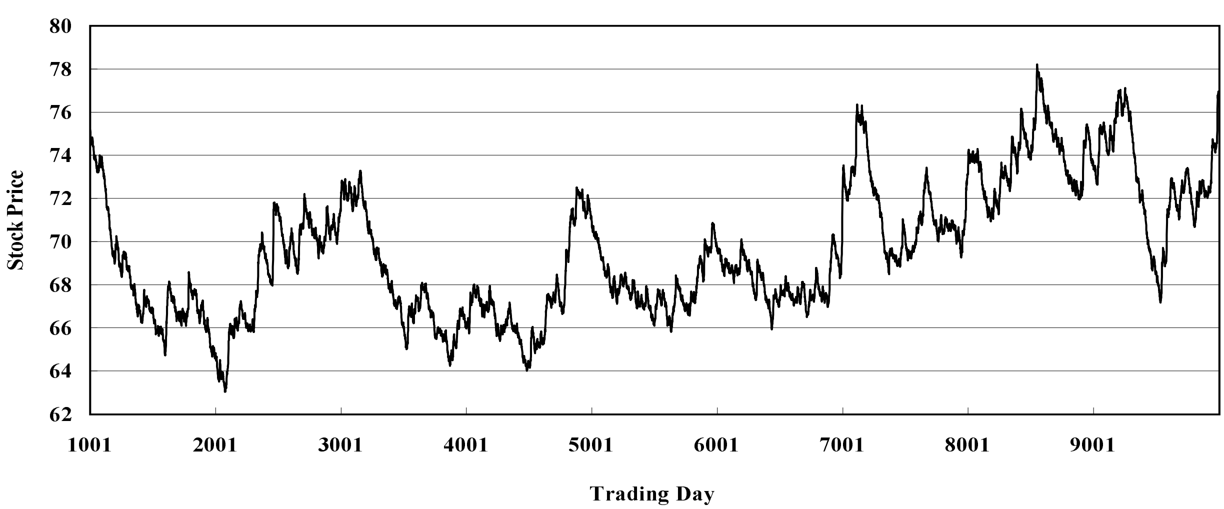


Figure 4 : Time Series Plot of the Stock Return (Market A)

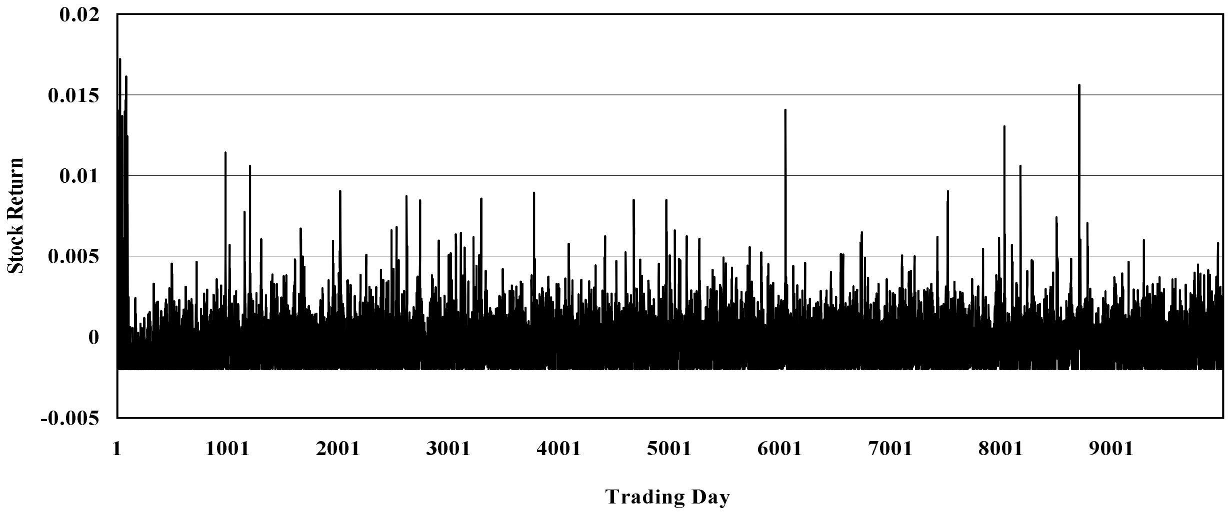


Figure 5 : Time Series Plot of the Stock Return (Market B)

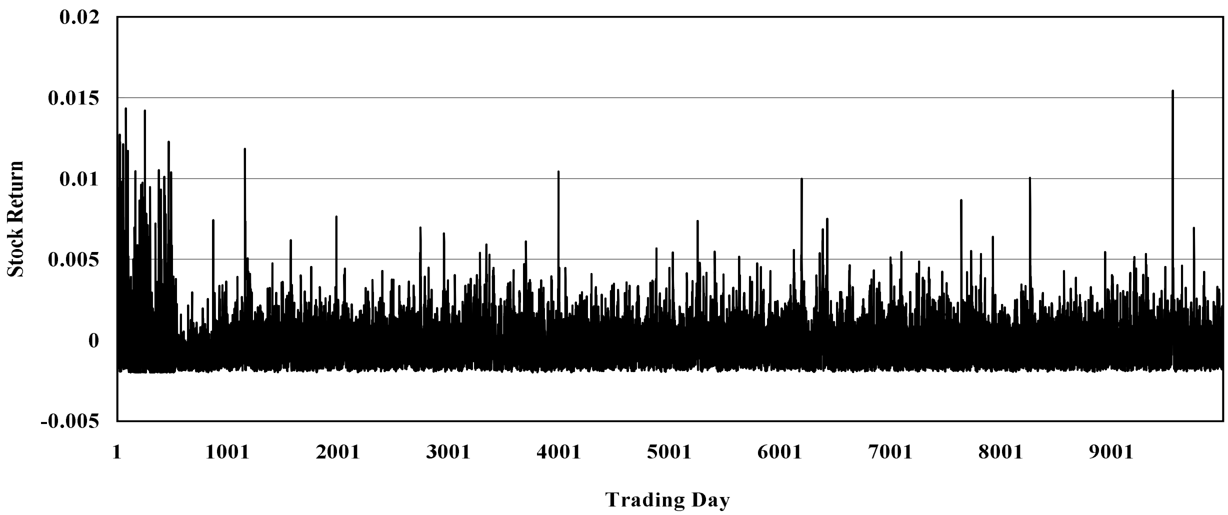


Figure 6 : Time Series Plot of the Stock Return (Market C)

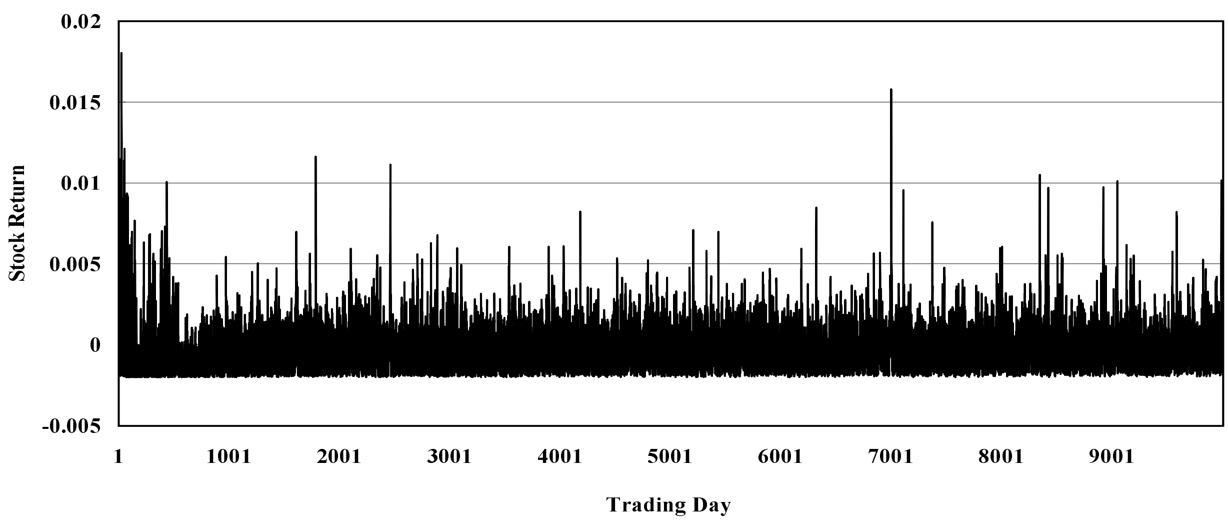


Table 5: Basic Statistics of the Artificial Stock Price Series

Market A						
Periods	$\overline{P}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	78.32788	13.62220	0.692494	2.200712	106.5439	0.000000
1001-2000	63.39534	1.357275	-0.354404	2.312132	40.64887	0.000000
2001-3000	61.91586	1.180842	0.917530	3.636987	157.2165	0.000000
3001-4000	61.86186	0.888477	-0.331832	2.725438	21.49304	0.000022
4001-5000	60.10845	1.181903	0.927996	3.232536	145.7824	0.000000
5001-6000	61.60884	0.896830	0.132566	2.951264	3.027935	0.220035
6001-7000	61.78188	1.633962	0.356041	2.035466	59.89105	0.000000
7001-8000	60.45598	1.156648	-0.521288	2.493351	55.98573	0.000000
8001-9000	61.87491	1.345820	0.902358	3.066857	135.8945	0.000000
9001-10000	60.10790	1.285679	0.461578	2.999811	35.50906	0.000000
Market B						
Periods	$\overline{P}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	108.0933	12.68692	-0.451207	1.650351	109.8293	0.000000
1001-2000	82.78370	3.343650	0.232044	2.110596	41.93407	0.000000
2001-3000	73.32885	2.196288	1.075537	3.077568	193.0472	0.000000
3001-4000	76.48211	2.449680	-0.572735	2.382626	70.55220	0.000000
4001-5000	72.99718	1.330041	1.273381	5.006009	437.9195	0.000000
5001-6000	74.01037	1.124712	-0.481808	2.604814	45.19695	0.000000
6001-7000	76.87204	2.054136	0.659705	2.813349	73.98679	0.000000
7001-8000	76.97926	1.258317	0.429422	2.765607	33.02311	0.000000
8001-9000	74.16528	1.437827	0.134599	2.422469	16.91711	0.000212
9001-10000	76.18609	1.726372	0.278032	2.009311	53.77799	0.000000
Market C						
Periods	$\overline{P}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	95.37813	12.08421	-0.435627	1.738648	97.92054	0.000000
1001-2000	68.12425	2.558067	1.192156	3.467249	245.9694	0.000000
2001-3000	68.60801	2.415664	-0.676687	2.251292	99.67445	0.000000
3001-4000	68.13919	2.498183	0.592067	2.058576	95.35216	0.000000
4001-5000	67.38430	2.232205	0.929403	2.813418	145.4156	0.000000
5001-6000	67.99319	1.176889	0.671658	2.598026	81.92002	0.000000
6001-7000	68.13972	0.931234	0.497421	3.292627	44.80586	0.000000
7001-8000	71.38652	1.850401	0.856242	3.030693	122.2310	0.000000
8001-9000	73.89949	1.524528	0.394129	2.855530	26.75926	0.000002
9001-10000	72.78556	2.353734	-0.239208	2.479806	20.81177	0.000030

Table 6: Basic Statistics of the Artificial Stock Return Series

Market A						
Periods	$\overline{r}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	-0.000334	0.002317	3.285575	18.63701	11987.34	0.000000
1001-2000	-0.000059	0.001681	1.118201	5.572807	484.2014	0.000000
2001-3000	0.000012	0.001765	1.167794	5.453934	478.1986	0.000000
3001-4000	0.000031	0.001692	0.891895	4.381115	212.0578	0.000000
4001-5000	-0.000022	0.001652	1.028622	4.881467	323.8405	0.000000
5001-6000	-0.000014	0.001635	0.728591	3.365408	94.03773	0.000000
6001-7000	-0.000039	0.001653	1.495067	10.35198	2624.689	0.000000
7001-8000	0.000019	0.001627	0.890006	4.498190	225.5423	0.000000
8001-9000	-0.000006	0.001846	2.006383	12.64014	4543.108	0.000000
9001-10000	0.000056	0.001567	0.418674	2.545671	37.77750	0.000000
Market B						
Periods	$\overline{r}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	-0.000020	0.002738	2.283185	8.642649	2195.468	0.000000
1001-2000	-0.000115	0.001555	1.308220	7.179182	1012.972	0.000000
2001-3000	-0.000070	0.001479	0.868873	3.959215	164.1606	0.000000
3001-4000	0.000063	0.001601	1.019151	5.090089	355.1311	0.000000
4001-5000	-0.000061	0.001404	0.556026	2.728130	54.60715	0.000000
5001-6000	0.000005	0.001504	0.844090	3.664948	137.1712	0.000000
6001-7000	0.000032	0.001638	1.074103	4.932855	347.9466	0.000000
7001-8000	0.000005	0.001541	0.814424	3.891260	143.6455	0.000000
8001-9000	-0.000025	0.001470	0.950137	5.776774	471.7296	0.000000
9001-10000	-0.000008	0.001642	2.076164	15.74482	7486.346	0.000000
Market C						
Periods	$\overline{r}$	σ	Skewness	Kurtosis	Jarqu-Bera	p-value
1-1000	-0.000176	0.002498	2.356092	10.60410	3334.458	0.000000
1001-2000	-0.000148	0.001514	1.115679	6.842539	822.6695	0.000000
2001-3000	0.000105	0.001710	1.116244	5.578395	484.6717	0.000000
3001-4000	-0.000087	0.001489	0.635668	3.135544	68.11116	0.000000
4001-5000	0.000071	0.001570	0.669616	3.427464	82.34453	0.000000
5001-6000	-0.000018	0.001497	0.730715	3.544839	101.3596	0.000000
6001-7000	0.000039	0.001636	1.990691	15.69916	7379.999	0.000000
7001-8000	0.000016	0.001587	1.045142	5.452793	432.7285	0.000000
8001-9000	-0.000001	0.001622	1.370530	7.217321	1054.134	0.000000
9001-10000	0.000042	0.001677	1.284784	6.277304	722.6417	0.000000

Table 7: Unit Root Test

	Market A	**Market B**	**Market C**
Periods	DF of P_t	DF of P_t	DF of P_t
1-1000	-3.743852	-0.205775	-1.629092
1001-2000	-1.102648	-2.378888	-3.287183
2001-3000	0.139497	-1.641350	1.907363
3001-4000	0.556529	1.176659	-1.993159
4001-5000	-0.452132	-1.490206	1.412270
5001-6000	-0.070920	0.119053	-0.387298
6001-7000	-0.775960	0.592499	0.773165
7001-8000	0.399396	-0.019152	0.194885
8001-9000	-0.163501	-0.550615	-0.163876
9001-10000	1.145653	-0.205708	0.787532

The MacKinnon critical values for rejection of hypothesis of a unit root at 99% (95%) significance level is -2.5668 (-1.9395).

(G)ARCH family of time series are designed to capture the regularities in the behavior of volatility which is the phenomenon of volatility clustering. Therefore, we carried out the Lagrange multiplier test for the presence of ARCH effects. If the null hypothesis of ARCH effect is rejected, we will further identify the GARCH structure by using the Bayesian Information Criterion (BIC). The results are exhibited in Table 9. Clearly, the presence of GARCH effects seems to be very robust compared with the BDS test.

4.2 Micro-structure

Based on the result described above, the difference between the three markets is not large. However, it doesn't imply anything about the micro-structure. We are interested in how the traders behave in the three markets. The basic questions proposed in the previous research (Chen and Yeh, 2000) would help us to examine this issue. Are they *martingale believers*? Do they search new ideas intensively? What kind of strategies do the traders employ?...., and so on.

In Table 10, it evidenced that, on the average, the martingale belief doesn't survive in the traders' mind. The time series plot of the number of martingale believers for each market is also given in Figure 7, 8 and 9 respectively. In the 10000 periods, there is no more than eight traders holding martingale believes in each period. Now, we may interested in what the traders actually do if they don't believe martingale hypothesis. In Table 11 and 12, we can get an impression about the traders' search and thinking activities. In each market, there is about 90% traders trying to change their ideas which means the price dynamics is not easily captured. However, the inside information in each market is different. In Market A, the traders follow the ideas from business school which is prediction accuracy oriented. There is about 50% traders who registered to business school benefit from their search. Clearly, search is useful. It also implies that the *useful forecasting models* change over time. There is no robust forecasting model in this environment. On the other hand, the business school updates knowledge every 20 periods. As time goes on, the updated models have been out-of-date before it updates once again. In this situation, even though the traders is very adaptive in terms of they modify strategies at each period, but they can only reuse the *old* ideas. Therefore, the chance they benefit from business school is getting lower. It is also exhibited in the decrease of the average of traders with successful

Table 8: PSC Filtering and BDS Test

Market A						
Periods	(p,q), (R^2)	DIM=2	DIM=3	DIM=4	DIM=5	Reject
1-1000	(1,0) (0.084586)	2.763187	3.355127	3.573923	3.801602	Yes
1001-2000	(1,2) (0.095338)	1.170001	1.536866	1.657808	1.732317	No
2001-3000	(1,2) (0.130978)	1.446862	1.503426	1.545025	1.537045	No
3001-4000	(1,0) (0.046633)	0.983322	1.267603	1.522006	1.634546	No
4001-5000	(2,0) (0.069238)	1.314665	1.588530	1.696342	1.709995	No
5001-6000	(0,3) (0.051332)	0.876030	1.318596	1.522116	1.618994	No
6001-7000	(1,0) (0.066648)	0.804488	0.863171	0.959267	1.000311	No
7001-8000	(0,2) (0.072331)	1.254773	1.393243	1.425195	1.392453	No
8001-9000	(1,2) (0.148256)	1.156910	1.209763	1.267034	1.342426	No
9001-10000	(1,0) (0.017866)	0.822629	0.955026	1.098682	1.245252	No
Market B						
Periods	(p,q), (R^2)	DIM=2	DIM=3	DIM=4	DIM=5	Reject
1-1000	(1,0) (0.120571)	2.631589	2.950451	3.164891	3.472194	Yes
1001-2000	(3,3) (0.086179)	0.910340	1.215087	1.292173	1.361617	No
2001-3000	(1,0) (0.029251)	1.436759	1.382518	1.465204	1.517484	No
3001-4000	(1,2) (0.079725)	0.992550	1.185630	1.262048	1.248313	No
4001-5000	(0,0) (0.000000)	1.028910	1.154727	1.229629	1.275487	No
5001-6000	(2,2) (0.048942)	1.465323	1.476508	1.532547	1.544393	No
6001-7000	(2,2) (0.086259)	1.217710	1.568579	1.670191	1.674020	No
7001-8000	(1,0) (0.036988)	0.993627	1.141134	1.259636	1.346128	No
8001-9000	(1,0) (0.048028)	0.777911	0.794150	0.835682	0.798633	No
9001-10000	(2,2) (0.083901)	1.003050	1.238080	1.438938	1.546275	No
Market C						
Periods	(p,q), (R^2)	DIM=2	DIM=3	DIM=4	DIM=5	Reject
1-1000	(1,2) (0.172231)	3.198414	3.851904	4.487734	5.187971	Yes
1001-2000	(1,0) (0.043393)	0.831786	1.056206	1.160091	1.175626	No
2001-3000	(0,2) (0.081779)	1.679190	2.057802	2.273284	2.336968	No
3001-4000	(2,2) (0.037457)	0.993473	1.261842	1.242837	1.188526	No
4001-5000	(3,2) (0.053997)	1.157223	1.210224	1.183651	1.155643	No
5001-6000	(4,0) (0.033742)	0.850240	0.984305	1.084103	1.078252	No
6001-7000	(1,0) (0.056553)	0.767305	0.884078	1.002873	1.017948	No
7001-8000	(1,0) (0.060121)	1.452094	1.545296	1.572037	1.598763	No
8001-9000	(1,0) (0.056991)	1.591326	1.694601	1.693094	1.603306	No
9001-10000	(3,3) (0.108706)	0.928695	1.148883	1.323939	1.417919	No

The BDS test statistic is asymptotically normal with mean 0 and standard deviation 1. The significance level of the test is set at 0.95.

Table 9: GARCH Modeling

Periods	**Market A**	**Market B**	**Market C**
1-1000	(1,1)	(2,2)	(1,2)
1001-2000	(1,1)	(1,2)	(1,1)
2001-3000	(0,1)	(1,1)	(1,1)
3001-4000	(1,1)	(1,1)	(1,1)
4001-5000	(1,1)	(1,1)	(1,1)
5001-6000	(1,1)	(0,1)	(0,1)
6001-7000	(1,1)	(1,1)	(1,1)
7001-8000	(1,1)	(1,1)	(1,1)
8001-9000	(1,1)	(0,1)	(0,1)
9001-10000	(0,1)	(1,1)	(1,1)

search on the h day after the business school has updated the information (See Table 13).

In Market B, the traders' action follows profit and they renew their ideas at each period. It induces a different phenomenon. On the average, there is about 17 traders (except first subperiod) who benefit from their thinking. The average of the ratio of traders with successful thinking is also lower. The reasons are as follows. First, the traders' action is to myopically maximize the one-period expected utility and they evaluate the ideas too frequently (at each period). It makes the strategies have lower chance to survive. Second, the traders' ideas in their mind also evolve at each period. Therefore, even though each trader has 20 ideas, these ideas easily tend to evolve similar structures.[8] It explains the low ratio of traders with successful thinking.

In the beginning of the simulation of Market C, there exists both types of traders, prediction accuracy and profit oriented traders. Due to both criteria coevolving in this market, it makes the traders more difficult to capture the price dynamics and make profit. Therefore, more traders intend to change their strategies (See the final column in Table 11), and the number of traders getting useful ideas gradually decreases (See the final column in Table 12). The interesting thing is that the traders' behavior tends to be profit oriented. The market dynamics is getting dominated by them, so the number of profit oriented traders with successful thinking increases. This makes the prediction accuracy oriented traders more difficult to survive. There are two possible reasons to explain this phenomenon.

- Profit oriented traders are more easier to survive.
- Profit oriented traders are more adaptive compared with business school.

In order to test the hypotheses, we can set the equal evaluation cycle for the traders and business school ($m_1 = n_2$), for example, 20. The second conjecture is related to the influence of speculators who care about short-term profits and investors who focus on long-term profits. These problems will be discussed in the future research.

The information about the complexity of evolving strategies also confirms the differences between these markets. The results are exhibited in Table 14. Figure 10-15 are the time series plots of the complexity of

[8] How to design the evolution of traders' mind is an important issue. It will influence the traders' adaptability. We may only evolve the realized strategies and keep the other strategies unchanged, or the synthesis of both methods. Of course, this problem is not easy to solve. It is left to the future research.

Table 10: Average of the Number of Martingale Believers ($\overline{N}_1$)

Periods	**Market A**	**Market B**	**Market C**
1-1000	0.323	0.812	0.501
1001-2000	0.191	0.547	0.437
2001-3000	0.133	0.625	0.552
3001-4000	0.229	0.617	0.411
4001-5000	0.224	0.715	0.460
5001-6000	0.148	0.519	0.443
6001-7000	0.191	0.583	0.523
7001-8000	0.206	0.487	0.535
8001-9000	0.112	0.562	0.493
9001-10000	0.090	0.653	0.616

Table 11: Average of the Number of Traders Registered to Business School and Himself

	Market A	**Market B**	**Market C**		
Periods	$\overline{N}_2$	$\overline{N}_4$	$\overline{N}_2$	$\overline{N}_4$	$\overline{N_2 + N_4}$
1-1000	88.762	87.196	47.280	45.452	92.732
1001-2000	88.956	87.400	40.054	53.035	93.089
2001-3000	88.877	87.282	36.092	56.763	92.855
3001-4000	88.756	87.310	30.314	62.666	92.980
4001-5000	88.689	87.258	26.297	66.627	92.924
5001-6000	88.814	87.247	22.560	70.390	92.950
6001-7000	88.859	87.088	20.106	72.809	92.915
7001-8000	88.982	87.432	16.052	76.799	92.851
8001-9000	88.915	87.465	14.264	78.799	93.063
9001-10000	88.884	87.312	10.546	82.429	92.975

Figure 7 : The Number of Traders with Martingale Strategies on Each Trading Day (Market A)

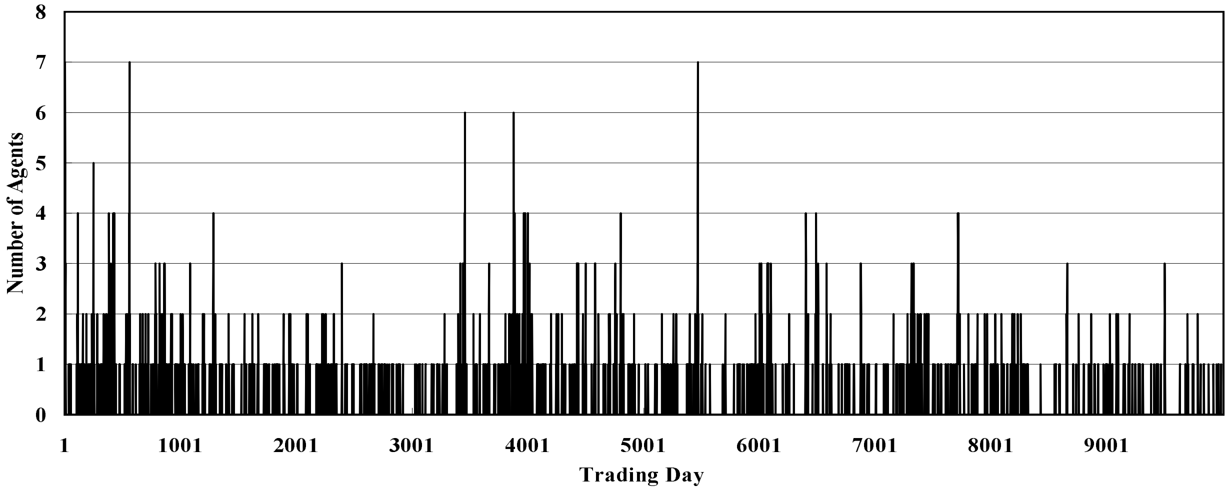


Figure 8 : The Number of Traders with Martingale Strategies on Each Trading Day (Market B)

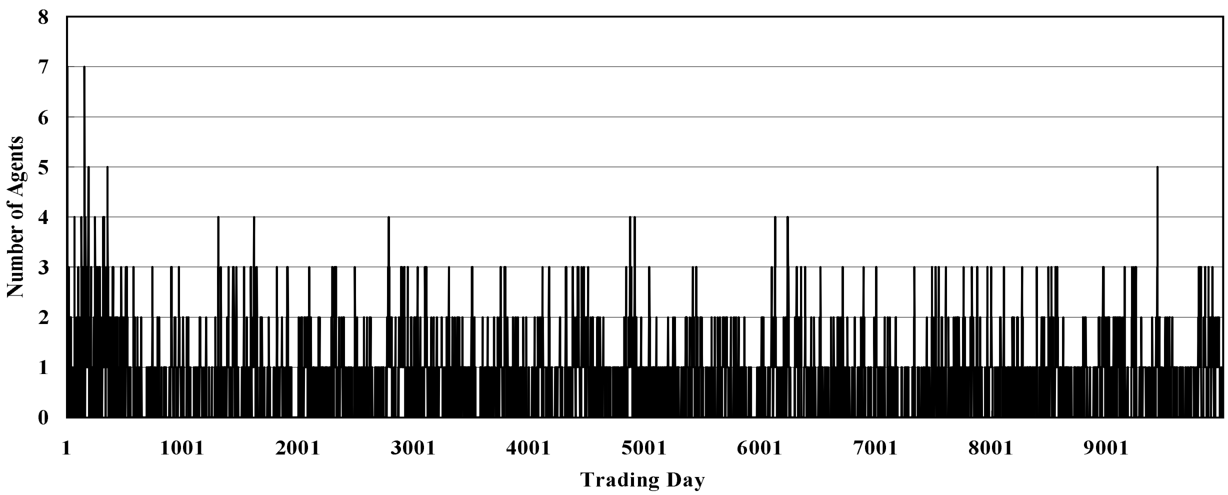


Figure 9 : The Number of Traders with Martingale Strategies on Each Trading Day (Market C)

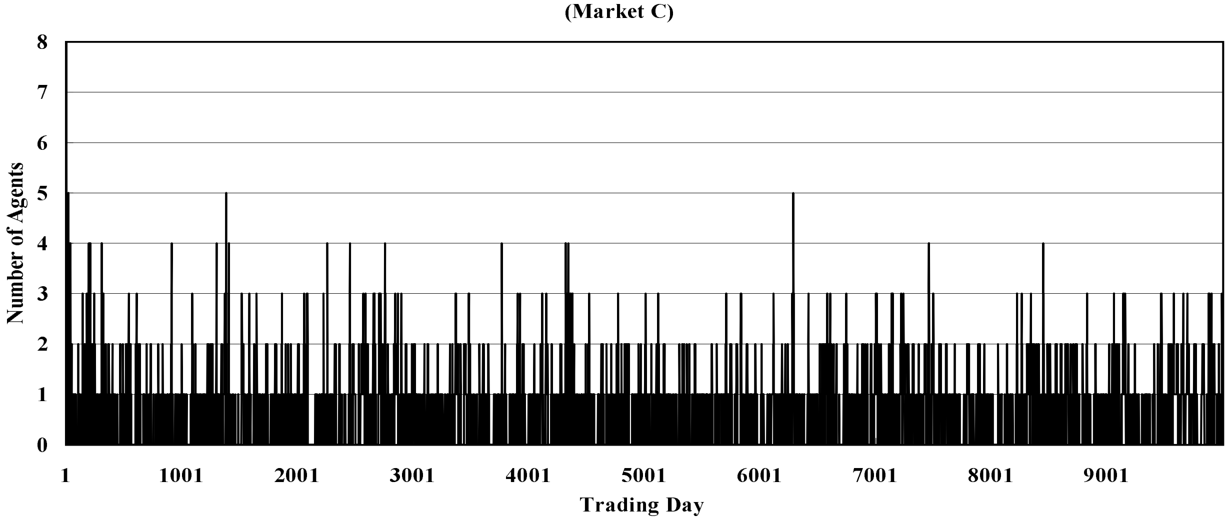


Table 12: Average of the Number of Traders with Successful Search and thinking

	Market A	Market B	Market C		
Periods	$\overline{N_3}$	$\overline{N_5}$	$\overline{N_3}$	$\overline{N_5}$	$\overline{N_3 + N_5}$
1-1000	43.048 (0.48417)	28.285 (0.32433)	28.987 (0.61098)	15.677 (0.35173)	44.664 (0.48132)
1001-2000	44.066 (0.49585)	17.455 (0.19970)	23.501 (0.58711)	10.656 (0.20121)	34.157 (0.36690)
2001-3000	42.588 (0.47887)	17.826 (0.20426)	21.140 (0.58583)	11.417 (0.20107)	32.557 (0.35061)
3001-4000	44.776 (0.50463)	17.470 (0.20007)	18.874 (0.62134)	12.598 (0.20096)	31.472 (0.33852)
4001-5000	43.284 (0.48778)	17.357 (0.19890)	15.657 (0.59712)	13.361 (0.20058)	29.018 (0.31234)
5001-6000	42.664 (0.47976)	17.322 (0.19853)	13.511 (0.59780)	14.234 (0.20217)	27.745 (0.29837)
6001-7000	45.340 (0.51040)	17.428 (0.20005)	12.052 (0.59938)	14.574 (0.20011)	26.626 (0.28653)
7001-8000	42.439 (0.47678)	17.497 (0.20014)	9.733 (0.60698)	15.305 (0.19935)	25.038 (0.26970)
8001-9000	42.650 (0.47960)	17.460 (0.19960)	8.571 (0.60027)	15.887 (0.20171)	24.458 (0.26289)
9001-10000	45.490 (0.51168)	17.337 (0.19860)	6.112 (0.58299)	16.589 (0.20128)	22.701 (0.24428)

The value shown in the parentheses is the average of the ratio of traders with successful search and thinking.

evolving strategies. In the business school, the strategies try to trace the price dynamics in the past 10 periods (m_2) over time. Therefore, they tend to become more complex in order to fit the nonlinear structure. However, as mentioned above, the traders' action in Market B is to myopically maximize the one-period expected utility. Therefore, it is not necessary to evolve complex structure. Moreover, the ideas is renewed at each period, it further makes the strategies have no chance getting complicated. On the other hand, in Market C, due to the increasing of the proportion of profit oriented traders, the complexity of strategies decreases gradually.

5 Concluding Remarks

In the primary research, we built an environment composed of multi-population genetic programming based traders. Besides replicating the stylized facts, the comparison between SGP-based and MGP-based simulations are also discussed. From the marco-phenomena point of view, we don't get too much difference, while the micro-structure does. The difference may come from:

- The different oriented traders, profit and prediction accuracy.
- The different evaluation cycle.
- The evolution of traders' mind.

Of course, the influence of these points will be discussed more detail in the future research. Moreover, the effect of the level of the traders' *intelligence* (the number of ideas for each trader) is also an important issue. However, due to highly computation-intensive, this problem can't be done easily at this moment.

Figure 10 : Trader's Complexity : The Average of Depth of GP-Trees (Market A)

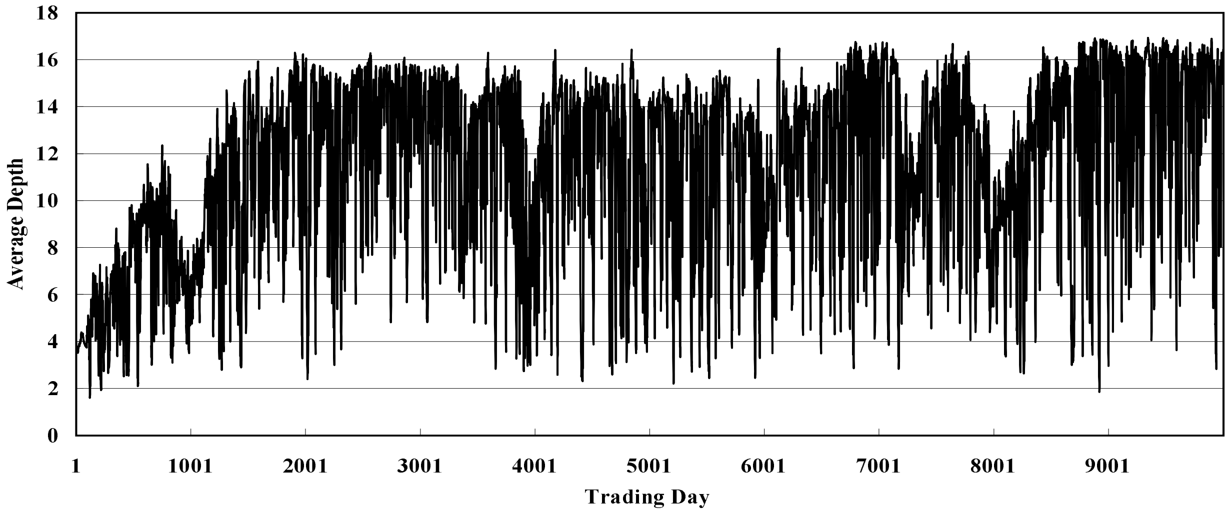


Figure 11 : Trader's Complexity : The Average of Depth of GP-Trees (Market B)

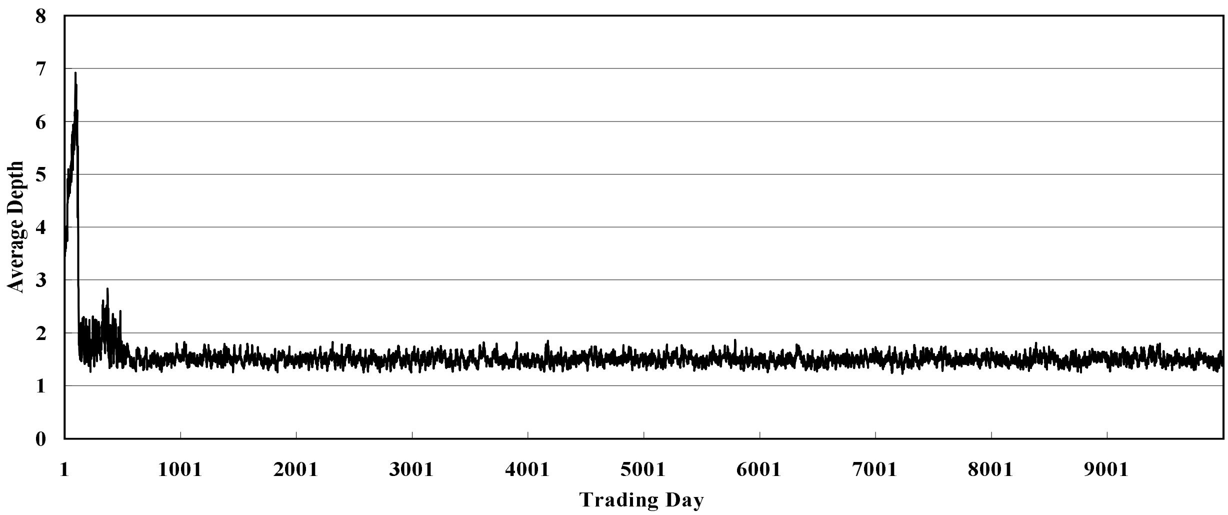


Figure 12 : Trader's Complexity : The Average of Depth of GP-Trees (Market C)

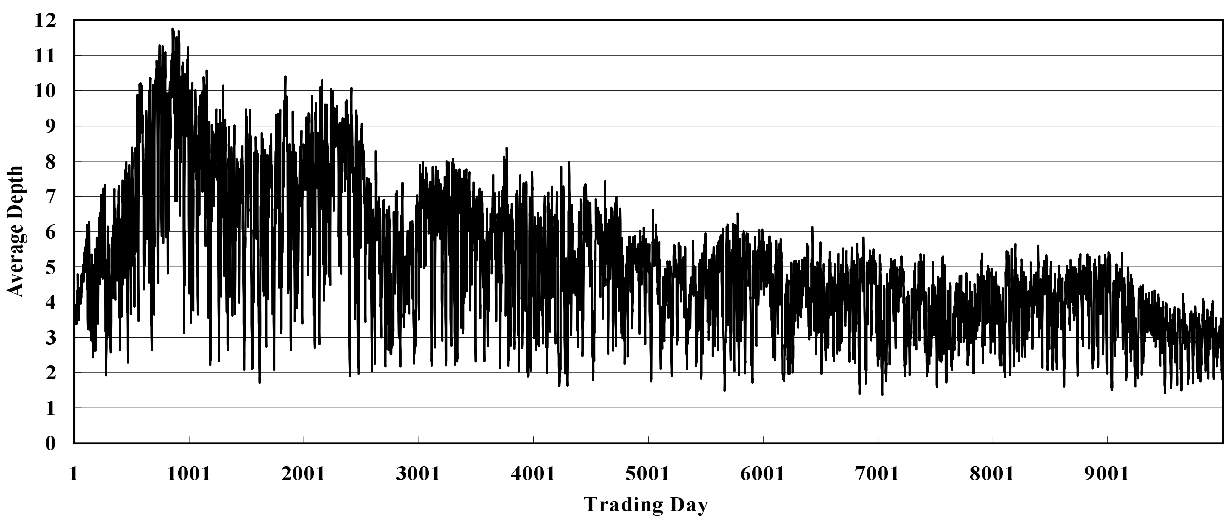


Figure 13 : Trader's Complexity : The Average of the Number of Nodes of GP-Trees (Market A)

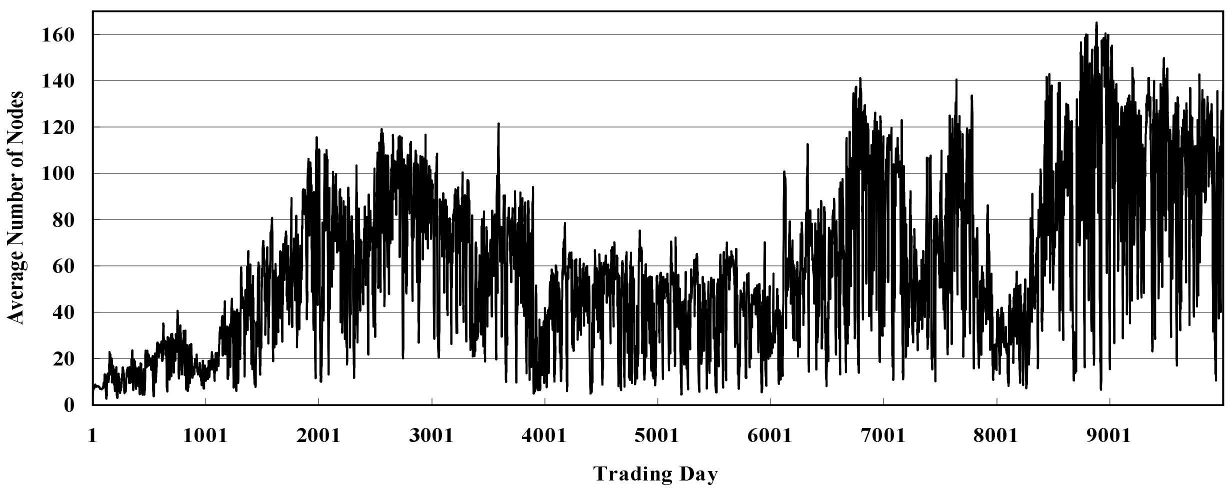


Figure 14 : Trader's Complexity : The Average of the Number of Nodes of GP-Trees (Market B)

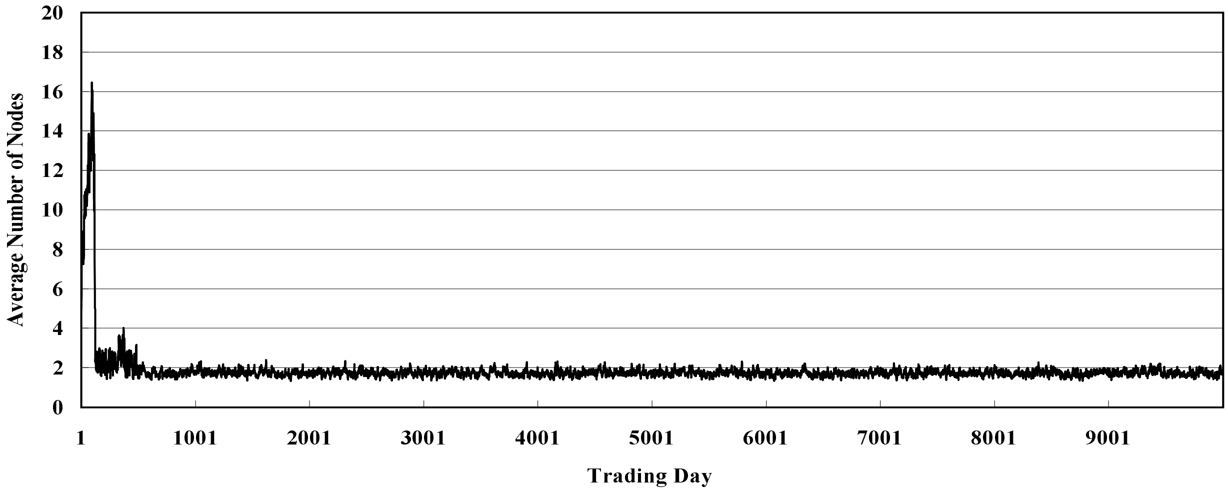


Figure 15 : Trader's Complexity : The Average of the Number of Nodes of GP-Trees (Market C)

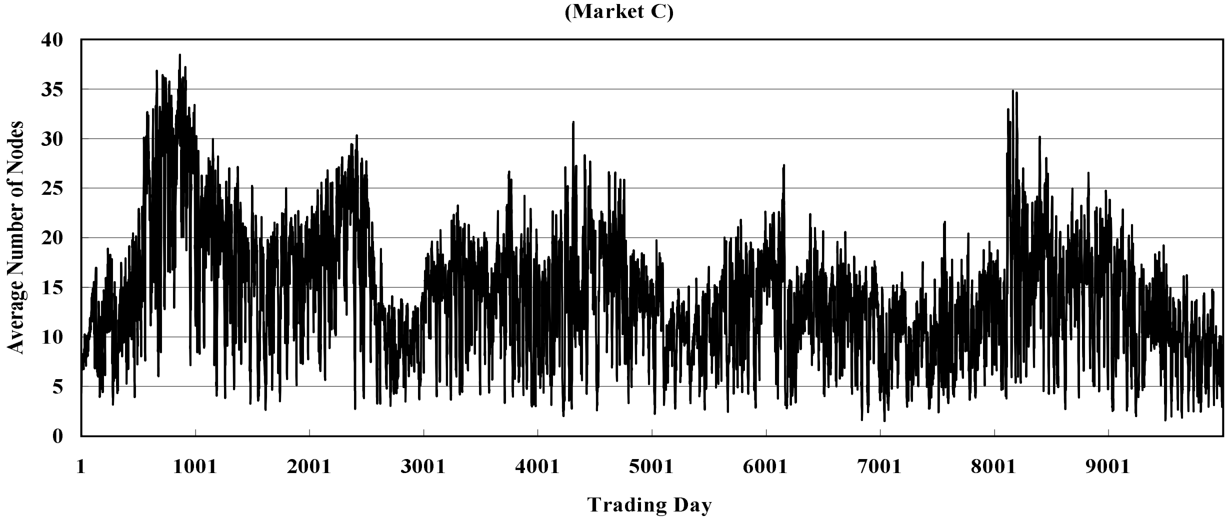


Table 13: Average of the Number of Traders with Successful Search on the h day after Business School Has Updated the Information

	Market A	Market B	Market C	
h	$\overline{N}_{3,h}$	$\overline{N}_{5,h}$	$\overline{N}_{3,h}$	$\overline{N}_{5,h}$
1	49.622 (0.56006)	18.76 (0.21546)	17.450 (0.66427)	13.798 (0.21455)
2	46.322 (0.52227)	18.78 (0.21613)	17.202 (0.65034)	13.988 (0.21654)
3	45.516 (0.51218)	19.07 (0.21768)	16.802 (0.63761)	14.108 (0.21862)
4	44.876 (0.50410)	18.78 (0.21525)	16.348 (0.62345)	13.856 (0.21362)
5	44.314 (0.49816)	18.64 (0.21367)	15.918 (0.60160)	14.072 (0.21569)
6	43.464 (0.48949)	18.39 (0.21079)	16.070 (0.61127)	13.912 (0.21334)
7	43.646 (0.49112)	18.36 (0.20980)	15.990 (0.60924)	14.160 (0.21701)
8	44.214 (0.49616)	18.28 (0.20994)	15.812 (0.60298)	13.940 (0.21304)
9	42.672 (0.48105)	18.08 (0.20739)	15.312 (0.57831)	14.276 (0.22227)
10	43.152 (0.48583)	18.44 (0.21139)	15.436 (0.58023)	14.098 (0.21655)
11	41.934 (0.47137)	18.50 (0.21193)	15.248 (0.58407)	14.166 (0.21728)
12	41.820 (0.47076)	18.49 (0.21151)	15.382 (0.57872)	13.904 (0.21358)
13	41.948 (0.47298)	18.27 (0.20932)	15.038 (0.56905)	14.230 (0.21954)
14	42.562 (0.47924)	18.42 (0.21081)	15.210 (0.57724)	13.718 (0.21103)
15	43.636 (0.48890)	18.73 (0.21448)	15.522 (0.58260)	13.830 (0.21356)
16	42.996 (0.48294)	18.47 (0.21094)	14.942 (0.57212)	14.244 (0.21901)
17	42.656 (0.48038)	18.58 (0.21294)	15.398 (0.58360)	13.918 (0.21322)
18	43.118 (0.48457)	19.21 (0.22044)	15.546 (0.58831)	13.944 (0.21273)
19	42.304 (0.47602)	18.45 (0.21136)	15.764 (0.59703)	14.210 (0.21941)
20	41.918 (0.47145)	18.09 (0.20715)	15.886 (0.58758)	14.224 (0.21972)

The values shown in the parentheses are the ratios of traders with successful search on the h day after business school has updated the information.

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Table 14: Complexity of Evolving Strategies

	Market A		Market B		Market C	
Periods	$\overline{k}$	$\overline{\kappa}$	$\overline{k}$	$\overline{\kappa}$	$\overline{k}$	$\overline{\kappa}$
1-1000	6.49306	15.45485	2.05669	3.07381	6.72513	18.38097
1001-2000	10.63684	45.34048	1.50741	1.74106	6.92334	16.87815
2001-3000	12.92895	76.73194	1.49331	1.72647	6.40866	15.28082
3001-4000	11.83218	60.33760	1.50069	1.72664	5.74555	14.30512
4001-5000	11.37810	43.41115	1.49665	1.73112	4.99250	14.59051
5001-6000	11.25262	40.83188	1.49975	1.73287	4.39418	11.63130
6001-7000	12.01427	66.41879	1.48403	1.69795	4.14494	12.61487
7001-8000	11.73566	65.89867	1.49715	1.72324	3.70228	10.19008
8001-9000	11.75256	74.46129	1.49465	1.71892	3.99040	15.93009
9001-10000	13.85391	100.87724	1.49338	1.73230	3.25615	10.64821

$\overline{k}$ and $\overline{\kappa}$ are the average of k_t and κ_t taken over each period.

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